Nuclear Polarization and the Lamb Shift in Light Muonic Atoms

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Proton Radius Puzzle - Mainz - June 3rd, 2014

TRIUMF   University of Manitoba   האוניברסיטה העברית בירושלים The Hebrew University of Jerusalem

Outline

- **Motivation**
  - Proton radius puzzle
  - Accurate nuclear radii
- **Background**
  - Lamb shift, charge radius & polarization
- **Derivation**
  - *Ab-initio* calculation of nuclear polarization
- **Results**
  - $\mu\mathbf{D}$
  - $\mu^{4}\mathbf{He}^{+}$
  - Error estimates
  - Preliminary: $A = 3$
- **Summary**
- **Outlook**

How large is the proton?

- r_p **from electron-proton interaction**
  1. e-p scattering: $r_p = 0.875(10)$ fm
  2. Hydrogen spectroscopy: $r_p = 0.8768(69)$ fm
  3. $\Longrightarrow$ CODATA-2010: $\boldsymbol{r_p = 0.8775(51)}$ fm

     Mohr *et al.*, Rev. Mod. Phys. (2012)

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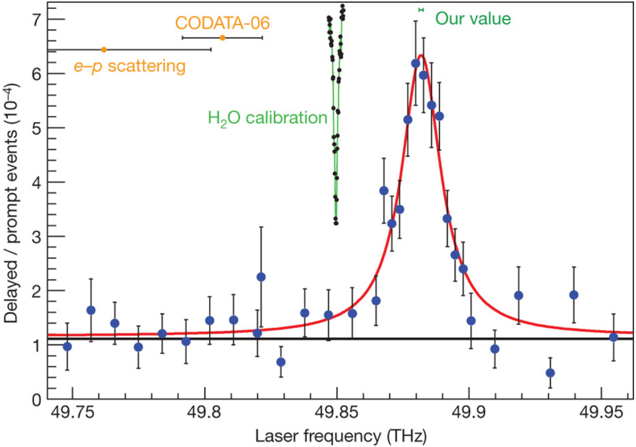
- r_p **from** μ**H Lamb shift (2S-2P)**
  1. μH 2S$^{F=1}_{1/2}$ - 2P$^{F=2}_{3/2}$: $r_p = 0.84184(67)$ fm (5σ)

     Pohl *et al.*, Nature (2010)
  2. Combined with

     μH 2S$^{F=0}_{1/2}$ - 2P$^{F=1}_{3/2}$: $\boldsymbol{r_p = 0.84087(39)}$ fm (7σ)

     Antognini *et al.*, Science (2013)

Proton radius puzzle



Origins of the discrepancy?

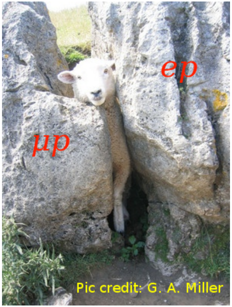
Study the discrepancy between r_p from ep and μp experiments

- accuracy in ep scattering measurements?
  - Q^2 not small enough / floating normalization
  - dispersion analysis:

    global fit of n & p EM form factors

    $\rightarrow r_p = 0.84(1)$ fm with $\chi^2_{\text{red}} \approx 2.2$

Lorenz, Hammer, Meißner, EPJA (2012)



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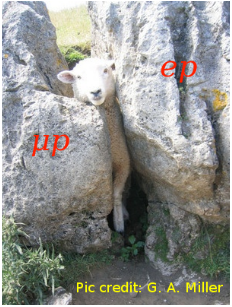
- exotic hadronic structure?

  Birse, McGovern, EPJA (2012) *vs* Miller, PLB (2013)



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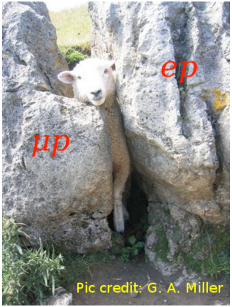

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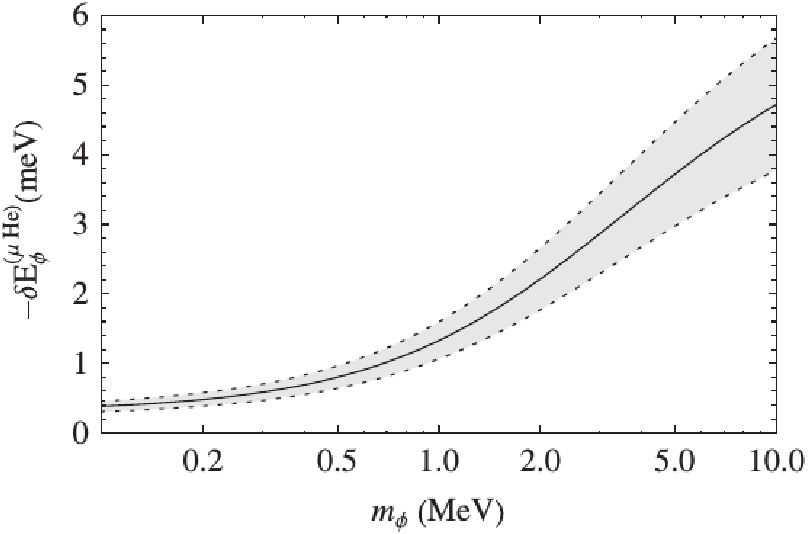
- exotic hadronic structure?
  Birse, McGovern, EPJA (2012) vs Miller, PLB (2013)
- beyond-standard-model physics?
  - new force carriers, e.g., dark photon: interact differently with e and μ
  - explain both the r_p puzzle & $(g-2)_\mu$ puzzle
    Tucker-Smith, Yavin, PRD (2011), Batell, McKeen, Pospelov, PRL (2011), Carlson, Rislow, PRD (2012).

