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Re-Examination of the Shape of G_E^p Data

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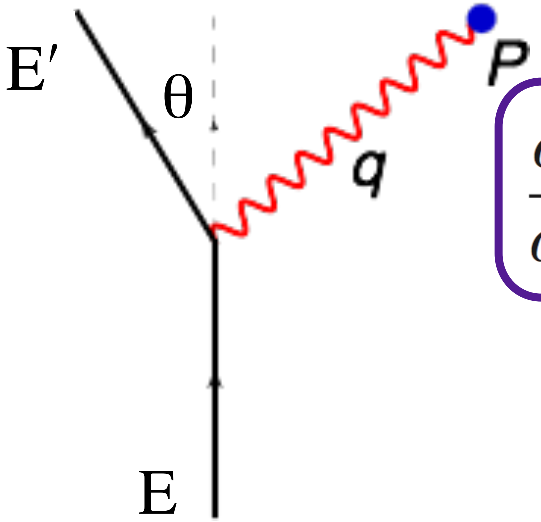
ep Elastic Scattering

- The Mainz *ep* elastic scattering data for $Q^2 < 1$ GeV^2 of Bernauer *et al.* are the best in the world.
- What do they tell us about the proton charge radius?
- Exploration by Carl Carlson, Sarah Maddox and KG

Elastic Scattering



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left\{ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2} \right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \cos^2\frac{\theta}{2}$$

$$\tau = \frac{\nu^2}{Q^2} = \frac{1}{\gamma^2} = \frac{Q^2}{4M^2} \qquad Q^2 = -q \cdot q = 4EE' \sin^2\frac{\theta}{2}$$

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \left\{ \frac{G_E^2 + \frac{\tau}{\epsilon} G_M^2}{1+\tau} \right\}$$

From σ to G_E

$$\sigma_r = \sigma/\sigma_D$$

$$G_D = \frac{1}{\left(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2}\right)^2}$$

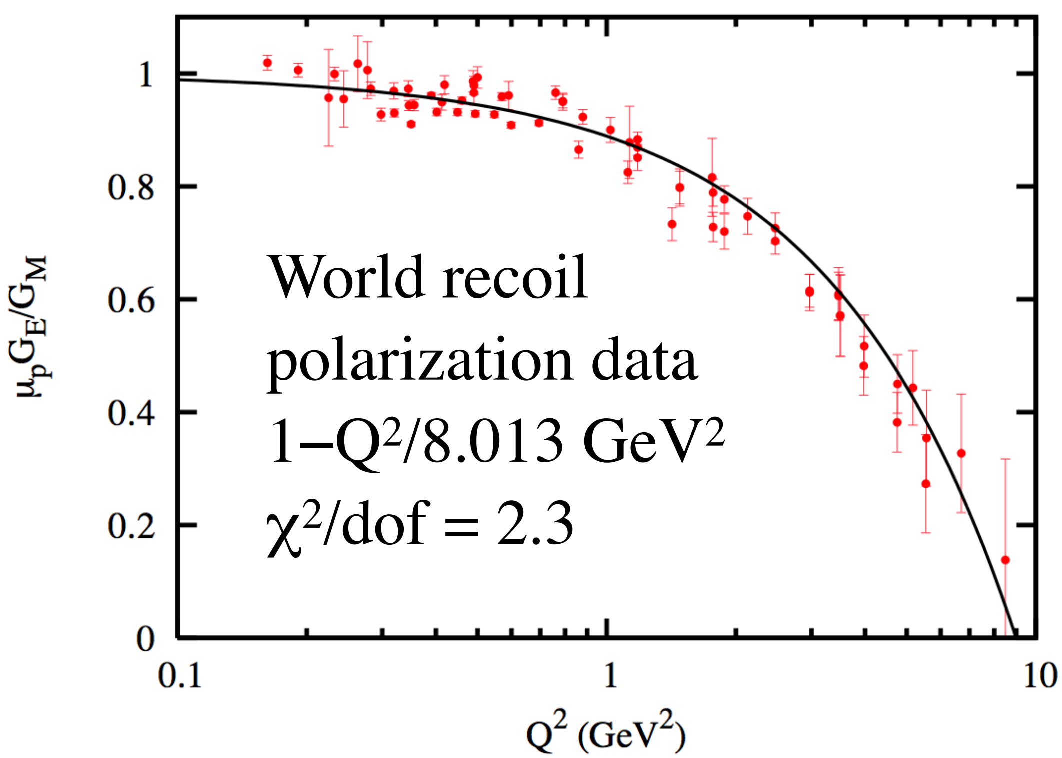
$$\frac{G_E}{G_M} \approx \frac{1}{\mu_p}\left(1 - \frac{Q^2}{8\ \mathrm{GeV}^2}\right)$$

$$G_E = G_D \sqrt{\frac{\sigma_r\left(1 + \frac{\tau}{\epsilon}\mu_p^2\right)}{1 + \frac{\tau}{\epsilon}\left(\frac{G_E}{G_M}\right)^{-2}}}$$

$$G_E = G_D \sqrt{\frac{\sigma_r\left(1 + \frac{\tau}{\epsilon}\mu_p^2\right)}{1 + \frac{\tau}{\epsilon}\mu_p^2\left(1 - \frac{Q^2}{8\ \mathrm{GeV}^2}\right)^{-2}}}$$

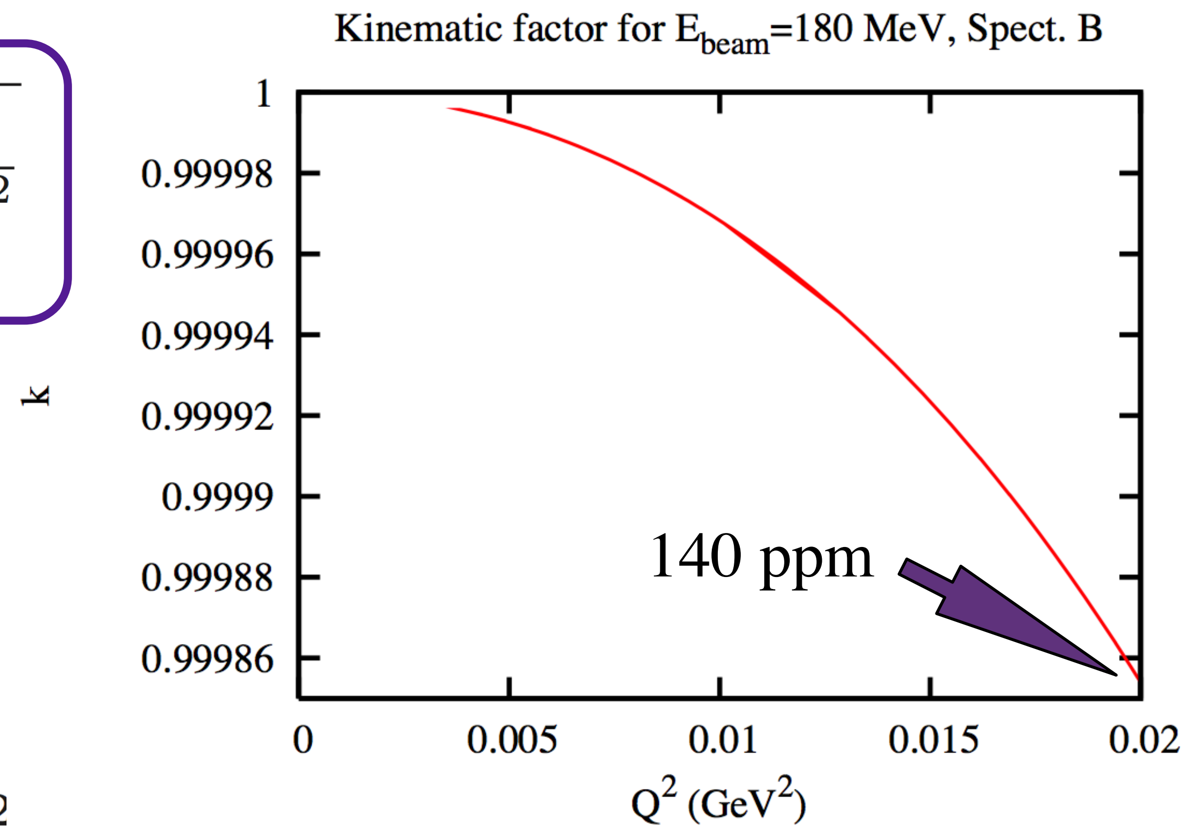
$$k = \sqrt{\frac{1 + \frac{\tau}{\epsilon}\mu_p^2}{1 + \frac{\tau}{\epsilon}\mu_p^2\left(1 - \frac{Q^2}{8\ \mathrm{GeV}^2}\right)^{-2}}}$$

$$G_E = G_D k \sqrt{\sigma_r}$$



Low Q^2

$$k = \sqrt{\frac{1 + \frac{\tau}{\epsilon}\mu_p^2}{1 + \frac{\tau}{\epsilon}\mu_p^2\left(1 - \frac{Q^2}{8\ \mathrm{GeV}^2}\right)^{-2}}}$$



