THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

NUCLEAR POLARIZABILITY CORRECTION TO THE MUONIC DEUTERIUM LAMB SHIFT

MISHA GORSHTEYN

UNIVERSITÄT MAINZ

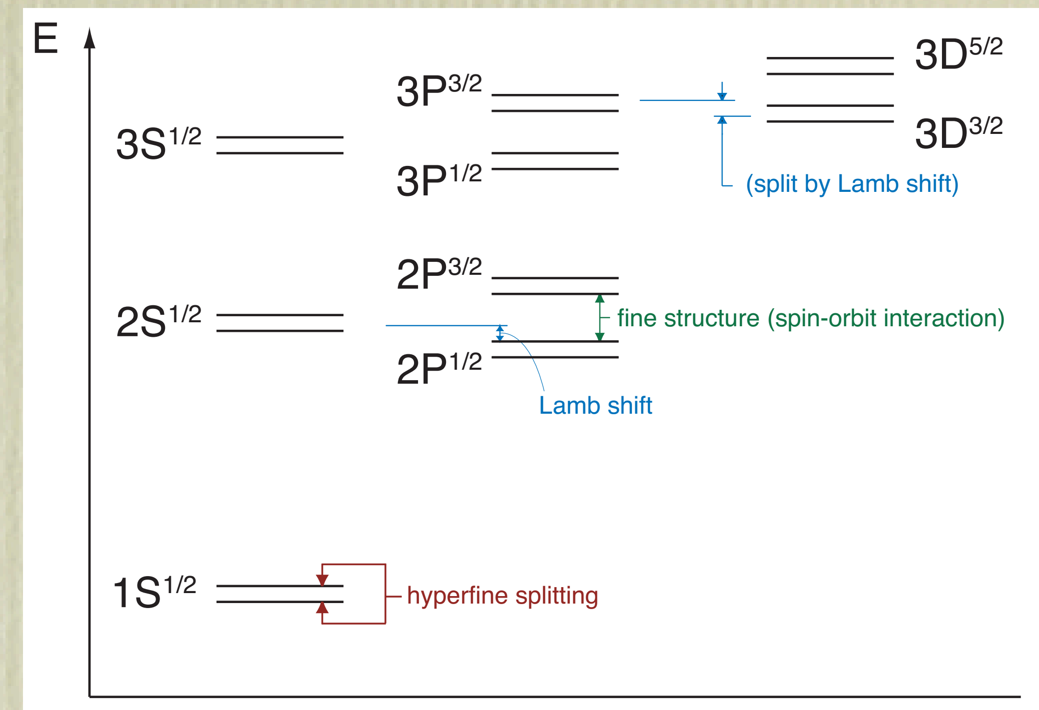
MITP WORKSHOP “PROTON RADIUS PUZZLE” - JUNE 2-6, 2014 SCHLOß WALDTHAUSEN, MAINZ

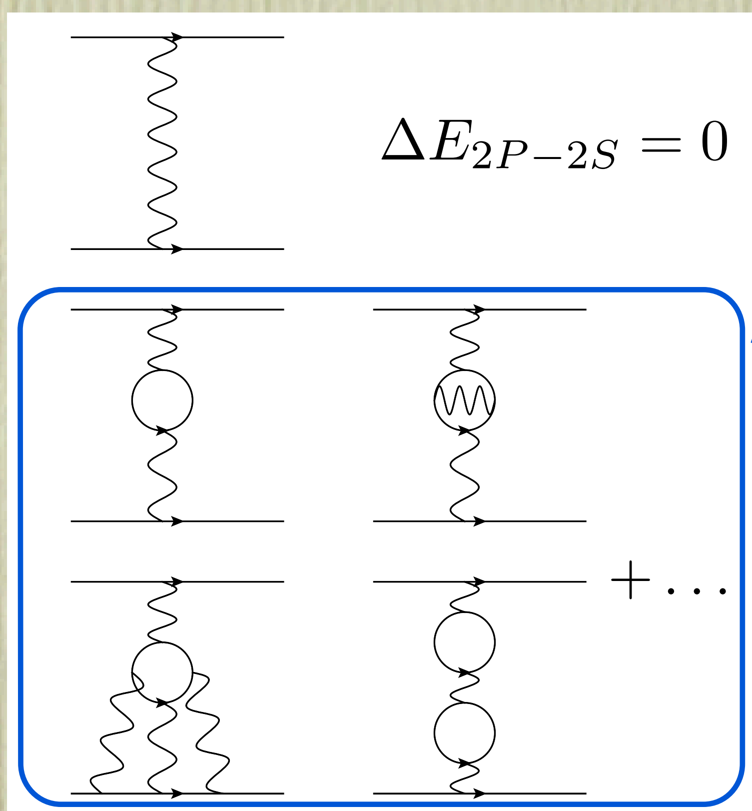
Deuteron Radius from Lamb Shift in D Atom

Deuterium Atom Spectrum

Leading order: degenerate

Rad. corr.: fine/hyperfine structure





Finite size correction

$$\Delta E_{nP-nS} = \Delta E^{QED}_{nP-nS} - \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_E^2 + \mathcal{O}(\alpha_{em}^5)$$

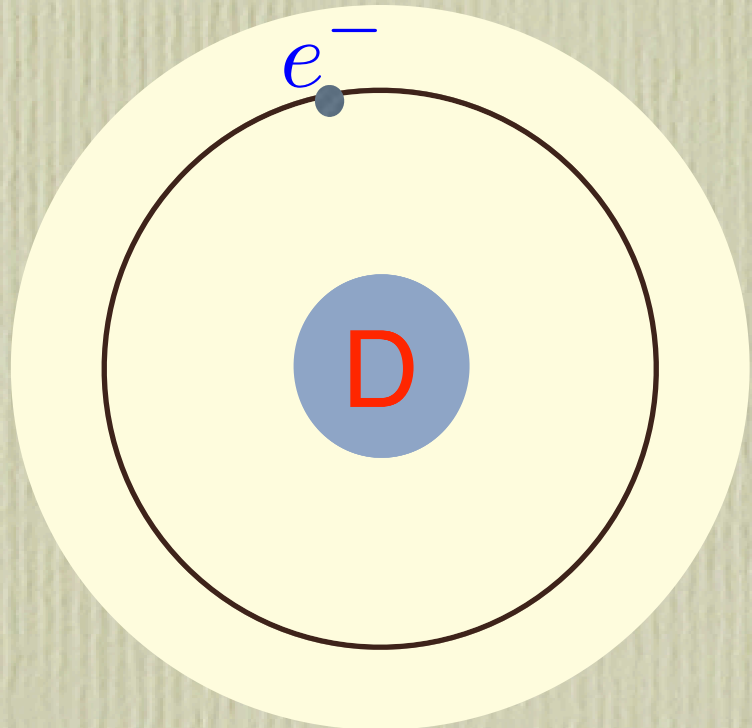
Further structure-dep. corrections - O(α^5)

CODATA 2010 (e-Deuterium): $r_E(d) = 2.1424(21)$ fm

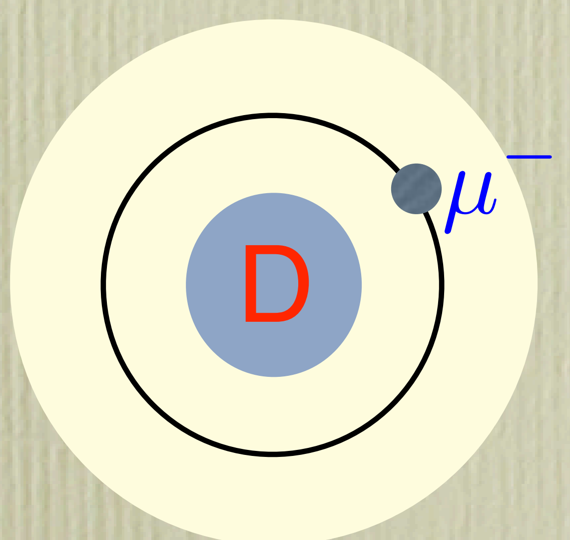
Agrees with e-D scattering $r_E(d) = 2.128(11)$ fm

