On the importance of the tail of proton charge density

or: how to get the *rms*-radius from (e,e) data?

Ingo Sick

Proton *rms*-radius

important quantity

traditionally determined via electron scattering, $q = 0$ slope of $G(q^2)$

analysis of world data yields R=0.886±0.008 fm

Recent result from Lamb-shift in muonic Hydrogen

very precise radius: R=0.8418±0.0007 fm

disagrees with (e,e) by many σ

Reasons for discrepancy?

many ideas discussed in literature

too many to detail here

no culprit identified

Purpose of talk:

scrutinize determination of *rms*-radius from (e,e)-data

understand anomalies

How to determine the *rms*-radius?

priori this looks simple:

fit data with parameterization for $G_e(q)$, $G_m(q)$

$q = 0$ slope of $G_e(q) \rightarrow$ *rms*-radius R

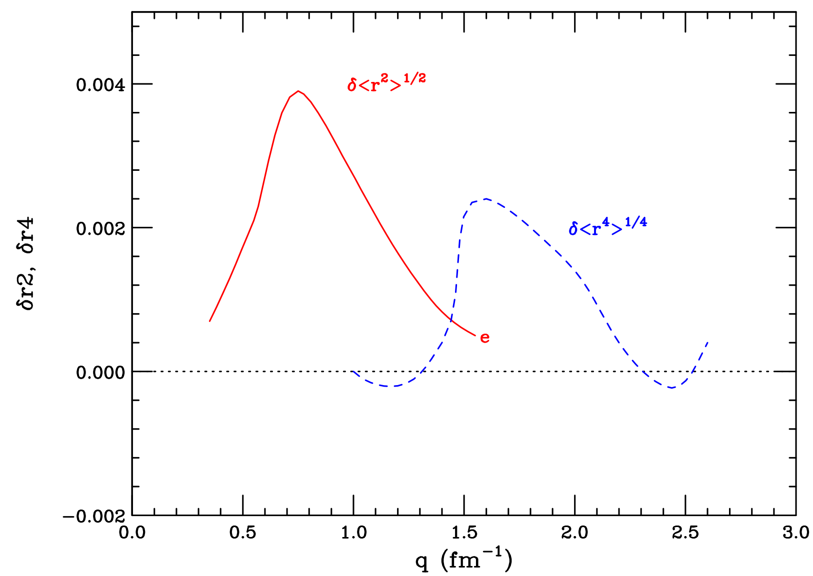
An unavoidable problem:

cannot measure down to $q = 0$

even if could, finite size effect would be too small: $G(q) = 1 - q^2R^2/6 + ...$

at very low q measure only the "1"

given exp. uncertainties δG



Important consideration

q-region sensitive to *rms*-radii

$0.5 < q < 1.3 fm^{-1}$

$0.01 < Q^2 < 0.06 GeV^2/c^2$

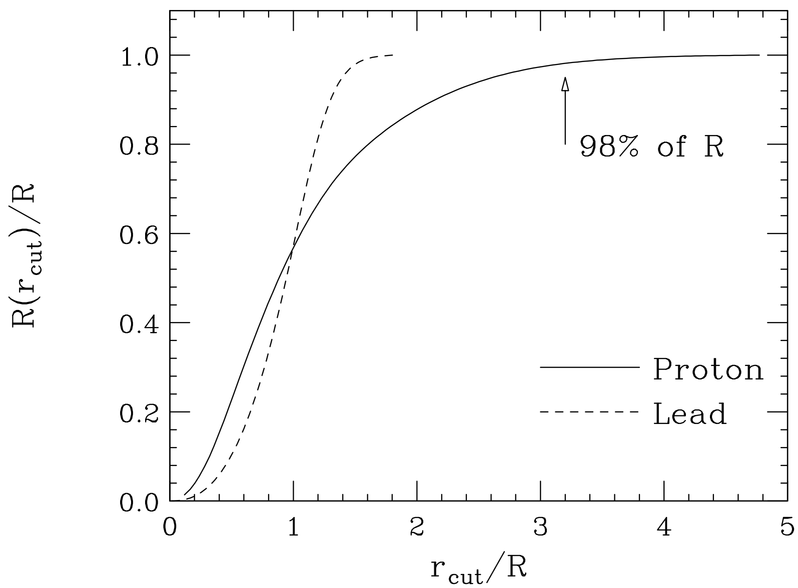
Data above $Q^2 \sim 0.06$ not relevant for R!

Extrapolation to $q = 0$ particularly difficult for proton

form factor $\sim$ dipole $1/(1+q^2c^2)^2$
$\rightarrow$ density $\sim$ exponential $\sim e^{-r/c}$
for *qualitative* discussion ignore rel. corr, 2γ, ... $\rightarrow$ G(q)=FT($\rho(r)$)

exponential density has very long tail!

Study $[\int_0^{r_{cut}} \rho(r)\ r^4 dr / \int_0^{\infty} \rho(r)\ r^4 dr]^{1/2}$ as function of cutoff r_{cut}



to get 98% of rms-radius R must integrate out to $r \sim 3.2 \cdot R \sim 3fm$

$\Longrightarrow$ R sensitive to very large r where $\rho(r)$ poorly determined

large r affect $G(q)$ at very low q, below q_{min}

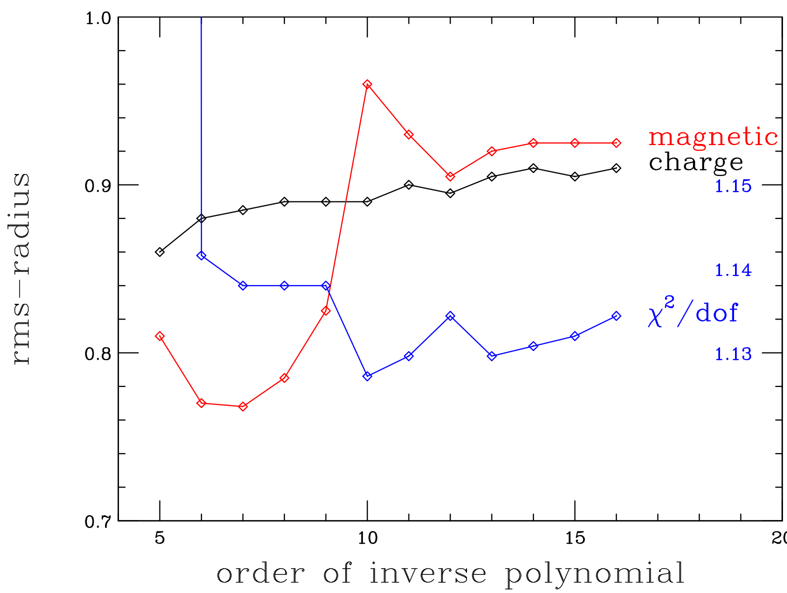
But there are worse pitfalls!

discuss starting from two recent results:

- Inverse Polynomial fit of Bernauer *et al.*
- Continued Fraction fit of Lorenz *et al.*

Inverse Polynomial Bernauer

$$G(q^2) = 1/(1 + a_1 q^2 + a_2 q^4 + ...)$$



Curious behavior:

between order N=7 and N=10 R^M jumps from $0.76 fm$ to $0.96 fm$

χ^2 best for N=10

would nominally be *the* best fit!

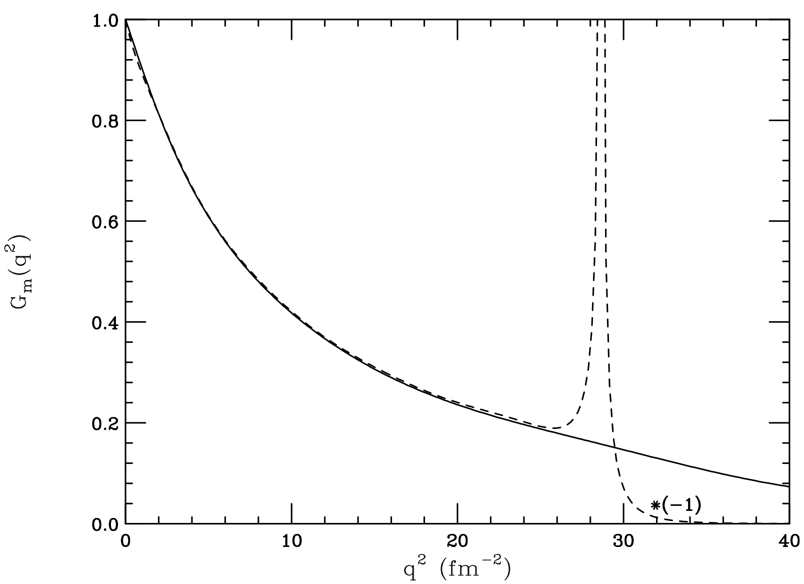
Bernauer *et al.* chose order N=7 (χ^2 ±stabilized)

Question remains:

what is responsible for jump?

how can the q^{20}-term affect the rms-radius?