$$G_E = 1 - \frac{1}{6} r_E^2 Q^2$$

$$\frac{1}{\mu_p} G_M = 1 - \frac{1}{6} r_M^2 Q^2$$

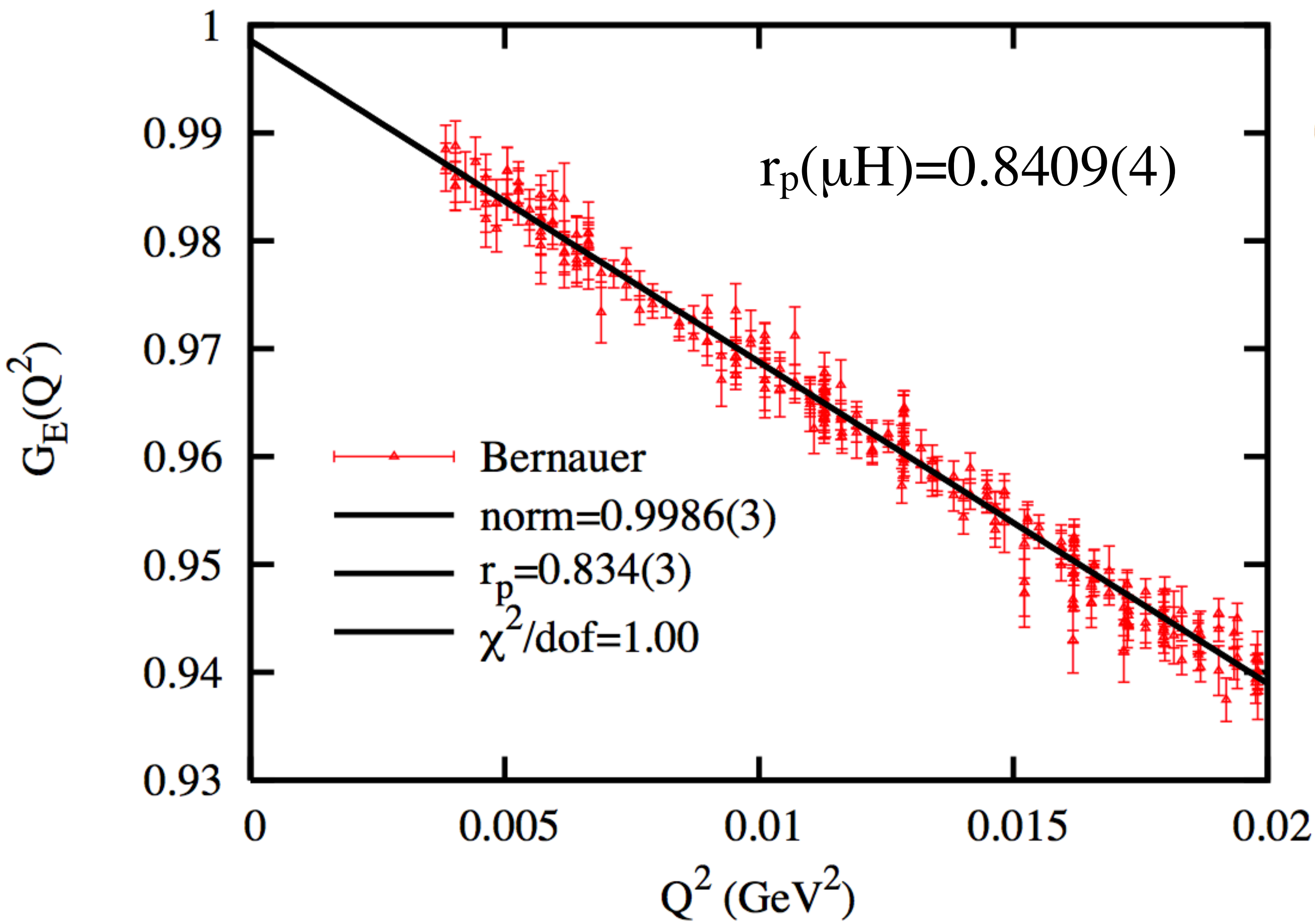
$$\mu_p \frac{G_E}{G_M} = 1 - \frac{1}{6}(r_E^2 - r_M^2) Q^2$$

$$G_{E,M}(Q^2) \propto 1 - \frac{1}{6}\langle r_{E,M}^2\rangle Q^2 + \frac{1}{120}\langle r_{E,M}^4\rangle Q^4 - \frac{1}{5040}\langle r_{E,M}^6\rangle Q^6 + \ldots$$

$$\sigma_r = \frac{G_E^2}{G_D^2}$$

Non-relativistic

Linear Fit at Low Q^2



$$G_E = 1 - \frac{r_p^2 Q^2}{6\hbar^2 c^2}$$

- Only B spectrometer
- 3 of 34 norm sets
- 166 points

- Extracted r_p is "too small"
- We cannot ignore curvature

$r_p=0.849\pm0.019$ with Q^4 term

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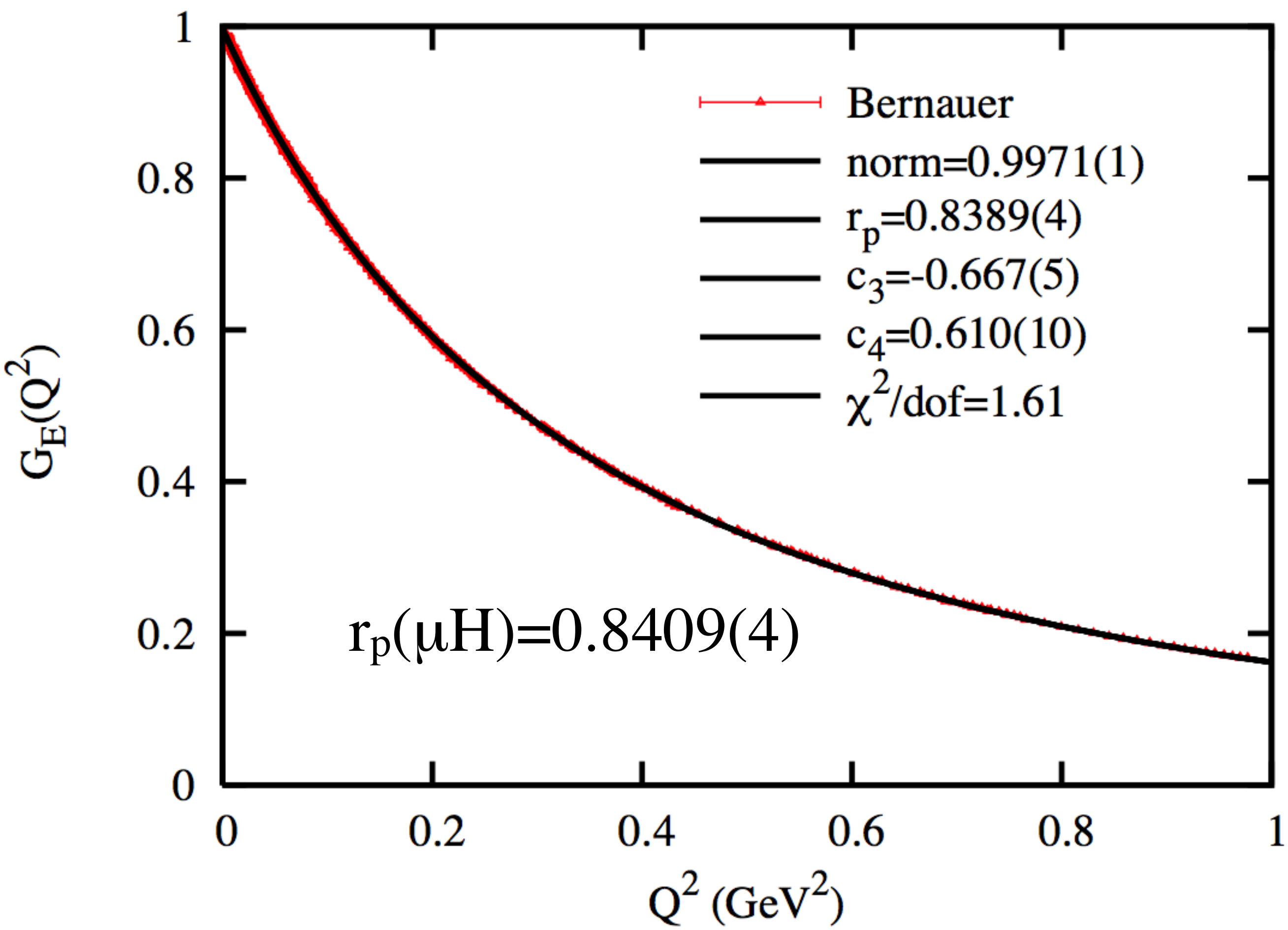
Continued Fraction Fit

$$\frac{dG_E}{dQ^2}\Big|_{Q^2=0} = -\frac{r_p^2}{6\hbar^2 c^2}$$

$$f(Q^2) = \frac{c_1}{1+\frac{c_2 Q^2}{1+\frac{c_3 Q^2}{1+\frac{c_4 Q^2}{1+\dots}}}}$$

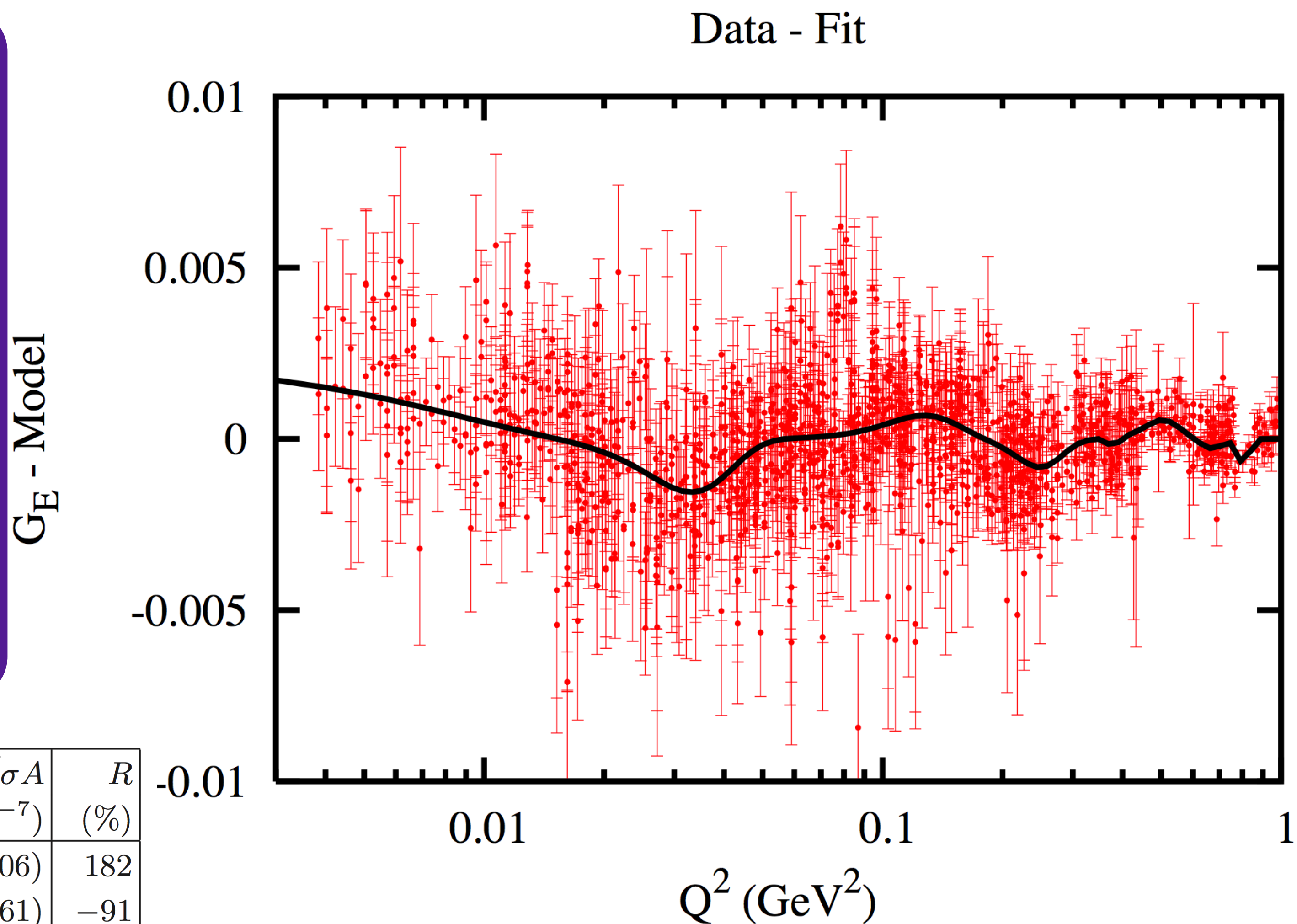


$$c_2 = (r_p \hbar c)^2/6$$

- Out of the box, the Mainz data (1422 points) yields a small radius
- Is χ^2 too big?

