JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Lattice form factor activities in Mainz

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Motivation

- Baryonic form factors
  - provide information on hadron structure
    - distribution of electric charge and magnetisation
    - charge radii
  - accurate experimental data available
  - relatively simple to compute on the lattice
    - large systematic uncertainties remain and need to be controlled

Form factors

- Rosenbluth formula describes electron-nucleon scattering

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \propto \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right], \quad \tau = \frac{Q^2}{4M^2}$$

- Form factors measured experimentally
  - e.g at MAMI here in Mainz

Form factors

- The matrix element of a nucleon interacting with an electromagnetic current is decomposed by the Dirac and Pauli form factors - F_1 and F_2 respectively

$$\langle N(p', s')|V_\mu|N(p, s)\rangle = \overline{u}(p', s') \left[\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N} F_2(Q^2)\right] u(p, s)$$

- These are related to the Sachs form factors G_E and G_M that are measured in scattering experiments

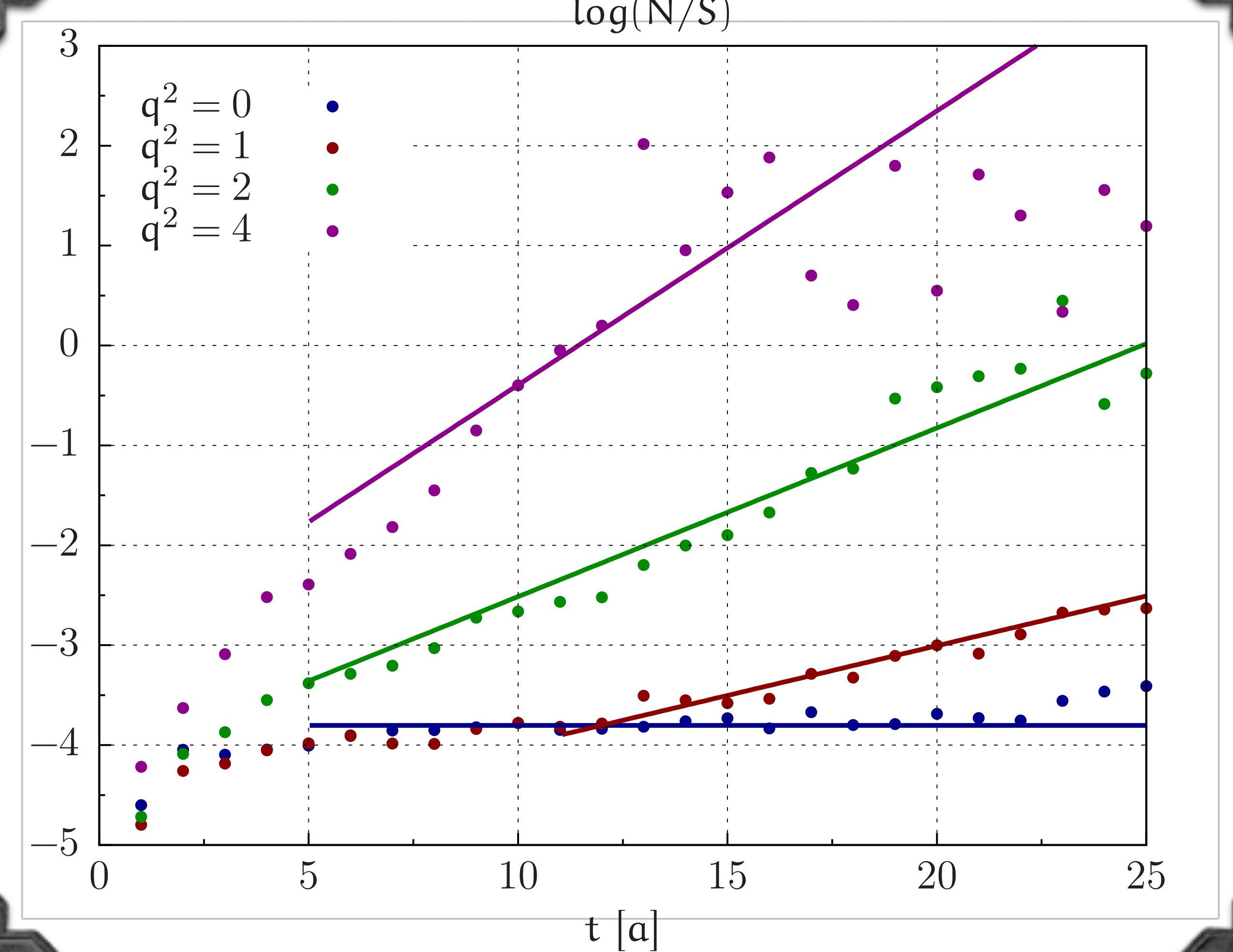
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

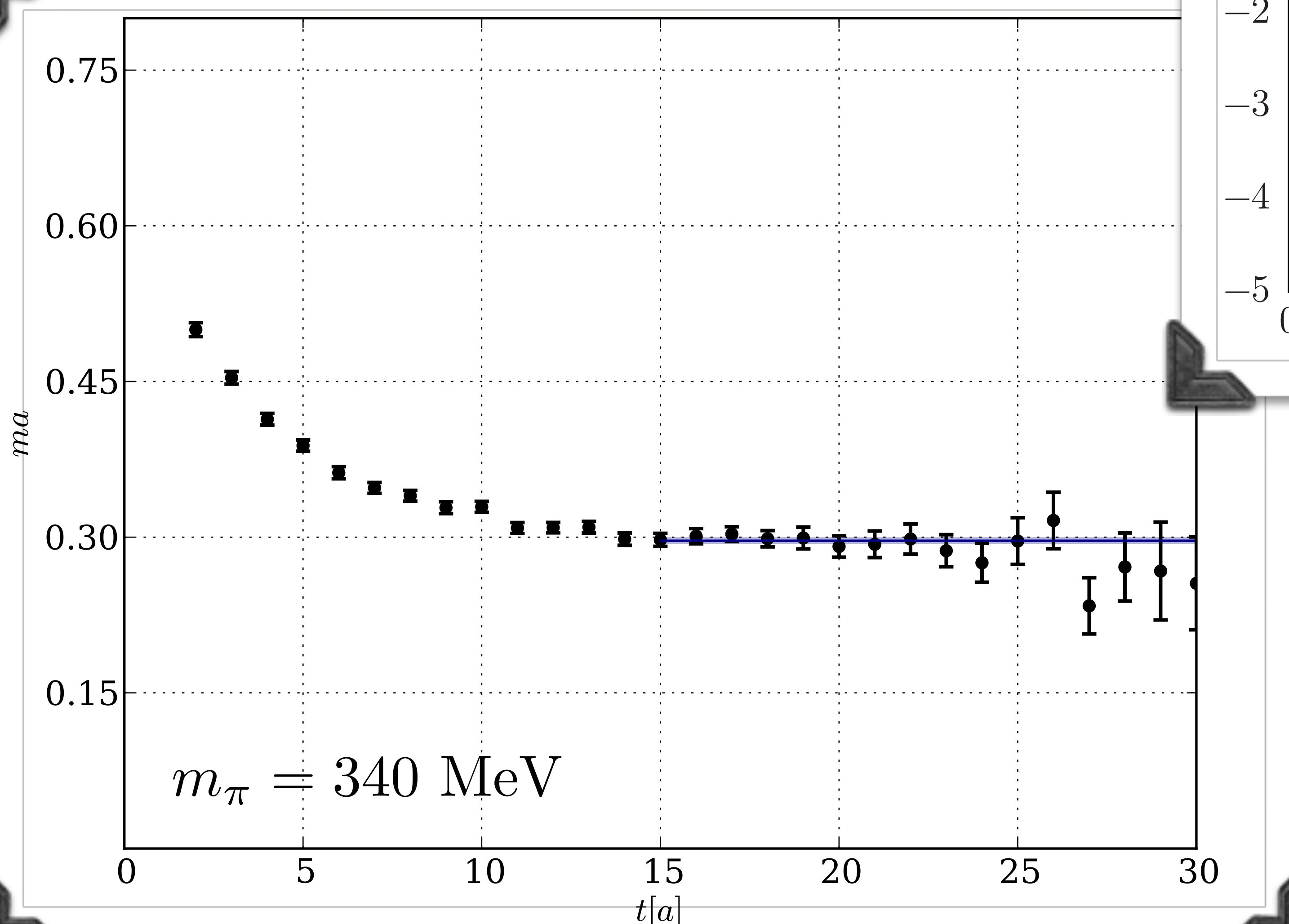
Understanding nucleon structure from first principles

- Systematic effects not fully controlled
  - Lattice artefacts
  - Chiral extrapolation to physical pion mass
  - Finite-volume effects
  - “Contamination” from excited states
  - Quark-disconnected diagrams ignored
- Perform a systematic study of the form factors with controlled systematics

Baryon correlation functions

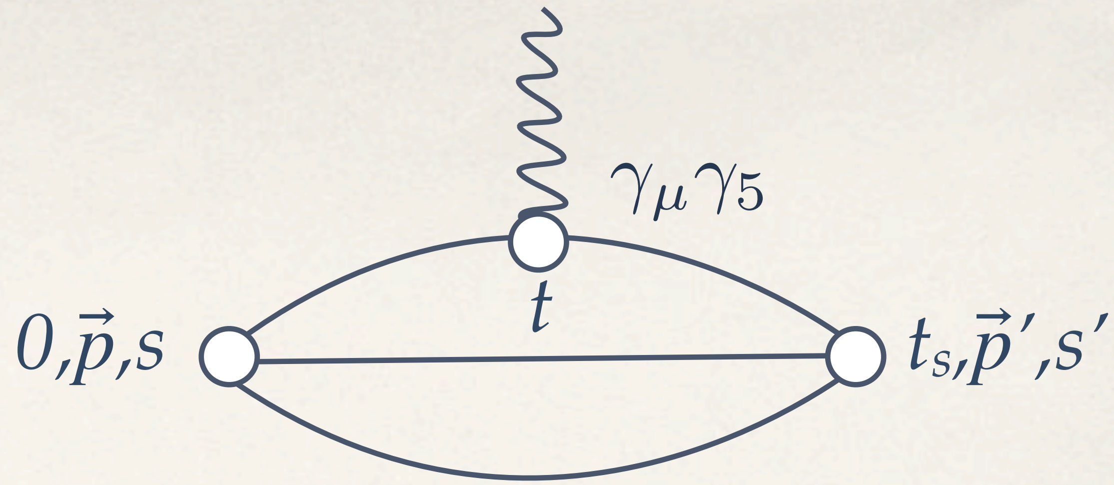
- Exponentially increasing noise-to-signal ratio





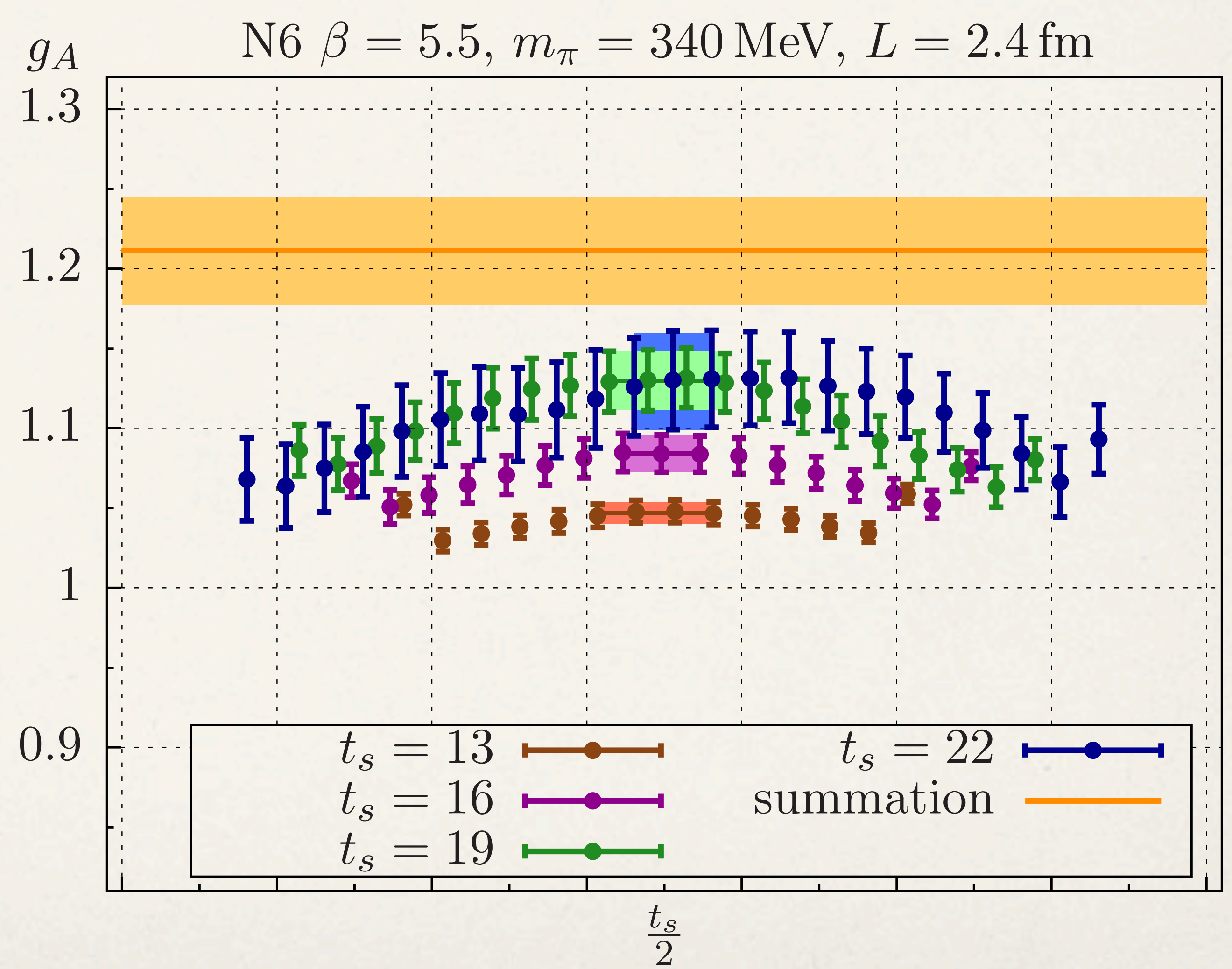
- Provides a challenge for accurate calculations of baryon form factors

