CHIRAL PERTURBATION THEORY OF MUONIC HYDROGEN LAMB SHIFT

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Vladimir Pascalutsa

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University of Mainz, Germany

@ MITP Workshop “Proton charge radius”
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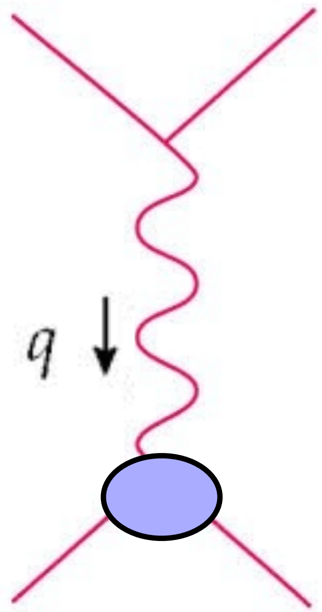
Proton structure in hydrogen spectrum



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$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm \right| \delta V^{(1\gamma)} \left| nlm \right\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, r_E^2\, \frac{\alpha^3 m_r^3}{\pi n^3} + O(\alpha^5)$$

wave function at origin

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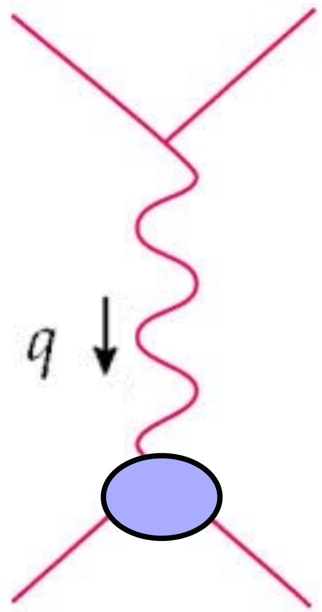

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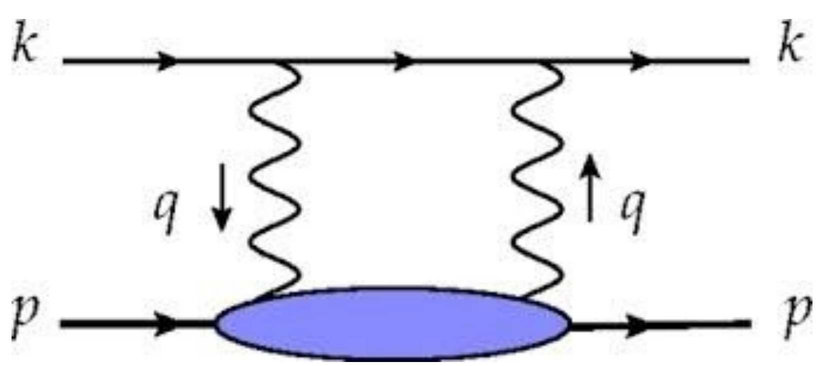
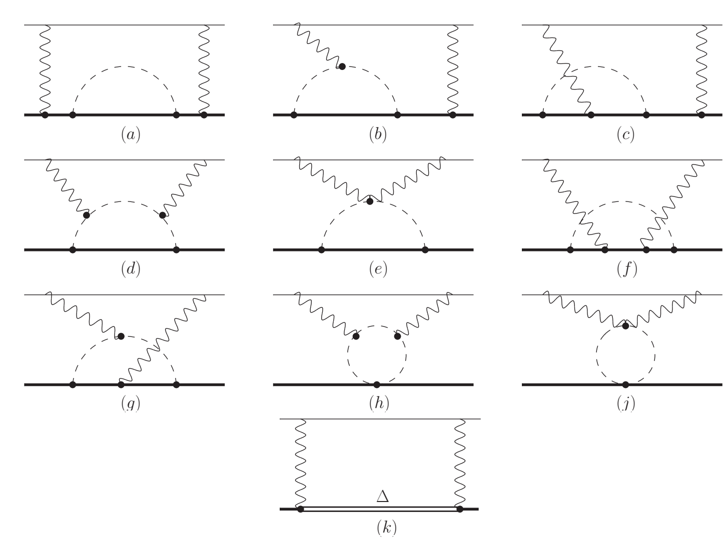


$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\mathrm{elastic}} + \delta V^{(2\gamma)}_{\mathrm{polariz.}}$$

ChPT prediction:
finite (free-LEC free)
result



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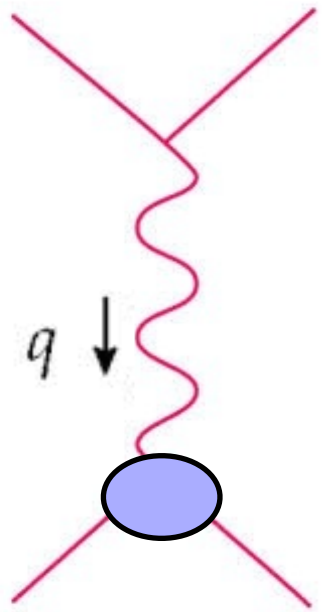

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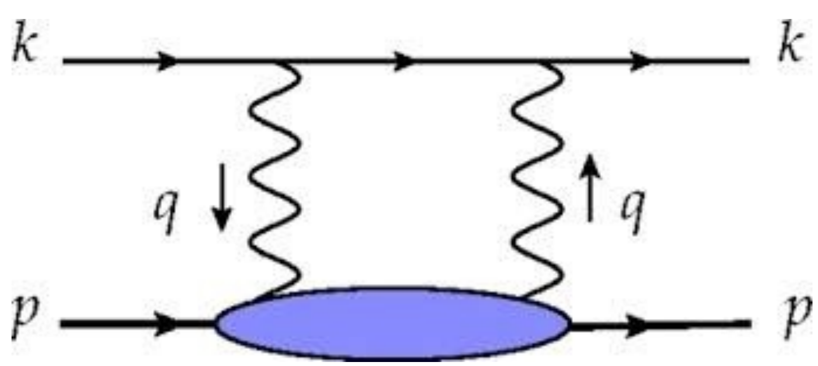

included in 3rd Zemach moment

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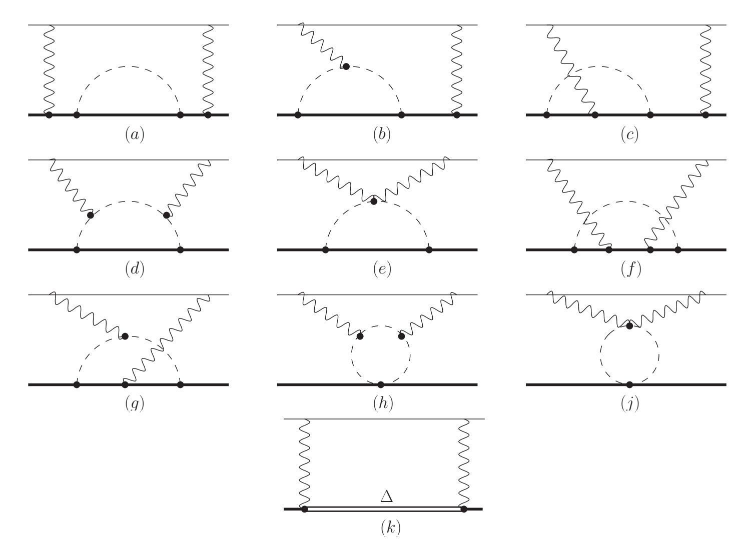

3rd Zemach moment (or Friar moment)

Correction to Coulomb due to proton's charge distribution:

$$\delta V_{\rm FF}(r) = -\int \frac{d\vec{q}}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{r}}\, \frac{4\pi\alpha}{\vec{q}^{\,2}} \left[G_E(-\vec{q}^{\,2}) - 1\right]$$
$$= \frac{\alpha}{\pi r}\int_{t_0}^{\infty} \frac{dt}{t}\, e^{-r\sqrt{t}}\, \mathrm{Im}\, G_E(t)$$

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1st order PT

$$\Delta E^{\rm FF(1)}_{2P-2S} = -\frac{\alpha^4 m_r^3}{2\pi}\int_{t_0}^{\infty} dt\, \frac{\mathrm{Im}\, G_E(t)}{(\sqrt{t}+\alpha m_r)^4} = -\frac{\alpha^4 m_r^3}{12}\sum_{N=2}^{\infty}\frac{(-\alpha m_r)^{N-2}}{(N-2)!}\left\langle r^N\right\rangle = -\frac{\alpha^4 m_r^3}{12}\Big(\left\langle r^2\right\rangle - \alpha m_r\left\langle r^3\right\rangle\Big) + O(\alpha^6).$$

Moments of charge distribution:

$$\left\langle r^N\right\rangle \equiv \int d\vec{r}\, r^N \rho(r) = \frac{(N+1)!}{\pi}\int_{t_0}^{\infty} dt\, \frac{\mathrm{Im}\, G_E(t)}{t^{1+N/2}}.$$

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2nd order PT

$$\Delta E^{\mathrm{FF}(2)}_{2S} = -\alpha^5 m_r^4\, \frac{2}{\pi}\int_0^{\infty} dk \left\{ \frac{1}{\pi}\int_{t_0}^{\infty} \frac{dt}{t}\, \frac{1}{t+k^2}\, \mathrm{Im}\, G_E(t) \right\}^2 + O(\alpha^6)$$
$$= -\alpha^5 m_r^4\, \frac{2}{\pi}\int_0^{\infty} \frac{dk}{k^4}\, \left\{G_E(-k^2) - 1\right\}^2 + O(\alpha^6).$$

$$\Delta E^{(2)}_{2P-2S} = -\frac{1}{12}\,\alpha^5 m_r^4 \left(\left\langle r^3 \right\rangle - \frac{1}{2}\left\langle r^3 \right\rangle_{(2)} \right)$$

Moments of charge distribution:

$$\left\langle r^N \right\rangle \equiv \int d\vec{r}\, r^N \rho(r) = \frac{(N+1)!}{\pi}\int_{t_0}^{\infty} dt\, \frac{\mathrm{Im}\, G_E(t)}{t^{1+N/2}}.$$

$$\left\langle r^3 \right\rangle = \frac{48}{\pi}\int_0^{\infty} \frac{dk}{k^4}\, \left\{G_E(-k^2) - 1 + \tfrac{1}{6}\left\langle r^2 \right\rangle k^2 \right\}$$

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$$\Delta E^{(2)}_{2P-2S} = -\frac{1}{12}\,\alpha^5 m_r^4\left(\langle r^3\rangle - \frac{1}{2}\langle r^3\rangle_{(2)}\right)$$

cancellation

Moments of charge distribution:

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Polarizability contribution in ChPT

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

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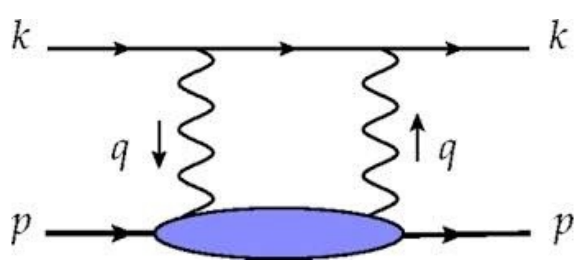
[2] Theoretical Physics Group, School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, UK
[3] Institute for Theoretical and Experimental Physics, Bol'shaya Cheremushkinskaya 25, 117218 Moscow, Russia



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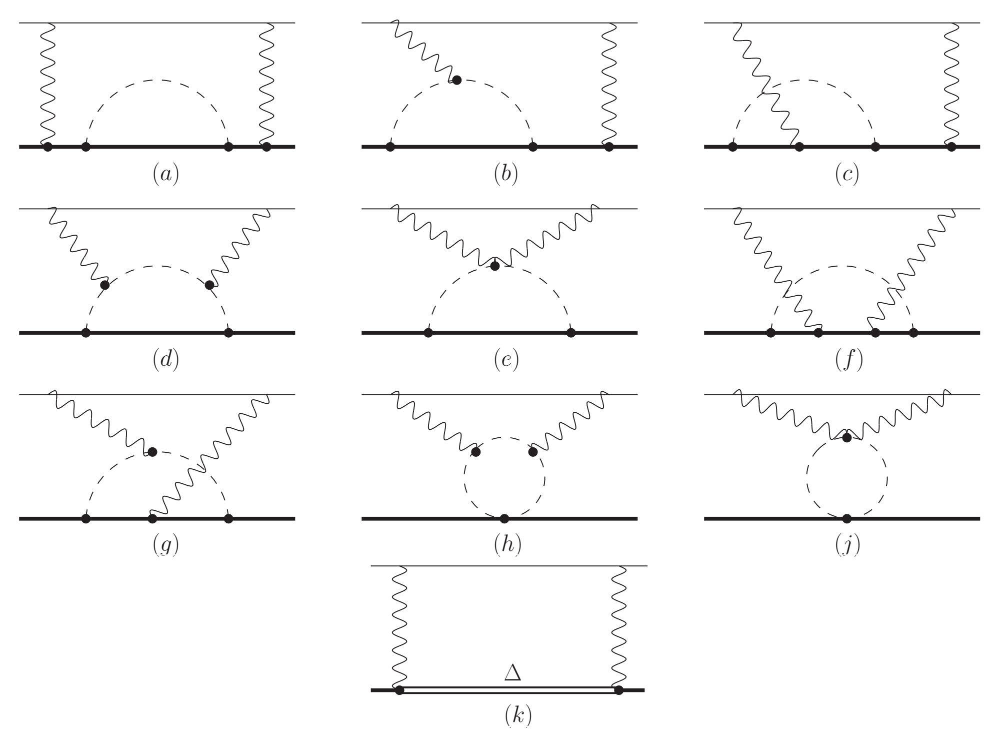

with corrections to elastic proton FFs subtracted, i.e. "polarizability" alone

Proton polarizability effect in mu-H

Heavy-Baryon (HB)ChPT

[Alarcon, Lensky & VP, EPJC (2014)]

(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\mathrm{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\mathrm{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\mathrm{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
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$G \simeq 0.9160$ is the Catalan constant.