Warts and All

- Wart-finding: fit a Gaussian with $0.006 < \sigma < 0.06$
- Peaks are 50−100% of errors
- Integrals are statistically significant



A ($\times 10^{-6}$)	Q_0^2 (GeV^2)	σ ($\times 10^{-4}$)	$\sqrt{2\pi}\sigma A$ ($\times 10^{-7}$)	R (%)
3092(16940)	−0.006(57)	87(164)	674(3906)	182
−1559(216)	0.033(1)	82(11)	−319(61)	−91
687(133)	0.127(6)	264(60)	456(137)	58
−832(158)	0.248(8)	326(75)	−608(203)	−76
−925(594)	0.380(2)	59(34)	−137(117)	−103
554(139)	0.504(16)	509(154)	707(279)	55
−281(103)	0.670(21)	414(201)	−292(177)	−56
735(270)	0.805(10)	276(84)	−508(243)	146

- $G(Q^2)=A\exp[(Q^2-Q_0^2)^2/2\sigma^2)]$
- $R = A / \sigma_{GE}$
- If one peak is real, all are real

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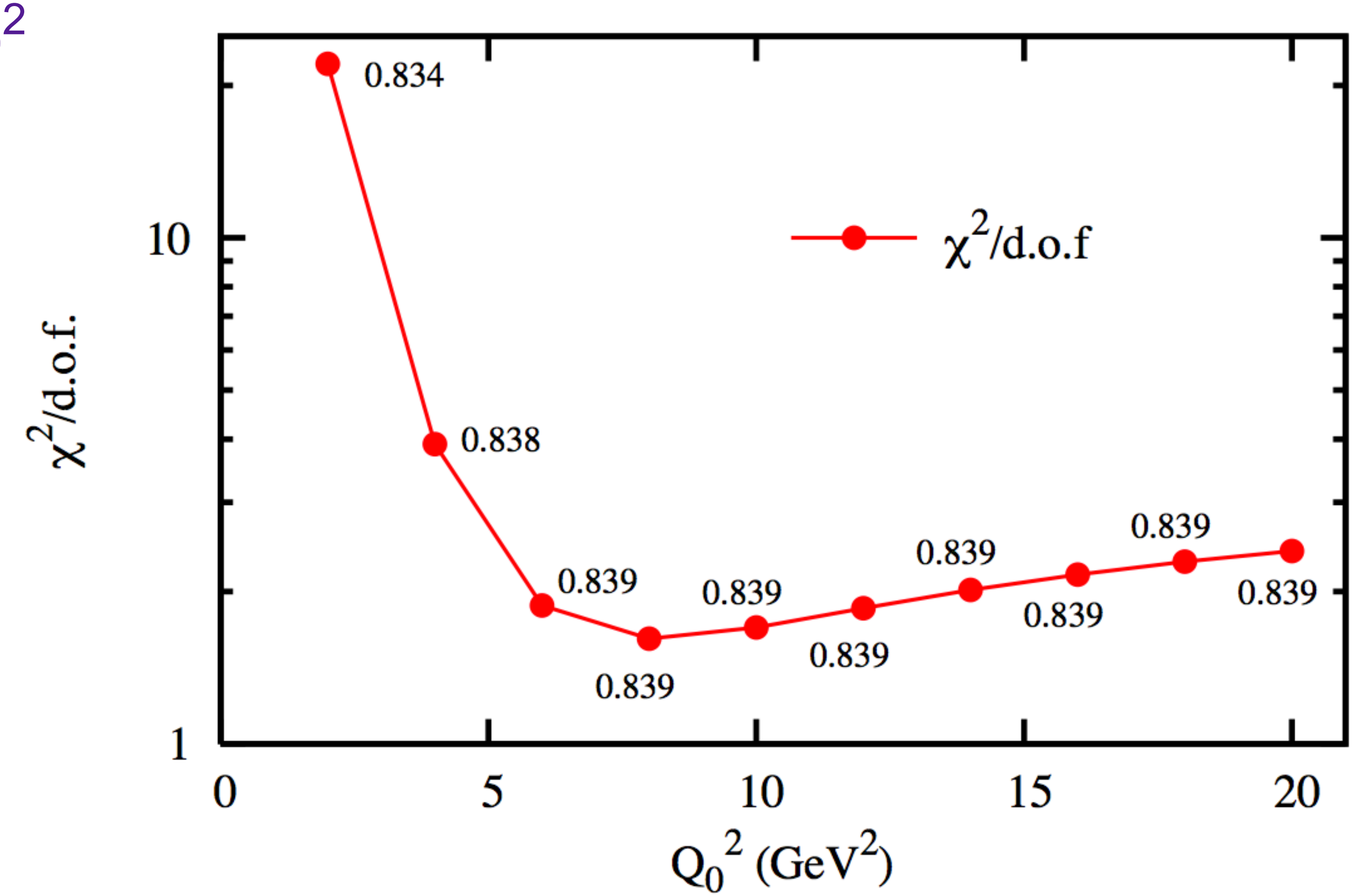
Rosenbluth?

What do the data tell us about G_E/G_M?

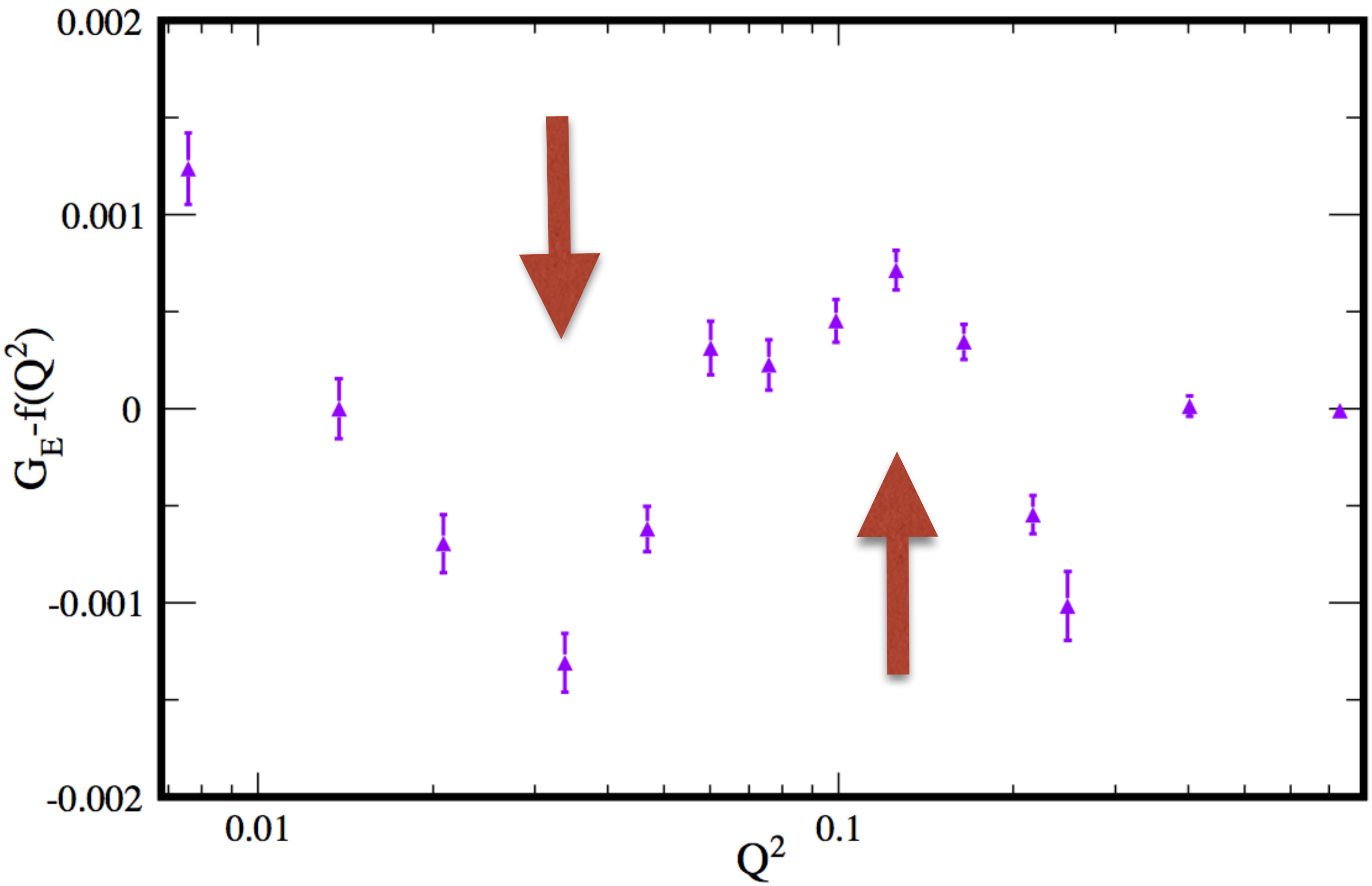
- Look at χ^2/dof for various Q_0^2
- Minimum for $Q_0^2 \approx 8$ GeV^2
- Implies that the applied 2-photon corrections are reasonable
- r_p is insensitive to Q_0^2

