Non-relativistic Supergravity

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Applied Newton-Cartan Geometry II

Based on work with R. Andringa, E. Bergshoeff, A. Chatzistavrakidis, J. Lahnsteiner, E. Sezgin, L. Romano, T. Zojer

Introduction

- Why combine non-relativistic gravity and supersymmetry?
- Relativistic supergravity is useful as a tool to study non-perturbative QFT:
  - AdS/CFT
  - Non-perturbative physics of supersymmetric theories on curved backgrounds can be studied via localization. Supergravity is a handy tool to construct such theories. (Festuccia, Seiberg)
- What about the non-relativistic regime?
  - Susy in non-relativistic holography where NC structures appear, e.g. Lifshitz holography? (Ross; Christensen, Hartong, Obers, Rollier)
  - Non-relativistic susy field theories on non-trivial backgrounds and localization? Precision tests of non-relativistic holography? (Knodel, Lisbao, Liu)
- Other applications where NC geometry shows up and including supersymmetry can be useful
  - Construction of (toy models for) effective condensed matter field theories (Abanov, Auzzi, Geracie, Gromov, Hoyos, Jensen, Prabhu, Roberts, Son,...).
  - Hořava-Lifshitz supergravity?
- Need coupling of non-relativistic susy field theories to non-relativistic supergravity (dynamical or not). One needs to know the structure of non-relativistic supergravity.

Introduction

- Also interesting from a theoretical point of view. Non-trivial construction.
    1. Non-relativistic superalgebras? (uniqueness?)
    2. Structure of non-relativistic supermultiplets (on-shell vs. off-shell)?
    3. Non-relativistic limit?
- So far, non-trivial examples that have been constructed are in $d = 3$. Interesting from the viewpoint of applications. Higher dimensions is technically more involved (structure of supergravity multiplets?).
- Here: discuss the pure supergravity multiplets in $d = 3$ with 4 supercharges that have been constructed. Techniques: gauging, superconformal methods, null reduction.
- Come in different kinds:
    1. Based on super-Bargmann algebra, on-shell/off-shell, without/with torsion
    2. Based on extension of super-Bargmann algebra
- Specific attention will be paid to the question whether it is possible to obtain non-relativistic curved supersymmetric backgrounds.

Outline

- $d = 3$ on-shell Newton-Cartan torsionless supergravity from gauging a super-Bargmann algebra.
- Extended Bargmann supergravity.
- Off-shell torsionfull Bargmann supergravities from superconformal methods.
- Curved backgrounds in torsionless off-shell Bargmann supergravity?
- Off-shell torsionfull Bargmann supergravity from null reduction.
- Conclusions and outline for future work

Gauging the $3D$, $\mathcal{N} = 2$ super-Bargmann algebra

- An easy way to obtain non-relativistic supergravity is by gauging, i.e. viewing the supergravity multiplet as a gauge multiplet of a non-relativistic superalgebra. Akin to how the vielbein formulation of GR can be viewed as a gauging of the Poincaré algebra (Chamseddine, West).
- Will only work in $d = 3$. In higher dimensions, extra non-gauge fields have to be added to the multiplet. Question of representation theory.
- Suitable super-algebra: the $d = 3$, $\mathcal{N} = 2$ super-Bargmann algebra. (Lukierski,Prochnicka,Stichel,Zakrzewski):

$$
\begin{aligned}
&[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,, && [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,, && [G_a, H] = -P_a\,,\\
&[G_a, P_b] = -\delta_{ab}M\,, && [J_{ab}, Q^{\pm}] = -\frac{1}{2}\gamma_{ab}Q^{\pm}\,, && [G_a, Q^{+}] = -\frac{1}{2}\gamma_{a0}Q^{-}\,,\\
&\{Q^{+}_{\alpha}\,, Q^{+}_{\beta}\} = 2\delta_{\alpha\beta}H\,, && \{Q^{+}_{\alpha}\,, Q^{-}_{\beta}\} = -[\gamma^{a0}]_{\alpha\beta}P_a\,, && \{Q^{-}_{\alpha}\,, Q^{-}_{\beta}\} = 2\delta_{\alpha\beta}M\,.
\end{aligned}
$$

- Contraction of $3d$, $\mathcal{N} = 2$ super-Poincaré algebra (including central charge).
- Bosonic part = $3d$ Bargmann algebra.
- $\mathcal{N} = 1$ subalgebra with Q^{-}. (Clark, Love)

