Nucleon Form Factors: Impact of Delta Resonance and Analyticity

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Proton charge radius

atomic energy splittings:

- in electronic hydrogen

- in muonic hydrogen: $r_E^p = 0.84087(39)$ fm

[conf.: Antognini et al., 2013]

$\longrightarrow$ CODATA value: $\longleftarrow$

$r_E^p = 0.8775(51)$ fm

electron-proton scattering:

- extrapolated without physical constraints

- here: combined with other processes using analyticity and unitarity

$\Rightarrow$ discrepancy due to new physics?

(violation of lepton universality, hidden photons, ...)

Theoretical framework

Basic definitions

- Nucleon matrix elements of the em vector current J_μ^I

$$\langle N(p')|J_\mu^I|N(p)\rangle = \bar{u}(p')\left[F_1^I(t)\,\gamma_\mu + i\,\frac{F_2^I(t)}{2m}\,\sigma_{\mu\nu}q^\nu\right]u(p)$$

⋆ isospin $I = S, V$ (isoScalar, isoVector) $[= (p \pm n)/2]$

⋆ four-momentum transfer $t \equiv q^2 = (p' - p)^2 \equiv -Q^2$

⋆ F_1 = Dirac form factor, F_2 = Pauli form factor

⋆ Normalizations: $F_1^V(0) = F_1^S(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

⋆ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2$, $G_M = F_1 + F_2$

⋆ Nucleon radii: $F(t) = F(0)\,\left[1 + t\langle r^2\rangle/6 + \ldots\right]$ [except for the neutron charge ff]

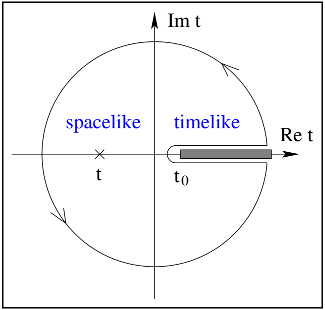
Dispersion relations

The form factors have cuts in the interval $[t_n, \infty[$ $(n = 0, 1, 2, ..)$ and poles

Federbush, Goldberger, Treiman, Drell, Zachariasen, Frazer, Fulco, Höhler, . . .

$\Rightarrow$ Dispersion relations for $F_i(t)$ $(i = 1, 2)$:

$$F_i(t) = \frac{1}{\pi}\int_{t_0}^{\infty} dt' \, \frac{\text{Im}\, F_i(t')}{t' - t}$$



- suppression of higher mass states
- central objects: spectral functions

$\text{Im}\ F_i(t)$

– cuts $\hat{=}$ multi-meson continua

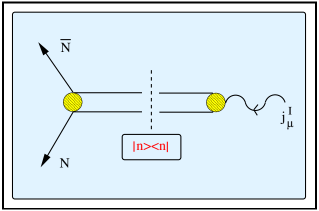
– poles $\hat{=}$ vector mesons

Spectral functions

- Spectral decomposition:

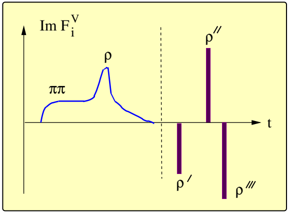
$$\mathrm{Im}\,\langle \bar{N}(p')N(p)|J^I_\mu|0\rangle \sim \sum_n \langle \bar{N}(p')N(p)|n\rangle\langle n|J^I_\mu|0\rangle \Rightarrow \mathrm{Im}\,F$$

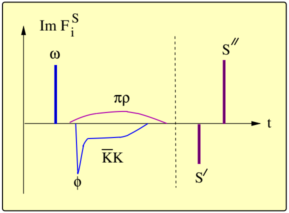
  - ⋆ on-shell intermediate states
  - ⋆ generates imaginary part
  - ⋆ accessible physical states



- *Isoscalar* intermediate states: $3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots +$ VMs $\rightarrow t_0 = 9M_\pi^2$
- *Isovector* intermediate states: $2\pi, 4\pi, \ldots +$ VMs $\rightarrow t_0 = 4M_\pi^2$
- Note that some poles are *generated* from the appropriate continua

Isovector/-scalar spectral functions





• 2π continuum is well-known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40\,M_\pi^2$

$$\mathrm{Im}\ F_i^V(t) = \frac{q_t^3}{\sqrt{t}} |F_\pi(t)| J_i(t)$$

⋆ $F_\pi(t)$ = pion vector form factor

⋆ $J_i \sim$ P-wave pion-nucleon partial waves in the t-channel $[\sim f_\pm^1(t)]$

Frazer, Fulco, Höhler, Pietarinen, . . .