Chiral Perturbation Theory

(low-energy EFT of QCD)

[Weinberg (1979), Gasser & Leutwyler (1984, 85)]

Schematically,

$$Z_{QCD} = \int \prod_x \left(dG\, dq \right) e^{i \int d^4x [-G\cdot G + \bar{q}(\not{D} - m) q + \dots]}$$

$$\stackrel{E \ll 1 GeV}{=} \int \prod_x \left(dU\, dN \dots \right) e^{i \int d^4x [\partial U^\dagger \partial U - m(U + U^\dagger) B_0 + \bar{N}(\not{D} - M_0) N + \dots]}$$

where $U(x) = e^{2i\pi(x)/f_\pi}, \quad m_\pi^2 = B_0(m_u + m_d) + O(m^2), \quad B_0 \simeq - < \bar{q}q > / f_\pi^2 \approx 3\, GeV$

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Consequence of chiral symmetry: pion fields enters with a derivative or mass, i.e. interactions have positive powers of pion 4-momentum

$$\frac{p^\mu}{4\pi f_\pi}, \quad \text{or} \quad \frac{|\vec p|}{4\pi f_\pi}, \frac{m_\pi}{4\pi f_\pi}$$

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Power-counting: how many powers of p will a given Feynman graph contribute

Baryon ChPT

pion cloud + Delta(1232) excitation

Jenkins & Manohar, PLB (1991)
Hemmert, Holstein, Kambor, JPhysG (1998)
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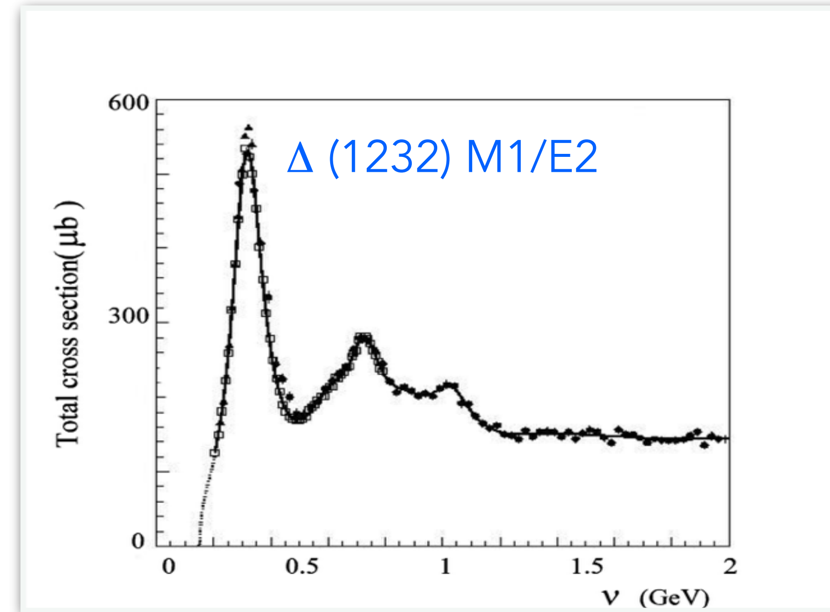

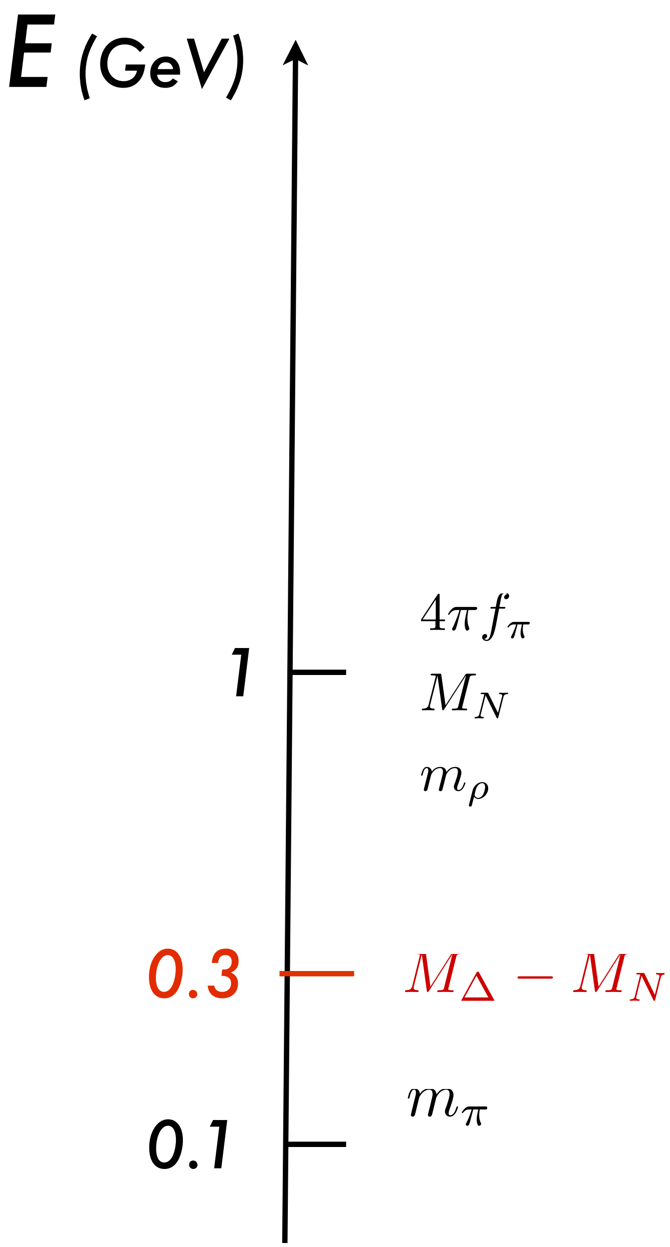

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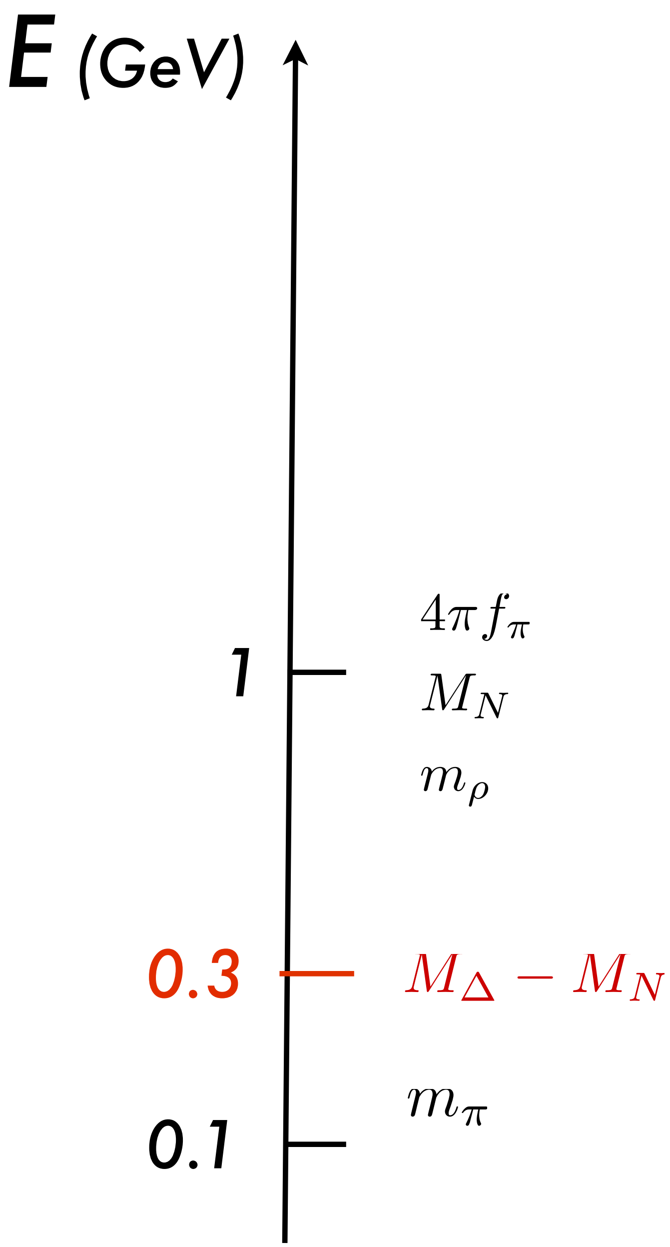
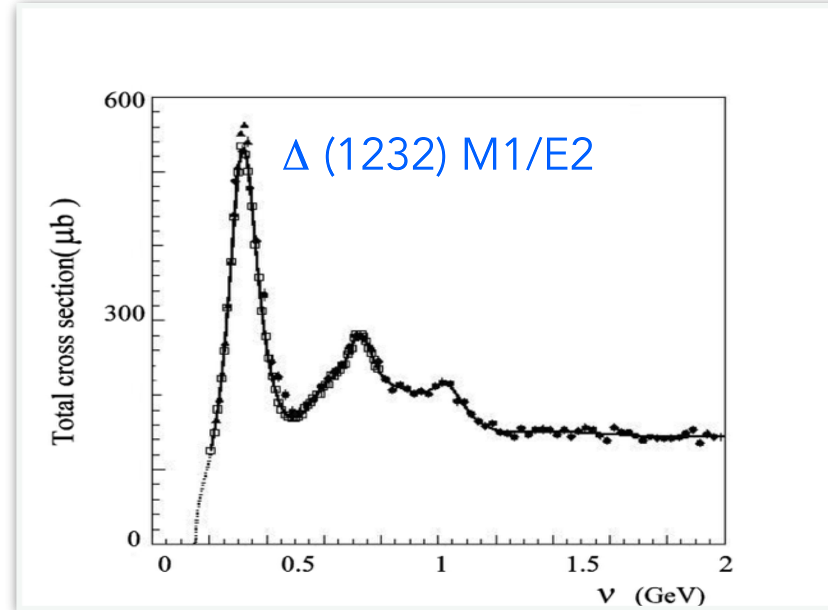
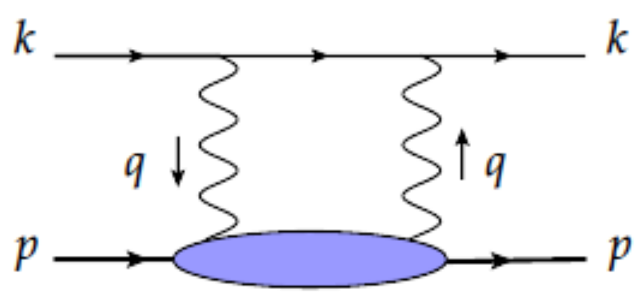




- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE, ``delta-counting") depends on what chiral order is assigned to the excitation scale.

Lamb shift in terms of VVCS amplitudes



empirically known 'inelastic'

unknown 'subtraction'

$$\Delta E^{(\mathrm{pol})}_{nS} = -4\alpha_{em}\phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w(Q^2/4m_\ell^2)\, \left[T_2^{(\mathrm{NB})}(0,Q^2) - T_1^{(\mathrm{NB})}(0,Q^2)\right]$$

where unpolarized, **forward** Doubly-Virtual Compton scattering (VVCS) amplitude:

$$\begin{aligned} T^{\mu\nu}(p,q) &= \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|T j^\mu(x) j^\nu(0)|p\rangle \\ &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) \\ &+ \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu,Q^2) \end{aligned}$$

NB stands for non-Born, i.e. w/o elastic FFs

$$T_1^{(\mathrm{NB})}(0,Q^2) \simeq Q^2\beta_{M1}$$
$$T_2^{(\mathrm{NB})}(0,Q^2) \simeq Q^2(\alpha_{E1}+\beta_{M1}), \quad \text{for low } Q$$

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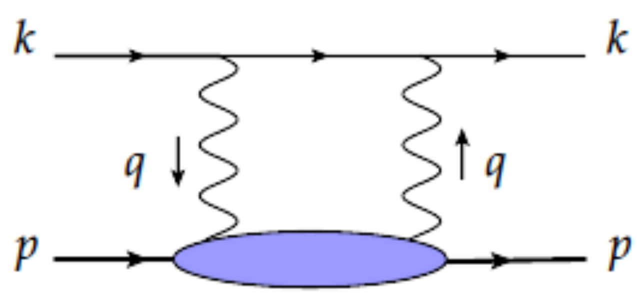

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$$\Delta E_{nS}^{(\mathrm{pol})} = -4\alpha_{em}\phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w(Q^2/4m_\ell^2)\, \left[T_2^{(\mathrm{NB})}(0,Q^2) - T_1^{(\mathrm{NB})}(0,Q^2)\right]$$

$$\phi_n^2(0) = m_r^3\alpha^3/(\pi n^3)$$

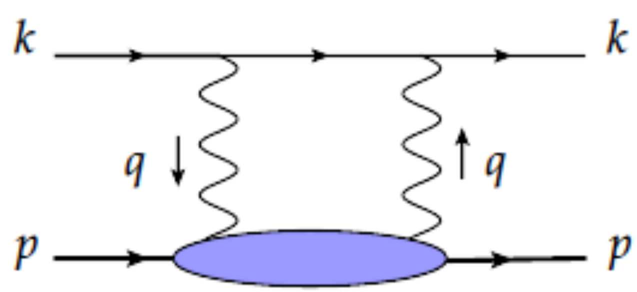
where unpolarized, **forward** Doubly-Virtual Compton scattering (VVCS) amplitude:

$$\begin{aligned} T^{\mu\nu}(p,q) &= \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|T j^\mu(x) j^\nu(0)|p\rangle \\ &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) \\ &+ \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu,Q^2) \end{aligned}$$

NB stands for non-Born, i.e. w/o elastic FFs

$$\begin{aligned} T_1^{(\mathrm{NB})}(0,Q^2) &\simeq Q^2\beta_{M1} \\ T_2^{(\mathrm{NB})}(0,Q^2) &\simeq Q^2(\alpha_{E1}+\beta_{M1}), \quad \text{for low } Q \end{aligned}$$

