MITP Workshop
Proton Radius Puzzle
Waldthausen Castle,
Mainz, Germany

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Two-photon exchange corrections in elastic ep and μp scattering. Elastic contribution. Dispersive framework

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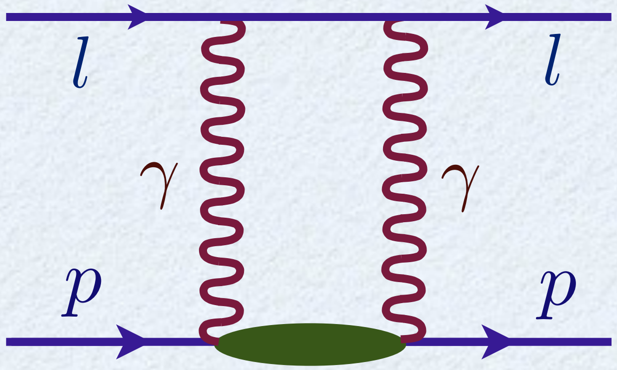
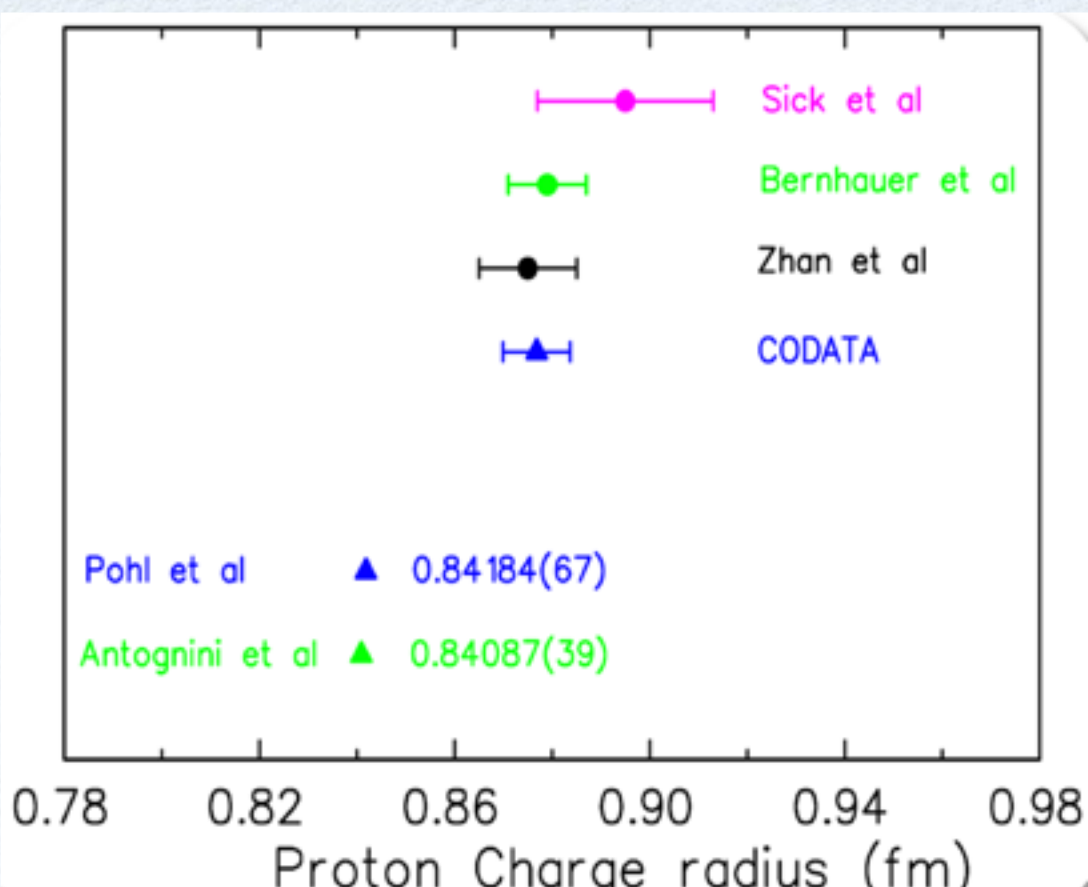
Outline

- Motivation
- Previous two-photon exchange (TPE) calculations
- Dispersion relation framework for 2γ corrections
- Results for elastic ep scattering and polarisation transfer
- Estimations for elastic μp scattering

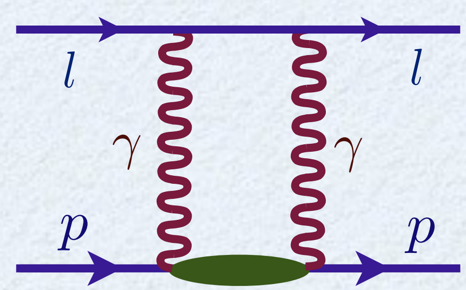


Proton radius puzzle

	e hydrogen	μ hydrogen
Lamb shift	$r_E = 0.8768 \pm 0.0069 fm$	$r_E = 0.8409 \pm 0.0004 fm$
ep-elastic scattering	$r_E = 0.870 \pm 0.025 fm$	7.7σ difference !

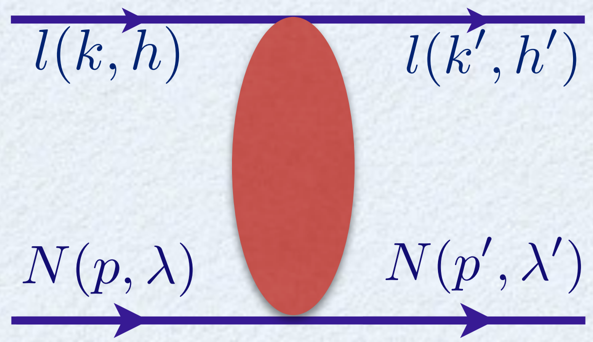


TPE hadronic correction is dominant uncertainty in scattering experiments



$$\sigma^{exp} \equiv \sigma_{1\gamma}(1 + \delta_{soft} + \delta_{2\gamma})$$

Structure amplitudes: TPE correction



$Q^2 = -(k-k')^2$ momentum transfer

$s = (p+k)^2$
$u = (k-p')^2$
$\nu = \frac{s-u}{4}$ crossing symmetric variable

ϵ photon polarization parameter

Discrete symmetries → **6 structure amplitudes**

$$T^{non-flip} = \frac{e^2}{Q^2}\bar{l}(k',h')\gamma_\mu l(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

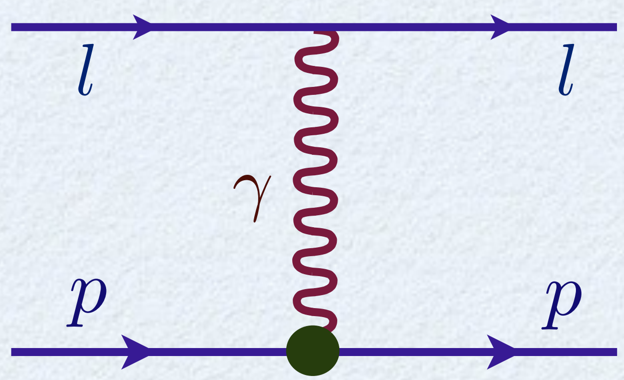
P.A.M. Guichon and M. Vanderhaeghen (2003)

$$T^{flip} \sim \mathcal{F}_4,\ \mathcal{F}_5,\ \mathcal{F}_6$$

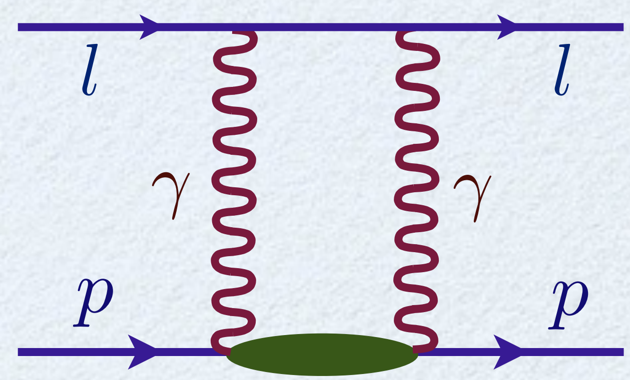
M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

Leading TPE contribution to cross section - interference term

1 photon diagram



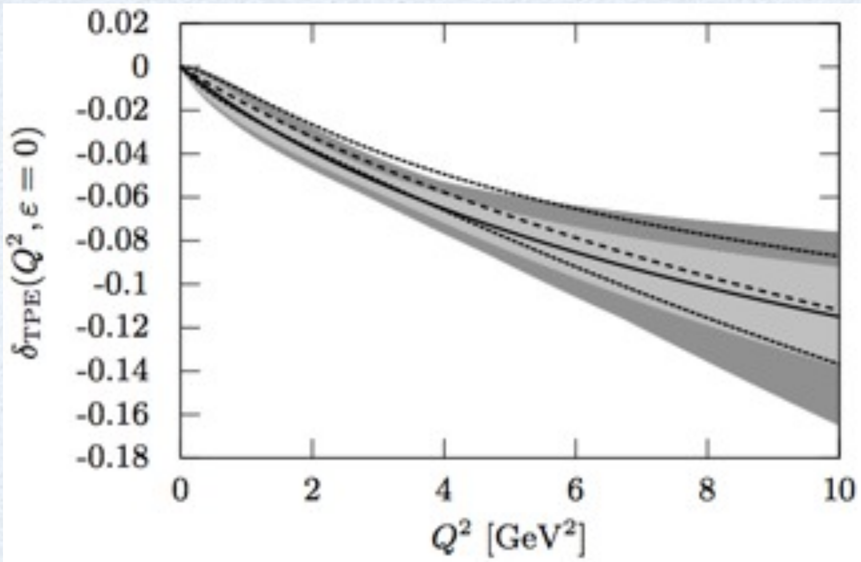
2 photon exchange diagram

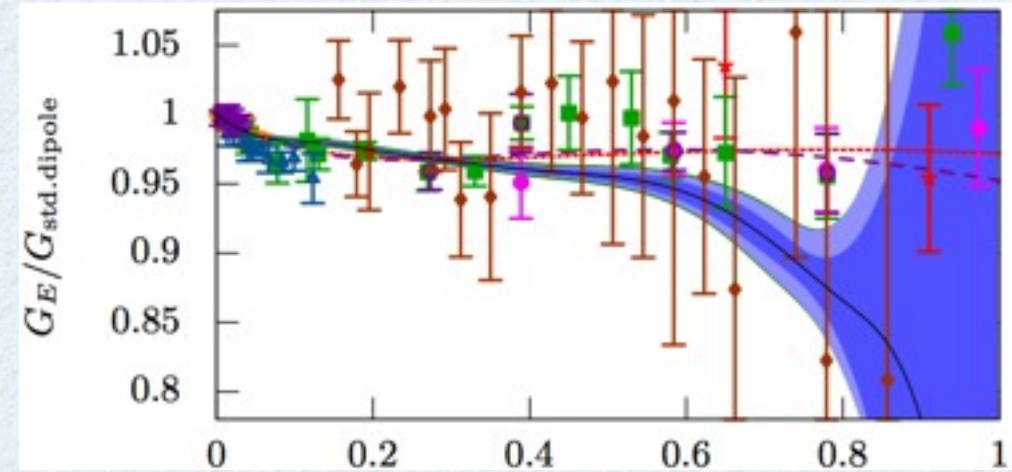


$$\delta_{2\gamma} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3, \Re\mathcal{F}_4, \Re\mathcal{F}_5$$

