Lattice QCD and Nucleon Form factors

C. Alexandrou
University of Cyprus, The Cyprus Institute and DESY-Zeuthen

Proton Radius Puzzle Workshop

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Outline

Quantum ChromoDynamics (QCD)

QCD-Gauge theory of the strong interaction
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t}\bar{\psi}_f\left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$$

Harald Fritzsch

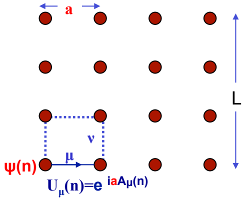
Murray Gell-Mann

Heinrich Leutwyler

This “simple” Lagrangian produces the amazingly rich structure of strongly interacting matter in the universe.

Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena
→ In this talk: **Nucleon Form Factors**

QCD on the lattice



Lattice QCD: K. Wilson, 1974 provided the formulation; M. Creutz, 1980 performed the first numerical simulation

- Discretization of space-time with lattice spacing a: quark fields $\psi(x)$ and $\bar{\psi}(x)$ on lattice sites and gauge field $U_\mu(x)$ on links
- Finite a provides an ultraviolet cutoff at π/a → non-perturbative regularization; Finite L → discrete momenta in units of $2\pi/L$ if periodic b.c.
- Construct an appropriate action S and rotate into imaginary time → Monte Carlo simulation to produce a representative ensemble of $\{U_\mu(x)\}$ using the largest supercomputers →
  Observables: $\langle\mathcal{O}\rangle = \sum_{\{U_\mu\}} O(D^{-1}, U_\mu)$, D^{-1} is the fermion propagator

5.0 Pflop/s, biggest in Europe and 7th in the world

Fermion action

Several $\mathcal{O}(a)$-improved fermion actions, K. Jansen, Lattice 2008

Action	Advantages	Disadvantages
Clover improved Wilson	computationally fast	breaks chiral symmetry needs operator improvement
Twisted mass (TM)	computationally fast automatic improvement	breaks chiral symmetry violation of isospin
Staggered	computational fast	four doublers (fourth root issue) complicated contractions
Domain wall (DW)	improved chiral symmetry	computationally demanding needs tuning
Overlap	exact chiral symmetry	computationally expensive

Several collaborations:

Clover	QCDSF, BMW, ALPHA, CLS, PACS-CS, NPQCD
Twisted mass	ETMC
Staggered	MILC
Domain wall	RBC-UKQCD
Overlap	JLQCD

Systematic uncertainties

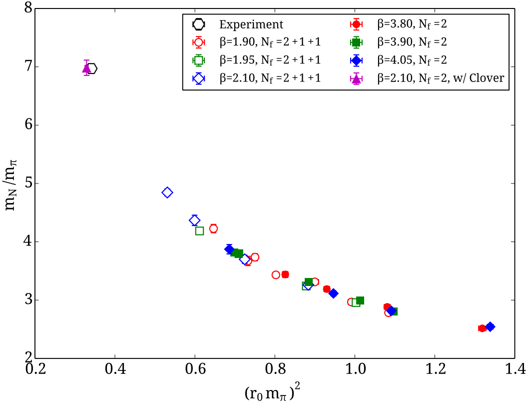
- Finite lattice spacing a - take the continuum limit $a \to 0$
- Finite volume L - take infinite volume limit $L \to \infty$
- Identification of hadron state of interest - g_A, σ-terms, EM form factors
- Simulation at physical quark masses - now feasible
- Inclusion of quark loop contributions - now feasible

Recent achievements

Simulation with physical quark masses

A number of collaborations are producing simulations with physical values of the quark mass

European Twisted Mass Collaboration (ETMC):



$L \sim 3$ fm and $a \sim 0.1$ fm; $r_0 \sim 0.5$ fm

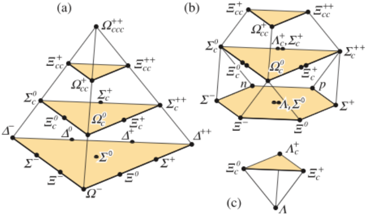
Noise to signal increases with t_s:$\sim e^{(m_h - \frac{3}{2} m_\pi) t_s}$

