Progress towards a new precise microwave measurement of the 2S-2P Lamb shift in atomic hydrogen

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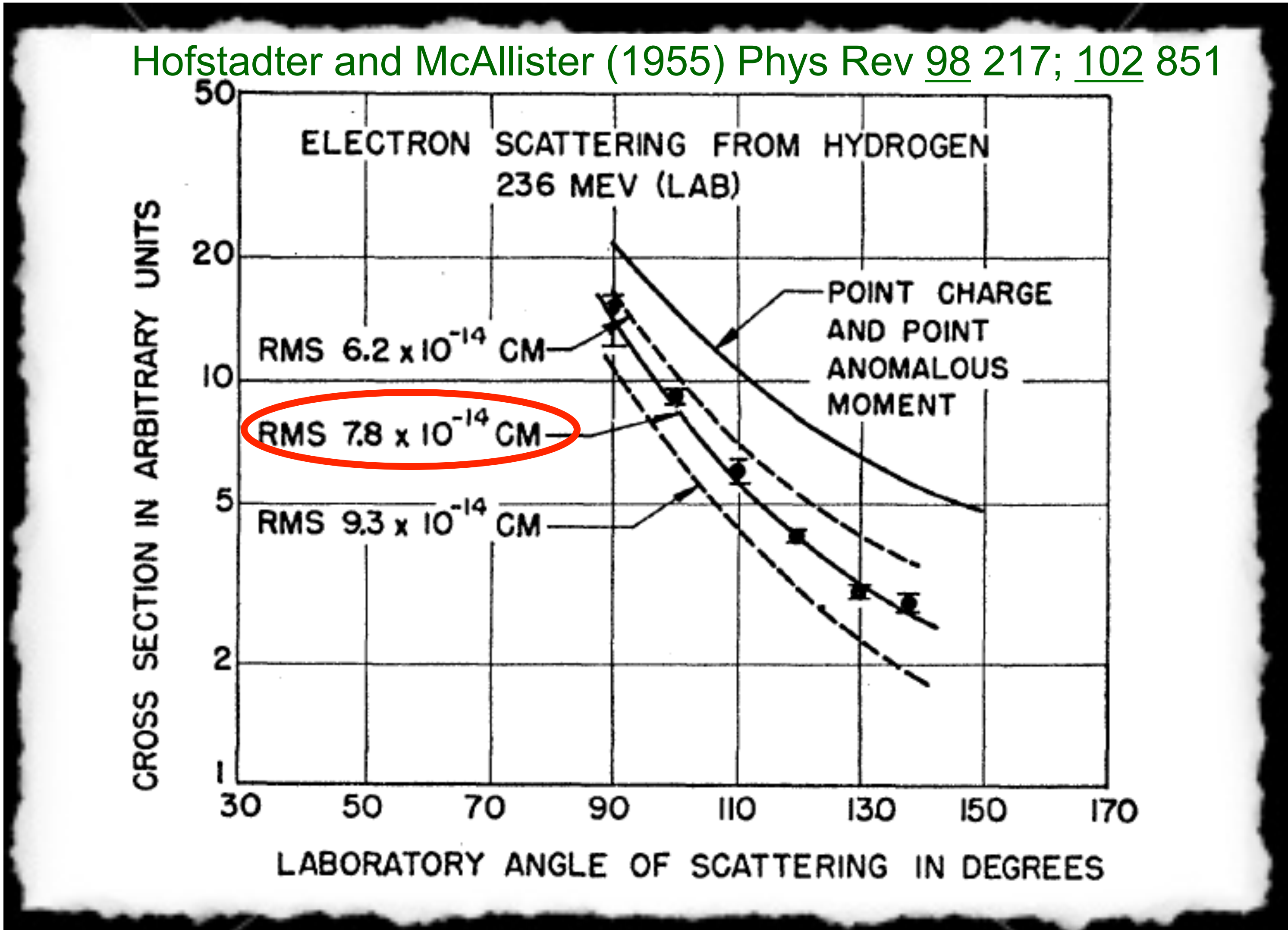
Support from **NSERC, CRC, CFI**

NIST

From e-p scattering, already in in 1955 the proton charge radius was known to one digit: 0.8 fm

59 years later, we still know it to one digit

Hofstadter and McAllister (1955) Phys Rev 98 217; 102 851



In 1956 it was suggested that the proton size should show up in the hydrogen $2S_{1/2}$-$2P_{1/2}$ interval (the Lamb shift)

Phys Rev 105 1681

Contribution to Lamb Shift Due to Finite Proton Size

WALTER ARON AND A. J. ZUCHELLI*

Department of Physics, University of Virginia, Charlottesville, Virginia

(Received December 28, 1956)

THE scattering of high-energy electrons by protons has recently been interpreted in terms of a finite spatial distribution of charge for the proton.[1] We have noticed that the resultant deviation from a pure Coulomb field is such as to reduce the existing discrepancy[2] between theoretical and experimental results for the hydrogen Lamb shift. Since the proton size is small compared to atomic dimensions, one easily finds, using nonrelativistic wave functions,

$$\Delta E=\tfrac{1}{6}|\psi(0)|^2e^2\langle R^2\rangle_{\text{Av}},$$

where $\langle R^2\rangle_{\text{Av}}$ is the mean square radius of the proton charge distribution and $\psi(0)$ is the amplitude of the hydrogen wave function at the origin. (A similar result was obtained by Salpeter[3] in discussing the effect of proton motion in the deuteron Lamb shift.) Taking the mean value given by Chambers and Hofstadter, $R_{\text{rms}}=(0.77\pm0.10)\times10^{-13}$ cm, one finds the energy shift for the $2S_{\frac{1}{2}}$ level:

$$\Delta E=0.118\pm0.03\ \text{Mc/sec}.$$

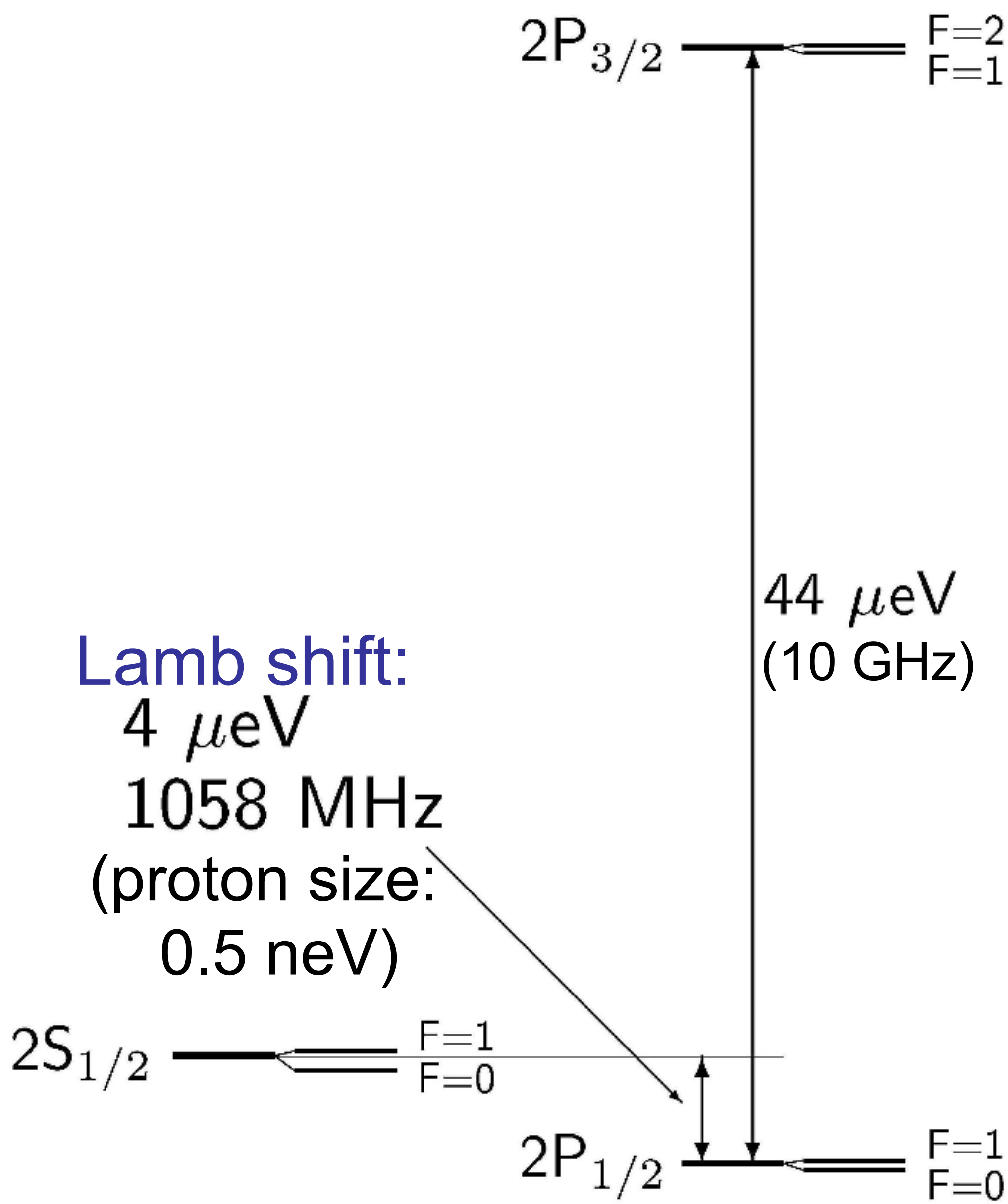
* National Science Foundation Predoctoral Fellow.

[1] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).

[2] Baranger, Bethe, and Feynman, Phys. Rev. **92**, 482 (1953).

[3] E. E. Salpeter, Phys. Rev. **89**, 92 (1953).

Hydrogen n=2 Energy levels

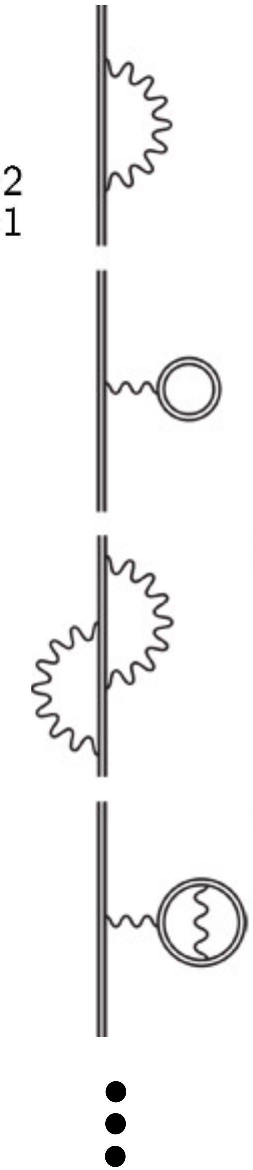


Lamb shift:

4 μeV

1058 MHz

(proton size: 0.5 neV)



QED contributions

$$E^{(2)}_{\mathrm{SE}} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}F(Z\alpha)m_e c^2, \qquad (19)$$

where

$$F(Z\alpha) = A_{41}\ln(Z\alpha)^{-2} + A_{40} + A_{50}(Z\alpha) + A_{62}(Z\alpha)^2\ln^2(Z\alpha)^{-2} + A_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + G_{\mathrm{SE}}(Z\alpha)(Z\alpha)^2. \qquad (20)$$

$$A_{41} = \frac{4}{3}\delta_{l0},$$

$$A_{40} = -\frac{4}{3}\ln k_0(n,l) + \frac{10}{9}\delta_{l0} - \frac{1}{2\kappa(2l+1)}(1-\delta_{l0}),$$

$$A_{50} = \left(\frac{139}{32} - 2\ln 2\right)\pi\delta_{l0},$$

$$A_{62} = -\delta_{l0},$$

$$A_{61} = \left[4\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right) + \frac{28}{3}\ln 2 - 4\ln n - \frac{601}{180} - \frac{77}{45n^2}\right]\delta_{l0} + \left(1-\frac{1}{n^2}\right)\left(\frac{2}{15}+\frac{1}{3}\delta_{j1/2}\right)\delta_{l1} + \frac{96n^2 - 32l(l+1)}{3n^2(2l-1)(2l)(2l+1)(2l+2)(2l+3)}(1-\delta_{l0}).$$

$$E_{\mathrm{RR}} = \frac{m_r^3}{m_e^2 m_N}\frac{\alpha(Z\alpha)^5}{\pi^2 n^3}m_e c^2\delta_{l0}\left[6\zeta(3) - 2\pi^2\ln 2 + \frac{35\pi^2}{36} - \frac{448}{27} + \frac{2}{3}\pi(Z\alpha)\ln^2(Z\alpha)^{-2} + \cdots\right]. \qquad (56)$$

$$E^{(2)}_{\mathrm{VP}} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}H(Z\alpha)m_e c^2$$