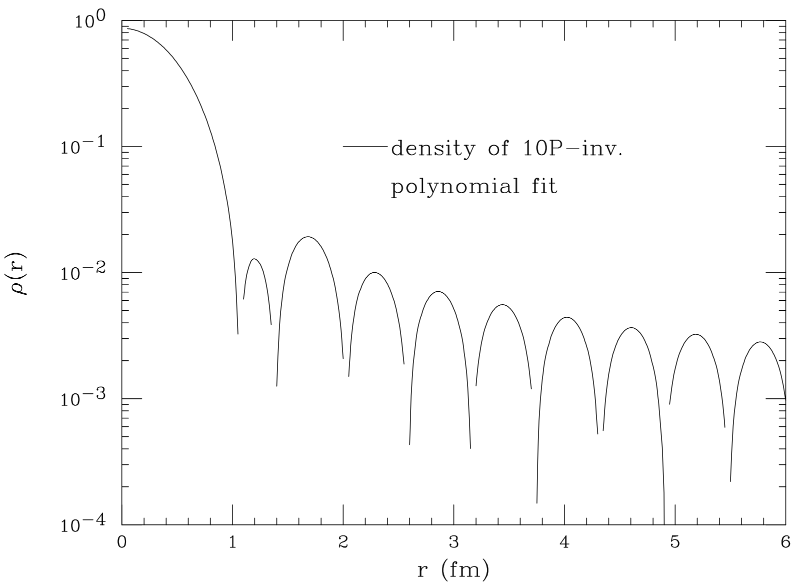
Understanding

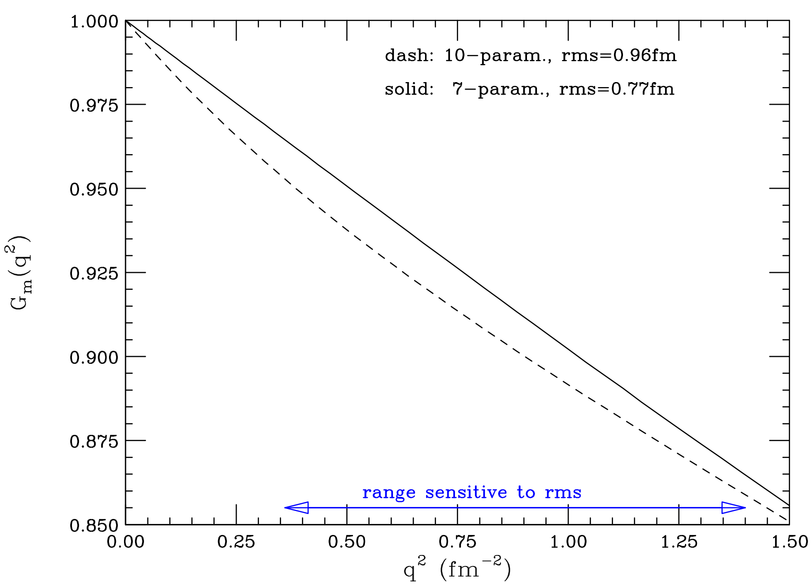


G_M for N=10 has pole at $q > q_{max}$

In $\rho(r)_m$ this leads to oscillatory tail extending to *very* large r, see next page

Density from $G(q)$ with pole





Tail affects $G_m(q^2)$ at *very low* q^2 <u>below</u> q^2_{min} of data

Structure at $q < q_{min}$ gives better χ^2 than N=7

note: data are floating

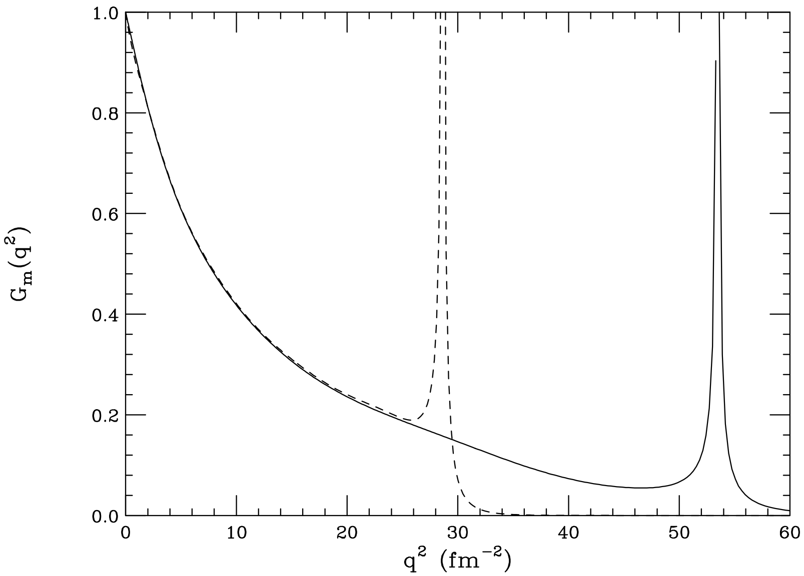
confirms old insight that absolute σ's *much* more valuable

Conclusion: N=10 fit is pathological.

but is N=7 better?

A priori: yes, since more 'reasonable'

however: N=7 has pole too!



but pole is at larger q, happens to have much smaller effect

Cannot believe either radius!

Continued Fraction fits by Lorenz *et al.*

$$G(q) = \cfrac{1}{1 + \cfrac{q^2 b_1}{1 + \cfrac{q^2 b_2}{1 + \cdots}}}$$

many fits of Bernauer data with variable q_{max}

for *e.g.* 5 terms and $q_{max} = 3.5 fm^{-1}$ find charge-rms-radius $0.84 fm$

disagrees with "accepted" result of $0.88 fm$

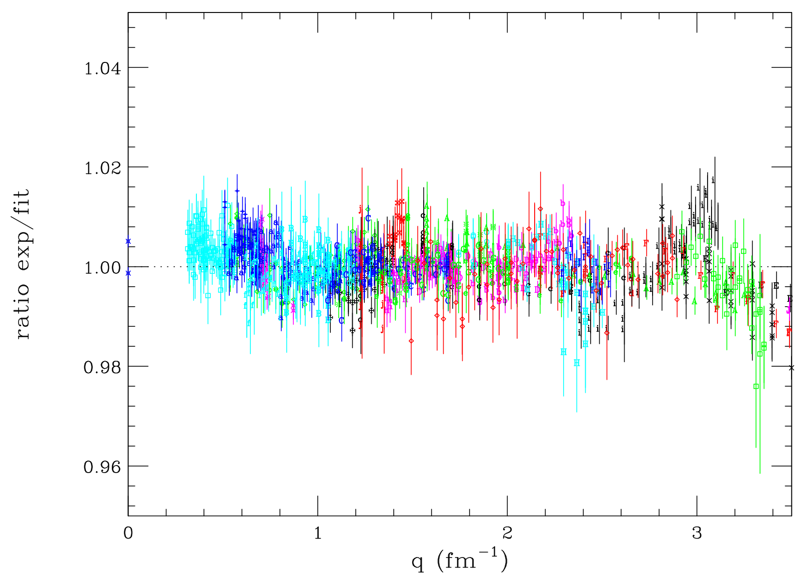
One reason

$\chi^2 \sim 1.4/dof$ not very good

$\rightarrow$ systematic deviations at low q

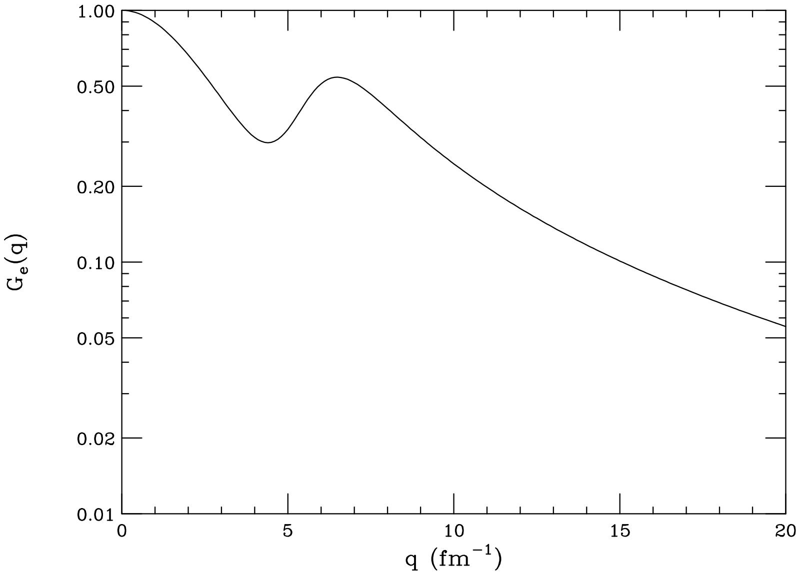
Spline fit gives $1.06/dof$

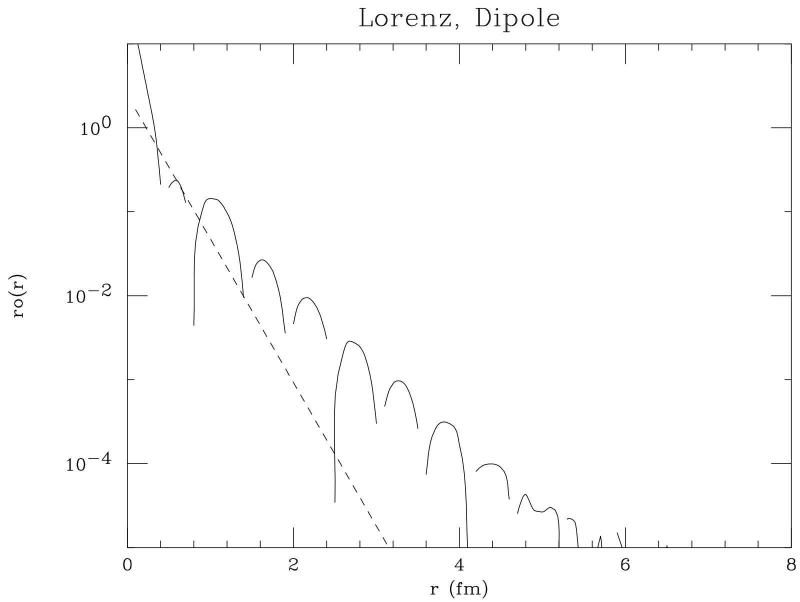
from such a fit cannot draw conclusions



Main problem of Lorenz *et al.*

Unphysical behavior of G at $q > q_{max} = 3.5 fm^{-1}$ (parameters courtesy H.-W. Hamme