Tucker-Smith & Yavin's prediction for $\mu^4\mathrm{He}^+$



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New experiments to shed light on the puzzle

- **Jefferson Lab**
  - ep scattering for Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2

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  - Lamb shift in muonic helium: μ^4He^+ (ongoing), μ^3He^+ (planned)

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high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

Extract nuclear charge radius

$\langle r^2 \rangle$ from Lamb shift

$$\Delta E_{LS} = \delta_{QED} + \delta_{size}(r) + \delta_{pol}$$

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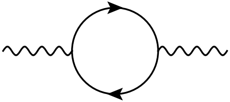
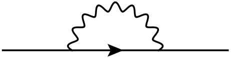
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects





- Two μ^4He^+ calculations differ

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$$\Delta E_{LS} = \delta_{QED} + \delta_{size}(r) + \delta_{pol}$$

- Nuclear finite-size corrections (elastic):
  - leading term: $\frac{m_r^3}{12}(Z\alpha)^4 \langle r^2 \rangle$
  - 3rd Zemach moment: $-\frac{m_r^4}{24}(Z\alpha)^5 \langle r^3 \rangle_{(2)} \quad \propto \langle r^2 \rangle^{3/2}$

Extract nuclear charge radius

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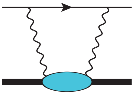
- Nuclear A polarization corrections (inelastic):
  - exchange of two virtual photons
  - dominant contribution $\sim (Z\alpha)^5$



- Nucleon p/n polarization corrections (inelastic)

Uncertainty in nuclear polarization

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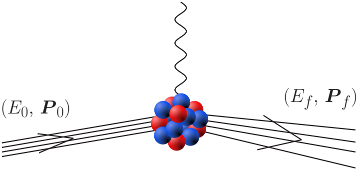
- Nuclear polarization $\Longrightarrow$ inputs from nuclear response functions

$$S_O(\omega) = \sum_f \! \! \! \! \! \! \! \int \; |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



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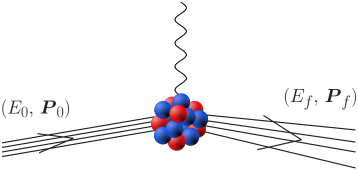

- Early calculations of δ_{pol} in muonic atoms:
  $\Rightarrow S_O(\omega)$ inputs were not accurate enough

Previous calculations of $S_O(\omega)$ & δ_{pol}

- μD
  - Modern forces: AV14 - Leidemann & Rosenfelder '95
  - State-of-the-art: AV18 - Pachucki '11
    - underestimates nuclear physics uncertainty
    - includes nucleon polarizability δ^p_{pol} (incorrect?)
  - $\not\pi$EFT: zero-range expansion - Friar '13
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- **Status of δ_{pol} in, *e.g.*, $\mu^{3,4}\mathrm{He}^+$**
  - experimental input for S_O is either too scattered or inexistent
  - need to calculate δ_{pol} using **modern potentials and *ab-initio* methods**

Nuclear polarization in $\mu^4\mathrm{He}^+$

We have performed the first *ab-initio* calculation of nuclear polarization in $\mu^4\mathrm{He}^+$ with state-of-the-art forces: AV18/UIX and χEFT

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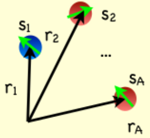
- **Our Goal**

  provide $\boldsymbol{\delta_{pol}}$ with accuracy comparable to the $\pm 5\%$ experimental needs

Nuclear potentials: two approaches



$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN} + V_{3N} + \dots$$

High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

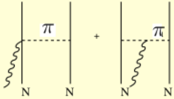
Traditional Nuclear Physics

AV18+UIX, ..., J_2

Effective Field Theory

N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \dots$$



two-body currents (or MEC)

subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$S(\omega) \propto |\langle \psi_f | J^\mu | \psi_0 \rangle|^2$$

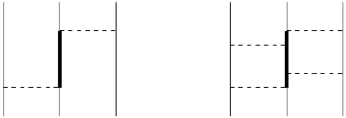
Exact Initial state &
Final state in the continuum at
different energies and for different A

Nuclear potentials: Traditional (phen.)

- **Argonne $v18$** fitted to
  - 1787 pp & 2514 np observables for $E_{lab} \leq 350\,\mathrm{MeV}$ with $\chi^2/\mathrm{datum} = 1.1$
  - nn scattering length & **D** binding energy