→ strong shoulder → isovector radii

Recent det. in LHM, EPJA **48** (2012) 151

• $K\bar{K}$ continuum can be extracted from analytically cont. KN scattering ampl.
→ generates most of the ϕ contribution

Hammer, Ramsey-Musolf, Phys. Rev. C **60** (1999) 045204, 045205

• Further strength in the ϕ-region generated by correlated $\pi\rho$ exchange
→ strong cancellations ($K\bar{K}, K^*K, \pi\rho$)
→ takes away sizeable strength from the ϕ

Meißner, Mull, Speth, van Orden, Phys. Lett. B **408** (1997) 381

Fit functions from dispersion relations

- Representation of the pole contributions: vector mesons
  [NB: can be extended for finite width]

$$\mathrm{Im}\; F_i^V(t) = \sum_v \pi a_i^v\, \delta(t - M_v^2)\;, \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \;\Rightarrow\; F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}$$

- *Isovector* spectral functions:

$$\mathrm{Im}\; F_i^V(t) = \mathrm{Im}\; F_i^{(2\pi)}(t) + \sum_{v=v_1,v_2,\ldots} \pi\, a_i^v \delta(t - M_v^2)\;, \quad (i = 1, 2)$$

- *Isoscalar* spectral functions:

$$\mathrm{Im} F_i^S(t) = \mathrm{Im} F_i^{(K\bar{K})}(t) + \mathrm{Im} F_i^{(\pi\rho)}(t) + \sum_{v=\omega,\phi,s_1,s_2,\ldots} \pi\, a_i^v \delta(t - M_v^2)$$

Parameters: 2 for the ω, ϕ, 3 for each other V-mesons minus # of constraints

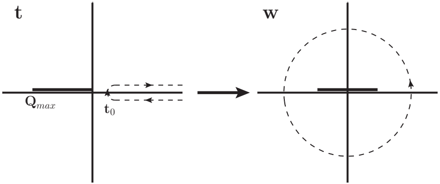
Ill-posed problem → extra constraint: min. # of poles to describe the data

Fit functions from analyticity

More elaborate in Hill, Paz, Phys. Rev. D **82**, 113005 (2010)

Here, take the simplest form:

In complex plane of $t = -Q^2$, map the cut conformally onto the unit circle in the w-plane



- Expand form factors in w
- Possible: constrain coeff. via integral over whole range of spectral function

Here mainly used for illustrative purposes.

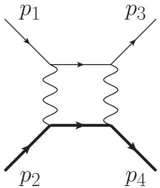
Cross sections with 1 or 2γ exchange

Born approx.: $\boxed{\frac{d\sigma_{1\gamma}}{d\Omega} = \frac{d\sigma_{point}}{d\Omega} \frac{\epsilon G_E(Q^2)^2 + \tau G_M(Q^2)^2}{\epsilon(1+\tau)}}$ $\quad \tau = \frac{Q^2}{4M_N^2}$

dep. on Q^2 and long. pol. of virtual photon $\epsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1}, \theta \sphericalangle p_1, p_3.$

$$\frac{d\sigma_{corr.}}{d\Omega} = (\mathcal{M}_{1\gamma}^\dagger + \mathcal{M}_{2\gamma}^\dagger + ...)(\mathcal{M}_{1\gamma} + \mathcal{M}_{2\gamma} + ...) = \frac{d\sigma_{1\gamma}}{d\Omega}(1 + \delta_{2\gamma} + ...)$$

$$\Rightarrow \delta_{2\gamma} \underbrace{\approx}_{\mathcal{O}(\alpha^3)} \frac{2Re(\mathcal{M}_{1\gamma}^\dagger \mathcal{M}_{2\gamma})}{|\mathcal{M}_{1\gamma}|^2}$$

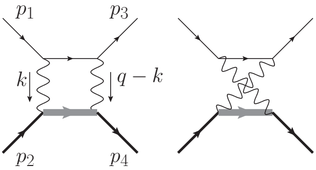


- For the same FFs in $\mathcal{M}_{1\gamma}$ and $\mathcal{M}_{2\gamma}$, the ϵ-dependence partly cancels