$G_E/G_M = 1 - Q^2/Q_0^2$



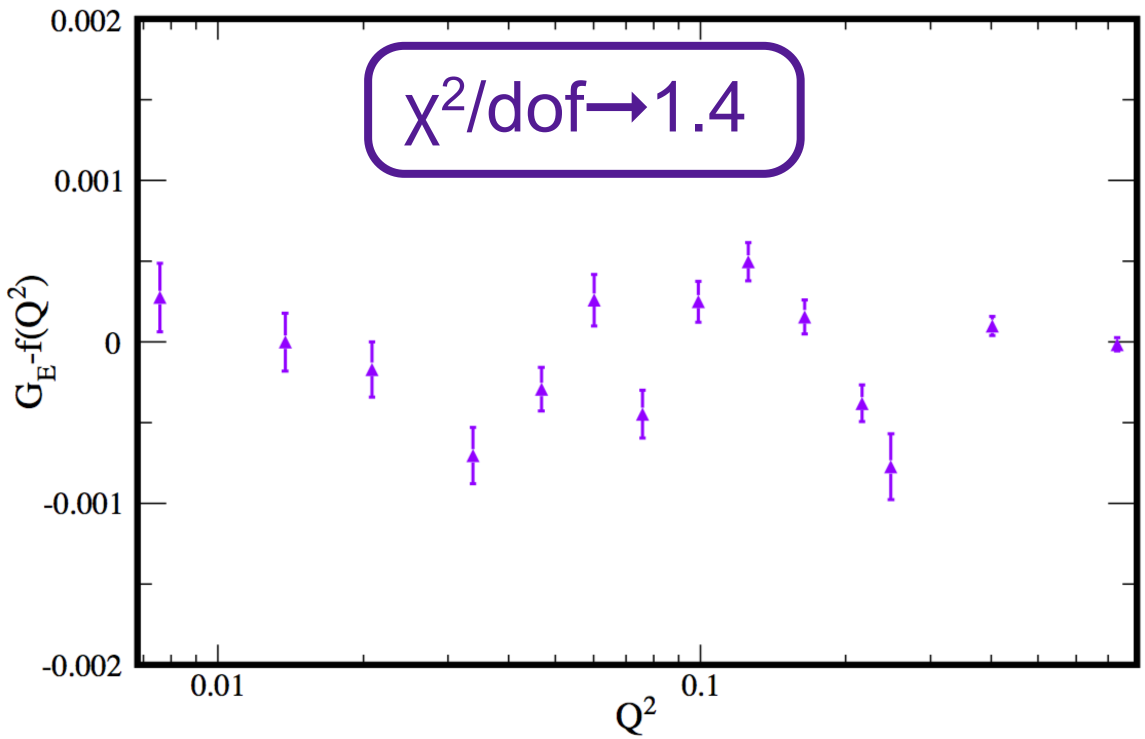
Warts Revisited

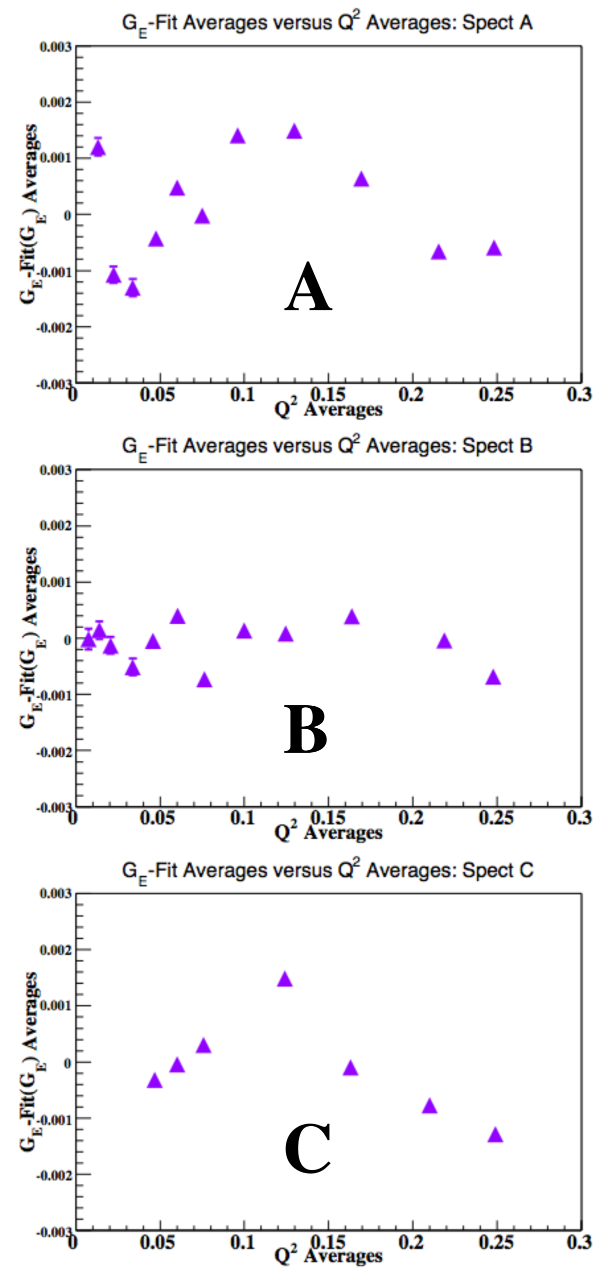
- Bin ~100 points together to visualize trends
- Systematic variations on the order of 0.001, on the order of but smaller than the individual error bars.
- Are these variations real?



Renormalization of 34 Sets

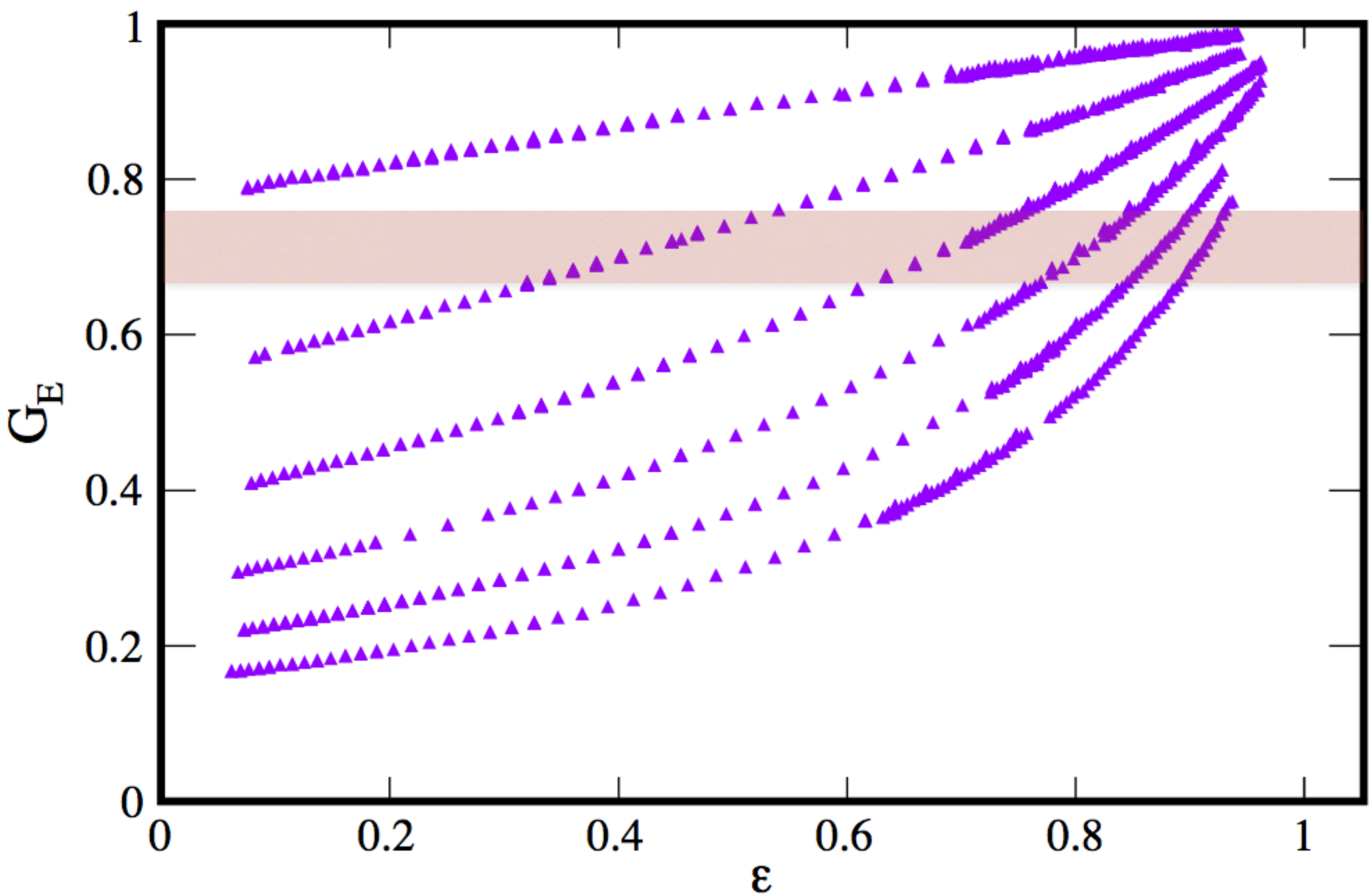
- For each set determine the weighted ratio of G_E to the fit; correct G_E by this ratio
- Ratios are ≲ 0.2% from unity, well within uncertainties of normalization
- G_E-f(Q^2) is now much flatter, especially for Spect. B (low Q^2).