Gauging the $3d$, $\mathcal{N} = 2$ super-Bargmann algebra

- Gauging (Andringa, Bergshoeff, Panda, de Roo; Andringa, Bergshoeff, J.R., Sezgin):

  Step 1: assign gauge fields to all generators, assign gauge transformation rules to gauge fields, construct gauge covariant field strengths

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$\hat{R}_{\mu\nu}(H)$
space translations	P_a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$\hat{R}_{\mu\nu}(P_a)$
boosts	G_a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$\hat{R}_{\mu\nu}(G_a)$
spatial rotations	J_{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$\hat{R}_{\mu\nu}(J_{ab})$
central charge transf.	M	m_μ	$\sigma(x^\nu)$	$\hat{R}_{\mu\nu}(M)$
two supersymmetries	$Q^\pm_\alpha$	$\psi_{\mu\pm}$	$\epsilon_\pm(x^\nu)$	$\hat{\psi}_{\mu\nu\pm}$

- Under boosts:

$$\delta\tau_\mu = 0\,, \qquad \delta e_\mu{}^a = \lambda^a\tau_\mu\,, \qquad \delta m_\mu = \lambda^a e_{\mu a}\,, \qquad \delta\psi_{\mu-} = -\frac{1}{2}\lambda^a\gamma_{a0}\psi_{\mu+}$$

- $\tau_{\mu\nu} = \tau_\mu\tau_\nu$, $h^{\mu\nu} = e^\mu{}_a e^\nu{}_b\delta^{ab}$ are Newton-Cartan metrics where

$$\tau^\mu\tau_\mu = 1\,, \quad \tau^\mu e_\mu{}^a = 0\,, \quad \tau_\mu e^\mu{}_a = 0\,, \quad e^a_\mu e^\nu{}_a = \delta^\nu_\mu - \tau^\nu\tau_\mu\,, \quad e_\mu{}^a e^\mu{}_b = \delta^a_b\,.$$

Gauging the $3d$, $\mathcal{N} = 2$ super-Bargmann algebra

- One has the following curvatures

$$\hat{R}_{\mu\nu}(H) = 2\partial_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^0\psi_{\nu]+}\,,$$
$$\hat{R}_{\mu\nu}(P^a) = 2\partial_{[\mu}e_{\nu]}{}^a - 2\omega_{[\mu}{}^{ab}e_{\nu]b} - 2\omega_{[\mu}{}^a\tau_{\nu]} - \bar{\psi}_{[\mu+}\gamma^a\psi_{\nu]-}\,,$$
$$\hat{R}_{\mu\nu}(M) = 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}{}^a e_{\nu]a} - \bar{\psi}_{[\mu-}\gamma^0\psi_{\nu]-}\,.$$

- Step 2: Impose 'conventional' curvature constraints

$$\hat{R}_{\mu\nu}(P^a) = 0\,, \qquad \hat{R}_{\mu\nu}(M) = 0\,.$$

Non-relativistic torsion constraints. Can be viewed as algebraic equations for $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$. Can be solved to uniquely determine

$$\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(\tau, e, m, \psi_\pm)\,, \qquad \omega_\mu{}^a = \omega_\mu{}^a(\tau, e, m, \psi_\pm)\,.$$

- $\omega_\mu{}^{ab}$, $\omega_\mu{}^a$ are therefore dependent fields. Correspond to usual Newton-Cartan spin connections for spatial rotations and boosts, with torsion delivered by $\psi_{\mu\pm}$.
- Once the spin connections are expressed as dependent fields, the conventional constraints are solved and hold identically.

Gauging the $3d$, $\mathcal{N}=2$ super-Bargmann algebra

- So far, like in the relativistic case.
- Step 3: Check closure of the algebra on the independent fields τ_μ, $e_\mu{}^a$, m_μ, $\psi_{\mu\pm}$. Closure requires extra constraints, that come in two kinds.
  1. The supersymmetric 'foliation or zero torsion constraint' and its consequences:

     $$\hat{R}_{\mu\nu}(H) = 2\partial_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^0\psi_{\nu]+} = 0\,.$$

     This constraint is required to ensure that the algebra closes into diffeomorphisms + local supersymmetry on τ_μ:

     $$[\delta_1,\delta_2]\,\tau_\mu = \partial_\mu\left(\xi^\nu\tau_\nu\right) = \mathcal{L}_\xi\tau_\mu + \delta_Q\left(-\xi^\mu\psi_{\mu+}\right)\tau_\mu + \xi^\nu\hat{R}_{\mu\nu}(H)\,,\quad \text{with}\;\; \xi^\mu = \xi^\mu(\epsilon_{1,2})\,.$$

