The ^{4}He radius
from
μ^4He$^+$ spectroscopy

Aldo Antognini
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for the
CREMA collaboration

μHe$^+$ Lamb shift

Measure ΔE(2S-2P) in $\mu\,^3\mathrm{He}^+$ and $\mu\,^4\mathrm{He}^+$ with 50 ppm

$\Downarrow$

$r_{^3\mathrm{He}}$ and $r_{^4\mathrm{He}}$ with $u_r = 3\times10^{-4}$ $\Longleftrightarrow$ 0.0005 fm

if polarisability contribution known with $u_r = 5\%$

Antognini et al., Can. J. Phys. 89, 47 (2011)

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- new muonic force?

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Benchmark for few-nucleon theories
- absolute radii of ^{3}He, ^{4}He
and ^{6}He, ^{8}He via isotopic shifts

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Cancio Pastor et al., arXiv:1201.1362
Müller, Wang, Shiner...

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Enhanced bound-state QED test when combined with He$^+$(1S-2S) [MPQ and Amsterdam]
- Finitite size $\sim Z^4 R^2$
- Bohr structure $\sim Z^2 R_\infty$
- Challenging QED contributions $\sim (Z\alpha)^{5\ldots6}$

Why testing bound-state QED?

• Free QED

$$a_e = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \Delta(\text{had.}, ...)$$

• Bound-state QED

- Binding effects ($Z\alpha$) — bad convergence, all-order approach/expansion
- Radiative corrections (α and $Z\alpha$)
- Recoil corrections (m/M and $Z\alpha$) — relativity $\not\Leftrightarrow$ two-body system
- Radiative–recoil corrections (α, m/M and $Z\alpha$)
- Nuclear structure corrections

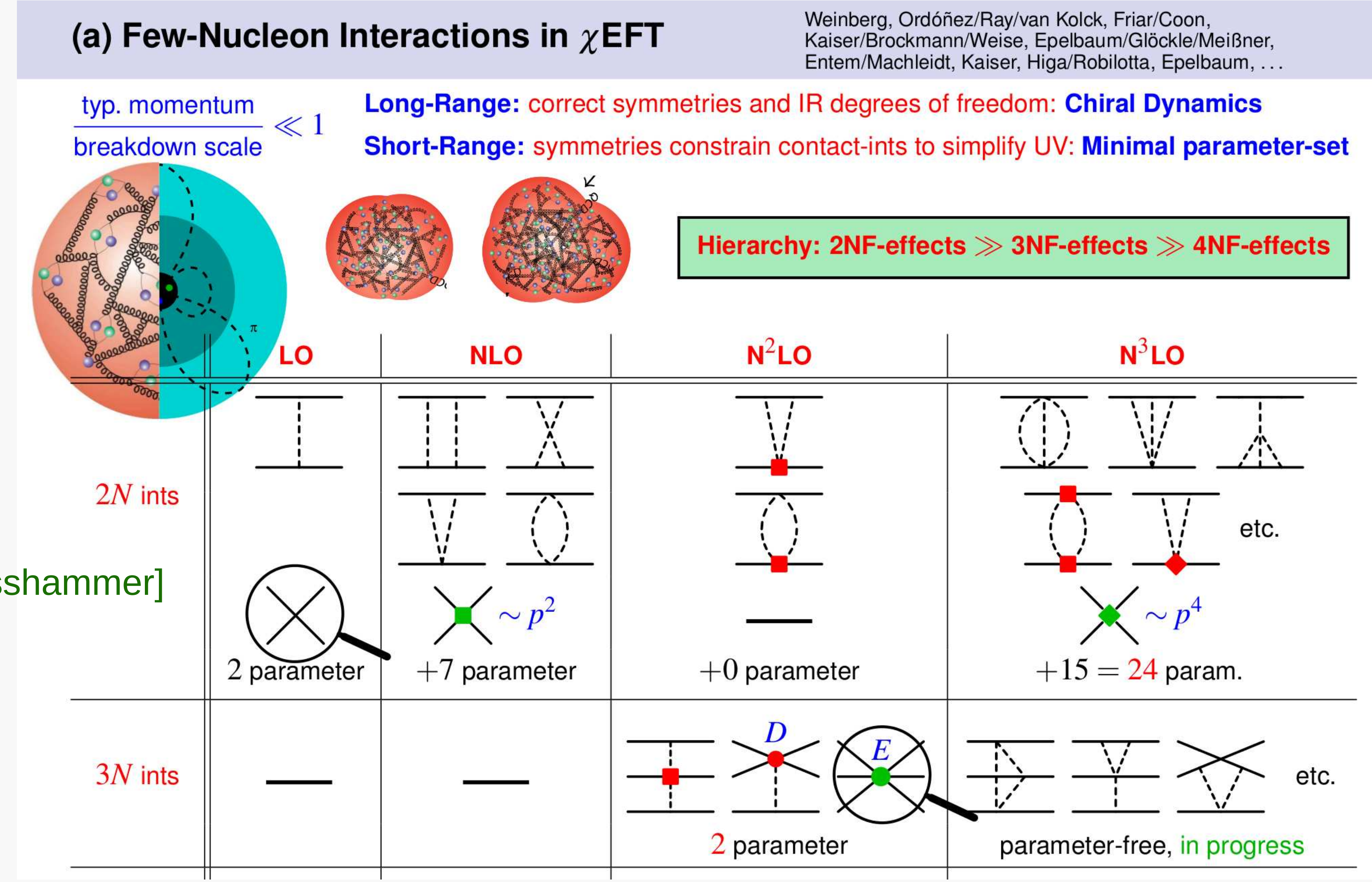
→ Cannot develop the calculation in a systematic way
→ Corrections are mixed up: $\alpha^x \cdot (Z\alpha)^y \cdot (m/M)^z$
→ Difficulty in finding out the desired order of corrections

New development: NRQED

[after Nio]

QED	$g-2$ free particle particle mass only perturbative around free particle	Lamb shift bound-state particle three scales, hierarchy non-perturbative
QCD	deep inelastic scattering pQCD	hadron lattice, Chiral perturbation

Few-nucleon theories and He-radius



[from Griesshammer]

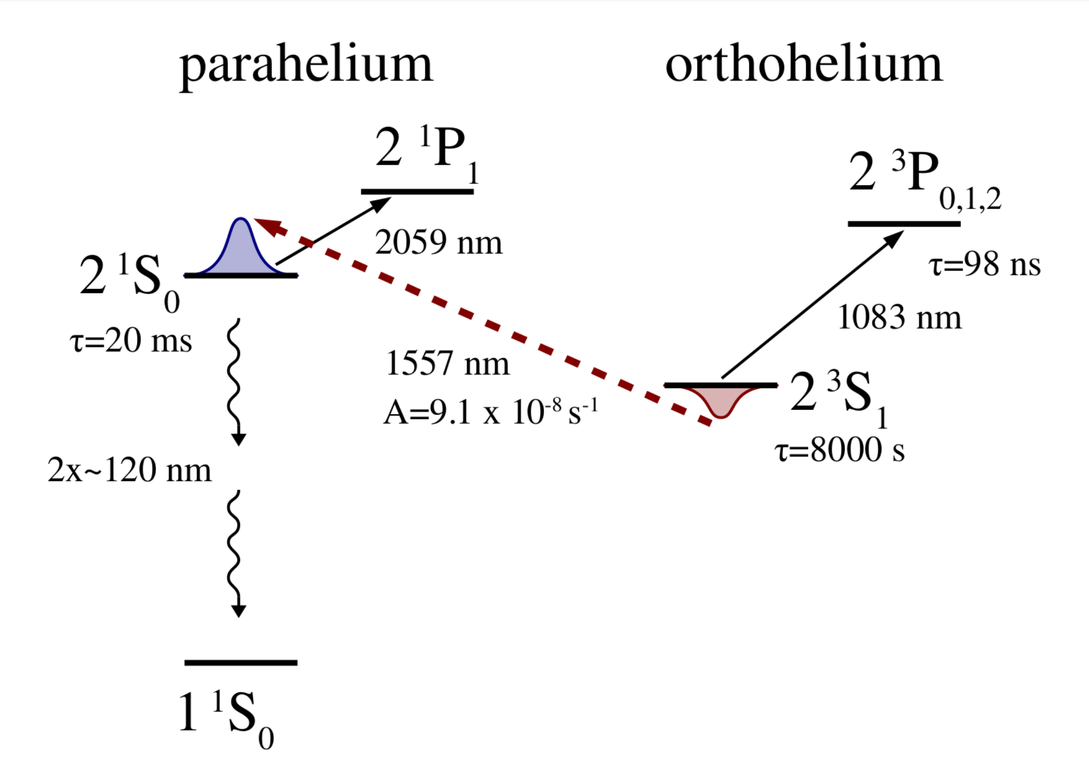
From $r_{\mathrm{He}} \to c_D$ or c_E.

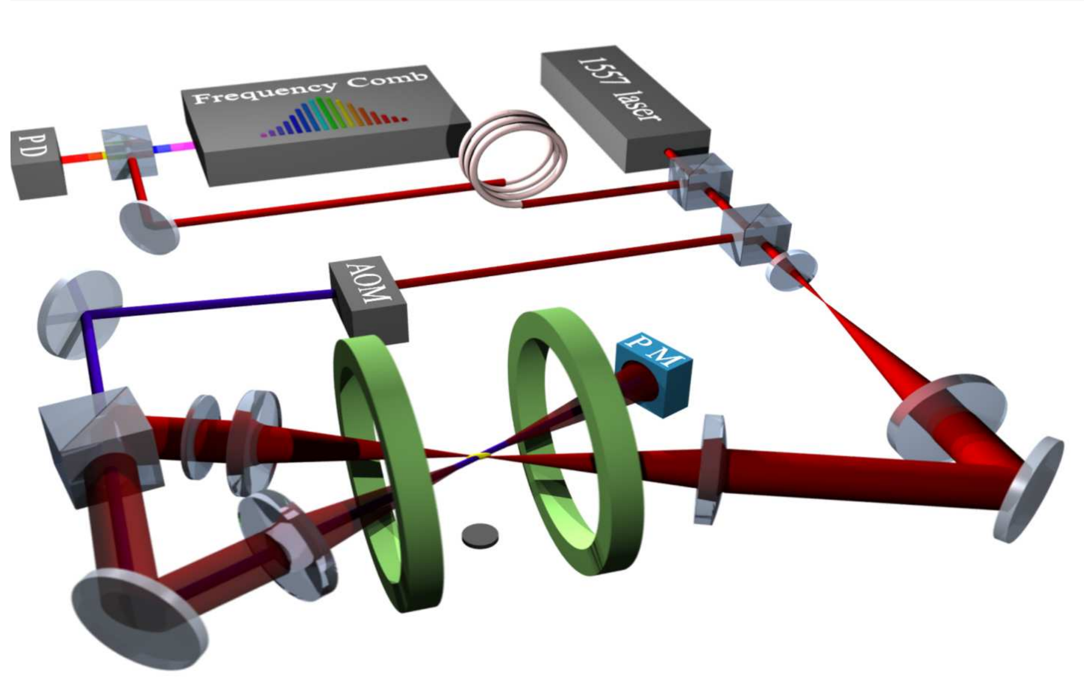
[Navratil et al., PRL99, 042501 (2007)]

Radii are “clean” benchmarks to test few-nucleons theories or to fix LEC

[Gazit, Marcucci, Forssen, Kievsky, Stadler, Krebs...]