G_E vs. ε

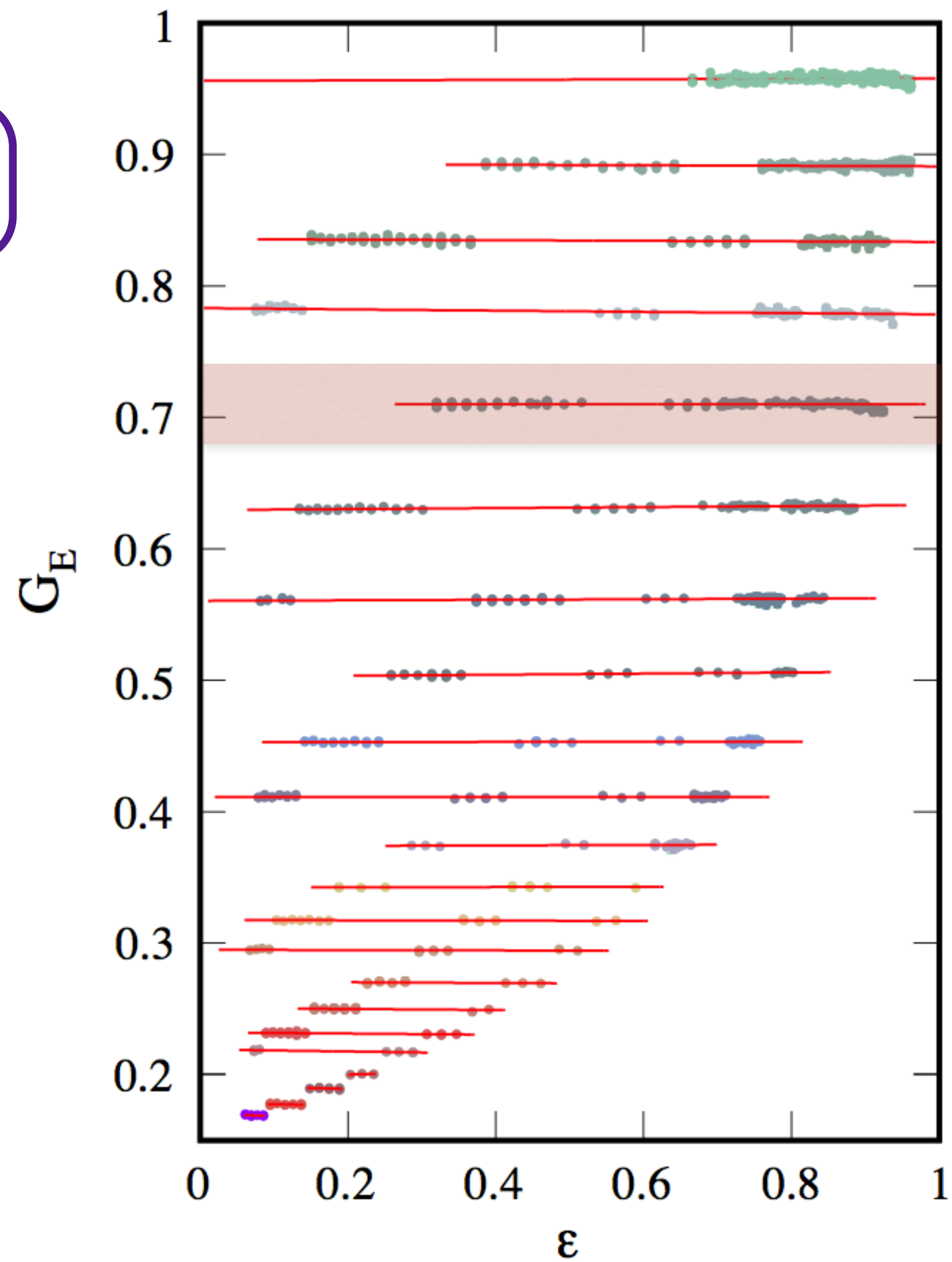
- 1422 extracted G_E values plotted versus ε
- Six curves correspond to the 6 beam energies



ε-Dependence

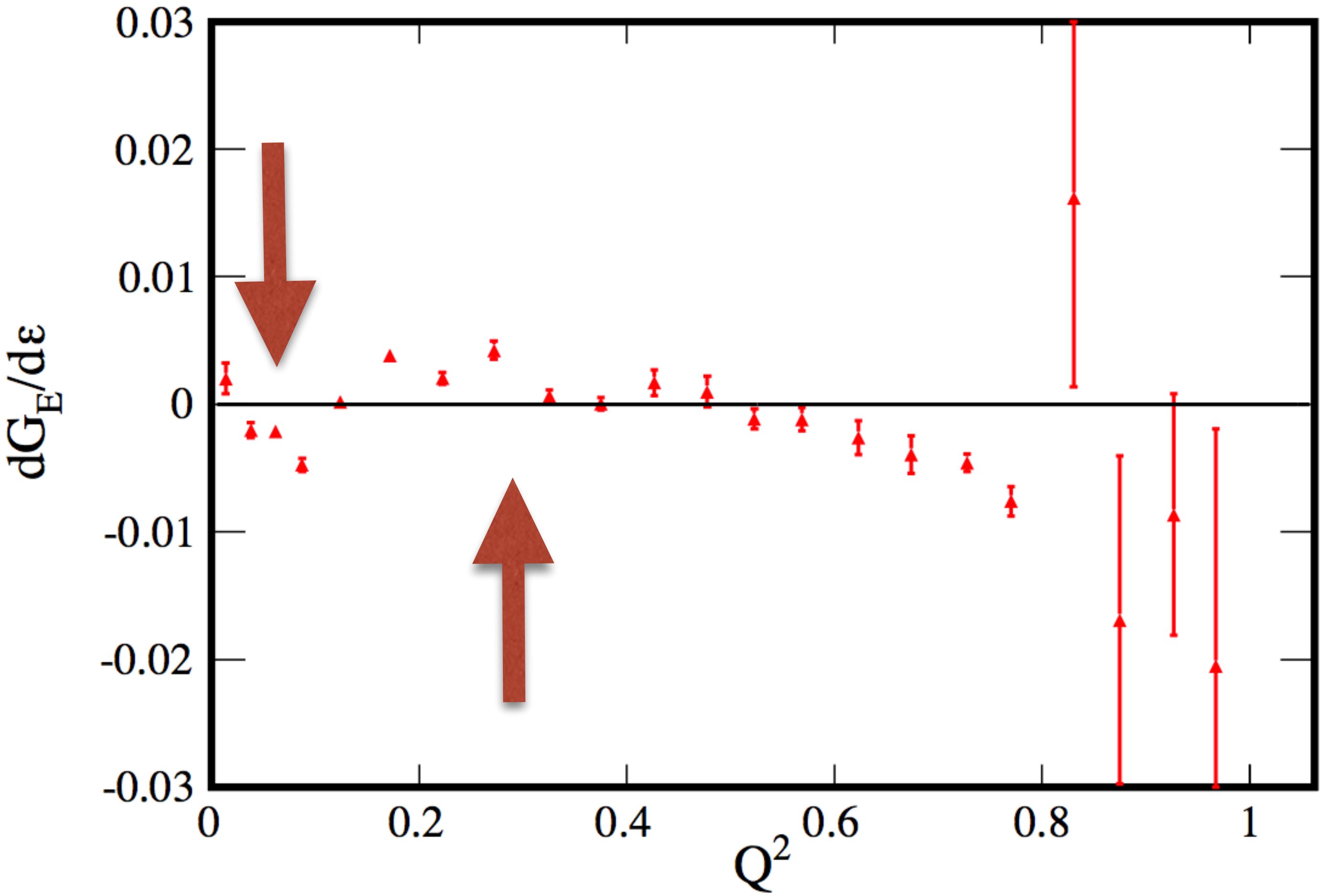
$$G_E(Q^2_{\text{average}}) = G_E(Q^2{}_{\text{measured}}\frac{f(Q^2{}_{\text{average}})}{f(Q^2{}_{\text{measured}})}$$

- Check for ε (beam energy) dependence in G_E
- For each bin in G_E, evolve to a common Q^2 using the fit
- Fit a line versus ε for each bin in G_E



ε or E_{beam}?

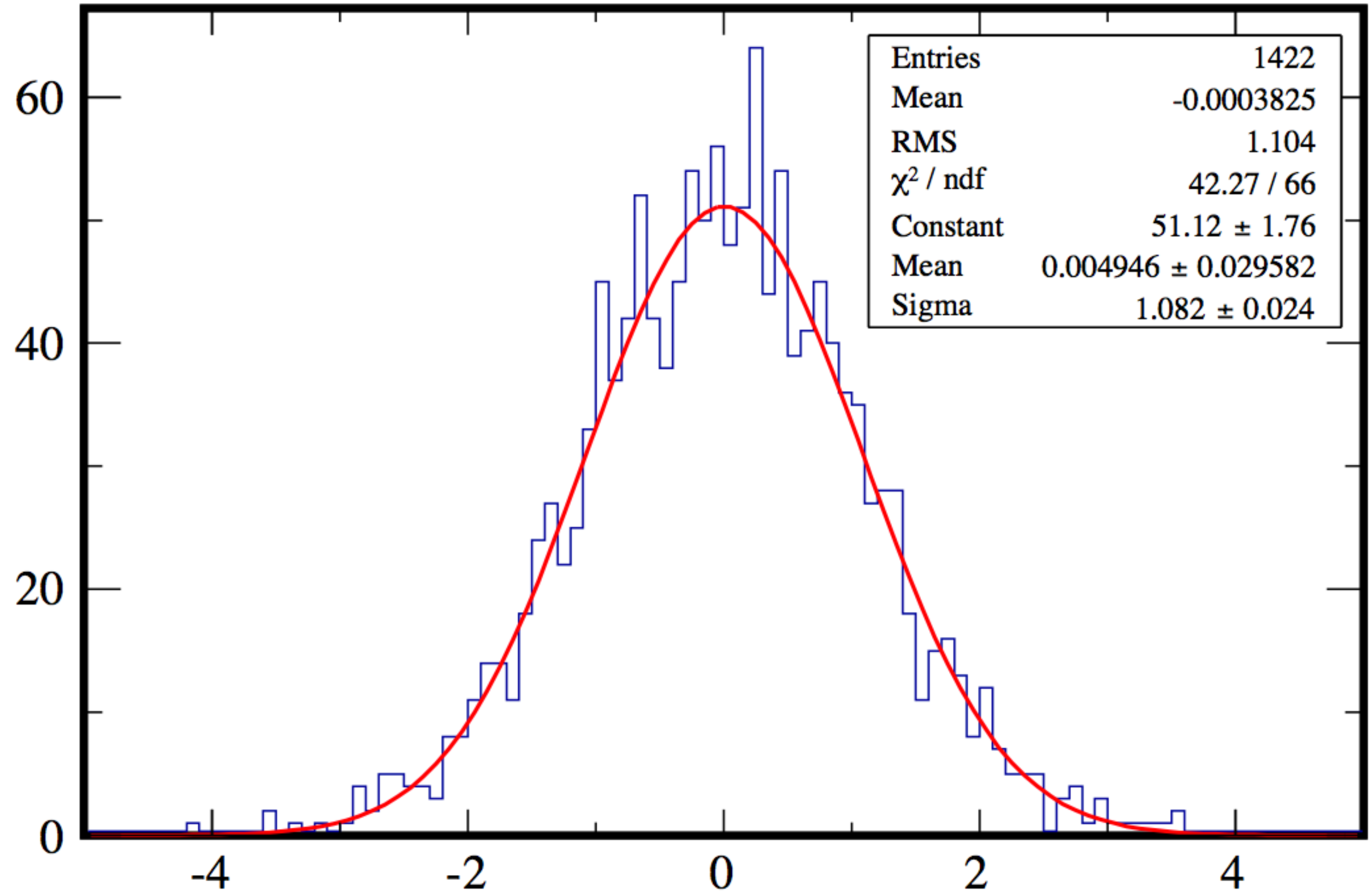
- |ε-slopes| ≲ 0.005
- At low Q^2 G_M does not contribute, so ε dependence is really an E_{beam} dependence
- Variations from zero are well within the error budget for absolute norms



Largest deviations in ε-slopes and in G_E occur at similar Q^2. Coincidence? Systematic wandering of the dataset?