     This is a non-trivial constraint, whose (supersymmetry) variation needs to be imposed as constraint as well:

     $$\hat{R}_{\mu\nu}(H) = 0 \qquad \Rightarrow \qquad \hat{\psi}_{\mu\nu+} = 0 \qquad \Rightarrow \qquad \hat{R}_{\mu\nu}(J^{ab}) = 0\,.$$

  2. Fermionic constraints, required to ensure algebra closure on $\psi_{\mu-}$:

     $$\hat{\psi}_{ab-} = e^\mu{}_a e^\nu{}_b\hat{\psi}_{\mu\nu-} = 0\,, \qquad \gamma^a\hat{\psi}_{a0-} = \gamma^\mu\tau^\nu\hat{\psi}_{\mu\nu-} = 0\,.$$

     Last constraint is fermionic e.o.m. and varies to bosonic e.o.m. of NC gravity

     $$\hat{R}_{0a}(G^a) = 0\,.$$

- Consistent set of constraints. Defines on-shell non-relativistic NC supergravity multiplet.

Curved supersymmetric backgrounds?

- For practical applications, one wants to allow for supersymmetric curved solutions and ideally one wants an off-shell multiplet.
- The above is on-shell in two senses:
  1. Fermionic equations are needed for closure
  2. $\hat{R}_{\mu\nu}(H) = 0$ needs to be imposed for closure.
- One of the fermionic equations varies to the NC gravity e.o.m. $\hat{R}_{0a}(G^a) = 0$. Consistency with boosts implies that one also has to impose that $\hat{R}_{\mu a}(J^a{}_b) = 0$. Coupling to matter doesn't source the latter e.o.m. in NC gravity, so hard to get curved space-time.
- Moreover, $\hat{R}_{\mu\nu}(H) = 0$ varies here to $\hat{R}_{\mu\nu}(J^{ab}) = 0$, implying flat space.
- One thus needs something better.
- Two strategies:
  1. Consider something different than NC gravity, in which matter can source spatial curvature (and/or the zero torsion constraint).
  2. Go as off-shell as possible and/or change the torsion constraint. What is the most off-shell multiplet that one can write down?

Extended Bargmann Supergravity

- In three dimensions, a non-relativistic gravity theory, different from NC gravity exists.
- The $d = 3$ Bargmann algebra admits an extra central charge:(Lévy-Leblond)

$$[G_a, P_b] = \epsilon_{ab} M \,, \qquad [G_a, G_b] = \epsilon_{ab} S \,.$$

- This 'extended Bargmann algebra' admits a non-degenerate bilinear form (trace)

$$< G_a, P_b >= \delta_{ab} \,, \quad < H, S >=< M, J >= -1 \,,$$

and so a Chern-Simons action

$$S_{\text{EBG}} = \frac{k}{4\pi} \int < A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A >$$

for

$$A_\mu = \tau_\mu H + e_\mu{}^a P_a + \omega_\mu J + \omega_\mu{}^a G_a + m_\mu M + s_\mu S$$

can be constructed: (Papageorgiou, Schroers; Hartong, Lei, Obers)

$$S = \frac{k}{4\pi} \int \mathrm{d}^3x \, \varepsilon^{\mu\nu\rho} \left(e_\mu{}^a R_{\nu\rho}(G_a) - m_\mu R_{\nu\rho}(J) - \tau_\mu R_{\nu\rho}(S) \right) \,.$$

- Can be obtained as a non-relativistic limit of a relativistic Chern-Simons action for Poincaré $\times$ $\mathrm{U}(1)^2$ (Bergshoeff, J.R.)

Extended Bargmann Supergravity

- Variation with respect to ω_μ, ${\omega_\mu}^a$ and s_μ leads to

$$R_{\mu\nu}(M) = 0\,, \qquad R_{\mu\nu}(P^a) = 0\,, \qquad R_{\mu\nu}(H) = 0\,.$$

- Since we have an action, matter coupling is straightforward. E.g.

$$S = S_{\mathrm{EBG}} + \int \mathrm{d}^3x\, e \left[\frac{\mathrm{i}}{2}\left(\Phi^* \tau^\mu D_\mu \Phi - \Phi \tau^\mu D_\mu \Phi^*\right) - \frac{1}{2m} {e^\mu}_a e^{\nu a} D_\mu \Phi^* D_\mu \Phi \right] .$$

with $D_\mu \Phi = \partial_\mu \Phi + \mathrm{i}\, m\, m_\mu \Phi$.

