Asymptotic symmetries of p-form theories

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Applied Newton-Cartan Geometry

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Motivation

- Supergravity (self-dual forms, extended objects, ...)
- Holography: Asymptotic symmetries are large gauge transformations with non-zero charges.
- Soft theorem & Memmory effect: In flat space, they are related to Weinberg's soft theorems and Memory effect. [Strominger et. al. '14]

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Introduction

- We consider $(p+1)$-form gauge fields in $d = 2p + 4$,

$$\mathcal{A}_{\mu_0 \mu_1 \cdots \mu_p}$$

- In this spacetime dimension the radiation and Coulomb falloff are the same,

$$r^{-\frac{d-2}{2}} , \qquad r^{-(d-p-3)} .$$

- Memory effect; the trace of radiation can be recorded in charges.

Maxwell-type p-form theories

- p-form gauge symmetry

$$S = -\frac{1}{2}\int_{\mathcal{M}} \boldsymbol{\mathcal{F}}_{p+2} \wedge \star \boldsymbol{\mathcal{F}}_{p+2} + \int_{\partial\mathcal{M}} \mathcal{L}_b$$

$$\boldsymbol{\mathcal{F}}_{p+2} = \mathrm{d}\boldsymbol{\mathcal{A}}_{p+1}\,, \qquad \boldsymbol{\mathcal{A}}_{p+1} \to \boldsymbol{\mathcal{A}}_{p+1} + \mathrm{d}\boldsymbol{\Lambda}_p$$

  In **flat spacetime** we find finite and well-defined expressions for the conserved surface charges in $d = 2p + 4$ dimensions.

- Final result: The algebra of asymptotic global charges is centrally extended (An infinite copies of Heisenberg algebra).

Flat space holography

- Flat-spacetime $ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
- BMS-slicing $\to$ Marika's talk $ds^2 = -du^2 - 2du dr + r^2 d\Omega_{d-2}^2$
- dS-slicing (Lorenz invariant):

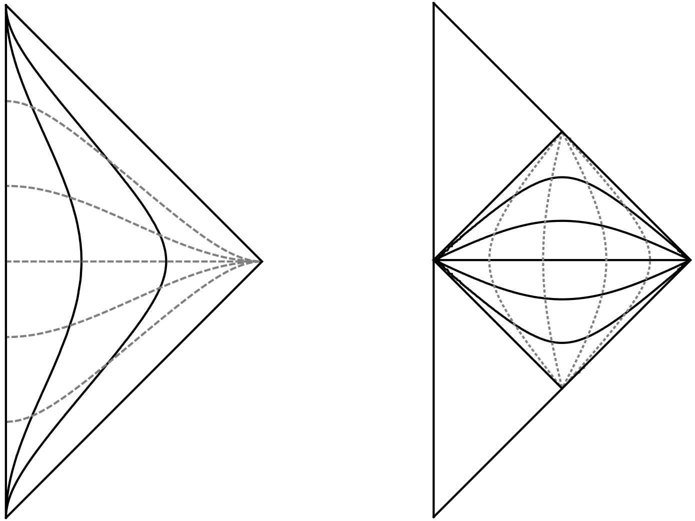
$$ds^2 = d\rho^2 + \rho^2 \left(-d\tau^2 + \cosh^2\tau \, d\Omega_{d-2}^2\right)$$

$$r = \rho\cosh\tau\,, \quad t = \rho\sinh\tau\,, \quad r,\rho \geq 0\,, \quad t,\tau \in \mathbb{R}\,.$$

- Scaling:

$$x^\mu \to \lambda x^\mu \; : \qquad \rho \to \lambda\rho\,.$$

Slices of flat space



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Outline

Boundary conditions at spatial infinity

Variational principle

Asymptotic symmetries

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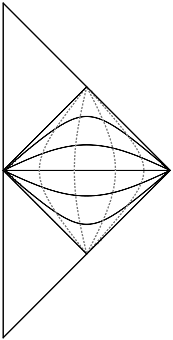
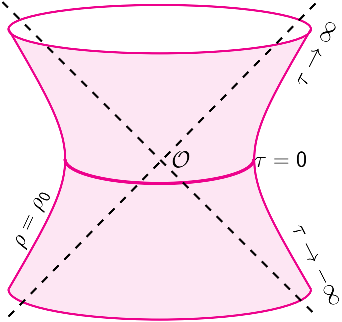
Boundary conditions at spatial infinity





- Falloff behaviour

$$\mathcal{A}_{\mu_0 \cdots \mu_p}(\rho, x^a) = A_{\mu_0 \cdots \mu_p}(x^a)\rho^n + \text{subleading}\,,$$

is fixed such that we find finite and well-defined expressions for the conserved surface charges in $d = 2p + 4$ dimensions.