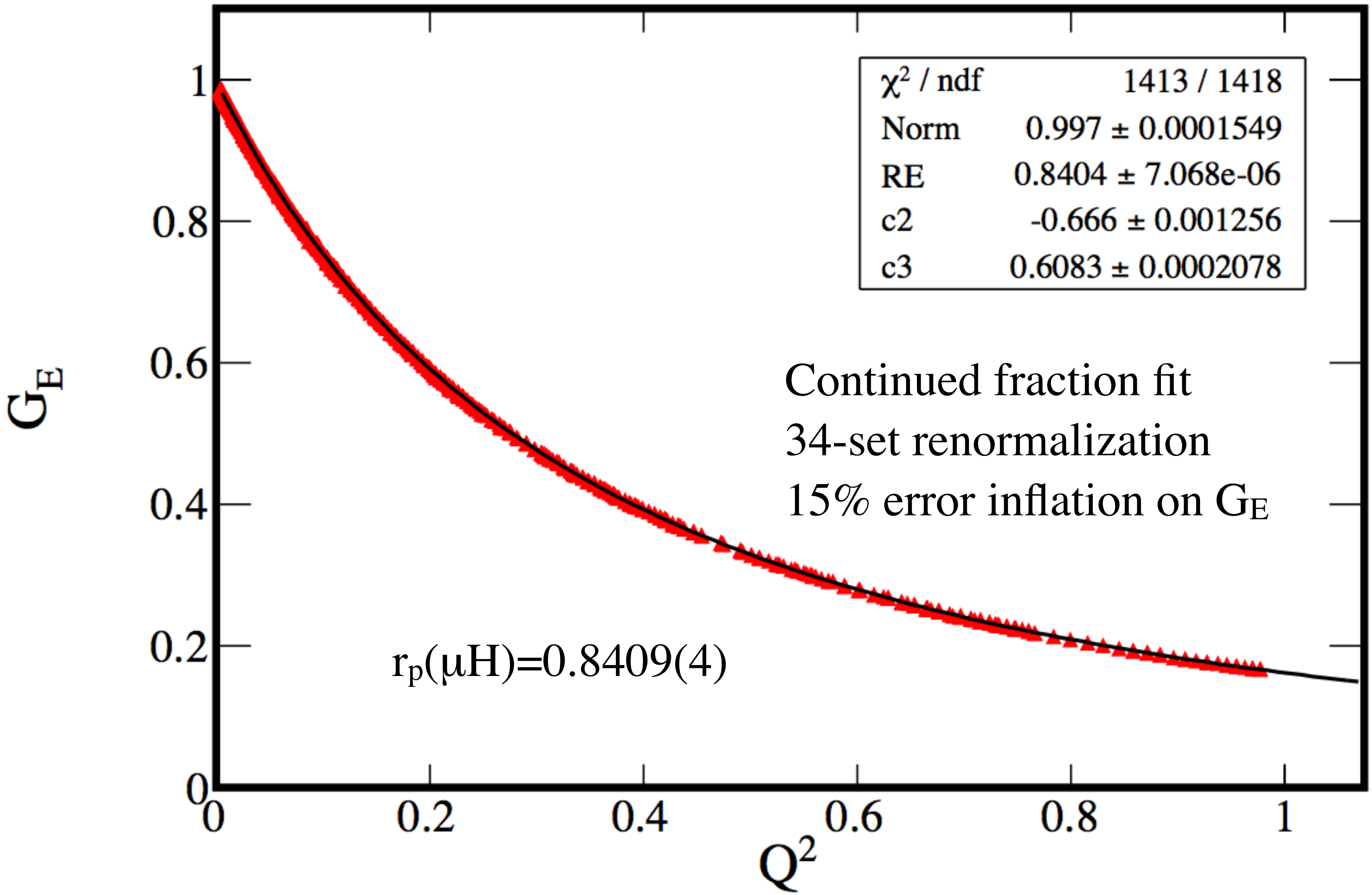
Error Statistics

- Data renormalization reduces the χ^2/dof of the global fit from 1.6 to 1.4
- Increasing the errors on G_E by 15% reduces the χ^2/dof to 1.0
- Statistics are now “perfect”



$[G_E(Q^2)-f(Q^2)]/1.15\sigma_{GE}$ is normally distributed

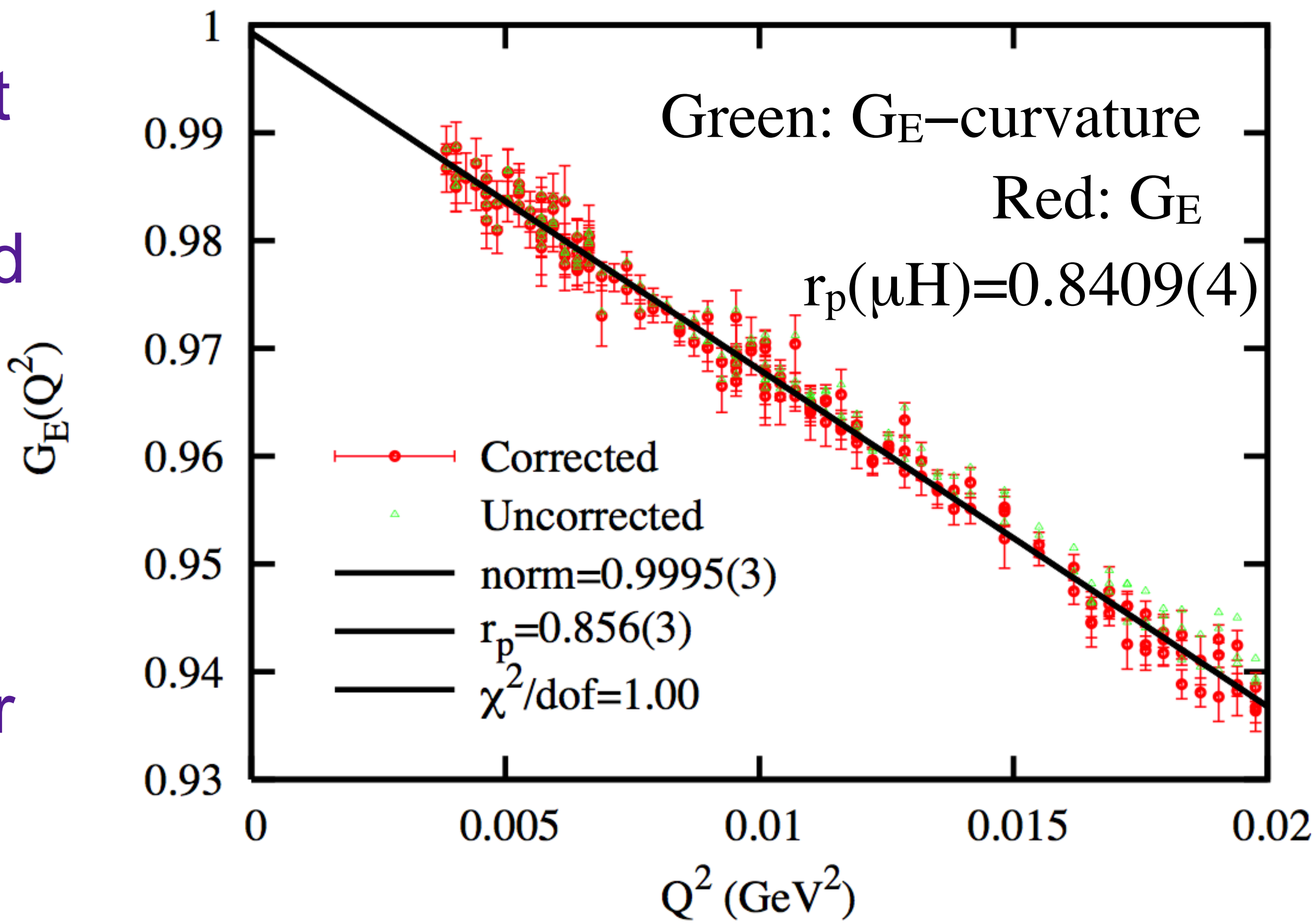
Final Fit



χ^2 / ndf 1413 / 1418
Norm 0.997 ± 0.0001549
RE 0.8404 ± 7.068e-06
c2 -0.666 ± 0.001256
c3 0.6083 ± 0.0002078
Continued fraction fit
34-set renormalization
15% error inflation on G_E
$r_p(\mu H)$=0.8409(4)
G_E
Q^2
0
0.2
0.4
0.6
0.8
1