Helium spectroscopy in Amsterdam



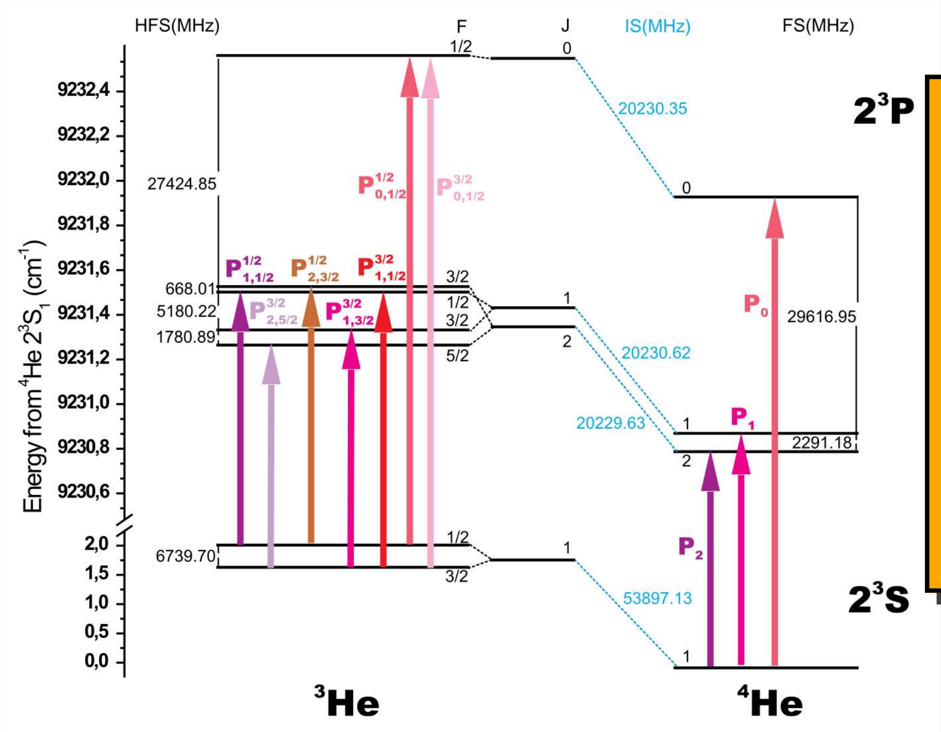


- Trap μK cold ^{4}He* and ^{3}He*.
- Measure the double forbidden 1557 nm line
  (M1 transition between two metastable states).
  (200'000 times narrower than 2^3P states
- Precision of $u_r = 8 \times 10^{-12}$ (1.5 kHz).

From isotope shift
$R^2_{^3\text{He}} - R^2_{^4\text{He}} = 1.028(11)$ fm^2

[R. van Rooij et al., Science 333, 196 (2011)]

2S-2P metrology of ^{3}He and ^{4}He in Florence

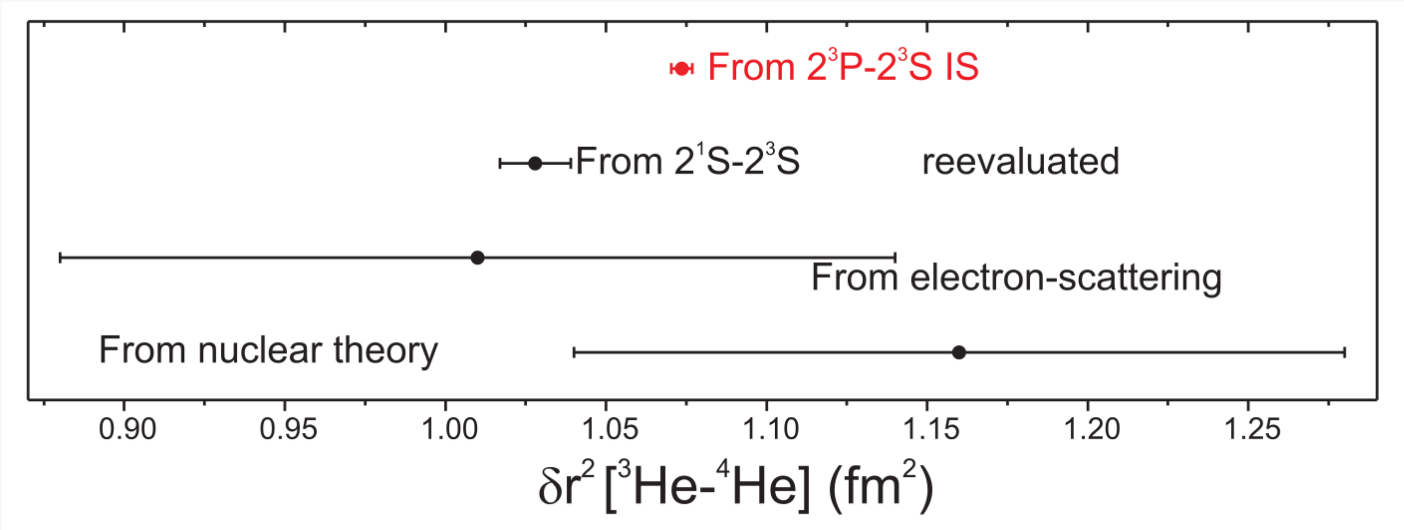


- Measure transitions between the 2S-2P hypefine mannifolds in metastable ^{3,4}He beams with $u_r \approx 5 \times 10^{-12}$ (2.5 kHz) using saturation spectroscopy at 1083 nm
- From isotope shift theory

  $$R^2_{^3\text{He}} - R^2_{^4\text{He}} = 1.074(3)\ \text{fm}^2$$

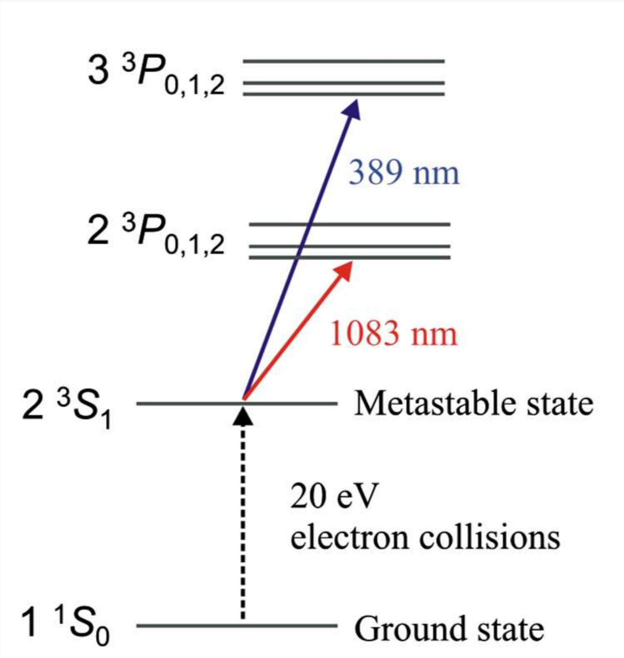
- Test of three-body bound-state QED

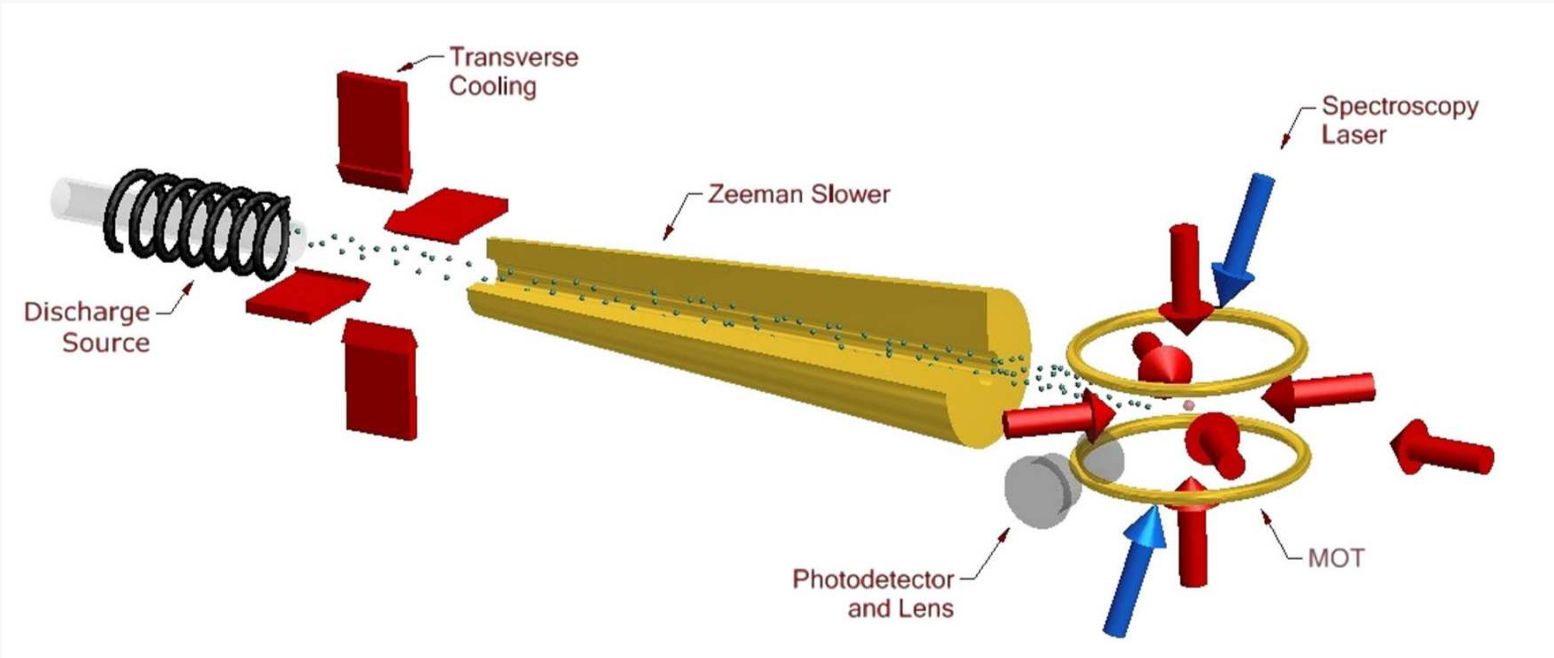
[Cancio Pastor et al., PRL 108, 143001 (2012)]