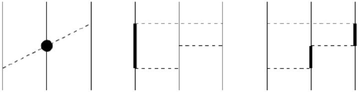
- **Urbana IX**

  $$V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$$



- **Illinois**

  $$+V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R}$$



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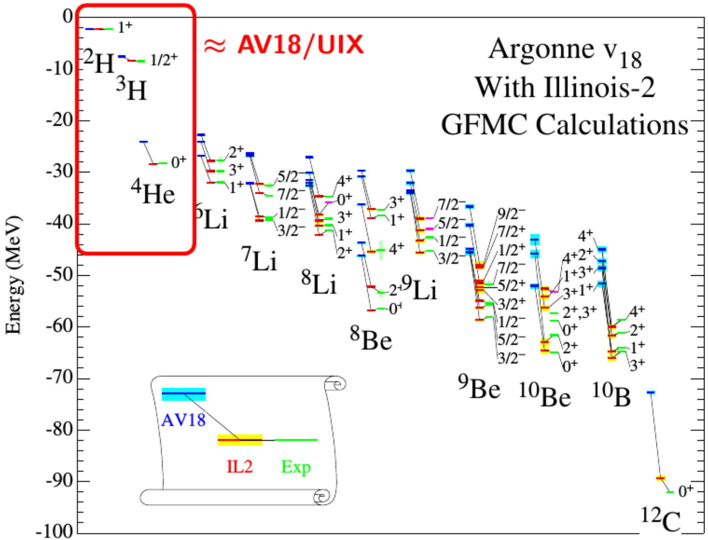
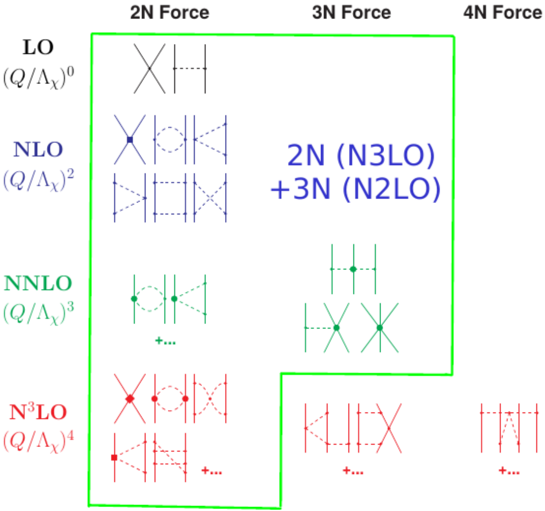


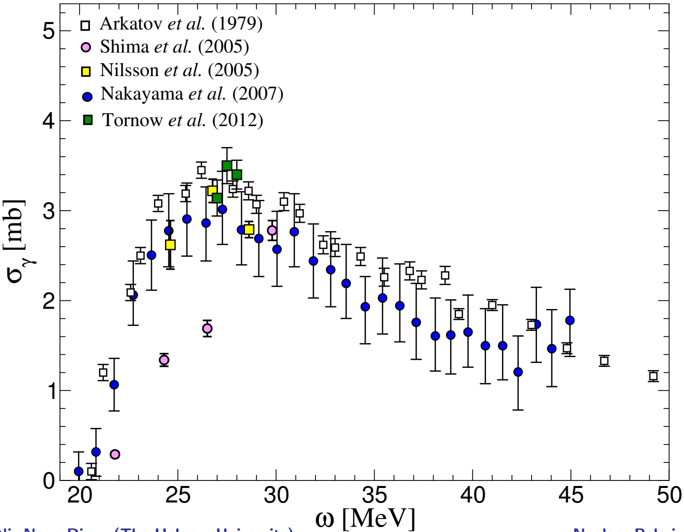
Nuclear potentials: Chiral-EFT

- **effective theory** of low-energy QCD
- **nuclear forces** are built in systematic expansions of Q/Λ
- **coupling constants** fitted to nuclear data



Nuclear potentials: ^{4}He Photoabsorption cross-section

electric dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



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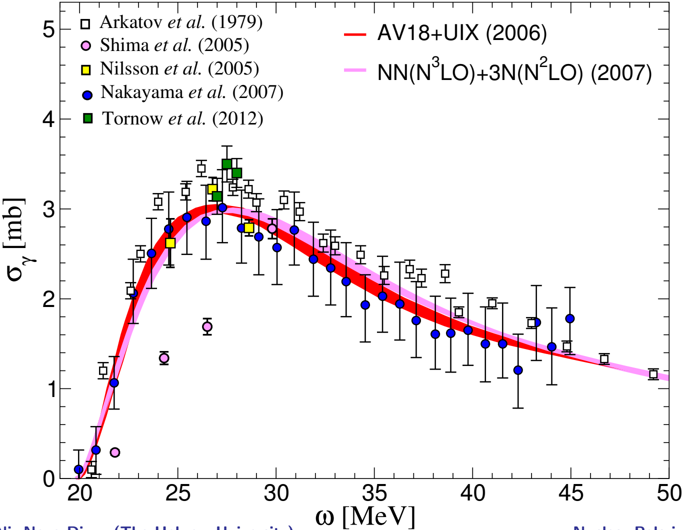
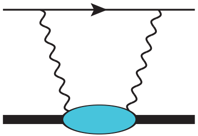


Nuclear polarization: basic idea

- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$
$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb from protons

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate inelastic effects of ΔH on muonic spectrum in 2^{nd}-order perturbation theory

$$\delta_{pol} = \sum_{N \neq N_0, \mu} \langle N_0 \mu_0 | \Delta H | N \mu \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - \epsilon_\mu} \langle N \mu | \Delta H | \mu_0 N_0 \rangle$$

$|\mu_0\rangle$: muon wave function for $2S/2P$ state

Nuclear polarization: contributions

Systematic contributions to nuclear polarization

- non-relativistic limit δ_{NR}
- longitudinal and transverse relativistic polarizations $\delta_L + \delta_T$
- Coulomb distortions δ_C
- corrections from finite nucleon size δ_{NS}

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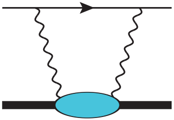
Non-relativistic limit

- **Neglect Coulomb interactions in the intermediate state**



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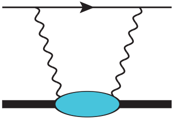