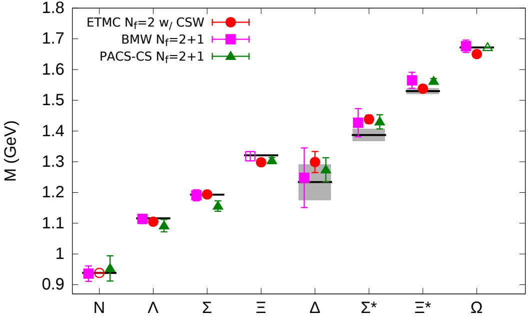
Hadron spectrum

SU(4) representations:

$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$$

$$\square \otimes \square \otimes \square = \square\square\square \oplus \ldots$$





Results by ETMC using simulations with physical pion mass, C.A., V. Drach, K. Jansen, Ch. Kallidonis and G. Koutsou

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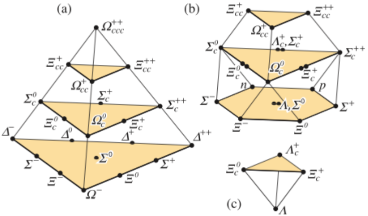

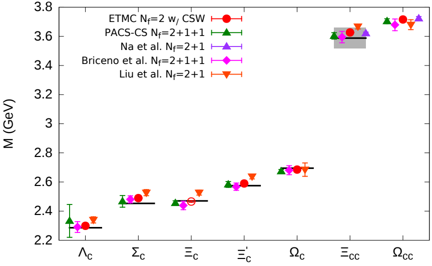

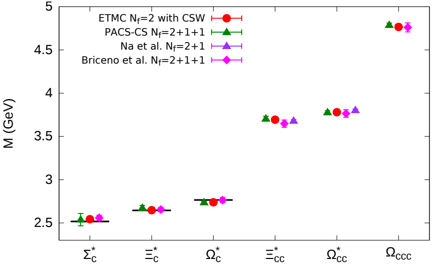

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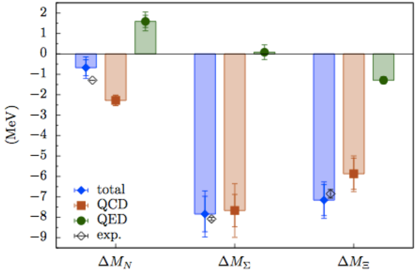
Isospin and electromagnetic mass splitting

RBC and BMW collaborations: Treat isospin and electromagnetic effects to LO



Baryon spectrum with mass splitting from BMW

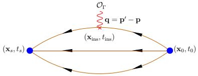
- Nucleon mass: isospin and electromagnetic effects with opposite signs
- Physical splitting reproduced

Nucleon Structure

Methods for hadron structure

Evaluation of three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins})\overline{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta\gg 1]{(t_{\rm ins}-t_0)\Delta\gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins}-t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

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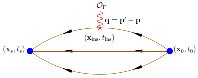

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- $\mathcal{M}$ the desired matrix element
- t_s, t_{ins}, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

Summing over t_{ins}:

$$\sum_{t_{ins}=t_0}^{t_s} R(t_s, t_{ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

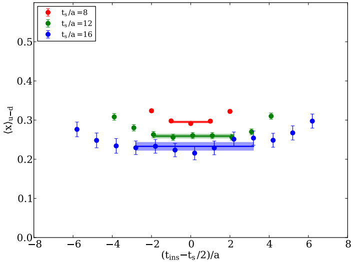
So the excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{ins}$ and/or $t_{ins} - t_0$.

However, one needs to fit the slope rather than to a constant

Connect lattice results to measurements:

$\mathcal{O}_{\overline{MS}}(\mu) = Z(\mu, a)\mathcal{O}_{latt}(a)$

$\Longrightarrow$ evaluate $Z(\mu, a)$ non-perturbatively

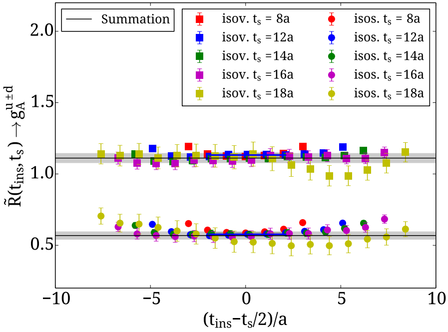


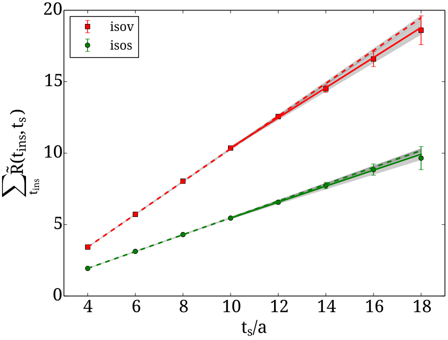
Axial charge g_A

The good news:

Axial-vector FFs: $A^3_\mu = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow \frac{1}{2}\bar{u}_N(\vec{p}')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]u_N(\vec{p})|_{q^2=0}$
$\rightarrow$ yields $G_A(0) \equiv g_A$: i) well known experimentally, & ii) no quark loop contributions

$N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, 1200 statistics





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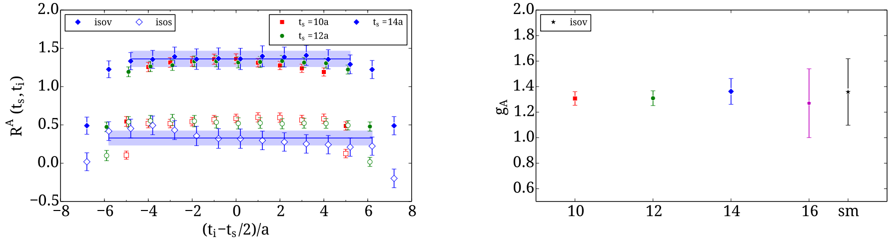
$N_f = 2$ twisted mass plus clover, a=0.091 fm, m_π=134 MeV, 1020 statistics



- No detectable excited states contamination
- Consistent results between summation and plateau methods

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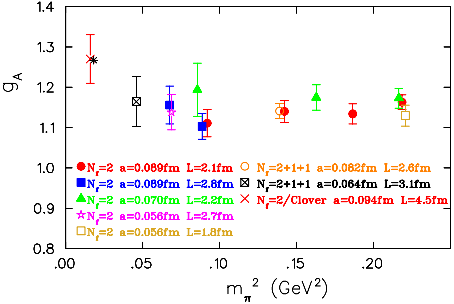

Results from ETMC at physical point mass

- Results at physical pion mass are now becoming available $\rightarrow$ need a dedicated study with high statistics, a larger volume and 3 lattice spacings

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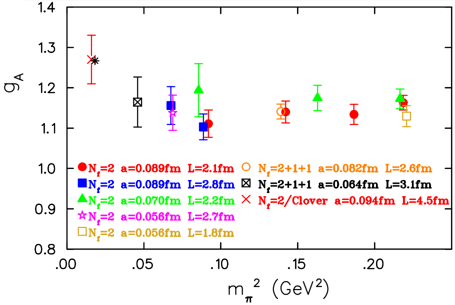
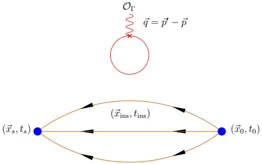


Results from ETMC at physical point mass

- Results at physical pion mass are now becoming available → need a dedicated study with high statistics, a larger volume and 3 lattice spacings
- A number of collaborations are engaging in systematic studies, e.g.
  - $N_f = 2+1$ Clover, J. R. Green *et al.*, arXiv:1209.1687
  - $N_f = 2$ Clover, R.Hosley *et al.*, arXiv:1302.2233
  - $N_f = 2$ Clover, S. Capitani *et al.* arXiv:1205.0180
  - $N_f = 2+1$ Clover, B. J. Owen *et al.*, arXiv:1212.4668
  - $N_f = 2+1+1$ Mixed action (HISQ/Clover), T. Bhattacharya *et al.*, arXiv:1306.5435
  - $N_f = 2+1$ Domain wall fermions, S. Ohta *et al.*, RBC-UKQCD