Dispersion relation framework

2γ corrections





$f(z)$

analyticity

cross section correction

exp. data/phenomenology

$$\Re\mathcal{F}(\nu) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{min}}^{\infty} \frac{\Im\mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

unitarity

DR

amplitudes: real parts

amplitudes: imaginary parts

D. Borisyuk, A. Kobushkin (2008)

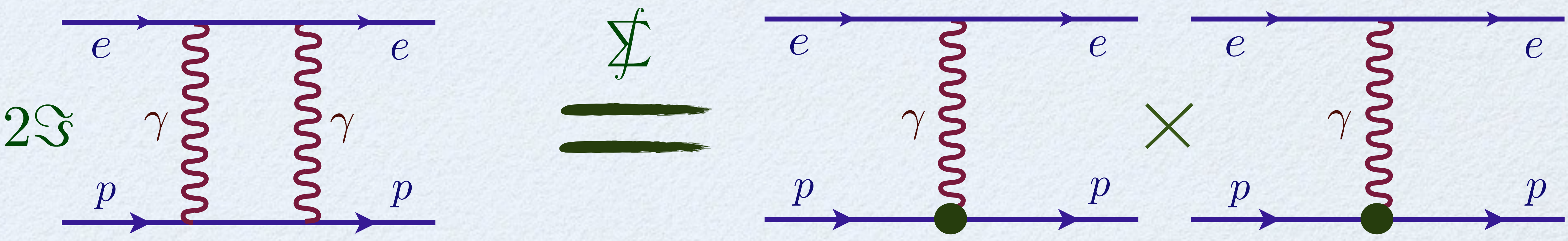
Unitarity relations. Imaginary part

$$S = 1 + iT \qquad S^+ S = 1 \qquad \longrightarrow \qquad \Im T_{h'\lambda',h\lambda}$$

only on-shell information is required

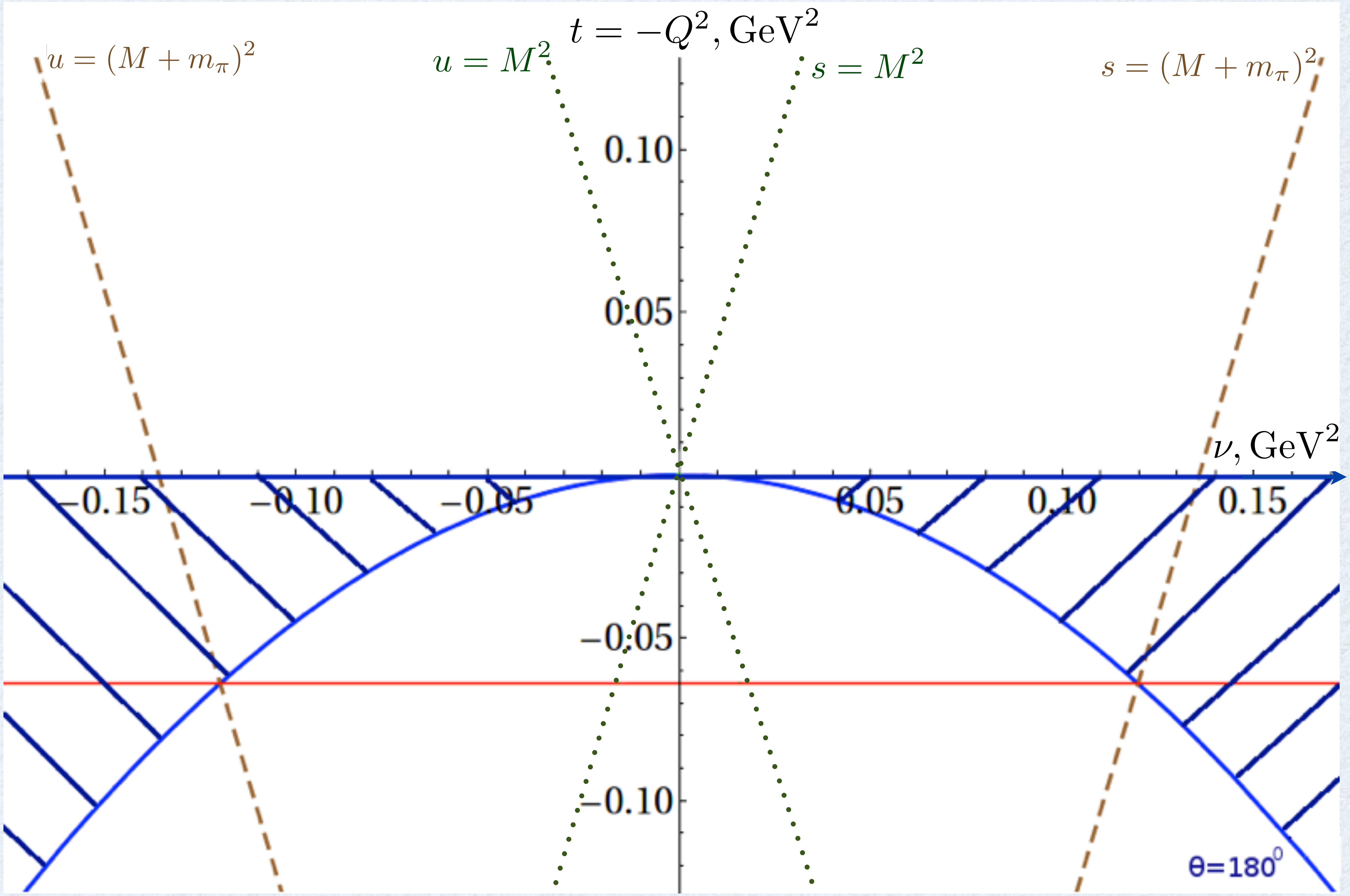
$$2\Im T_{h'\lambda',h\lambda} = \sum\!\!\!\!\!\!\!\int d\Pi'' T^+_{h'\lambda',\mu} T_{\mu,h\lambda} (2\pi)^4 \delta^4(k + p - \sum_i q_i)$$

e and N intermediate state



on-shell one-photon amplitudes

Kinematic regions (e⁻p)



$$Q^2 = m_\pi^2 \left(\frac{2M + m_\pi}{M + m_\pi}\right)^2 \sim 3.5 m_\pi^2$$

- intersection of phys. region and inelastic threshold

Proton intermediate state is outside physical region

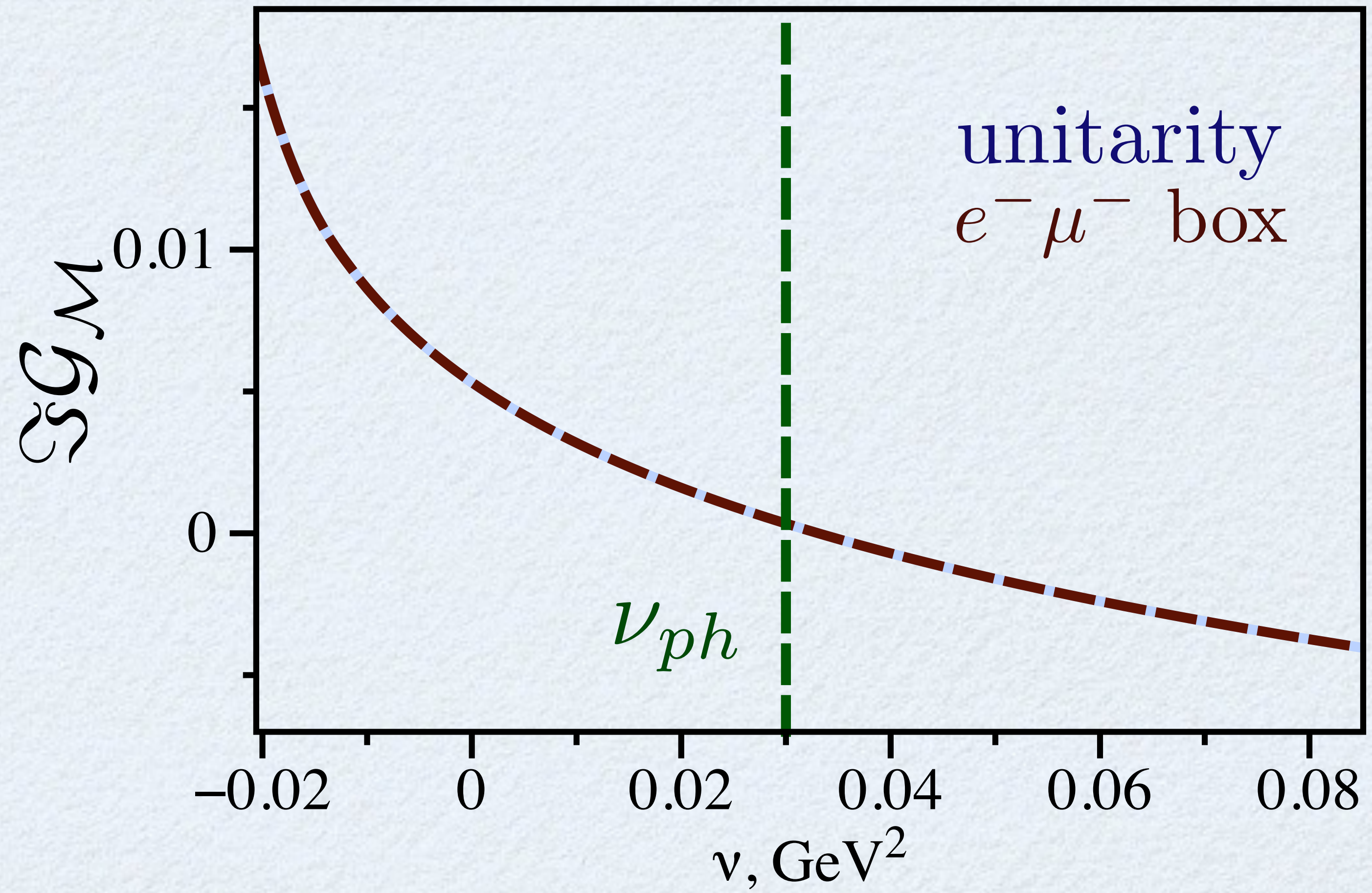
Analytical continuation

$\int d\Omega$ symmetric coordinates wrt electron momentum transfer

$$\cos\theta_1 = \sqrt{1-\alpha^2}(b\cos\phi + c\sin\phi) \qquad \cos\theta_2 = \sqrt{1-\alpha^2}(b\cos\phi - c\sin\phi)$$