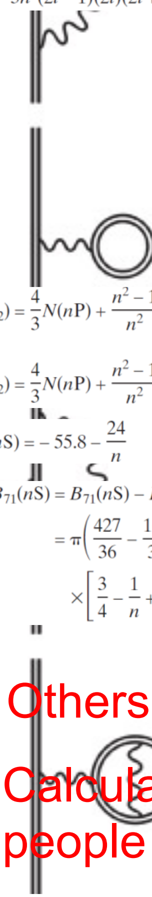
$$V_{40} = -\tfrac{4}{15}\delta_{l0} \qquad V_{50} = \tfrac{5}{48}\pi\delta_{l0} \qquad V_{61} = -\tfrac{2}{15}\delta_{l0}$$

$$E^{(2)}_{\mu\mathrm{VP}} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}\left(-\frac{4}{15}\right)\left(\frac{m_e}{m_\mu}\right)^2\left(\frac{m_r}{m_e}\right)^3 m_e c^2$$

$$E^{(2)}_{\mathrm{had\ VP}} = 0.671(15)E^{(2)}_{\mu\mathrm{VP}}$$

$$H^{(1)}(Z\alpha) = V_{40} + V_{50}(Z\alpha) + V_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + G^{(1)}_{\mathrm{VP}}(Z\alpha)(Z\alpha)^2,$$

$$H^{(R)}(Z\alpha) = G^{(R)}_{\mathrm{VP}}(Z\alpha)(Z\alpha)^2, \qquad G^{(R)}_{\mathrm{VP}}(Z\alpha) = \left(\frac{19}{45}-\frac{\pi^2}{27}\right)\delta_{l0} + \left(\frac{1}{16}-\frac{31\pi^2}{2880}\right)\pi(Z\alpha)\delta_{l0}$$



$$B_{60}(nS) - B_{60}(1S) = b_L(nS) - b_L(1S) + A(n), \qquad (40)$$

where

$$A(n) = \left(\frac{38}{45} - \frac{4}{3}\ln 2\right)[N(nS) - N(1S)] - \frac{337\,043}{129\,600} - \frac{94\,261}{21\,600n} + \frac{902\,609}{129\,600n^2} + \left(\frac{4}{3} - \frac{16}{9n} + \frac{4}{9n^2}\right)\ln^2 2 + \left(-\frac{76}{45} + \frac{304}{135n} - \frac{76}{135n^2}\right)\ln 2 + \left(-\frac{53}{15} + \frac{35}{2n} - \frac{419}{30n^2}\right)\zeta(2)\ln 2 + \left(\frac{28\,003}{10\,800} - \frac{11}{2n} + \frac{31\,397}{10\,800n^2}\right)\zeta(2) + \left(\frac{53}{60} - \frac{35}{8n} + \frac{419}{120n^2}\right)\zeta(3) + \left(\frac{37\,793}{10\,800} + \frac{16}{9}\ln^2 2 - \frac{304}{135}\ln 2 + 8\zeta(2)\ln 2 - \frac{13}{3}\zeta(2) - 2\zeta(3)\right)[\gamma + \psi(n) - \ln n]. \qquad (41)$$

$$E^{(4)} = \left(\frac{\alpha}{\pi}\right)^2\frac{(Z\alpha)^4}{n^3}m_e c^2 F^{(4)}(Z\alpha)$$

$$F^{(4)}(Z\alpha) = B_{40} + B_{50}(Z\alpha) + B_{63}(Z\alpha)^2\ln^3(Z\alpha)^{-2} + B_{62}(Z\alpha)^2\ln^2(Z\alpha)^{-2} + B_{61}(Z\alpha)^2\ln(Z\alpha)^{-2} + B_{60}(Z\alpha)^2 + \cdots. \qquad (30)$$

$$B_{40} = \left[\frac{3\pi^2}{2}\ln 2 - \frac{10\pi^2}{27} - \frac{2179}{648} - \frac{9}{4}\zeta(3)\right]\delta_{l0} + \left[\frac{\pi^2\ln 2}{2} - \frac{\pi^2}{12} - \frac{197}{144} - \frac{3\zeta(3)}{4}\right]\frac{1-\delta_{l0}}{\kappa(2l+1)}$$

$$B_{50} = -21.5561(31)\delta_{l0} \qquad B_{63} = -\tfrac{8}{27}\delta_{l0}$$

$$B_{62} = \frac{16}{9}\left[\frac{71}{60} - \ln 2 + \gamma + \psi(n) - \ln n - \frac{1}{n} + \frac{1}{4n^2}\right]$$

$$B_{61} = \frac{413\,581}{64\,800} + \frac{4N(nS)}{3} + \frac{2027\pi^2}{864} - \frac{616\ln 2}{135} - \frac{2\pi^2\ln 2}{3} + \frac{40\ln^2 2}{9} + \zeta(3) + \left(\frac{304}{135} - \frac{32\ln 2}{9}\right) \times \left[\frac{3}{4} + \gamma + \psi(n) - \ln n - \frac{1}{n} + \frac{1}{4n^2}\right], \qquad (36)$$

$$B_{61}(nP_{1/2}) = \frac{4}{3}N(nP) + \frac{n^2-1}{n^2}\left(\frac{166}{405} - \frac{8}{27}\ln 2\right)$$

$$B_{61}(nP_{3/2}) = \frac{4}{3}N(nP) + \frac{n^2-1}{n^2}\left(\frac{31}{405} - \frac{8}{27}\ln 2\right)$$

$$b_L(nS) = -55.8 - \frac{24}{n}$$

$$\Delta B_{71}(nS) = B_{71}(nS) - B_{71}(1S) = \pi\left(\frac{427}{36} - \frac{16}{3}\ln 2\right) \times \left[\frac{3}{4} - \frac{1}{n} + \frac{1}{4n^2} + \gamma + \psi(n) - \ln n\right]$$

$$E^{(6)} = \left(\frac{\alpha}{\pi}\right)^3\frac{(Z\alpha)^4}{n^3}m_e c^2[C_{40} + C_{50}(Z\alpha) + \cdots].$$

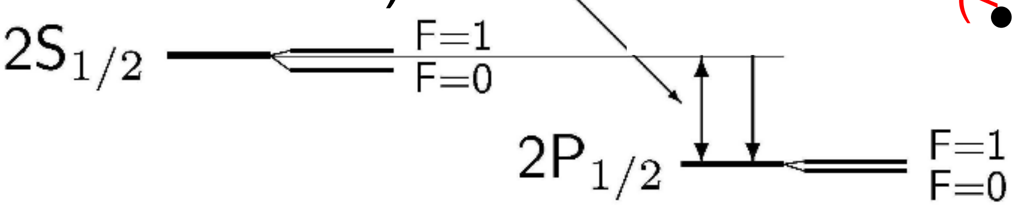
$$C_{40} = \left[-\frac{568a_4}{9} + \frac{85\zeta(5)}{24} - \frac{121\pi^2\zeta(3)}{72} - \frac{84\,071\zeta(3)}{2304} - \frac{71\ln^4 2}{27} - \frac{239\pi^2\ln^2 2}{135} + \frac{4787\pi^2\ln 2}{108} + \frac{1591\pi^4}{3240} - \frac{252\,251\pi^2}{9720} + \frac{679\,441}{93\,312}\right]\delta_{l0} + \left[-\frac{100a_4}{3} + \frac{215\zeta(5)}{24} - \frac{83\pi^2\zeta(3)}{72} - \frac{139\zeta(3)}{18} - \frac{25\ln^4 2}{18} + \frac{25\pi^2\ln^2 2}{18} + \frac{298\pi^2\ln 2}{9} + \frac{239\pi^4}{2160} - \frac{17\,101\pi^2}{810} - \frac{28\,259}{5184}\right]\frac{1-\delta_{l0}}{\kappa(2l+1)}, \qquad (48)$$

$$a_4 = \Sigma_{n=1}^{\infty}1/(2^n n^4)$$

4 μeV

1058 MHz

(proton size: +0.5 neV)



Others here can explain the details

Calculated and recalculated by many people over the past 5 decades

- Uncertainty is <1 part per million
- (<1% of proton size contribution)

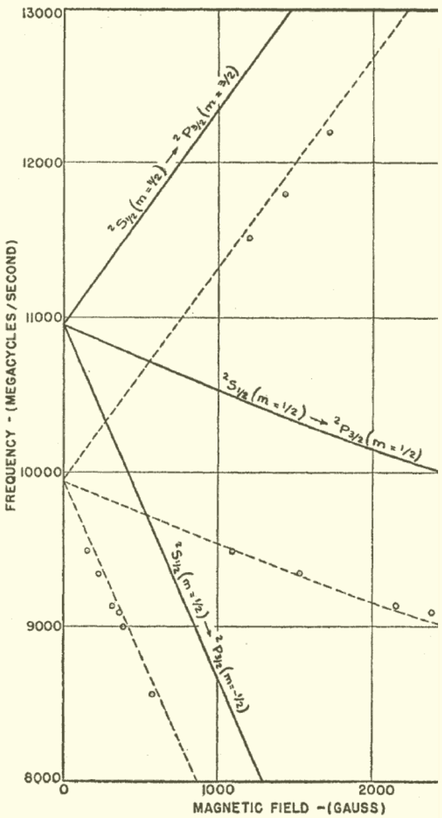
Theory has been stable at this level of precision for ~20 years

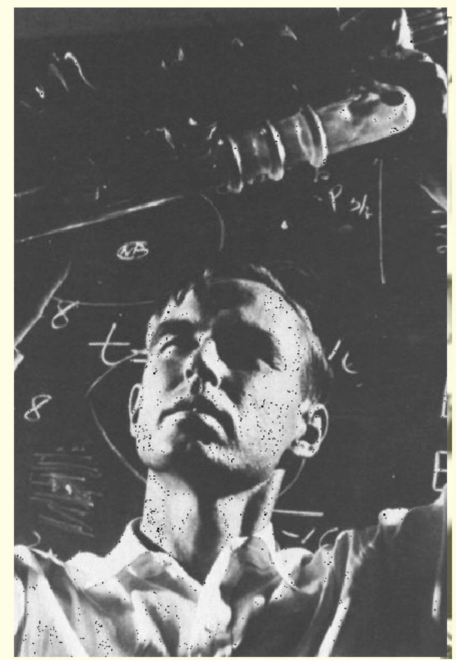
PHYSICAL REVIEW VOLUME 72, NUMBER 3 AUGUST 1, 1947

Fine Structure of the Hydrogen Atom by a Microwave Method* **

WILLIS E. LAMB, JR. AND ROBERT C. RETHERFORD
Columbia Radiation Laboratory, Department of Physics, Columbia University, New York, New York

(Received June 18, 1947)



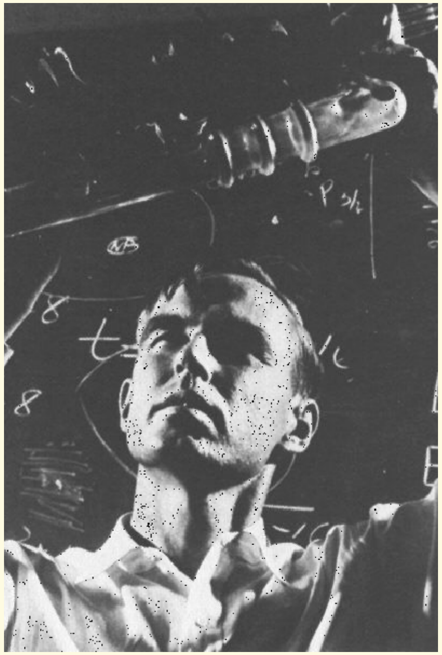


PHYSICAL REVIEW LETTERS

Measurement of the Lamb Shift in Hydrogen, $n=2$

S. R. Lundeen and F. M. Pipkin
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 7 August 1980)

A measurement based on the fast-atomic-beam separated-oscillatory-field method of sub-natural-linewidth spectroscopy gives, for the Lamb shift in hydrogen, $\mathcal{S}(n=2) = 1057.845(9)$ MHz. The result is not in good agreement with theory.



9 part-per-million measurement of Lamb shift

Determines the proton size to an accuracy of 3%

Still the most precise determination of this interval

33 years between Lamb and Lundeen & Pipkin.

Now another 33 years have passed and it is time for another measurement.

Other precise hydrogen measurements

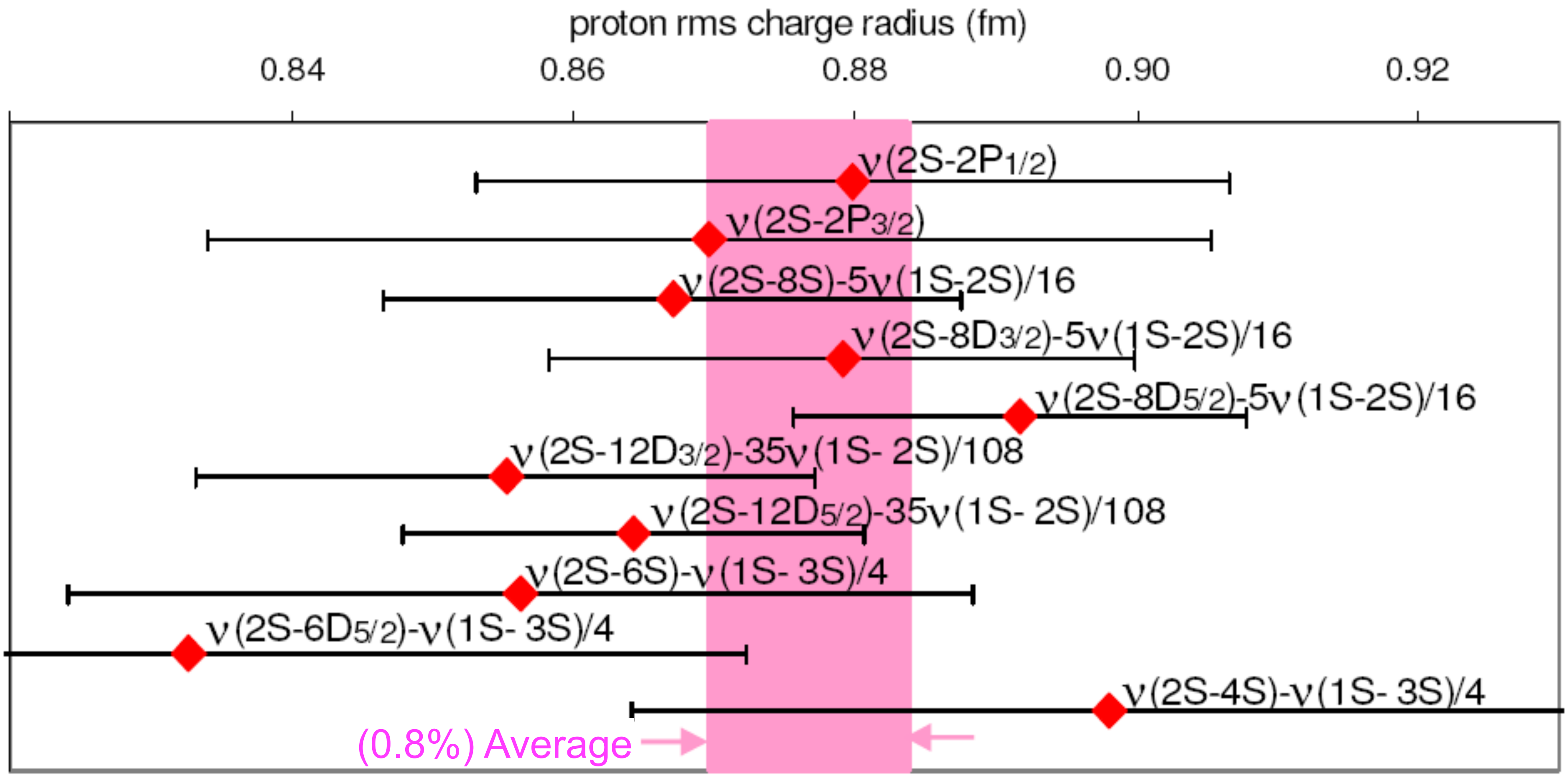
Cannot be used directly for r_p determination (Ry)