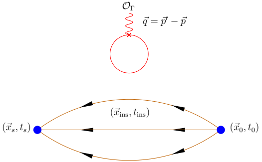
Disconnected quark loop contributions



The good news

- Notoriously difficult:
  - $L(x_{\rm ins}) = Tr\,[\Gamma G(x_{\rm ins}; x_{\rm ins})]$ $\rightarrow$ need quark propagators from all $\vec{x}_{\rm ins}$ or L^3 more expensive as compared to the calculation of hadron masses
  - Large gauge noise $\rightarrow$ large statistics
- Use special techniques that utilize stochastic noise on all spatial lattice sites $\rightarrow$ N_r more expensive that hadron masses with $N_r \ll L^3$
- Reduce noise by increasing statistics
  $\Longrightarrow$ take advantage of graphics cards (GPUs) $\rightarrow$ need to develop special multi-GPU codes

Disconnected quark loop contributions



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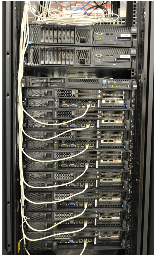
A Fermi card



Cluster of 8 nodes of Fermi GPUs at the Cyprus Institute

C. A., M. Constantinou, S. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, G. Koutsou, A. Strelchenko, A. Vaquero arXiv:1211.0126
C.A., K. Hadjiyiannakou, G. Koutsou, A. O'Cais, A. Strelchenko, arXiv:1108.2473

Axial charge g_A

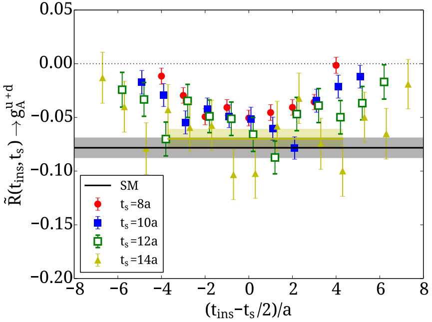
The good news

To compute $\Delta\Sigma^q$ we need also the isoscalar g_A^{u+d}

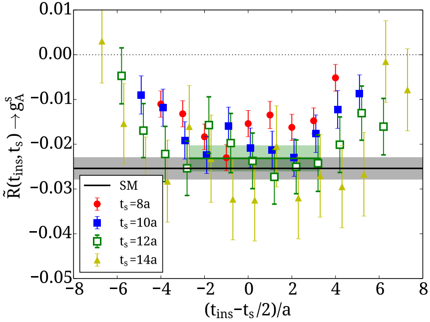
Dedicated high statistics study

Choose one ensemble to perform a high statistics analysis for all disconnected contributions to nucleon observables

$N_f = 2 + 1 + 1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, $\sim$ **150, 000** statistics (on 4700 confs)



Disconnected isoscalar, agrees with Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001