Lattice formulation

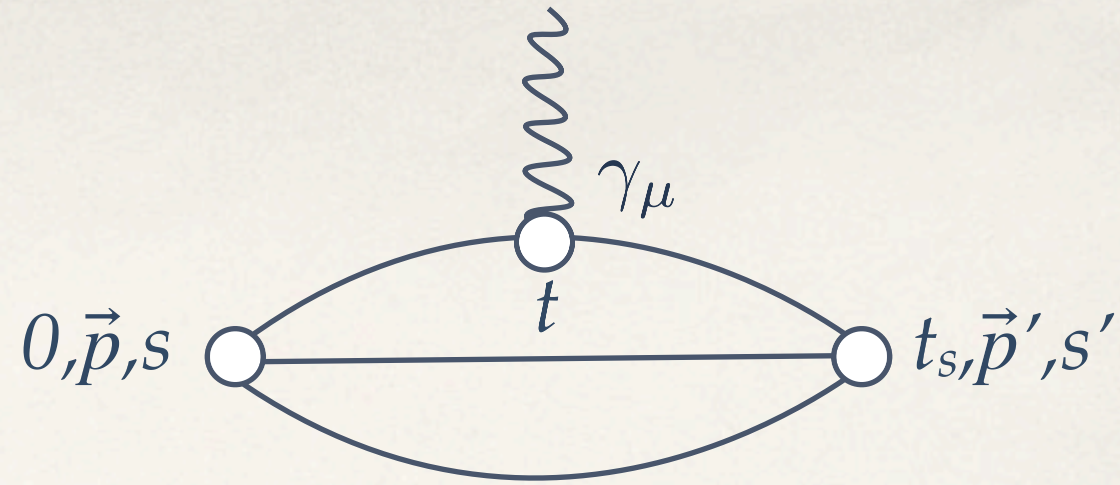


$$R_A(\vec{q}=0,t,t_s) = \frac{C_3(\vec{q}=0,t,t_s)}{C_2(\vec{q}=0,t,t_s)} \propto g_A + \mathcal{O}(e^{-\Delta t}, e^{-\Delta(t-t_s)})$$

- Plateau method
- Extract nucleon hadronic matrix elements from ratios of three- and two-point functions
- Form factors should be independent of time and source position

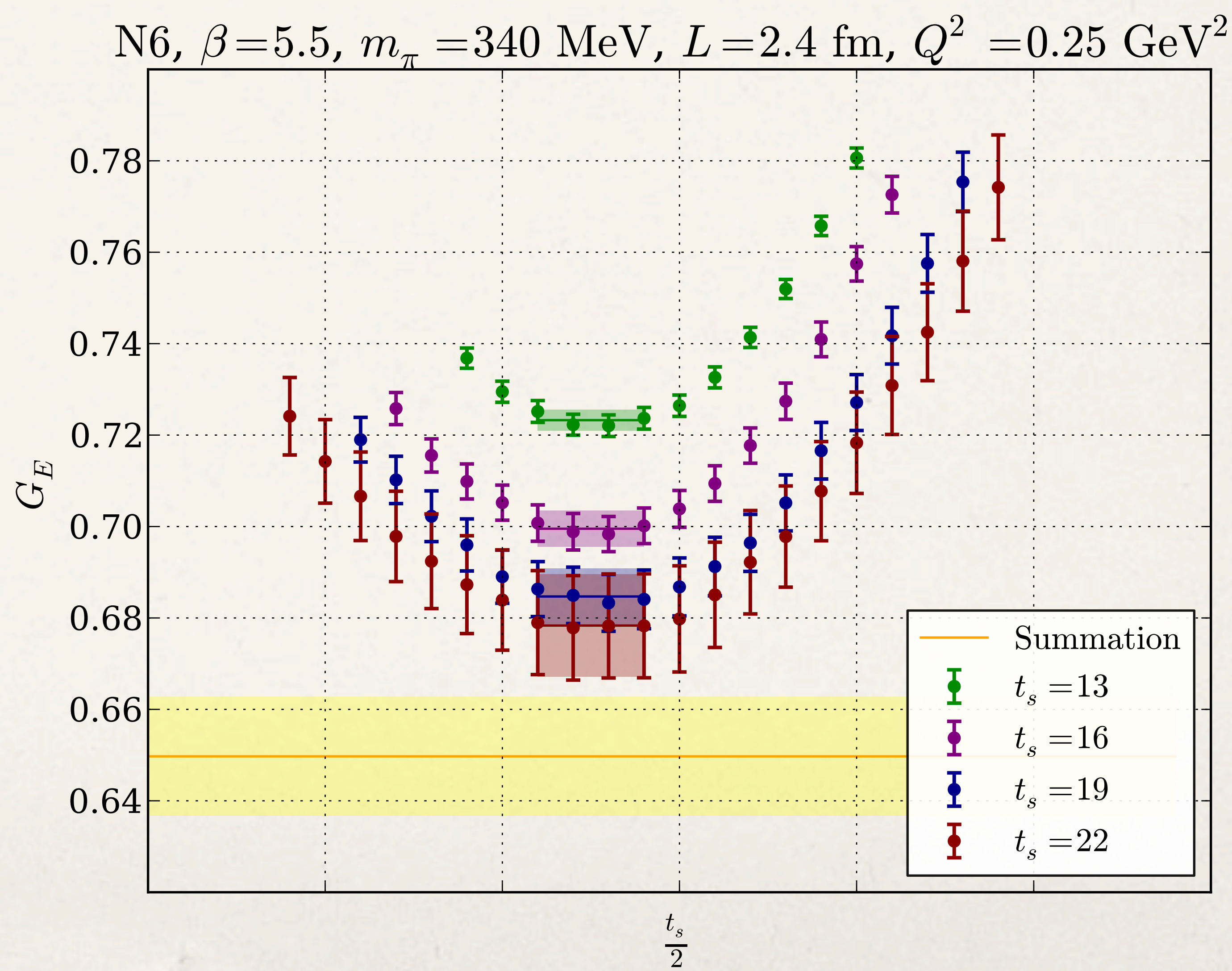


Lattice formulation



$$R_V(\vec{q},t,t_s) = \frac{C_3(\vec{q},t,t_s)}{C_2(\vec{q},t,t_s)}\sqrt{\frac{C_2(\vec{q},t_s-t)C_2(\vec{0},t)C_2(\vec{0},t_s)}{C_2(0,t_s-t)C_2(\vec{q},t)C_2(\vec{q},t_s)}} \propto G_E(Q^2), G_M(Q^2)$$

- Plateau method
- Extract nucleon hadronic matrix elements from ratios of three- and two-point functions
- Form factors should be independent of time and source position