Lamb shift in terms of VVCS amplitudes



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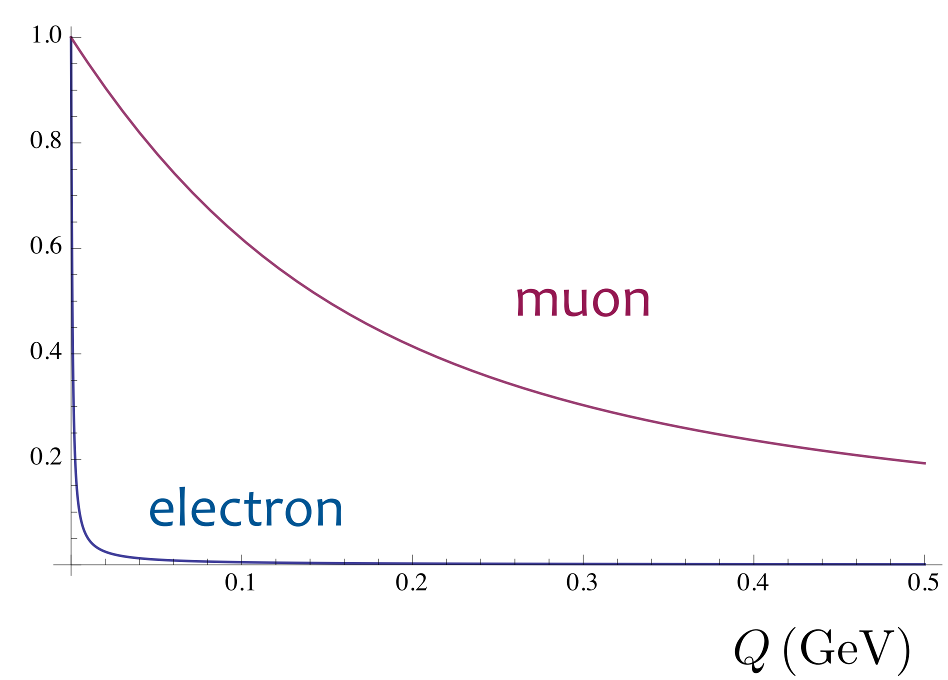
$$w_\ell(Q) = \sqrt{1+\frac{Q^2}{4m_\ell^2}} - \frac{Q}{2m_\ell}$$



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Effectiveness of BChPT vs. HBChPT

Heavy-Baryon is an expansion of Baryon ChPT in powers of m_π/M_N
and keeping only the LO term — approximation.

HB result has high-momentum contribution greater than expected uncertainty..

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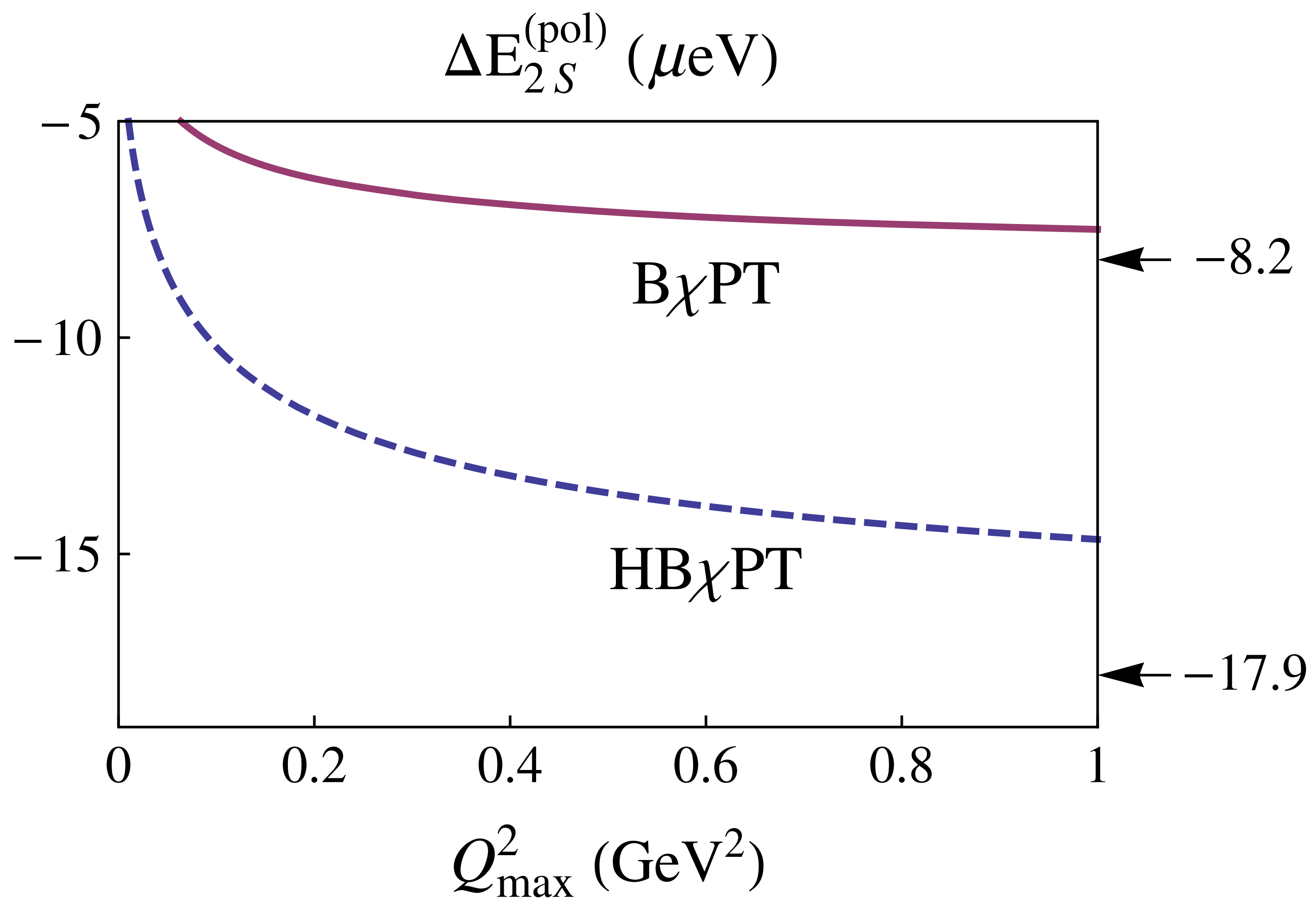
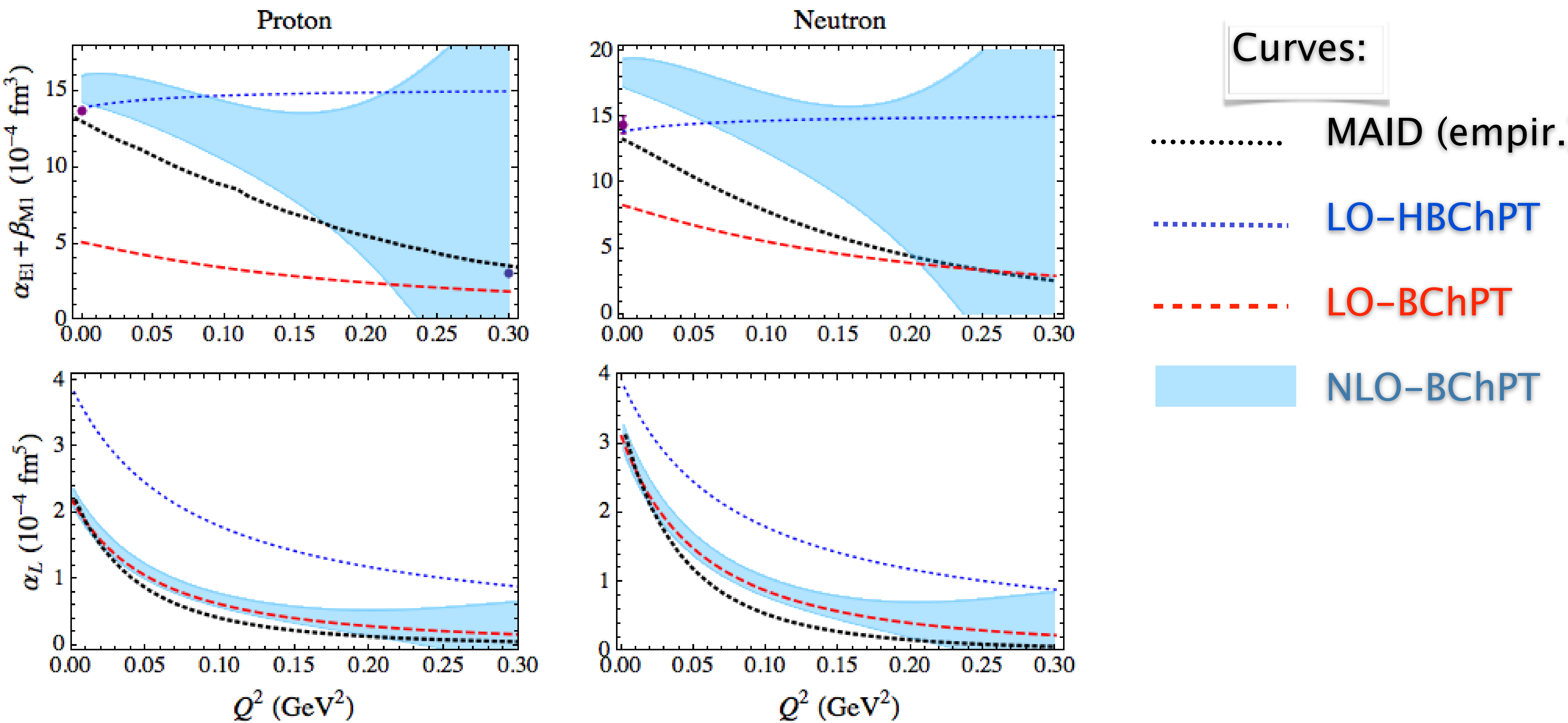


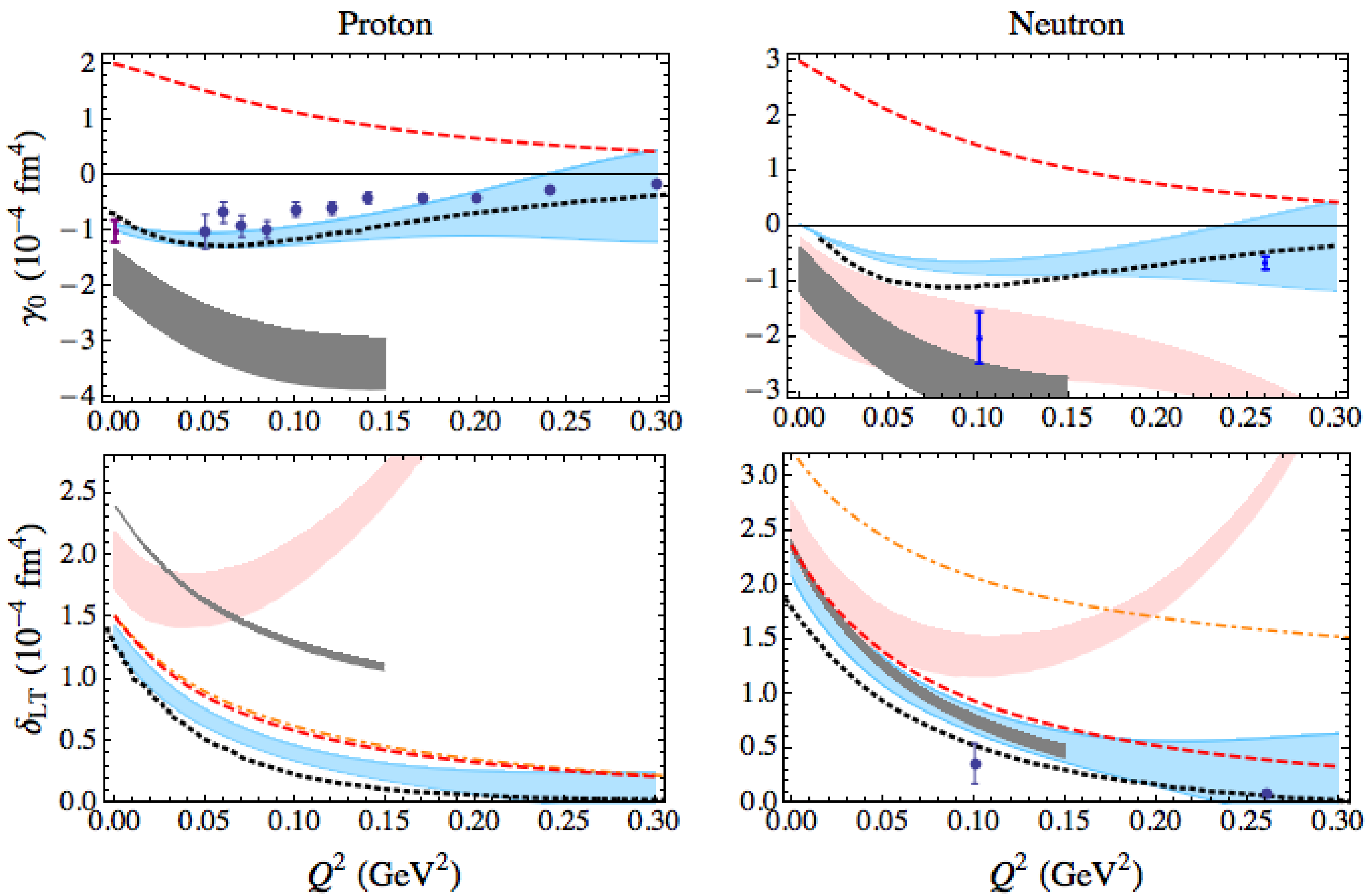
Predictions of BChPT for VVCS

Alarcon, Lensky & VP, in preparation



BChPT for polarised VVCS (deltaLT puzzle)

Alarcon, Lensky & VP, in preparation



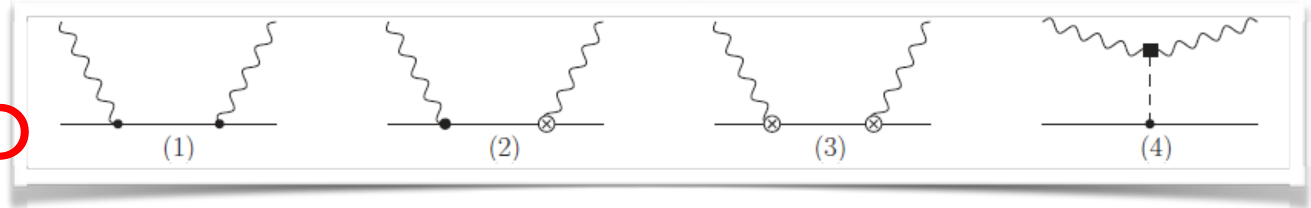
Curves:

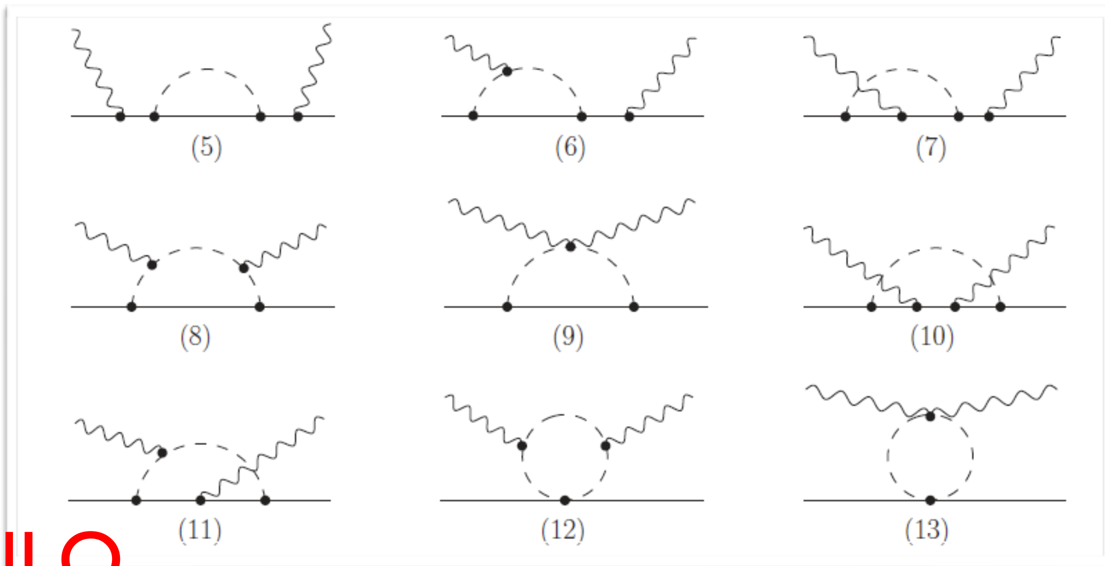
- MAID (empir.)
- NLO–HBChPT
- LO–BChPT
- NLO–BChPT
- NLO–BChPT [Krebs et al 2013)]

Data points:
JLab spin program

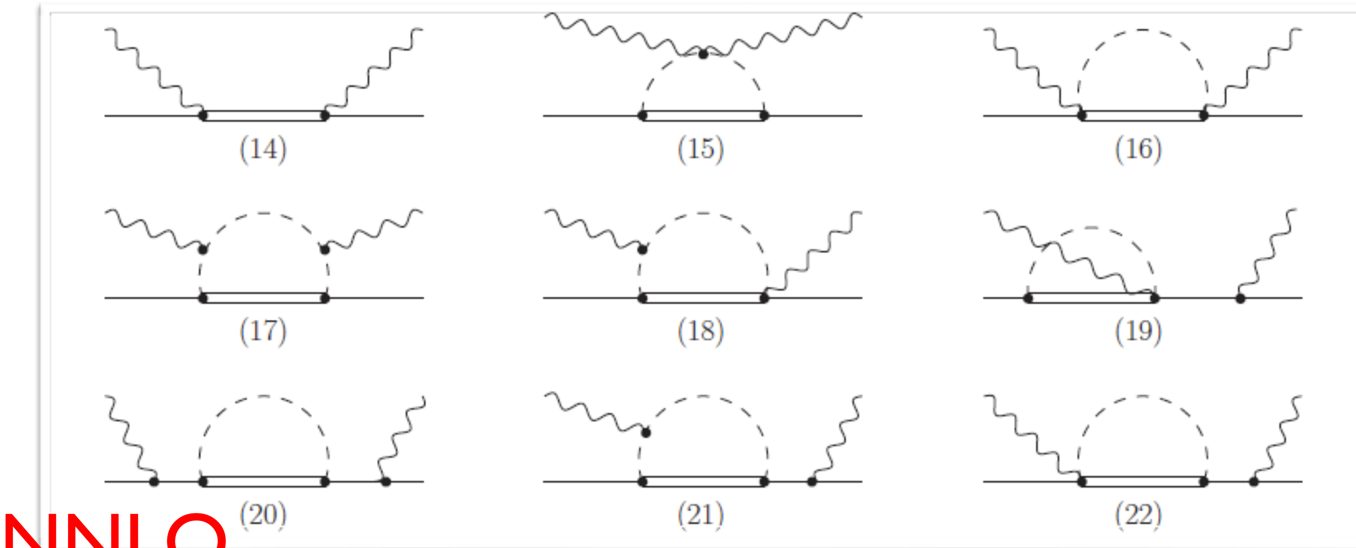
ChPT of Compton scattering off protons

LO



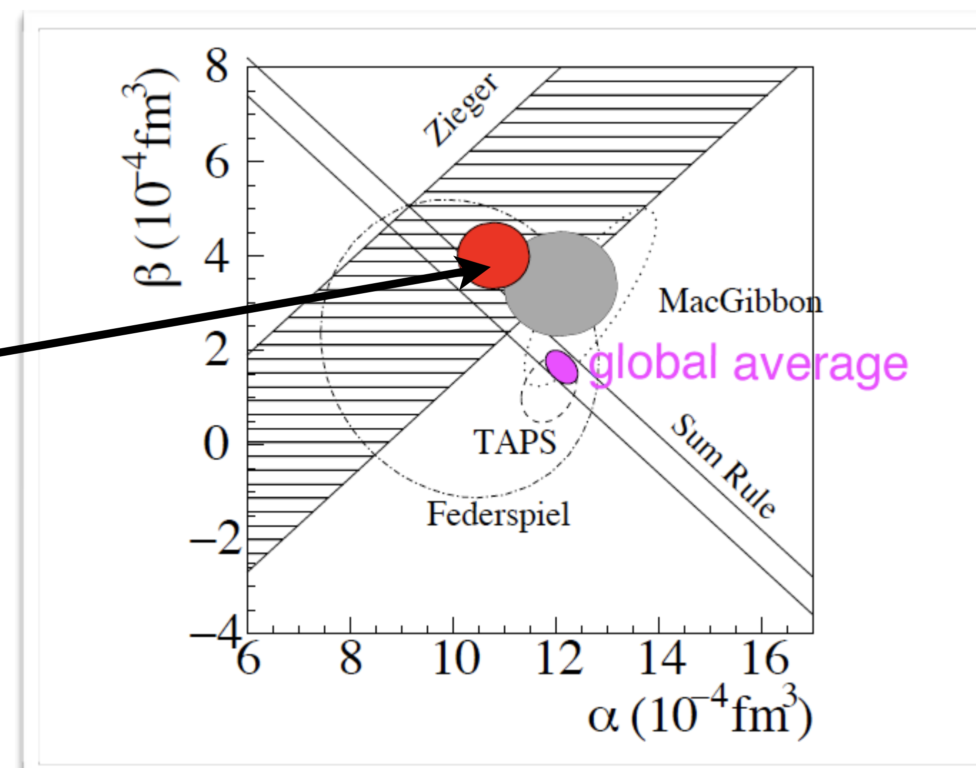


NLO



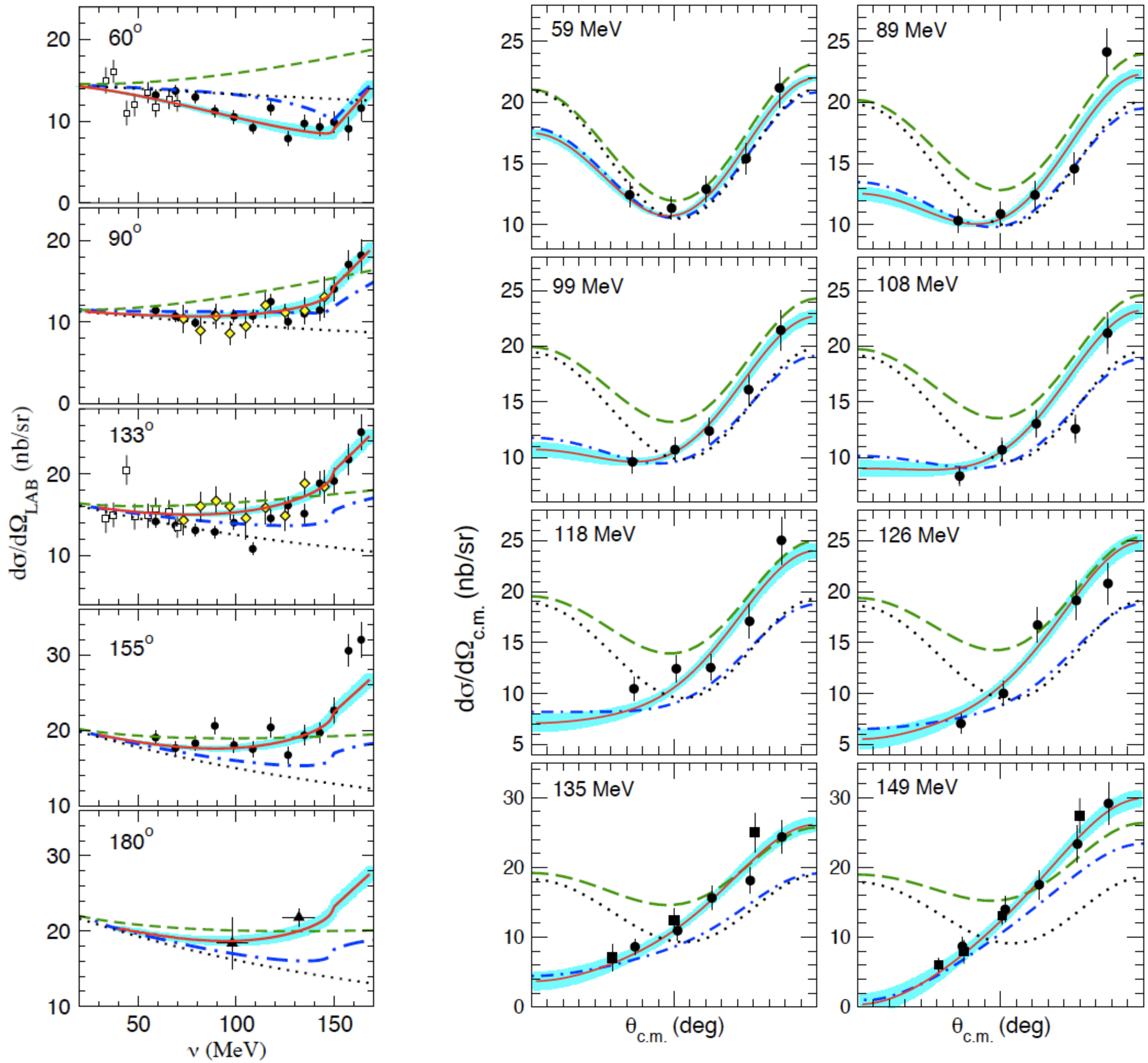
NNLO

$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$ size of the red blob



Lensky & V.P., EPJC (2010)

Unpolarized cross sections for RCS



Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

Curves:

.... Klein–Nishina

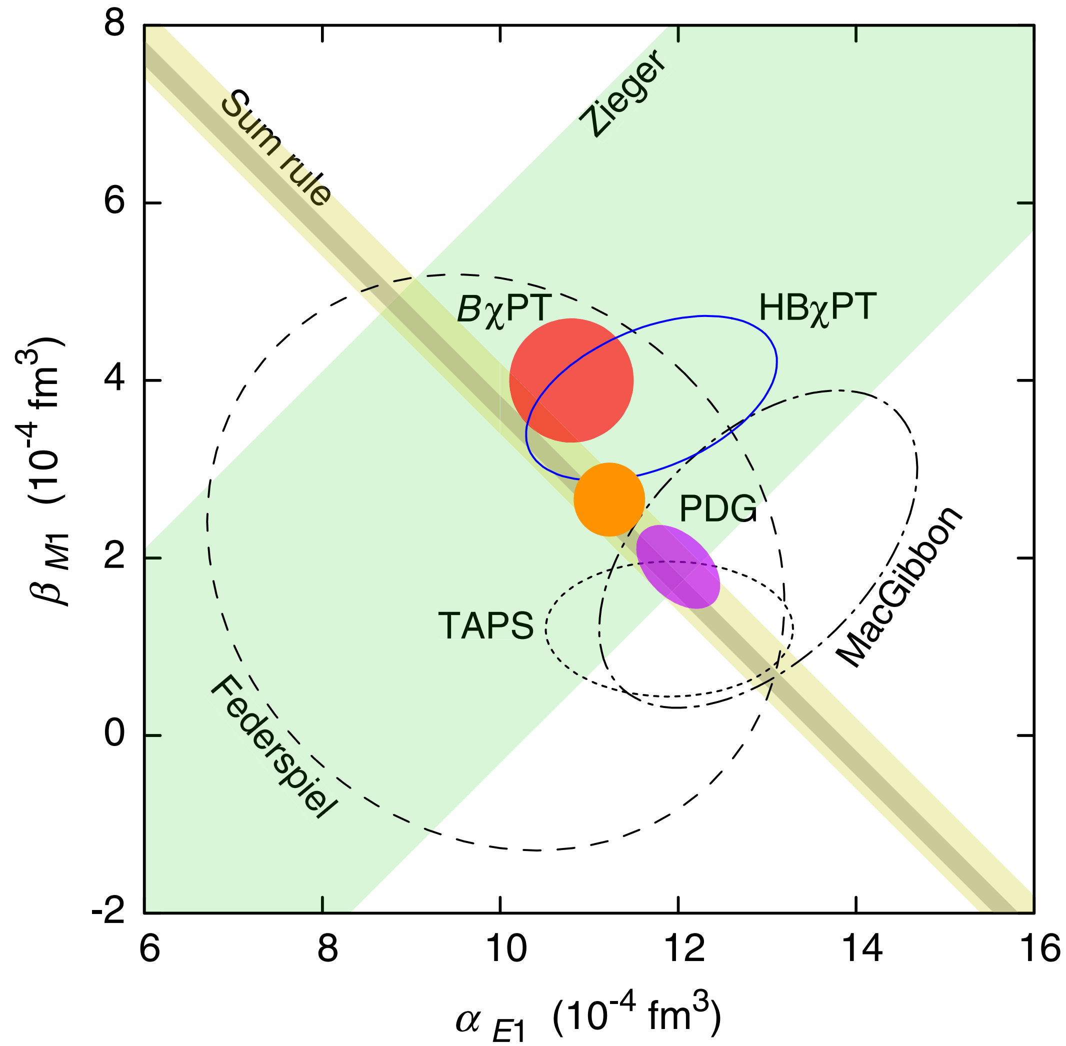
– – – – Born + WZW

– · – · – + p-qube

Total NNLO

Lensky & V.P.,EPJC (2010)