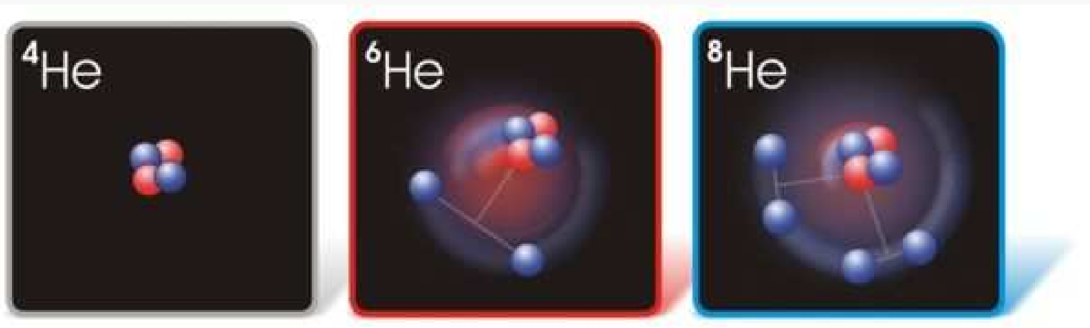
4σ discrepancy

^{6}He and ^{8}He spectroscopy at GANIL





- Finite size shift: 1 MHz
- Mass shift: 50 GHz

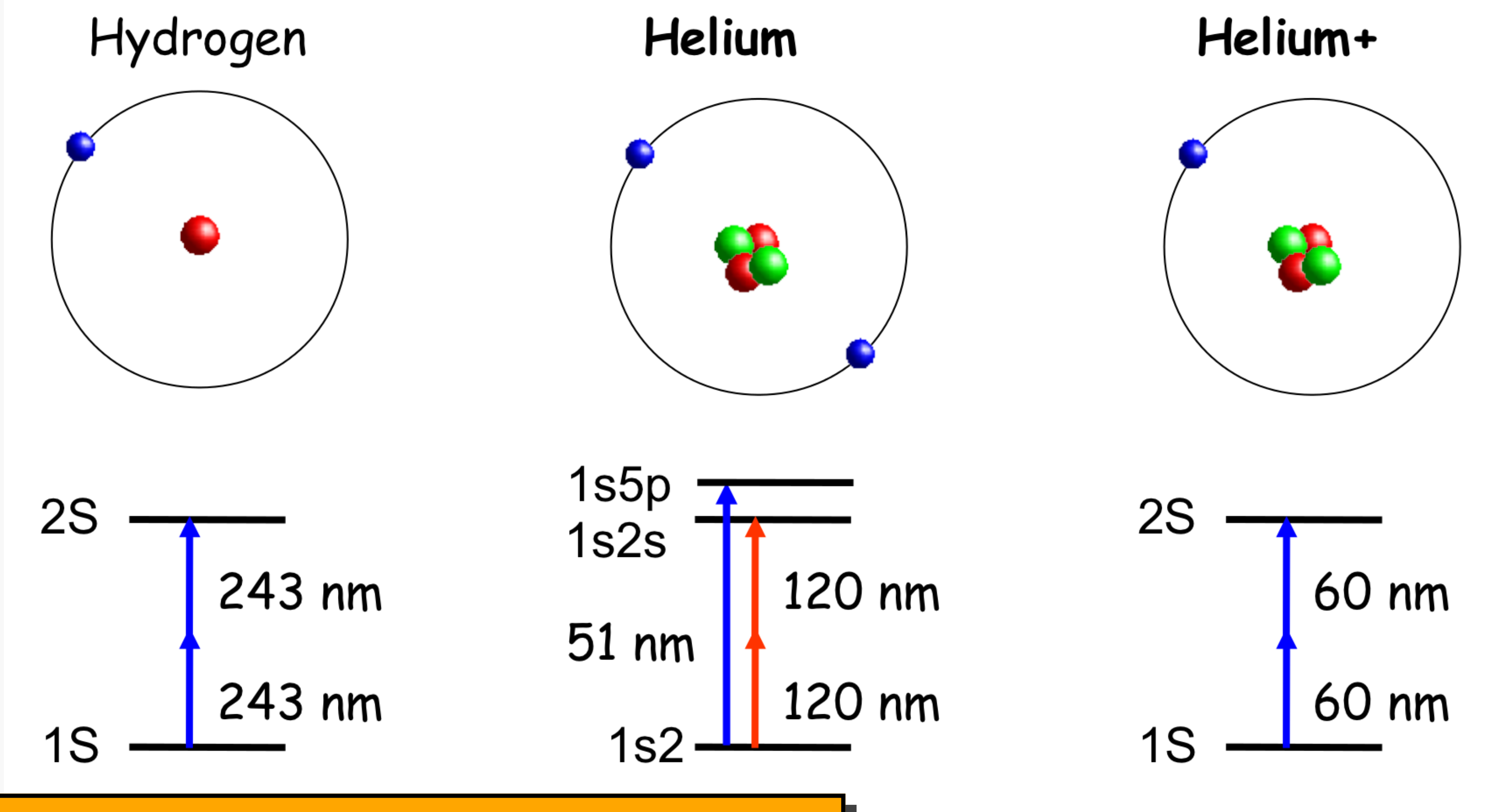


- Measure the 389 nm transitions with 10...70 kHz precison.
- From isotope shift theory and knowledge of ^{4}He charge radius

$$R_{^6\mathrm{He}} = 2.059(8)\ \text{fm}$$
$$R_{^8\mathrm{He}} = 1.958(16)\ \text{fm}$$

[Lu, Müller, Drake et al., RMP 85 1383 (2013)]

He^{+}(1S-2S) and He(1S2-1S5P)



Both experiments are performing XUV comb spectrocopy:

- two-photon, on a trapped He^{+} ion (MPQ)
- one-photon Ramsey technique, on a He jet (Amsterdam)

From these measuremsnts

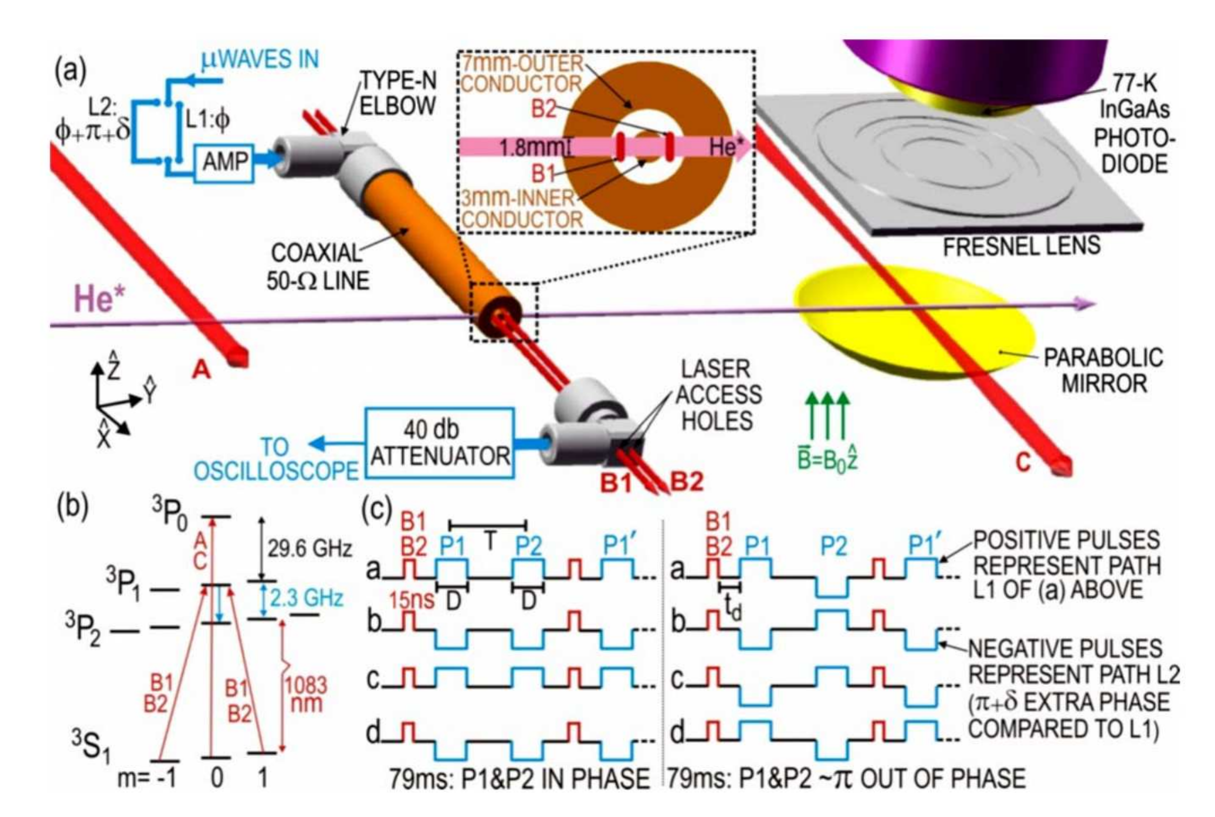
$\rightarrow R_{^4\mathrm{He}}$ or one/two-electrons bound-state QED test

[Hermann et al., PRA 79, 052505 (2009)]

[Kandula et al., PRA 84, 062512 (2011)]

The fine structure of He

Determine centroid position with 10^{-4} linewidth accuracy



Measure the fine structure intervals:

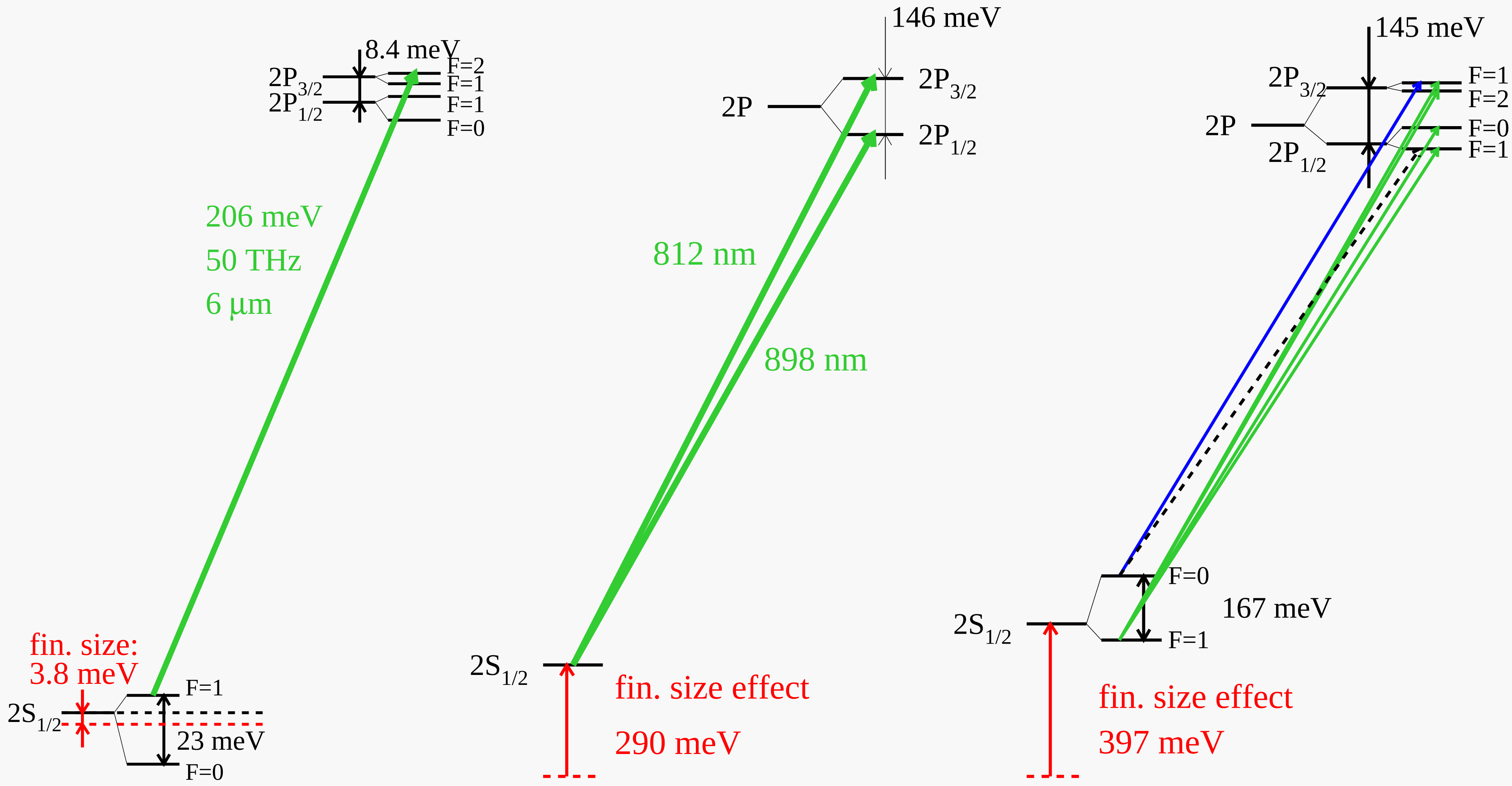
$2^3P_1 - 2^3P_2$ [Borbely et al., PRA 79, 060503(R) (2009)]

$2^3P_0 - 2^3P_2$ [Smiciklas and Shiner, PRL 105, 123001 (2010)]

of atomic helium. Compare with theory [Pachucki and Yerokhin, arXiv:1011.2467v2]

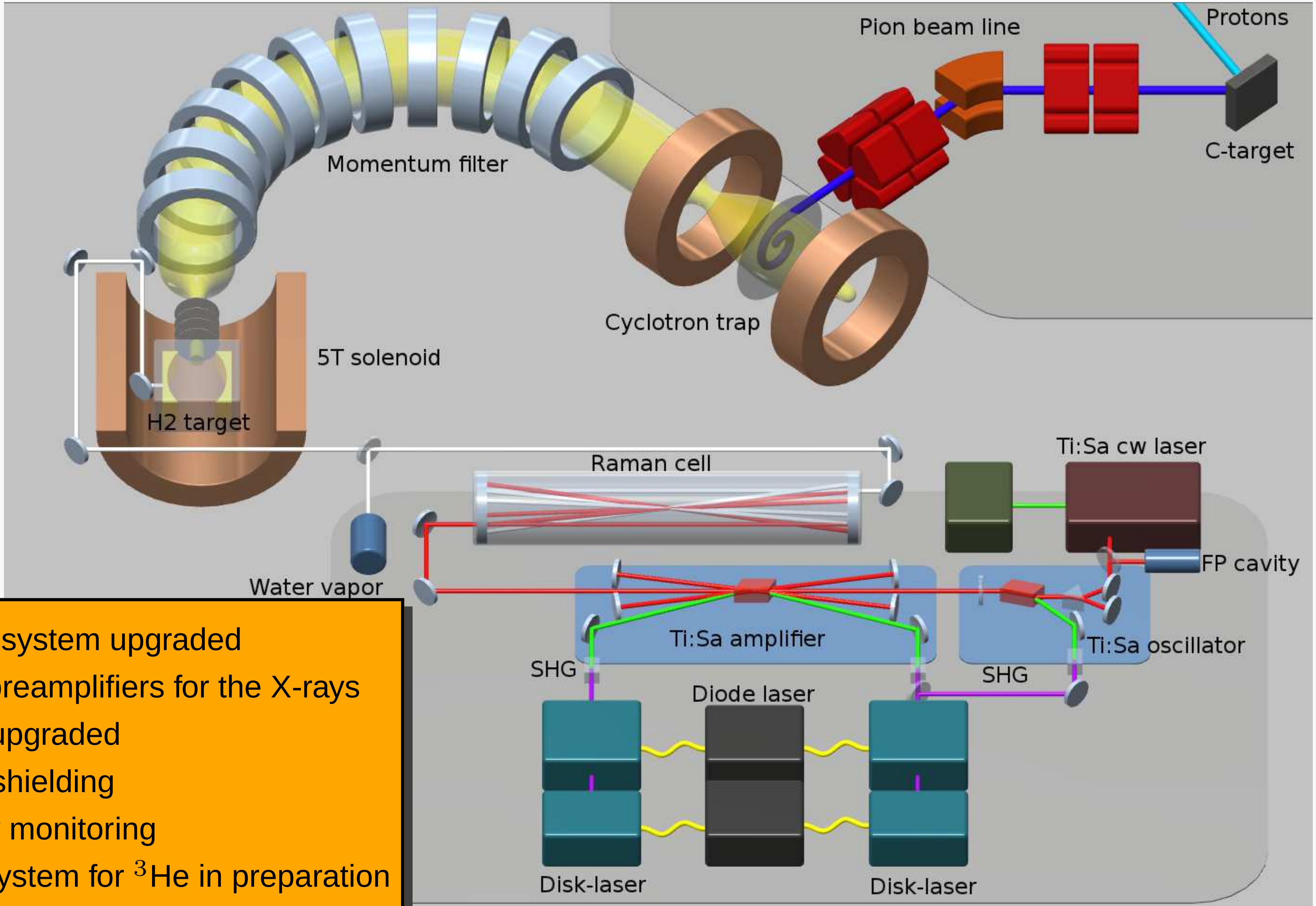
$$\rightarrow \alpha^{-1} = 137.03599955(64)(4)(368) \text{ with } u_r \sim 27 \times 10^{-9}$$

Muonic helium transitions



μp	$\mu\,{}^4He^+$	$\mu\,{}^3He^+$
$\lambda = 5.5 - 6\mu m$	$\lambda = 812$ and 898 nm	$\lambda = 850 - 1100$ nm
finite size: 2%	finite size: 20%	finite size: 25%

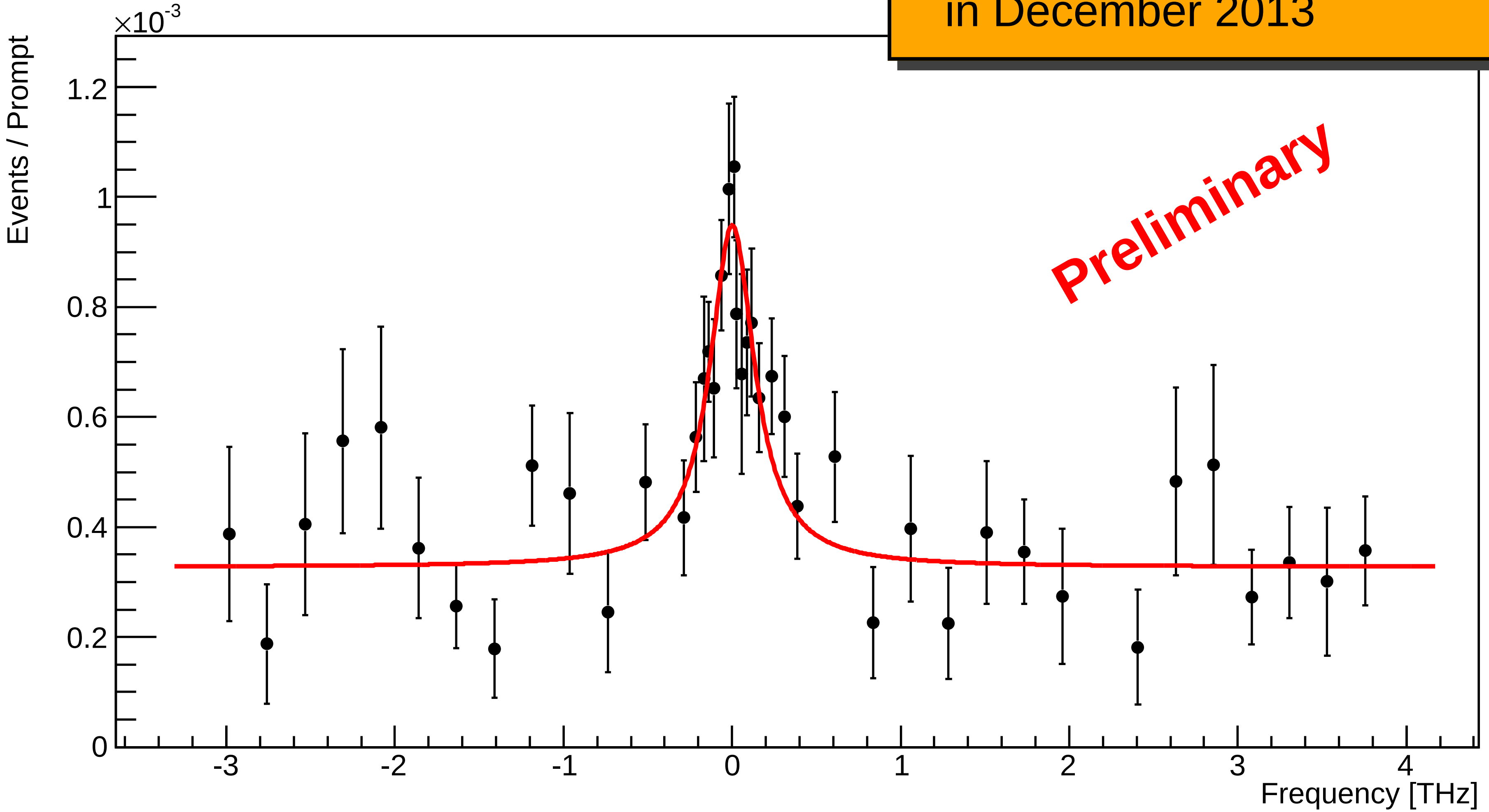
The setup for $\mu\,\mathrm{He}^+$ is similar to μp