$2\int_0^1 d\alpha \int_0^{2\pi} d\phi$ → angular integration to integration on curve in complex plane → deform contour keeping poles inside → after transition to unph. region

unphysical region physical region

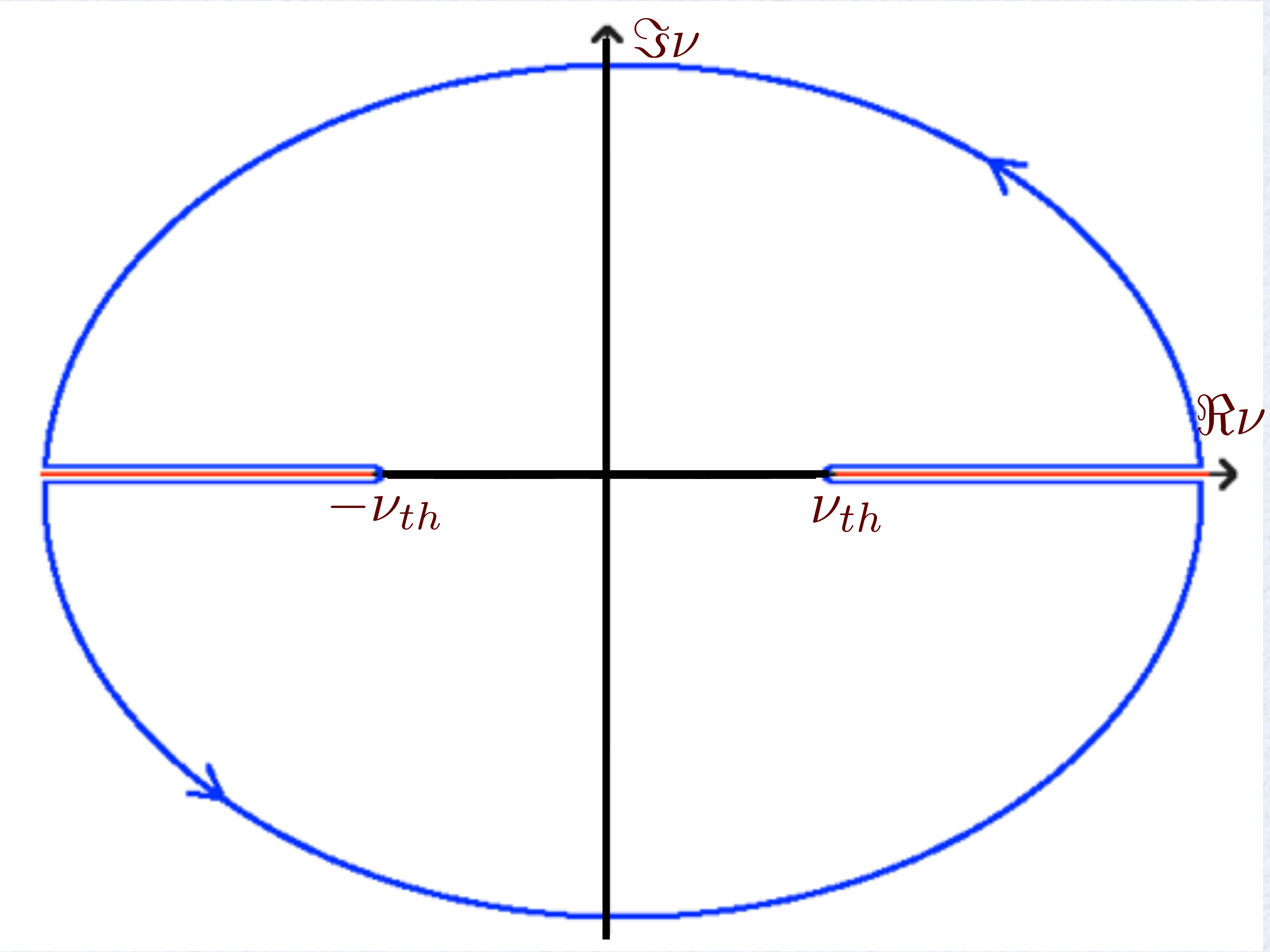


Analytical continuation reproduces results in unphysical region

This work

$$Q^2 = 0.1\ GeV^2$$
$$\nu_{ph} = 0.03\ GeV^2$$

Fixed-t dispersion relation. Real part



Under crossing $\nu \to -\nu$
two-photon exchange amplitudes

$\mathcal{G}_{\mathcal{M}}(\nu,t), \mathcal{F}_2(\nu,t) \Leftrightarrow \mathcal{G}_1(\nu,t), \mathcal{G}_2(\nu,t)$ are odd

$\mathcal{F}_3(\nu,t)$ is even

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3$$

$$\mathcal{G}_2 = \mathcal{G}_E + \frac{\nu}{M^2}\mathcal{F}_3$$

→ good HE behavior

Real part can be reconstructed with DRs

$$\Re\mathcal{G}^{odd}(\nu,t) = \frac{2\nu}{\pi}\mathcal{P}\int_{\nu_{th}}^{\infty} \frac{\Im\mathcal{G}^{odd}(\nu'+i0,t)}{\nu'^2-\nu^2}d\nu'$$

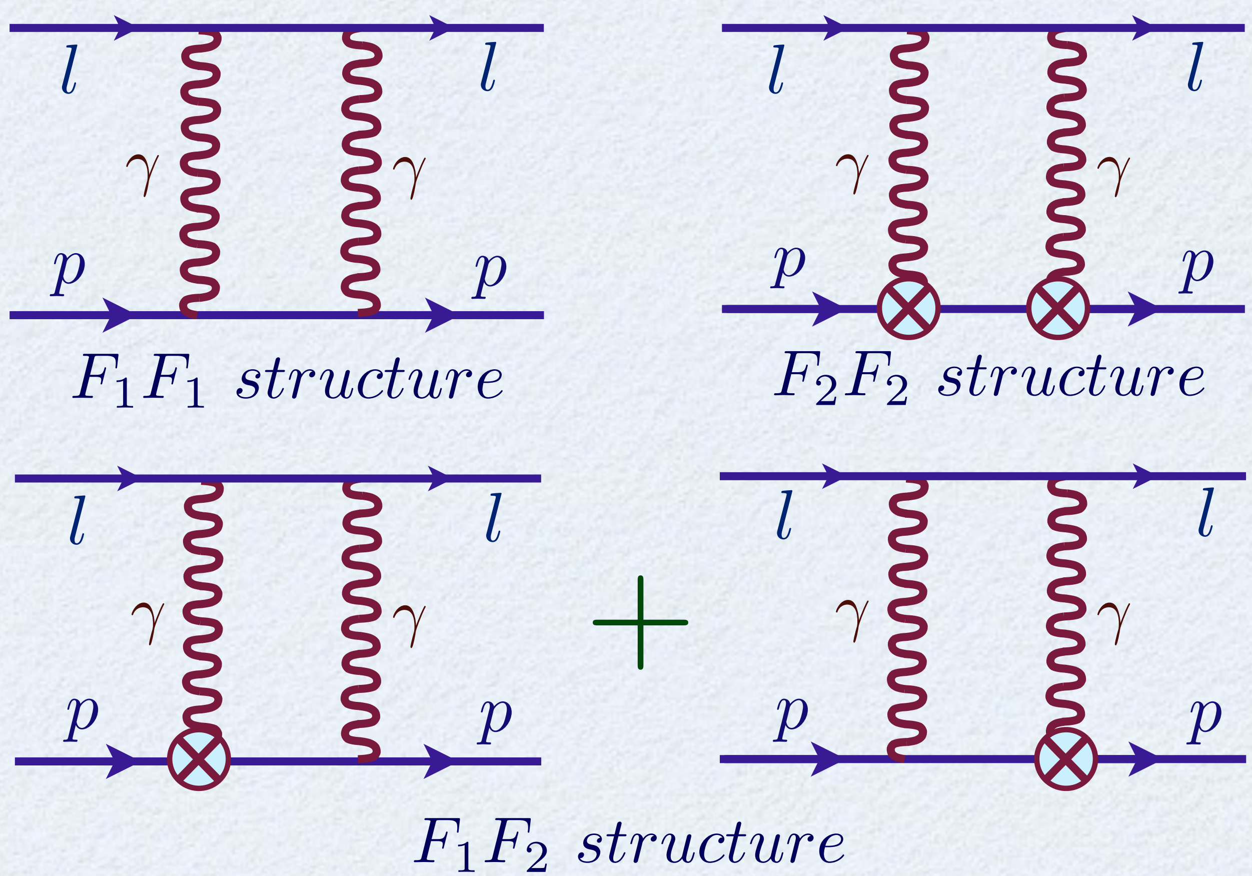
$$\Re\mathcal{G}^{even}(\nu,t) = \frac{2}{\pi}\mathcal{P}\int_{\nu_{th}}^{\infty} \nu'\frac{\Im\mathcal{G}^{even}(\nu'+i0,t)}{\nu'^2-\nu^2}d\nu'$$

Hadronic model

The one-photon exchange on-shell vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(Q^2)$$

P. G. Blunden, W. Melnitchouk, and J. A. Tjon (2003)



IR divergencies are subtracted

L.C. Maximon and J. A. Tjon (2000)

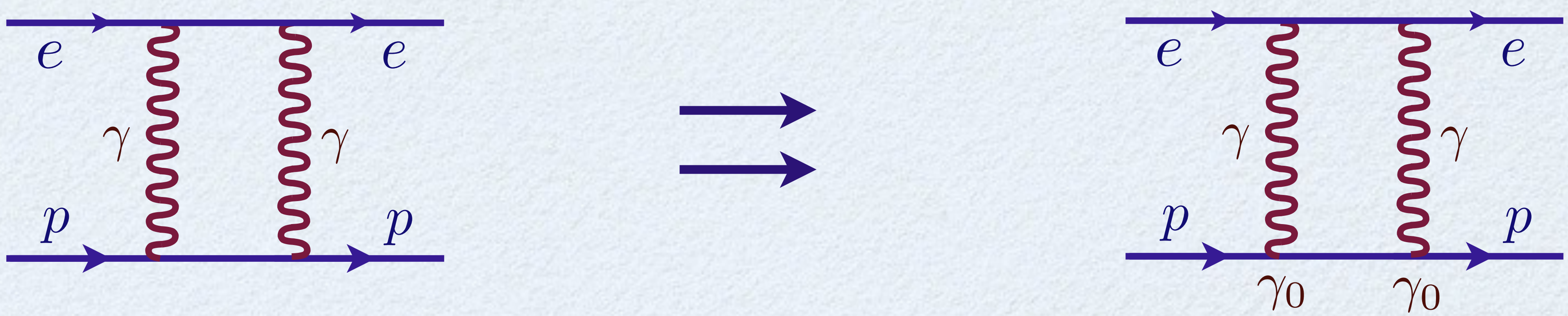
Point-like couplings → $F_1 = 1$ $\quad F_2 = \mu_p - 1$

Dipole FFs for G_M, G_E → $G_E = F_1 - \tau F_2$ $\quad G_M = F_1 + F_2$

$$(\tau = \frac{Q^2}{4M^2})$$

Small momentum transfer limit (e^-p)

Feshbach correction - scattering correction in Dirac theory (HE)