Muonic vs Electronic Hydrogen

Deuterium atom



muonic Deuterium atom

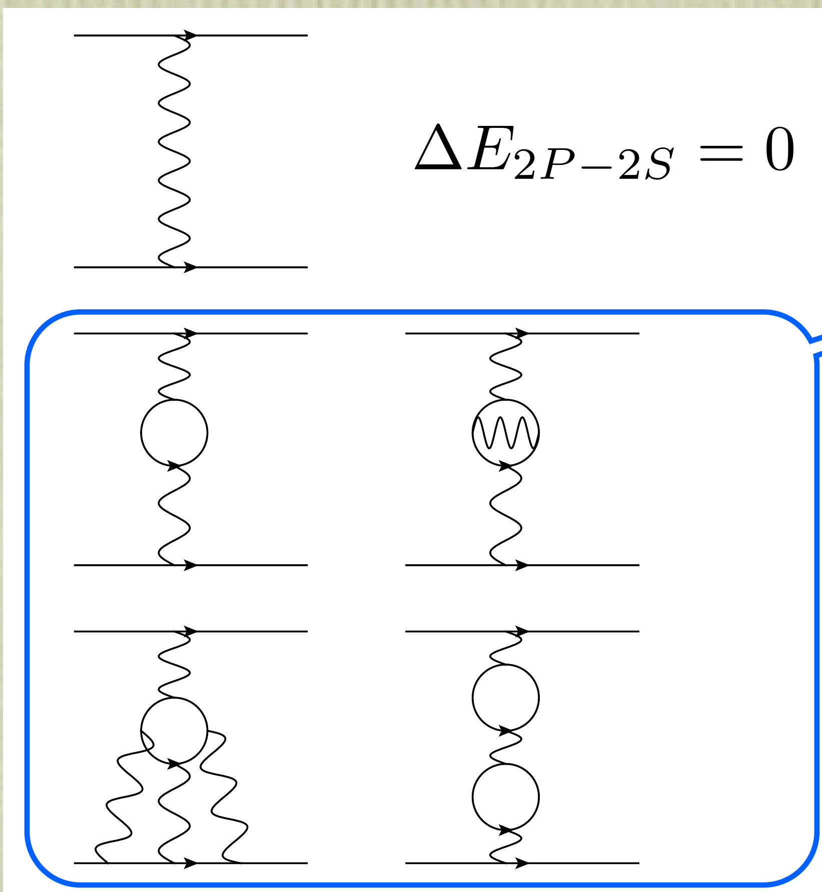


SM: the only difference is the mass

Bohr radius $$\frac{R_{\mu-H}}{R_{e-H}} = \frac{m_e}{m_\mu} \approx \frac{1}{200}$$

$$\Delta E^{FS,\, e-D}_{2P-2S} \approx 10^{-7} \times \Delta E^{FS,\, \mu-D}_{2P-2S}$$

Deuteron Radius from Lamb Shift



$$\Delta E_{nP-nS} = \Delta E^{QED}_{nP-nS} - \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_E^2 + \mathcal{O}(\alpha_{em}^5)$$

Modern QED calculations:

Borie, Annals Phys. 327 (2012) 733;
Eides et al., Phys.Rept. 342 (2001) 63;
Indelicato, arXiv:1210.5828,
...

$$\Delta E_{2P-2S} = 230.2972(400) - 6.10940\, r_E^2(d) + E_{\mathrm{TPE}}$$

Isotopic shift: 2S-1S transition in eD vs. eH

Huber et al. 1998;
Parthey et al. 2010

$$R_d^2 - R_p^2 = \frac{\Delta E_{\text{finite size}}}{\frac{7}{12}\alpha^4 \mu_d^3} \frac{2\pi}{c} (\hbar c)^3 - \left(1 - \frac{\mu_p^3}{\mu_d^3}\right) R_p^2$$

$$R_d^2 - R_p^2 = 3.819\,94(65)\ \mathrm{fm}^2$$

Very high precision;
Consistency check if Rp is known

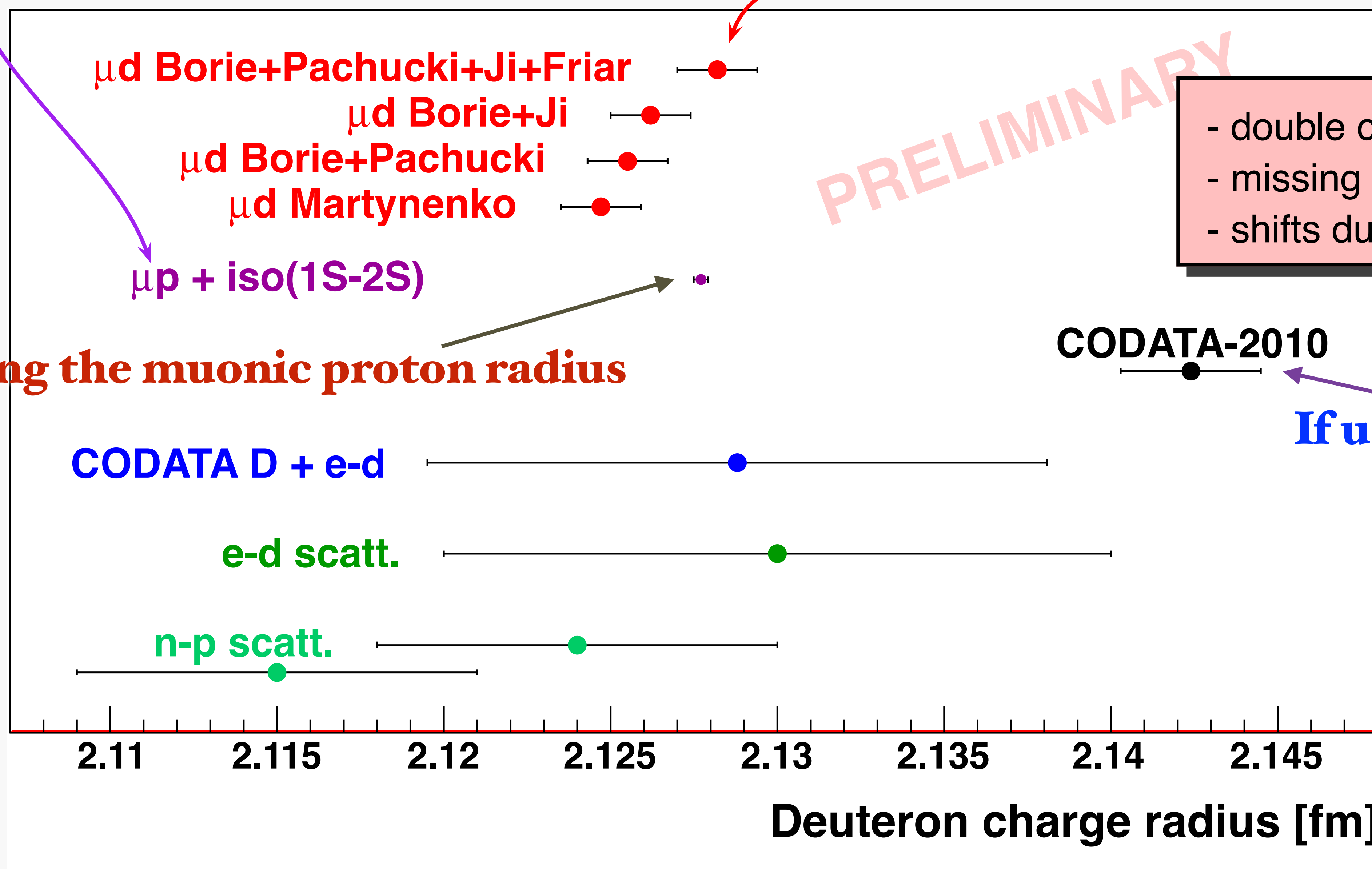
Deuteron radius from μd and μp (preliminary)

H-D iso-shift: $r_d^2 - r_p^2 = 3.820\,07(65)$ fm^2

μp : $r_p = 0.84087(39)$ fm

$\Rightarrow r_d = 2.12771(22)$ fm

Directly from μd spectroscopy using μd polarizabiliy with ± 0.03 meV



- double counting (th)?
- missing terms (th)?
- shifts due to close levels (exp)?