Proton polarizabilities



BChPT - Lensky & V.P., EPJC(2010)
HBChPT - Griesshammer, McGovern, Phillips, EPJA (2013)

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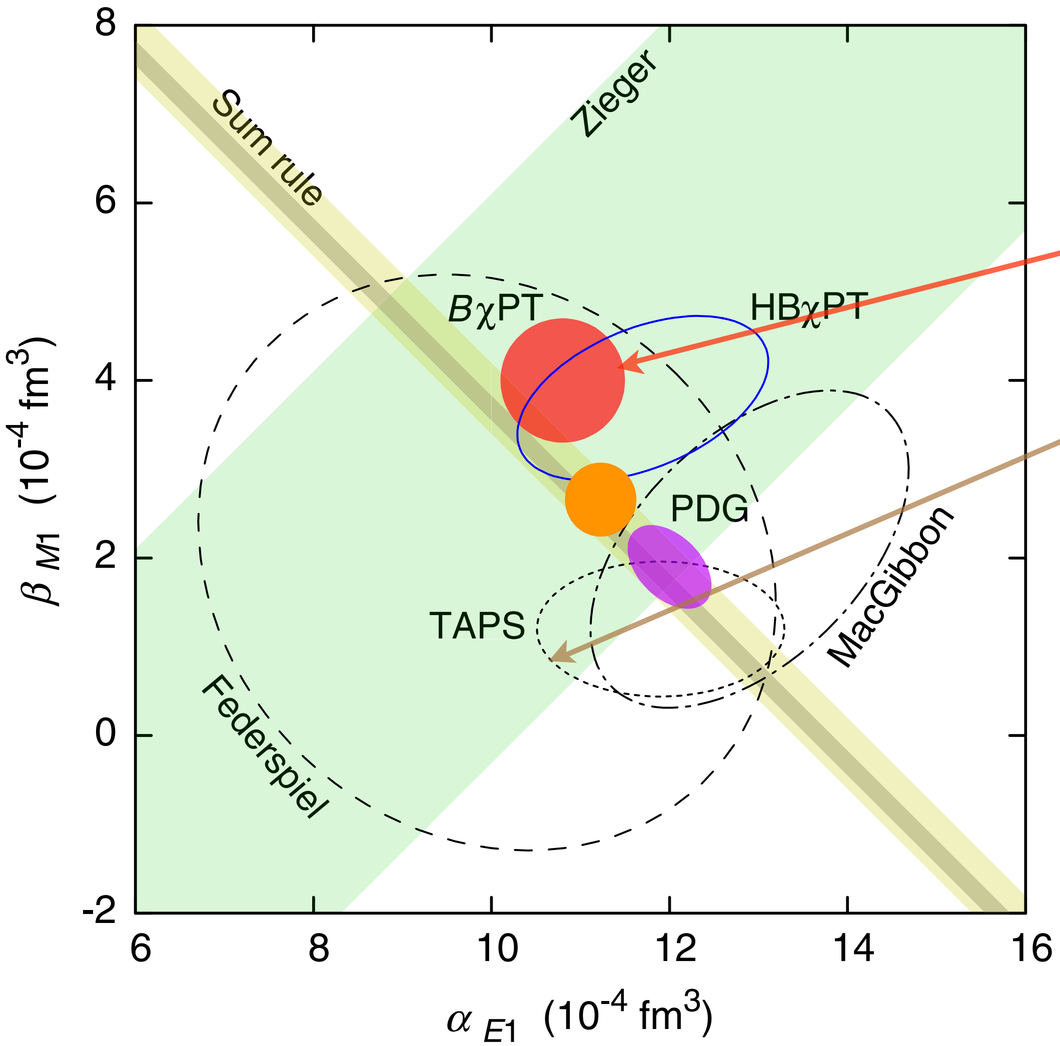

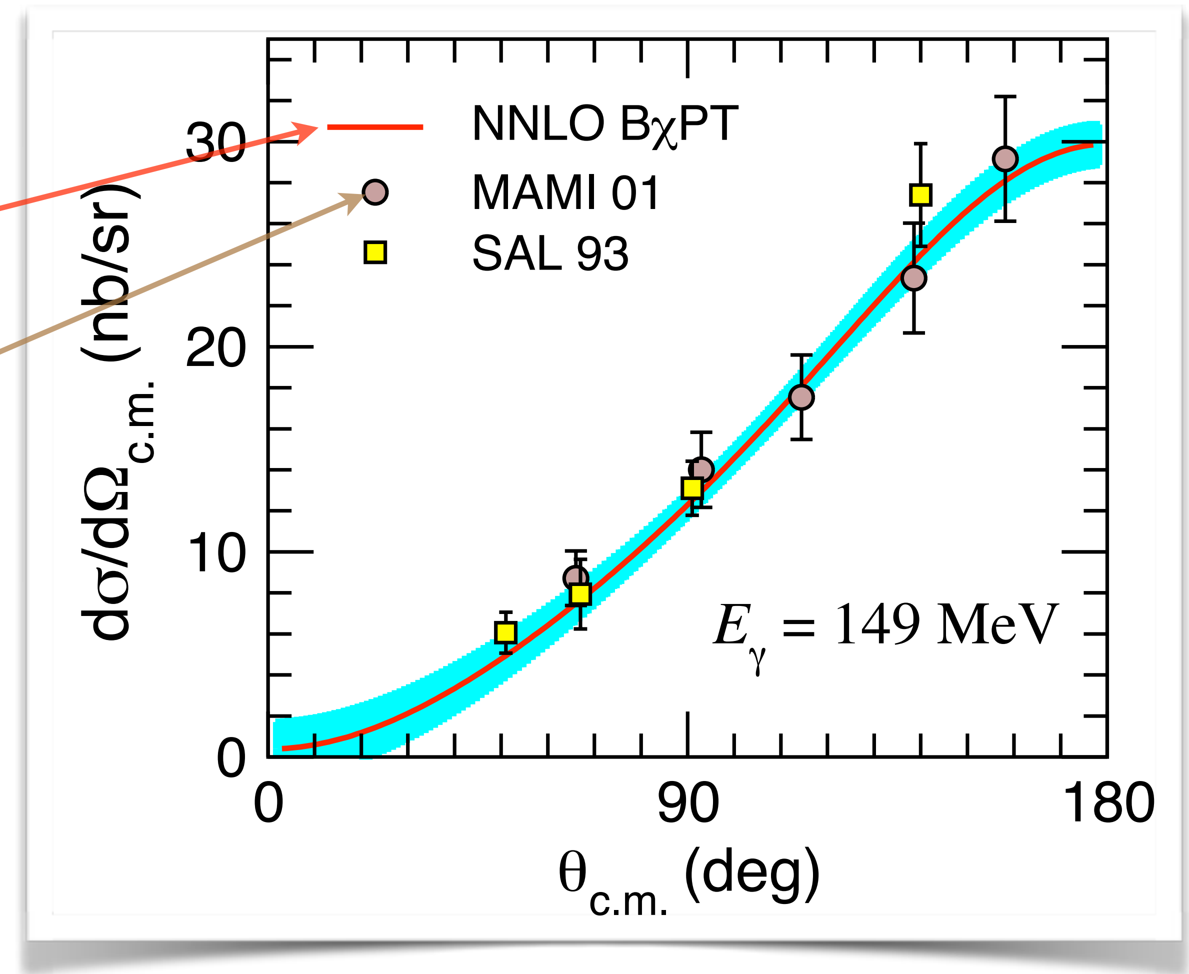

$$\beta_{M1} = (1.9 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{PDG}]$$

$$\beta_{M1} = (4.0 \pm 0.7) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{BChPT@NNLO}]$$

BChPT - Lensky & V.P., EPJC(2010)

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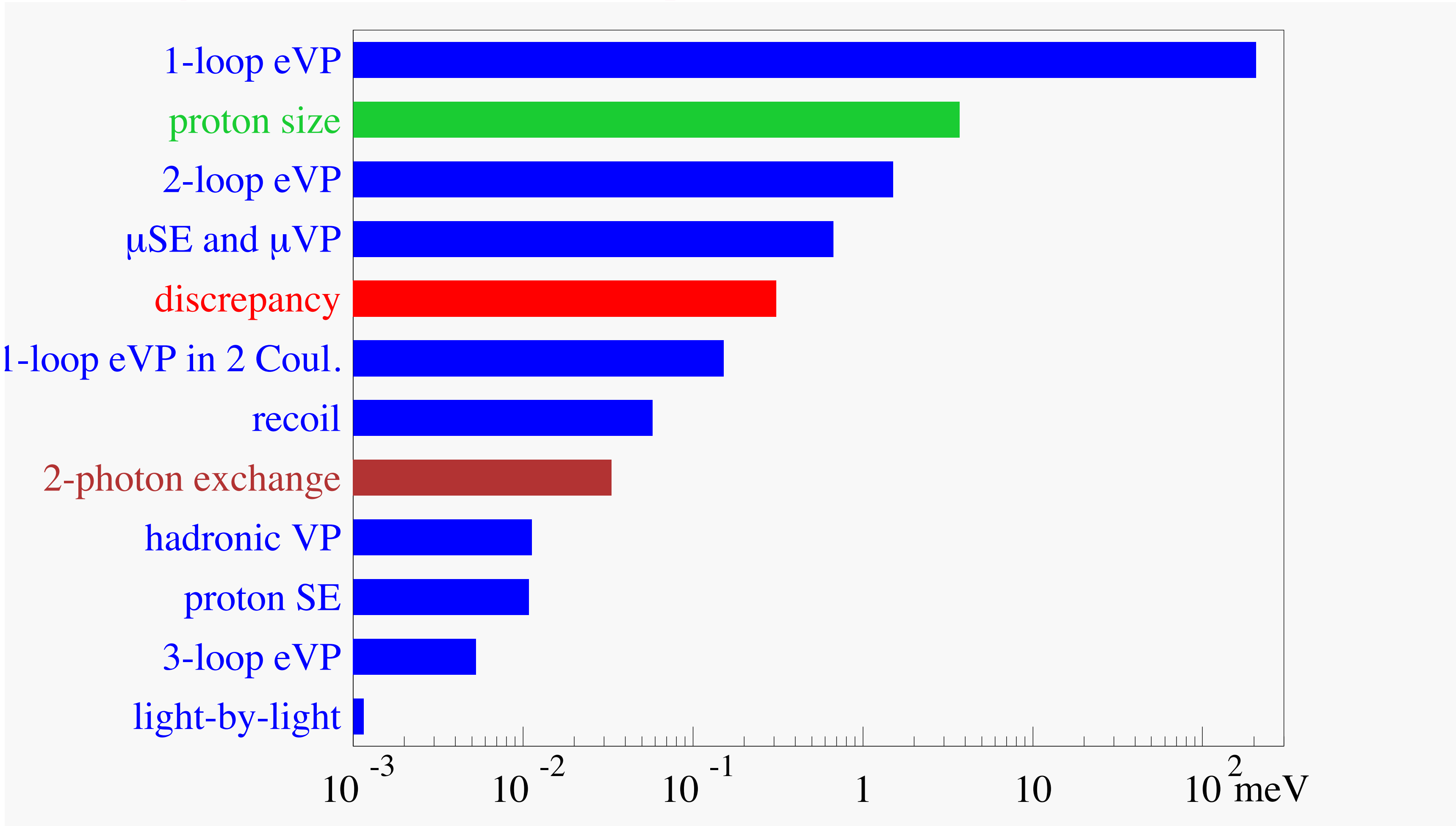
PDG adjusted values from 2012 edition (**purple**) to 2013 on-line edition (**orange**)

Proton structure in muonic hydrogen Lamb shift

Antognini et al, Ann Phys (2013):