large $G(q)$ at large q

falls *very* slowly

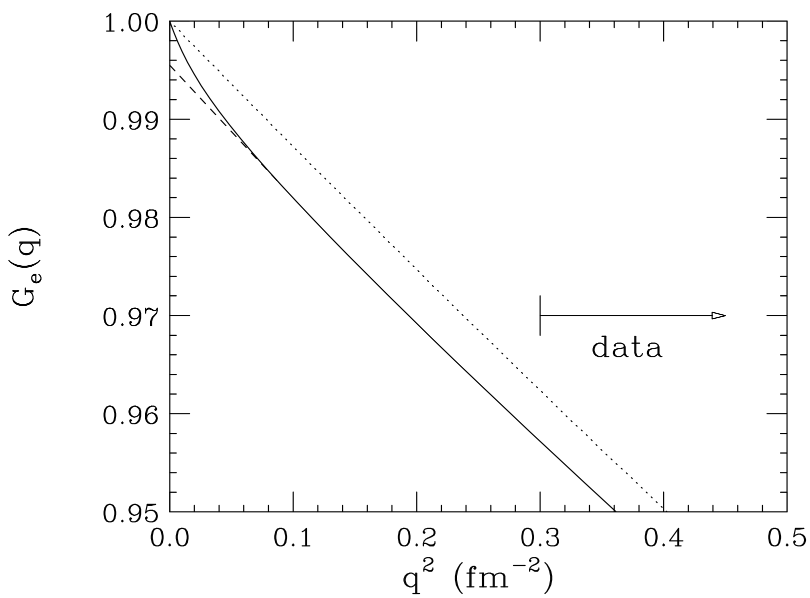
$\rightarrow$ structure of $\rho(r)$ at very large r

large contribution to *rms*-radius

affecting $G(q < q_{min})$

Extreme demonstration case

own 4-parameter Pade-fit of Bernauer data, $q < 2fm^{-1}$
excellent $\chi^2 \sim 1.06/dof$ (as good as Spline fit)
no pole



——— Pade-fit
.... Fit with 'normal' rms-radius

rms-radius = $1.49fm$!!

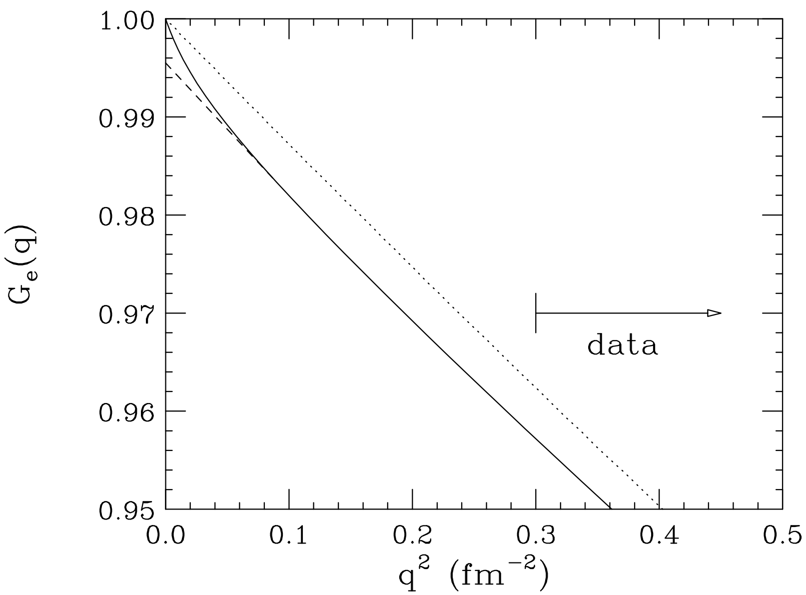
visible by naked eye in $G(q)$ at *very* low q

Interpretation

split fit into two contributions $G_1 + G_2$:

$G_1 =$ **Pade for** $q^2 > 0.06$ **plus dashed line for** $q^2 < 0.06$

$G_2 =$ **Pade -** G_1



G_1 **has 'normal' q=0 slope, norm of 0.995**

$G_2 \sim e^{-q^2/(0.02 fm^2)}$

corresponding to $\rho \sim e^{-r^2/(200 fm^2)}$

G_2 **leads to large rms-radius despite small norm** ~ 0.005

Note: data are floating, solid and dotted curve give both excellent χ^2

absolute **cross sections would have been much better**

As for previous examples:

problem occurs due to uncontrolled behavior of $G(q > q_{max})$

leads to structure of $G(q < q_{min})$ **affecting** $q = 0$ **slope**

For understanding: compare approach for $A > 2 \quad \Longleftrightarrow \quad A \leq 2$

- for A>2 parameterize $\rho(r)$, fit data, get rms-radius from integral over r, or $q = 0$ slope
- for A$\leq$ 2 parameterize $G(q)$, fit to data, get rms-radius from $q = 0$ slope

Not equivalent !!

- $\rho(r)$ automatically confined to $r < r_{max}$ by parameterization
  Fermi density, Gauss density, Fourier-Bessel, SOG, ...
  physics constraint: $\rho(r)$ must fall like $W(\kappa r)^2/r^2$
    κ given by removal energy of lightest, least bound charged constituent

- constraint is missing when parameterizing $G(q)$
  $G(q > q_{max})$ *can* imply $\rho(r)$ which is large at large r
  allows for unphysical structure of $G(q)$ below q_{min}
  can falsify rms-radius

This is a generic problem. MUST be avoided.

...... and unfortunately concerns most current fits

Least affected: fits including all data up to maximal q measured
in this case data fix large-r behavior to some degree
$G(q > q_{max})$ constrained by small $G(q \sim q_{max})$ if $G \sim q^{-4}$ ($\rightarrow$ regular $\rho(0)$)

"Solutions"

1. Parameterize $G(q)$, always compute $\rho(r)$, check large-r behavior
   difficult as above examples show, not possible for parameterizations without FT

2. Parameterize $\rho(r)$ with sensible large-r fall-off
   FT$\rightarrow$ $G(q)$, fit parameters to σ's
   complicates life, but only a bit. Tricky point: definition of "sensible"

Solution

- parameterize $\rho(r)$ in basis with analytic FT
  SOG, Hermite, Laguerre, ...
  then easy to simultaneously consider $\rho(r >>)$

- constrain $\rho(r \gg)$ using *physical* model, for r where $\rho(r) < 0.01 \cdot \rho(0)$
  fall-off of ρ given by least-bound Fock component of proton = n+π^+
  (+complications, see I.S., Prog.Part.Nucl.Phys. 67 (2012)473)
  $\rightarrow$ adds physics explicitly, safest choice!
  replaces model dependence due to parameterization by physics input

- fit data up to maximal q, so data constrain tail of ρ as well
  straightforward with above bases

To conclude

Bad news

parameterized $G(q)$'s *may* have problems

very difficult to identify if this is the case or not

particularly if $G(q)$ has no FT (such as popular sum of powers of q^2)

$q = 0$ slope could be right or wrong

$q = 0$ slope could be sensible or not

can be believed only if $\rho(r)$ at large r has been studied!

and behaves reasonably ..... which is never done

Good news

fit with large-r constraint gives stable radii, free of diseases discussed

For data=world, or =world+Bernauer(+.4%), with fixed or floating norm, find

$R_{ch} = 0.886 \pm .008 fm, \qquad R_m = 0.858 \pm .024 fm$

see I.S., Prog. Part. Nucl. Phys. 67 (2012) 473

.......unfortunately it does not help with μH discrepancy

Backup

Physical model for large r

least-bound Fock state: $\mathrm{p} = \mathrm{n} + \pi^+$, $\mathrm{n} = \mathrm{p} + \pi^-$
dominates $\rho(r)$ completely at large-enough r ($> 0.8 fm$ in cloudy bag model)
will use as constraint

To exploit need relation $\mathrm{G}_e(q) \leftrightarrow \rho(r)$

for accurate shape need data up to largest q's
must account for relativistic corrections

$\rho(r)_{exp}$ from (e,e) *vs* relativistic corrections

non-relativistic: $\rho(r)$ = Fourier-transform of $G_e(q)$

Relativistic corrections:

1. Determine $\rho(r)$ in Breit-frame, accounts for Lorentz contraction

   use as momentum transfer $\kappa^2 = q^2/(1+\tau), \quad \tau = q^2/4M^2$

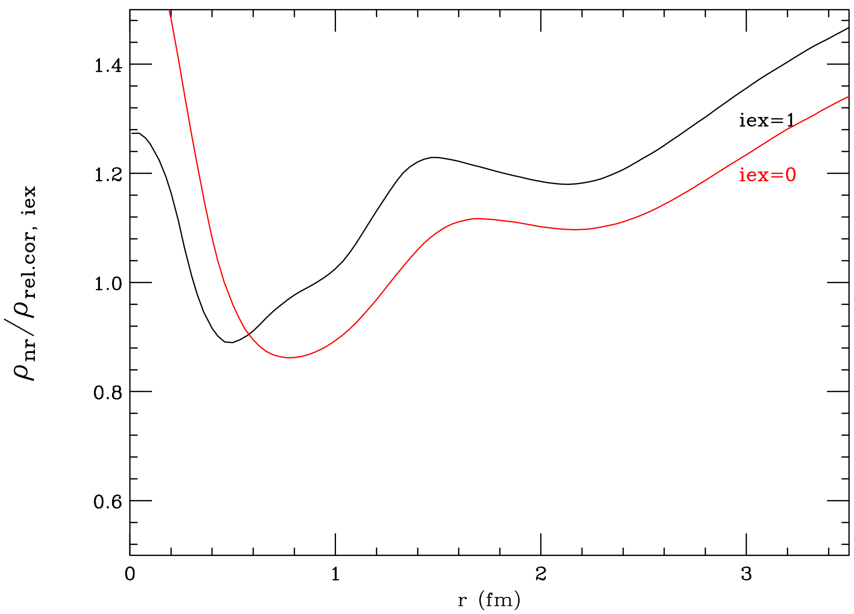
2. For composite systems boost operator depends on structure

   various prescriptions (Licht, Mitra, Ji, Holzwarth,...), all of form
   $G_e(q) \rightarrow G_e(q)(1+\tau)^\lambda$, λ=0 or 1

   de facto λ=0 or 1 makes little difference for ρ(large r)