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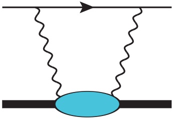

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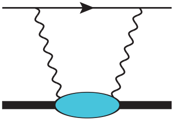
  - $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ "virtual" distance the proton travels in 2γ exchange
  - uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
  - $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \approx 0.17$ for $\boldsymbol{\mu}^4\mathbf{He}^+$

Non-relativistic limit



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  - $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \approx 0.17$ for $\boldsymbol{\mu}^4\mathbf{He}^+$

$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

- $\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} \Longrightarrow$ LO + NLO + N^{2}LO

NR limit at LO: $\delta_{NR}^{(0)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

$$\delta_{D1}^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

- $S_{D_1}(\omega) \Longrightarrow$ electric dipole response function $[\ \hat{D}_1 = R Y_1(\hat{R})\ ]$
- $\delta_{D1}^{(0)}$ is the dominant contribution to δ_{pol}

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- $\delta_{D1}^{(0)}$ is the dominant contribution to δ_{pol}
- $\Longrightarrow$ Rel. and Coulomb corrections added at this order

NR limit at NLO: $\delta_{NR}^{(1)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$

$$\boxed{\delta_{NR}^{(1)} = \delta_{R3pp}^{(1)} + \delta_{Z3}^{(1)}}$$

$$\delta_{R3pp}^{(1)} = -\frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R}d\boldsymbol{R}'|\boldsymbol{R} - \boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle$$

$$\delta_{Z3}^{(1)} = \frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R}d\boldsymbol{R}'|\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\rho_0(\boldsymbol{R}')$$

- $\delta_{R3pp}^{(1)} \Rightarrow$ 3rd-order proton charge correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd-order Zemach moment

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- $\delta_{R3pp}^{(1)} \Rightarrow$ 3rd-order proton charge correlation

- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd-order Zemach moment
  cancels Zemach moment in finite-size corrections
  c.f. Pachucki '11 & Friar '13 (μD)

NR limit at N^{2}LO: $\delta^{(2)}_{NR}$

$$\delta_{NR} = \delta^{(0)}_{NR} + \delta^{(1)}_{NR} + \delta^{(2)}_{NR}$$

- $\delta^{(2)}_{NR} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

$$\delta^{(2)}_{NR} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S_{R^2}(\omega) + \frac{16\pi}{25} S_Q(\omega) + \frac{16\pi}{5} \mathcal{S}_{D_1 D_3}(\omega) \right]$$

- $S_{R^2}(\omega) \Longrightarrow$ monopole response function
- $S_Q(\omega) \Longrightarrow$ quadrupole response function
- $S_{D_1 D_3}(\omega) \Longrightarrow$ interference between D_1 and D_3 $\quad [\ \hat{D}_3 = R^3 Y_1(\hat{R})\ ]$

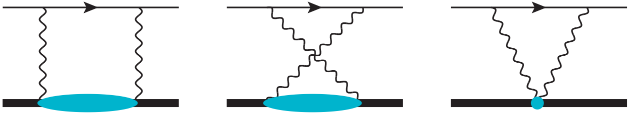
Nuclear polarization: contributions

Systematic contributions to nuclear polarization

- non-relativistic limit δ_{NR}
- **longitudinal and transverse relativistic polarizations** $\boldsymbol{\delta_L + \delta_T}$
- Coulomb distortions δ_C
- corrections from finite nucleon size δ_{NS}

Relativistic dipole polarization

We use the formalism of forward Compton scattering



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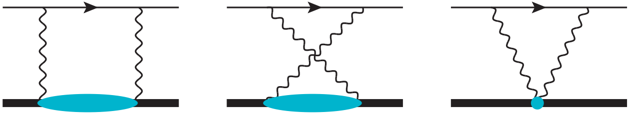

- Longitudinal contributions $\delta_L^{(0)}$
  - exchange Coulomb photon

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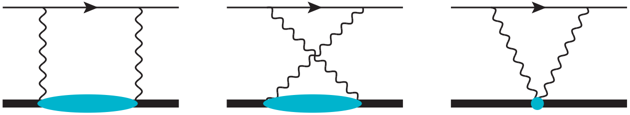

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- Transverse contributions $\delta_T^{(0)}$
  - convection current & spin current
  - seagull term: cancels infrared divergence restore gauge invariance

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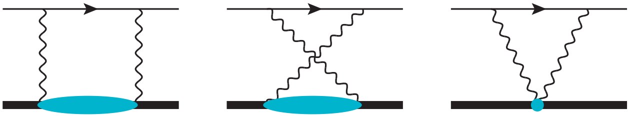

- Longitudinal contributions $\delta_L^{(0)}$
  - exchange Coulomb photon
- Transverse contributions $\delta_T^{(0)}$
  - convection current & spin current
  - seagull term: cancels infrared divergence restore gauge invariance
- $\delta_{L(T)}^{(0)}$ are sum rule of dipole response with different weights

$$\delta_{L(T)}^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}(\frac{\omega}{m_r})\, S_{D_1}(\omega)$$

Nuclear polarization: contributions

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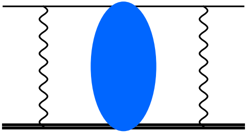
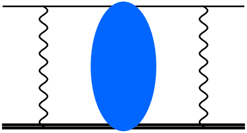
- **Coulomb distortions** $\boldsymbol{\delta_C}$
- corrections from finite nucleon size δ_{NS}