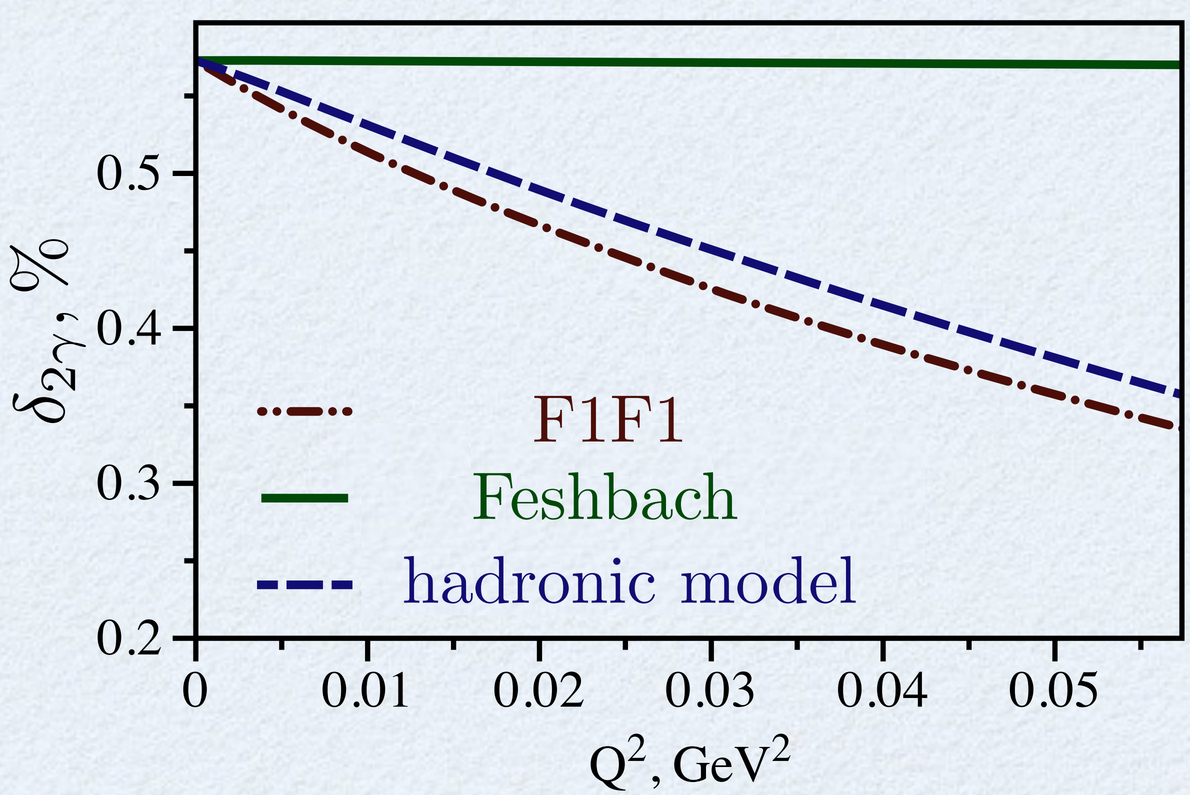


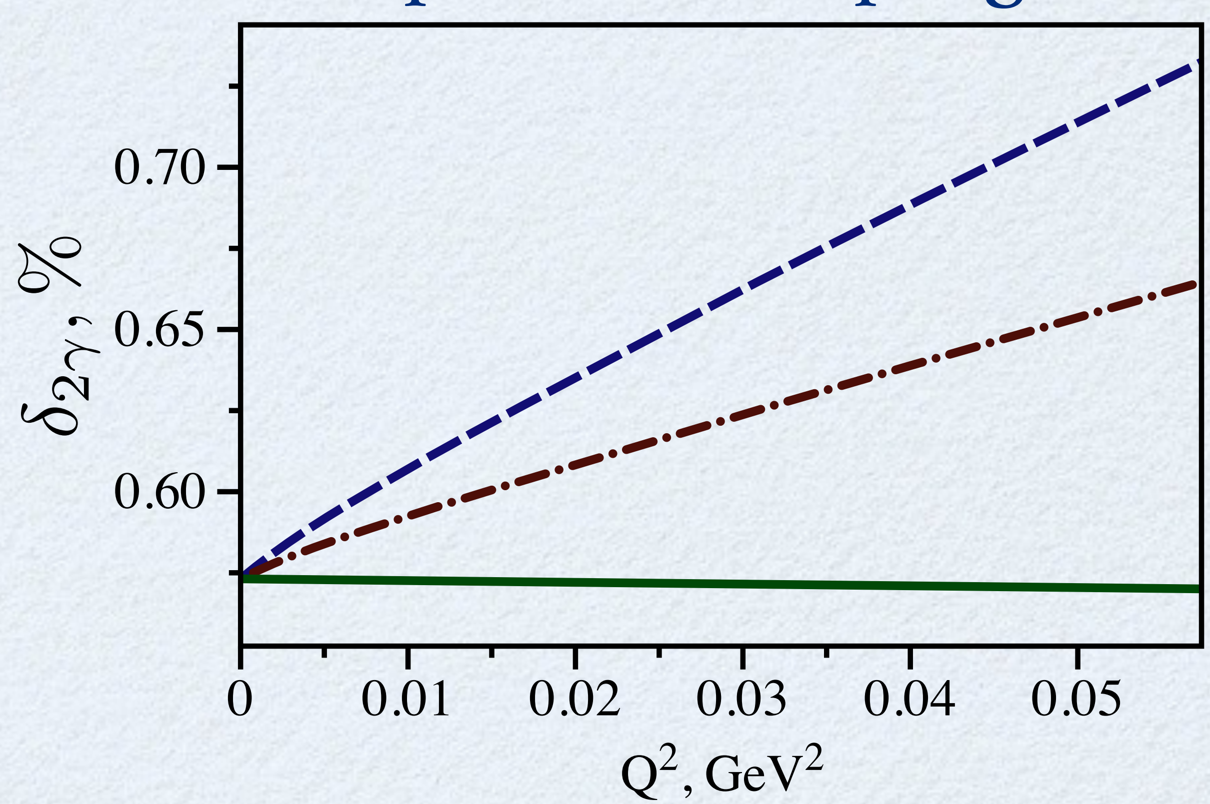
F_1F_1 structure | *Coulomb photons*

$$\delta_F = \pi\alpha\frac{\sqrt{1-\epsilon}}{\sqrt{1-\epsilon}+\sqrt{1+\epsilon}} = \pi\alpha\frac{sin(\frac{\theta}{2})-sin(\frac{\theta}{2})^2}{cos(\frac{\theta}{2})^2}$$