Test:

calculate $\rho(r)$ from given $G_e(q)$ with/without relativistic corr., take ratio



find: ambiguity in relativistic effects important for ρ at small r

but unimportant for large-r. Reason: large-r $\equiv$ low momenta

λ affects only normalization of large-r density, not shape

normalization *not* used in constraint

desirable side-effect: $\rho(r=0)$ flat after application of relativistic corrections

Calculation of density at very large r

a priori: asymptotic form = Whittaker function $W_{-\eta,3/2}(2\kappa r)/r$
with physical masses m_N, m_π, l=1
with separation energy = m_π, include CM-correction

makes sense only at *large* n-π relative distance: $R^p_{rms} = 0.89 fm$, R^π_{rms}=0.66fm
only at large r overlap of n and π small

potential difficulty
need to fold W^2/r^2 with charge distribution of n, π
could get into trouble with $r = 0$ divergence of W/r

In practice
calculate w.f. in square well potential, $V(r > R) = 0$ (courtesy D.Trautmann)
radius $R = 0.8 fm$ (not important), depth adjusted to separation energy

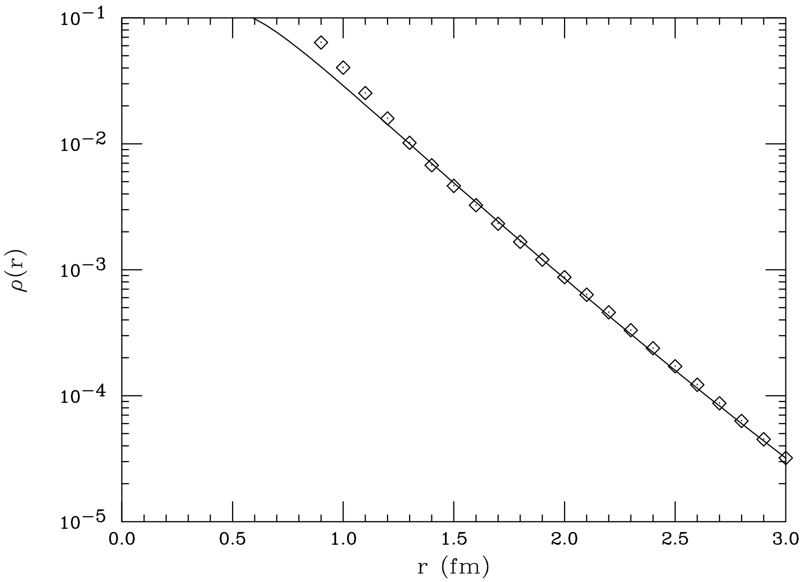
for $r > R$ shape of $\psi^2 \equiv$ shape of Whittaker function
can easily fold

Result

excellent agreement with shape of $\rho_{exp}(r)$

(= fit *world* data with [3][5] Pade) ◇

norm fit to ρ_{exp}



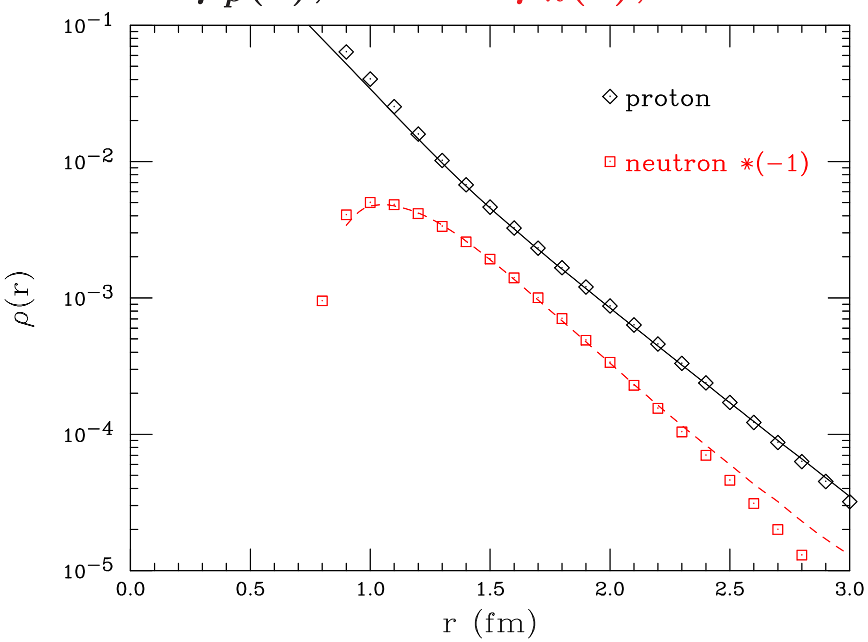
"Refinements" of model

allow also for $\Delta + \pi$ contribution
coefficients of various terms from Dziembowski,...,Speth
'Pionic contribution to nucleon EM properties in light-front approach'
include all states: $\pi^+ n$, $\pi^- p$, $\pi^- \Delta^{++}$, $\pi^+ \Delta^0$, $\pi^- \Delta^+$, $\pi^+ \Delta^-$
calculate similarly

effect on p-tail: small, tail even a bit closer to ρ_{exp} at small r

effect on n-tail: larger, gets close to ρ_{exp} with exactly *same* parameters
will ignore n since components $\neq \pi^- p$ too important

◇ $\rho_p(r)$, ◇ $-\rho_n(r)$, ——— shape tail



Problems background subtraction

Bernauer result

$$R^{ch} = 0.879 \pm 0.007\ fm$$
$$R^{m} = 0.777 \pm 0.02\ fm$$

At first sight nice confirmation of previous R^{ch}

(although I find larger model dependence)

Problematic: disagreement with *world* value $R^m = 0.855 \pm 0.035 fm$

Understanding

effect of R^m-discrepancy only 0.3% at q of maximal sensitivity to rms-radius

(Bernauer data oriented towards determination of R^{ch}!)

At this level background subtraction no good

background from Havar target-window 4 ... 10%

not measured!

primitive model: radiative tail Havar + quasi-elastic scattering in Fermigas model