Boundary conditions

- Lorenz invariant division of components;

$$\mathcal{A}^{\rho a_1 \cdots a_p} = A^{\rho a_1 \cdots a_p}(x)\, \rho^{-d+3} + \cdots$$
$$\mathcal{A}^{a_0 \cdots a_p} = A^{a_0 \cdots a_p}(x)\, \rho^{-d+2} + \cdots$$

being preserved by the following gauge transformations;

$$\Lambda^{\rho a_2 \cdots a_p} = \lambda^{\rho a_2 \cdots a_p}(x)\, \rho^{-d+5} + \cdots$$
$$\Lambda^{a_1 \cdots a_p} = \lambda^{a_1 \cdots a_p}(x)\, \rho^{-d+4} + \cdots$$

Maxwell case ($p = 0$)

- Asymptotic symmetries for this case has been studied by [Strominger Kapec Pate ... ,Campiglia Laddha Eyheralde, Seraj, ...]
- In the BMS-gauge;

$$A_r \sim \mathcal{O}(r^{-2}), \quad A_u \sim \mathcal{O}(r^{-1}), \quad A_z \sim \mathcal{O}(1),$$

- In the dS-gauge;

$$A_\rho \sim \mathcal{O}(\rho^{-1}), \qquad A_a \sim \mathcal{O}(1), \qquad \lambda \sim \mathcal{O}(1).$$

- Asymptotic charge;

$$Q_\lambda[A] = \int_{S^2} \left(\lambda \partial^\tau A_\rho - A_\rho \partial^\tau \lambda\right) .$$

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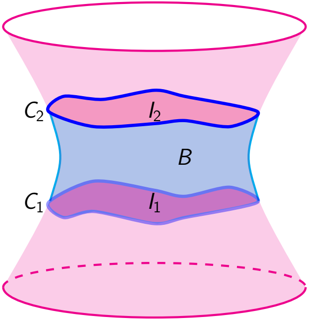
Variational principle



$$\partial\mathcal{M} = B \cup I_1 \cup I_2$$

$$\delta S = (-1)^{p+1} \int_{\mathcal{M}} \delta\boldsymbol{\mathcal{A}} \wedge \mathrm{d}{\star}\boldsymbol{\mathcal{F}} + \int_{\partial\mathcal{M}} (\delta\mathcal{L}_b - \delta\boldsymbol{\mathcal{A}} \wedge {\star}\boldsymbol{\mathcal{F}})\,.$$

E.O.M. $\mathrm{d}{\star}\boldsymbol{\mathcal{F}} = 0\,,\qquad (\nabla_\alpha \mathcal{F}^{\mu_0\cdots\mu_p\alpha} = 0)\,.$

Action principle

$$S = S_0 + S_b = -\frac{1}{2}\int_{\mathcal{M}} \mathcal{F} \wedge \star\mathcal{F} + S_b$$

Variation is vanishing on-shell upon a suitable fixation of initial and final conditions under generic field variations obeying our b.c.

$$\delta S_0 \sim \int_B \delta A^{a_0\cdots a_p}\left[(2p+4-d)A_{a_0\cdots a_p} - (p+1)\partial_{a_0}A_{\rho a_1\cdots a_p}\right]$$

In $d = 2p+4$ by using **Lorenz gauge** ($\mathrm{d}^\dagger\boldsymbol{\mathcal{A}} = 0$) the boundary term is fixed and respects the **residual gauge transformation**;

$$S_b = -\int_B \mathbf{A}_\rho \wedge \star\mathbf{A}_\rho = -\frac{1}{p!}\int_B A^{\rho a_1\cdots a_p}A_{\rho a_1\cdots a_p}$$

Conserved charges

- **Gauge** symmetries of the *well-defined* action lead to *conserved* **surface** charges,

$$\delta S \approx \int_{C_1} - \int_{C_2} = 0\,.$$

- Lorentz gauge fixing ($\mathrm{d}^\dagger \boldsymbol{\mathcal{A}} = 0$) leaves a residual part,

$$\mathrm{d}^\dagger \mathrm{d} \boldsymbol{\Lambda} = 0\,.$$

- **Conserved** charges ae associated to those **residual** gauge transformations which preserve our **boundary conditions**.