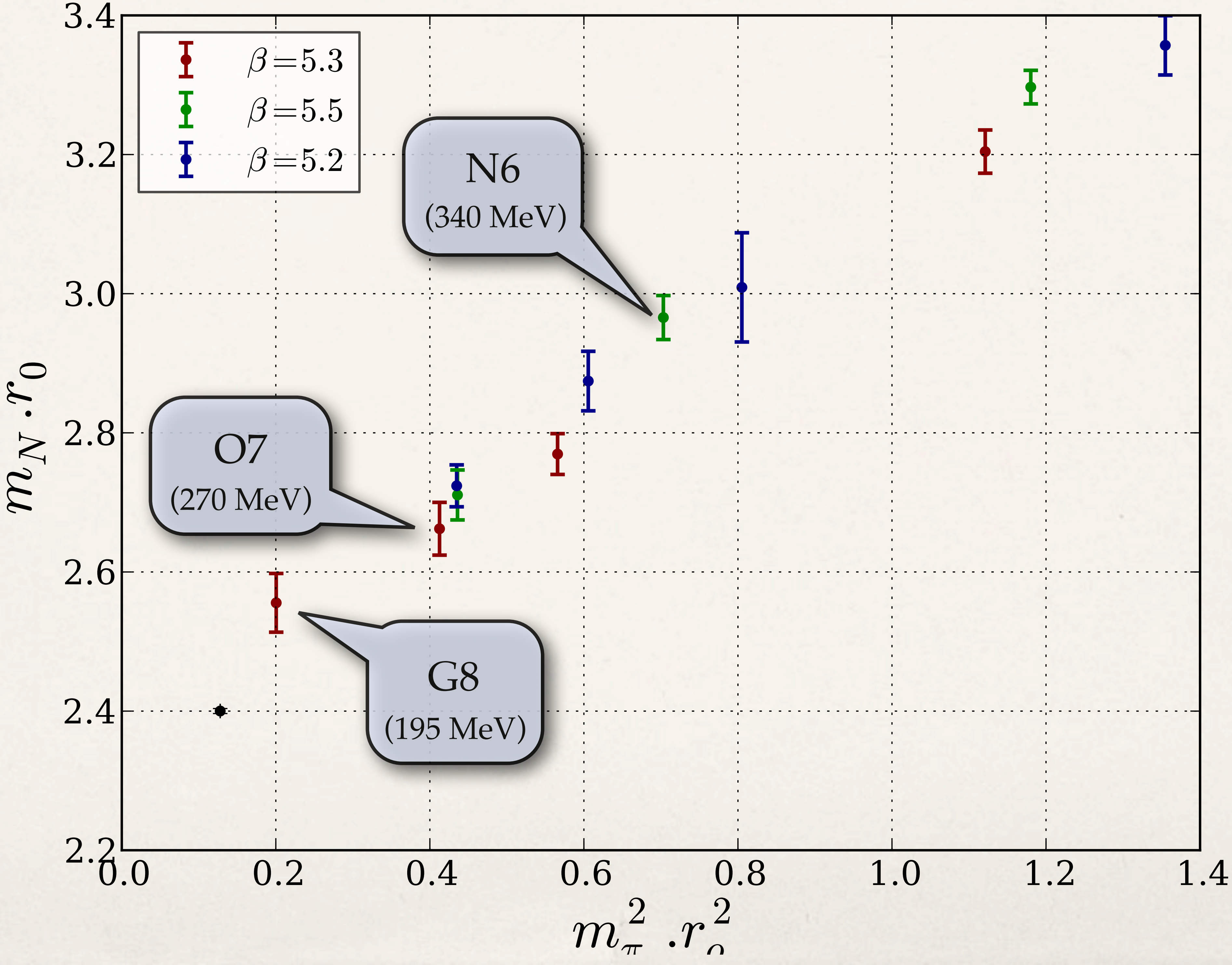


Lattice ensembles

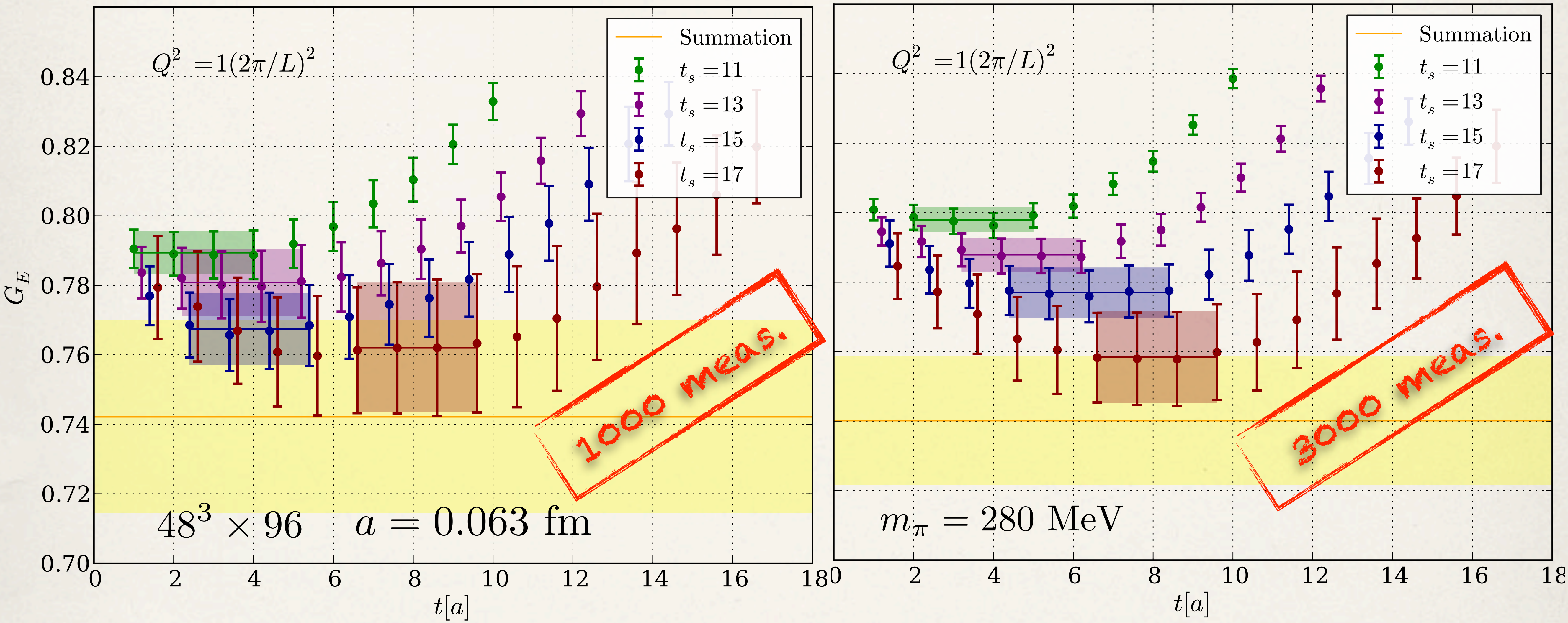
Run	β	a [fm]	$L^3 \times T$	m_π [MeV]	L [fm]	$m_\pi L$ [MeV]	N_{meas}
A3	5.2	0.079	$32^3\times64$	473	2.5	6.0	2128
A4				363	2.5	4.7	3200
A5				312	2.5	4.0	4000
B6			$48^3\times96$	262	3.8	5.0	2544
E5	5.3	0.063	$32^3\times64$	451	2.0	4.7	4000
F6			$48^3\times96$	324	3.0	5.0	3600
F7				277	3.0	4.2	3000
G8			$64^3\times128$	195	4.0	4.0	4176
N5	5.5	0.050	$48^3\times96$	430	2.4	5.2	1908
N6				340	2.4	4.0	3784
O7			$64^3\times128$	270	3.2	4.4	1960

- $N_f = 2$ non-perturbatively $O(a)$ improved Wilson fermions

Lattice ensembles



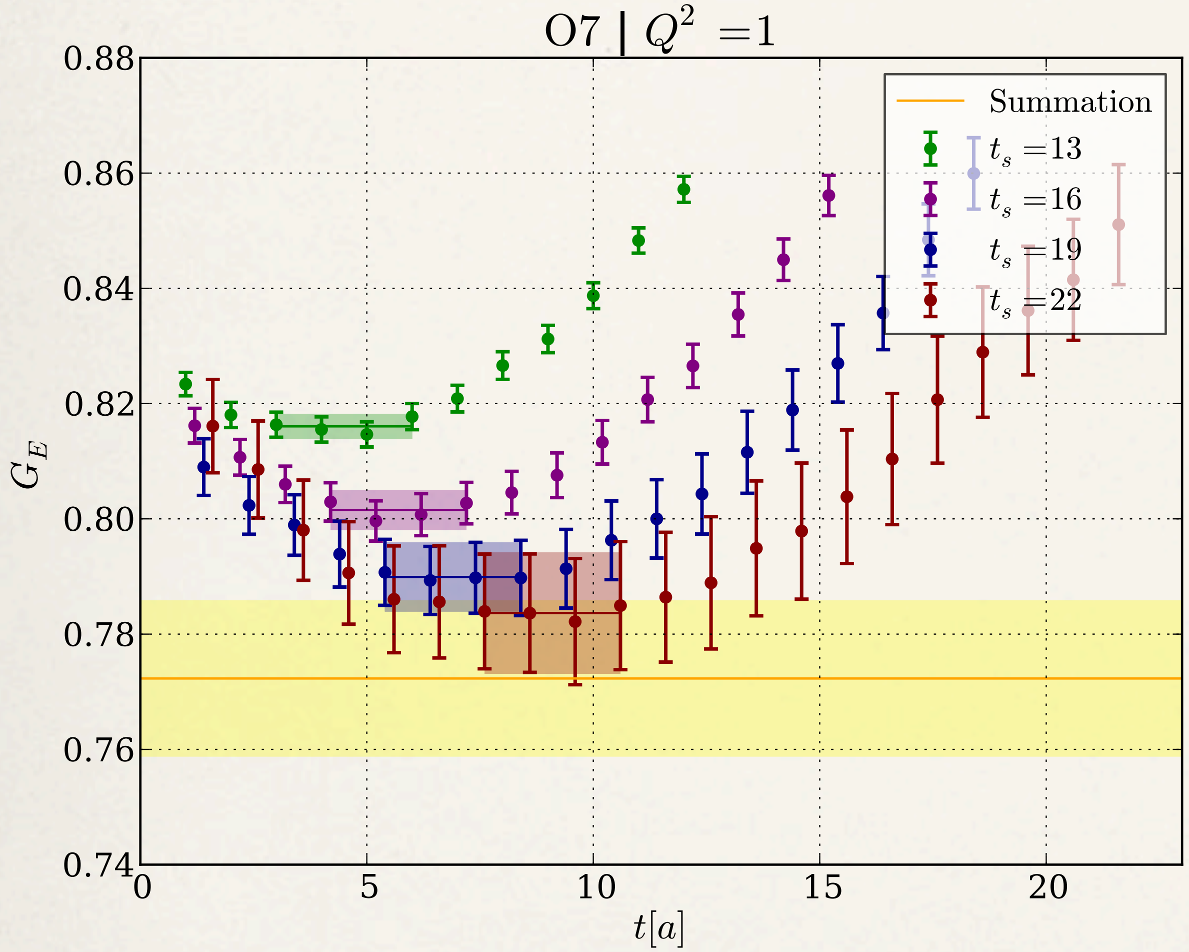
Form factor extraction

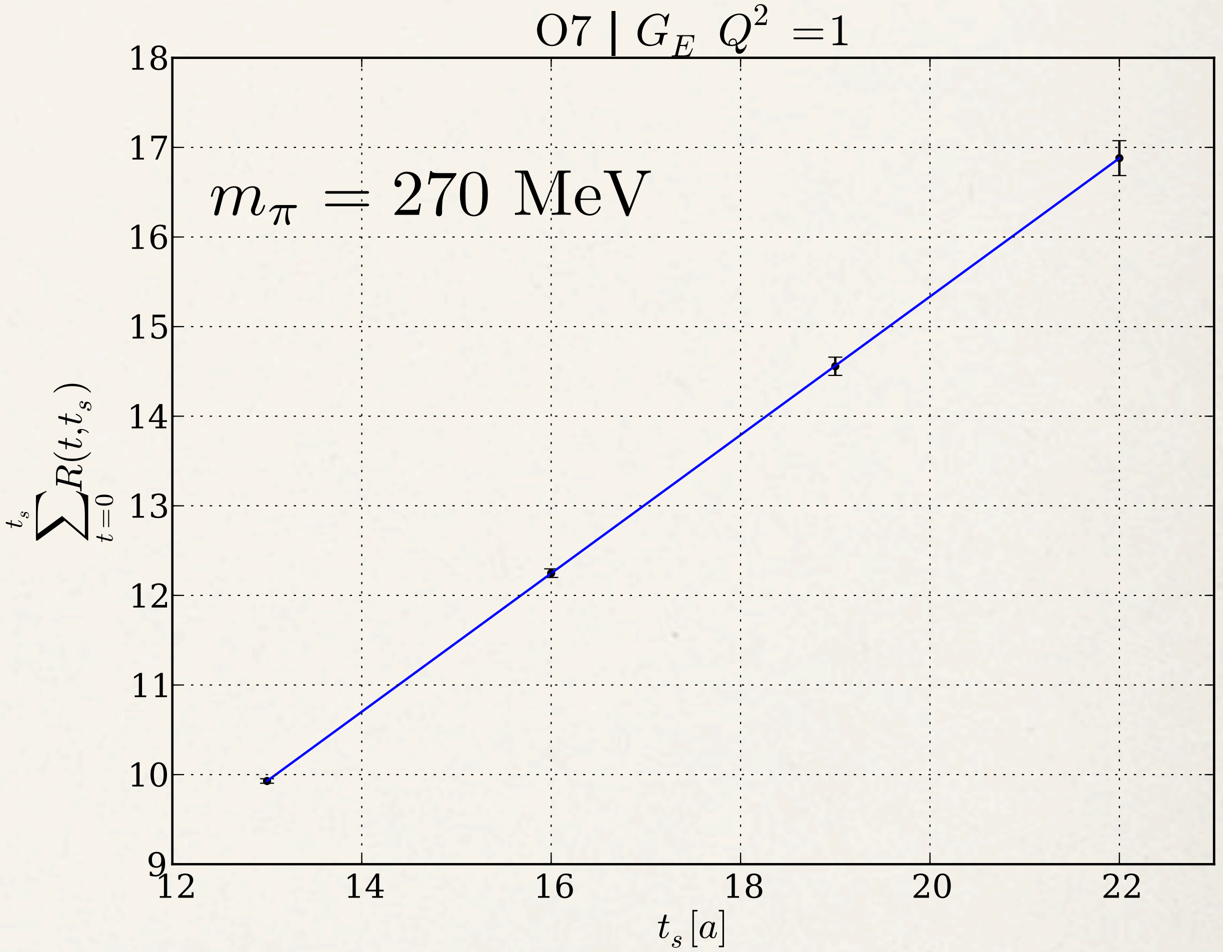


- Statistically demanding calculation - requires many measurements
- Unclear as to whether t_s=1.1 fm is sufficient to rule out bias

Summation method

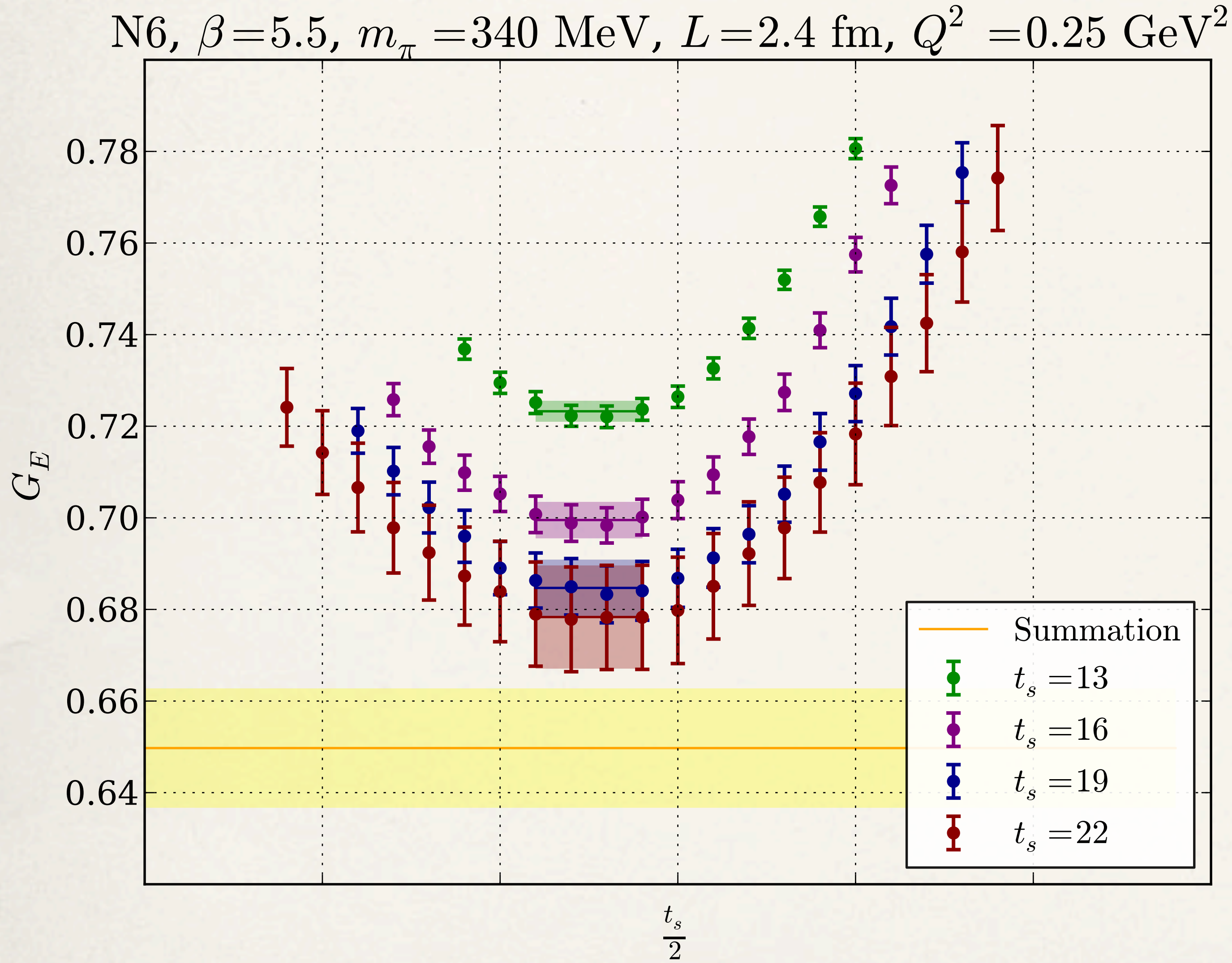
$$R(t,t_s) = G + \mathcal{O}(e^{-\Delta t}, e^{-\Delta(t-t_s)})$$