- Laser system upgraded
- New preamplifiers for the X-rays
- DAQ upgraded
- Light shielding
- Cavity monitoring
- Gas system for ^{3}He in preparation

The $2S_{1/2} - 2P_{3/2}$ resonance in $\mu\,^4\mathrm{He}^+$

Two weeks of measurements in December 2013



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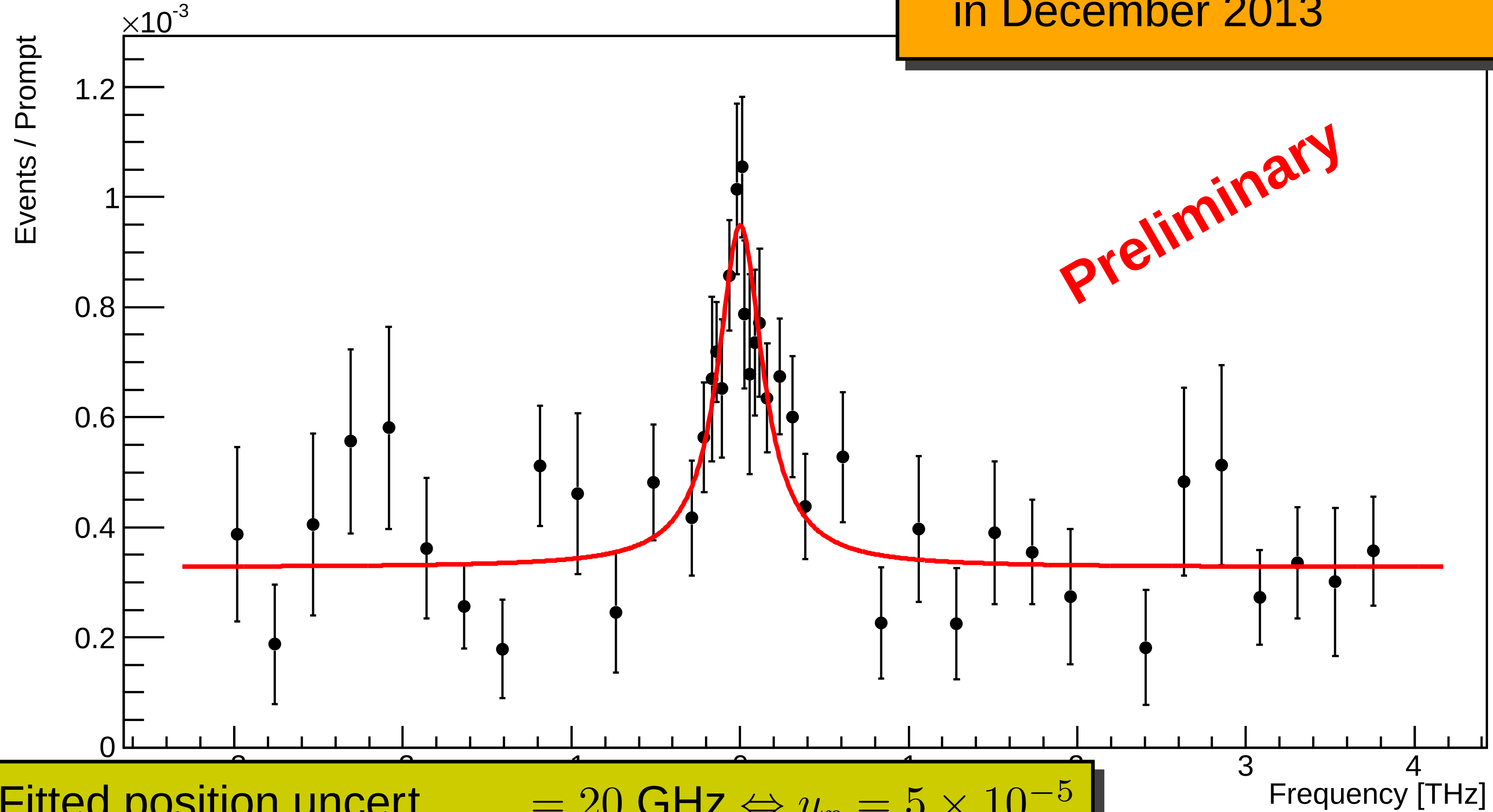
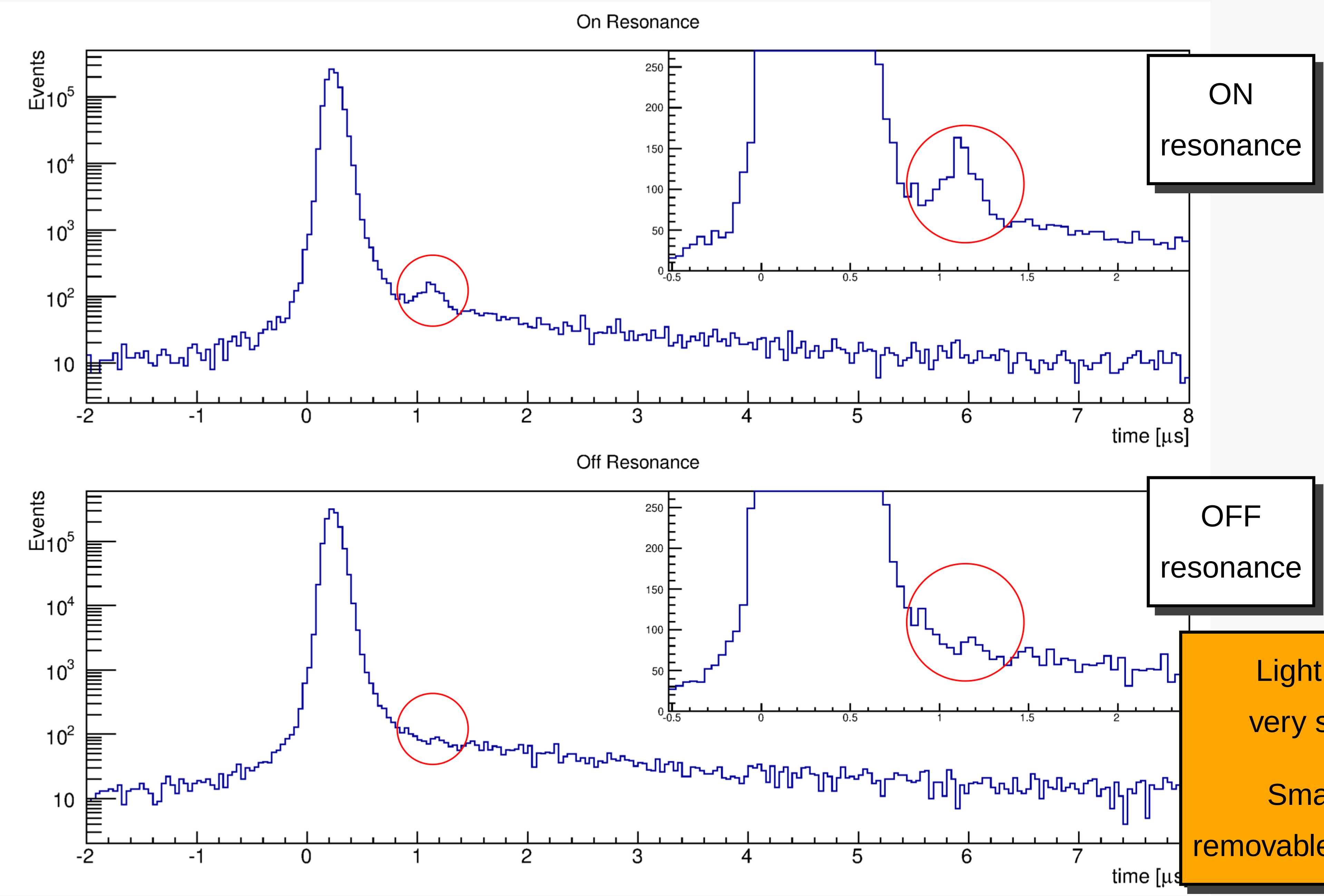


Fitted position uncert.	$= 20$ GHz $\Leftrightarrow u_r = 5 \times 10^{-5}$
Laser frequency uncert.	< 100 MHz
Systematics	< 10 MHz