Coulomb distortion



- **Non-perturbative Coulomb interaction in intermediate state**

Coulomb distortion



- **Non-perturbative Coulomb interaction in intermediate state**
  - naive estimation: $\delta_C^{(0)} \sim (Z\alpha)^6$

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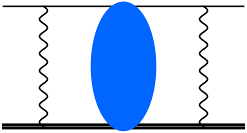

- **Non-perturbative Coulomb interaction in intermediate state**
  - naive estimation: $\delta_C^{(0)} \sim (Z\alpha)^6$
  - full analysis: logarithmically enhanced $\delta_C^{(0)} \sim (Z\alpha)^5 \ln(Z\alpha)$

Friar '77 & Pachucki '11

Nuclear polarization: contributions

Systematic contributions to nuclear polarization

- non-relativistic limit δ_{NR}
- longitudinal and transverse relativistic polarizations $\delta_L + \delta_T$
- Coulomb distortions δ_C
- **corrections from finite nucleon size** $\boldsymbol{\delta_{NS}}$

Finite nucleon size corrections

- In point-nucleon limit

$$\Delta H = -\alpha \sum_{i}^{Z} \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} + \frac{Z\alpha}{r}$$

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- consider finite nucleon sizes $\Longrightarrow$ include nucleon charge density

$$\Delta H = -\alpha \sum_{i}^{Z} \int d\boldsymbol{R}' \, \frac{n_p(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|} - \alpha \sum_{j}^{N} \int d\boldsymbol{R}' \, \frac{n_n(\boldsymbol{R}' - \boldsymbol{R}_j)}{|\boldsymbol{r} - \boldsymbol{R}'|} + \frac{Z\alpha}{r}$$

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- low-Q approximations of nucleon form factors

$$G_p^E(q) \simeq 1 - \frac{\langle r_p^2 \rangle}{6} q^2$$

$$G_n^E(q) \simeq -\frac{\langle r_n^2 \rangle}{6} q^2$$

LSR: Lanczos sum rule method

- Nuclear polarization $\Longrightarrow$ energy-dependent sum rules of the response functions

$$\delta_{pol} \propto I_O = \int_0^\infty d\omega \, S_O(\omega) \, g(\omega)$$

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- With the Lanczos sum rule (LSR) method, we directly calculate I_O, without explicitly solving S_O.

- The calculated I_O converges as the LIT of S_O, if $g(\omega)$ is smooth.

NND, Ji, Bacca, Barnea, arXiv:1403.7651 (2014)

Nuclear polarization in μD - Preliminary !!!

	Pachucki '11 (AV18)	our work (AV18)
$\delta^{(0)}_{D1}$	-1.910	-1.907
$\delta^{(0)}_{L}$	0.035	0.029
$\delta^{(0)}_{T}$	—	-0.012
$\delta^{(0)}_{C}$	0.261	0.262
$\delta^{(2)}_{R2}$	0.045	0.042
$\delta^{(2)}_{Q}$	0.066	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139
$\delta^{(1)}_{NS}$	—	0.017
$\delta^{(2)}_{NS}$	—	-0.015
δ_{M}	0.016	0.008
$\delta^{\star} = \delta^{A}_{pol} + \delta^{A}_{Zem}$	**-1.638**	**-1.656**

- compare: $\delta^{(0)}_{L}$ & $\delta^{(0)}_{T}$; $\delta^{(2)}$ & δ_M ; δ_{NS}

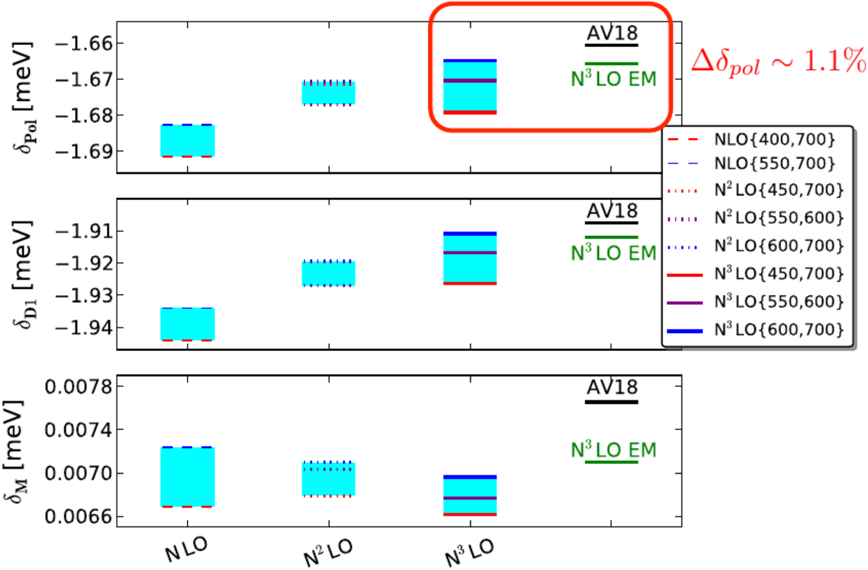
Nuclear polarization in μD - Preliminary !!!