- Defining currents

$$t^\mu = \frac{4\pi}{k} \frac{\delta}{\delta \tau_\mu} (e\mathcal{L}_{\mathrm{m}})\ , \qquad {t^\mu}_a = \frac{4\pi}{k} \frac{\delta}{\delta {e_\mu}^a} (e\mathcal{L}_{\mathrm{m}})\ , \qquad j^\mu = \frac{4\pi}{k} \frac{\delta}{\delta m_\mu} (e\mathcal{L}_{\mathrm{m}})\ ,$$

one finds

$$\epsilon^{\mu\nu\rho} R_{\nu\rho}(S) = t^\mu\ , \qquad \epsilon^{\mu\nu\rho} R_{\nu\rho}(J) = j^\mu\ , \qquad \epsilon^{\mu\nu\rho} R_{\nu\rho}(G_a) = -{t^\mu}_a\ .$$

- Currents obey conservation laws and ‘symmetry properties’ as a consequence of Bianchi identities.
- All components of the curvature tensors are sourced by matter, unlike in NC gravity.

Extended Bargmann Supergravity

- Supersymmetric version exists, based on super-algebra with non-degenerate supertrace

$$[J, Q^{\pm}] = -\frac{1}{2}\gamma_0 Q^{\pm}\,, \qquad [J, R] = -\frac{1}{2}\gamma_0 R\,, \qquad [G_a, Q^{+}] = -\frac{1}{2}\gamma_a Q^{-}\,,$$
$$[G_a, Q^{-}] = -\frac{1}{2}\gamma_a R\,, \qquad [S, Q^{+}] = -\frac{1}{2}\gamma_0 R\,, \qquad \{Q^{+}_{\alpha}, Q^{+}_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} H\,,$$

  and

$$\{Q^{+}_{\alpha}, Q^{-}_{\beta}\} = -(\gamma^a C^{-1})_{\alpha\beta} P_a\,, \qquad \{Q^{-}_{\alpha}, Q^{-}_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} M\,,$$
$$\{Q^{+}_{\alpha}, R_{\beta}\} = (\gamma_0 C^{-1})_{\alpha\beta} M\,.$$

- Good starting point to study matter couplings and see whether non-relativistic supersymmetric field theories on non-trivial supersymmetric backgrounds can be obtained.
- Non-trivial: might require matter couplings to s_μ and the gravitino belonging to R. Unclear how to do this.
- Requires more knowledge about the representations of the above super-algebra.

Superconformal methods

- Off-shell supergravity can be obtained via superconformal methods.
- Relativistic gravity is equivalent to a real, compensating scalar ϕ coupled to conformal gravity (gauging of conformal algebra)
  1. Gauge the conformal algebra $\{P_A, M_{AB}, D, K_\mu\}$, with associated gauge fields $\{E_M{}^A, \Omega_M{}^{AB}, b_M, f_M{}^A\}$. $E_M{}^A$ is independent, $\Omega_M{}^{AB}$ and $f_M{}^A$ are dependent and b_M is a Stückelberg field for K_M.
  2. Couple a real 'compensating' scalar ϕ to a conformal gravity background
     $$\delta\phi = w\lambda_D\phi\,, \qquad D_M\phi = (\partial_M - wb_M)\,\phi\,.$$
     Action
     $$S = -\frac{1}{2}\int \mathrm{d}^D x\, E\,\phi\Box^C\phi\,,$$
     $$\Box^C\phi = E^{AM}\left(\partial_M D_A\phi - (w+1)b_M D_A\phi + \Omega_{MAB}D^B\phi + 2wf_{MA}\phi\right)\,,$$
     $$E_A{}^M f_M{}^A = -\frac{1}{4(D-1)}R\,.$$
  3. The Einstein-Hilbert action is then obtained by gauge fixing dilatations via the choice $\phi = $ constant. GR = gauge equivalent to a conformally coupled scalar.
- Can be used for supergravity, by gauging the superconformal algebra and coupling a compensating multiplet, to fix superfluous symmetries and end up with Poincaré supergravity. Different compensating multiplets give different off-shell formulations.