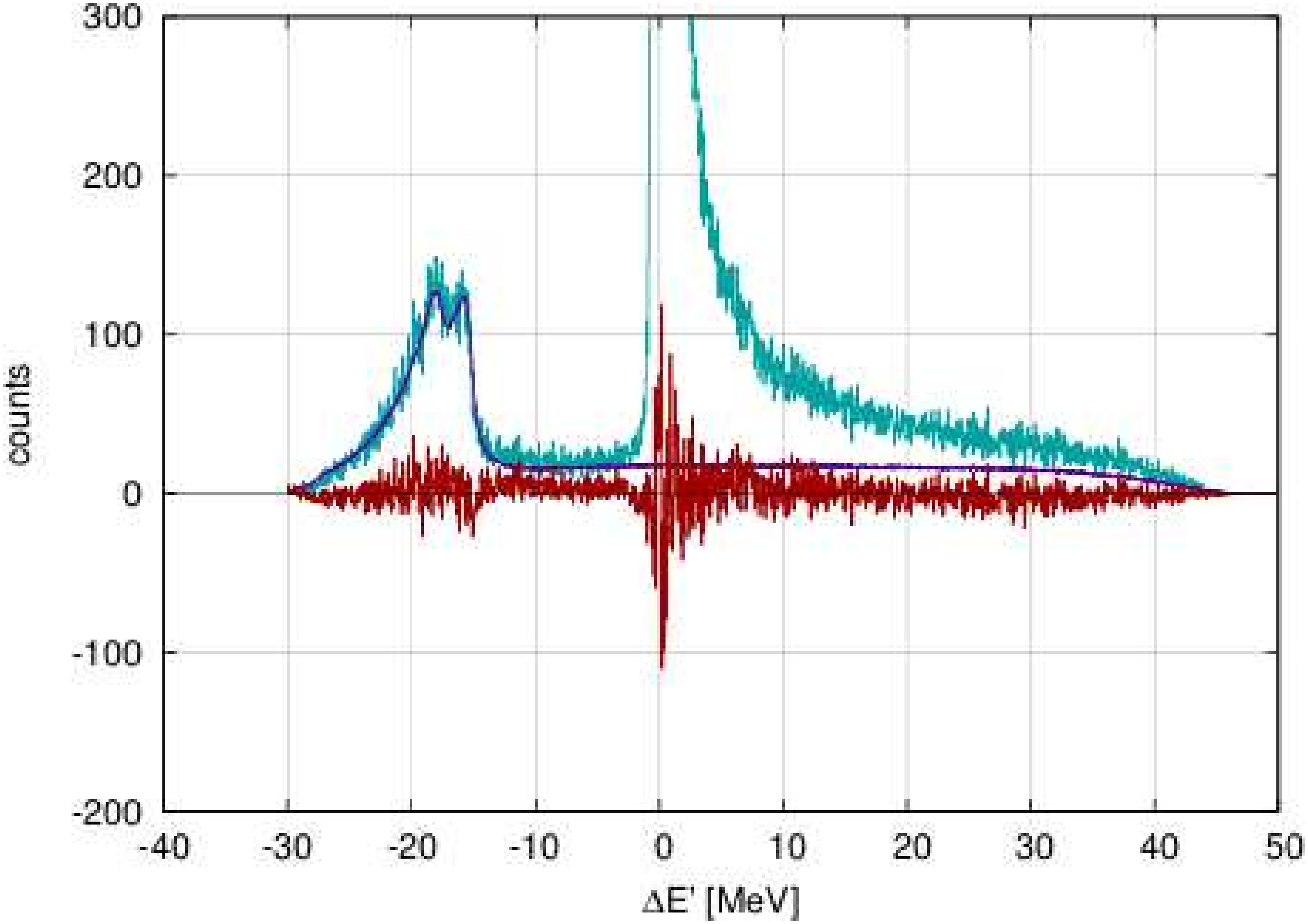
no *inelastic* scattering on Havar

Fermi-gas model in threshold region *very* poor

model does not account for deficiencies of detector

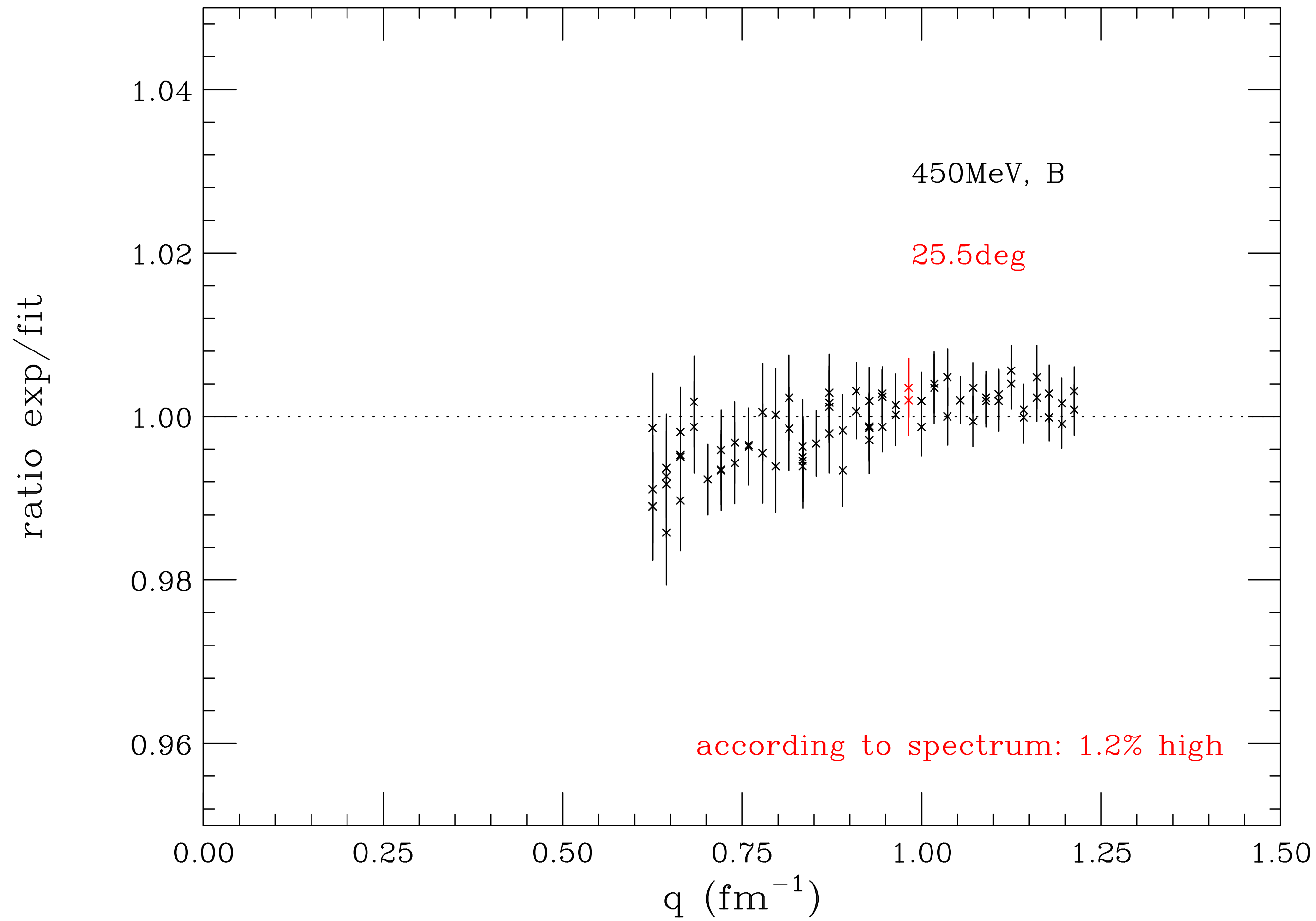
Spectrum shown in thesis

shows misfit amounting to 1.2% in cross section! Large compared to 0.3%!



Must be fixed before can believe results

1.2% problem is systematic

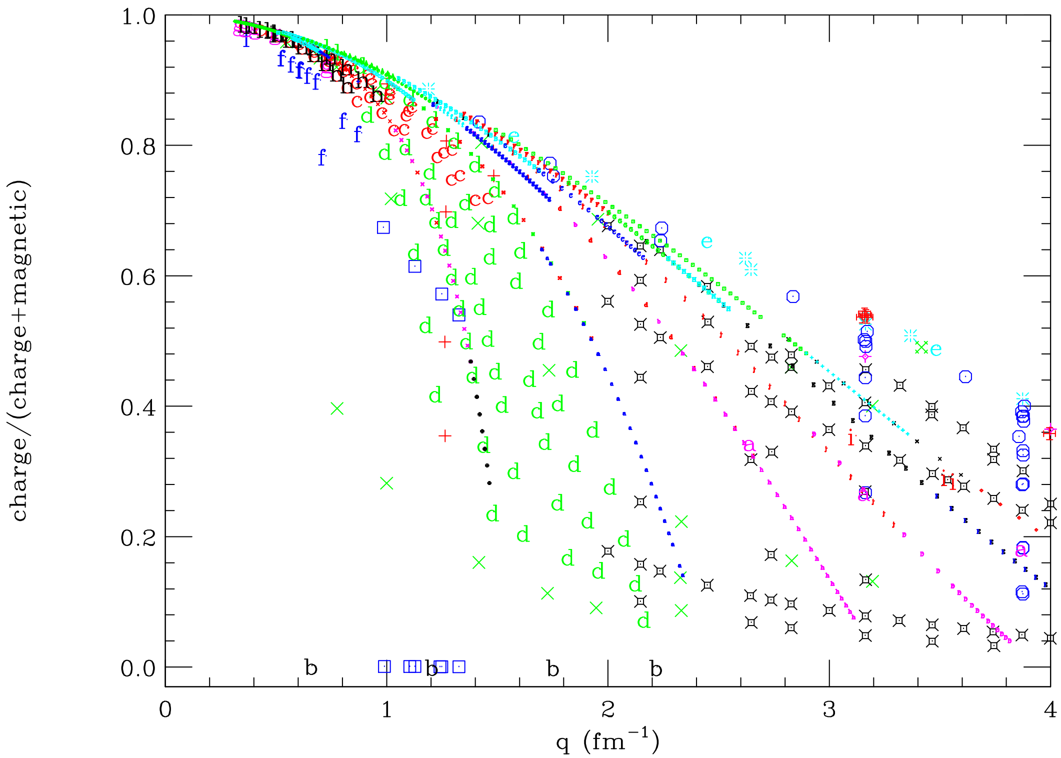


→ concerns entire *region* of q

≡ region of maximal sensitivity to *rms*-radius

Which experiments are sensitive to R^m?