	Pachucki '11	our work		
	(AV18)	(AV18)	N^3LO-EM	N^3LO-EGM
$\delta^{(0)}_{D1}$	-1.910	-1.907	-1.912	(-1.911,-1.926)
$\delta^{(0)}_{L}$	0.035	0.029	0.029	(0.029, 0.030)
$\delta^{(0)}_{T}$	–	-0.012	-0.012	-0.013
$\delta^{(0)}_{C}$	0.261	0.262	0.262	(0.262, 0.264)
$\delta^{(2)}_{R2}$	0.045	0.042	0.041	0.041
$\delta^{(2)}_{Q}$	0.066	0.061	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.151	-0.139	-0.139	(-0.139,-0.140)
$\delta^{(1)}_{NS}$	–	0.017	0.017	0.017
$\delta^{(2)}_{NS}$	–	-0.015	-0.015	-0.015
δ_M	0.016	0.008	0.007	0.007
$\delta^{\star} = \delta^A_{pol} + \delta^A_{Zem}$	**-1.638**	**-1.656**	**-1.661**	**(-1.660,-1.674)**

- compare: $\delta^{(0)}_L$ & $\delta^{(0)}_T$; $\delta^{(2)}$ & δ_M ; δ_{NS}

Improved nuclear uncertainty in μD

Hernandez, Ji, S.B., Nevo-Dinur, Barnea, in preparation



$\Delta\delta_{pol} \sim 1.1\%$

Nuclear polarization in $\mu^4\mathrm{He}^+$

[meV]		AV18/UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546

★ NN: N^{3}LO-EM
$3N$: N^{2}LO (c_D=1, c_E=-0.029)

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	$\delta^{(2)}_{Q}$	0.484	0.561
	$\delta^{(2)}_{D1D3}$	-0.666	-0.784

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δ_{NS}	$\delta^{(1)}_{R1pp}$	-1.036	-1.071
	$\delta^{(1)}_{Z1}$	1.753	1.811
	$\delta^{(2)}_{NS}$	-0.200	-0.210

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δ_{pol}		-2.408	-2.542

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Nuclear polarization in $\mu^4\mathrm{He}^+$

[meV]	AV18/UIX	χEFT*
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$
  in a systematic expansion of
  $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{m_r/M_N} \approx 0.17$

$\star$ NN: N^{3}LO-EM
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- δ_{pol} with AV18/UIX & χEFT differ: $\sim 5.5\%$ (0.134 meV)

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Nuclear physics uncertainty

$^4\mathrm{He}$ obesrvable		AV18/UIX	χEFT	Difference
$\boldsymbol{\mu^4\mathrm{He}^+}$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

Nuclear physics uncertainty

$^4\mathrm{He}$ obesrvable		AV18/UIX	χEFT	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton nuclear radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\boldsymbol{\mu}^4\mathbf{He}^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

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- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

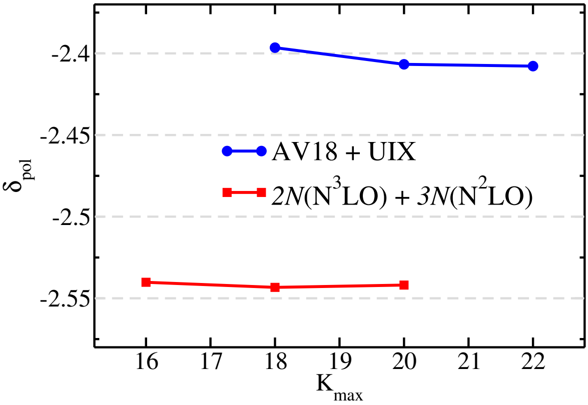
- systematic uncertainty in δ_{pol} from nuclear physics:
  $\frac{5.5\%}{\sqrt{2}} \implies \pm 4\% \ (1\sigma)$

Numerical accuracy

- **Convergence with model space size**

  Compare $\delta_{pol}^{(K_{max})}$ with $\delta_{pol}^{(K_{max}-4)}$

  - AV18/UIX $\sim 0.4\%$
  - χEFT $\sim 0.2\%$



Atomic physics uncertainty

- **Additional corrections**
  - $(Z\alpha)^6$ effects (beyond 2nd-order perturbation theory)
  - relativistic & Coulomb corrections to other multipoles (other than dipole)
  - higher-order nucleon-size corrections
  - combine these corrections $\Longrightarrow$ an additional few percent error

Nuclear polarization in $\mu^4\mathrm{He}^+$

Final result - δ_{pol} in $\mu^4\mathrm{He}^+$

- combine all errors in a quadratic sum
- our prediction: $\delta_{pol} = -2.47$ meV $\pm 6\%$
- more accurate than early calculations: $\delta_{pol} = -3.1$ meV $\pm 20\%$
  Bernabeu & Jarlskog '74; Rinker '76; Friar '77
- our accuracy is comparable to the 5% requirement for current $\boldsymbol{\mu^4\mathrm{He}^+}$ Lamb shift measurement
  Antognini *et al.* '11

Work in progress

The work is not completed yet ...

Preliminary: δ_{pol} in $\mu^3\mathrm{He}^+$

[meV]	AV18/UIX	χEFT★
$\delta^{(0)}$	-5.361	-5.468
$\delta^{(1)}$	-0.460	-0.387
$\delta^{(2)}$	0.841	0.887
δ_{NS}	0.797	0.805
δ_{Mag}	0.078	(0.066)
δ_{pol}	-4.104	-4.097

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?
- δ_{pol} with AV18/UIX & χEFT agree:
  $< 1\%$ (~ 0.01 meV)
  c.f. $\mu^4\mathrm{He}^+$:
  -2.475 ± 0.134 meV

★ NN: $\mathrm{N^3LO}$-EM
$3N$: $\mathrm{N^2LO}$ (c_D=1, c_E=-0.029)

Preliminary: δ_{pol} in μT

[meV]	AV18/UIX
$\delta^{(0)}$	-0.680
$\delta^{(1)}$	0.178
$\delta^{(2)}$	-0.025
δ_{NS}	0.053
δ_{Mag}	0.010
δ_{pol}	-0.465

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$?
- δ_{pol} with χEFT ?