Gauging the super-Schrödinger algebra

- One needs a conformal version of the super-Bargmann algebra.
- Exists: a ($z = 2$) super-Schrödinger algebra (Leblanc, Lozano, Min; Duval, Horvathy; Sakaguchi, Yoshida).

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$\hat{R}_{\mu\nu}(H)$
space translations	P_a	$e_\mu{}^a$	$\zeta^a(x^\nu)$	$\hat{R}_{\mu\nu}(P^a)$
boosts	G_a	$\omega_\mu{}^a$	$\lambda^a(x^\nu)$	$\hat{R}_{\mu\nu}(G^a)$
spatial rotations	J_{ab}	$\omega_\mu{}^{ab}$	$\lambda^{ab}(x^\nu)$	$\hat{R}_{\mu\nu}(J^{ab})$
central charge transf.	M	m_μ	$\sigma(x^\nu)$	$\hat{R}_{\mu\nu}(Z)$
dilatations	D	b_μ	$\Lambda_D(x^\nu)$	$\hat{R}_{\mu\nu}(D)$
spec. conf. transf.	K	f_μ	$\Lambda_K(x^\nu)$	$\hat{R}_{\mu\nu}(K)$
R-symmetry	R	r_μ	$\rho(x^\mu)$	$\hat{R}_{\mu\nu}(R)$
Q-supersymmetries	$Q_\pm$	$\psi_{\mu\pm}$	$\epsilon_\pm(x^\mu)$	$\hat{\psi}_{\mu\nu\pm}$
S-supersymmetry	S	ϕ_μ	$\eta(x^\mu)$	$\hat{R}_{\mu\nu}(S)$

- Bosonic part = Bargmann algebra + $\{D, K\}$

$$[D,H] = -2H\,, \quad [H,K] = D\,, \quad [D,K] = 2K\,,$$
$$[D,P_a] = -P_a\,, \quad [D,G_a] = G_a\,, \quad [K,P_a] = -G_a\,.$$

Gauging the super-Schrödinger algebra

- Gauging as before. Now e.g.

$$\hat{R}_{\mu\nu}(H) = 2\partial_{[\mu}\tau_{\nu]} - 4b_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^0\psi_{\nu]+}\,.$$

- Impose set of conventional constraints that make $\omega_\mu{}^{ab}$, $\omega_\mu{}^a$, b_a, f_μ, ϕ_μ dependent. One of these is

$$\hat{R}_{a0}(H) = 0 \qquad \Rightarrow \qquad b_a = e^\mu{}_a\tau^\nu\partial_{[\mu}\tau_{\nu]} - \frac{1}{4}e^\mu{}_a\tau^\nu\bar{\psi}_{\mu+}\gamma^0\psi_{\nu+}\,.$$

- Impose unconventional constraint $\hat{R}_{ab}(H) = 0$ to obtain algebra closure. Supersymmetry variations then lead to a chain of other unconventional constraints

$$\hat{R}_{ab}(H) = 0 \qquad \Rightarrow \qquad \frac{3}{4}\epsilon^{ab}\hat{R}_{\mu\nu}{}^{ab}(J) = \hat{R}_{\mu\nu}(R)\,.$$

- To regain Bargmann supergravity, one needs to fix D, K and S.
- Can be done by putting $\tau^\mu b_\mu = 0$ and coupling to a compensating multiplet $\{\phi, \lambda, S\}$ and fixing $\phi = 1$, $\lambda = 0$.
- Leads to a Newton-Cartan supergravity multiplet with bosonic (twistless) torsion

$$\tau_\mu\,, e_\mu{}^a\,, m_\mu\,, r_\mu\,, S\,, \psi_{\mu\pm}$$

Non-relativistic supersymmetry in curved backgrounds?

- Can one find supersymmetric curved backgrounds?
- So far, this question has only been considered for 'torsionless off-shell NC sugra'.
- Obtained from the above torsionfull multiplet by making a truncation:

$$b_a = 0\,, \qquad r_\mu = -\frac{3}{w}\tau_\mu S\,.$$

  Left over with $\{\tau_\mu, e_\mu{}^a, m_\mu, \psi_{\mu\pm}, S\}$.
- S ensures algebra closure without using the NC sugra e.o.m.s $\gamma^a\hat{\psi}_{a0-} = 0$ and $\hat{R}_{0a}(G^a) = 0$.
- One still needs $\hat{R}_{\mu\nu}(H) = 2\partial_{[\mu}\tau_{\nu]} - \frac{1}{2}\bar{\psi}_{[\mu+}\gamma^0\psi_{\nu]+} = 0$ that now varies to

$$\hat{R}_{\mu\nu}(J^{ab}) = \frac{4}{w}\epsilon^{ab}\tau_{[\mu}\hat{D}_{\nu]}S\,.$$

- Maximally and half supersymmetric solutions can be found. They however all have flat spatial sections. (Knodel, Lisbao, Liu)
- Need to introduce torsion and/or more auxiliary fields. Not been investigated yet.