If using the electronic proton radius

D Polarizability Correction to μD Lamb Shift

Polarizability correction in μH - talks by Jerry, Mike, Vladimir

Polarizability correction will have a larger impact in μD

$$\alpha_E^p = 1.1 \times 10^{-3}\mathrm{fm}^3 \qquad \alpha_E^d = 0.633(1)\mathrm{fm}^3$$

$$\beta_M^p = 3 \times 10^{-4}\mathrm{fm}^3 \qquad \beta_M^d = 0.072(5)\mathrm{fm}^3$$

$$\Delta E_{\mu H}^{pol} \approx 13\,\mu\mathrm{eV} \qquad \rightarrow \Delta E_{\mu D}^{pol} \approx \mathrm{few}100 \times 13\,\mu\mathrm{eV} \sim \mathrm{few\ meV}$$

Non-relativistic nuclear calculations

$$\Delta E_{\mathrm{pol}}^{\mathrm{NR}} = -8\alpha^2|\phi_n(0)|^2 \sum_{N\neq 0} \int \frac{d^3q}{4\pi} \frac{\langle 0|\rho_{\mathrm{ch}}(-\mathbf{q})|N\rangle\langle N|\rho_{\mathrm{ch}}(\mathbf{q})|0\rangle}{q^2\ (\omega_N + \frac{q^2}{2\,m_r})\,q^2}$$

Various calculations compare well

$$\Delta E_{pol} = 1.941(0 - \mathrm{range} + \mathrm{finite\,size})$$ Friar 2013

$$\Delta E_{pol} = 1.680(0 - \mathrm{range} + \mathrm{Coulomb} + \mathrm{ret.} + \mathrm{nucleon\,pol.})$$ Pachucki 2013

$$\Delta E_{pol} = 1.698(0 - \mathrm{range} + \mathrm{Coulomb} + \mathrm{rel.\,corr.})$$ Ji et al. 2013

Leidemann, Rosenfelder 1995 $\Delta E_{pol} = 1.500\,\mathrm{meV}$

Polarizability Correction to Lamb Shift

Questions to be raised:

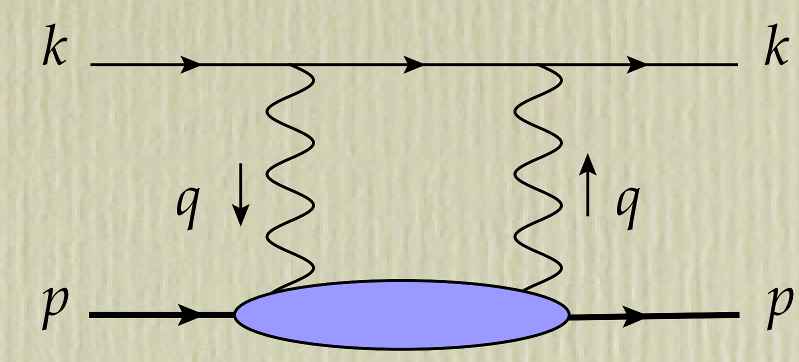
- what's the model dependence of NR calculations?

- is the claimed 1% uncertainty feasible?
  - NR models work reasonably well (how well, exactly?)
  - Sum Rules (TRK, Coulomb) obey at 10-15% level

- can the pol. correction be constrained from data?

Two photon exchange contribution to Lamb shift

Kinematics: 2 loop variables
q^2 and ν=(pq)/M



$$\mathcal{M} = e^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4} \bar{u}(k) \left[\gamma^\nu \frac{1}{\not{k} - \not{q} - m_l + i\epsilon} \gamma^\mu + \gamma^\mu \frac{1}{\not{k} + \not{q} - m_l + i\epsilon} \gamma^\nu \right] u(k) T_{\mu\nu}$$

Forward virtual Compton amplitude

$$T^{\mu\nu} = \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T j^\mu(x) j^\nu(0) | p \rangle$$
$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} (p - \frac{pq}{q^2} q)^\mu (p - \frac{pq}{q^2} q)^\nu T_2(\nu, Q^2)$$

Lamb shift (nS-nP)

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d} \phi_n^2(0) \int d^4q \frac{(q^2 + 2\nu^2) T_1(\nu, q^2) - (q^2 - \nu^2) T_2(\nu, q^2)}{q^4 [(q^2/2m_l)^2 - \nu^2]}$$

Two photon exchange contribution to Lamb shift

T_1, T_2 - the imaginary parts known (Optical theorem)

$$\mathrm{Im}T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2)$$

$$\mathrm{Im}T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2)$$

Inelastic structure functions = data

(real and virtual photoabsorption, FF)

Real parts - from forward dispersion relation

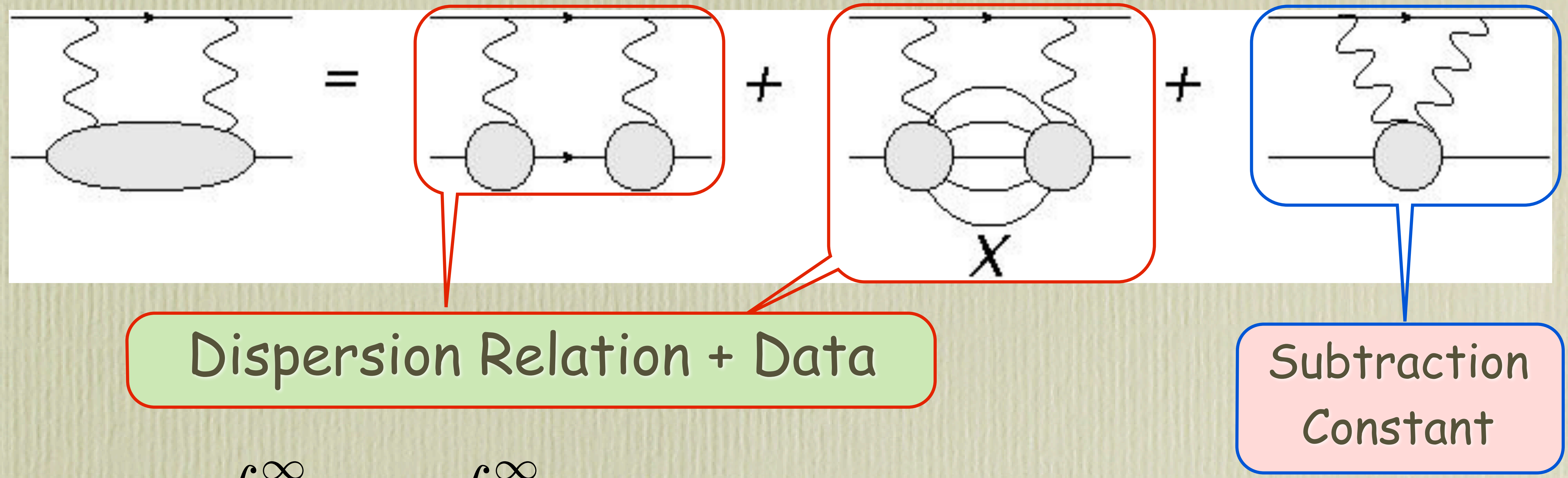
$F_1(\nu \to \infty, q^2) \sim \nu^{1+\epsilon}$ - subtraction needed