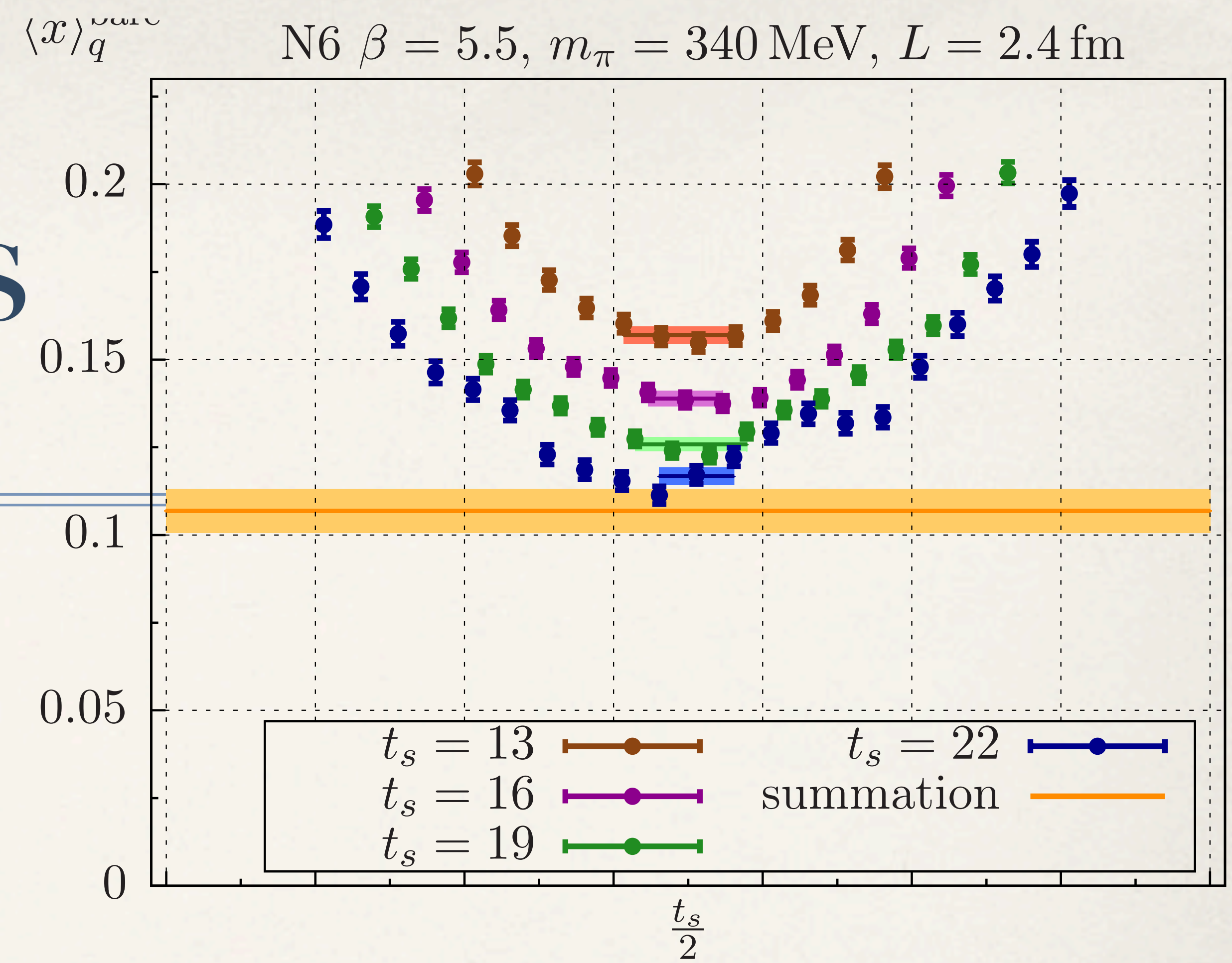


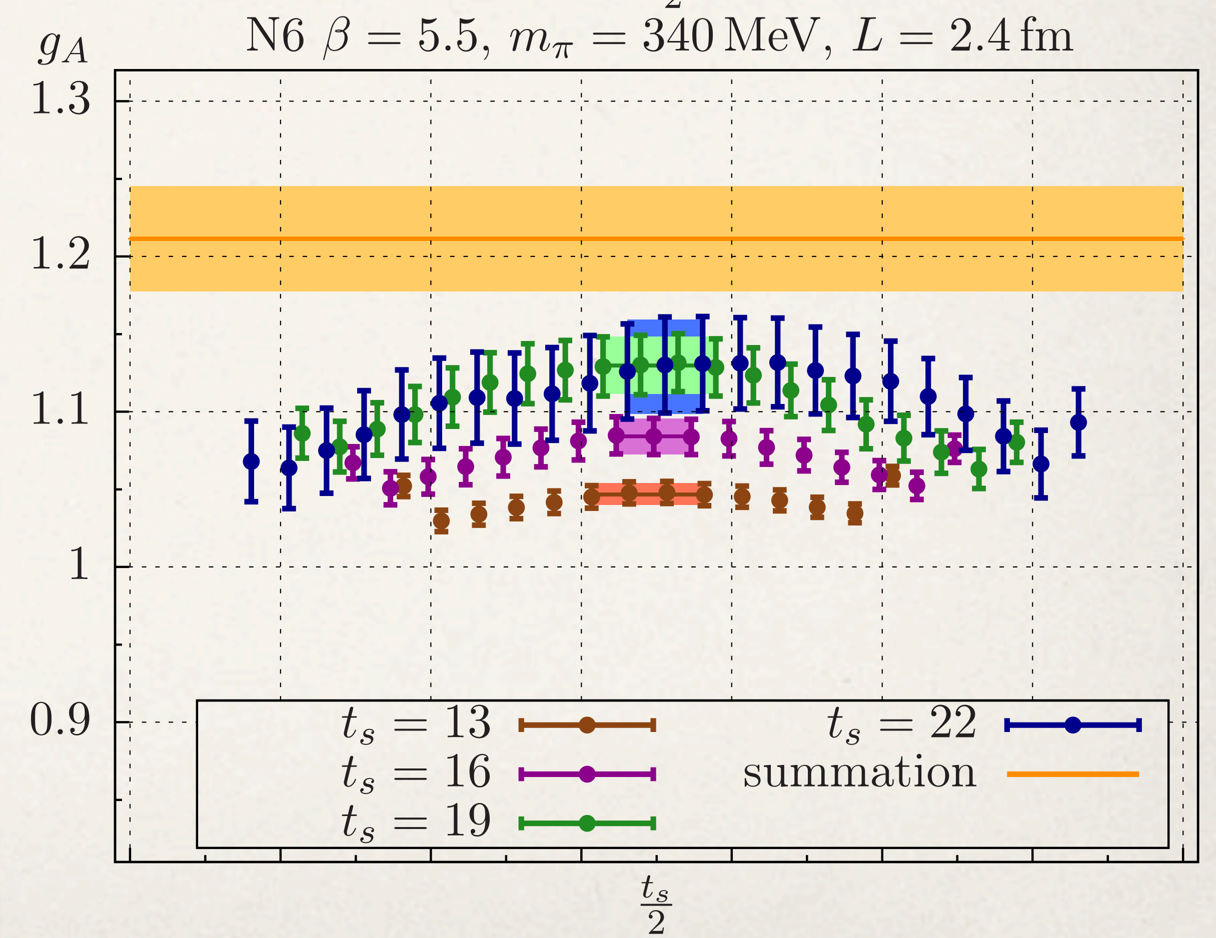


$$S(t_s) = \sum_{t=0}^{t_s} R(\vec{q},t,t_s) \to c(\Delta,\Delta') + t_s\left(G + \mathcal{O}\left(e^{-\Delta t_s}\right) + \mathcal{O}\left(e^{-\Delta' t_s}\right)\right)$$

Summation results







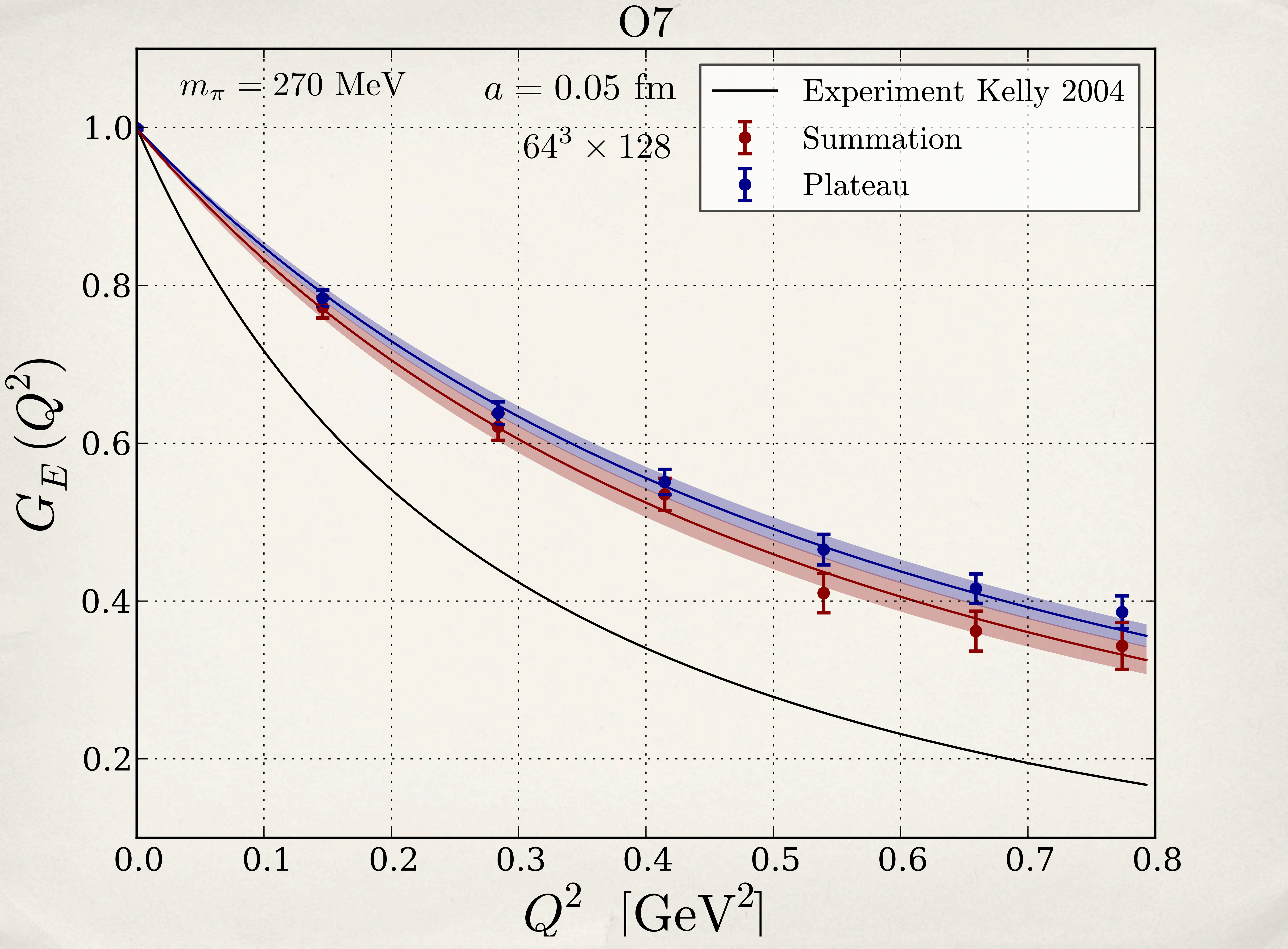
Vector form factors

- Model the Q^2 dependence
  - dipole ansatz:

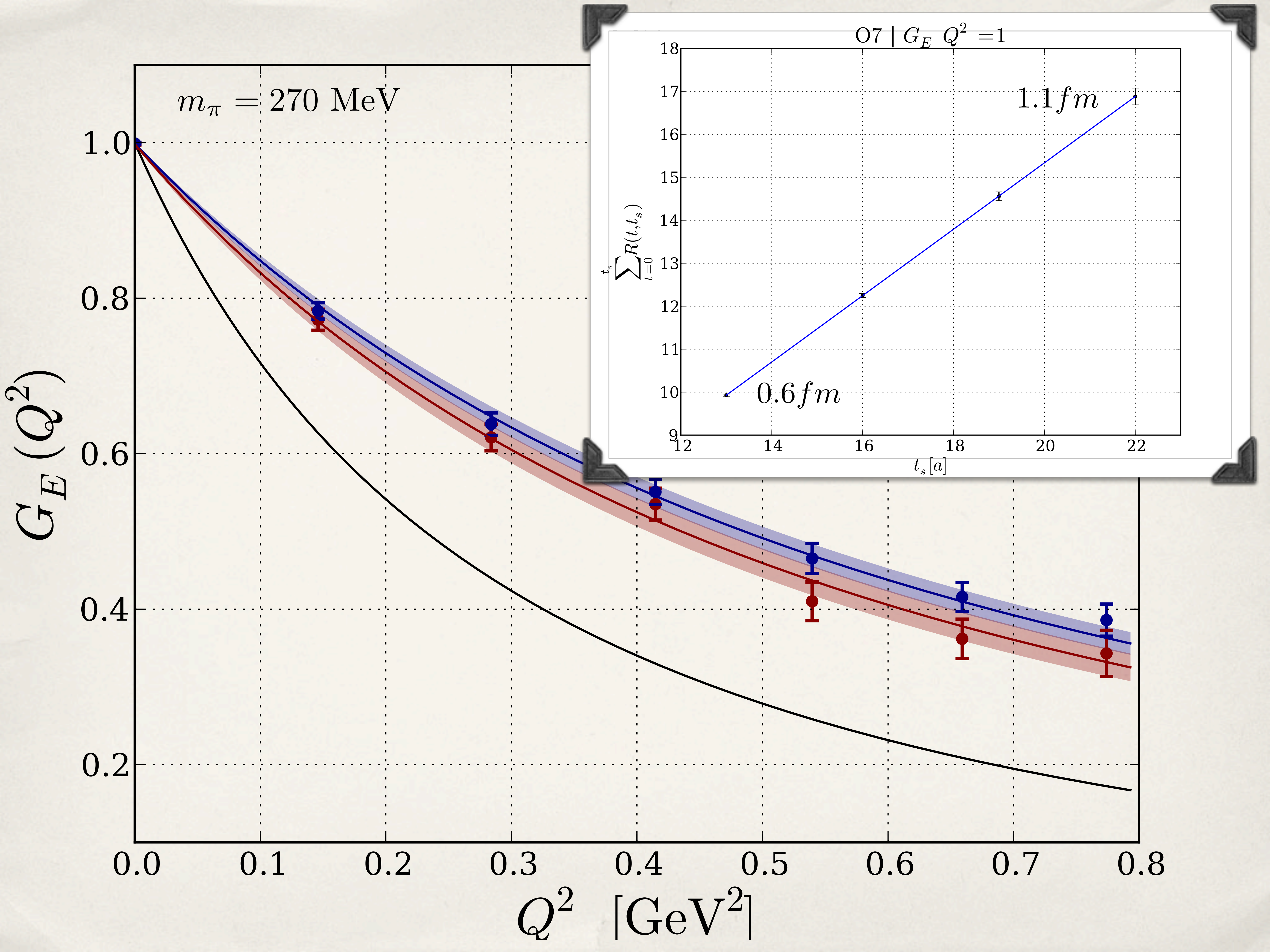
$$G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{\left(1 + \frac{Q^2}{M_{E,M}^2}\right)}$$

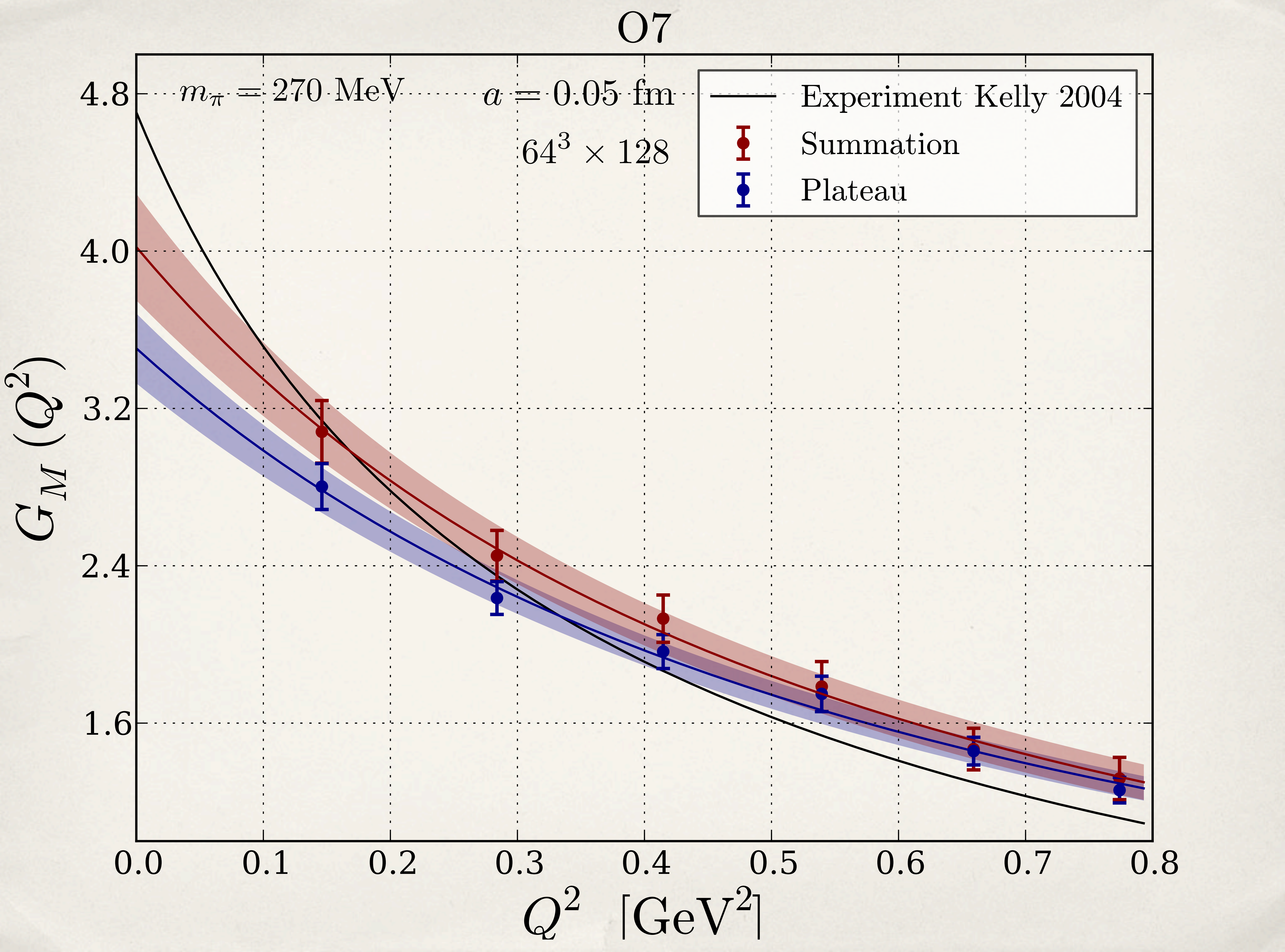
  - used to determine the radius
  - and to determine the magnetic moment, $\mu = G_M(0)$

$$\mu = \lim_{Q^2 \to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$$

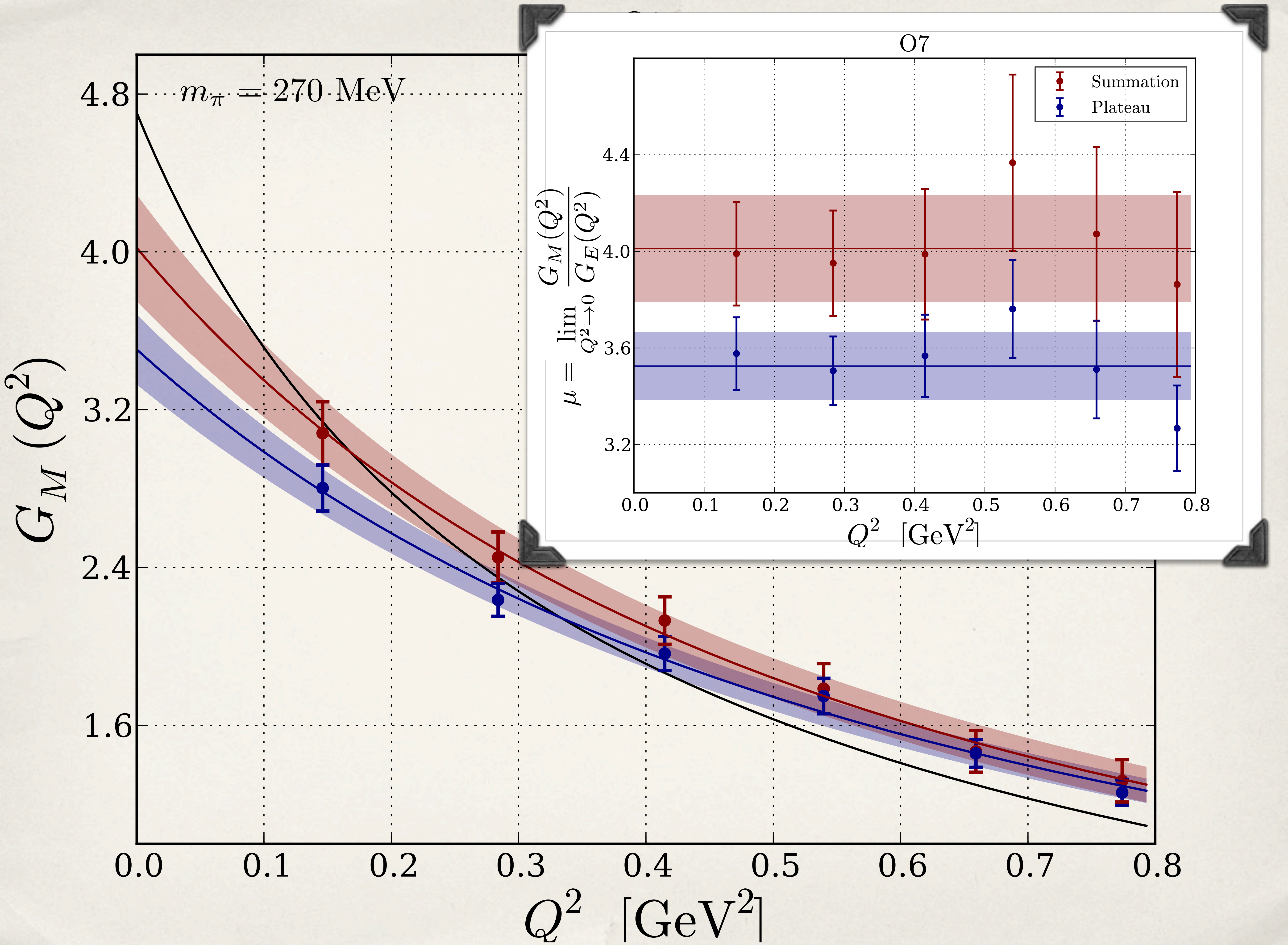


O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
Experiment Kelly 2004
Summation
Plateau
$G_E(Q^2)$
Q^2 [GeV^2]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.2
0.4
0.6
0.8
1.0