for all data plot ratio of contribution to cross section.
(charge)/(charge+magnetic)



maximal sensitivity to *rms*-radius at $q \sim 0.9 fm^{-1}$

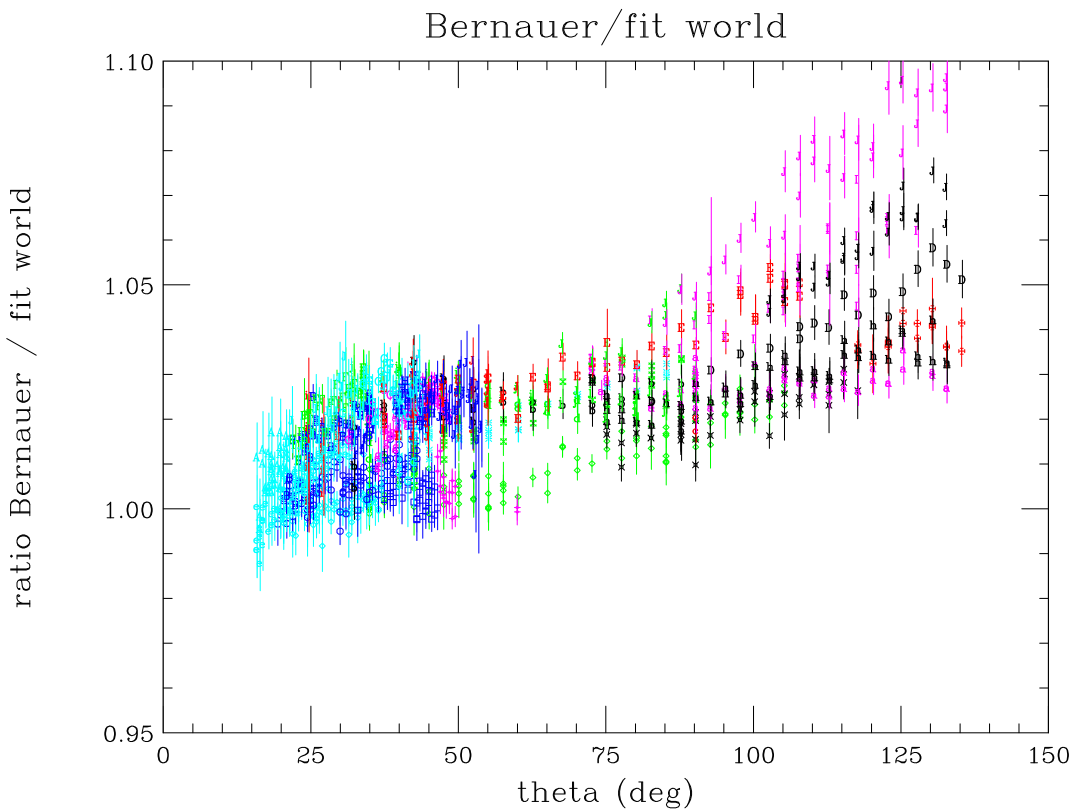
Target off-sets

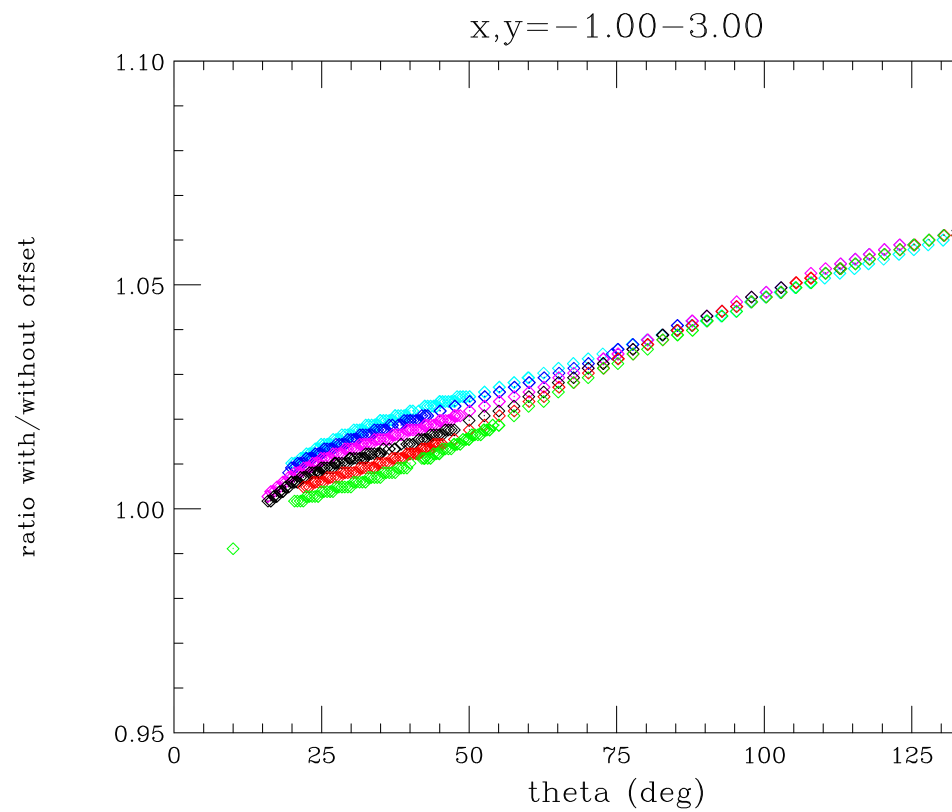
Disagreement Bernauer ↔ world data: very poor χ^2 of common fit

disagreement studied as function of different variables

systematics of difference most clearly seen in ratio as function of angle

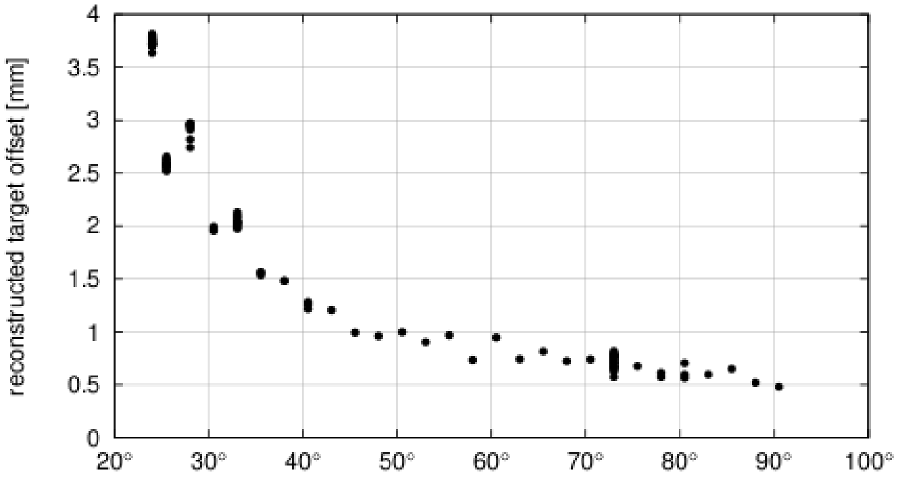
unlikely to be problem of world data (∼20 independent sets)





Most likely reason: x/y-offset of target from center of rotation

Off-set indeed seen by Bernauer et al.



according to thesis: corrected for in data analysis

BUT: there is a possible *different* reason

magnetic asymmetry of spectrometer relative to midplane would lead to incorrect reconstruction of target position

$\rightarrow$ different correction to σ: basically none

One A1 spectrometer now known to have asymmetry (partial short in coil)