The K_α time spectra



Light shielding very successful

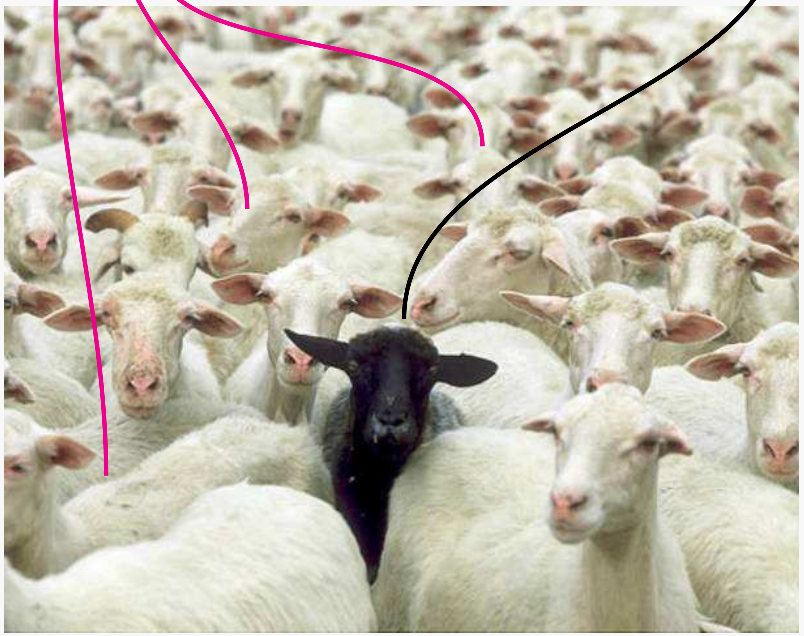
Small pickup removable with analysis

Proton radius from muonic hydrogen

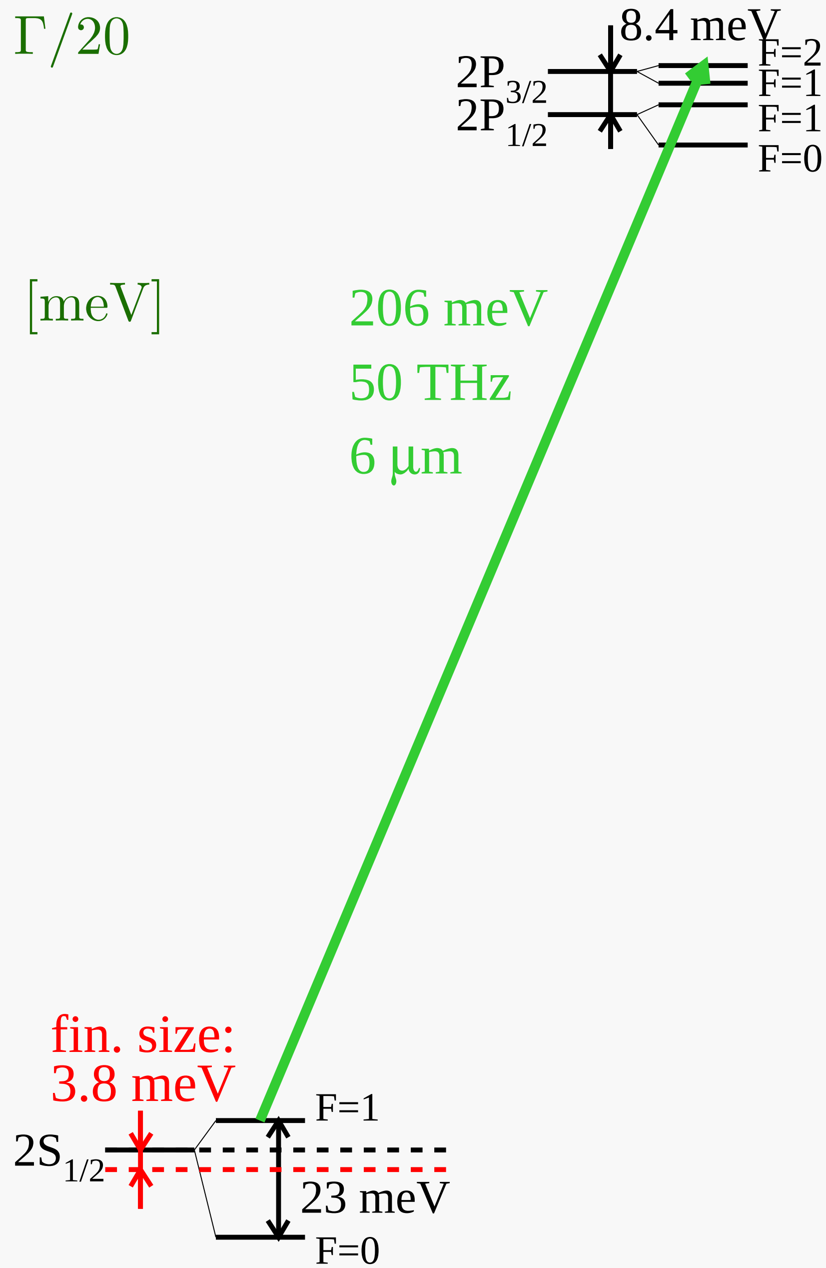
- Measure $\Delta E^{\rm exp}_{2P-2S}$ in μp with $u_r = 10^{-5} \leftrightarrow 0.5$ GHz $= \Gamma/20$

- Compute theoertical prediction

$$\Delta E^{\rm th}_{2P-2S} = 206.0336(15) - 5.2275(10)\, r_{\rm p}^2 + 0.0332(20) \quad [\rm meV]$$



The Lamb shift contributions



Comparing theory with experiment $\Longrightarrow r_{\rm p}$

μp, μd **and** μ He$^+$ **measurements/theory (Prel.!!)**

Measurements in muonic atoms

μp:	$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.3706(23)$ meV	
μd:	$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.8xx(34)$ meV	(preliminary !)
$\mu\,^4\mathrm{He}^+$:	$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 1524.xx(8)$ meV	(preliminary !)

Pachucki, Borie, Eides, Karshenboim, Jentschura, Indelicato, Miller, Martynenko, Carlson, Birse, Gorshteyn, Paz Hill, Pascalutsa, Pineda, Bacca Friar, Nir, Pascalutsa... Einstein, Schrödinger

Theory			QED		Finite size $[R^2]$		TPE $[R^3_{(2)}$ + Pol. contr.]
μp	$\Delta E_{\mathrm{LS}}^{\mathrm{th}}$	=	206.0336(15)	-	5.2275(10) r_{p}^2	+	0.0332(20) meV
μd	$\Delta E_{\mathrm{LS}}^{\mathrm{th}}$	=	228.7972(15)	-	6.1094(10) r_{d}^2	+	1.6910(160) meV
$\mu\,^4\mathrm{He}^+$	$\Delta E_{\mathrm{LS}}^{\mathrm{th}}$	=	1668.598(100)	-	106.340(xx)r_{He}^2	+	1.40(4)r_{He}^3 + 2.470(150) meV

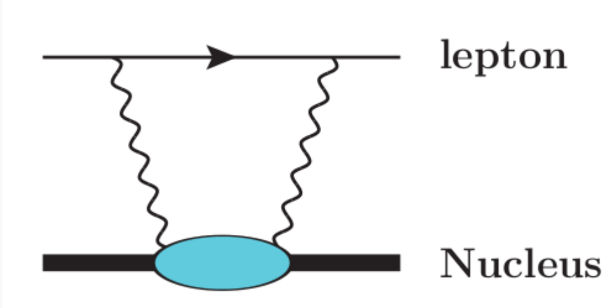
- recoil correction to two-photon with finite size: 0.266 meV (Borie). Is this included already in the TPE?
- intrinsic polarizability of the nucleons has not been yet accounted.
- shape dependence of the finite size corrections.

$r_{\mathrm{He}} = 1.681(4)$ fm [Sick]

$1\sigma_r \rightarrow \Delta E_{\mathrm{LS}}^{\mathrm{th}}$ changes by 1.4 meV

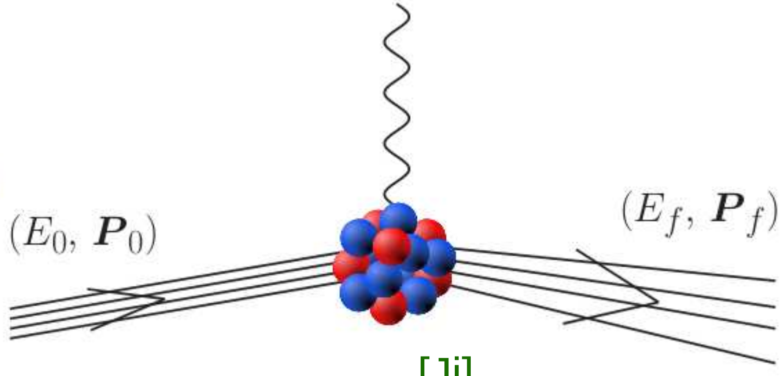
Nuclear polarization contribution in $\mu\,\mathrm{He}^+$

$$\Delta E_{\rm LS}^{\rm th} = \Delta E_{\rm QED} - \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle + \frac{m_r^3}{12}(Z\alpha)^4\langle r^3\rangle_{(2)} + \delta_{\rm pol}$$



- From nuclear response function $S_0(\omega) \rightarrow$ nuclear polarization contribution

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2\delta(E_f - E_0 - \omega)$$



[Ji]

- Two ways to get the response function:

- From photo-absorption [Bernabeau & Jarlskog, Rinker, Friar]
  $\delta_{\rm pol} = 3.1$ meV $\pm 20\%$

- From state-of-the-art potentials (chiral EFT, AV18/UIX) [Nevo Dinur talk!!]
  $\delta_{\rm pol} = 2.47$ meV $\pm 6\%$

Nuclear and nucleon polarizabilities in $\mu\,\mathrm{He}^+$

Nuclear polarizability has been corrected for
intrinsic nucleons finite size
but not for the
intrinsic nucleon polarizabilities

- Estimate [following in part Carlson, Gorchthein and Vanderhaegen, PRA 89, 022504 (2014)]
  - $\delta_{\rm pol}(p) \approx \delta_{\rm pol}(n) \approx 13.5\ \mu$eV
  - it is scaling with the number of nucleons $N = 4$
  - it is scaling with $|\Phi(0)|^2 \sim m^3 Z^3$

$\Rightarrow$ $\delta_{\rm pol}(\rm nucleons) \approx 4 \cdot 8 \cdot 13.5\ \mu eV = 0.4$ meV

$\Rightarrow$ To be compared with $\delta_{\rm pol}(\rm nuclear) = 2.47(15)$ meV

Is the nucleons polarization contribution so large?
(only a factor 6 smaller than the nuclear contribution)

A more precise quantification is urgently needed!!