W. A. McKinley and H. Feshbach (1948)

dipole FFs $\epsilon = 0.8$ **point-like coupling**



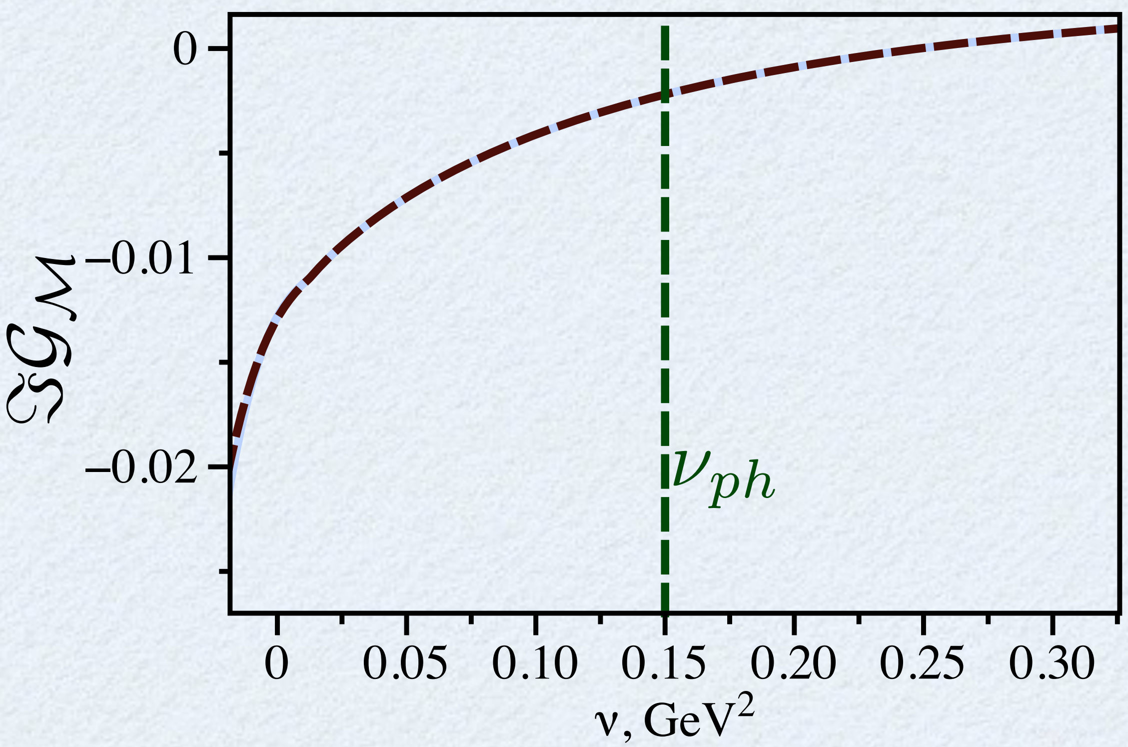


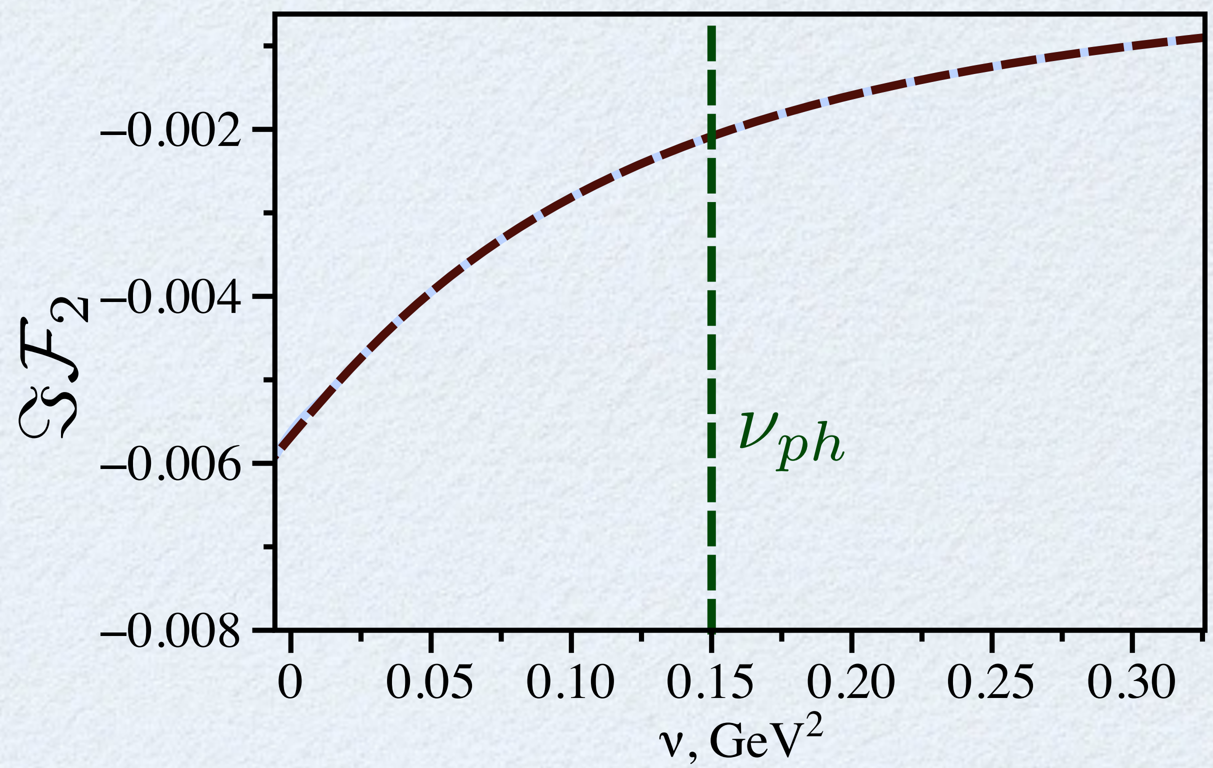
Hadronic model vs. dispersion relations

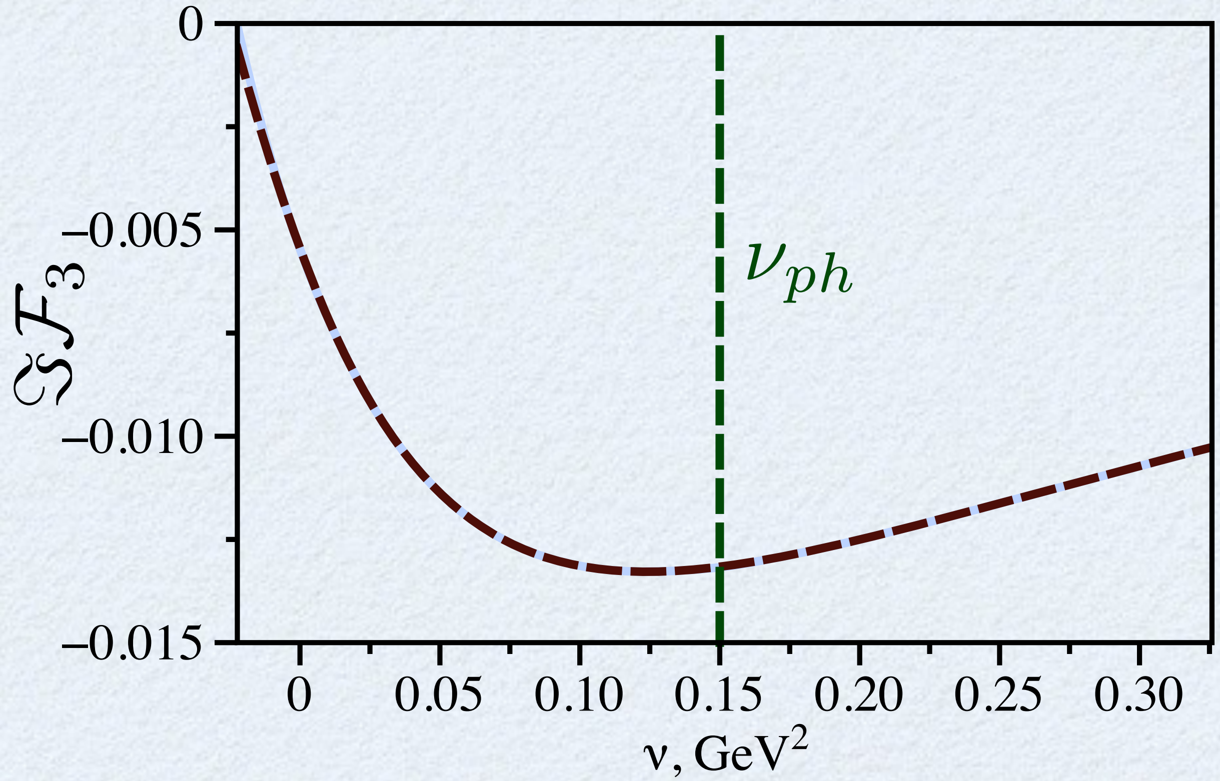
- Imaginary parts

Amplitudes imaginary parts

Dipole form of G_M, G_E







$F_1 F_1$ *structure*

hadronic model

unitarity relations

$Q^2 = 0.1\ GeV^2$

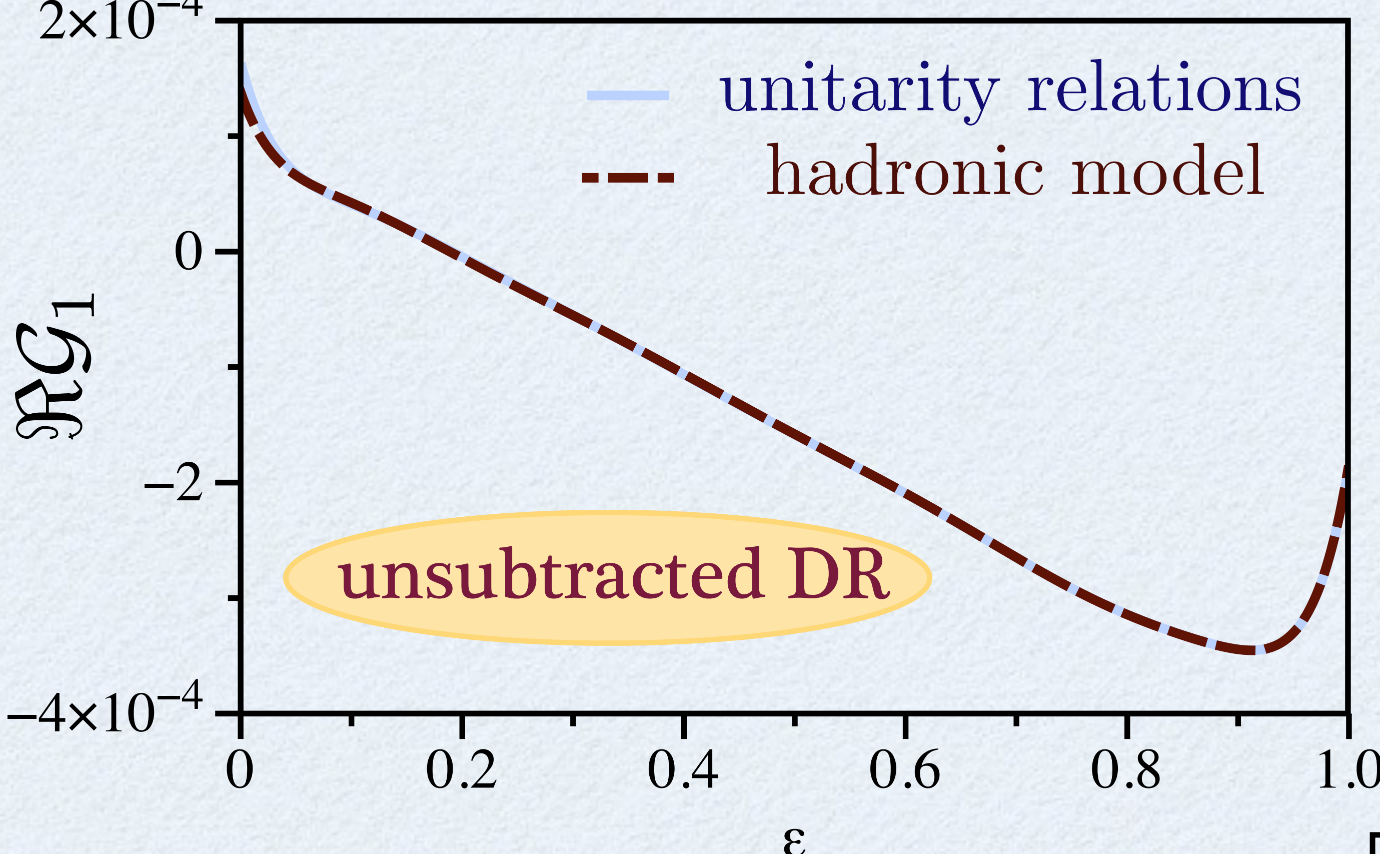
$\nu_{ph} = 0.15\ GeV^2$

Hadronic model vs. dispersion relations

- Imaginary parts are the same
- Real parts

Amplitudes real parts

Dipole form of G_M, G_E



F_2F_2 *structure*

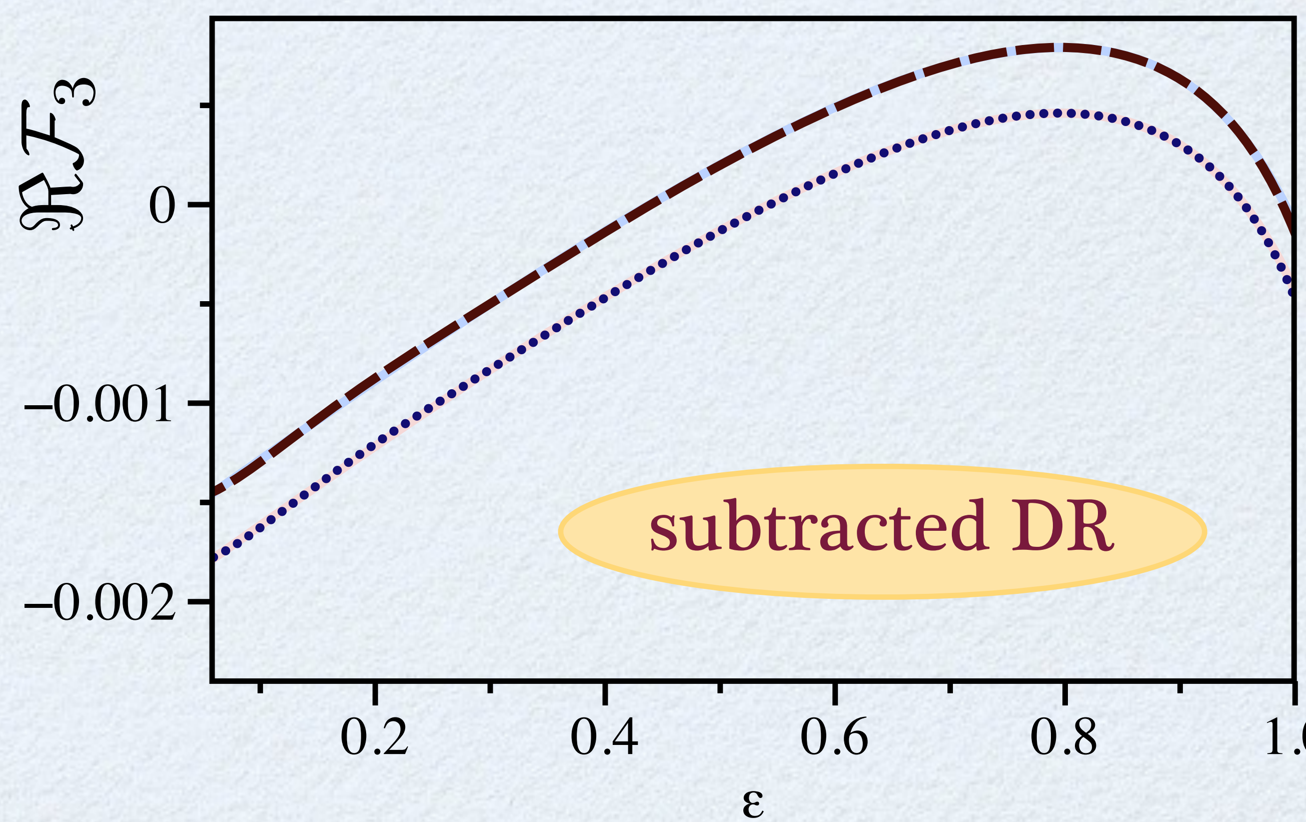
$Q^2 = 0.1 GeV^2$

exact

$\mathrm{DR}(\nu_0 = 2\mathrm{GeV}^2)$

exact

$\mathrm{DR}(\nu_0 = 1\mathrm{GeV}^2)$



Hadronic model vs. dispersion relations

- Imaginary parts are the same

- Real parts are the same for

all F1F1 amplitudes $\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3$

all F1F2 amplitudes $\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3$

F2F2 amplitudes $\mathcal{F}_2 \quad \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3$

Fixed-t subtracted dispersion relation works

F2F2 amplitudes $\mathcal{G}_M \quad \mathcal{F}_3$

Hadronic model vs. dispersion relations

- Imaginary parts are the same

- Real parts are the same for

all F1F1 amplitudes	all F1F2 amplitudes	F2F2 amplitudes
$\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3$	$\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3$	$\mathcal{F}_2 \quad \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3$