$F_2(\nu \to \infty, q^2) \sim \nu^{\epsilon}$ - no subtraction

$$\mathrm{Re}T_1(\nu, Q^2) = \bar{T}_1(0, Q^2) + T_1^{pole}(\nu, Q^2) + \frac{\nu^2}{2\pi M} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu(\nu'^2 - \nu^2)} F_1(\nu', Q^2)$$

$$\mathrm{Re}T_2(\nu, Q^2) = T_2^{pole}(\nu, Q^2) + \frac{1}{2\pi} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} F_2(\nu', Q^2)$$

TPE from Dispersion Relations

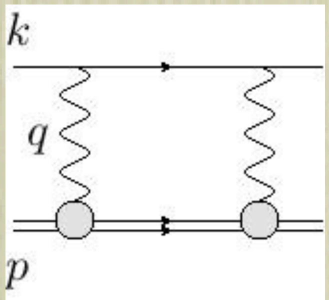


$$\Delta E = \int_0^\infty dQ^2 \int_{\nu_0}^\infty d\nu\ [\mathrm{DATA}]$$

Model + data

Data with real and virtual photons exist; feature elastic peak, QE peak, nucleon excitations, continuum

TPE: elastic contribution to Lamb shift

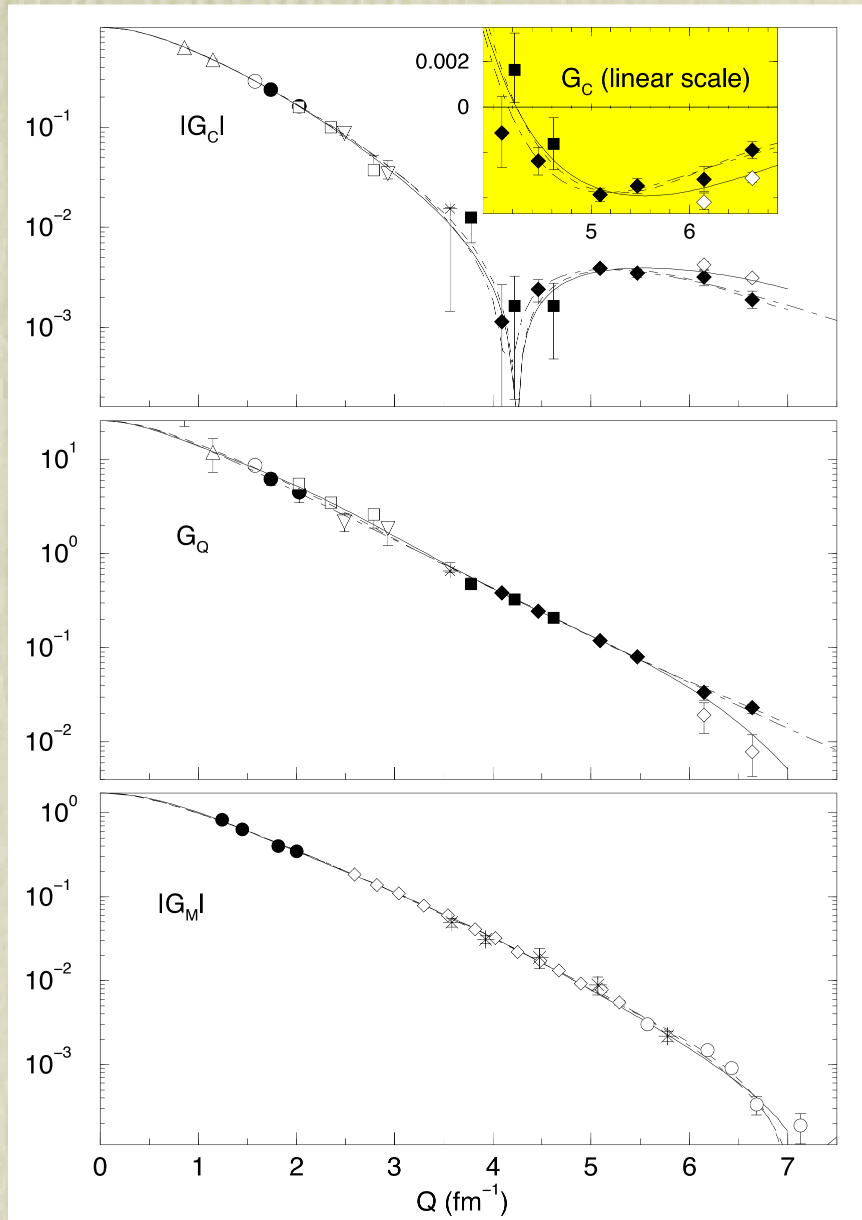


On-shell Deuteron electromagnetic vertex

$$\langle d(p')|J^\mu(q)|d(p)\rangle = G_2(Q^2)[\xi'^{*\mu}(\xi q) - \xi^\mu(\xi'^* q)] - \left[G_1(Q^2)(\xi'^*\xi) - G_3(Q^2)\frac{(\xi'^* q)(\xi q)}{2M_d^2}\right](p+p')^\mu,$$

Charge, magnetic and quadrupole FF's

$$\begin{aligned} G_M &= G_2, \\ G_C &= G_1 + \frac{2}{3}\tau_d G_Q, \\ G_Q &= G_1 - G_2 + (1+\tau_d)G_3, \end{aligned}$$



Deuteron form factors parametrization from:

Abbott et al., EPJ A7 (2000) 421

TPE: elastic contribution to Lamb shift

$$\Delta E^{el} = \alpha^2\phi^2(0)\int_0^\infty \frac{dQ^2}{Q^2}$$
$$\left\{\frac{16Mm}{(M+m)Q}G'_C(0) - \frac{m}{M(M^2-m^2)}\left(\frac{\gamma_2(\tau_d)}{\sqrt{\tau_d}} - \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}\right)\left[\frac{G_C^2(Q^2)-1}{\tau_d} + \frac{8}{9}\tau_d(G_Q^2(Q^2) - G_{Q,pt}^2)\right]\right.$$
$$\left. + \frac{m}{M(M^2-m^2)}\frac{2}{3}(G_M^2(Q^2) - G_{M,pt}^2)\left[(1+\tau_d)\left(\frac{\gamma_1(\tau_d)}{\sqrt{\tau_d}} - \frac{\gamma_1(\tau_l)}{\sqrt{\tau_l}}\right) - \frac{\gamma_2(\tau_d)}{\sqrt{\tau_d}} + \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}\right]\right\}$$

Point charge in the WF - subtract

$$\Delta E_C^{el} = 0.4162\,\mathrm{meV}$$
$$\Delta E_M^{el} = -0.00008\,\mathrm{meV}$$
$$\Delta E_Q^{el} = 0.0007\,\mathrm{meV}$$

Total elastic $\Delta E_{tot}^{el} = 0.417(2)\,\mathrm{meV}$

Compare to $\Delta E_{tot}^{el} = 0.370\,\mathrm{meV}$ Martynenko, Faustov, 2004

Including the -0.023 meV from Thomson term (see below) here would make 0.395 meV compare to 0.37 meV better (but not great)