Third Zemach contribution in $\mu\,\mathrm{He}^+$

- The third Zemach contribution can be computed:

  - assuming a charge distribution (Gaussian) [Borie]

$$\delta_{\rm Zem} = 1.40(4)\ \langle R_c^2\rangle^{3/2}$$
$$= 6.65(19)\ \text{meV} \quad \text{(using Sick } ^4\text{He radius)}$$

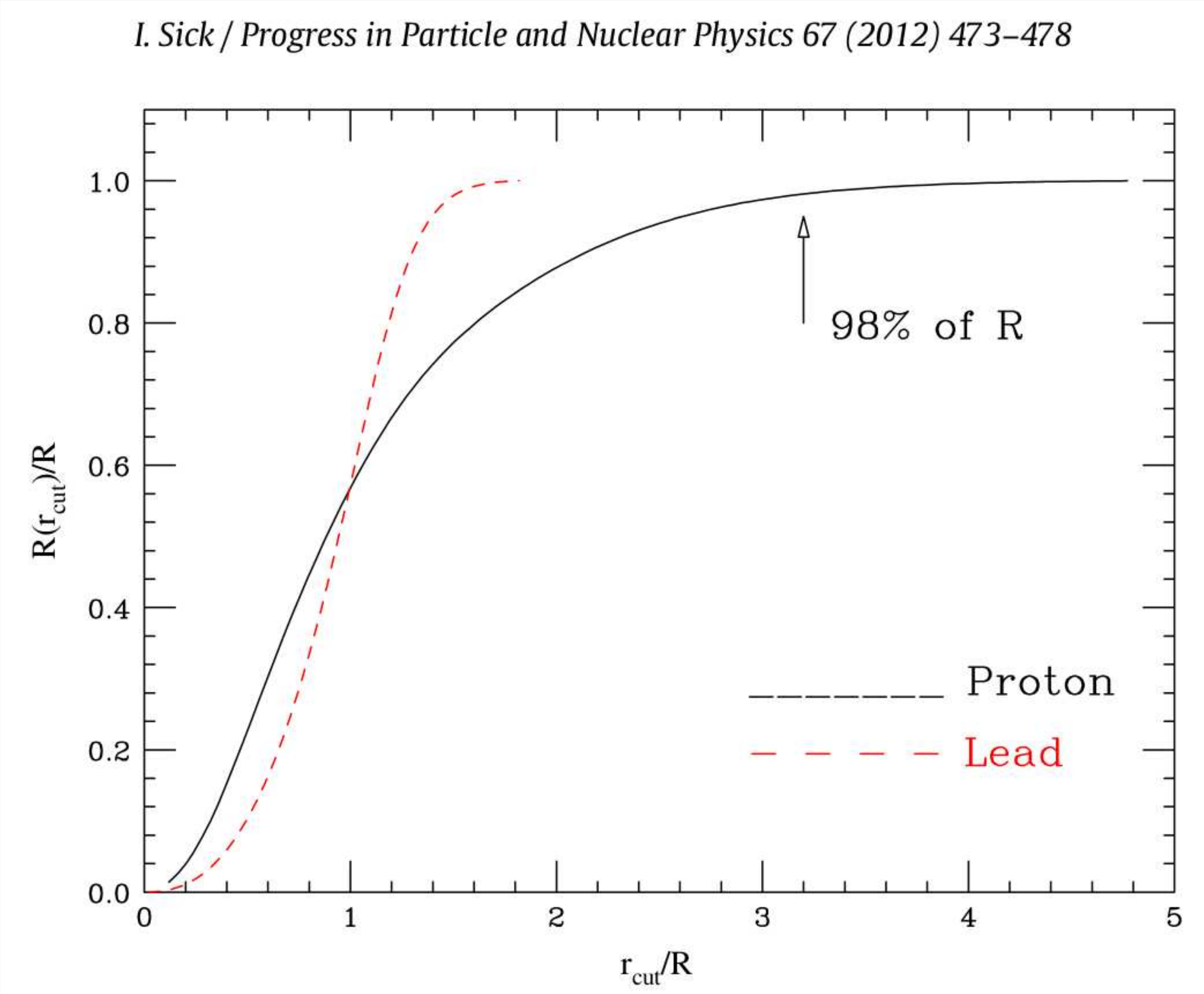
  - using state-of-the-art potentials [Ji, Nevo Dinur et al., arXiv:1311.0938]

$$\delta_{\rm Zem} = 6.12/5.94\ \text{meV} \quad \text{(AV18/UIX-potential)}$$
$$= 6.53/6.34\ \text{meV} \quad \text{(EFT-potential)}$$
$$\text{using}\quad r_p = 0.88/0.84\ \text{fm}$$

Agreement $\rightarrow$ the charge distribution seems to be under control

BUT it would be interesting to
- compute the finite size contributions using the measured form factors
- demonstrate the convergence of the higher charge moments contributions
- determine the third Zemach radius from e-He scattering

Difficulties due to large-r tail (from I. Sick)



The extrapolation from finite q to $q = 0$ is much more difficult for p than for nuclei with $A > 2$

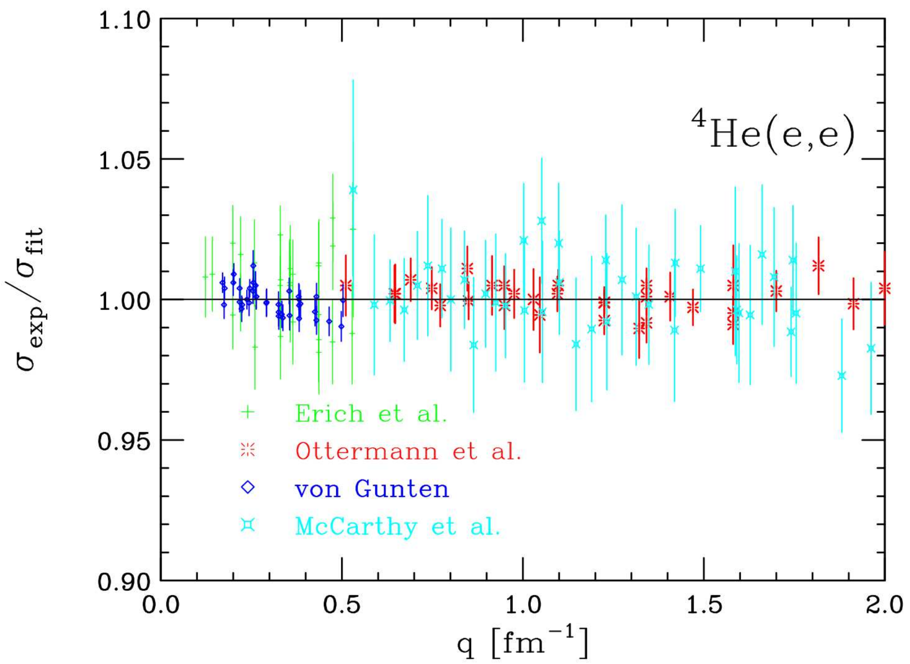
Extrapolation of $G(q)$ is not fully reliable. Needs to consider $\rho(r)$ at large r.

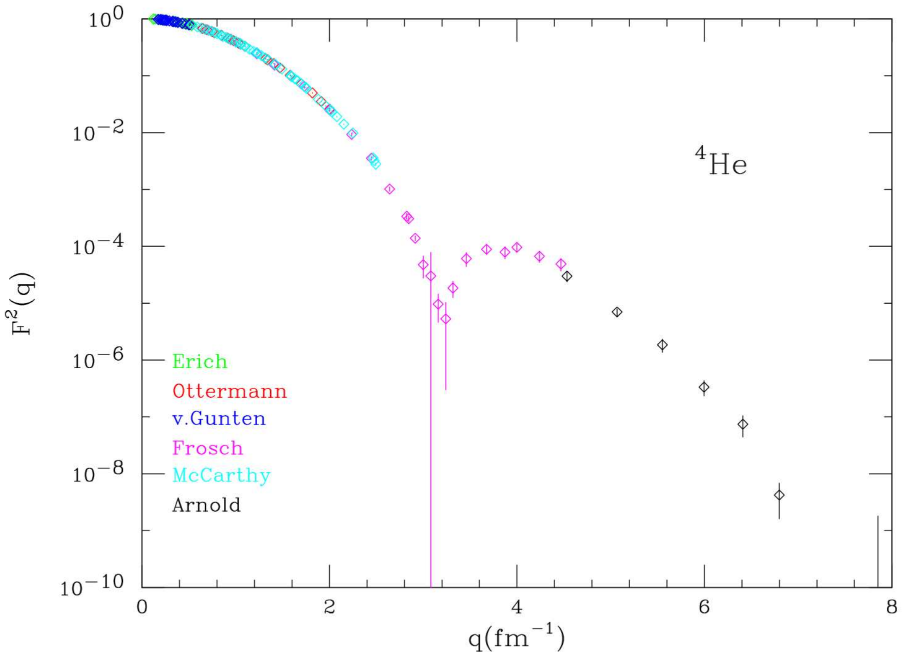
Most $e - p$ scattering fits have not been checked for large-r behavior

Need a physical model to constrains the large-r behavior

Slow convergence of the p rms radius vs upper cutoff $r_{\rm cut}$ calculated over the integral of the charge density $\rho(r)$

He radius from e-scattering





- world data of e-scattering.
- constraints density at large r:
  - shape: from p-wavefunction $\sim$ Whittaker.
  - absolute density: from p-He scattering + FDR.
- point density from potential + GFMC (small r) + FDR (large r).
- fold point density with charge density distribution of p and n.
- include Coulomb distortions.