Off-shell non-relativistic supergravity from null reduction

- An off-shell non-relativistic supergravity multiplet can also be obtained by doing a null reduction of off-shell $d=4, \mathcal{N}=1$ supergravity. Work in progress (Bergshoeff, Chatzistavrakidis, Lahnsteiner, Romano, J.R.)
- NC gravity can be obtained from a null reduction of (the e.o.m.s) of GR. (Julia, Nicolai)
- Field content of off-shell $d=4, \mathcal{N}=1$ supergravity

$$E_M{}^A\,,\quad \Psi_M\,,\quad A_M\,,\quad F$$

- Null reduction Ansatz: $A=\{a,+,-\}, M=\{\mu,z\}$

$$E_M{}^A = \begin{matrix} \\ \mu \\ z \end{matrix}\begin{pmatrix} a & - & + \\ e_\mu{}^a & \tau_\mu & -m_\mu \\ 0 & 0 & 1 \end{pmatrix}, \qquad A_z = a\,,\ \ A_\mu = a_\mu - m_\mu a\,.$$

- For spinors

$$\begin{aligned} \varepsilon &= \epsilon_- \otimes \chi_+ + \epsilon_+ \otimes \chi_-\,, \\ \Psi_z &= \psi_{z-} \otimes \chi_+ + \psi_{z+} \otimes \chi_-\,, \\ \Psi_\mu &= (\psi_{\mu-} - m_\mu \psi_{z-}) \otimes \chi_+ + (\psi_{\mu+} - m_\mu \psi_{z+}) \otimes \chi_-\,. \end{aligned}$$

Off-shell non-relativistic supergravity from null reduction

- Diffeomorphisms ξ^M give $3d$ diffeomorphisms ξ^μ and central charge ξ^z. In the vielbein Ansatz, $E_z{}^a = 0$ and $E_z{}^- = 1$ fix part of the $4d$ Lorentz transformations ($\Lambda^a{}_+$ and $\Lambda^+{}_+$). Left over with $3d$ spatial rotations and Galilean boosts.

- $E_z{}^+ = 0$ is a non-trivial constraint (necessary to have a null Killing vector). Consistency with supersymmetry demands

$$E_z{}^+ = 0 \qquad \Rightarrow \qquad \psi_{z+} = 0 \qquad \Rightarrow \qquad \epsilon^{ab}\hat{\tau}_{ab} = 2\bar{\psi}_{z-}\psi_{z-} + 12a\,.$$

  The latter constraint no longer varies to other constraints.

- One ends up with a multiplet with field content

$$\tau_\mu\,,\;\; e_\mu{}^a\,,\;\; m_\mu\,,\;\; a_\mu\,,\;\; a\,,\;\; F\,,\;\; \psi_{\mu\pm}\,,\;\; \psi_{z-}\,.$$

- Algebra closes upon using only the constraint $\epsilon^{ab}\hat{\tau}_{ab} = 2\bar{\psi}_{z-}\psi_{z-} + 12a$.

- Potentially allows for construction of supersymmetric curved backgrounds.

Conclusions

- Non-relativistic supergravity theories could be useful as a tool to study non-perturbative aspects of non-relativistic field theory.

- An interesting class of applications requires knowing whether curved supersymmetric solutions of non-relativistic supergravity exist.

- On-shell Bargmann supergravity in $d = 3$ easily constructed via gauging. Zero bosonic torsion and NC e.o.m.s imply flat space.

- Finding curved supersymmetric backgrounds thus requires either:
  1. Going beyond Bargmann supergravity. E.g. extended Bargmann supergravity, where matter couplings might lead to curved backgrounds.
  2. Going as off-shell as possible and/or introducing bosonic torsion. Auxiliary fields and torsion can evade the flat space constraints. Via superconformal methods or null reduction.

- Outlook:
  - Matter couplings?
  - Higher dimensions?
  - Superspace techniques?
  - Concrete applications to non-perturbative susy field theory?
  - $c \to 0$ limit?