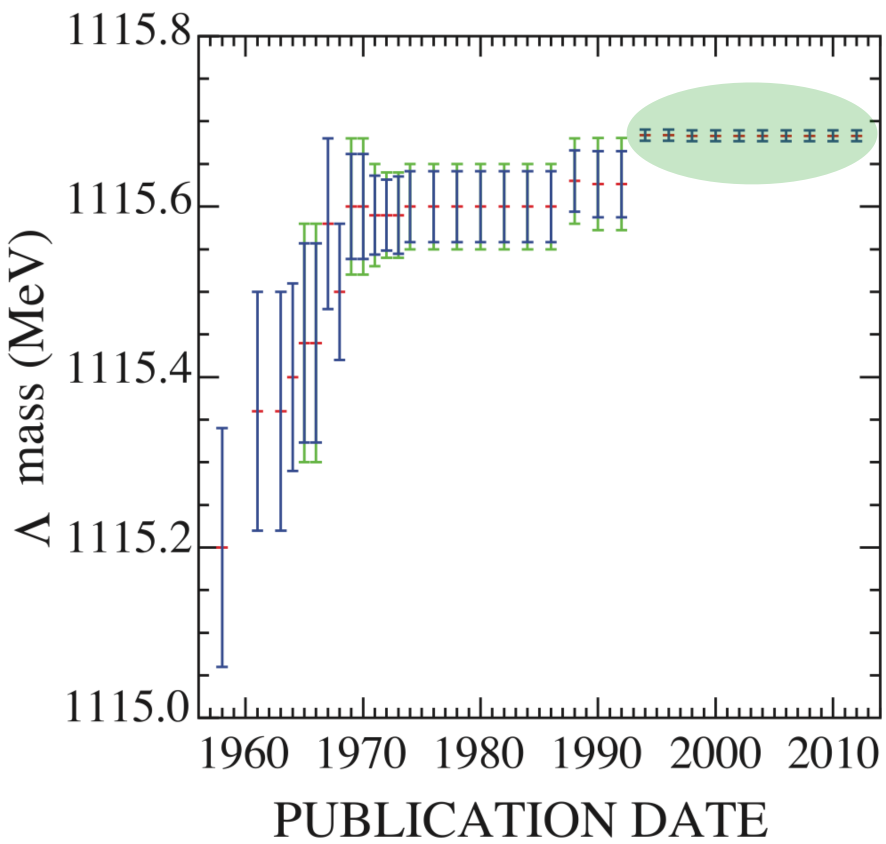
Curvature Subtraction

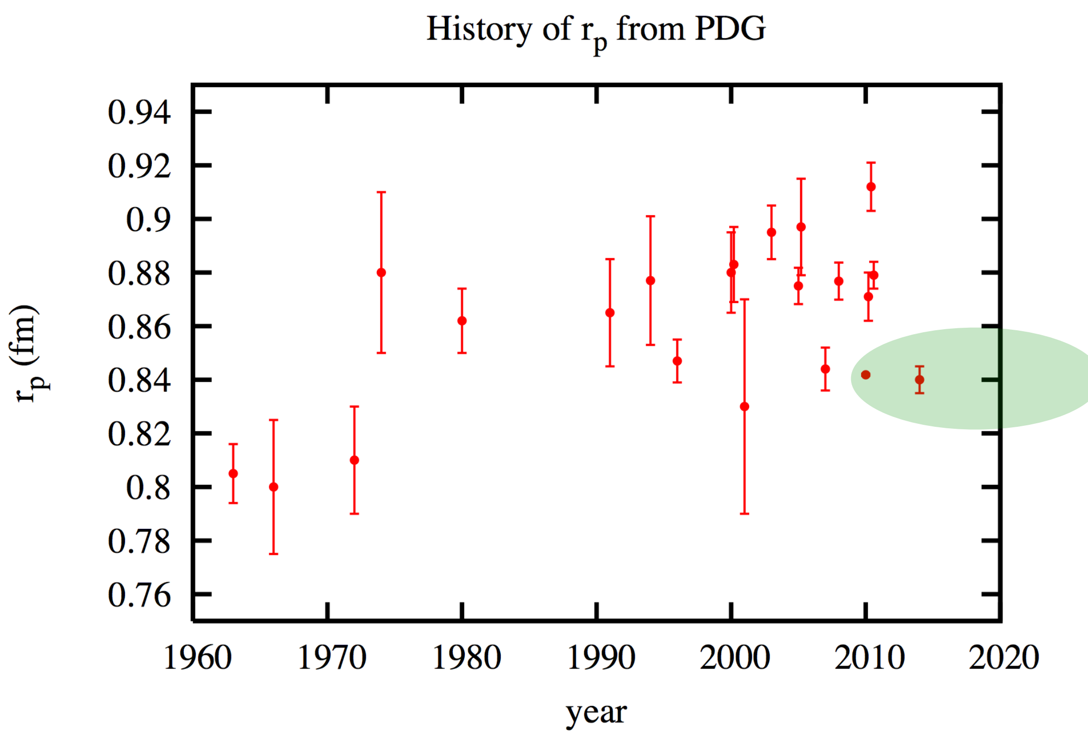
- What if we subtract the deduced curvature of G_E and fit the result to a straight line for $Q^2<0.02$ GeV^2?
- Now r_p is "large"
- However, the linear term is over half of the contribution to G_E up to $Q^2=0.3$ and should not be "wasted"



Either way you look at it, fitting only to Q^2=0.02 is dangerous

Perspective





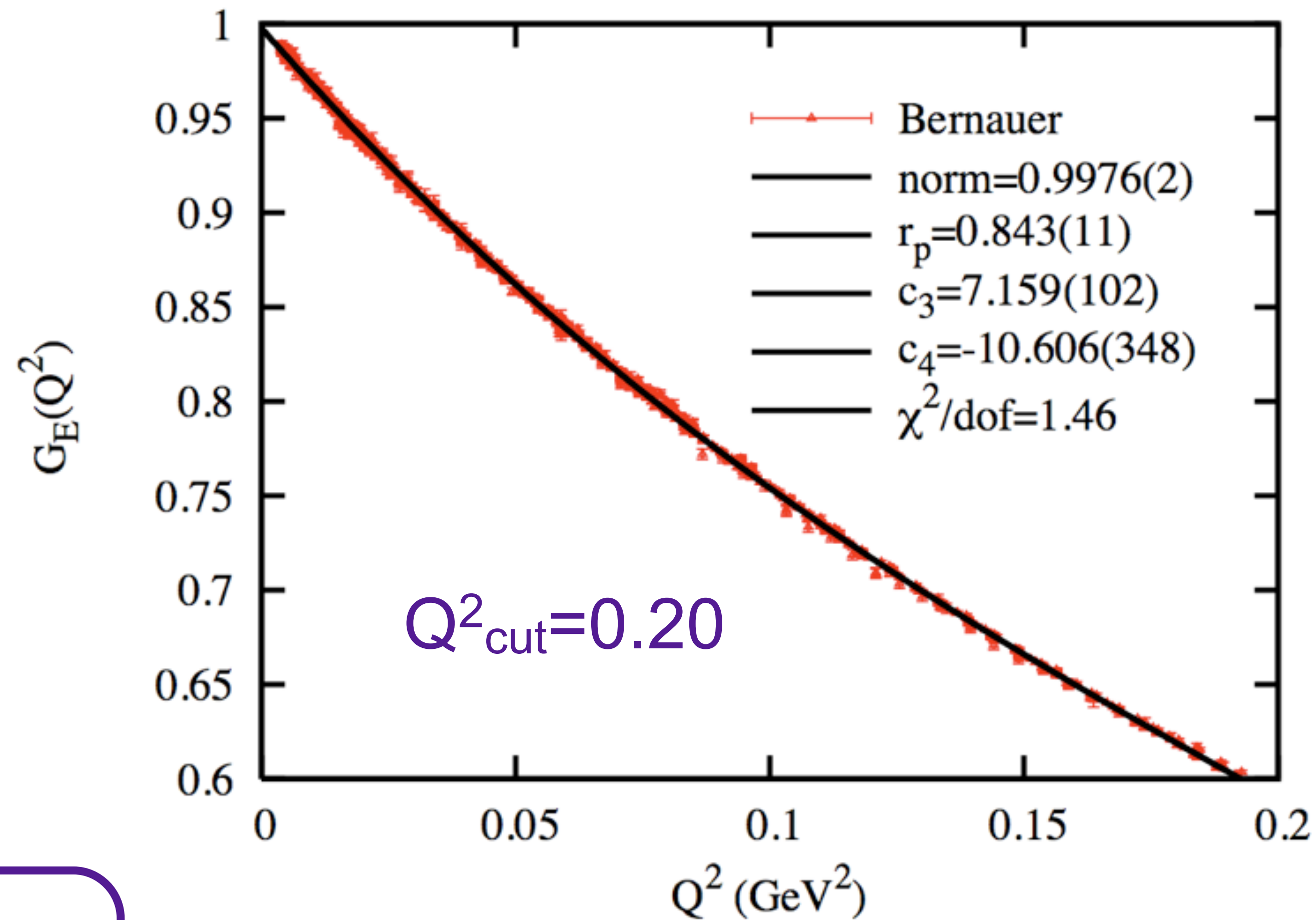
There still needs to be many consistent and accurate points for r_p on the graph above. Bring on the new measurements!

Conclusions

- The Mainz *ep* elastic scattering dat set of 1422 points, covering the range $0.004 < Q^2 < 1.0$ GeV^2 is the most precise and accurate data set available.
- If G_E is a monotonically falling function of Q^2, then these data yield $r_p = 0.804$ fm.
- Systematic variations (not fit statistics) suggest an error of about 0.004.
- Fitting the ratio σ/σ_{dipole} directly requires a fit-form with inflections which may bias a global fit to favor systematic fluctuations of G_E on a scale of $\Delta Q^2 \sim 0.05$ GeV^2.
- Bernauer's analysis is sound under the assumptions that fluctuations of G_E on a scale of $\Delta Q^2 \sim 0.05$ GeV^2 are real.
- Only an independent measurement of equivalent or better accuracy will be able to empirically address this.