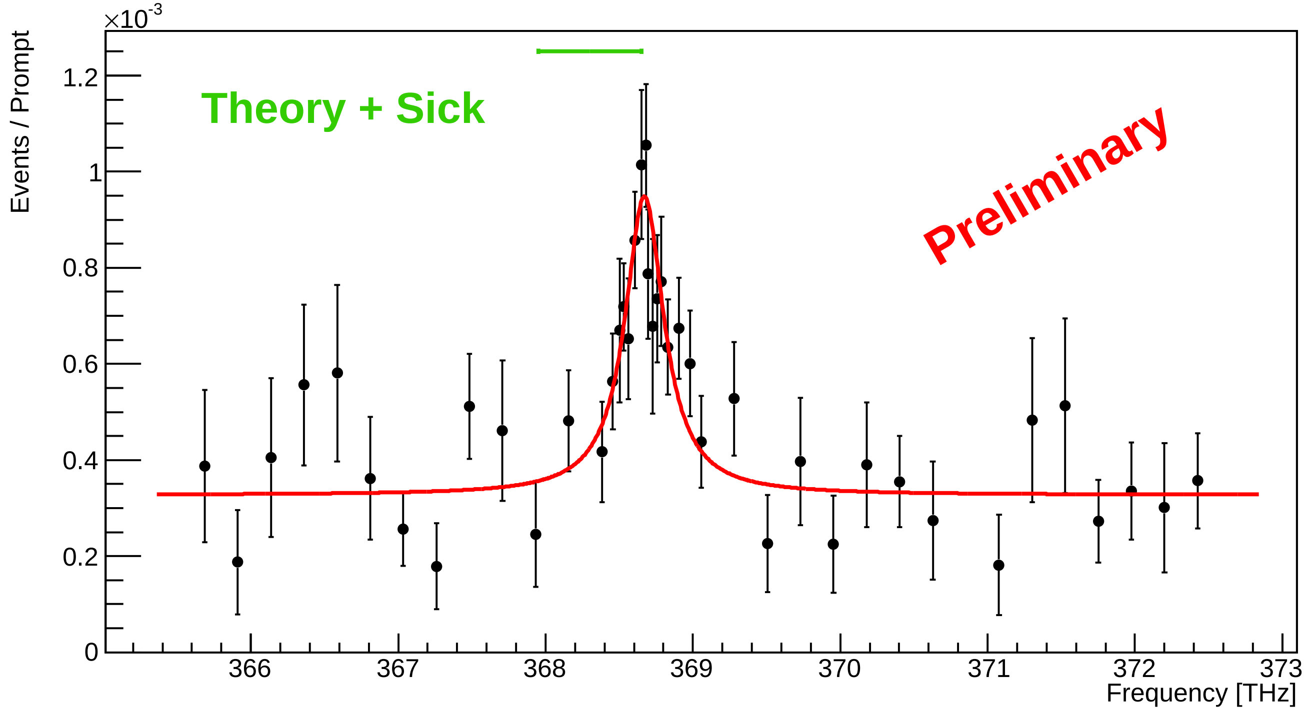
Fit with SOG

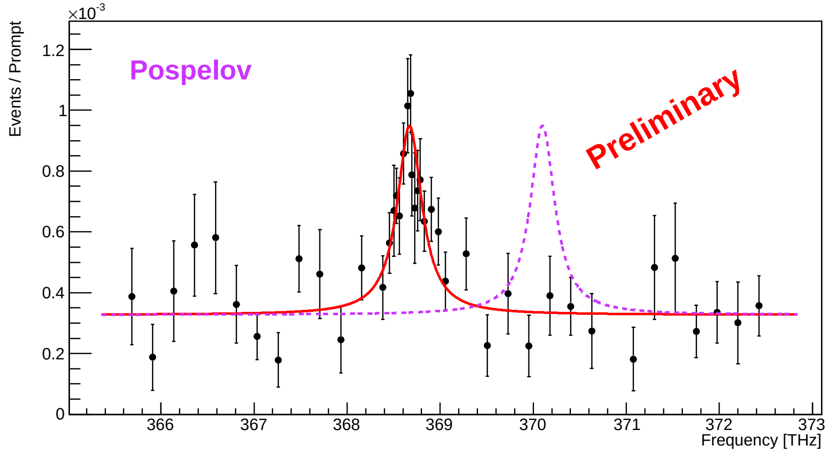
$\rightarrow R = 1.681(4)$ fm

(best known radius from e-scattering)

[Sick, PRC 77, 941392(R) (2008)]

Secret results!





- The transition has been found at the expected position i.e., whithin the uncert. given by r_{He} from $e-$He scattering.
- New physics model of Pospelov excluded
- Zavattini value from old $\mu\,\mathrm{He}^+$ experiment excluded

Need to summarize all 2S-2P contributions

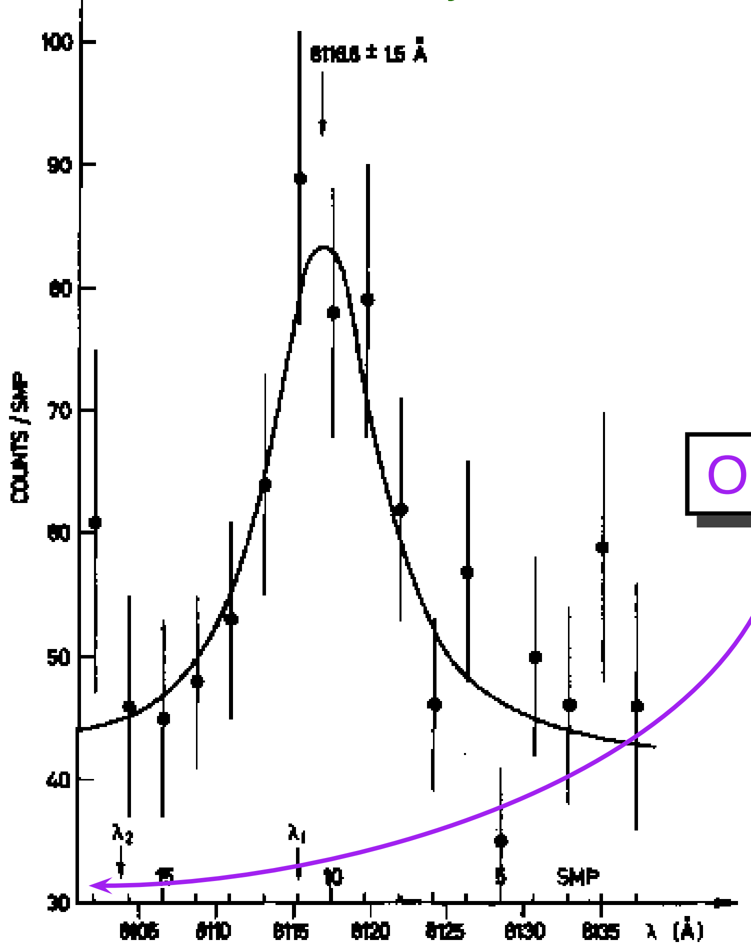
^{4}He nuclear charge radius

$1.681(4)$ fm	$u_r = 2 \times 10^{-3}$	[Sick]
$1.677(1)$ fm	(VERY preliminary)	$[\mu\,\mathrm{He}^+]$

Zavattini “resonance”

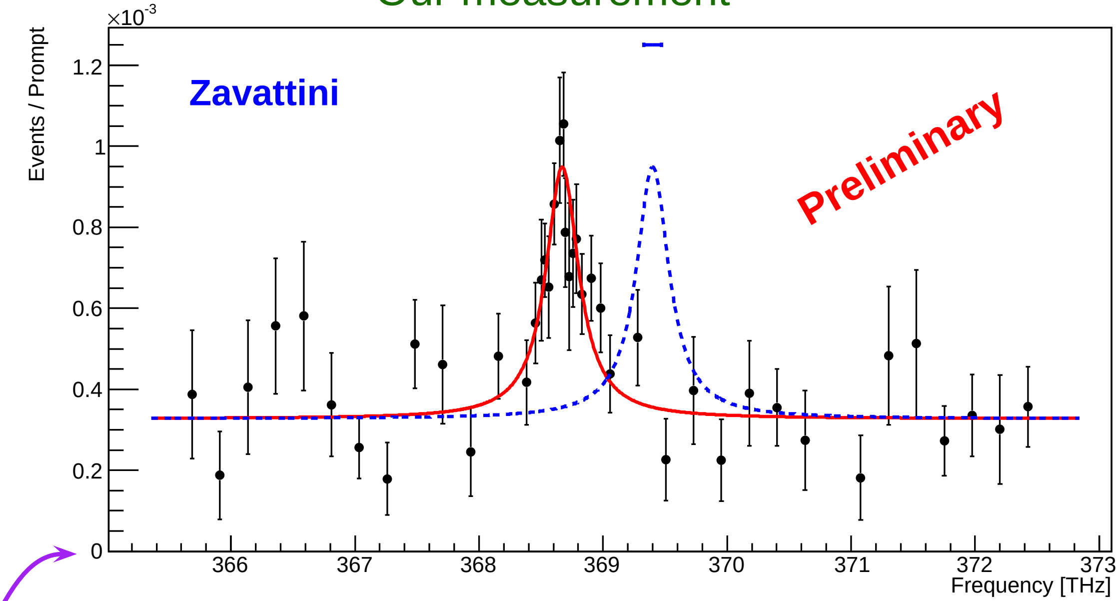
Zavattini radius seems apparently correct
but it results from a wrong experiment
combined with an incomplete theory!

[Carboni et al., Nucl. Phys. A 278, 381 (1977)]



OFFSET

Our measurement



Zavattini experiment was performed at 50 bar pressure:

$\Rightarrow$ 2S-population is collisionally quenched.

$\Rightarrow$ No population left for a laser experiment.

(for comparison: we are measuring $\mu\,\mathrm{He}^+$ at 2 mbar)

[Hauser et al., PRA 46, 2363 (1992)]

Conclusions

We have measured the $2S_{1/2} - 2P_{3/2}$ transition in $\mu\,{}^4\mathrm{He}^+$ with $u_r = 5 \times 10^{-5}$.
$\longrightarrow$ extract ^{4}He charge radius with $u_r = 3 \times 10^{-4}$
$\longrightarrow$ agreement with the e-scattering value ($u_r = 2 \times 10^{-3}$)
$\longrightarrow$ important information for the proton puzzle (spin-, isospin-dependence etc.)
$\longrightarrow$ interesting information for few-nucleons theory, to disentangle potentials....

2S-2P th. is converging.

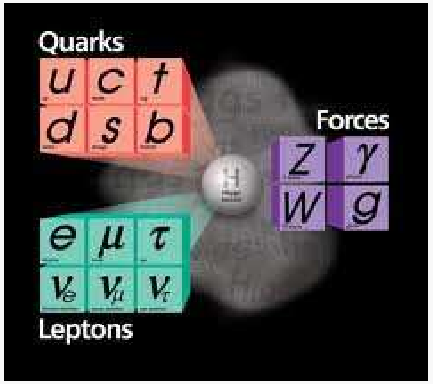
Statistical accuracy of $\mu\,\mathrm{He}^+$ meas.	20 GHz
Systematics	<0.1 GHz
Natural linewidth	320 GHz
Uncertainty third Zemach	50 GHz
Uncertainty nucl. pol.	36 GHz

Missing:

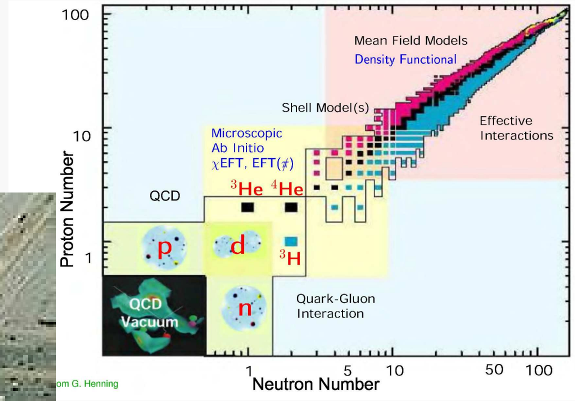
- Intrinsic nucleon polarizability for $\mu\,\mathrm{He}^+$
- Charge distribution dependence of theory?
- Polarizability contr. to Lamb shift for $\mu\,{}^3\mathrm{He}^+$
- Polarizability contr. to HFS for $\mu\,{}^3\mathrm{He}^+$ and $\mu\mathrm{d}$

- Would be interesting to have a determinations of the He charge radius from few-nucleon th.
- Would be interesting to have calculations of polarizability using He breakup data
- Would be interesting to have a better ^{3}He charge radius from scattering.
- Would be advantageous/possible in e-scattering to measure cross sections ratio of H/He?

Motivation, summary, outlook



Test of H energy levels
Bound-state QED



New physics?

Low-energy QCD
EFT, χpt, lattice
strong bound-state
p-structure
few-nucleon th.

Scattering
$e + p \to e + p$
$e + d \to e + d$
$\mu + p \to \mu + p$
$\gamma + p \to \gamma + p$
...

H, He, He$^+$,
$\mu^+ e^-$, $e^+ e^-$
spectroscopy

μp, μd, μHe$^+$

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