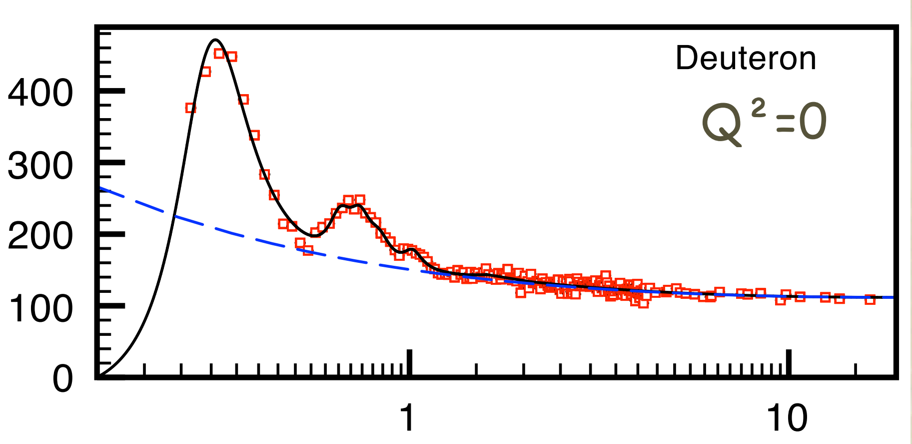
TPE: hadronic contribution to Lamb shift

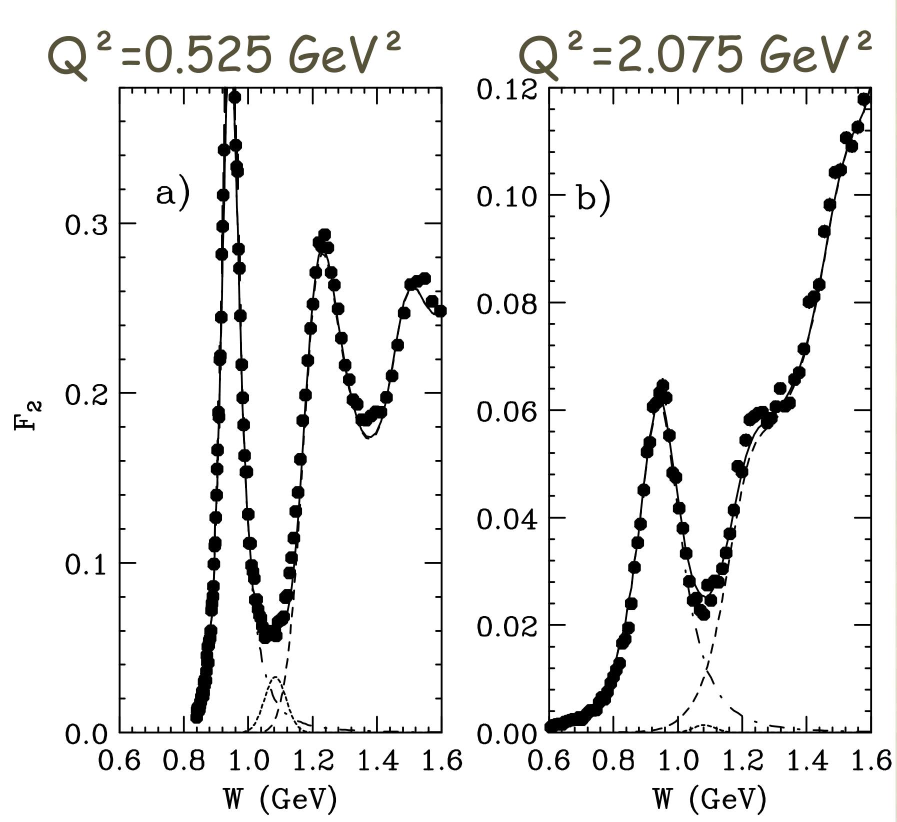
$$\Delta E^{inel} = \frac{2\alpha^2}{m_l M_d}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{thr}}^\infty \frac{d\nu}{\nu}\left[\frac{\sqrt{\tau_l}\gamma_1(\tau_l)-\sqrt{\tau}\gamma_1(\tau)}{\tau_l-\tau}F_1(\nu,Q^2)+\frac{M_d\nu}{Q^2}\frac{\frac{\gamma_2(\tau)}{\sqrt{\tau}}-\frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}}{\tau_l-\tau}F_2(\nu,Q^2)\right]$$

$$\tau_l = \frac{Q^2}{4m_l^2}, \qquad \tau = \frac{\nu^2}{Q^2}$$

Hadronic contribution:
fit to deuteron resonance data
matched to Regge-behaved background

Bosted and Christy, PR C77 (2008) 065206;
MG et al, PR C84 (2011) 065202





Contribution to the Lamb shift

$$\Delta E^{hadr} = 0.028\,\mathrm{meV}$$

Compare to $$\Delta E^{hadr} = 0.043\,\mathrm{meV}$$

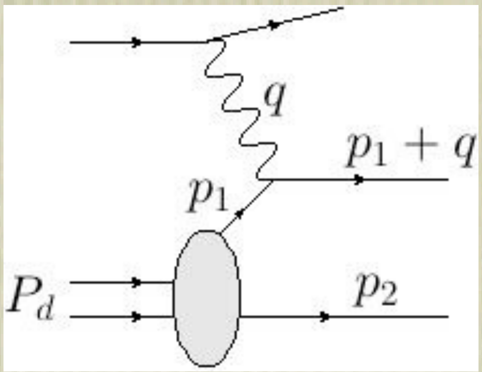
Pachucki, 2013

The difference is 15 μeV
Claimed uncertainty is 16 μeV

TPE: quasielastic contributions to Lamb shift

QE in the Plane-Wave Born Approximation

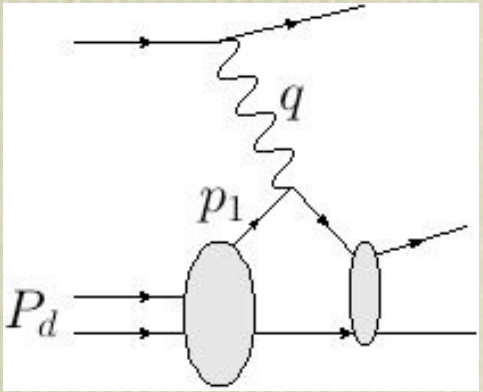
$$F_{1,2}^{d,QE}(\nu, Q^2) = \frac{1}{4\pi}\int d^3\vec{k}\phi^2(\vec{k})\left[F_{1,2}^p(\nu', Q^2) + F_{1,2}^n(\nu', Q^2)\right]$$



Deuteron momentum distribution

$$S(\nu, Q^2) = \frac{1}{2}\int_{k_{min}}^{k_{max}} k dk \phi^2(k)$$

Paris NN potential
Lacombe et al. 1981



At low relative knock-out nucleon momenta:
strong rescattering effects in the I=1 channel
Input: p-n scattering lengths (both I=0,1)

Works fine at substantial photon virtualities, not so fine at low Q^2.
But we just need to parametrize data - rescale by a function of Q^2
that will be obtained from a fit to all available data.