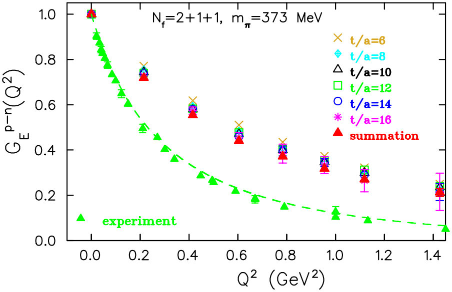


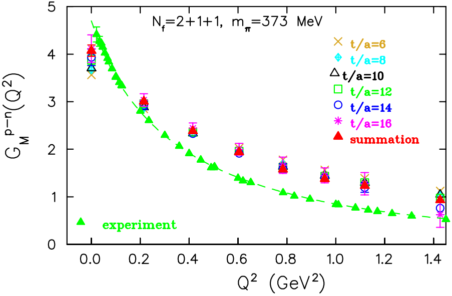
Strange quark loop

Electromagnetic form factors

$N_f = 2 + 1 + 1$ twisted mass, a = 0.082 fm, m_π = 373 MeV

Connected → isovector: ∼ **1200** statistics





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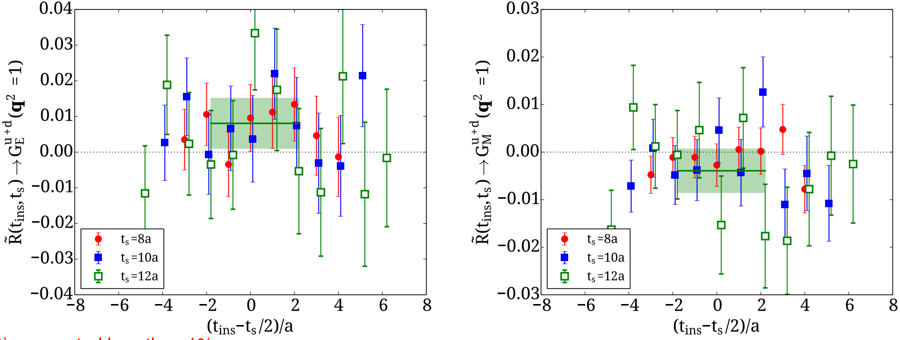


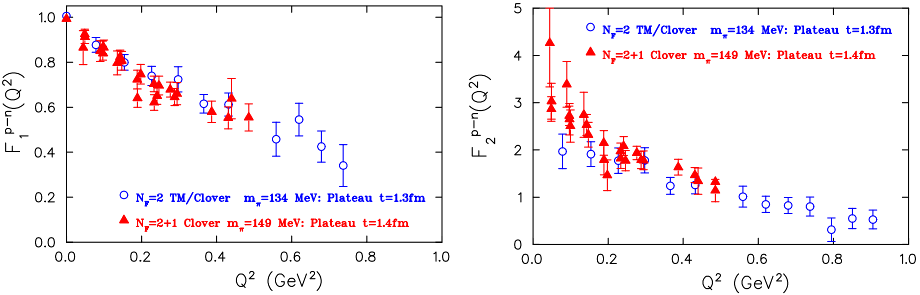
Disconnected less than 1%

Electromagnetic form factors

The good news

Two studies at near physical pion mass:

- ETMC: $N_f = 2$ twisted mass with clover, a = 0.091 fm, m_π = 134 MeV, 1020 statistics
- MIT: $N_f = 2 + 1$ clover produced by the BMW collaboration, $a = 0.116$ MeV, $m_\pi = 149$ MeV, ∼7750 statistics, J.M. Green *et al.* 1404.4029

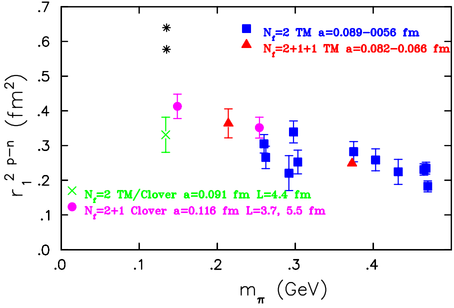


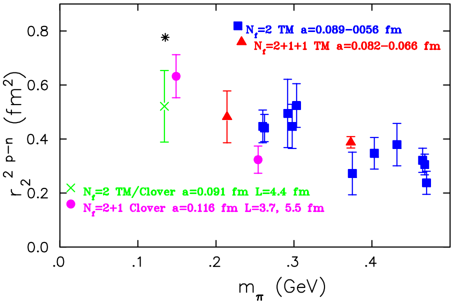
Agreement even before taking the continuum limit

BUT...

Dirac and Pauli radii

Dipole fits: $\frac{G_0}{(1+Q^2/M^2)^2} \Rightarrow \langle r_i^2 \rangle = -\frac{6}{F_i}\frac{dF_i}{dQ^2}|_{Q^2=0} = \frac{12}{M_i^2}$





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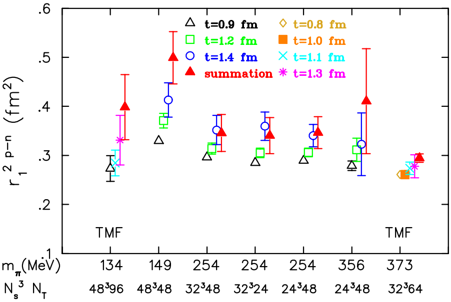