Combinations of measurements can eliminate Ry dependence and determine r_p

For example:
ν(2S-8D5/2)-(5/16)ν(1S-2S)
=5 369 962(6) kHz
is independent of Ry and determines r_p to 2%

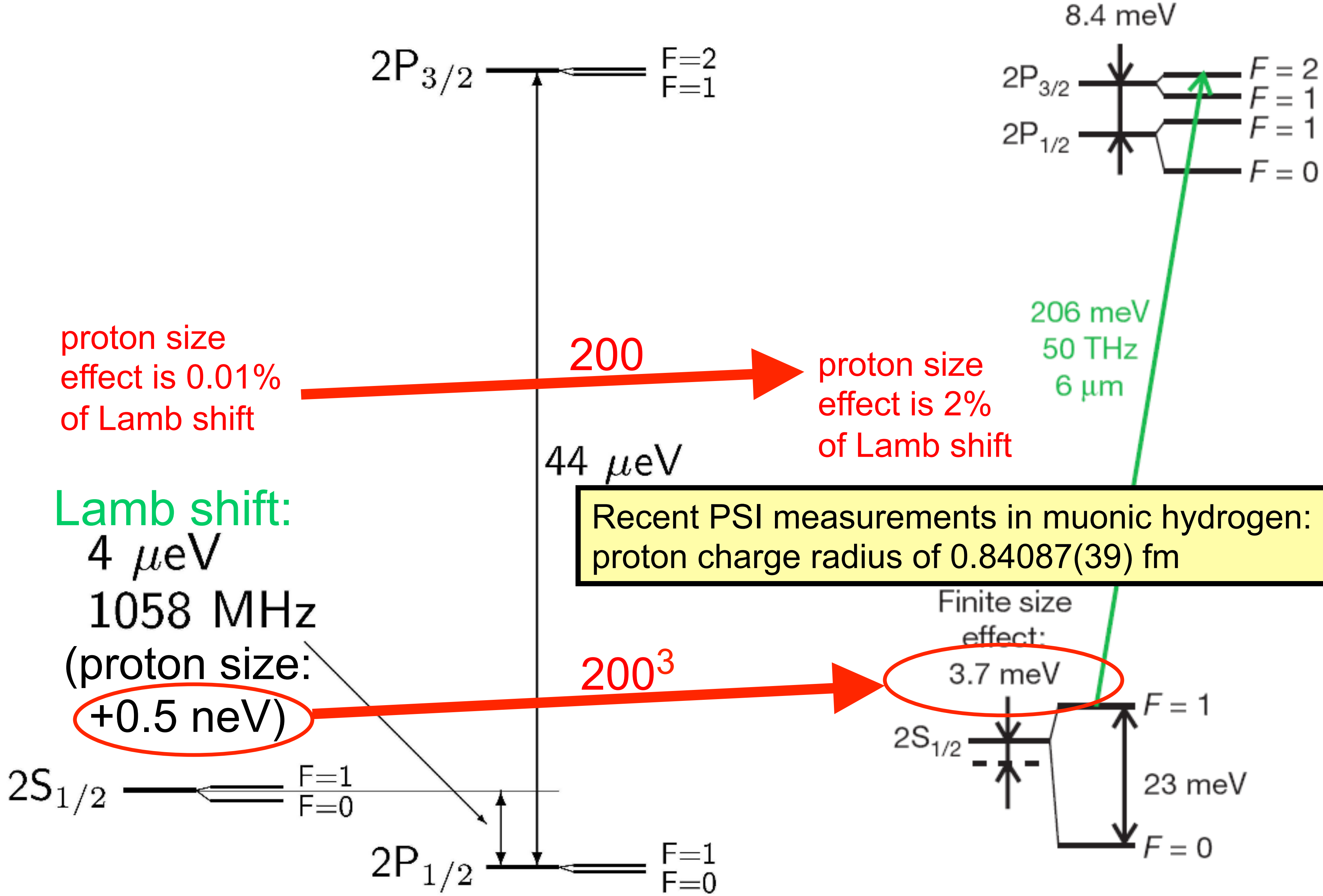
Laboratory	Frequency interval(s)	(ν/kHz)
MPQ	$\nu_H(1S_{1/2}–2S_{1/2})$ >14-digits	2 466 061 413 187.035(10)
MPQ	$\nu_H(2S_{1/2}–4S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2}–2S_{1/2})$	4 797 338(10)
	$\nu_H(2S_{1/2}–4D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2}–2S_{1/2})$	6 490 144(24)
	$\nu_D(2S_{1/2}–4S_{1/2}) - \frac{1}{4}\nu_D(1S_{1/2}–2S_{1/2})$	4 801 693(20)
	$\nu_D(2S_{1/2}–4D_{5/2}) - \frac{1}{4}\nu_D(1S_{1/2}–2S_{1/2})$	6 494 841(41)
MPQ	$\nu_D(1S_{1/2}–2S_{1/2}) - \nu_H(1S_{1/2}–2S_{1/2})$	670 994 334.64(15)
LKB/SYRTE	$\nu_H(2S_{1/2}–8S_{1/2})$	770 649 350 012.0(8.6)
	$\nu_H(2S_{1/2}–8D_{3/2})$	770 649 504 450.0(8.3)
	$\nu_H(2S_{1/2}–8D_{5/2})$	770 649 561 584.2(6.4)
	$\nu_D(2S_{1/2}–8S_{1/2})$	770 859 041 245.7(6.9)
	$\nu_D(2S_{1/2}–8D_{3/2})$	770 859 195 701.8(6.3)
	$\nu_D(2S_{1/2}–8D_{5/2})$	770 859 252 849.5(5.9)
LKB/SYRTE	$\nu_H(2S_{1/2}–12D_{3/2})$	799 191 710 472.7(9.4)
	$\nu_H(2S_{1/2}–12D_{5/2})$	799 191 727 403.7(7.0)
	$\nu_D(2S_{1/2}–12D_{3/2})$	799 409 168 038.0(8.6)
	$\nu_D(2S_{1/2}–12D_{5/2})$	799 409 184 966.8(6.8)
LKB	$\nu_H(2S_{1/2}–6S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2}–3S_{1/2})$	4 197 604(21)
	$\nu_H(2S_{1/2}–6D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2}–3S_{1/2})$	4 699 099(10)
Yale	$\nu_H(2S_{1/2}–4P_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2}–2S_{1/2})$	4 664 269(15)
	$\nu_H(2S_{1/2}–4P_{3/2}) - \frac{1}{4}\nu_H(1S_{1/2}–2S_{1/2})$	6 035 373(10)
Harvard	$\nu_H(2S_{1/2}–2P_{3/2})$	9 911 200(12)

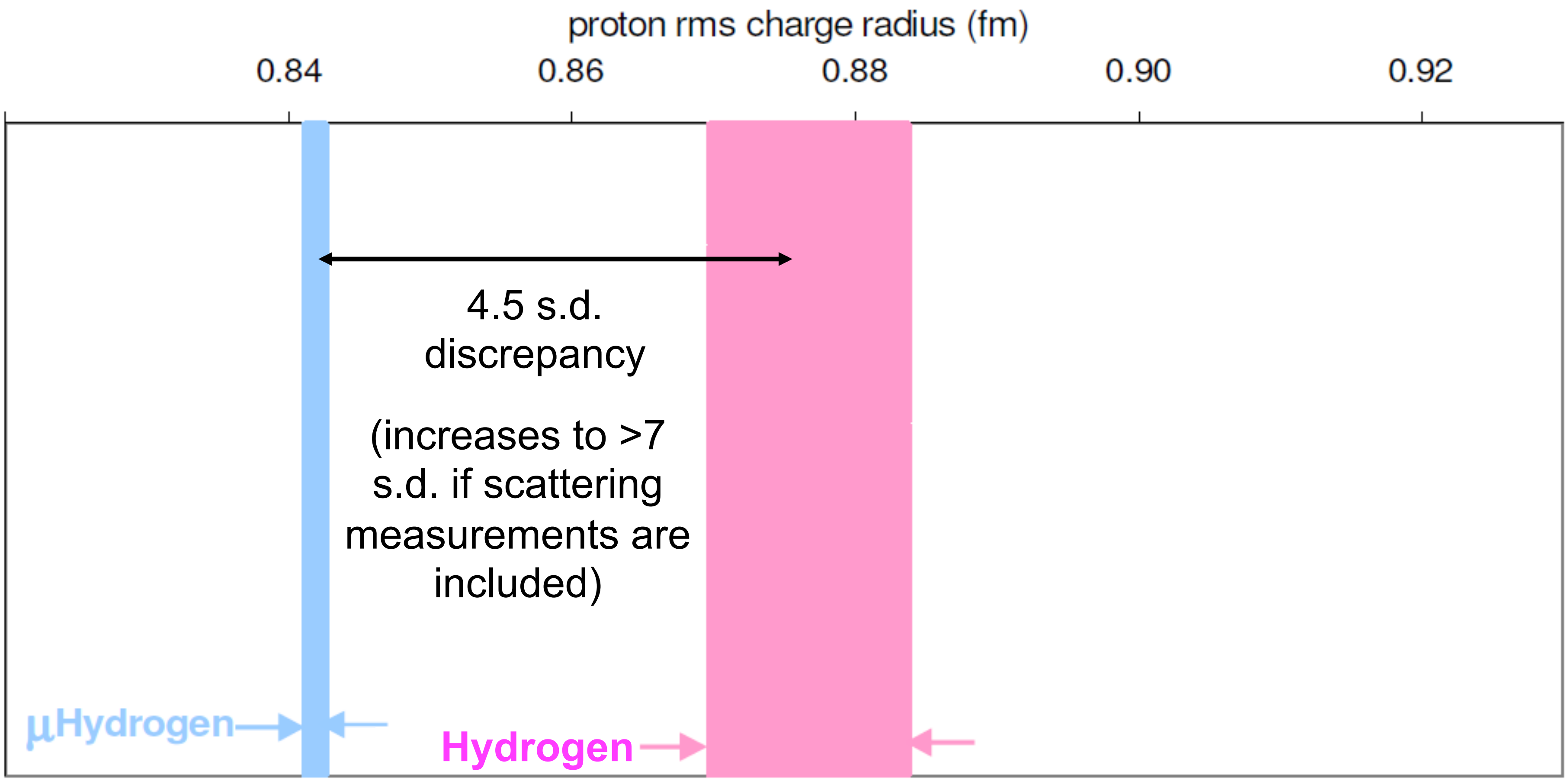
Ten r_p determinations from combinations of H intervals:



Hydrogen

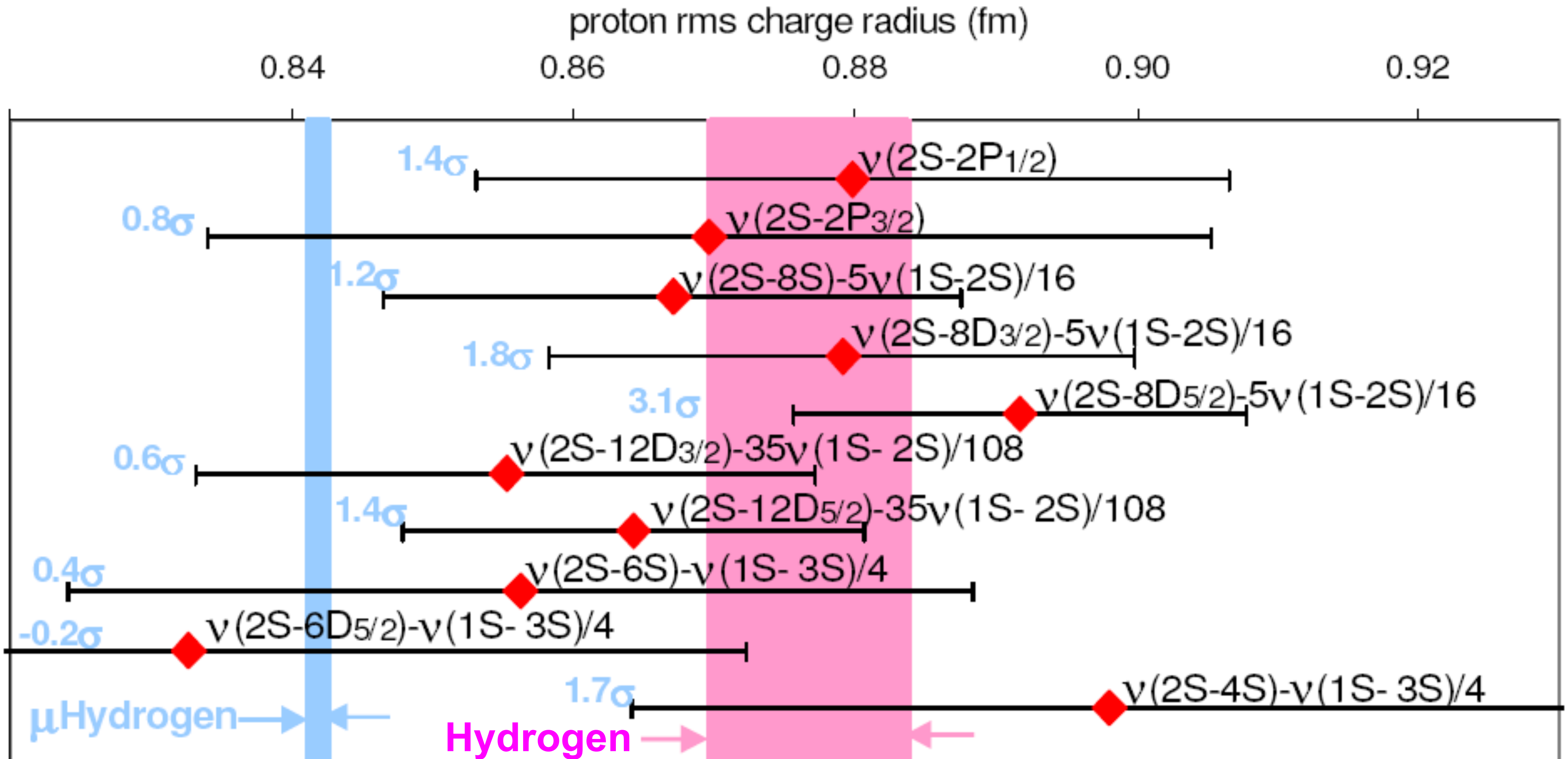
Muonic hydrogen




proton rms charge radius (fm)
0.84
0.86
0.88
0.90
0.92
4.5 s.d.
discrepancy
(increases to >7
s.d. if scattering
measurements are
included)
μHydrogen
Hydrogen

Comparing muonic hydrogen to the individual measurements makes the conflict seem not as big: all but one agree with μp to within 2 s.d.