Fit to Photo- and Electro disintegration data

Fit function of the following form:

$$F_{1,2}(\nu, Q^2) = f_{1,2}^{FSI}(Q^2)\, F_{1,2}^{FSI}(\nu, Q^2) + f_{1,2}^{PW}(Q^2)\, F_{1,2}^{PW}(\nu, Q^2) + f_{1,2}^{Real}(Q^2)\, F_{1,2}^{Real}(\nu, Q^2)$$

Obtain from a fit

Take from Model

Constraint from real photon data and Baldin Sum Rule

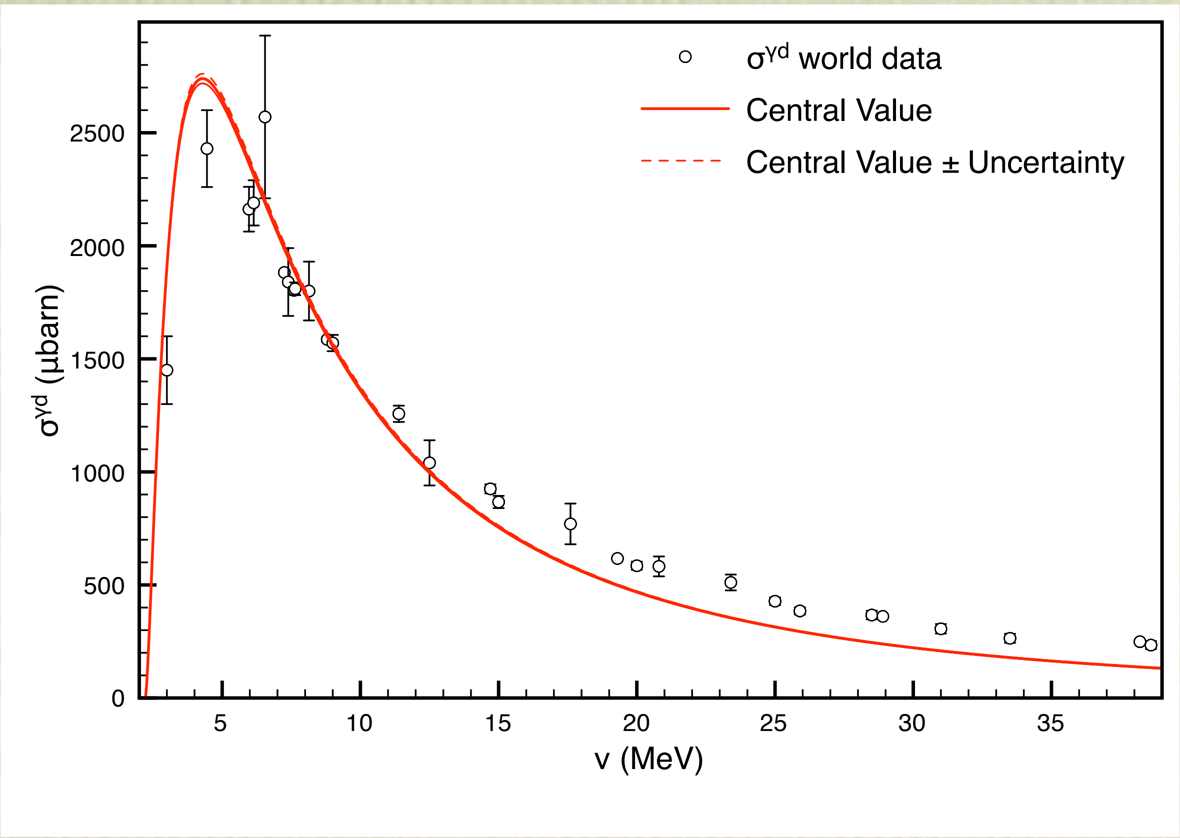
$$\alpha + \beta = \frac{2\alpha}{M_d} \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu^3} F_1(\nu, 0)$$

EFT: Chen et al.

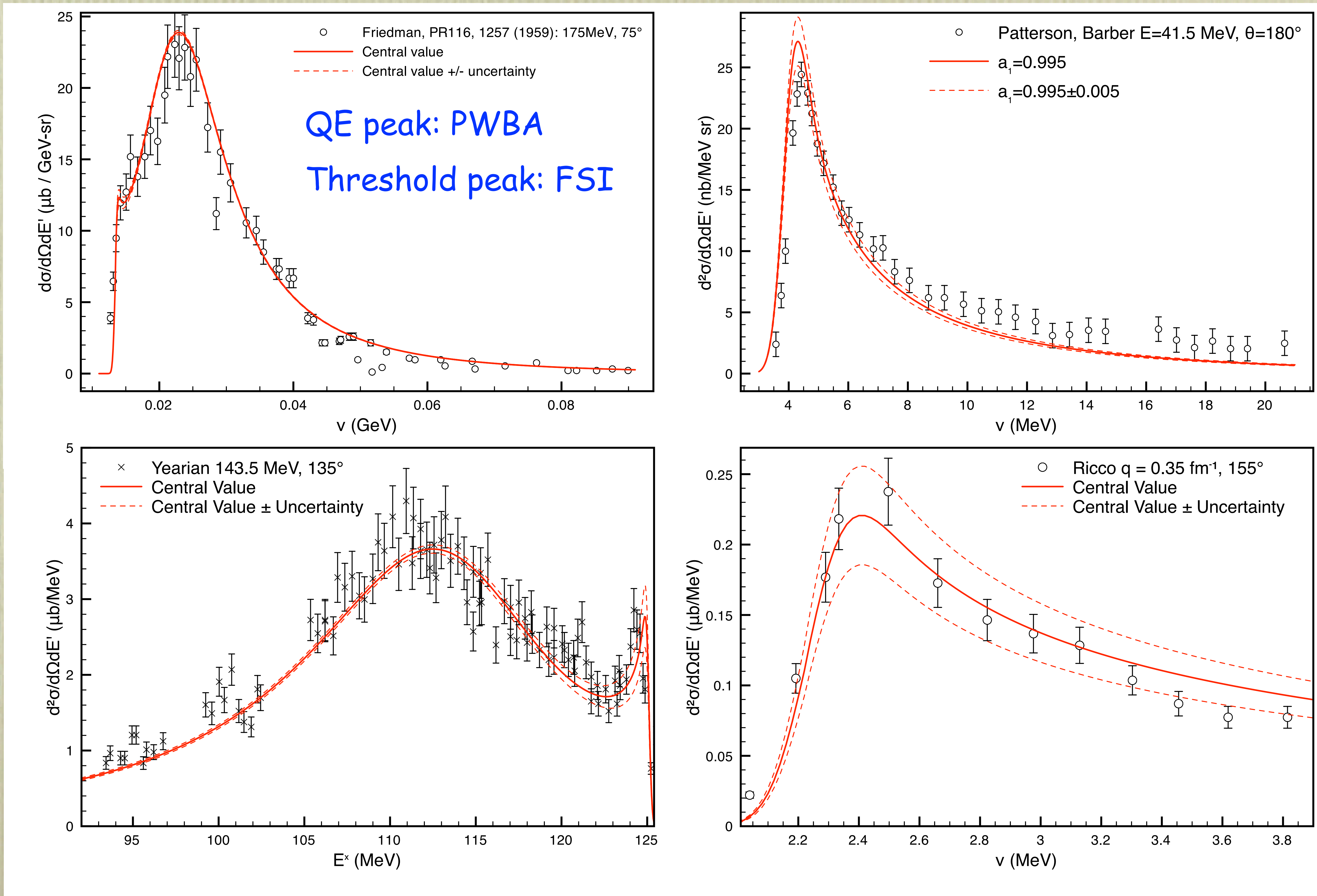
$$\alpha_E = 0.634\,\mathrm{fm}^3 \quad \beta_M = 0.067\,\mathrm{fm}^3$$