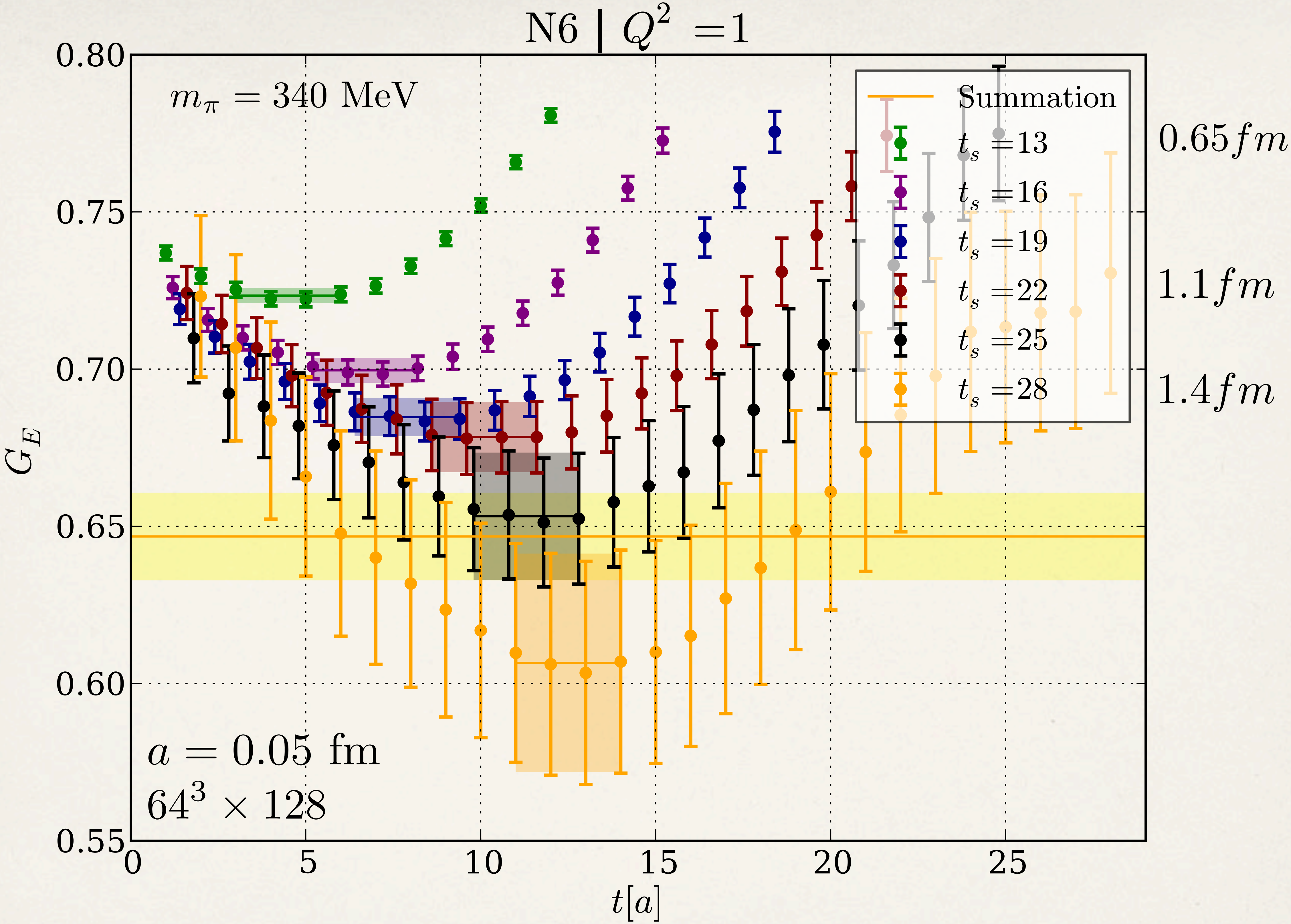

$m_\pi = 270$ MeV
1.0
0.8
0.6
0.4
0.2
$G_E(Q^2)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q^2 [GeV^2]
O7 | G_E Q^2 =1
18
17
16
15
14
13
12
11
10
9
$\sum_{t=0}^{t_s} R(t,t_s)$
12
14
16
18
20
22
$t_s\,[a]$
$1.1fm$
$0.6fm$



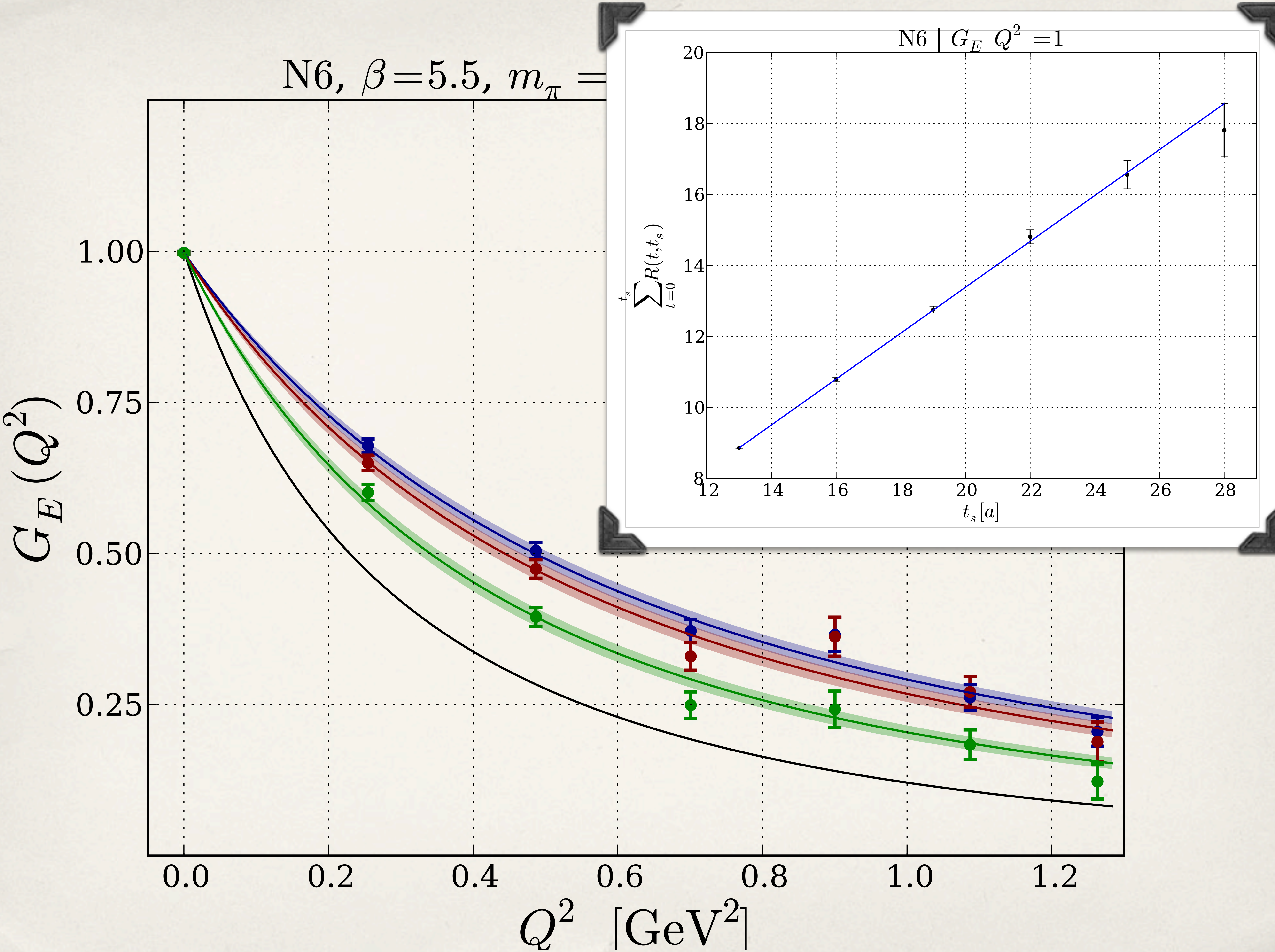
O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
Experiment Kelly 2004
Summation
Plateau
$G_M\,(Q^2)$
Q^2 [GeV^2]
4.8
4.0
3.2
2.4
1.6
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

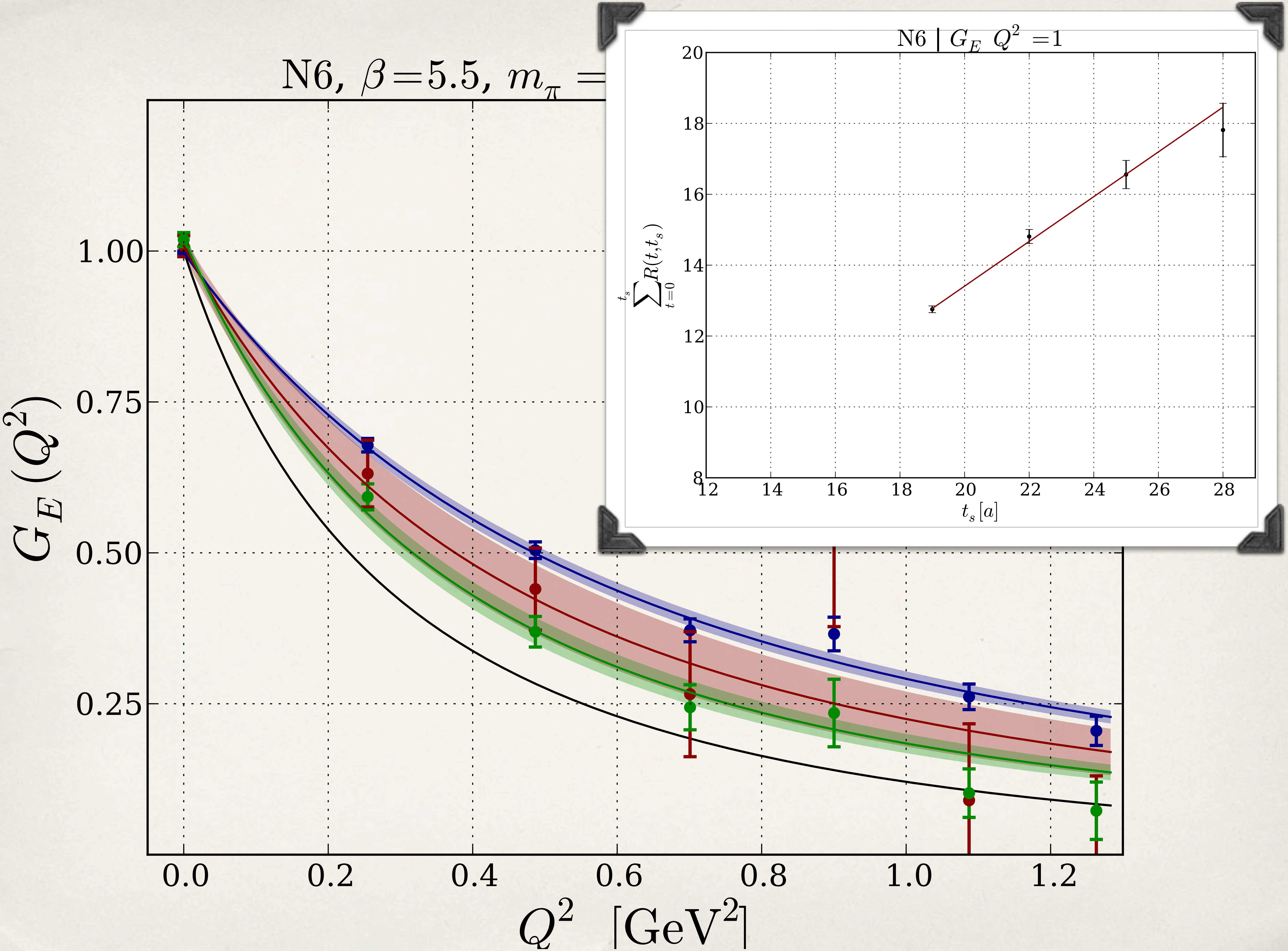


$m_\pi = 270$ MeV
$G_M\,(Q^2)$
4.8
4.0
3.2
2.4
1.6
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q^2 [GeV^2]
O7
Summation
Plateau
$\mu = \lim_{Q^2 \to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$
4.4
4.0
3.6
3.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q^2 [GeV^2]



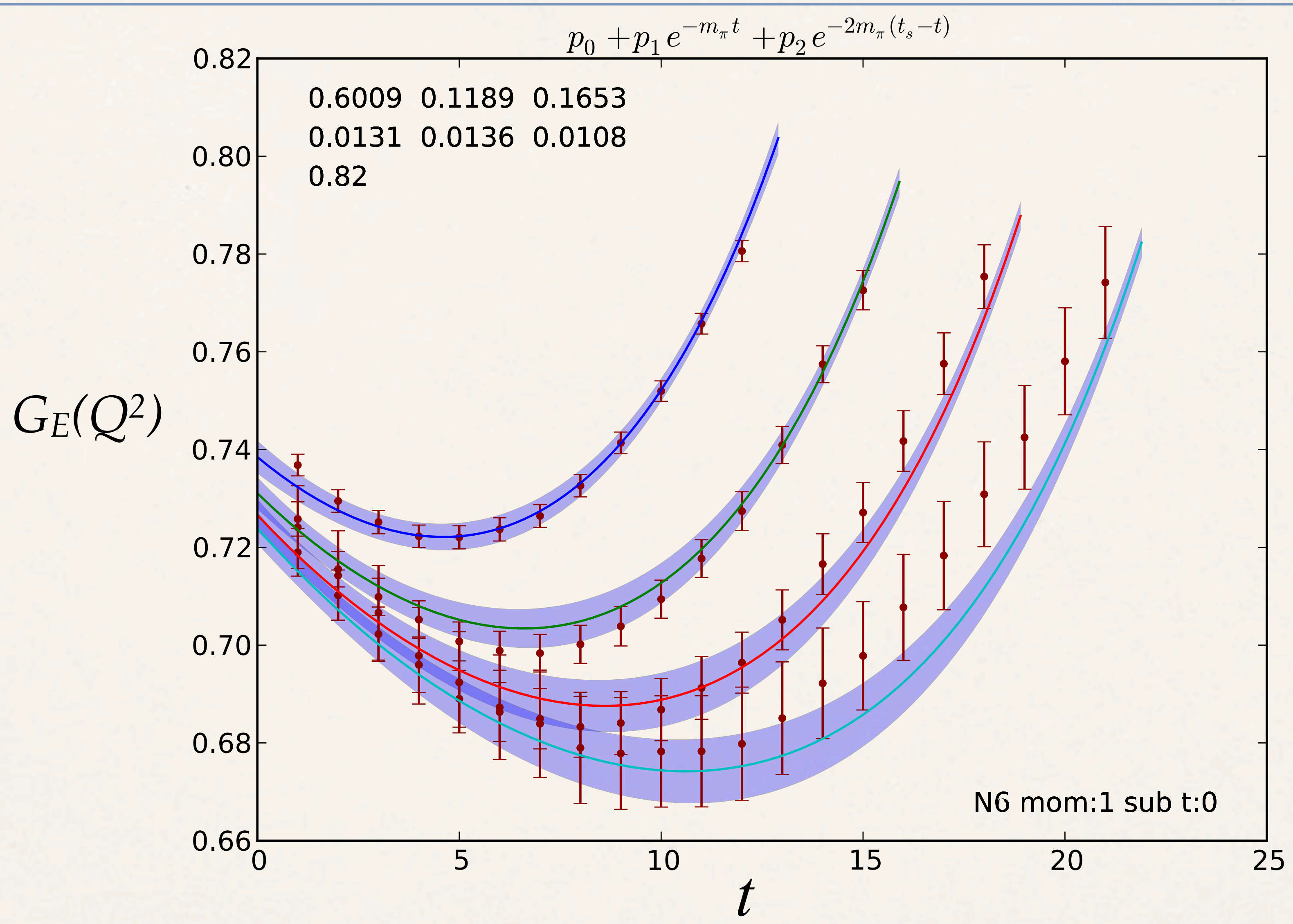
- N6: measured 6 different source-sink separations