Fixed-t subtracted dispersion relation works

F2F2 amplitudes $\mathcal{G}_M \quad \mathcal{F}_3$

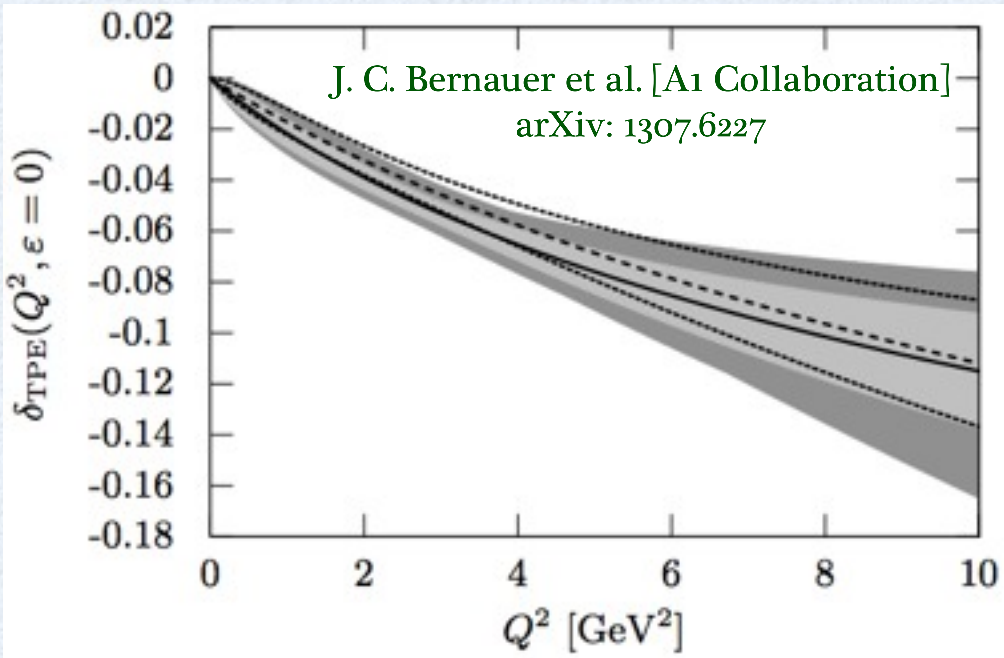
- Calculation based on DR

This work

- for amplitudes $\mathcal{G}_1, \ \mathcal{G}_2$ unsubtracted DR can be used

- for amplitude $\mathcal{F}_3$ subtracted DR should be used

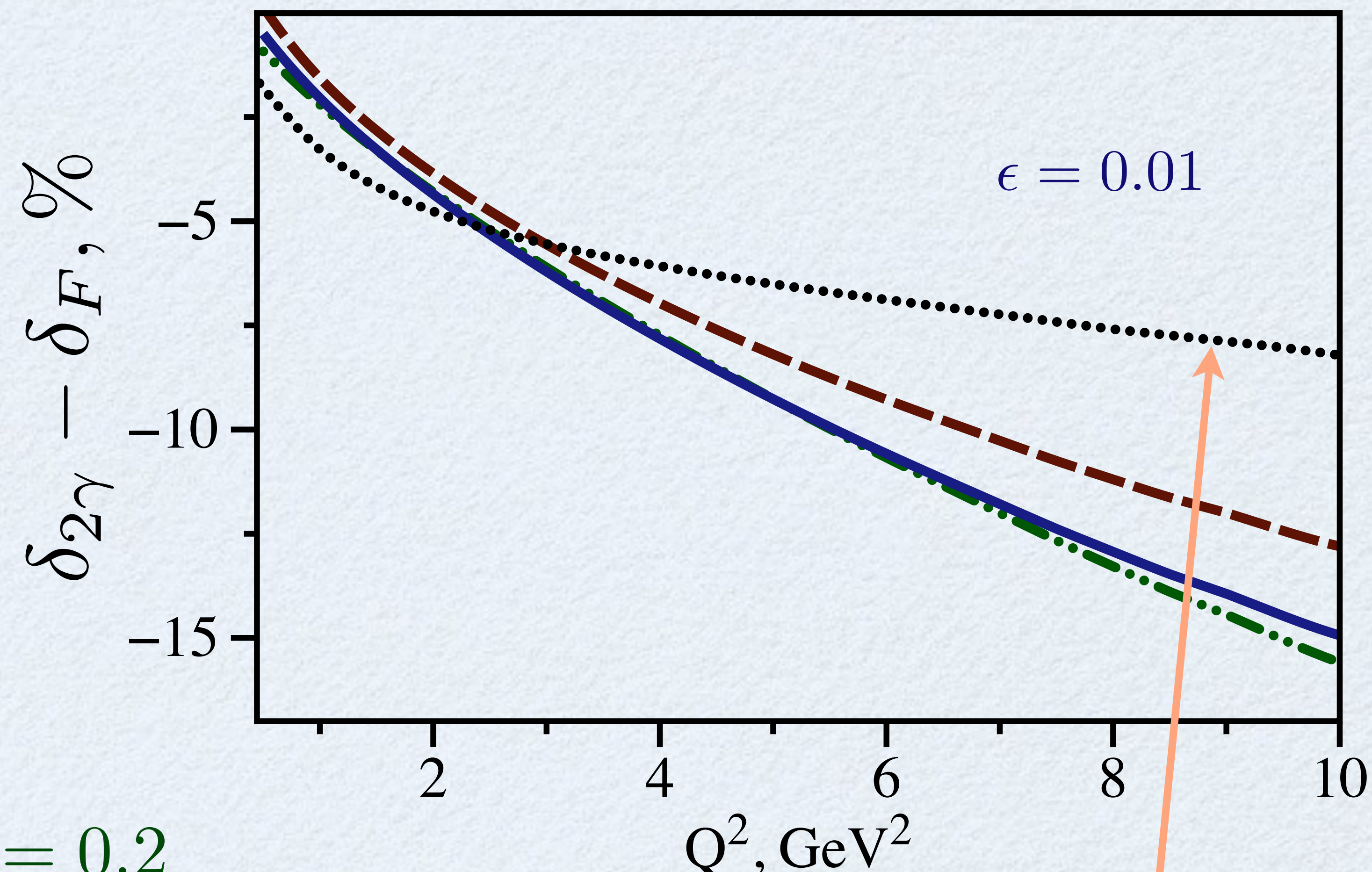
2ɣ in e⁻p elastic scattering



subtracted DR

subtraction point $\Re\mathcal{F}_3^{F_2 F_2}(\nu_0, Q^2)$

fixed from $\delta_{2\gamma}(\nu_0, Q^2)$ data