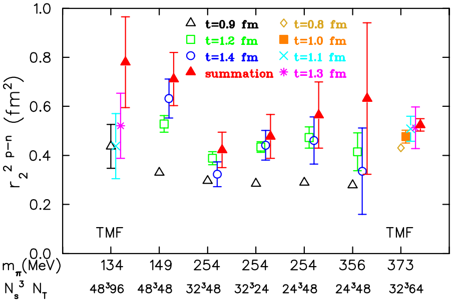

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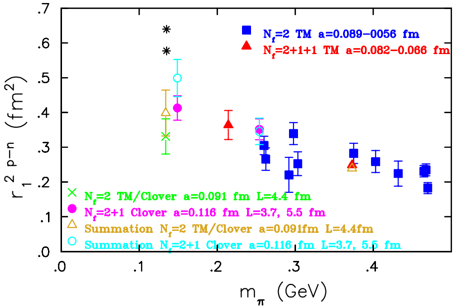

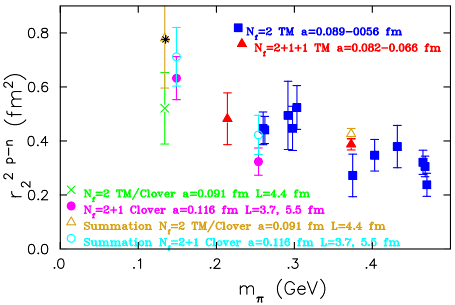

Need better accuracy at the physical point

Dirac and Pauli radii

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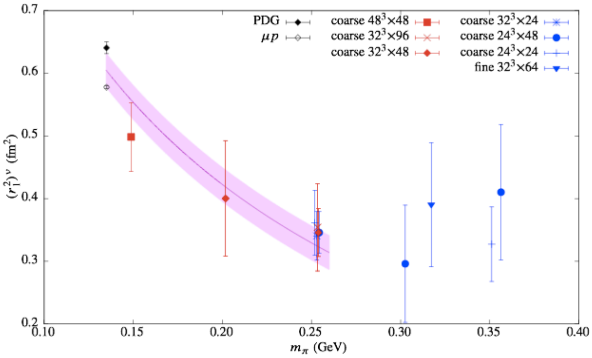
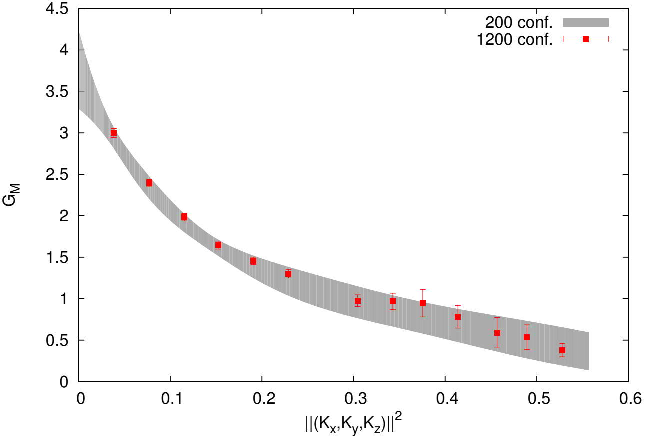


Using results from summation method

Momentum dependence of form factors

Avoid model dependence-fits: As a first step we calculated $G_M(0)$ or $F_2(0)$ at $m_\pi = 373$ MeV



Work in progress, C.A., G. Koutsou, K. Ottnad, M. Petschlies

Conclusions

Nucleon structure is a benchmark for the Lattice QCD approach

- Simulations at the physical point reproduce g_A $\rightarrow$ need high statistics and careful cross-checks
- Evaluation of disconnected quark loop diagrams has become feasible addressing an up to now unknown systematic error $\rightarrow$ but need high statistics and access to exascale computer resources
- The study of excited states and resonances is under way
- Errors are large $\rightarrow$ noise reduction techniques crucial

Many challenges still ahead but...

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Many challenges still ahead but...
as simulations at the physical pion mass and more computer resources are becoming available we expect many physical results on key hadron observables with increased accuracy

Acknowledgments

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
A. Abdel-Rehim, M. Constantinou, V. Drach, K. Hadjiyiannakou, K.Jansen
Ch. Kallidonis, G. Koutsou, M. Petschlies, A. Strelchenko, A. Vaquero

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