Summary

- **Lamb shifts in muonic atoms**
  - raise interesting questions about lepton universality
  - probe isospin dependence of the proton radius puzzle
  - allow high precision determination of the nuclear charge radius $\langle \boldsymbol{r^2} \rangle$
  - For $A > 1$ the precision of $\langle \boldsymbol{r^2} \rangle$ is bound by the nuclear polarization δ^A_{pol}

- **We perform the first *ab-initio* calculation of $\boldsymbol{\delta^A_{pol}}$ in $\boldsymbol{\mu^{3,4}\mathrm{He}^+}$ & $\boldsymbol{\mu\mathrm{T}}$, and improve the nuclear uncertainty in $\boldsymbol{\mu}$ D**

$\mu\mathrm{D}$	$\delta^\star = -1.66$	meV	$\pm 1.5\%$	(preliminary)
$\mu\mathrm{T}$	$\delta_{\mathrm{pol}} = -0.46$	meV	$\pm 3\%$	(preliminary)
$\mu^3\mathrm{He}^+$	$\delta_{\mathrm{pol}} = -4.1$	meV	$\pm 3\%$	(preliminary)
$\mu^4\mathrm{He}^+$	$\delta_{\mathrm{pol}} = -2.47$	meV	$\pm 6\%$	(Ji *et al.* PRL '13)

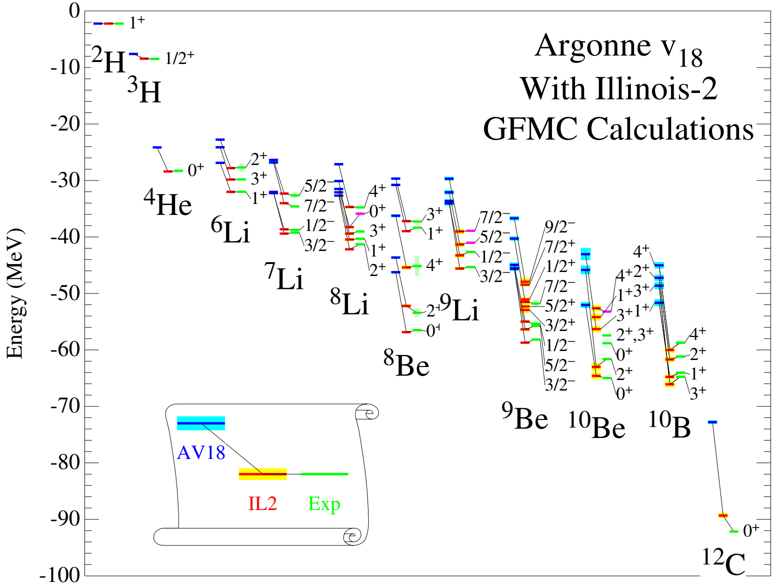
  - more accurate than previous calculations
  - will significantly improve the precision of $\langle \boldsymbol{r^2} \rangle$ extracted from ongoing $\boldsymbol{\mu^{3,4}\mathrm{He}^+}$ Lamb shift measurements

Outlook

- Study higher-order terms
- Reduce nuclear physics uncertainty
  - understand why various nuclear potentials differ
  - further explore the various parameterizations (3NF?)
  - include higher-order or otherwise improved χEFT forces
- Investigate nuclear polarization in *e.g.* $\boldsymbol{\mu^6\mathrm{Li}^{+2}},\ \boldsymbol{\mu^6\mathrm{He}^{+}}, \cdots$
- Investigate nuclear polarization in HFS of electronic and muonic atoms

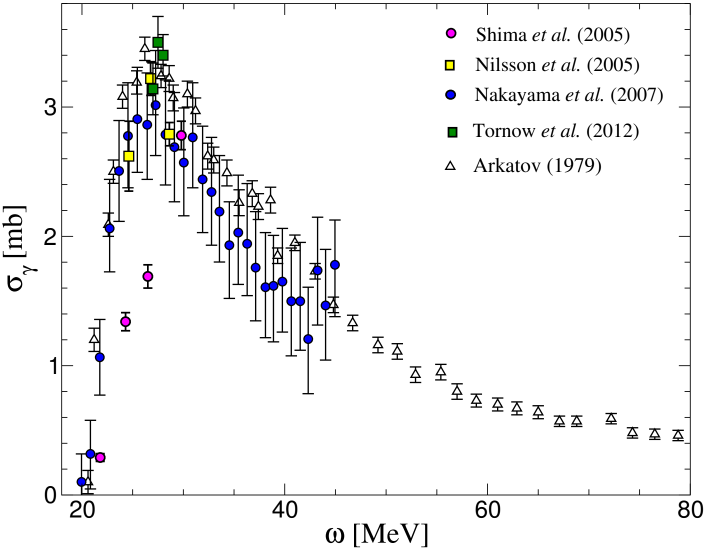
Happy Shavuot!