All Mainz Data to Q^2=0.2

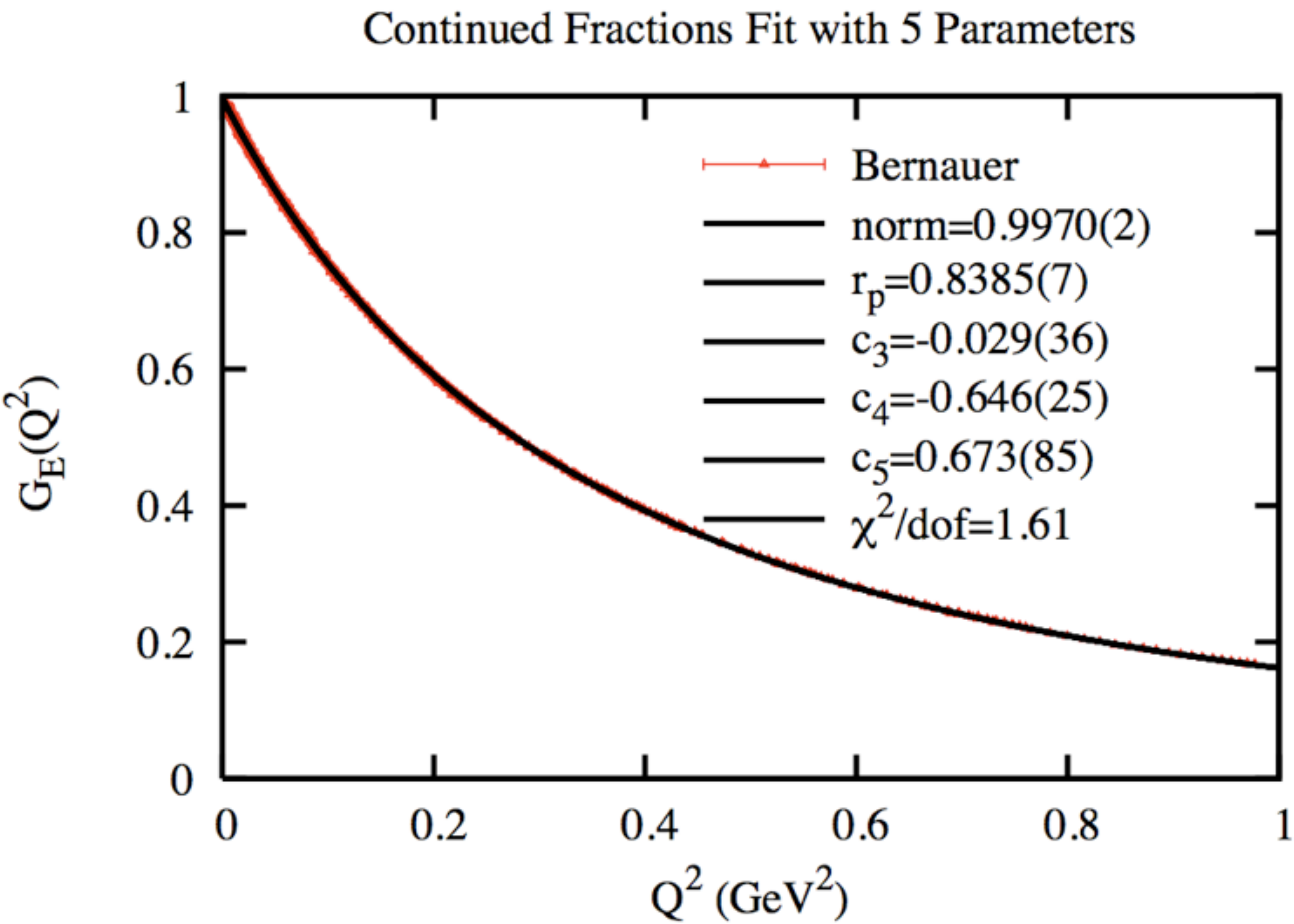
Radius of 0.84 fm persists with 4-parameter polynomial fit up to Q^2=0.2



$$G_E = G_D \sqrt{\frac{\sigma_r(1+\frac{\tau}{\epsilon}\mu_p^2)}{1+\frac{\tau}{\epsilon}\mu_p^2\left(1-\frac{Q^2}{8\mathrm{GeV}^2}\right)^{-2}}}$$

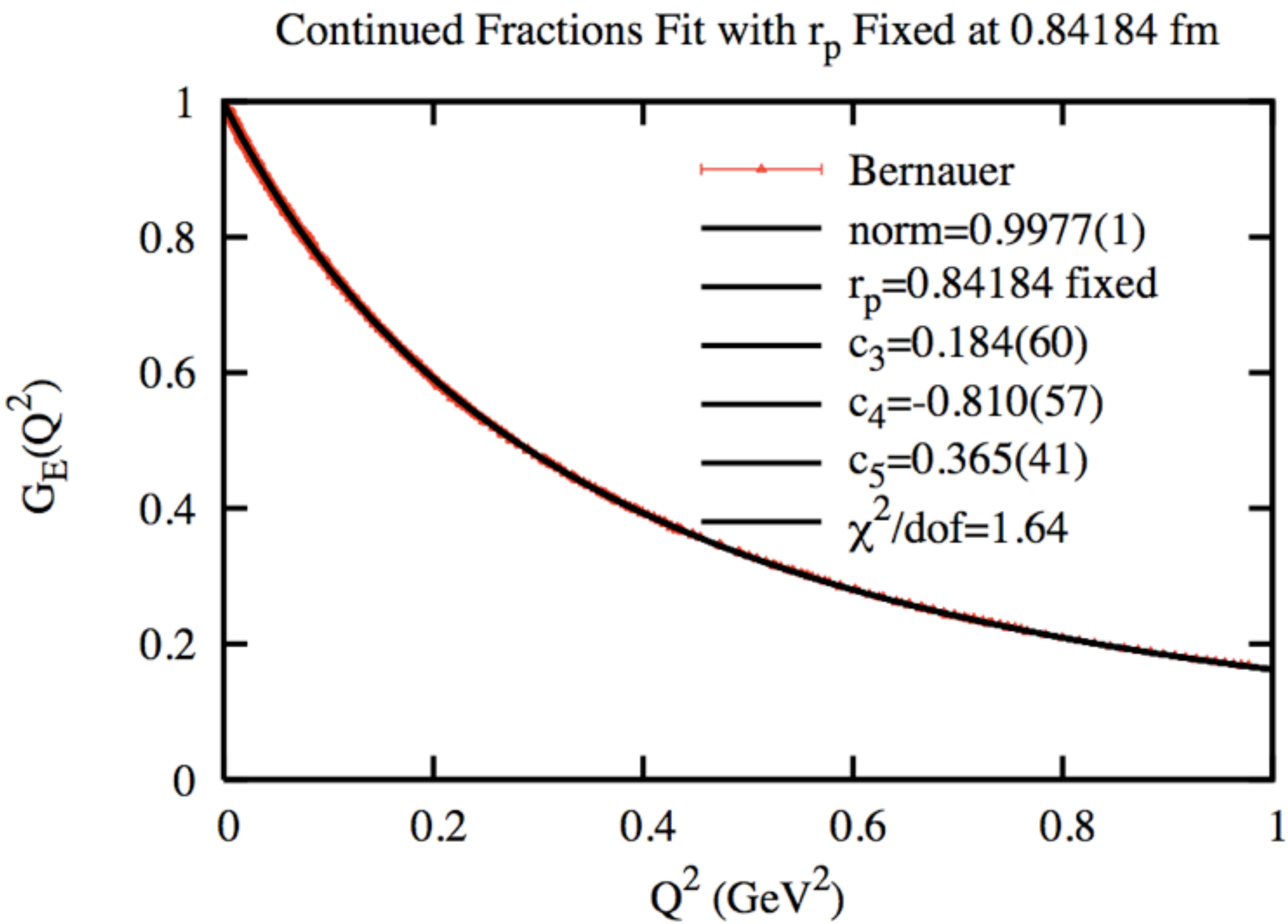
$$\frac{G_E}{G_M} \approx \frac{1}{\mu_p}\left(1-\frac{Q^2}{8\ \mathrm{GeV}^2}\right)$$

Global Fit



Continued Fractions Fit with 5 Parameters
Bernauer
norm=0.9970(2)
r_p=0.8385(7)
c_3=-0.029(36)
c_4=-0.646(25)
c_5=0.673(85)
χ^2/dof=1.61
G_E(Q^2)
Q^2 (GeV^2)
0
0.2
0.4
0.6
0.8
1

Global fit with r_p fixed



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Polynomial Fits

0<x<0.02

40 points

- $P(x) = a(1+bx+cx^2+dx^3+ ex^4)$
- $C(x) = a(1+bx/(1+cx/(1+dx)))$

- $P(x) = 0.9(1- 1.0x + 0.5x^2 - 0.5x^3 + 0.5x^4)$
- $C(x) = 0.9971(1+3.02229x/(1 - 0.667x/(1+0.610x)))$ (r_p=0.8389 fm)

C(x):

- 0.99668 −2.88986
- 0.99710 −3.01902 6.68471
- 0.99710 −3.02223 7.10326 −14.3212

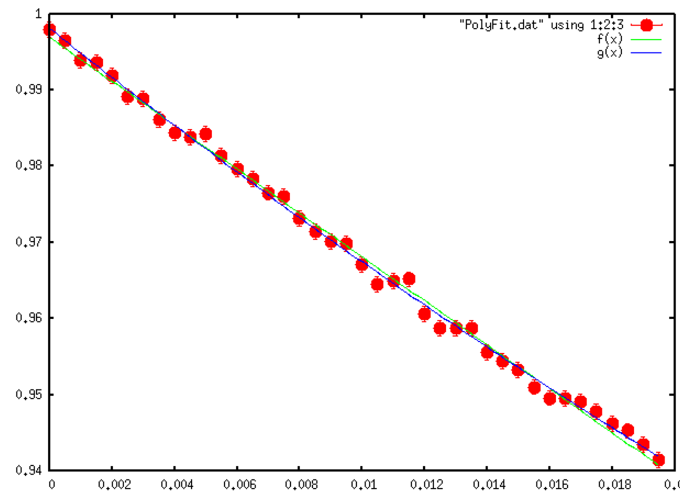
P(x):

- 0.89997 −0.99045
- 0.90000 −0.99999 0.48570
- 0.90000 −1.00000 0.49976 −0.4805

too small

Polynomial Fits

- Generate pseudodata from:
- $C(x) = 0.9971(1+3.02229Q^2/(1 - 0.667Q^2/(1+0.610Q^2)))$
- Corresponds to r_p=0.8389 fm
- $0<Q^2<0.02$; 40 points; $\sigma = 0.001$
- Fit to: $P(x) = a(1+bQ^2+cQ^4+dQ^6)$ X 25
- Last term d is too small in magnitude
- First two terms a, b are within statistics



- a =   0.99711 (48)   (46)   [ave. fit error] [σ for 25 trials]
- b = −3.003   (111) (107)
- Polynomial fits on [0,0.02] are OK except for the last term