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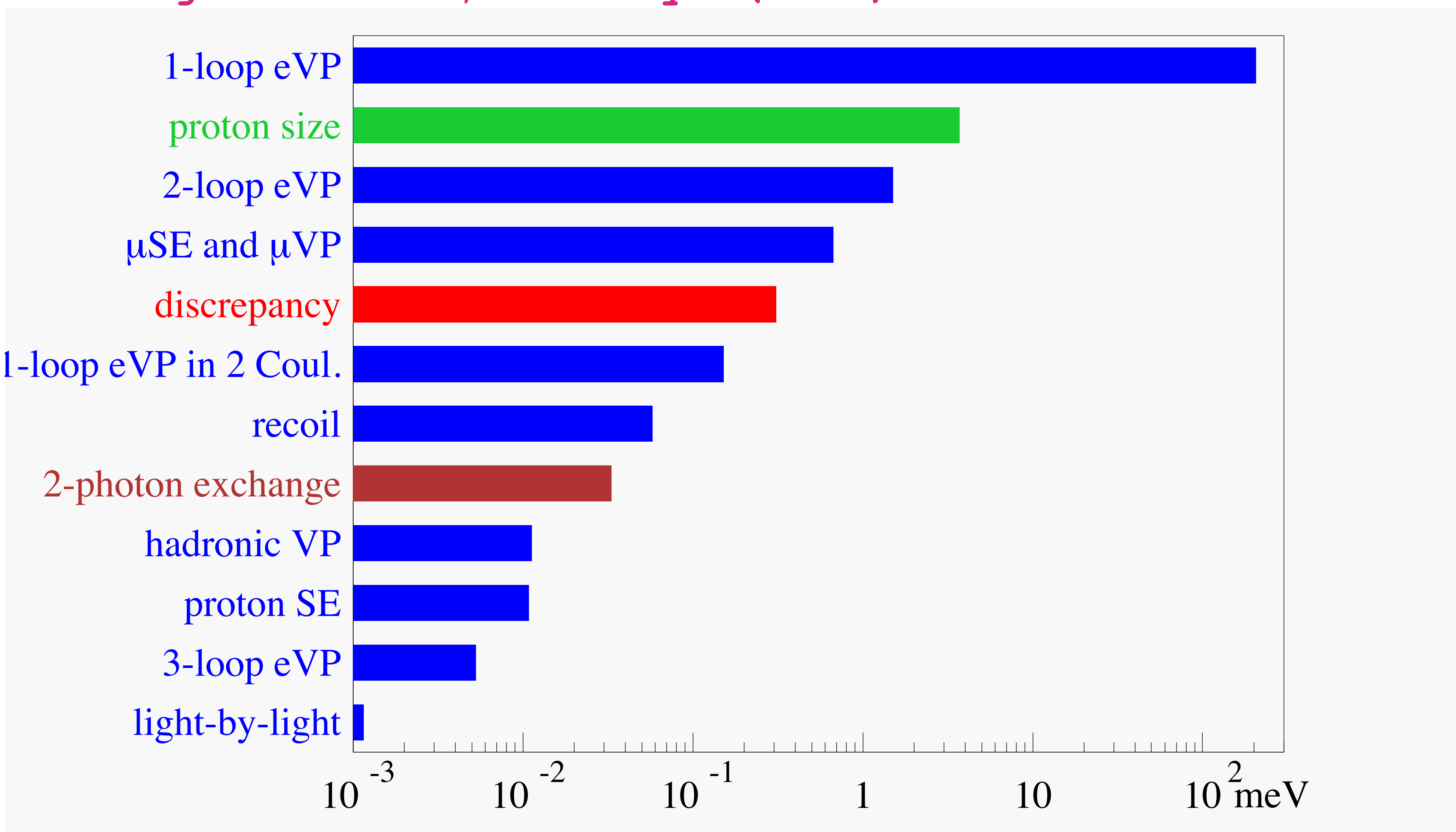

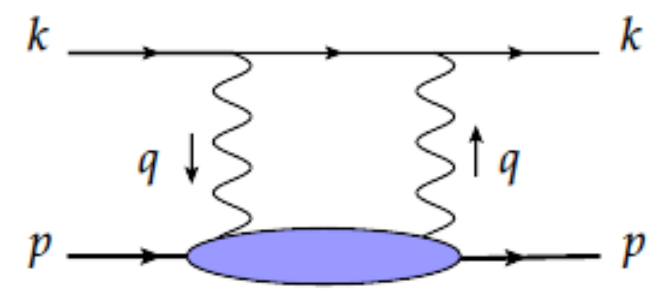

proposed to resolve the puzzle

De Rujula, PLB (2010)
Miller, PLB (2013)

calculable in ChPT @ LO

HB ChPT:

Nevado & Pineda, PRC (2008)

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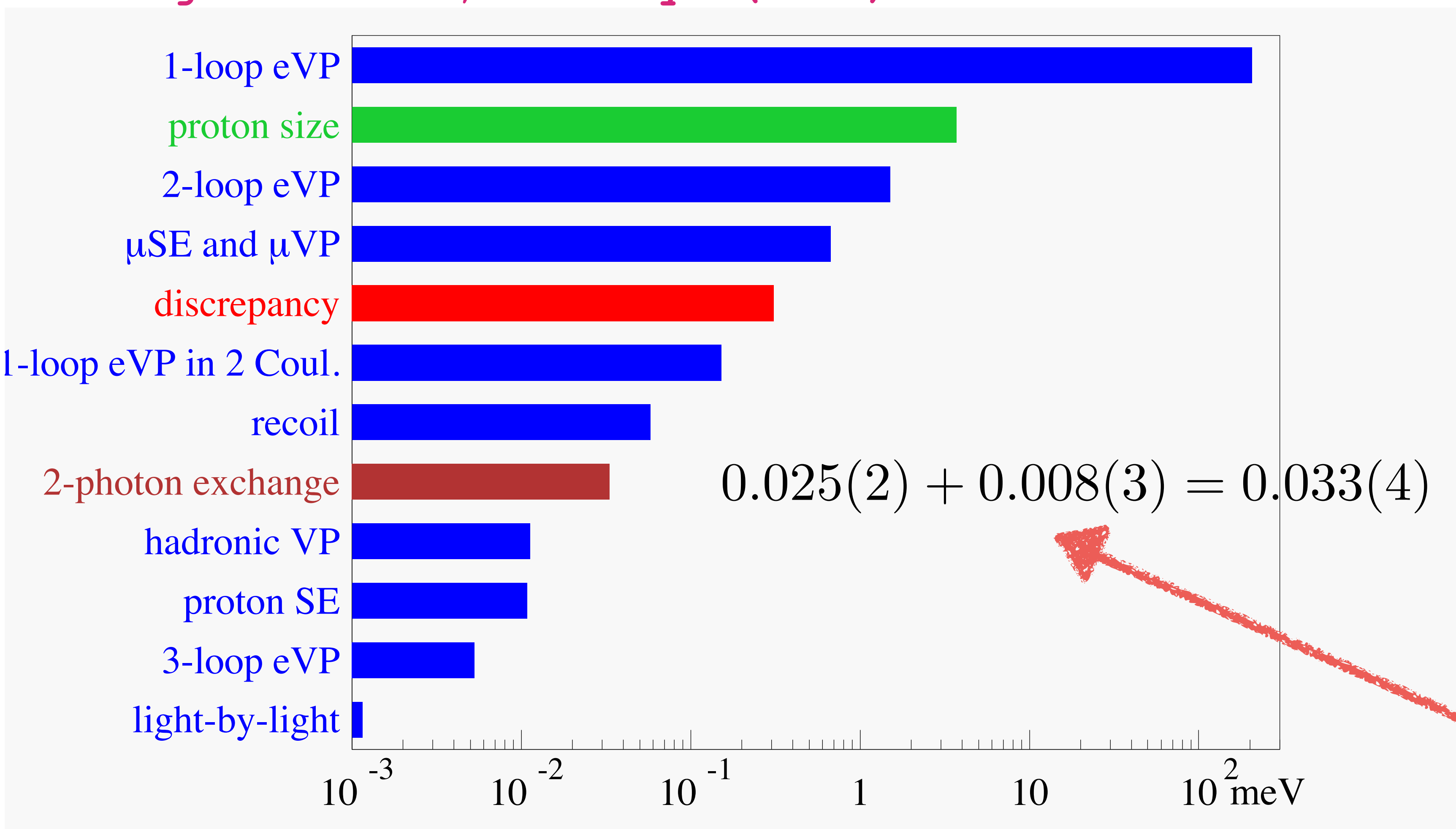
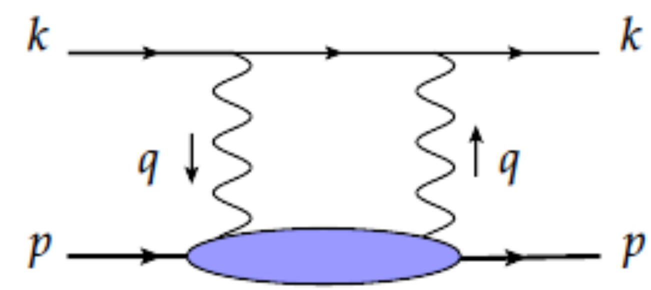


$$0.025(2) + 0.008(3) = 0.033(4)$$



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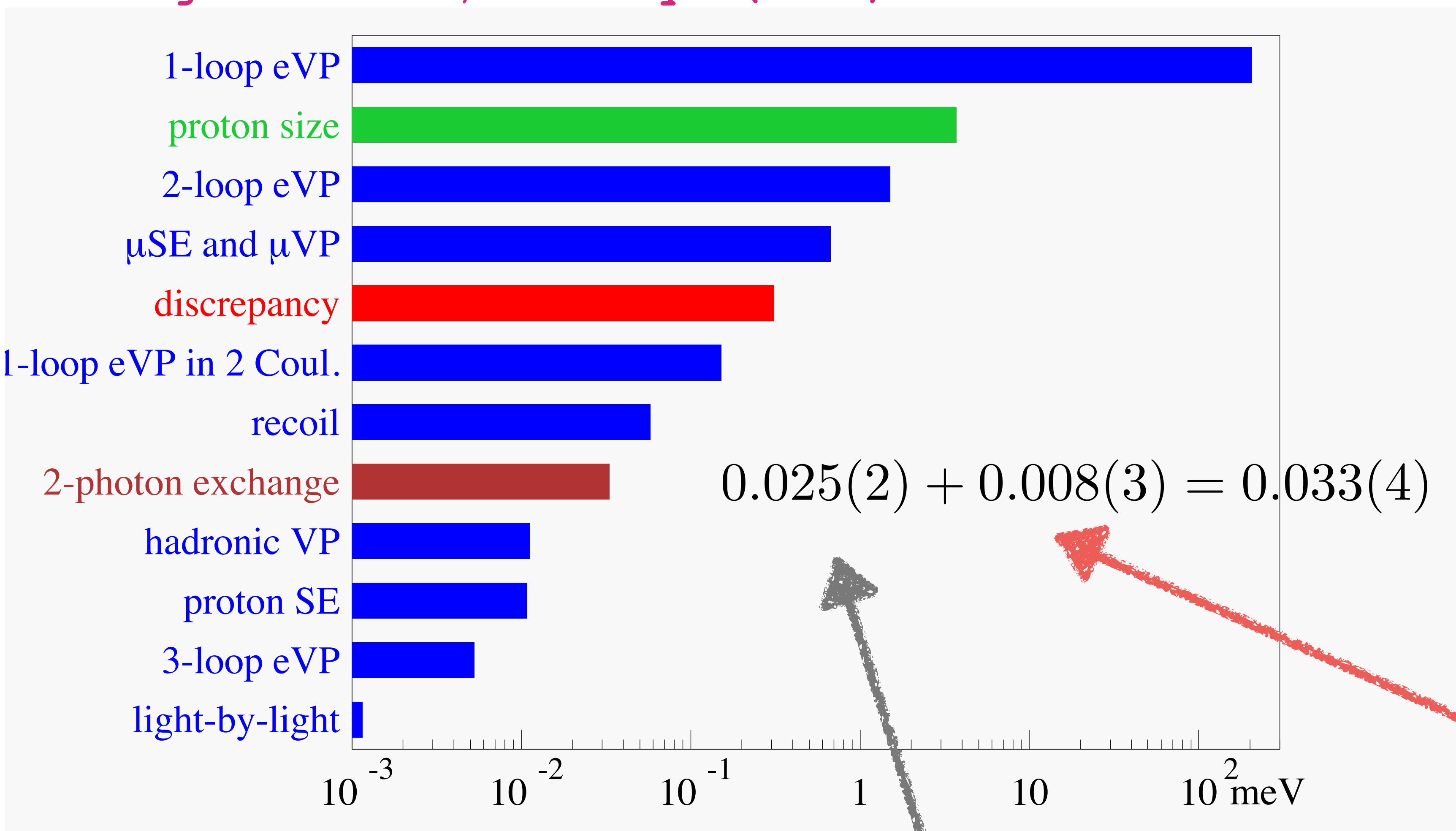

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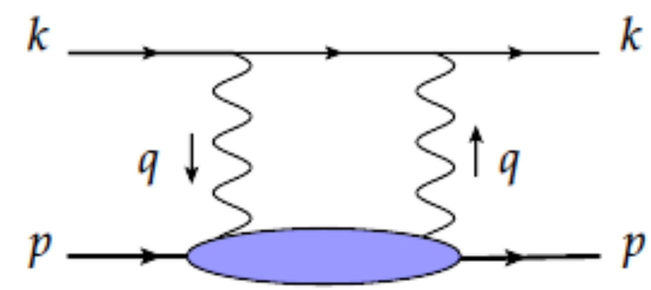

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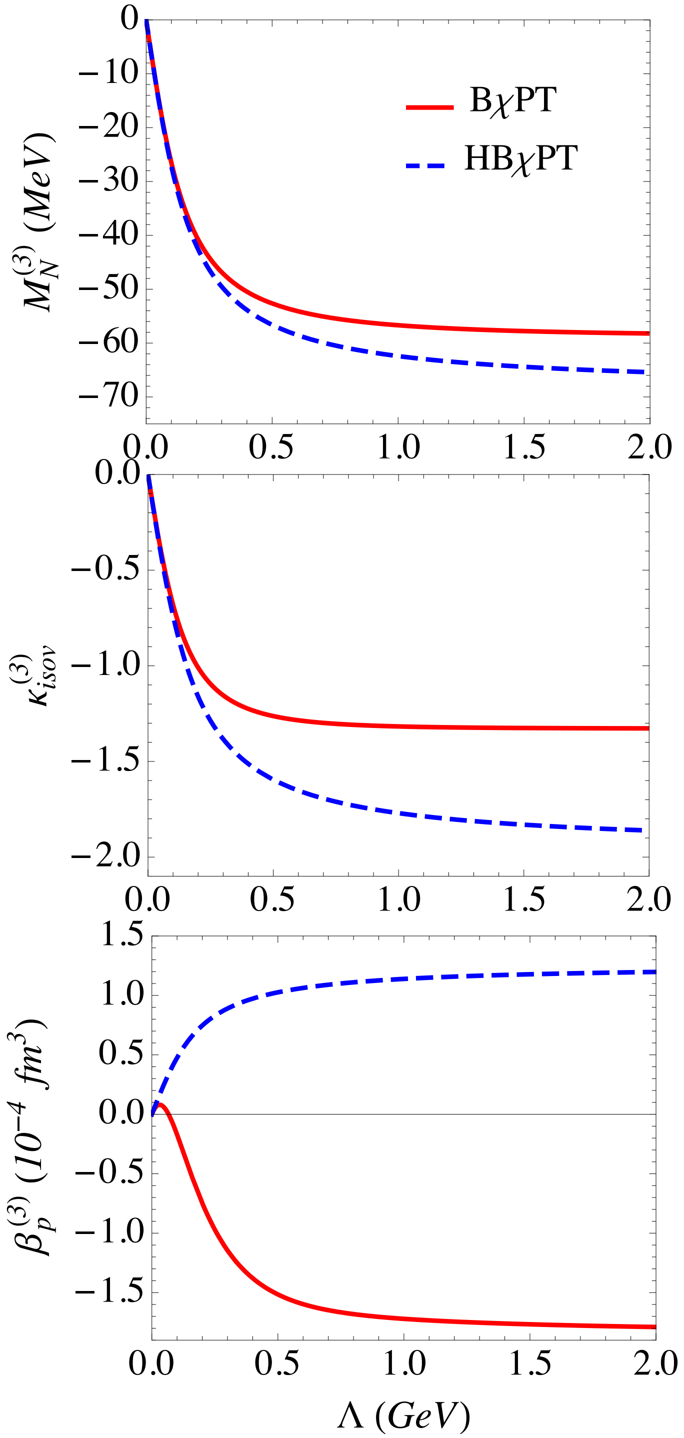
elastic (3rd Zemach moment):