not enough information available to make true correction

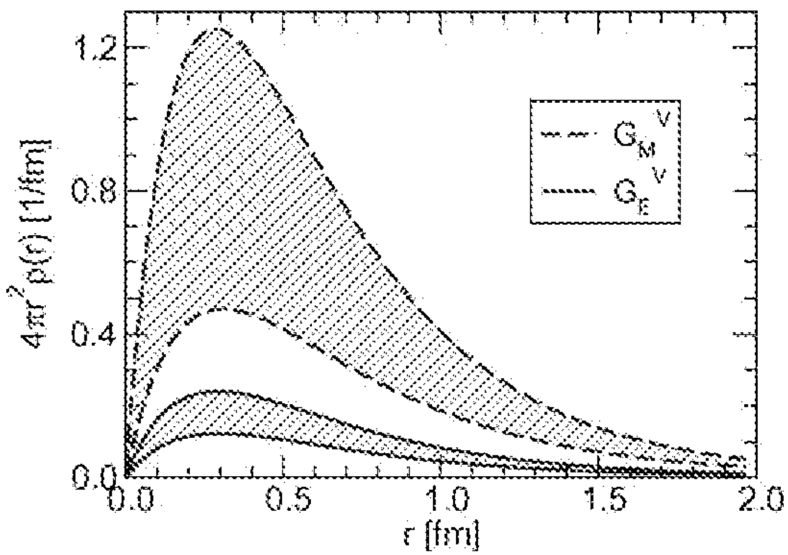
Tail from VDM

Alternative look at π-tail: N spectral function

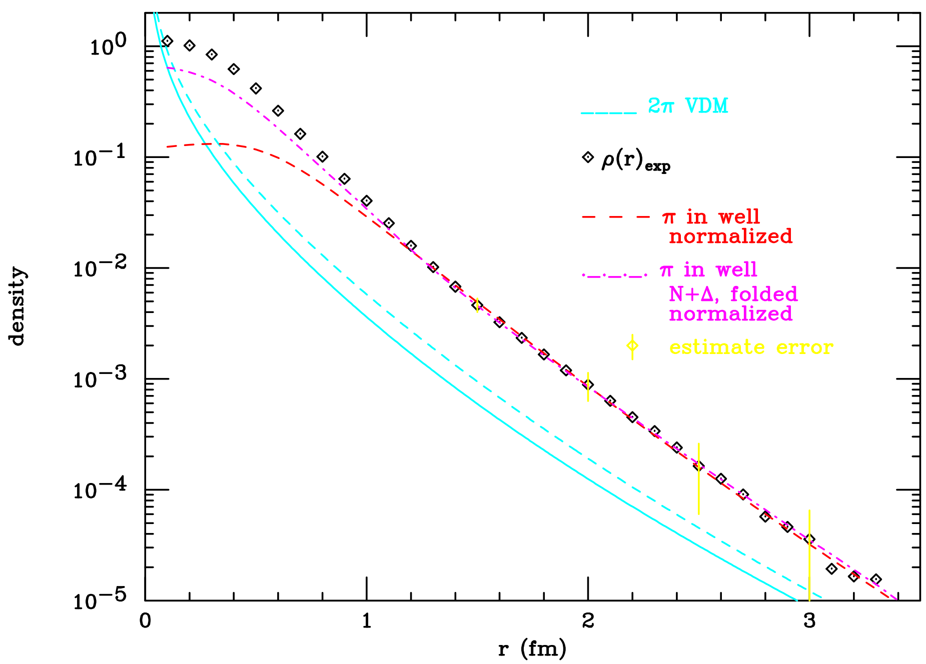
Hammer *et al.*, 2003

analysis of Hoehler *et al.* spectral function

after removal of ρ-peak



Compare



Comparison triggers questions

Shape of tail at large r

fall-off of W^2/r^2 agrees better with data

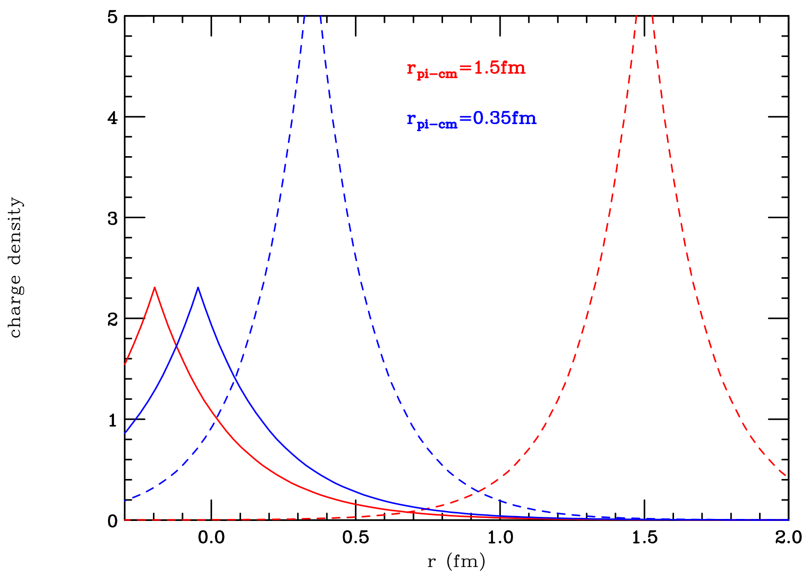
Size of π-contribution

a factor of 5 too low
at radii where $\rho < 0.01\ \rho(0)$ where would expect total π-dominance
what else could contribute there?

Maximum of ρr^2 at 0.35 fm!

does not make any sense

overlap n...π much too large to speak of "π" (see blue curves)



my conclusion: identification of π-tail in VDM-fit problematic

Reason:

1. **difficult to find spectral function that represents 1π-tail**

2. **has VDM enough degrees of freedom to fit (e,e)?**

Plausibility checks

fraction of norm in π-tail

experimental charge distribution

$\int_{1.}^{\infty} = 0.17$ $\int_{1.3}^{\infty} = 0.08$

Myhrer+Thomas, cloudy bag model ($\sim$ tail)

important to reduce spin sum rule, from value for relativistic quarks, 0.65 by factor 0.7-0.8 down to exp. value of 0.33$\pm$0.06

$P_{n\pi} = 0.2 - 0.25,$ $P_{\Delta\pi} = 0.05 - 0.1$

Bunyathyan+Povh, Deep inelastic scattering

reaction p + e $\rightarrow$ n(forward) + e' + X (only integral information)

$P_{n\pi} = 0.24 - 0.39$

Nikolaev *et al.* Drell-Yan (integral)

$P_{\pi n} = 0.21$ - 0.28

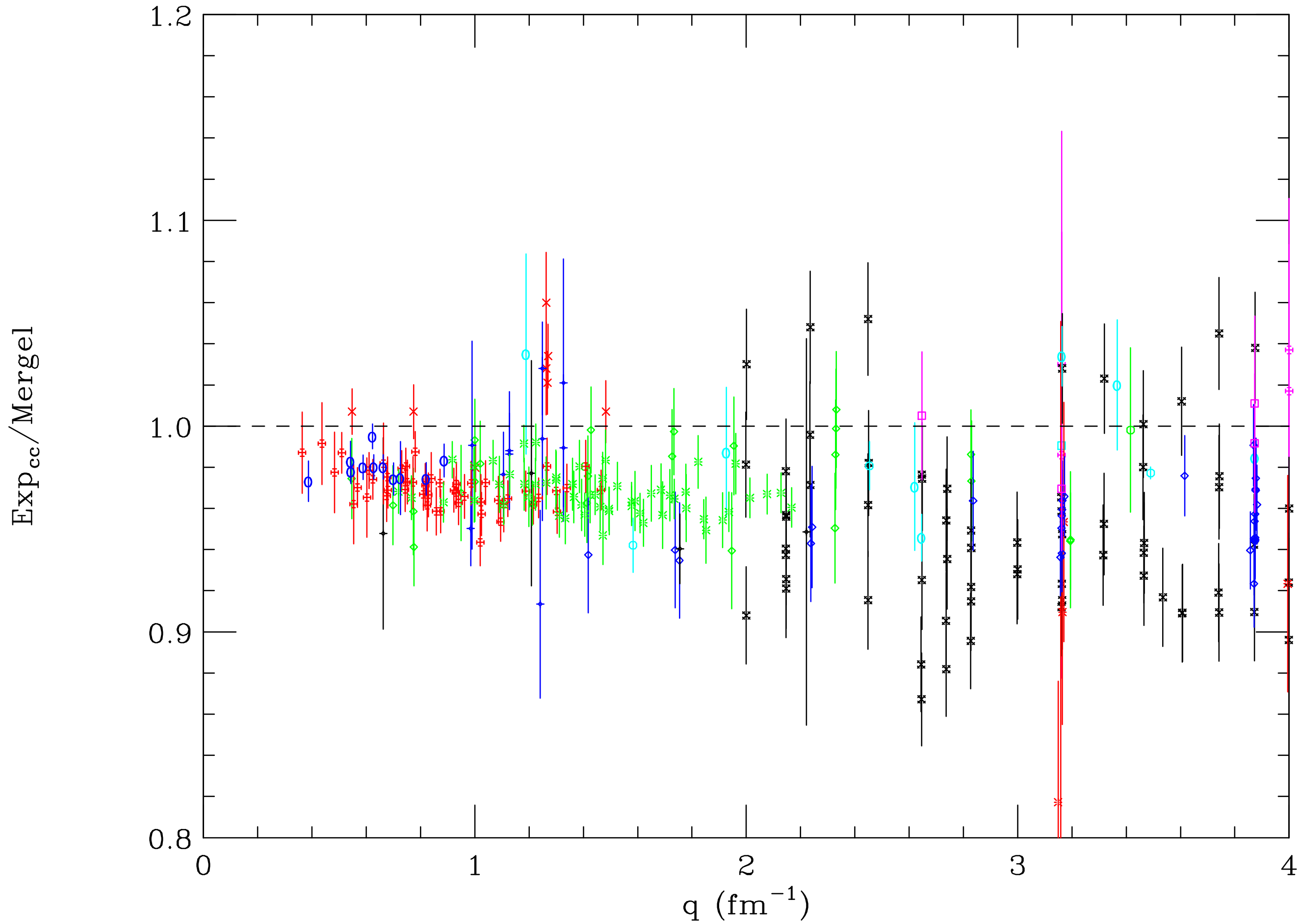
Hammer et al, VDM

$\int_{1.}^{\infty} = 0.03$ $\int_{1.3}^{\infty} = 0.017$

unrealistic

VDM-fits give too low *rms* since the time of Hoehler *et al.*!

Example: Mergell *et al.*



Details of SOG fit

Data used in fit

- *world* (e,e) data up to 12 fm^{-1}
  both cross sections and polarization data, 605 data points

- for some fits add Bernauer σ with 0.4% quadr. added to $\delta\sigma$
- two-photon exchange corrections
  needed to make G_{ep} from σ and P agree
  includes both soft+hard photons
  uses phenomenological modification for very large q
  Melnitchouk+Tjon

- (relative) tail density for $r > 1.3 fm$

Parameterization for G_e and G_m

use r-space parameterization to implement constraint
Sum-Of-Gaussians (SOG) parameterization: flexible + convenient
(equivalent results with Laguerre)

Detail

placed every $\sim 0.3 fm$, for $r < 3.3 fm$
amplitudes fit to σ, P, constraint
30 parameters

Results

average over various data sets and treatments of normalization

$R^{ch} = .886 \pm 0.008\ fm$ $\qquad$ $R^{m} = .858 \pm .024\ fm$

Great feature

result much less sensitive to use of absolute vs. floated data

Conclusion: disagreement with μ-H confirmed.