$$\delta_{2\gamma} = \delta_F + \delta_{TPE}$$

$$\delta_{TPE} = -(1-\epsilon) a ln(bQ^2 + 1)$$

subtraction points

$\epsilon_0 = 0.2$

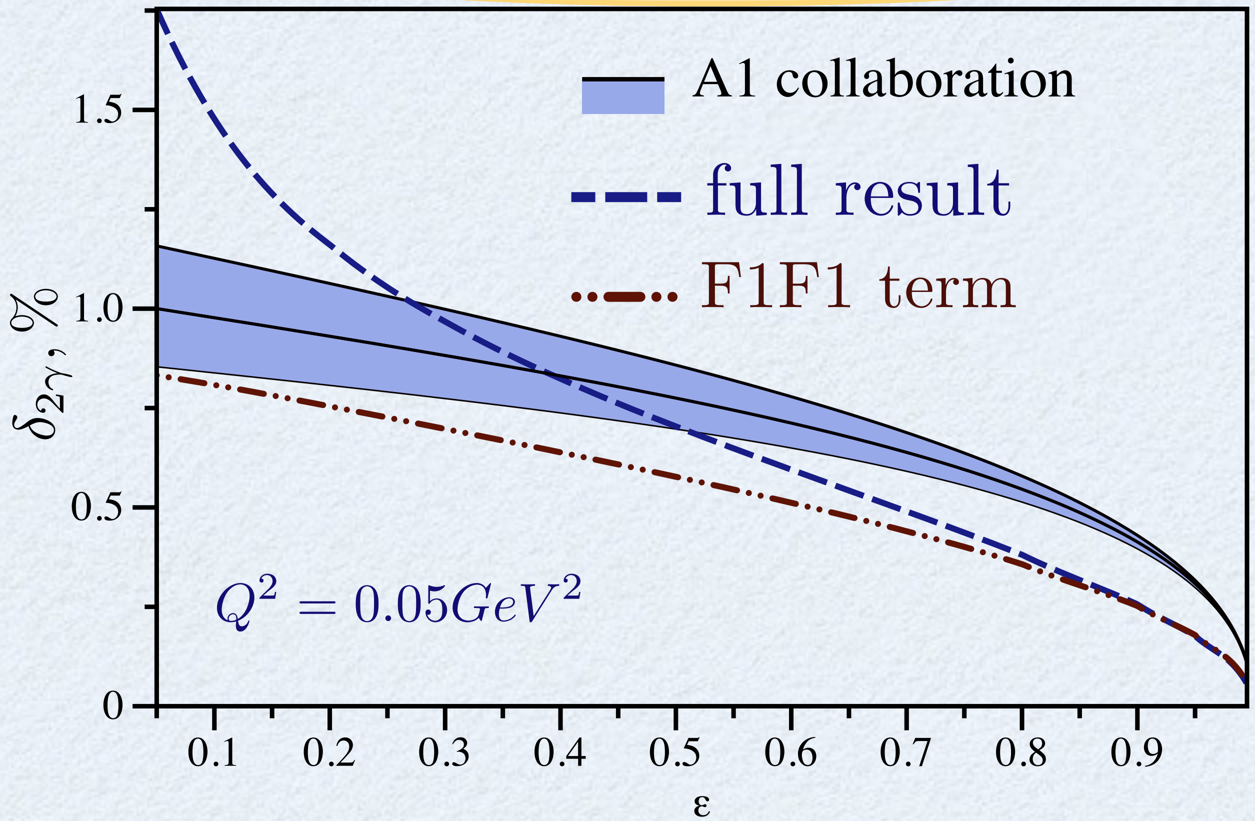
$\epsilon_0 = 0.5$

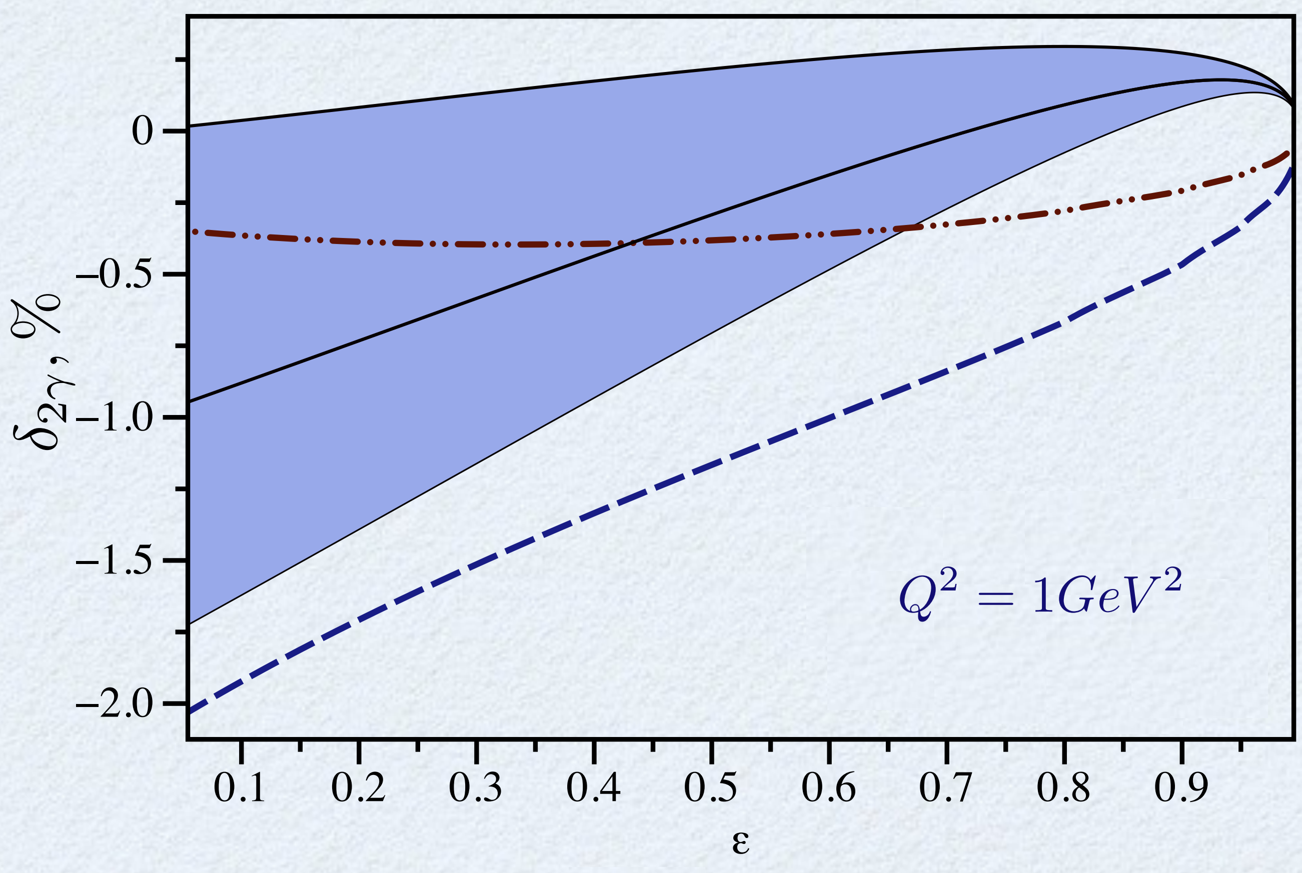
$\epsilon_0 = 0.8$

box diagram model calculation

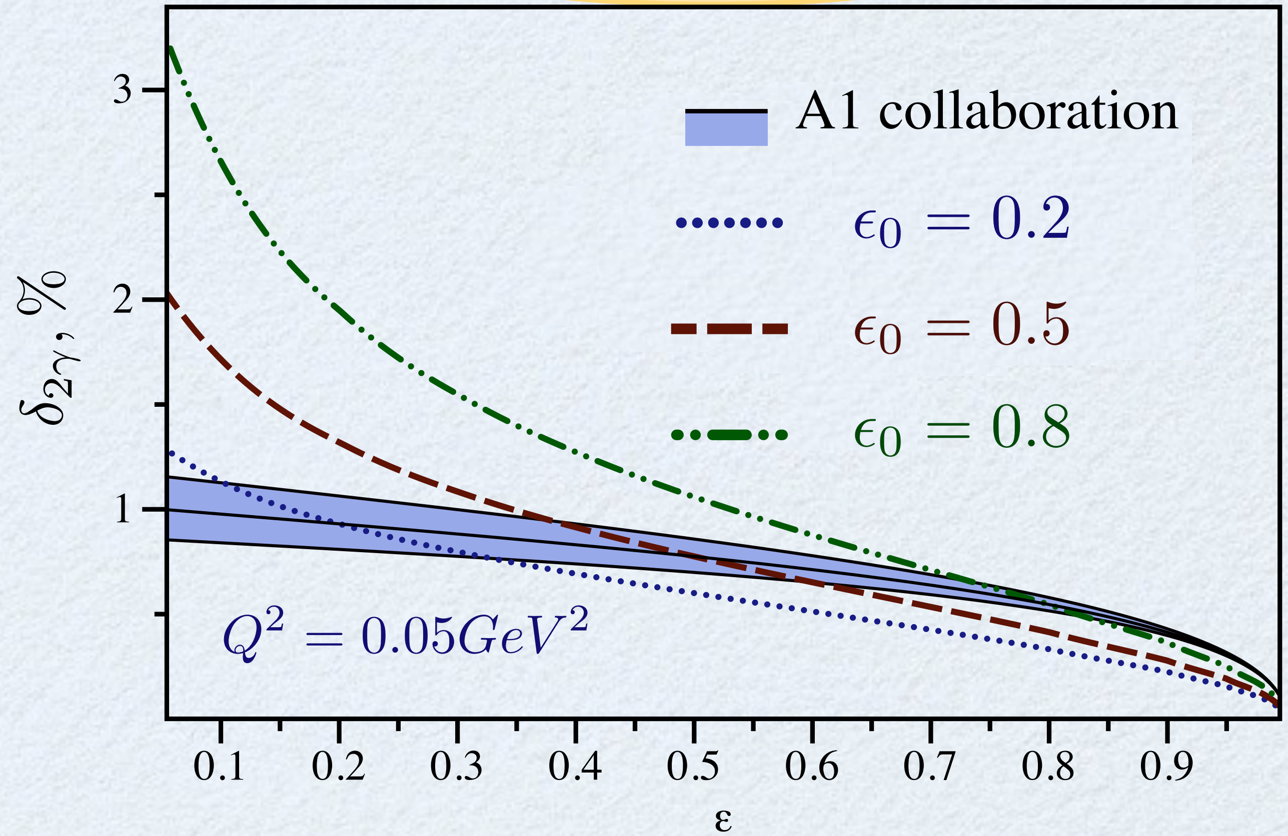
2ɣ in e⁻p elastic scattering

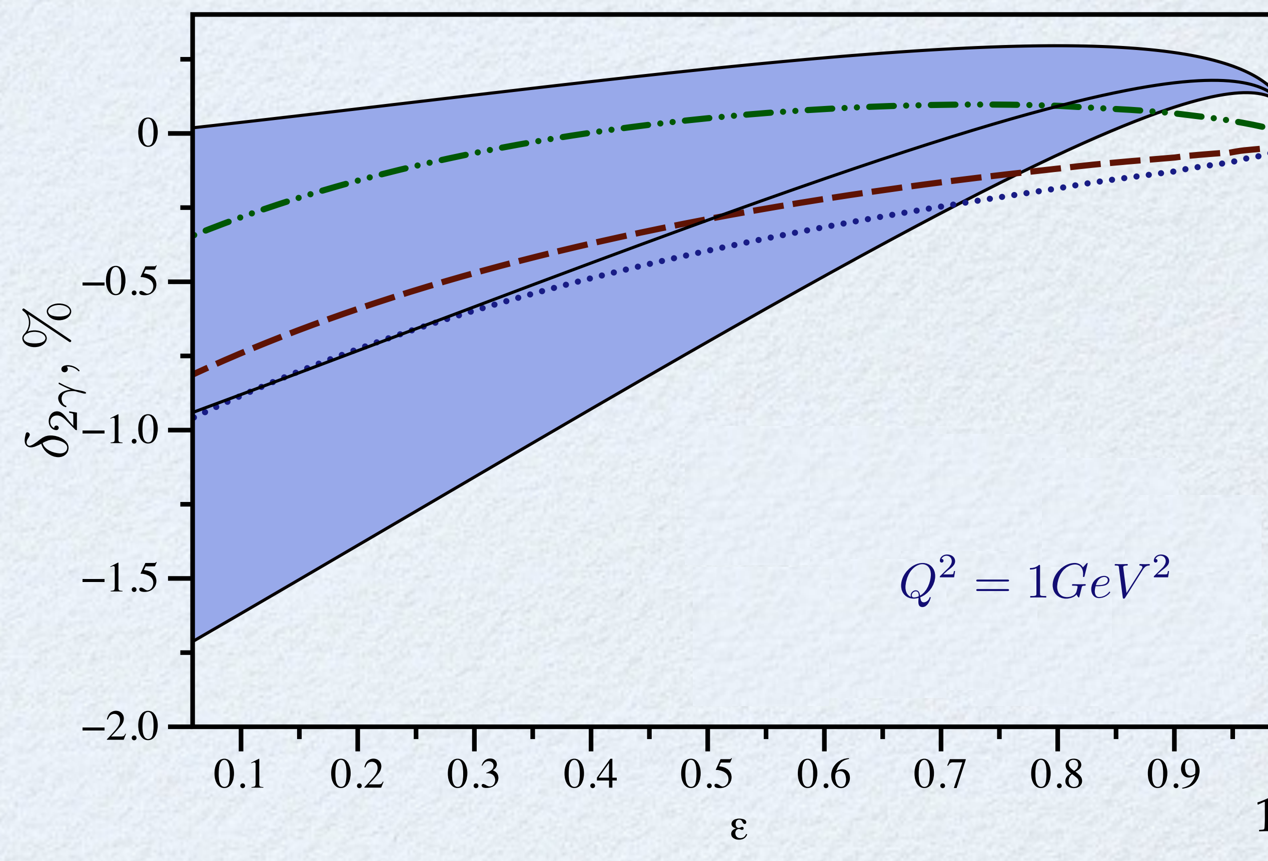
box diagram model calculation





subtracted DR

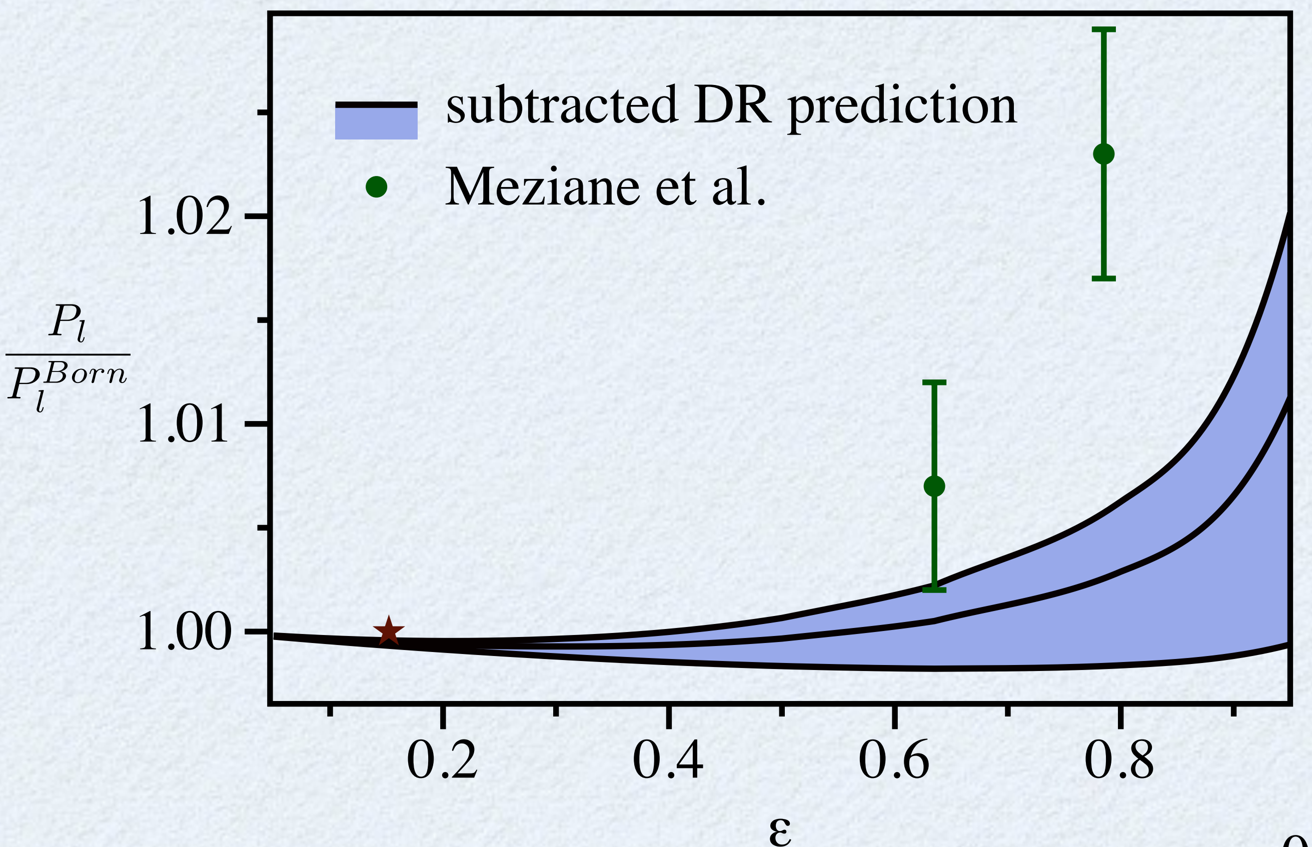




Polarization transfer observables (e^-p)