We need more measurements in hydrogen



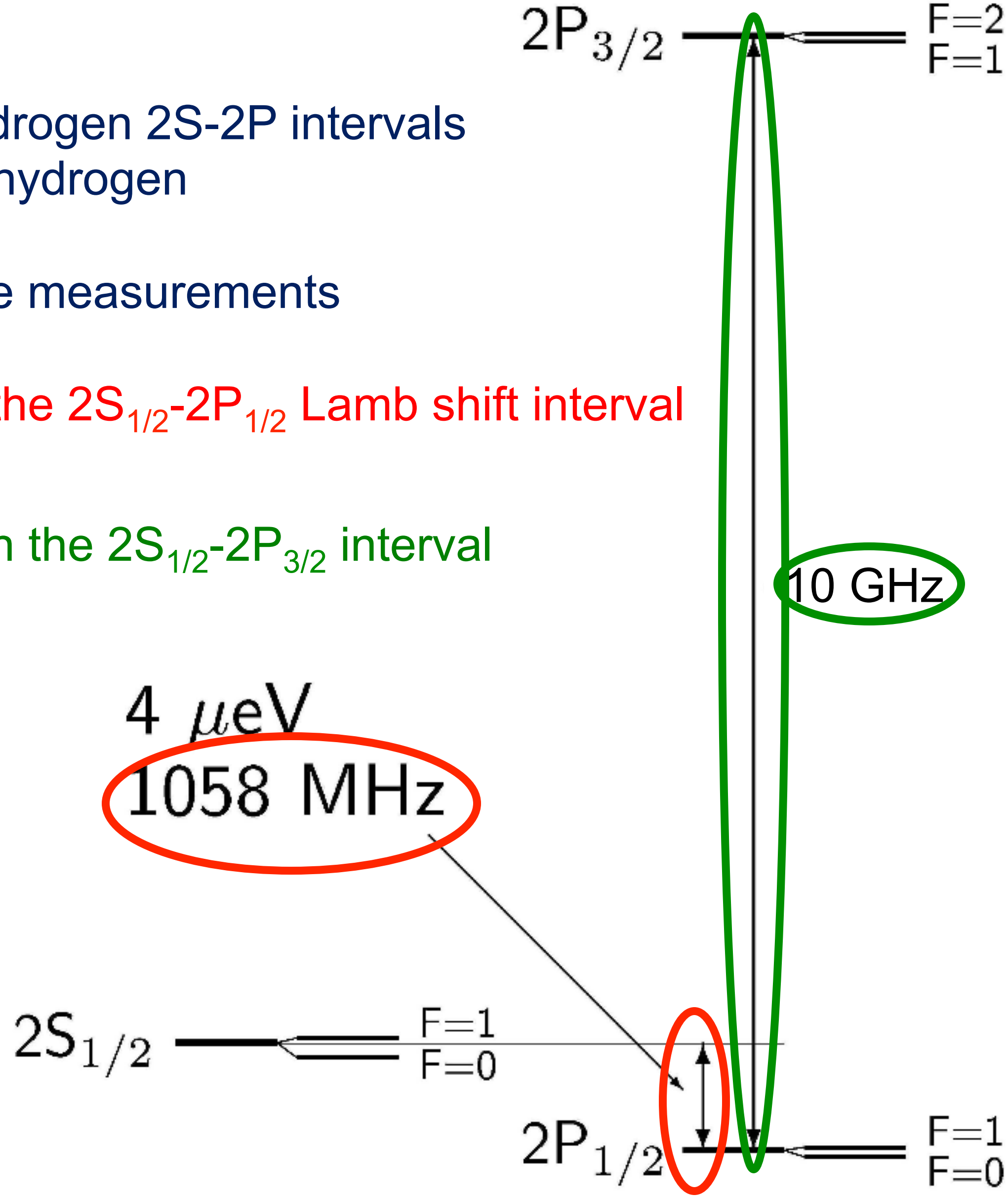
Our Experiment

Remeasure hydrogen 2S-2P intervals
in ordinary hydrogen

SOF microwave measurements

We will start with the $2S_{1/2}$-$2P_{1/2}$ Lamb shift interval

And follow up with the $2S_{1/2}$-$2P_{3/2}$ interval



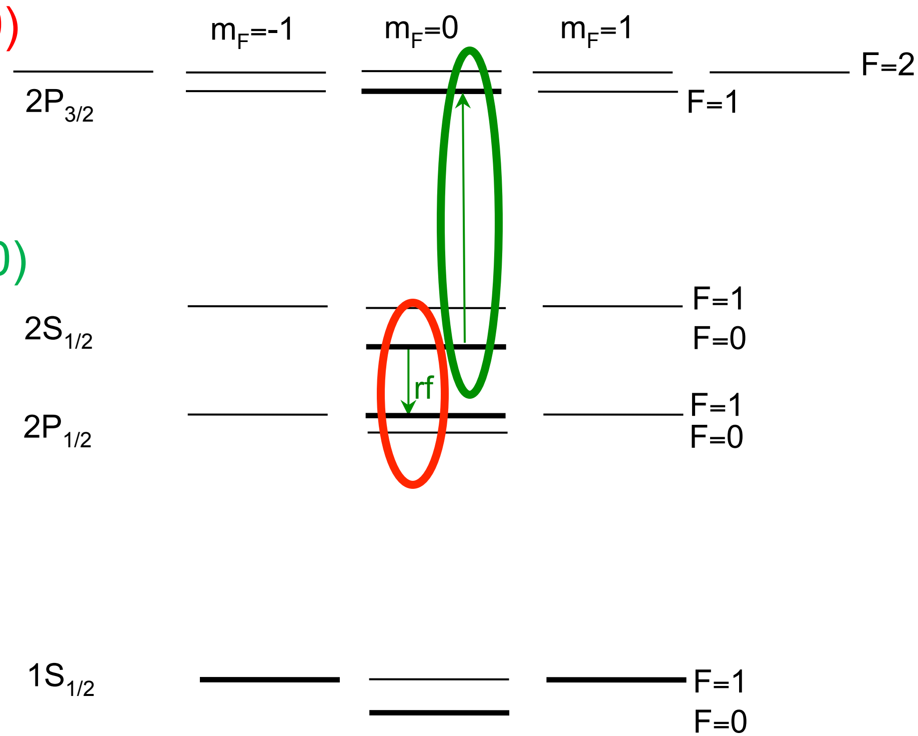
Our Experiment

Remeasure hydrogen 2S-2P intervals
in ordinary hydrogen

SOF microwave measurements

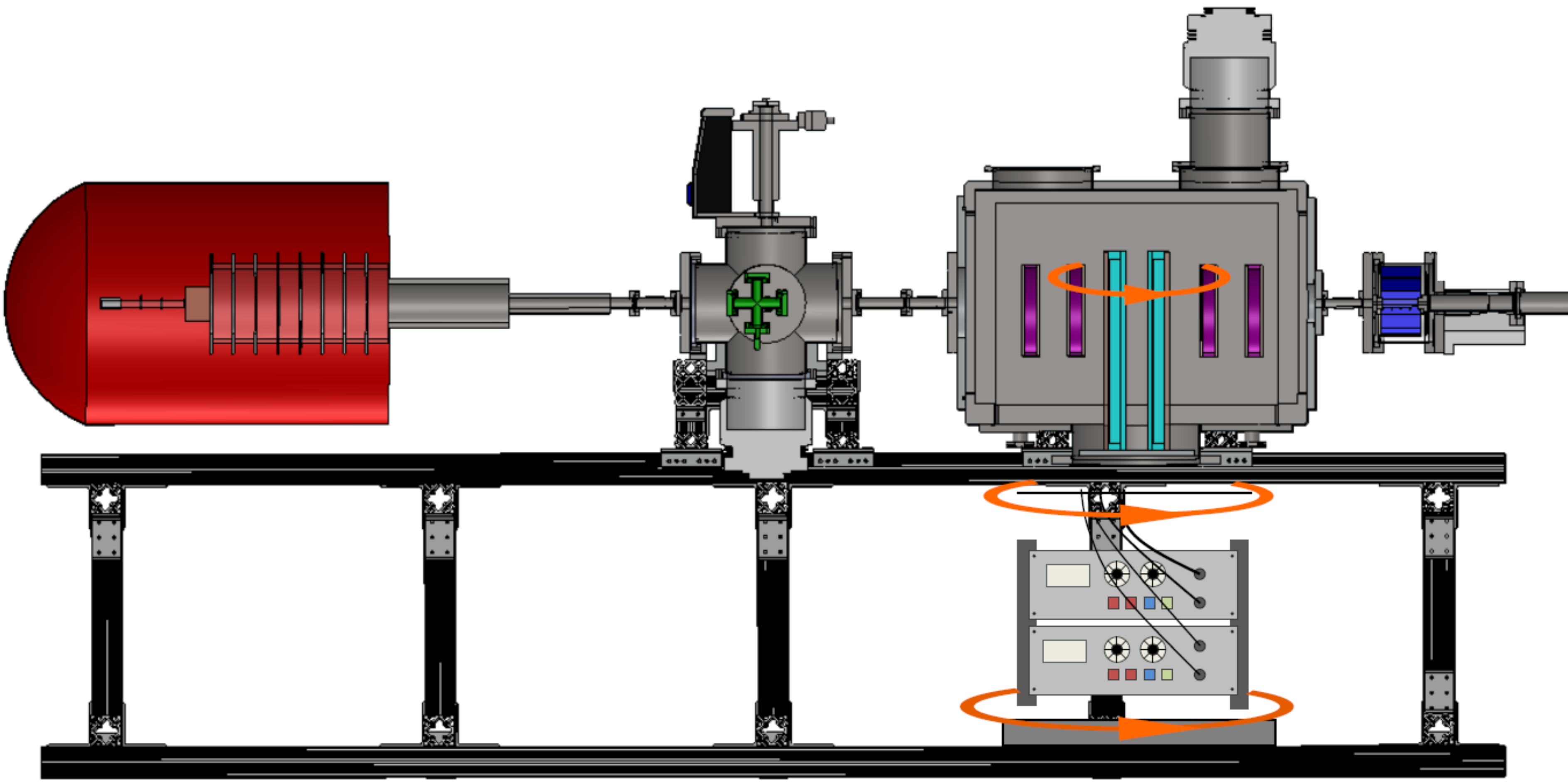
More specifically, we will start with the $2S_{1/2}$ ($F=0$, $m_F=0$) to $2P_{1/2}$ ($F=1$, $m_F=0$) interval

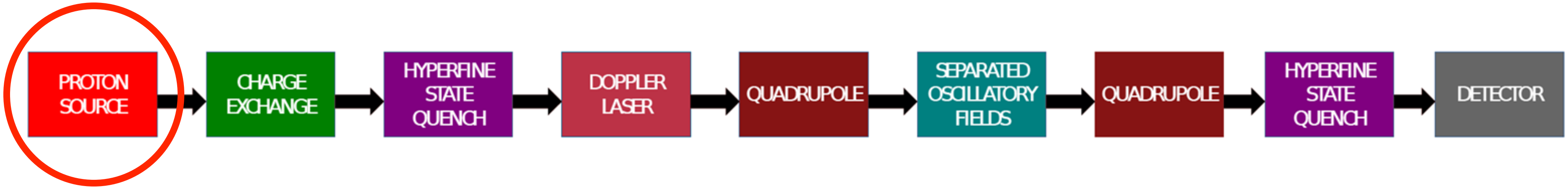
And later we will measure the $2S_{1/2}$ ($F=0$, $m_F=0$) to $2P_{3/2}$ ($F=1$, $m_F=0$) interval



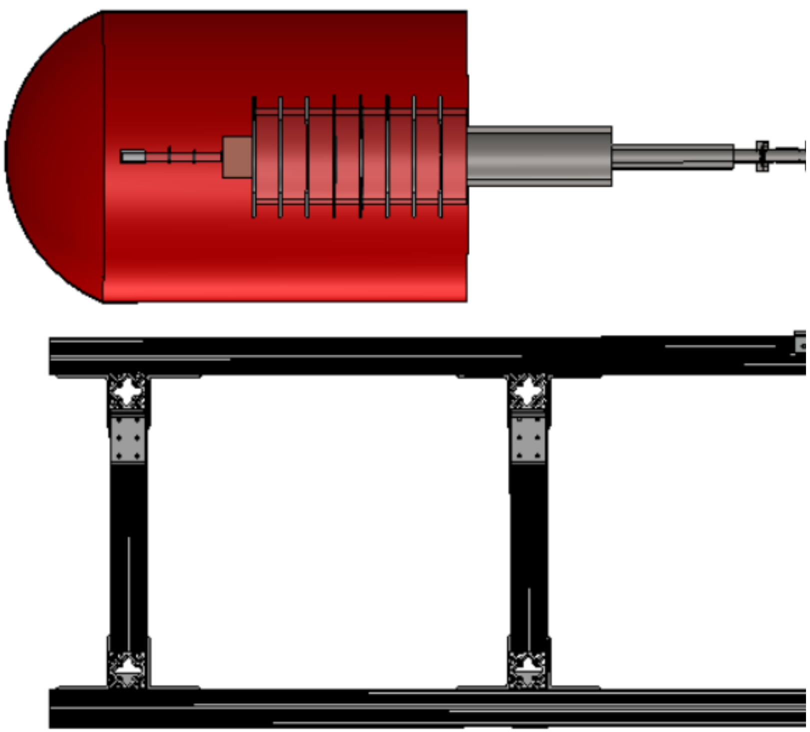
Will form a direct test of proton radius without the need for a precise Rydberg constant