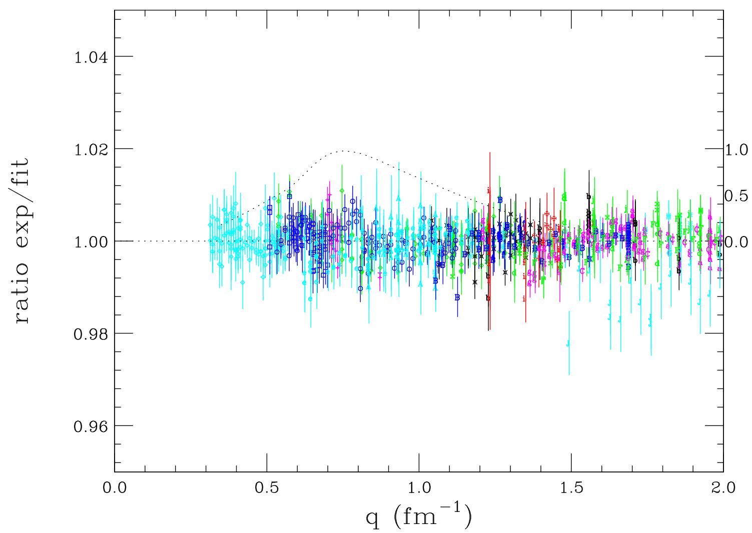
Question: to which degree could fit (e,e) with muonic R as constraint?

redo analysis with various combinations of data sets
floated or fixed normalization
constraint $R^{ch} = 0.84\ fm$

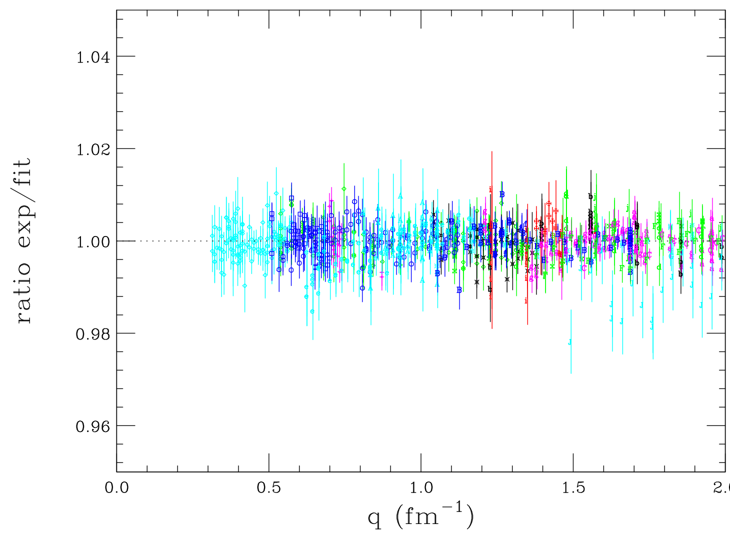
Increase in χ^2 due to constraint

Bernauer	5%
world floated + Bernauer	8%
world floated + tail	10%
world + tail	24%
world + Bernauer + tail	24%

R=0.84fm



R=0.88fm



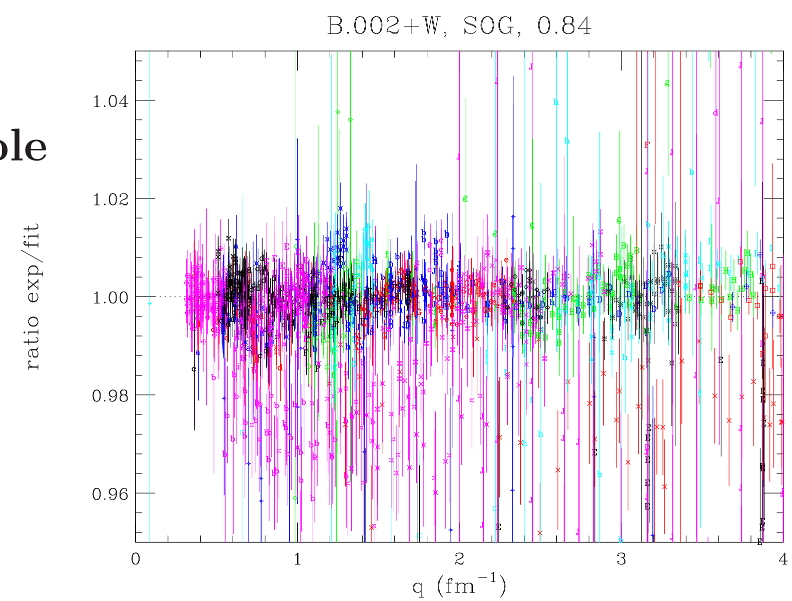
Results show that

1. **With floating data and no tail constraint:**
   can change R^{ch} with modest effect upon χ^2
   for Bernauer data effect on $\sigma_{exp}/\sigma_{fit}$ not visible

2. **With tail constraint: get larger increase**

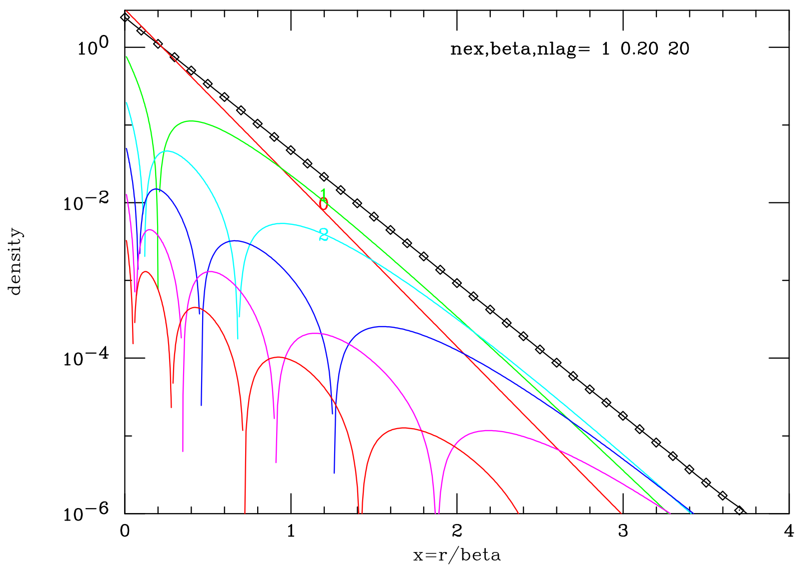
3. **Absolute σ + tail: fixes *rms*-radius best**
   gives also *visible* disagreement in data/fit
   world data 2-3% below fit

world+Bernauer, 0.84fm



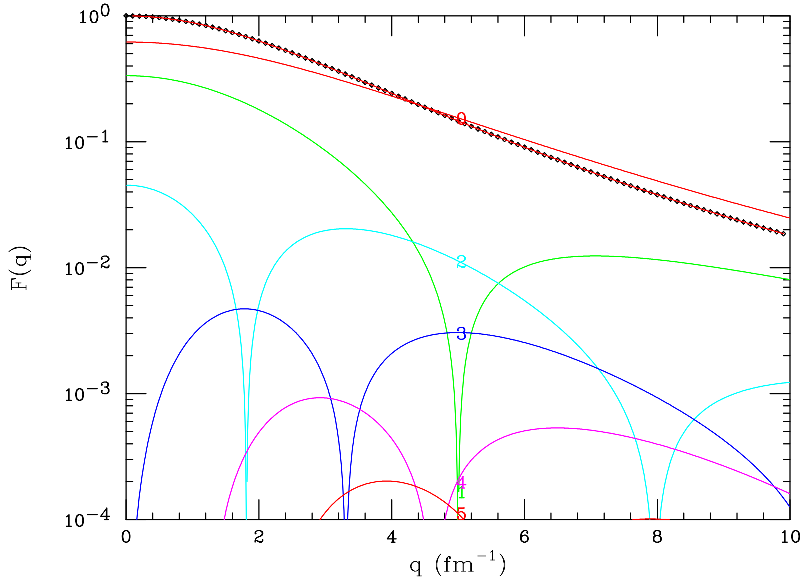
Laguerre as basis

With Laguerre basis approximate $e^{-r/\cdots}$ behavior optimally included in basis



excellent fit of exponential density

get best χ^2 for fit of *world* data
with fewest parameters



Floating data?

Should data be floated or not?

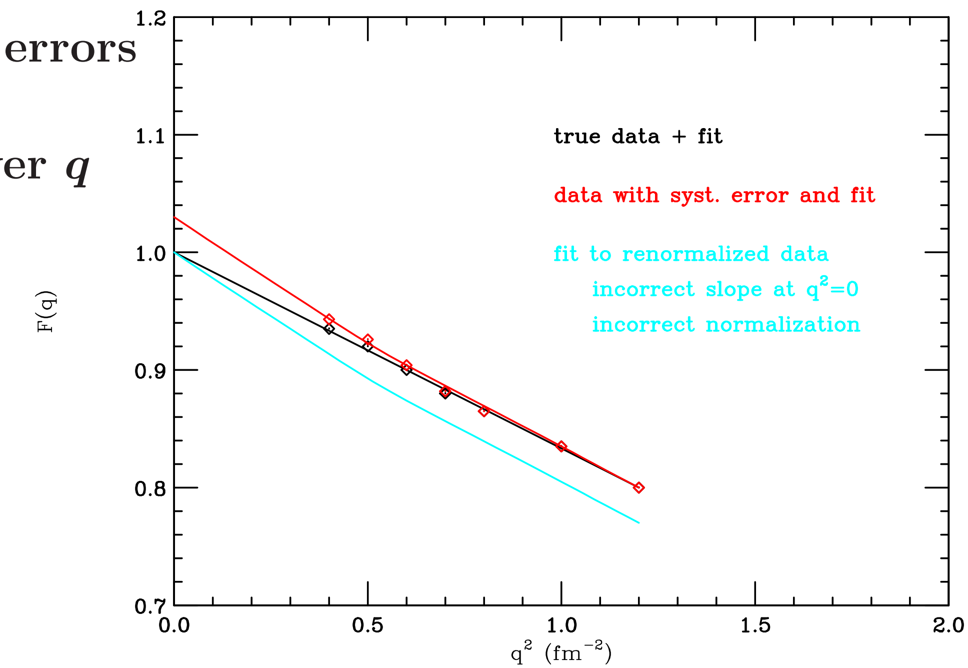
traditional: people float data.
'justification': absolute norm difficult to determine
main purpose: get good-looking χ^2

Two problems

1. Ignores > 50% of effort of experimentalists
   great effort made to get *absolute* normalization
   dont want to throw away

2. Floating greatly enhances problems with extrapolation to $q = 0$

become sensitive to q-dependent systematic errors
errors obviously increase with decreasing q
otherwise data could have been taken at lower q
extrapolation increases effect
affects in particular extracted rms-radius



absolute data much more valuable
than floating ones