Potential Models: Friar, Payne

$$\alpha_E = 0.633\,\mathrm{fm}^3 \quad \beta_M = 0.077\,\mathrm{fm}^3$$



Fit of QE Data $0.005\ GeV^2 < Q^2 < 3\ GeV^2$



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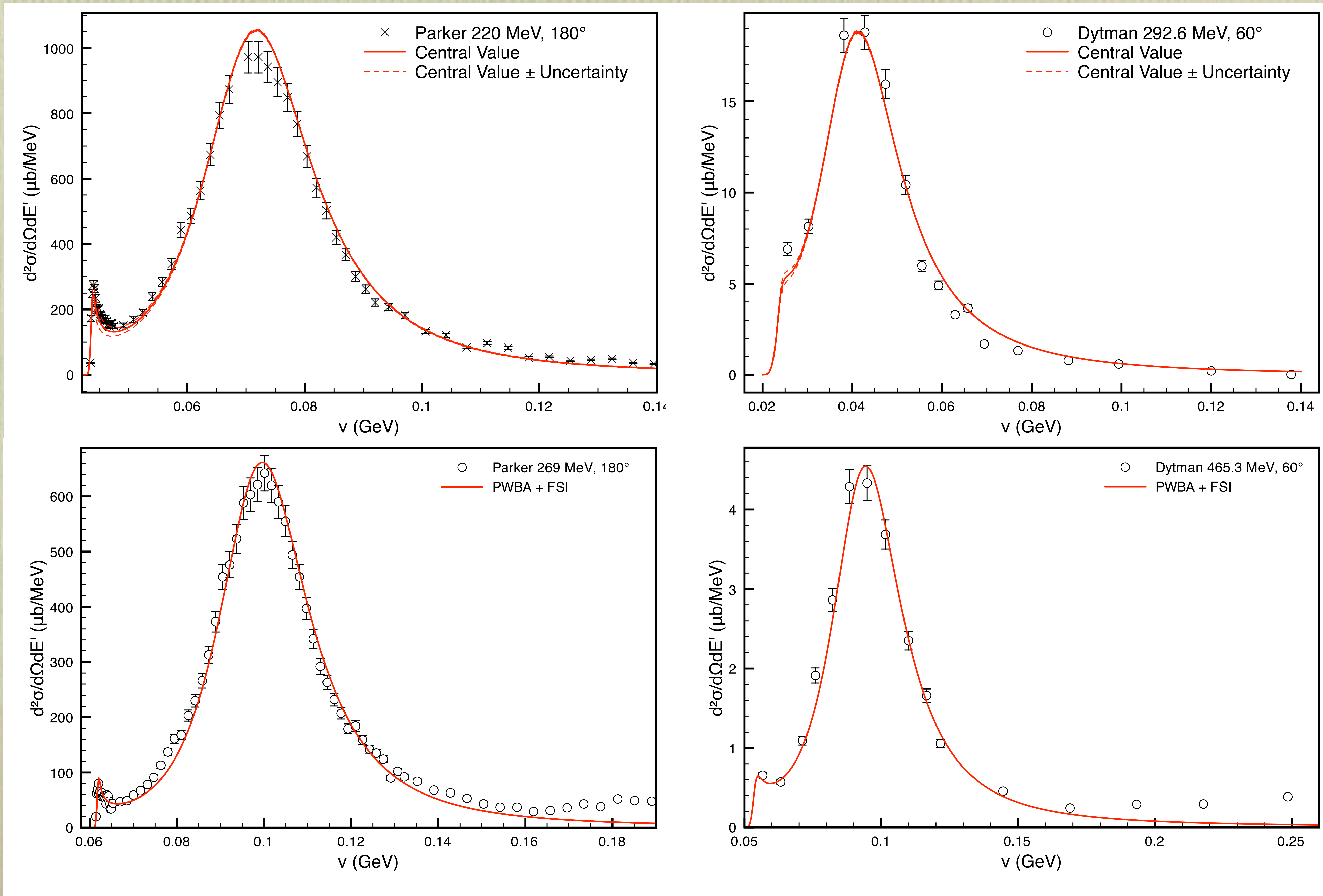
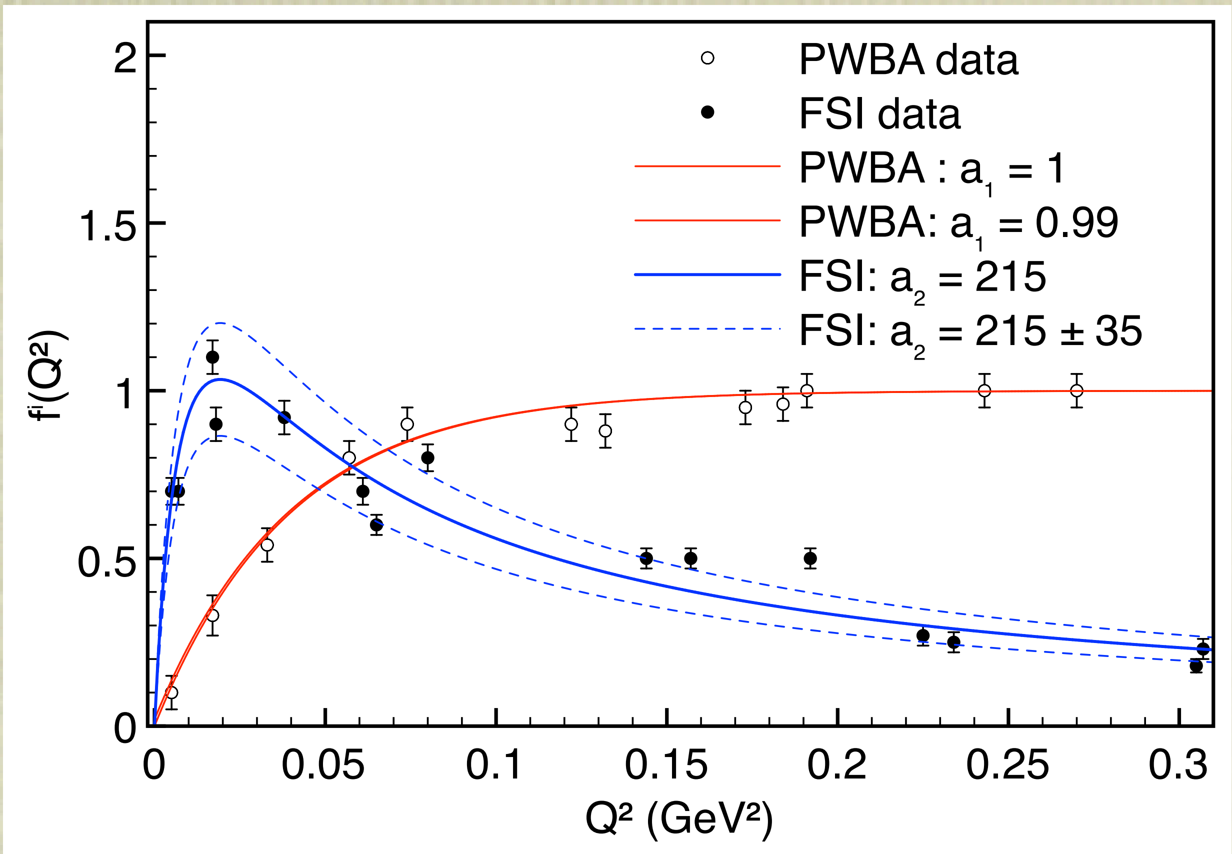