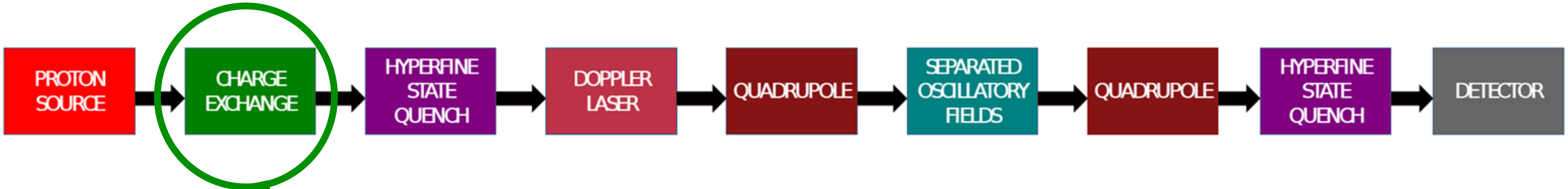
Our Experiment and progress to Date



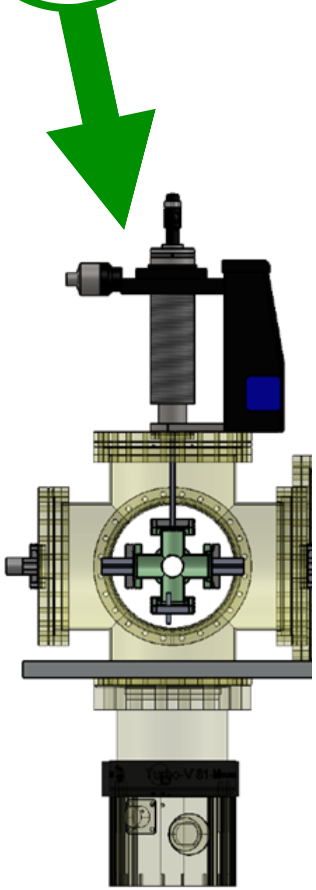


Stable ions source
with 10 μA of 50-keV

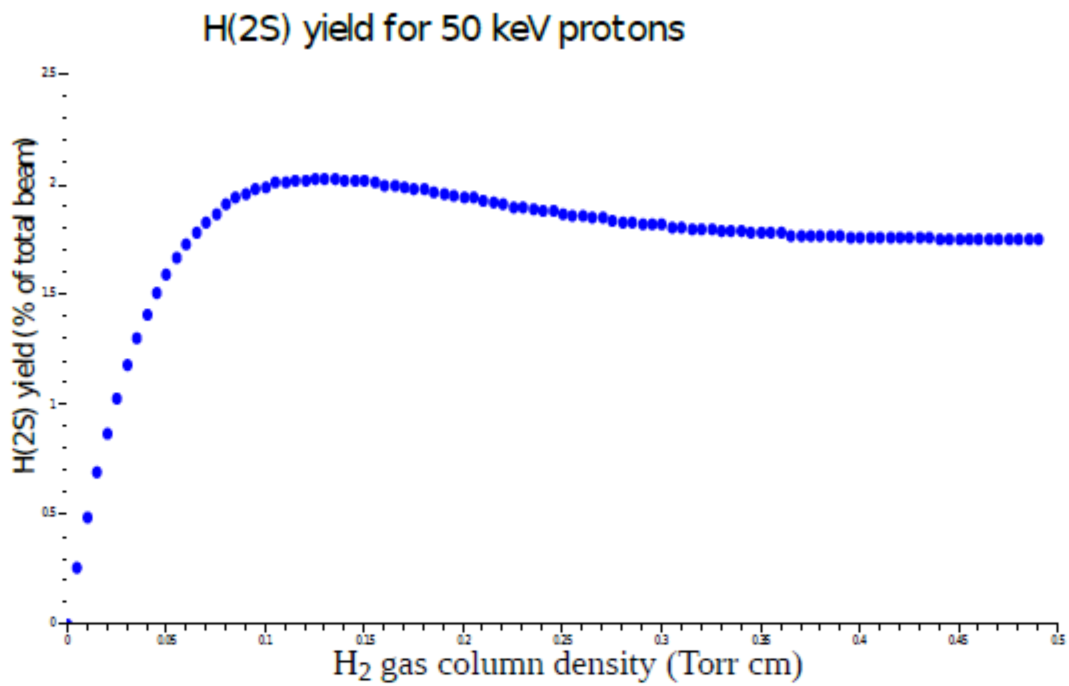




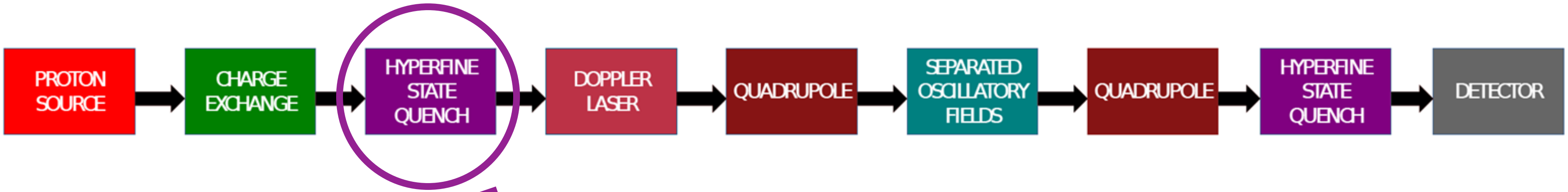
10 μA
50-keV to
100-keV
protons



protons
charge
exchange
with H_2 gas



- Protons at 50 keV undergo charge exchange with hydrogen molecules in cell.
- ~2% of protons estimated to be converted to metastable hydrogen: H(2S).

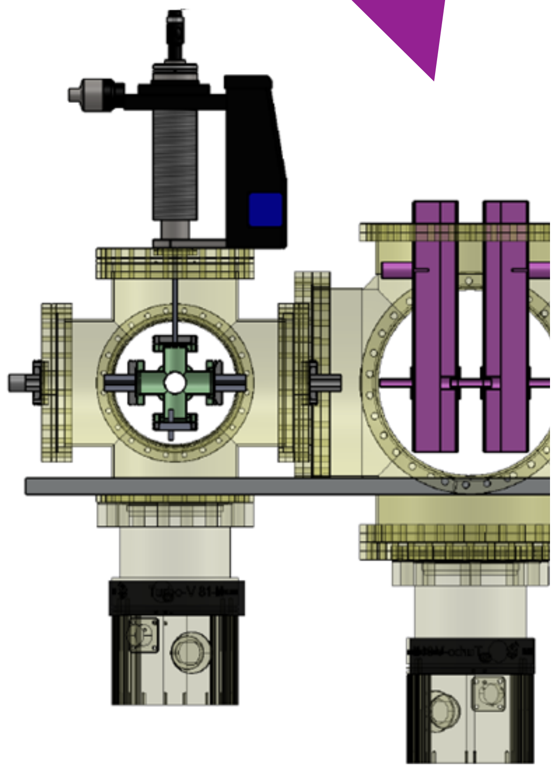


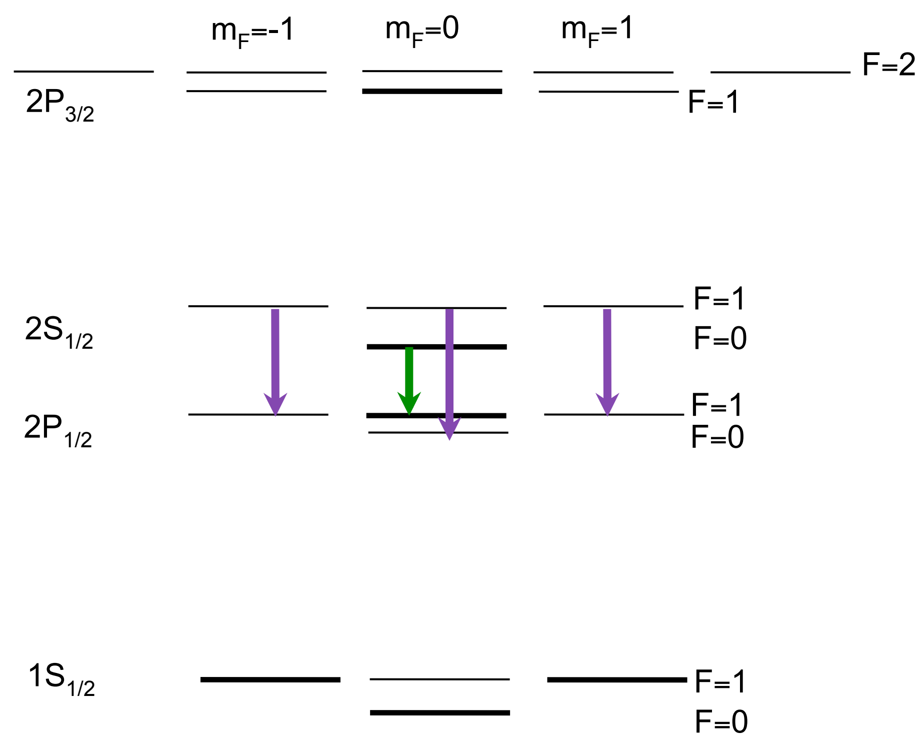
Charge exchange
2% H(2S)

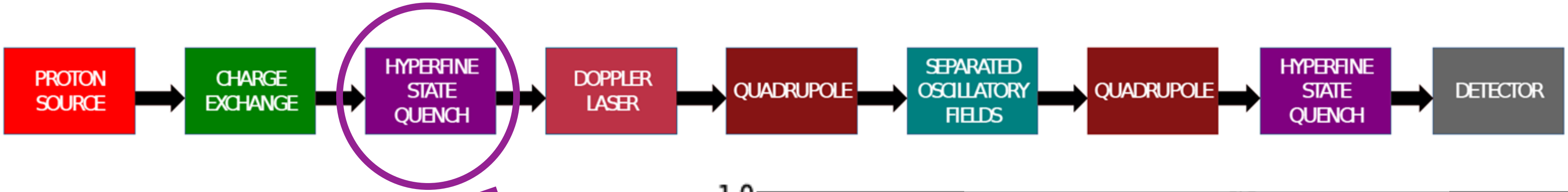
We empty the $2S_{1/2}$ F=1 states using 2 rf cavities that drive them down to short-lived $2P_{1/2}$ states

With F=1 states empty, can make a measurment of the isolated transition from the $2S_{1/2}$ F=0 transition

10 μA
50-keV to
100-keV
protons

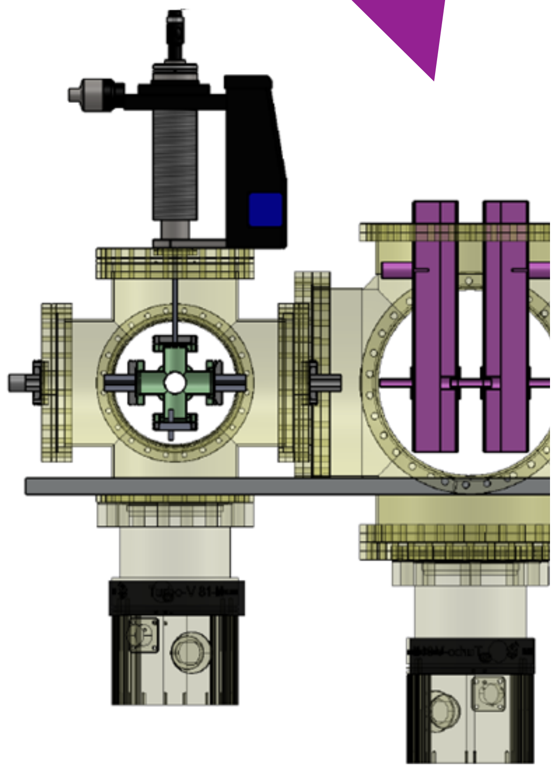


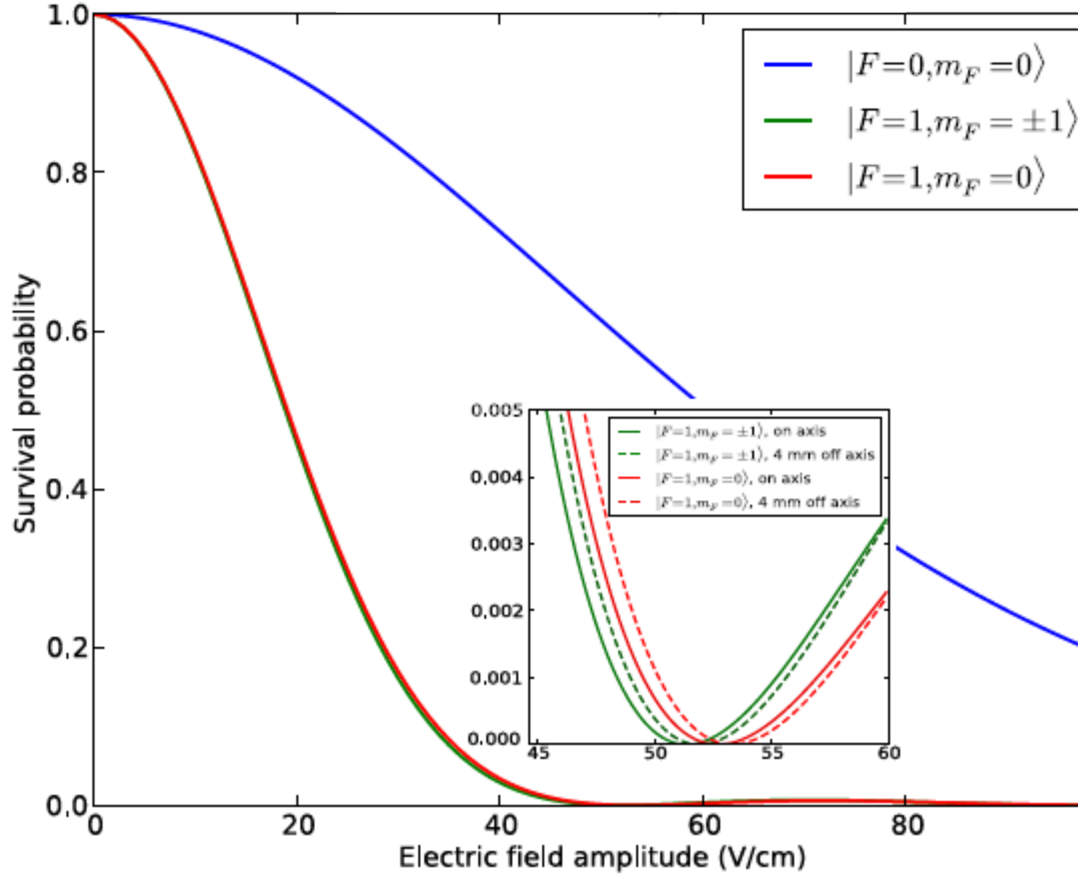




Charge exchange 2% H(2S)