N6, $\beta=5.5$, $m_\pi =$
$G_E(Q^2)$
1.00
0.75
0.50
0.25
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Q^2 [GeV^2]
N6 | G_E Q^2 =1
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s\,[a]$
8
10
12
14
16
18
20
12
14
16
18
20
22
24
26
28



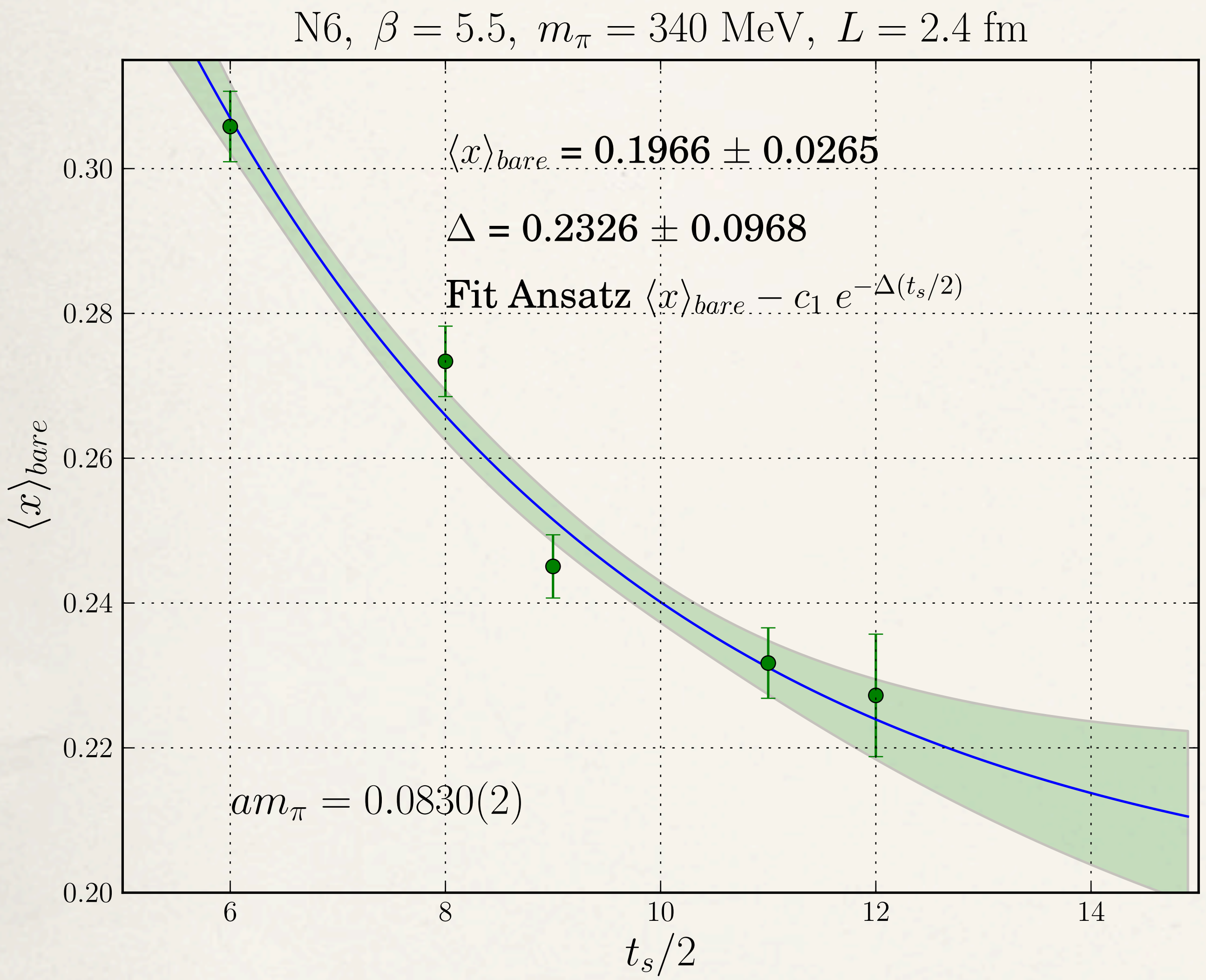
N6, β=5.5, m_π =
$G_E\,(Q^2)$
1.00
0.75
0.50
0.25
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Q^2 [GeV^2]
N6 | G_E Q^2 =1
$\sum_{t=0}^{t_s} R(t,t_s)$
20
18
16
14
12
10
8
12
14
16
18
20
22
24
26
28
$t_s\,[a]$

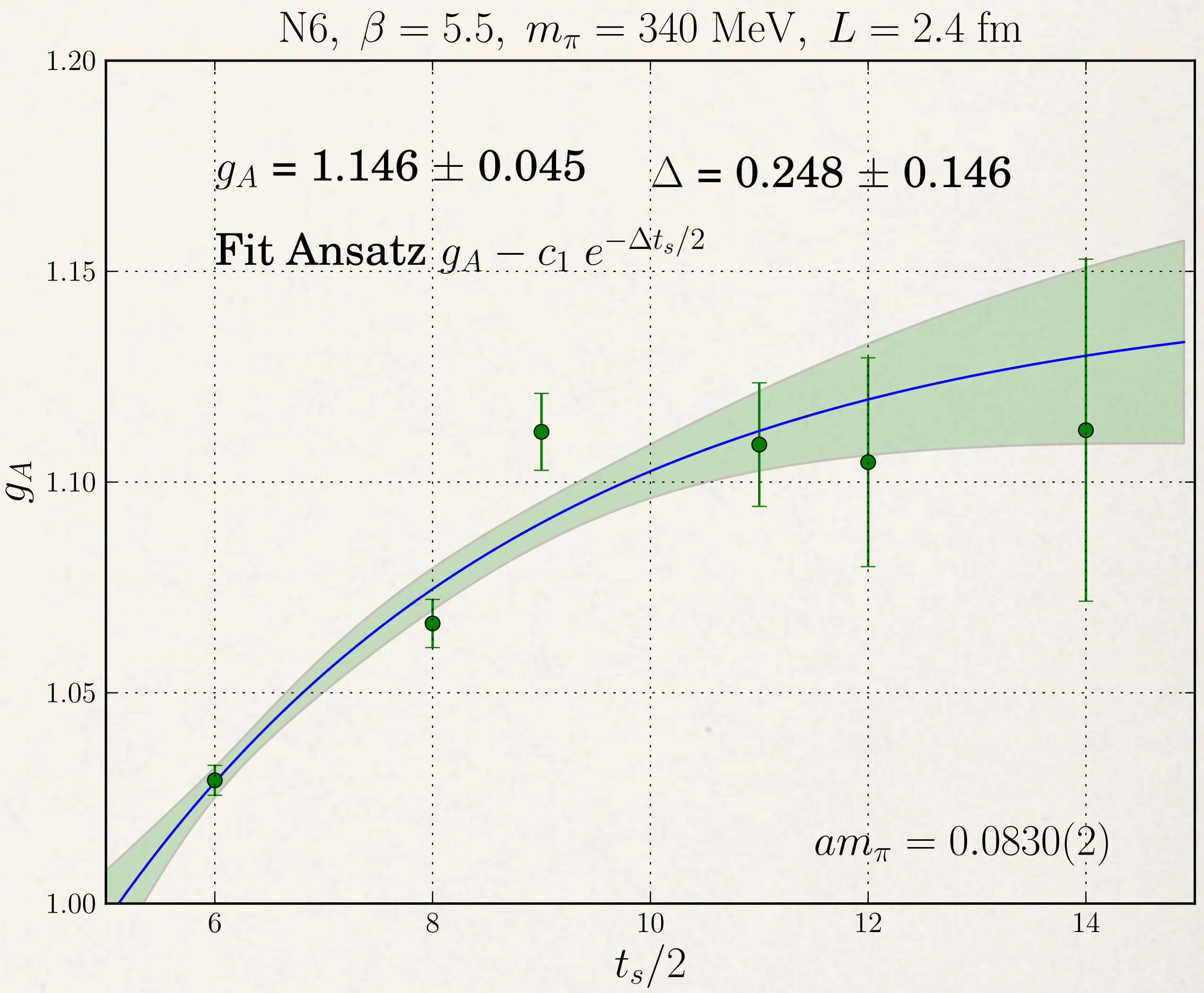
Excited state fits



- alternative to summation method
  - explicit excited state fits $R_V(t, t_s) \propto G_E + p_1 e^{-m_\pi t} + p_2 e^{-2m_\pi (t_s - t)}$