$\Rightarrow$ Allow the first nucleon resonance, with lowest mass and strongest coupling as intermediate state: Δ resonance

Explicit calculation for e-p scattering with Δ-state

$$\mathcal{M}_{2\gamma}^{box} \propto \int \frac{d^4k}{(2\pi)^4} L_{\mu\nu}^{box} H_{N/\Delta}^{\mu\nu} D(k) D(q-k)$$



$L_{\mu\nu}^{box}$: leptonic tensor, $H_{\Delta}^{\mu\nu}$: hadronic tensor, $\mathbf{D(k)}$: photon propagator

- dependence on IR-cutoff cancelled by term in bremsstrahlung cross section
- UV divergences avoided by appropriate form factors (FFs)

Cancellations between numerators, denominators and FFs by hand:
⇒ Used 2 indep. symbolic manipulation programs + 1 for integrals:

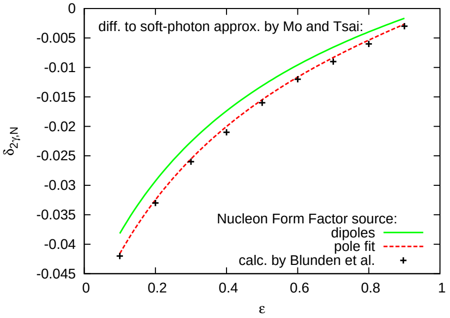
- FORM (Vermaseren, math-ph/0010025)
- FeynCalc (Mertig et al., Comp. Phys. Comm. 64, 345)
- LoopTools (Hahn et al., Comput.Phys.Comm.118, 153)

Results

(here all preliminary!)

TPE-correction with nucleon states

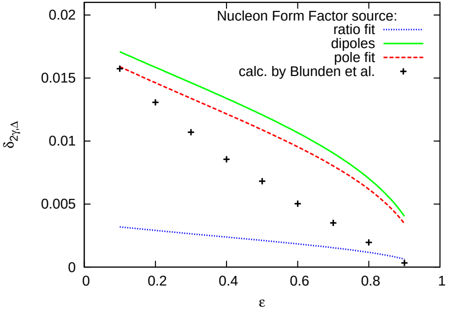
$Q^2 = 3\ \text{GeV}^2$:



- **diff. to soft-photon approx.: IR-cutoff-independent**
- **agreement with Blunden et al.**
- **non-linear ϵ-dependence**

TPE-correction with Δ states

$Q^2 = 3\ \text{GeV}^2$:



- **non-linear ϵ-dependence**
- **disagreement with Blunden et al.**
- **dependence on NFFs**
- **advantage of our calc.: correct extrapolation** $\delta_\Delta \underset{\epsilon\to 1}{=} 0$

Application to cross sections

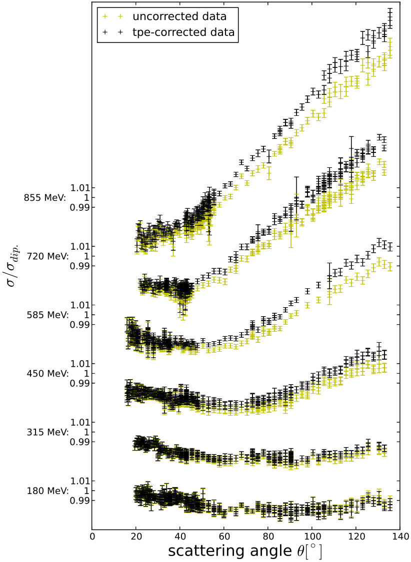
e-p scatt. by A1 coll. (Mainz):
highest quoted precision

- 6 energy settings for the incoming electron beam
- 3 spectrometers

Contains TPE-limit for $Q^2 \to 0$:
wrong sign for some kinematics
(Arrington, PRL 107, 119101 (2011))

We

- **subtract the orig. approx.**
- **include our full TPE calc. with N and Δ intermediate states and realistic FFs**



Illustrative fits

Start with 1 constraint:

Analyticity

Obeyed by conformal mapping:

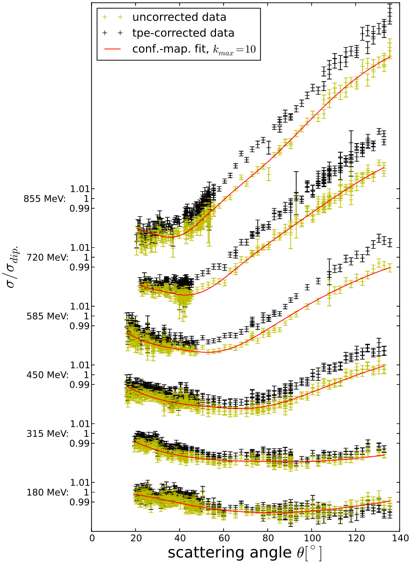
$$z(t, t_{cut}) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut}}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut}}}$$

$t_{cut} = 4m_\pi^2$: lowest singularity

Fit: $$G_{E/M}(t) = \sum_{k=0}^{k_{max}} a_k z(t)^k$$

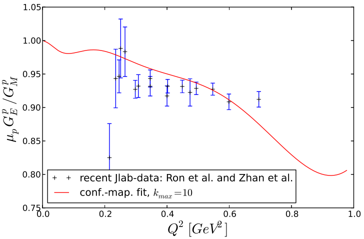
to the original data:

k_{max}	χ^2	r_E^P [fm]
5	1.230	0.892
6	1.137	0.868
7	1.126	0.867
8	1.122	0.876
9	1.114	0.849
10	**1.115**	**0.843**



Prediction from illustrative fit

Compare to recent polarization measurements:



- very similar to spline fit by Bernauer:

same wiggle from the magnetic form factor

- lower χ^2 than conventional fits, small r_E^p

$\Rightarrow$ **consider more physical constraints**

Fits with more physical constraints

Add constraints from:

Unitarity

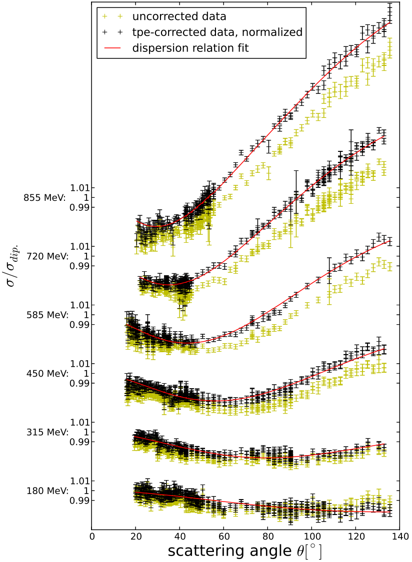
One possibility: bounds on coeff. in conf.-map. (increase χ^2)

Or: use all information on the spectral function

$\Rightarrow$ **via dispersion relations**

- increases impact of normalization
- increases $\frac{\chi^2}{ndf}$ to 1.43

Lower χ^2 inconsistent with spectral function



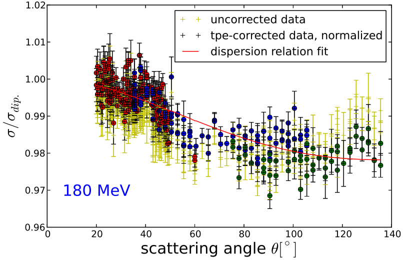
Fits with more physical constraints

- for every spectrometer:

  (here red, blue, green)

  systematic error bands growing linearly in θ, up to $\sim$0.5 %

- **systematic deviations minimized by full TPE**



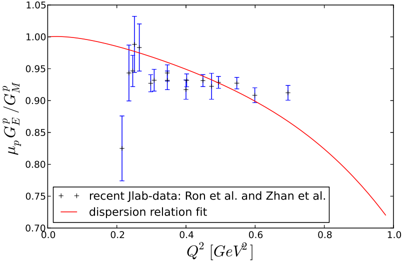
prediction from phys. constraints:

$\Rightarrow$no wiggles in FF ratio

$r_E^p = 0.843$ (0.830-0.857) fm

$r_M^p = 0.849$ (0.847-0.854) fm

(3-σ bootstrap-errors for DR fit)



Summary/Outlook

We

- calculated TPE-corr. with **N/Δ** intermediate states & **realistic FFs**
- fitted e-p **cross sections** w/ and w/o TPE using:
  - ⋆ a flexible function, only obeying analyticity, allowing better description of the data than original analysis
  - ⋆ a constrained function, using available information from other data via unitarity

⇒ **BOTH FITTING PROCEDURES YIELD GOOD AGREEMENT WITH r_E^p FROM MUONIC HYDROGEN**

To be done:

- consider our TPE corrections in **regular hydrogen hfs**, ...