10 μA 50-keV to 100-keV protons



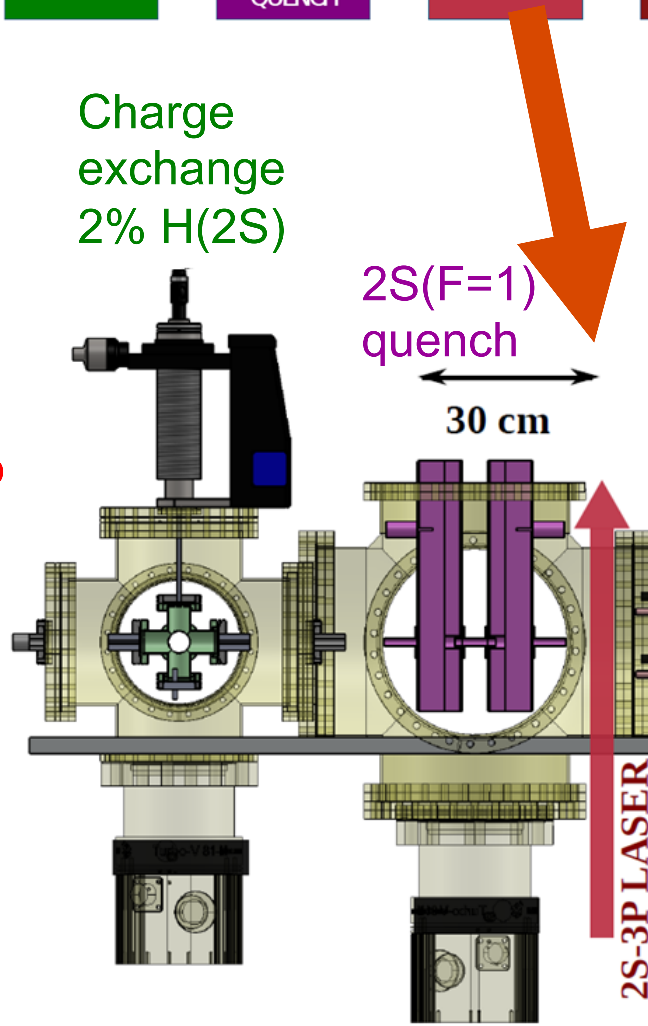


- Unwanted hyperfine states (2S, F=1) quenched using resonant microwaves (1088 MHz and 1147 MHz).
- Can retain 60% of 2S, F=0 population while emptying > **99.9%** of 2S, F=1

Charge exchange 2% H(2S)

2S(F=1) quench

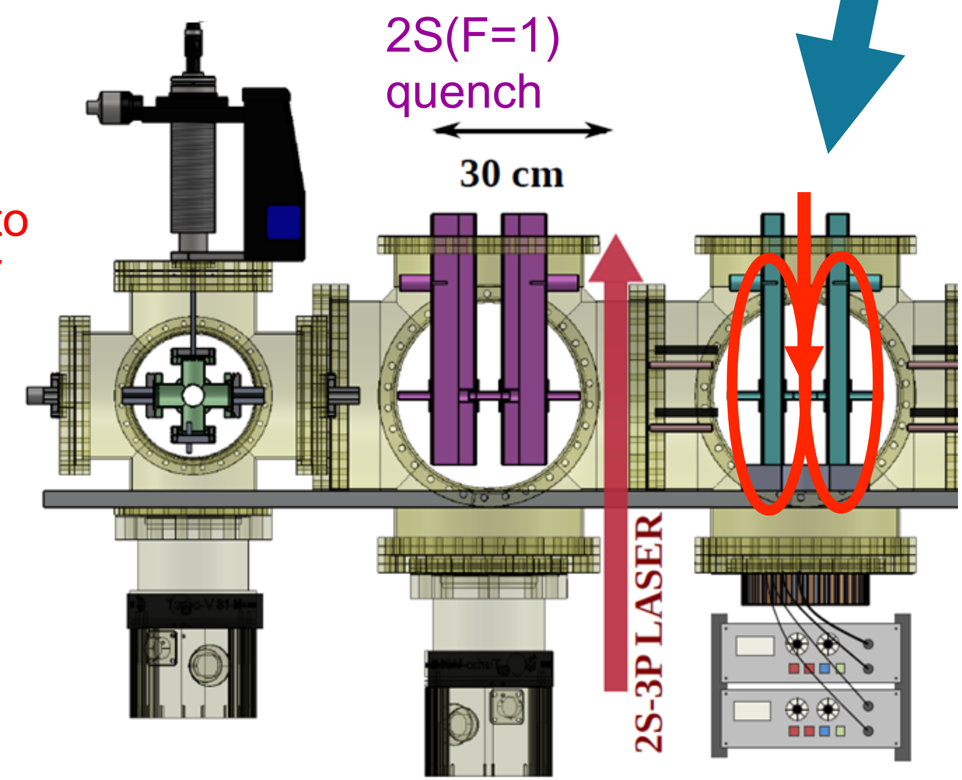
10 μA 50-keV to 100-keV protons



Amplified diode 2S-3P 656-nm laser system

To measure the speed, direction and angular spread of the H(2S) beam

10 µA
50-keV to
100-keV
protons

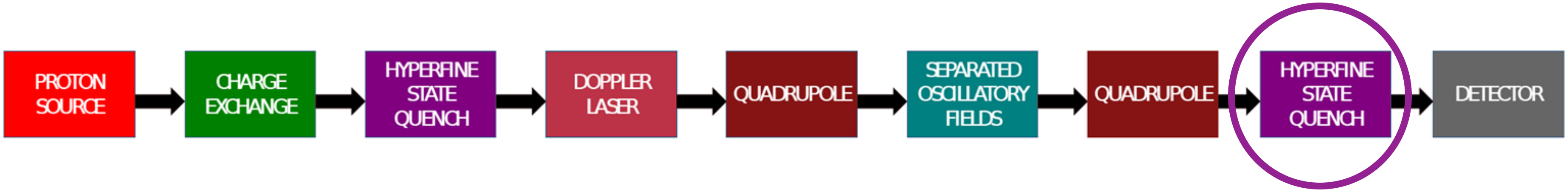


low-Q microwave cavities to create standing waves which drive the main SOF fields

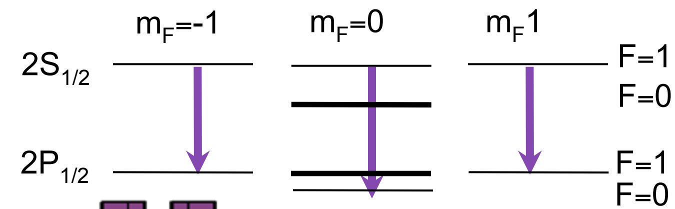
Critical parameter for the SOF measurement is the relative phase of the microwaves in the two cavities

Relative phase is measured by a pickup observing small interference signal in a tube connecting the two regions

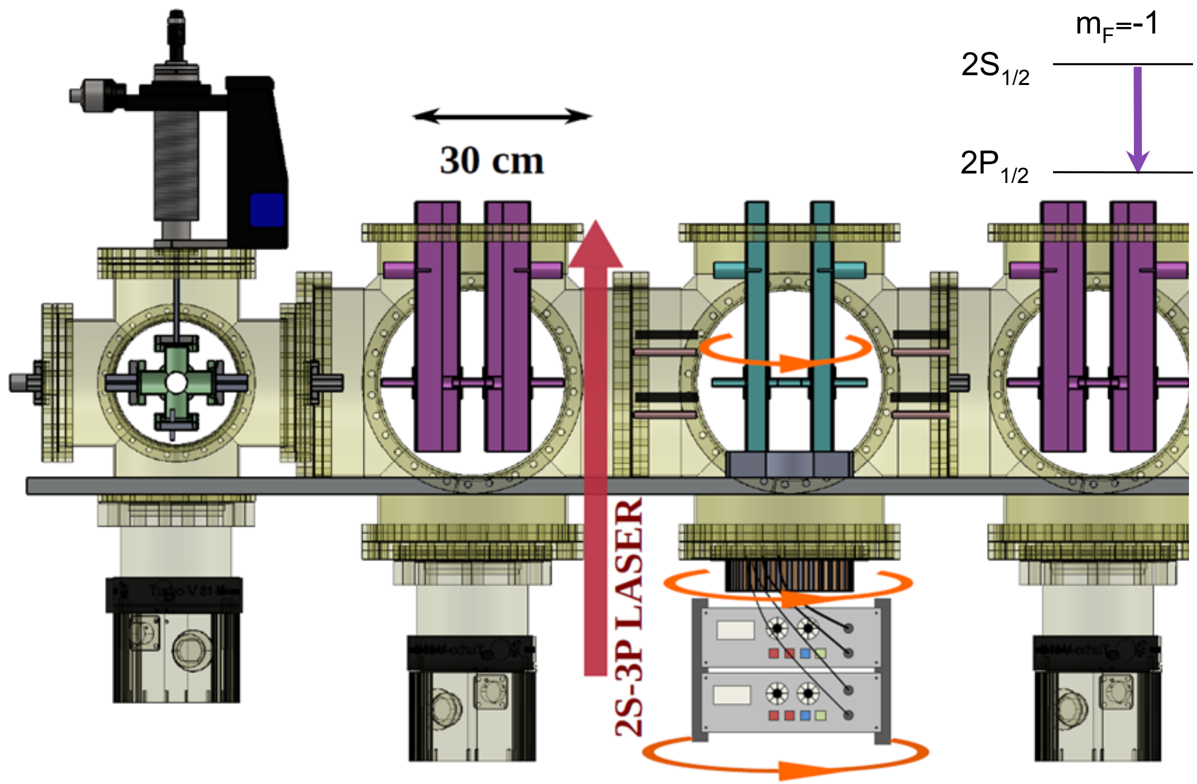
Any unanticipated error in relative phase is reversed by rotating **entire** microwave system by 180° – all in situ



a 2nd 2S(F=1) quench region assures that atoms that cascade into the 2S(F=1) atoms that decay from Rydberg states do not contribute to the signal



10 μA
50-keV to
100-keV
protons

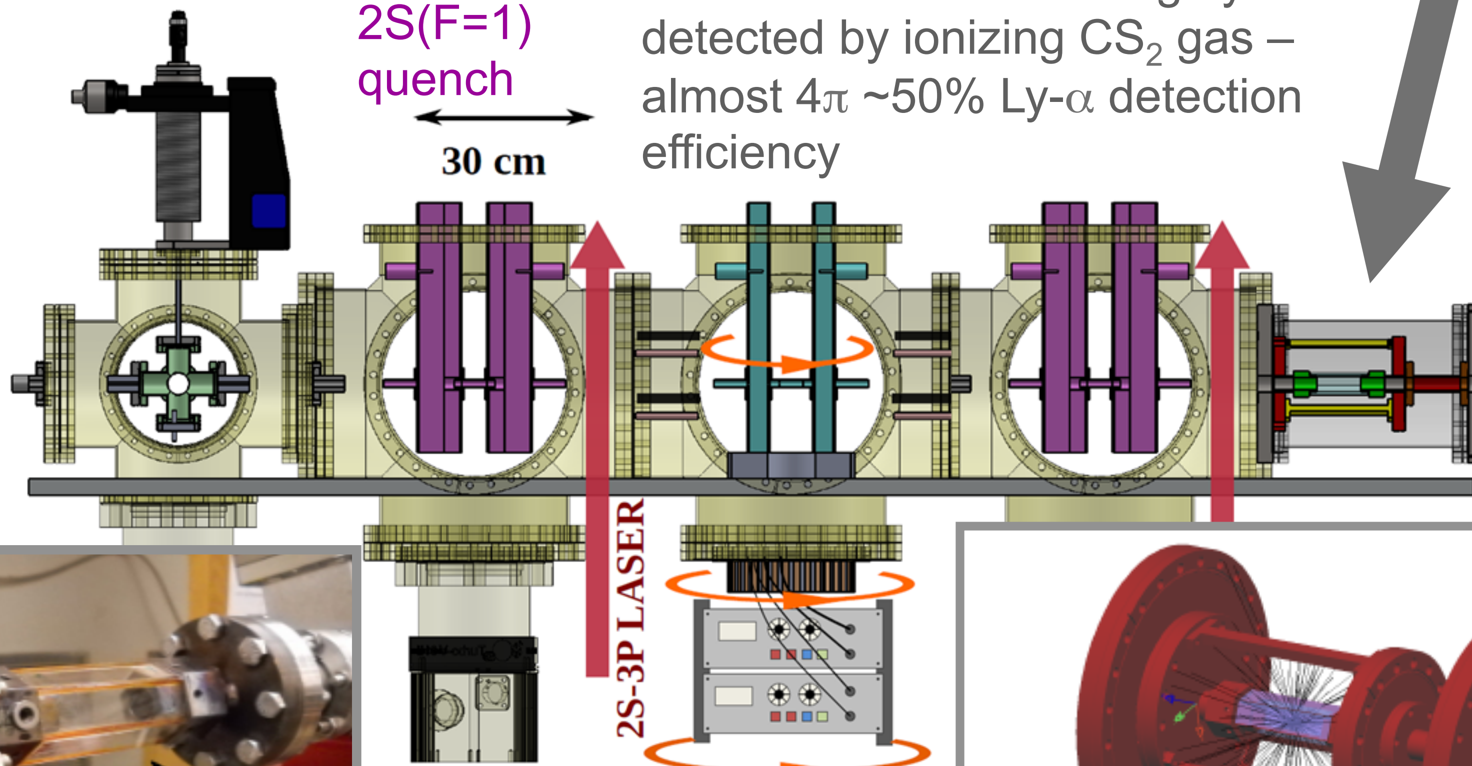


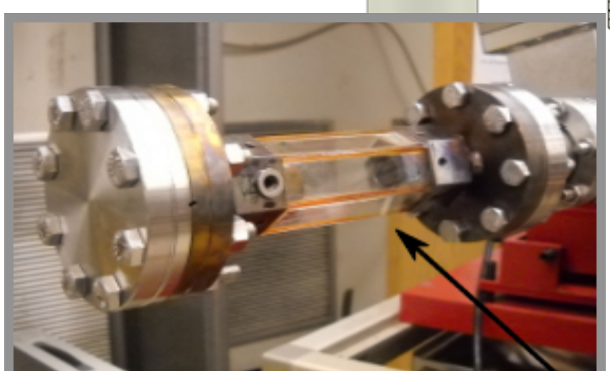
PROTON SOURCE → CHARGE EXCHANGE → HYPERFINE STATE QUENCH → DOPPLER LASER → QUADRUPOLE → SEPARATED OSCILLATORY FIELDS → QUADRUPOLE → HYPERFINE STATE QUENCH → DETECTOR