TPE: QE contribution to Lamb shift

Parametrization of world QE data at $0.005 < Q^2 < 3\ \mathrm{GeV}^2$;
Output of the fit:
rescaling functions for PWBA, FSI



$$\Delta E^{inel} = \frac{2\alpha^2}{m_l M_d}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{thr}}^\infty \frac{d\nu}{\nu}\left[\frac{\sqrt{\tau}_l\gamma_1(\tau_l)-\sqrt{\tau}\gamma_1(\tau)}{\tau_l-\tau}F_1(\nu,Q^2)+\frac{M_d\nu}{Q^2}\frac{\frac{\gamma_2(\tau)}{\sqrt{\tau}}-\frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}}{\tau_l-\tau}F_2(\nu,Q^2)\right]$$

$$\Delta E^{PWBA} = 1.616(739)\ \mathrm{meV}$$

$$\Delta E^{FSI} = 0.391(44)\ \mathrm{meV}$$

$$\Delta E^{Real} = 0.322(3)\ \mathrm{meV}$$

1% uncertainty in one parameter - 50% uncertainty in Lamb shift

Subtraction Contribution to Lamb Shift

$$\Delta E^{\beta}_{n0} = 2\alpha\phi^2_{n0}(0)\beta^d_M(0)\int\limits_0^{\infty} dQ^2 \frac{\gamma_1(\tau_l)}{\sqrt{Q^2}} F_\beta(Q^2)$$

What is β? - input from theory

EFT: Chen et al. $\alpha_E = 0.634\,\mathrm{fm}^3 \quad \beta_M = 0.067\,\mathrm{fm}^3$

Potential Models: Friar, Payne $\alpha_E = 0.633\,\mathrm{fm}^3 \quad \beta_M = 0.077\,\mathrm{fm}^3$

$$\beta^d_M = 0.072(5)\,\mathrm{fm}^3$$

What is F_β? - input from theory is needed!

Uncertainty estimate:

$F_\beta(Q^2) = G^d_C(Q^2)$

$F_\beta(Q^2) = G^d_M(Q^2)$

$\Delta E^{\beta} = -0.740(40)\,\mathrm{meV}$ $\Delta E^{\mathrm{Thomson}} = -0.023(1)\,\mathrm{meV}$

Total subtraction: $\Delta E^{\mathrm{Subt}} = -0.763(40)\,\mathrm{meV}$

Putting Pieces Together

Elastic	ΔE^{el}	0.417(2) meV	Constrained by data
Nuclear	ΔE^{PWBA}	1.616(739) meV	NOT Constrained by data
	ΔE^{FSI}	0.391(44) meV	
	$\Delta E^{\perp}$	0.322(3) meV	Constrained by data
Hadronic	ΔE^{hadr}	0.028(2) meV	Constrained by data
Subtraction	ΔE^{subt}	-0.740(40) meV	NOT related to data
	$\Delta E^{Thomson}$	-0.023(1) meV	Constrained by data
Total	ΔE_{total}	2.011(740) meV	

Compare to Pachucki's NR calculation

$$\Delta E_{pol} = 1.680(16)\,\text{meV}$$

Effect of the TPE on the Isotopic Shift

New evaluation of the polarizability correction in eD

$$\Delta E^{e-D}_{2S-1S} = 28.8 \pm 12.0\,\mathrm{kHz}$$

$$r_E(d) = 2.1442(29)\,\mathrm{fm}$$

The D radius extraction uncertainty is dominated by that in the proton radius

Previous evaluation: Friar, Payne 1997

$$\Delta E^{e-D}_{2S-1S} = 19.04(7)\,\mathrm{kHz}$$

$$r_E(d) = 2.1424(21)\ \mathrm{fm}$$

Total uncertainty in isotopic shift was 0.89 kHz not dominated by the polarizability

Only a mild effect in eD if eH proton radius is used;
huge effect if μH proton radius is used
(is the former way the right way?)

TPE: QE contribution to Lamb shift

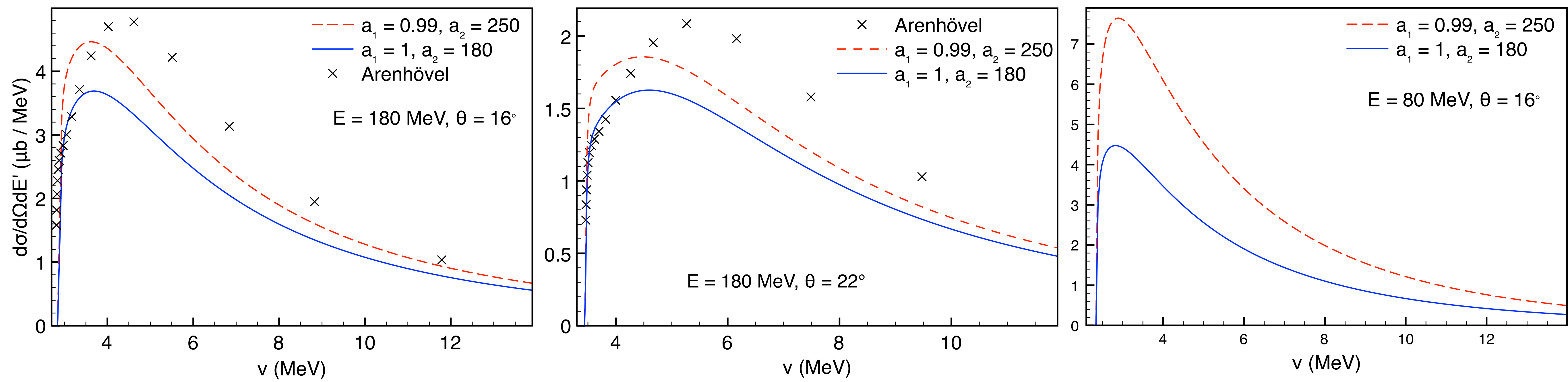
Where does the bulk of the uncertainty come from?

All kinematics contribute;
not all are weighted equally:

$$\langle Q^2 \rangle = 0.003 - 0.006\ (\mathrm{GeV/c})^2$$
$$\langle \nu \rangle = 6 - 10\ \mathrm{MeV}$$

At low Q^2 longitudinal cross section dominates

At lowest Q^2 only backward data available: F_2 unconstrained



Sensitivity in MAMI (180MeV - 16°, 22°) and P2 (80 MeV - 16°) kinematics

A1@MAMI: 180 MeV run for θ ≥ 15° - in early 2014; more if needed (MESA/P2)
Will test EFT and potential mod. calculations in their validity domain

New QE data to constrain the TPE correction

E_{lab}, θ_{lab}	Exp. precision	$\delta(\Delta E^{\mu D}_{2S-2P})$	$\delta(\Delta E^{eD}_{1S-2S})$
180 MeV, 30°	2%	740 μeV	12 kHz
	1%	370 μeV	6 kHz
180 MeV, 22°	2%	390 μeV	6.32 kHz
	1%	195 μeV	3.16 kHz
180 MeV, 16°	2%	211 μeV	3.36 kHz
	1%	110 μeV	1.68 kHz
80 MeV, 16°	2%	67 μeV	1.078 kHz
	1%	48 μeV	0.780 kHz

A1@MAMI: 180 MeV run for $\theta \geq 16^\circ$ - under analysis (Michael Distler's talk);
already these data may help reducing the uncertainty significantly!
more if needed with the new LINAC MESA/P2)
Will test EFT and potential mod. calculations in their validity domain

SUMMARY & OUTLOOK

- Dispersion Relations: adequate tool to compute structure-dependent corrections with reliable uncertainty estimate

- If necessary data are available: firm prediction for the polarizability correction to μH Lamb Shift

- For deuteron lack of data in kinematics that matters - large uncertainty in μD Lamb Shift

- Magnetic polarizability - input from EFT desirable

- New measurements of QE eD scattering at low Q^2 and forward angles - to reduce the uncertainty

- Polarizability correction for He-3, He-4?