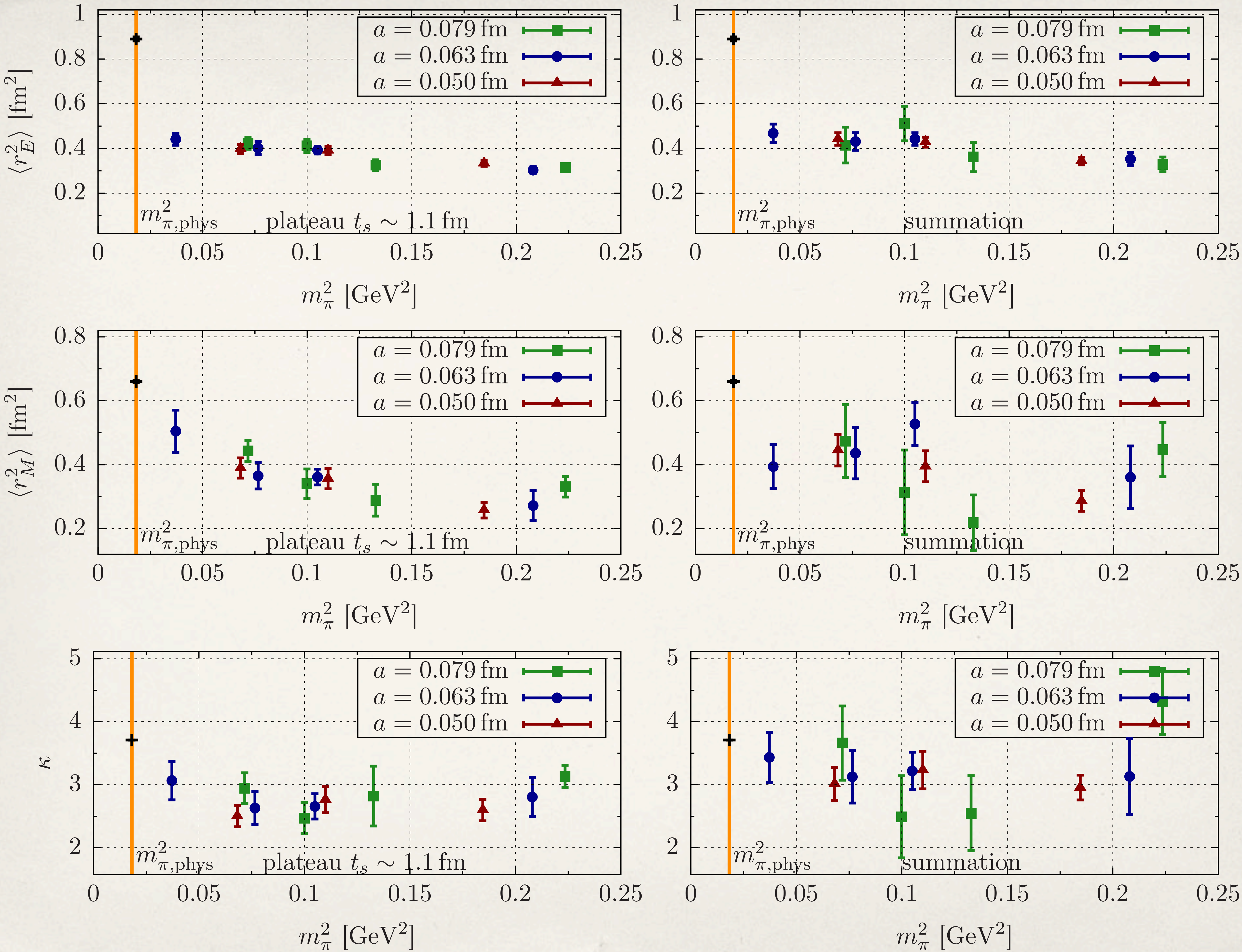
Mid-point/fit method





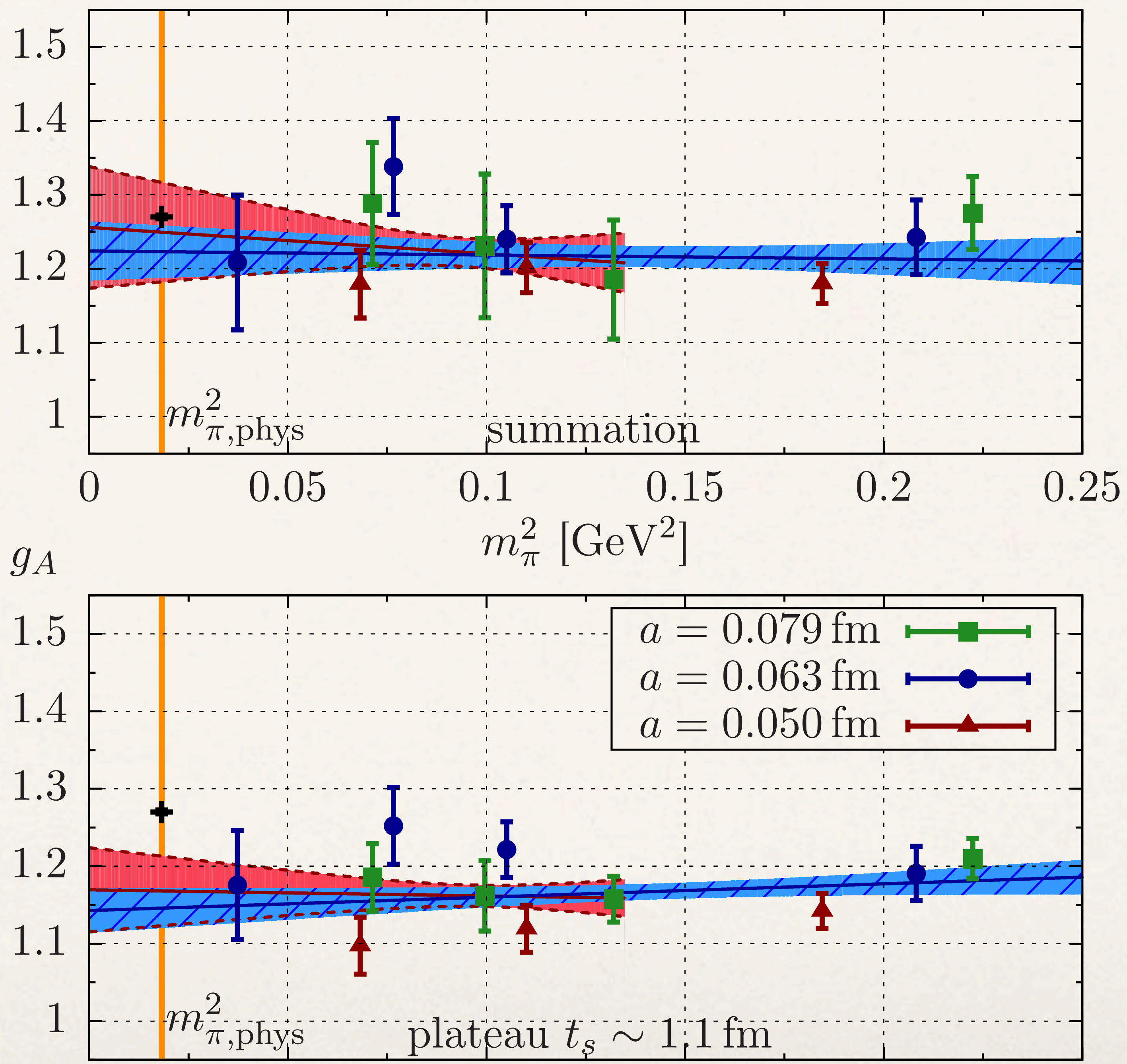
- alternative to summation method
  - extrapolate from plateau fits

Chiral extrapolations

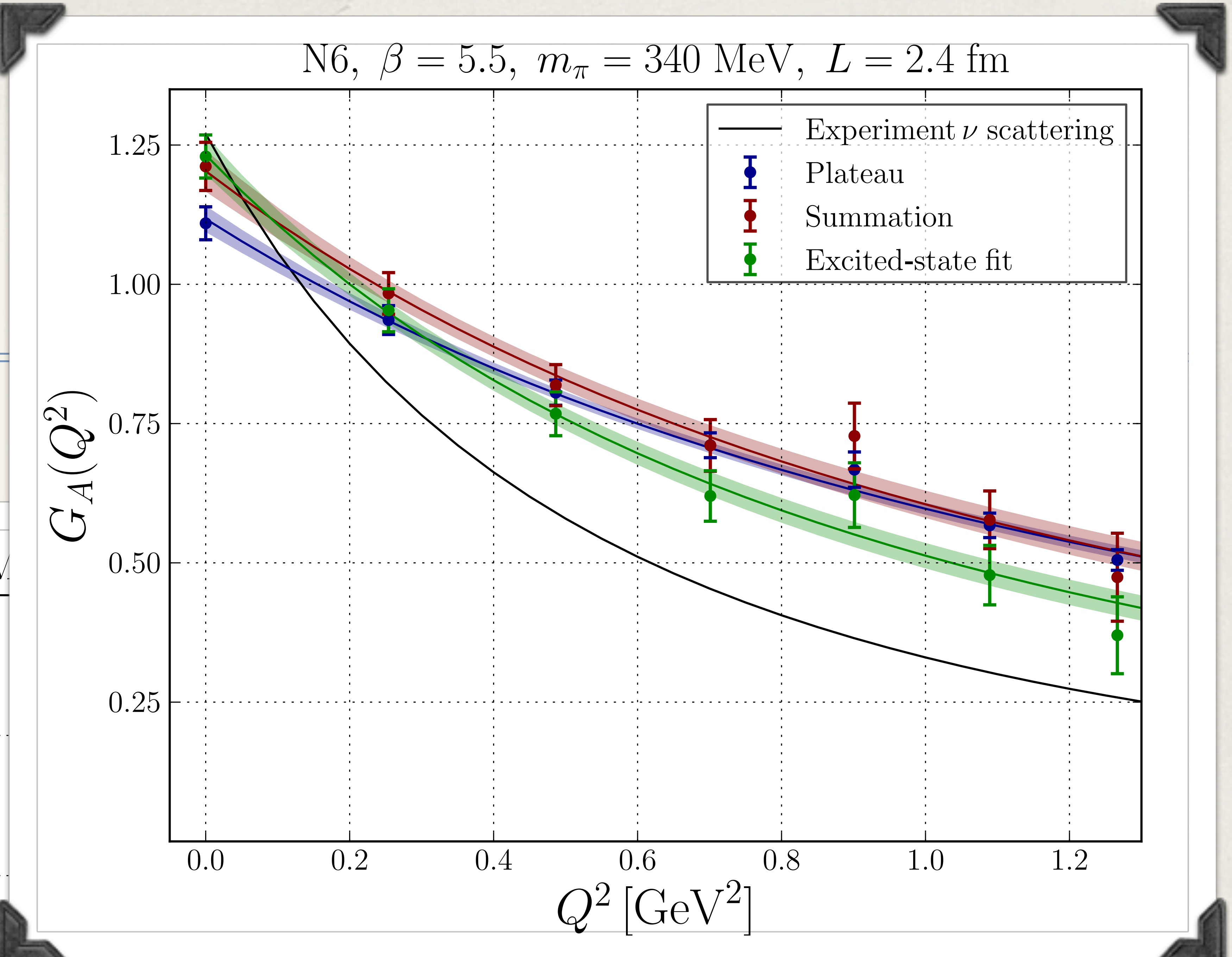


a = 0.079 fm
a = 0.063 fm
a = 0.050 fm
$m^2_{\pi,\mathrm{phys}}$
plateau $t_s \sim 1.1$ fm
summation
$\langle r_E^2 \rangle$ [fm^2]
$\langle r_M^2 \rangle$ [fm^2]
κ
m_π^2 [GeV^2]

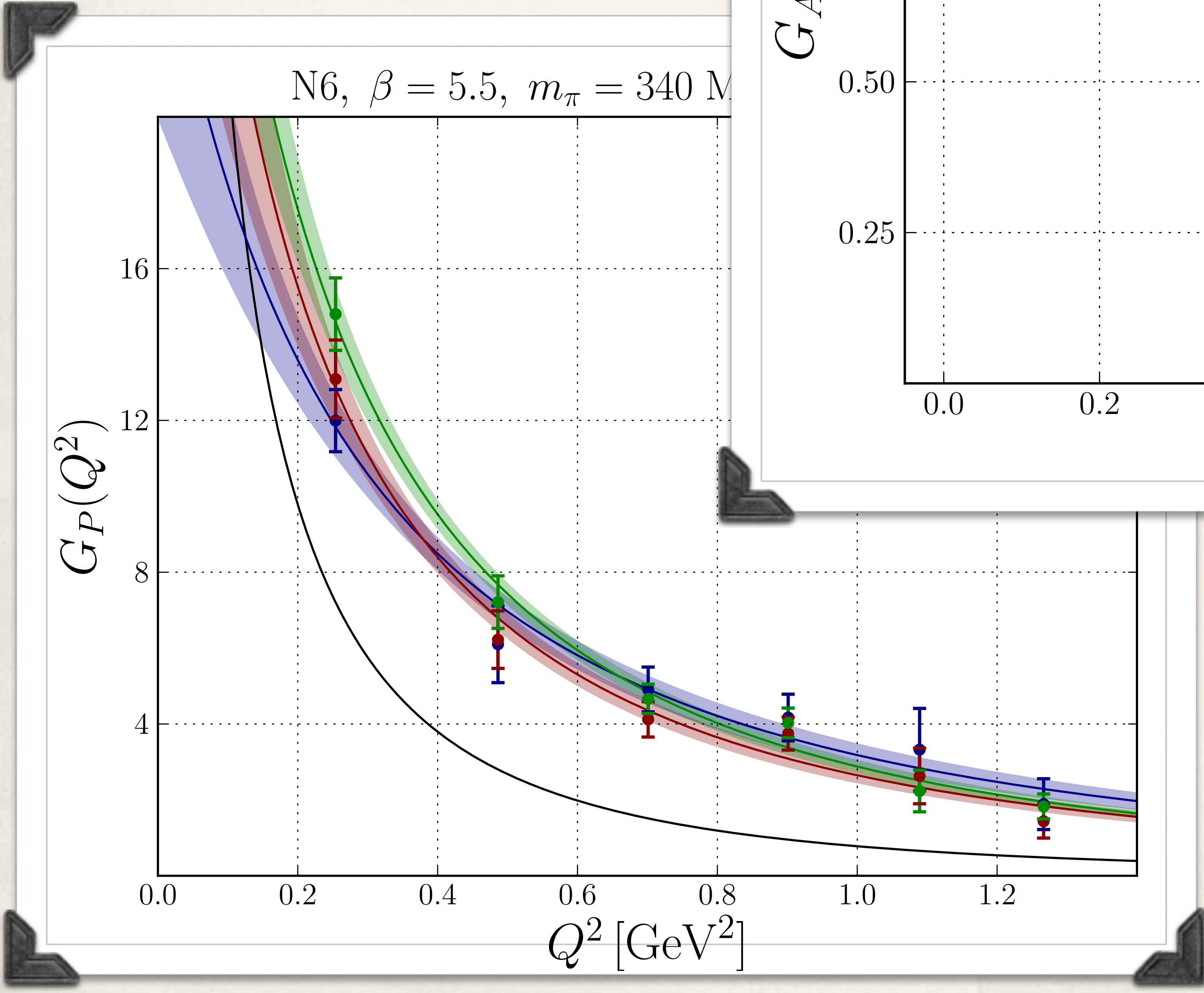
Axial Charge



G_A & G_P



N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm
Experiment ν scattering
Plateau
Summation
Excited-state fit
$G_A(Q^2)$
Q^2 [GeV^2]



N6, $\beta = 5.5$, $m_\pi = 340$ M
$G_P(Q^2)$
Q^2 [GeV^2]

Summary

- We observe a systematic variation in the form factors for the plateau method for different source-sink separations t_s
    - clearer for large statistics
- Summed insertions help control excited state contamination
    - remove the need to fit plateaus
- Explicit excited states fits may help further reduce contamination
- Important to consider range of t_s
    - small t_s have smallest statistical errors but most effected by excited state contamination

Outlook

- N6 ensemble hints at bias for plateau method even for t_s=1.1fm
  - check for the most chiral ensembles
- Finalise treatment of chiral behaviour of the form factors and derived quantities
- Continue axial form factors $G_P(Q^2)$ and $G_A(Q^2)$ study
- Introduce dynamical strange quark
- Simulations at the physical pion mass

Thank you