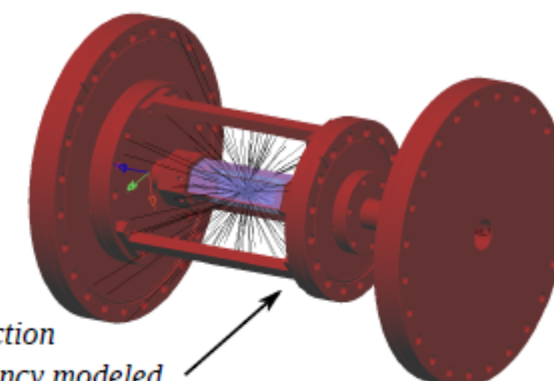
We detect the 2S atoms that remain by mixing 2S with 2P with a DC electric field and resulting Ly-α is detected by ionizing CS_2 gas – almost 4π ~50% Ly-α detection efficiency

10 μA
50-keV to
100-keV
protons

2S(F=1)
quench

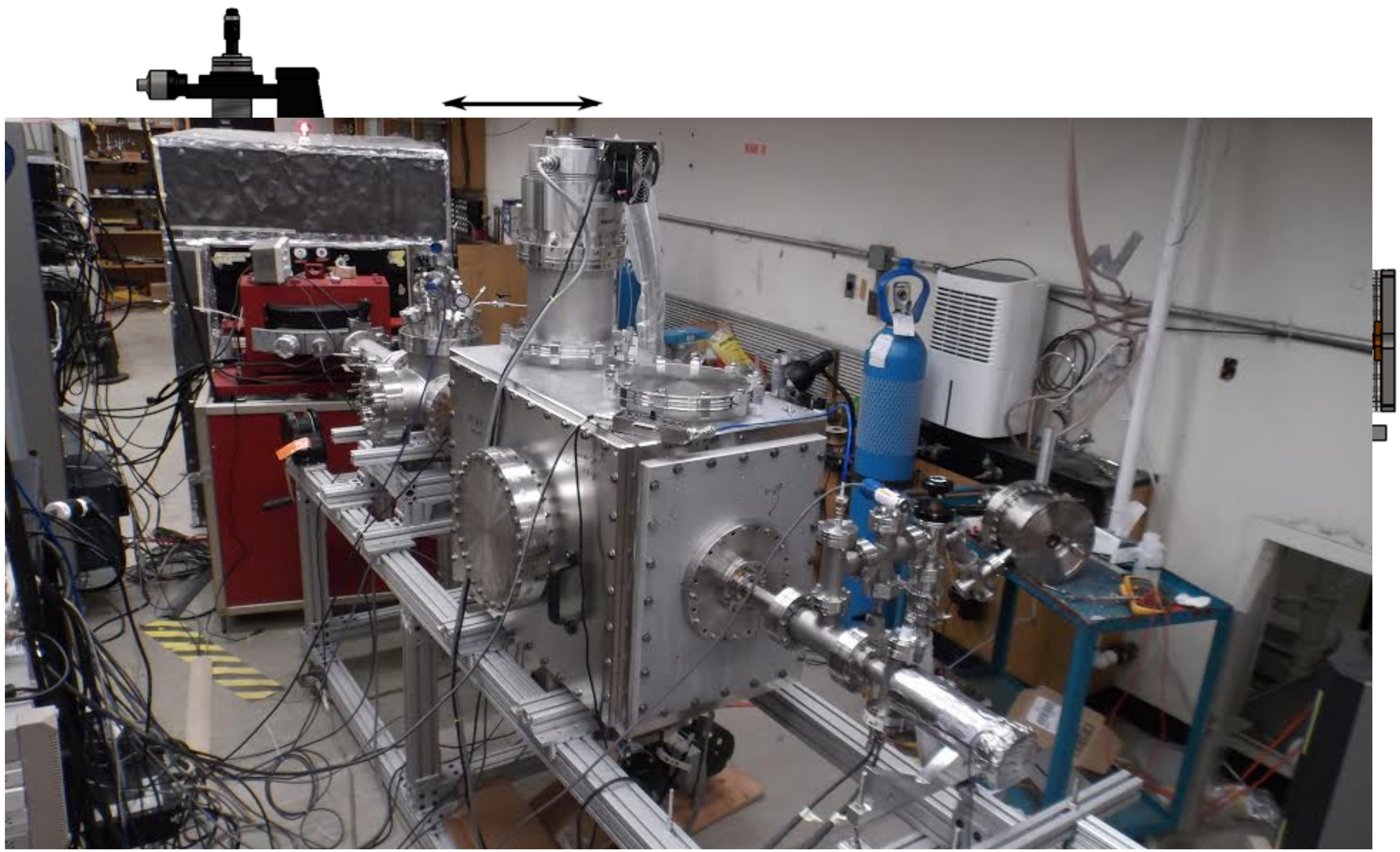




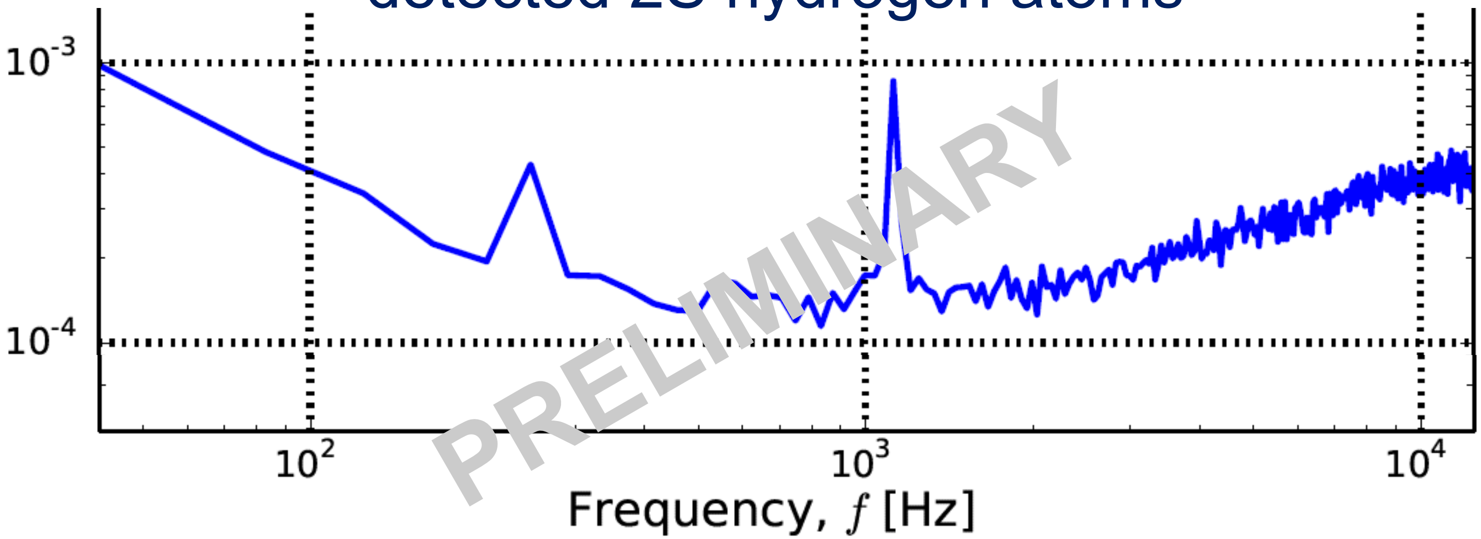


Collection efficiency modeled with ray-tracing software (LightTools)


PROTON SOURCE
CHARGE EXCHANGE
HYPERFINE STATE QUENCH
DOPPLER LASER
QUADRUPOLE
SEPARATED OSCILLATORY FIELDS
QUADRUPOLE
HYPERFINE STATE QUENCH
DETECTOR

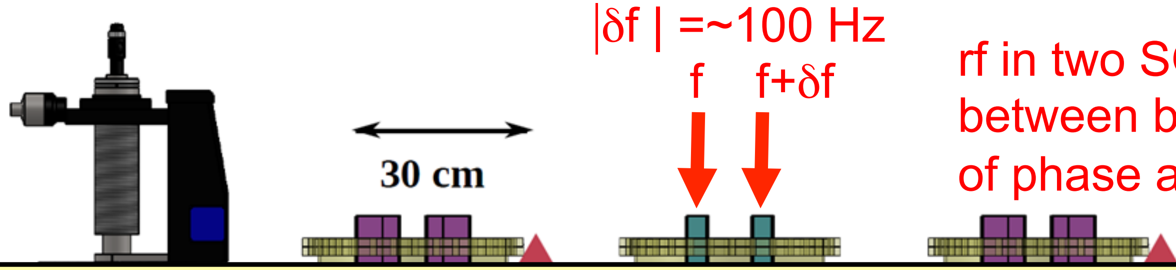


Noise-to-signal ratio (per root Hz) of detected 2S hydrogen atoms



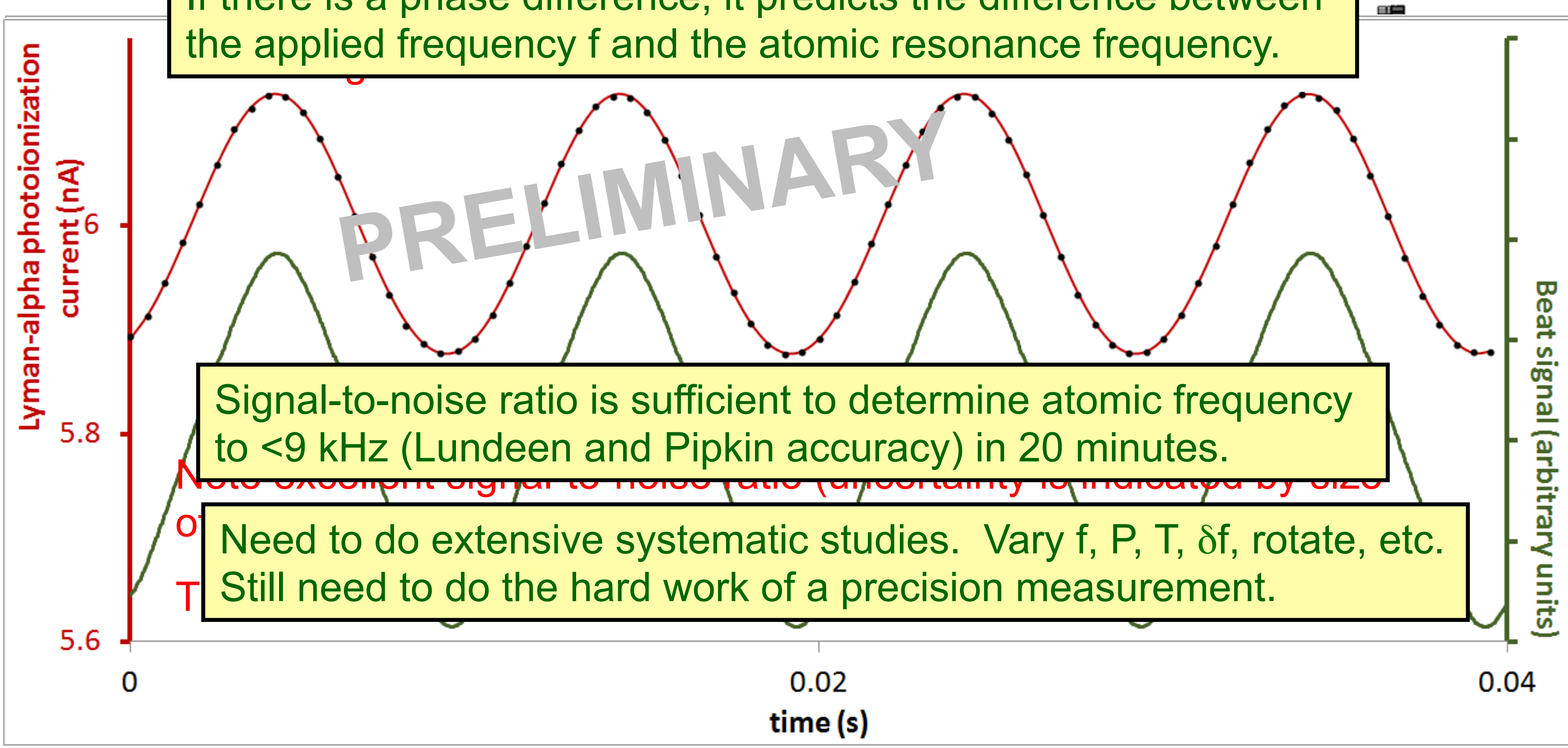
Very good signal-to-noise ratio (approaching 10^4 in 1 second) at most frequencies between 100 Hz and 10 kHz

Diffference in phase between beat signal and SOF signal is zero if rf frequency (f) is in resonance with the atomic transition.



rf in two SOF regions oscillate between being in phase and out of phase at δf beat frequency

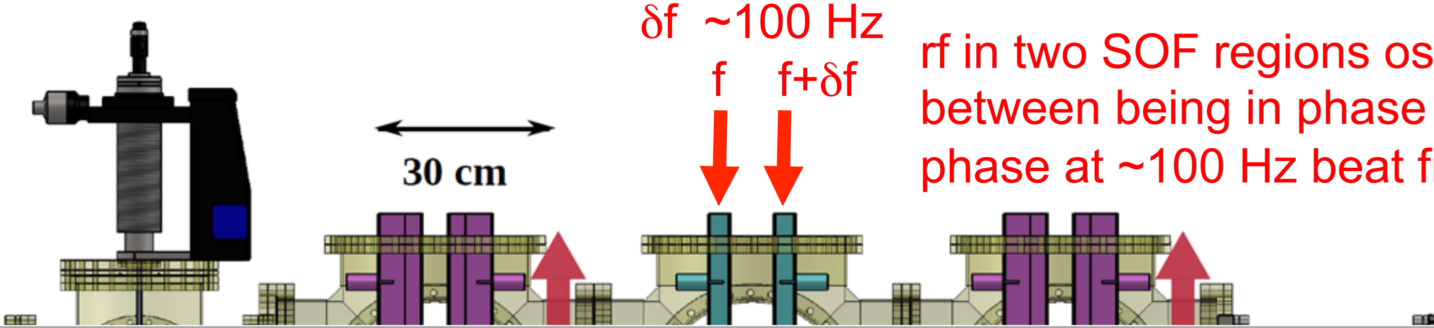
If there is a phase difference, it predicts the difference between the applied frequency f and the atomic resonance frequency.