Projection on sphere

The $(p+1)$-form gauge fields of the bulk $\boldsymbol{\mathcal{A}}$ are decomposed into one $(p+1)$-form, two p-forms and one $(p-1)$-form on S^{d-2};

$$\hat{\mathbf{A}} \equiv \mathbf{A} - \mathrm{d}\tau \wedge \hat{\mathbf{A}}_\tau \,, \qquad \hat{\mathbf{A}}_\tau \equiv \boldsymbol{\tau} \cdot \mathbf{A} \,,$$
$$\hat{\mathbf{A}}_\rho \equiv \mathbf{A}_\rho - \mathrm{d}\tau \wedge \hat{\mathbf{A}}_{\rho\tau} \,, \qquad \hat{\mathbf{A}}_{\rho\tau} \equiv \boldsymbol{\tau} \cdot \mathbf{A}_\rho \,,$$

Conserved charges corresponding to gauge transformation

$$\mathbf{A} \to \mathbf{A} + \mathrm{d}\boldsymbol{\lambda} \quad \text{and} \quad \mathbf{A}_\rho \to \mathbf{A}_\rho + \mathrm{d}\boldsymbol{\lambda}_\rho$$

$$Q_\lambda[\mathcal{A}] = -\int_{S^{d-2}} \left(\hat{\boldsymbol{\lambda}} \wedge \star \hat{\mathbf{F}}_{\rho\tau} + \hat{\mathbf{A}}_\rho \wedge \star (\widehat{\mathrm{d}\boldsymbol{\lambda}})_\tau - 2\hat{\boldsymbol{\lambda}}_\rho \wedge \star \hat{\mathbf{A}}_{\rho\tau} \right)$$

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Classification of charges

Charges are classified according to the **Hodge decomposition** of their gauge parameter p-forms on **sphere**;

$$\hat{\boldsymbol{\lambda}} = \hat{\boldsymbol{\lambda}}^{\text{exact}} + \hat{\boldsymbol{\lambda}}^{\text{coexact}} + \hat{\boldsymbol{\lambda}}^{\text{harmonic}} .$$

(harmonic forms on sphere exist only for functions and top forms)

- **Coexact** $\hat{\boldsymbol{\lambda}} = \mathrm{d}^\dagger \hat{\boldsymbol{\psi}}$. Generalization of (**Abelian**) charges in 4d Maxwell case ($p = 0$).
- **Exact** $\hat{\boldsymbol{\lambda}} = \mathrm{d}\hat{\boldsymbol{\epsilon}}$. Present in $p \geq 1$. (**non-Abelian**)
- **Zero-mode** Brane charges as generalization of electric charge.

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Algebra of charges

$$\{Q_\epsilon^{\text{coexact}}, Q_\lambda^{\text{coexact}}\} = -\delta_\varepsilon Q_\lambda^{(\text{coexact})} = 0\,,$$

$$\{Q_\epsilon^{\text{exact}}, Q_\lambda^{\text{exact}}\} = -\delta_\varepsilon Q_\lambda^{(\text{exact})} = \text{Heisenberg algebra on } S^{2p+2}\,,$$

$$\{Q_\epsilon^{\text{coexact}}, Q_\lambda^{\text{exact}}\} = 0\,.$$

The zero mode charges $Q_\epsilon^{\text{zero-mode}}$ commute with all other charges.

$$\{Q_{\boldsymbol{\lambda}}, Q_{\boldsymbol{\epsilon}}\} = 2\int_C \left(\hat{\boldsymbol{\lambda}}_\rho \wedge \star\partial_\tau \hat{\boldsymbol{\epsilon}}_\rho - \hat{\boldsymbol{\epsilon}}_\rho \wedge \star\partial_\tau \hat{\boldsymbol{\lambda}}_\rho\right).$$

Summary of 2-form theory in $6d$

Mode expansion of the gauge parameters and gauge fields on S^4: The τ dependence will be given by the two type Legendre functions $P^{1,2}_{\ell+1}$ while the spacial dependence is determined by spherical harmonics $Y_{\ell m_\alpha}$ with $\alpha = 1, 2, 3$. So the algebra of exact charges are;

$$\{Q^{(a)}_{lm_\alpha}, Q^{(b)}_{l'm'_\alpha}\} = 4\delta_{ll'}\delta_{m_\alpha,-m'_\alpha}\epsilon^{ab}, \qquad l \geq 1, \quad a, b = 1, 2,$$

- These charges are conserved even offshell.
- The bulk action for the exact sector vanishes.

Outlook

- Non-critical dimensions $d \neq 2p + 4$.
- (Anti-)self dual cases in $d = 6, 10$.
- Generalized global symmetries.