Carlson & Vanderhaeghen, PRA (2011)
Birse & McGovern, EPJA (2012)

newer analysis:
Karshenboim,1405.6039

Backup slides

UV dependence in HB- vs B-ChPT



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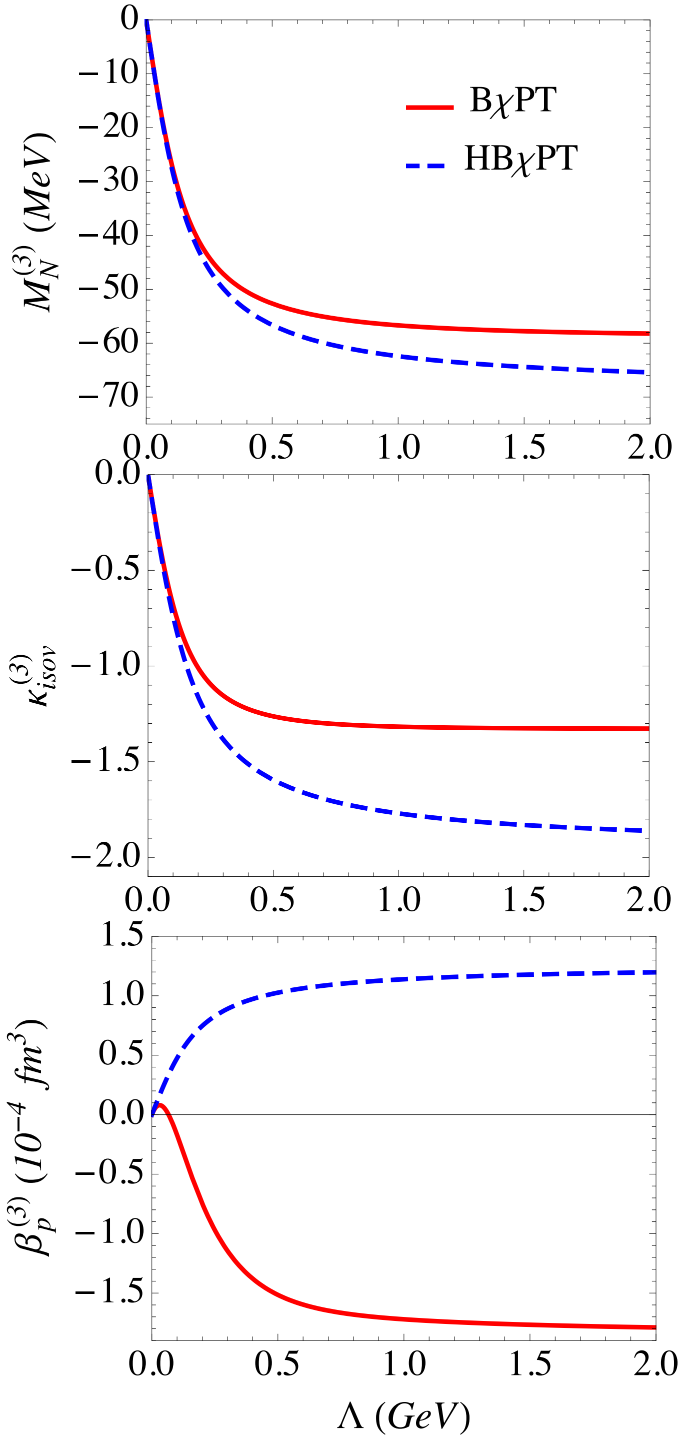

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$$\kappa \sim m_\pi$$

$$\beta_M \sim \frac{1}{m_\pi}$$

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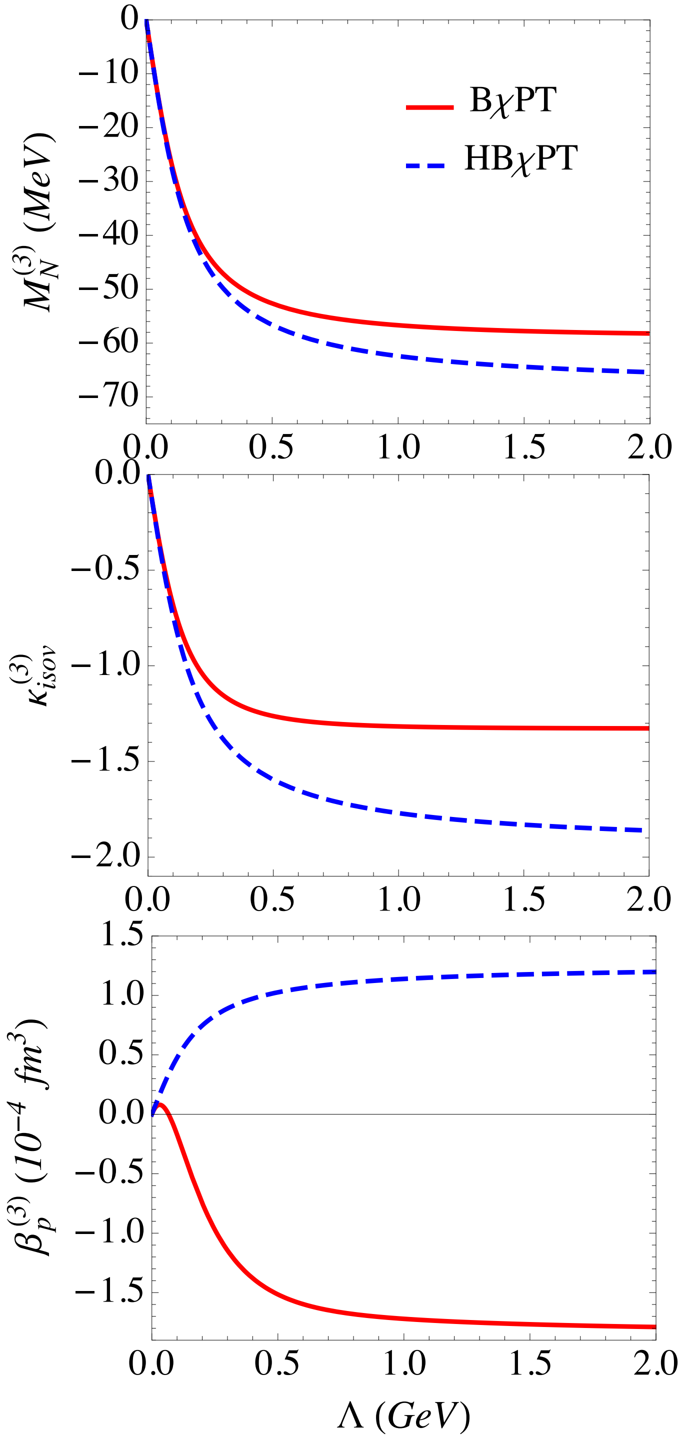

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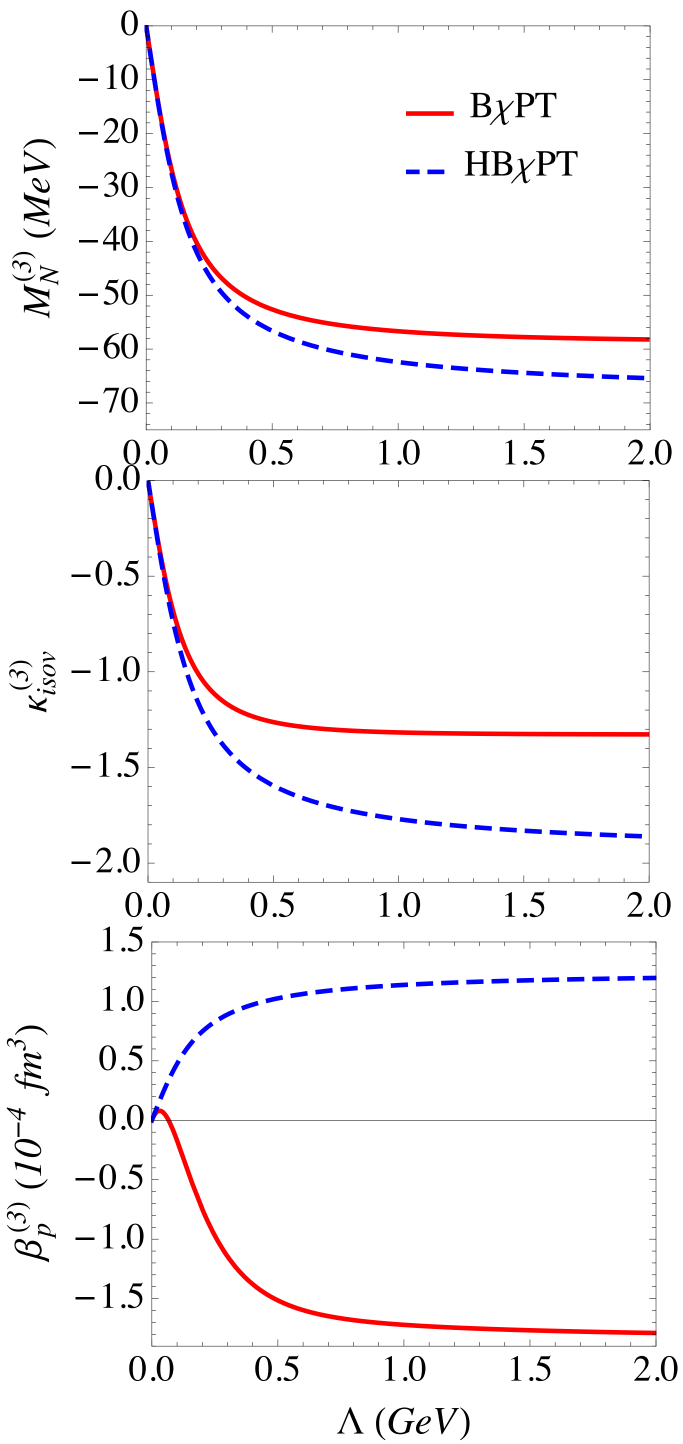

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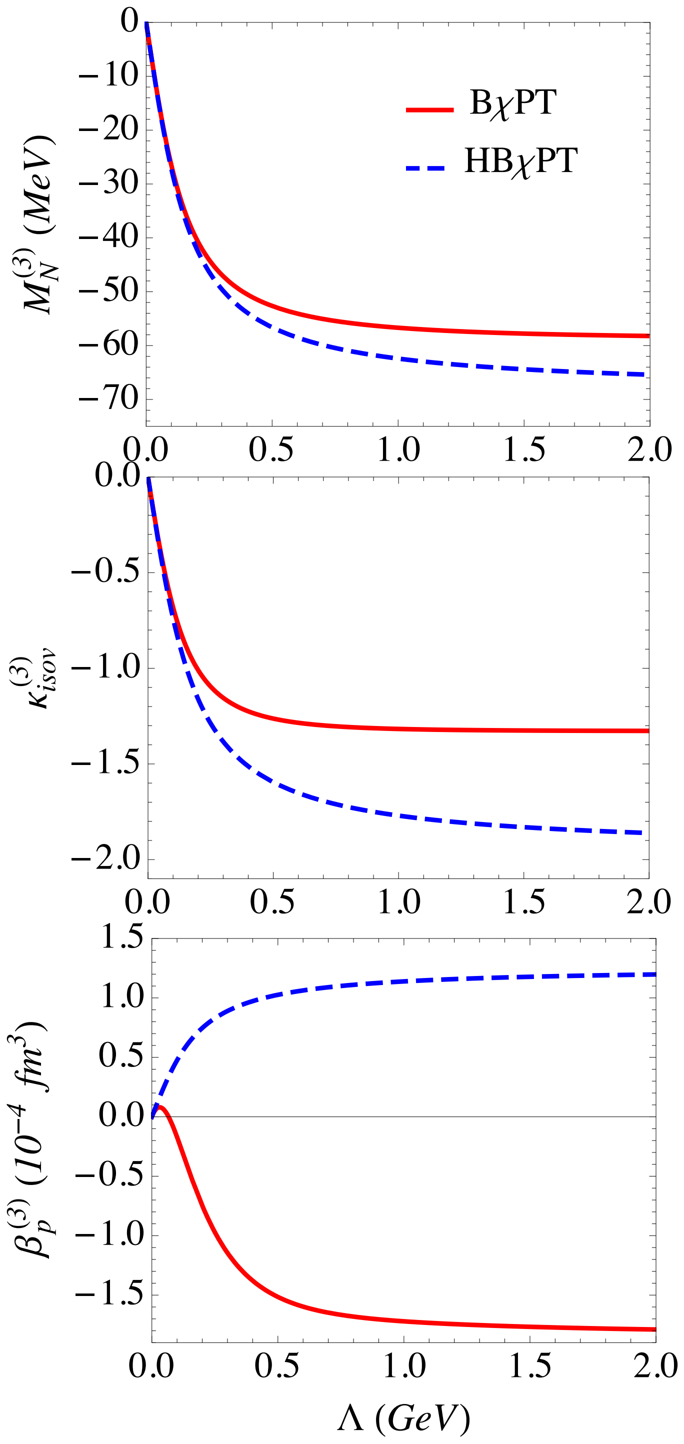