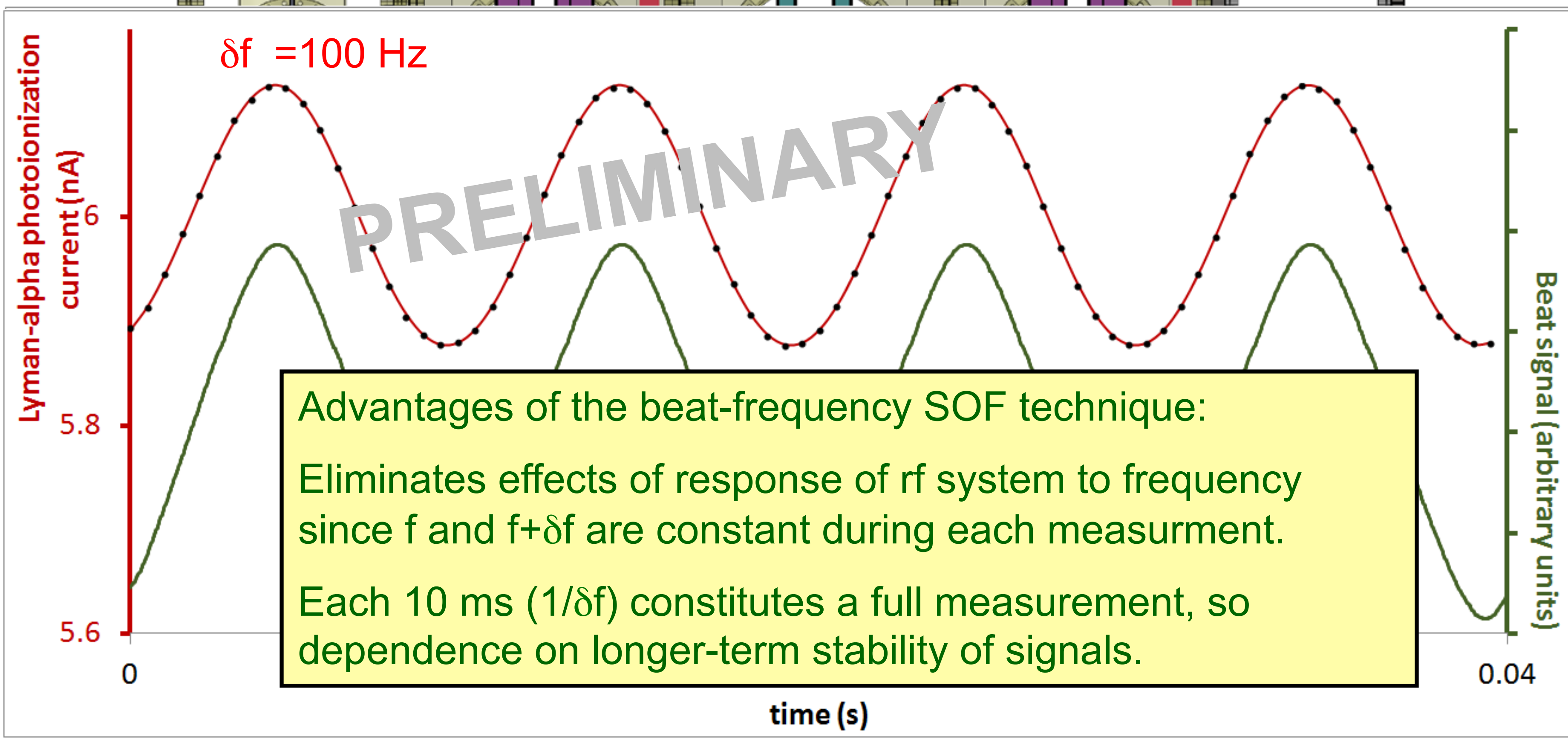
Signal-to-noise ratio is sufficient to determine atomic frequency to <9 kHz (Lundeen and Pipkin accuracy) in 20 minutes.

Need to do extensive systematic studies. Vary f, P, T, δf, rotate, etc.
Still need to do the hard work of a precision measurement.

We are using a new beat-frequency SOF technique



rf in two SOF regions oscillate between being in phase and out of phase at ~100 Hz beat frequency

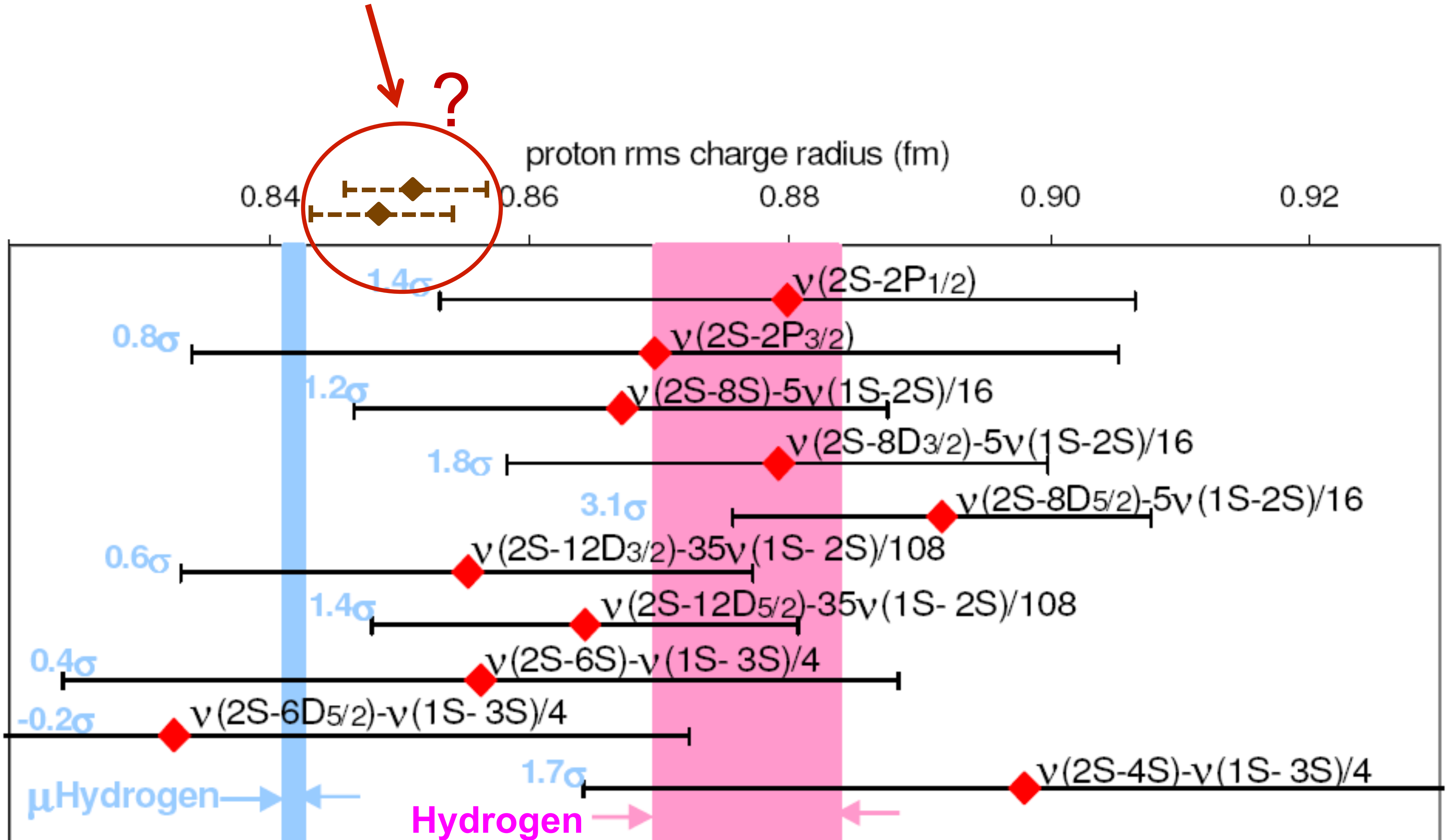


Advantages of the beat-frequency SOF technique:

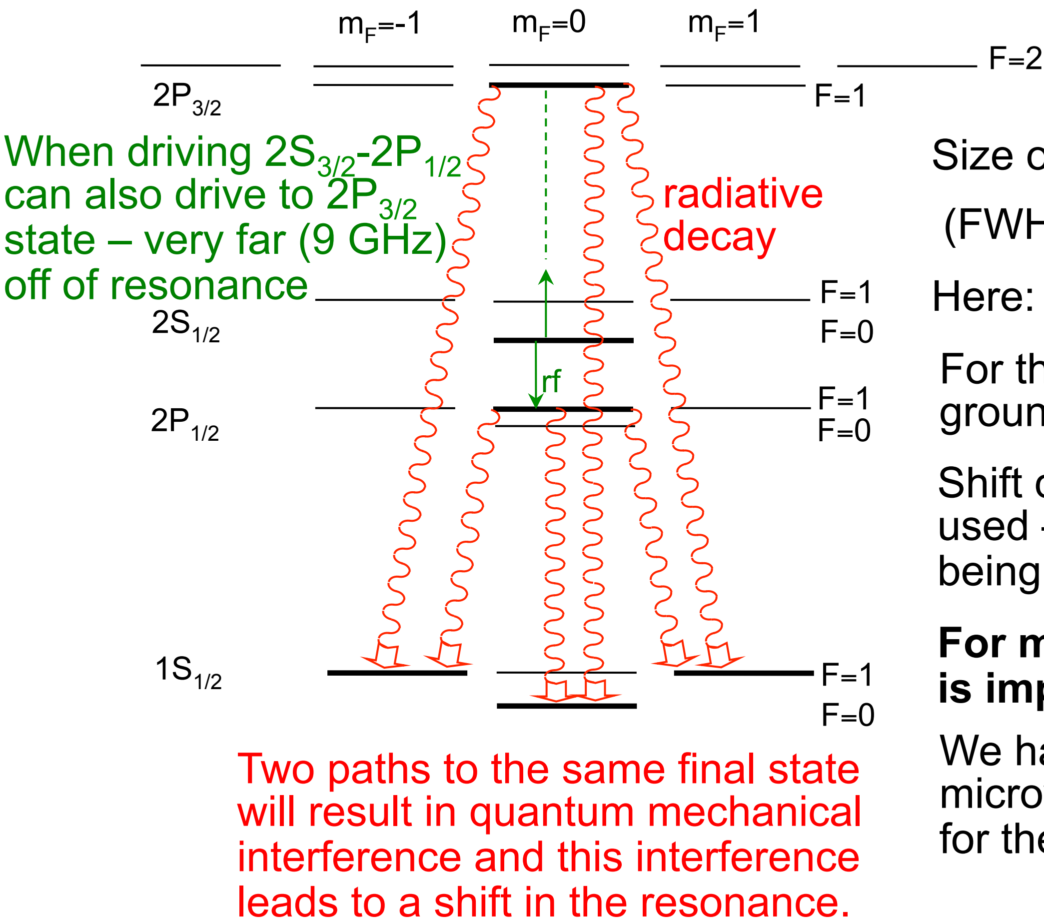
Eliminates effects of response of rf system to frequency since f and f+δf are constant during each measurment.

Each 10 ms (1/δf) constitutes a full measurement, so dependence on longer-term stability of signals.

Our (eventual) aim is an accuracy of 2 kHz for each fo the 2S-2P intervals, which would provide two new measurements of the proton radius with uncertainties indicated



Interference Shifts – an important systematic for precision measurements



Size of shift scales as

(FWHM) * (FWHM)/detuning

Here: (40 MHz) * (40 MHz) / (9 GHz) ~ 200 kHz

For this experiment, shift cancels exactly if all ground states are included.

Shift depends on the experimental technique used – the interference depends on what is being measured and what paths can interfere.

For most precision measurements this effect is important.

We have calculated shifts in detail for several microwave and laser measurement techniques for the n=2 triplet states of helium.

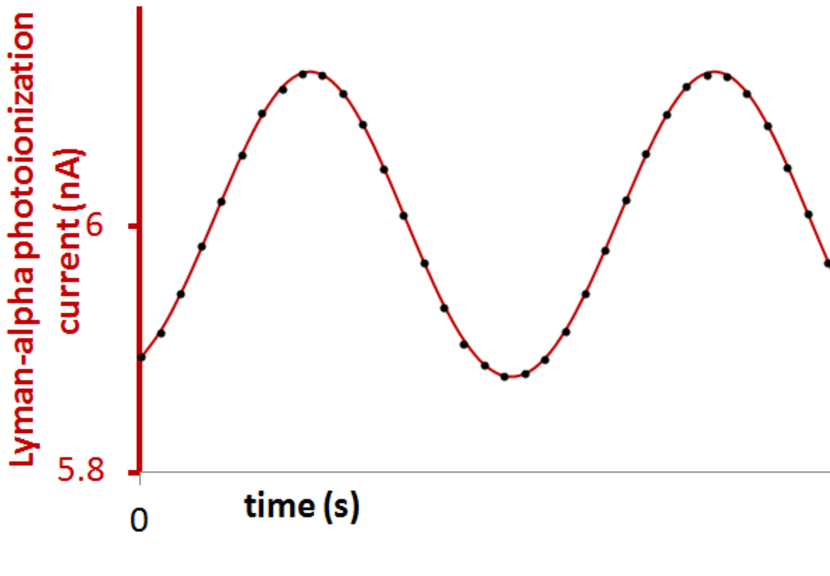
Conclusions:

We are measuring the n=2 Lamb shift of Hydrogen

We see excellent signal-to-noise

We need to do extensive tests for systematics to complete the measurement (6 months?)

Measurement will make a significant determination of the proton charge radius.



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