Experimental points at $Q^2 \sim 2.5 \mathrm{GeV}^2$

M. Meziane et al. [GEp2gamma Collaboration] (2011)



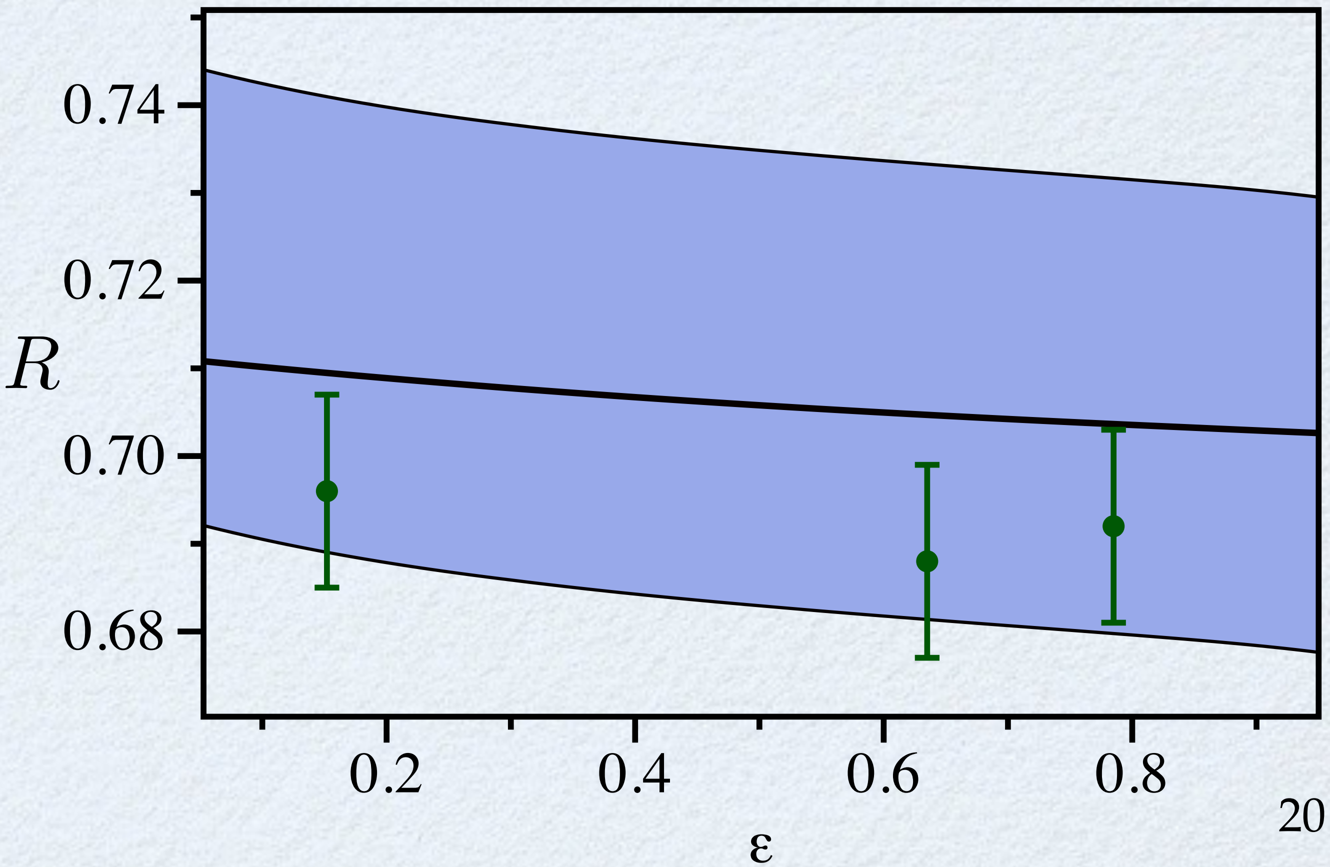
Longitudinal polarization transfer

$$\frac{P_l}{P_l^{Born}} - 1 \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3$$

Longitudinal to transverse polarization transfer

$$R = -\mu_p \sqrt{\frac{(1+\epsilon)\tau}{2\epsilon}} \frac{P_t}{P_l}$$

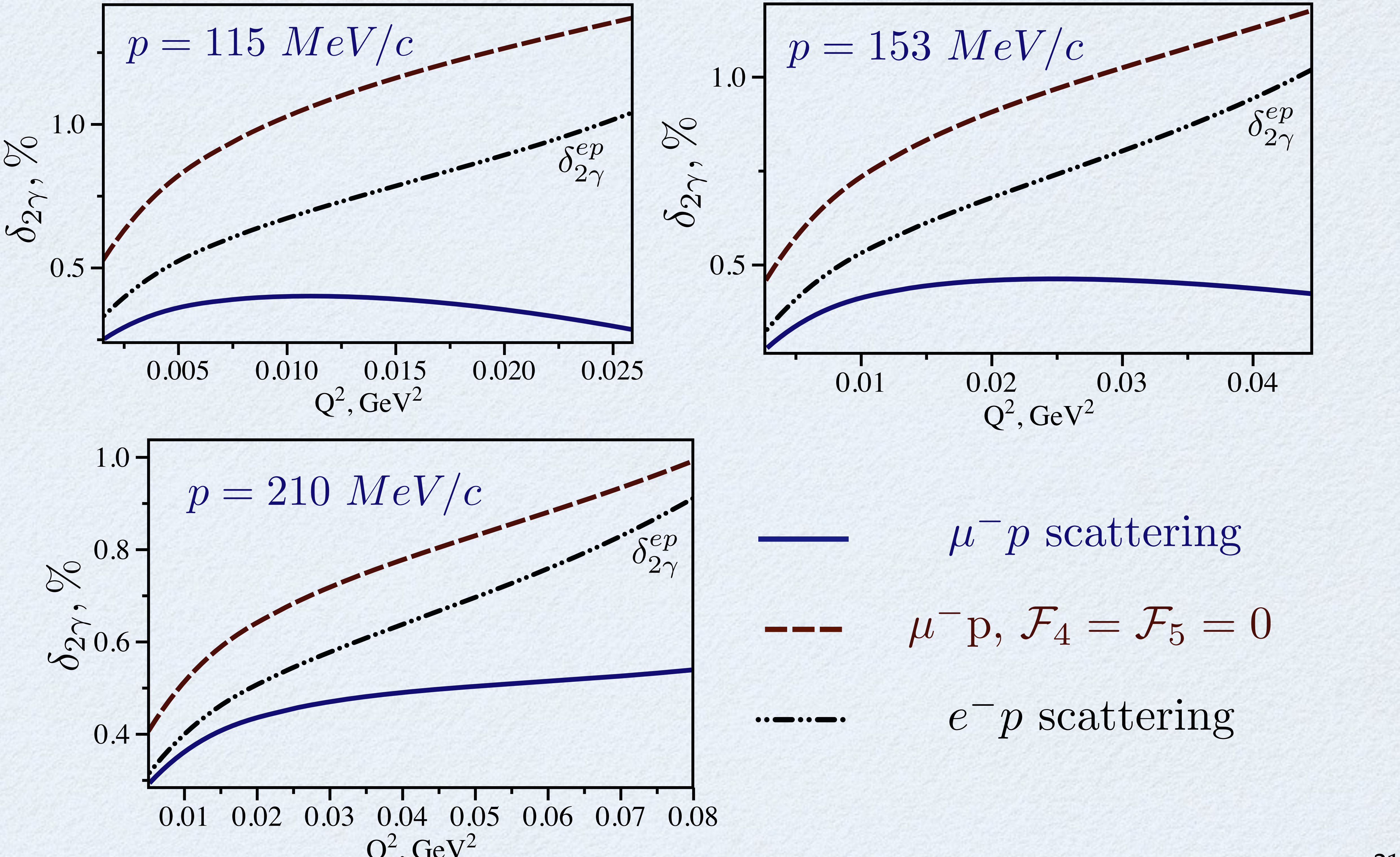
$$R - \mu_p \frac{G_E}{G_M} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3$$



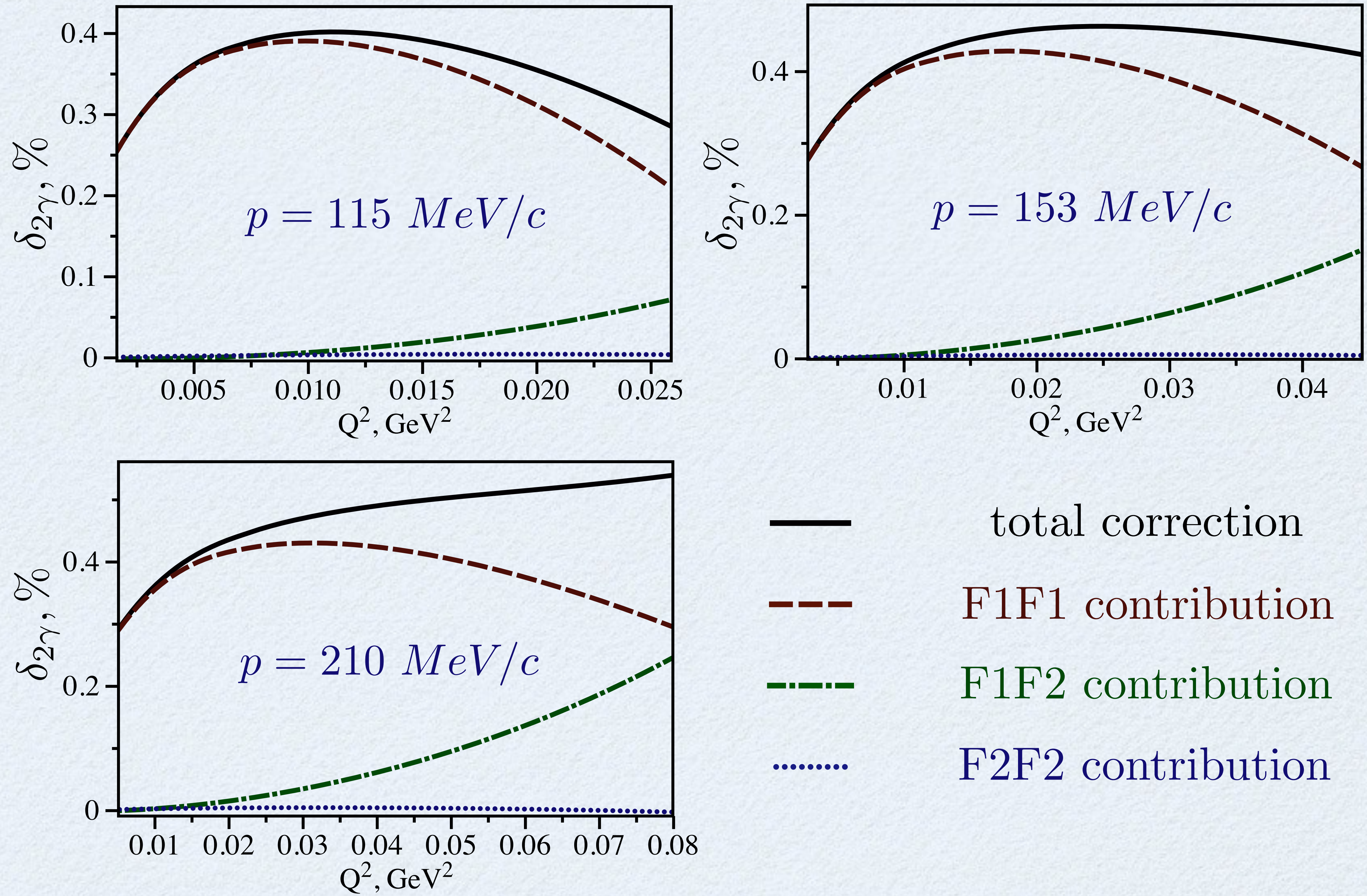
μ^-p experiments estimates

TPE correction

$\delta_{2\gamma} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3, \Re\mathcal{F}_4, \Re\mathcal{F}_5$



μ^-p experiments estimates



F1F1 contribution dominates

DRs approach requires two subtraction points

Conclusions and outlook

- Dispersion relations framework was developed
  - DR for ep scattering require 1 exp. point as input
  - DR for μp scattering require 2 exp. points as input
- DR checked vs. hadronic model calculation (ep):

F1F1, F1F2: agreement   F2F2 : on-shell model violates DR

- Theoretical estimates for elastic (ep and μp) cross section and polarization transfer observables were made
- Next step: inclusion of inelastic intermediate states (πN)

Thanks for your attention !!!