BACK UP


Argonne v18
With Illinois-2
GFMC Calculations
Energy (MeV)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
2H
3H
4He
6Li
7Li
8Li
8Be
9Li
9Be
10Be
10B
12C
AV18
IL2
Exp

^{4}He photoabsorption cross sections

electric-dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



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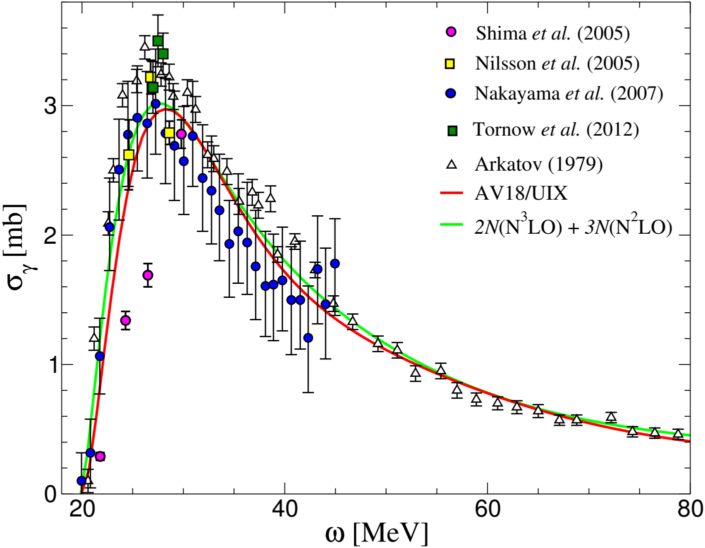

Non-Relativistic Approximation

- Coulomb interactions in the (virtual) intermediate state are neglected
- Results are expressed in an expansion of $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

  $\Delta E_{NR} = \Delta E_{NR}^{(2)} + \Delta E_{NR}^{(3)} + \Delta E_{NR}^{(4)}$

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1. $\sim R^2$ **term:**

- $\Delta E_{NR}^{(2)}$ **is the dominant polarizability contribution**

$$\Delta E_{NR}^{(2)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\mathrm{th}}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S^{D_1}(\omega)$$

- $S^{D_1}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{D}_1 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$
- $\hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

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2. $\sim R^3$ **term:**

$$\Delta E_{NR}^{(3)} = -\frac{m_r^4}{24}(Z\alpha)^5\left[\iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \langle N_0|\hat{\rho}^\dagger(\boldsymbol{R})\hat{\rho}(\boldsymbol{R}')|N_0\rangle - \langle r^3\rangle_{(2)}\right]$$

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- 2nd term: Zemach moment
  $\langle r^3\rangle_{(2)} = \iint d^3R d^3R' |\boldsymbol{R}-\boldsymbol{R}'|^3 \rho_0(\boldsymbol{R})\, \rho_0(\boldsymbol{R}')$
  $\rho_0(\boldsymbol{R}) = \langle N_0|\hat{\rho}(\boldsymbol{R})|N_0\rangle$

  cancels exactly the **Zemach term** in (elastic) finite-size corrections
  c.f. Pachucki PRL 2011 (μD)

Non-Relativistic Approximation

3. $\sim R^4$ **term:**

- $\Delta E_{NR}^{(4)}$ **corresponds to higher-multipole corrections**

$$\Delta E_{NR}^{(4)} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S^{Q_0}(\omega) + \frac{16\pi}{25} S^{Q_2}(\omega) + \frac{16\pi}{5} S^{D_{13}}(\omega) \right]$$

- $S^{R^2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{R}^2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$

  $S^{Q_2}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle N_0 J_0 || \hat{Q}_2 || N J \rangle|^2 \delta(\omega - E_N + E_{N_0})$

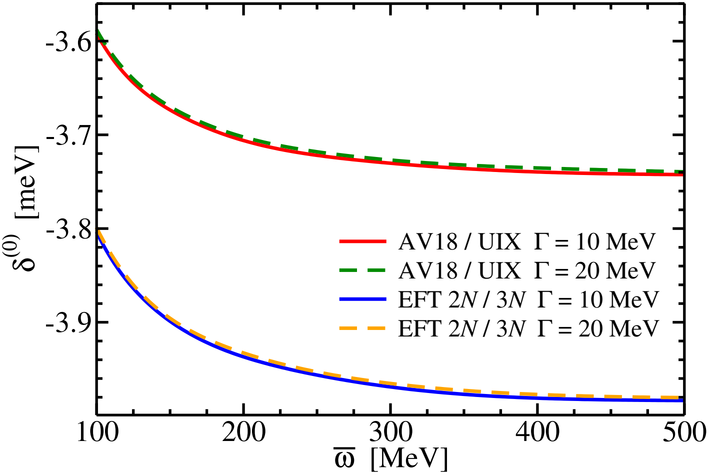
  $$S^{D_{13}}(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} (-1)^{J_0 - J} \times \mathrm{Re} \left(\langle N_0 J_0 || \hat{D}_3 || N J \rangle \langle N J || \hat{D}_1 || N_0 J_0 \rangle \right) \delta(\omega - E_N + E_{N_0})$$

- $\hat{R}^2 = \frac{1}{Z} \sum_i^Z R_i^2$ $\qquad$ $\hat{D}_1 = \frac{1}{Z} \sum_i^Z R_i Y_1(\hat{R}_i)$

  $\hat{Q}_2 = \frac{1}{Z} \sum_i^Z R_i^2 Y_2(\hat{R}_i)$ $\qquad$ $\hat{D}_3 = \frac{1}{Z} \sum_i^Z R^3 Y_1(\hat{R}_i)$

Convergence of Ab-initio calculations

Convergence of $\bar{\omega}$



Convergence of Ab-initio calculations

$\delta^{(0)}$ convergence with the largest model space K_{max}