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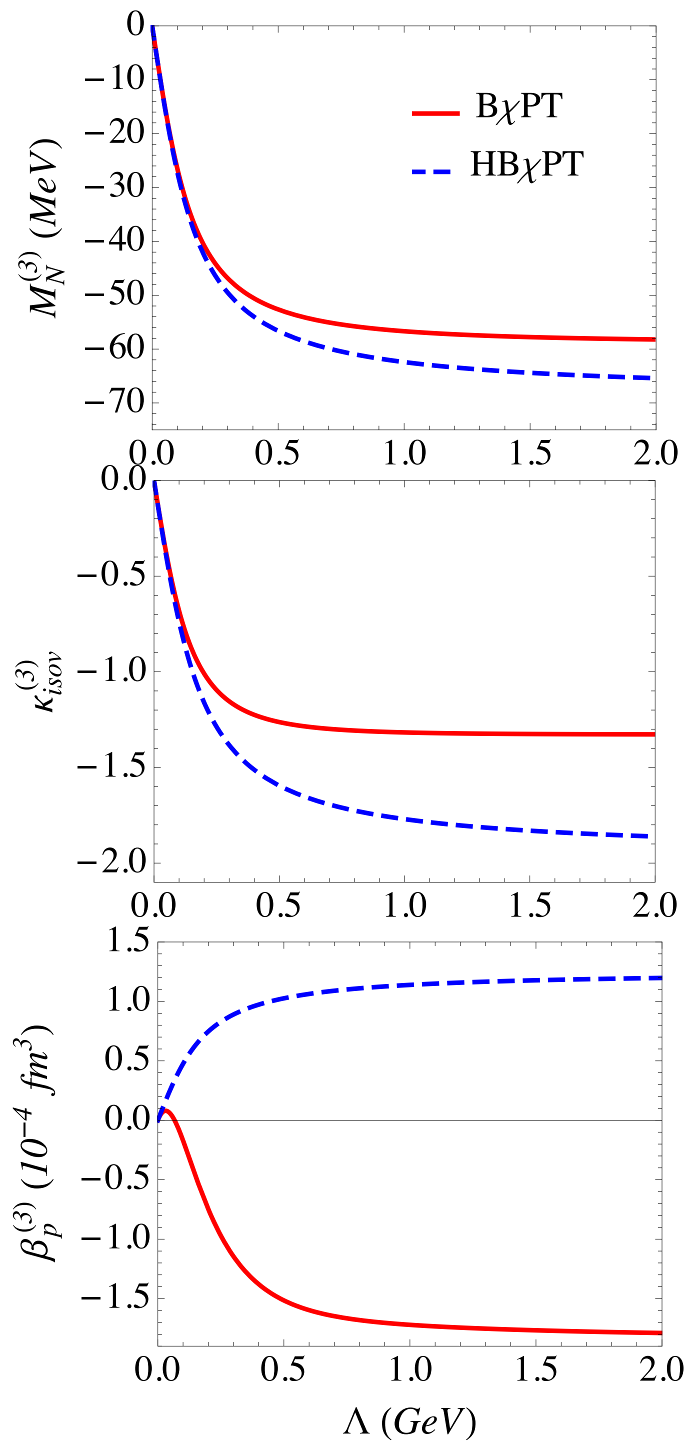

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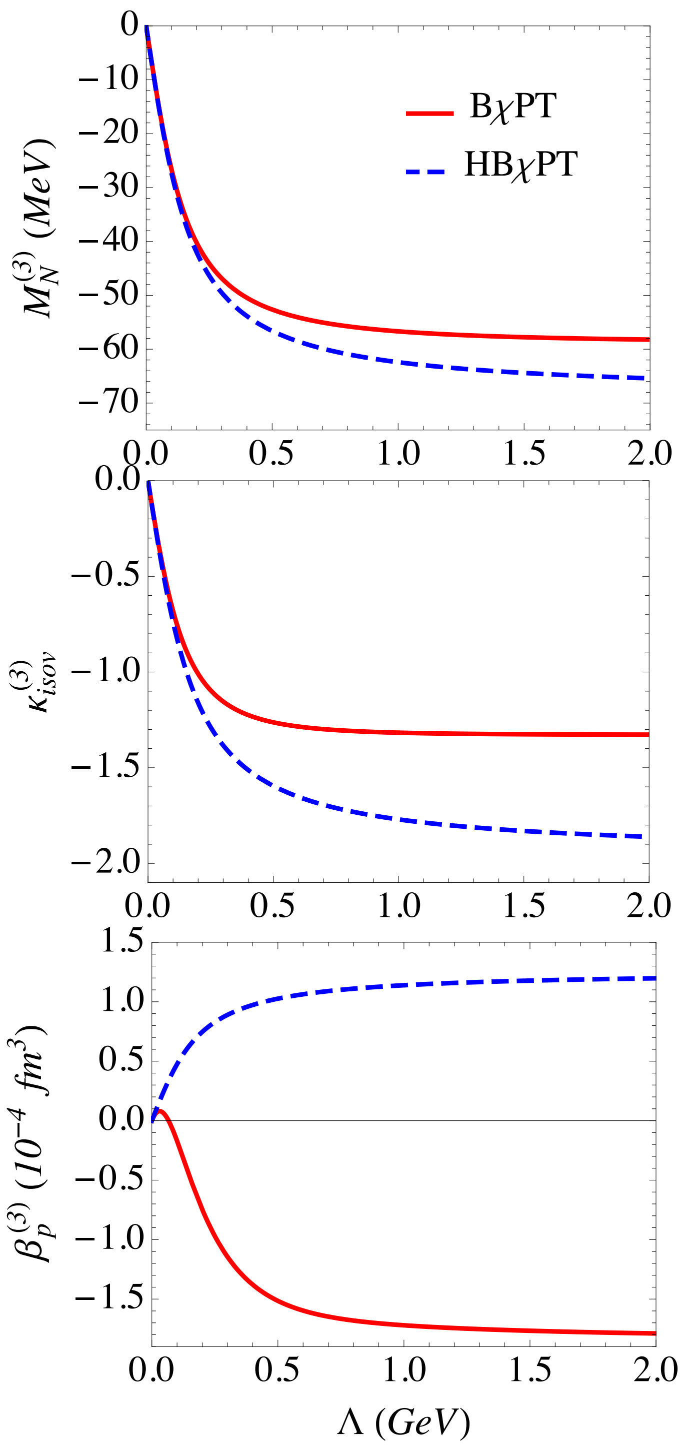

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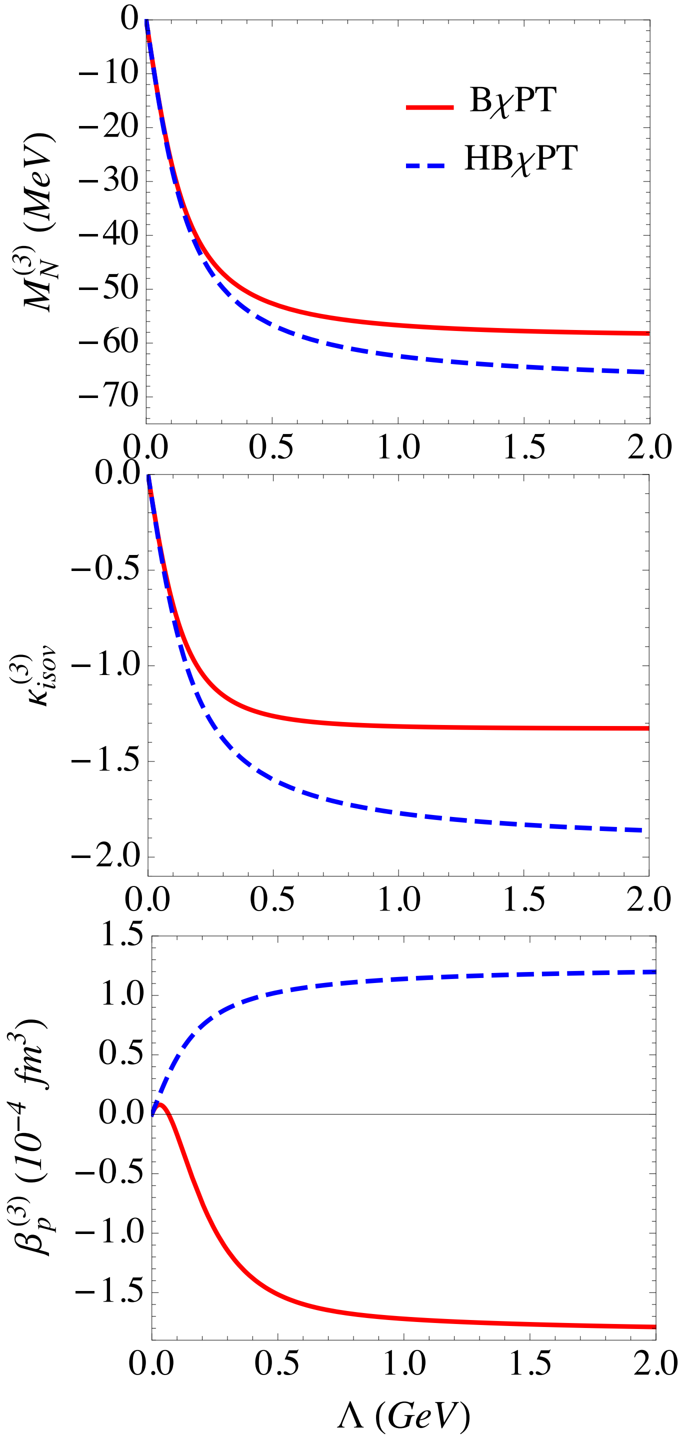
E.g.: the effective range parameters of the NN force

UV dependence in HB- vs B-ChPT



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E.g.: the effective range parameters of the NN force are such quantities -- hope for "perturbative pions" (KSW)

From beam asymmetry

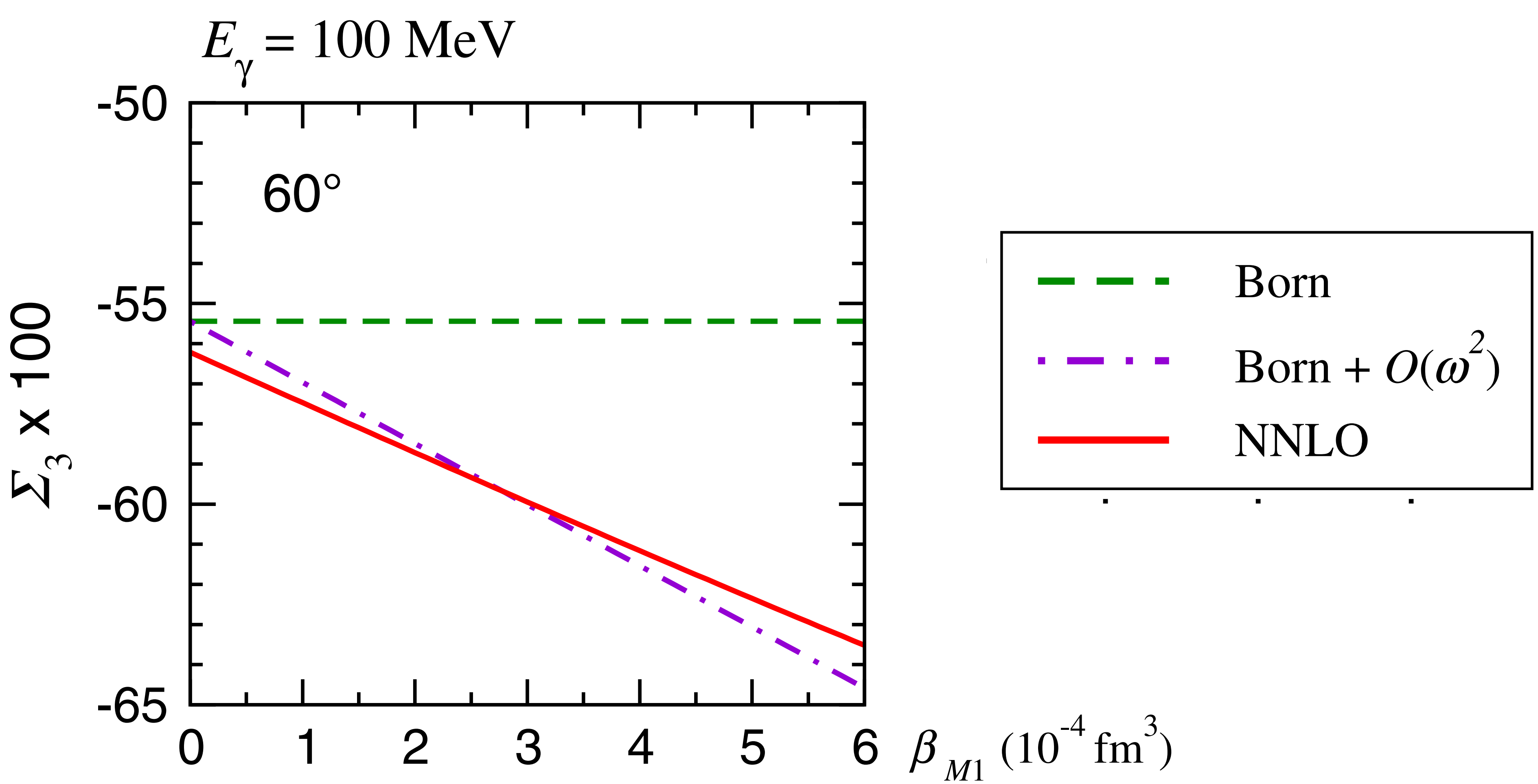
PRL **110,** 262001 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Separation of Proton Polarizabilities with the Beam Asymmetry of Compton Scattering

Nadiia Krupina and Vladimir Pascalutsa
PRISMA Cluster of Excellence Institut für Kernphysik, Johannes Gutenberg–Universität Mainz, 55128 Mainz, Germany

(Received 3 April 2013; published 25 June 2013)

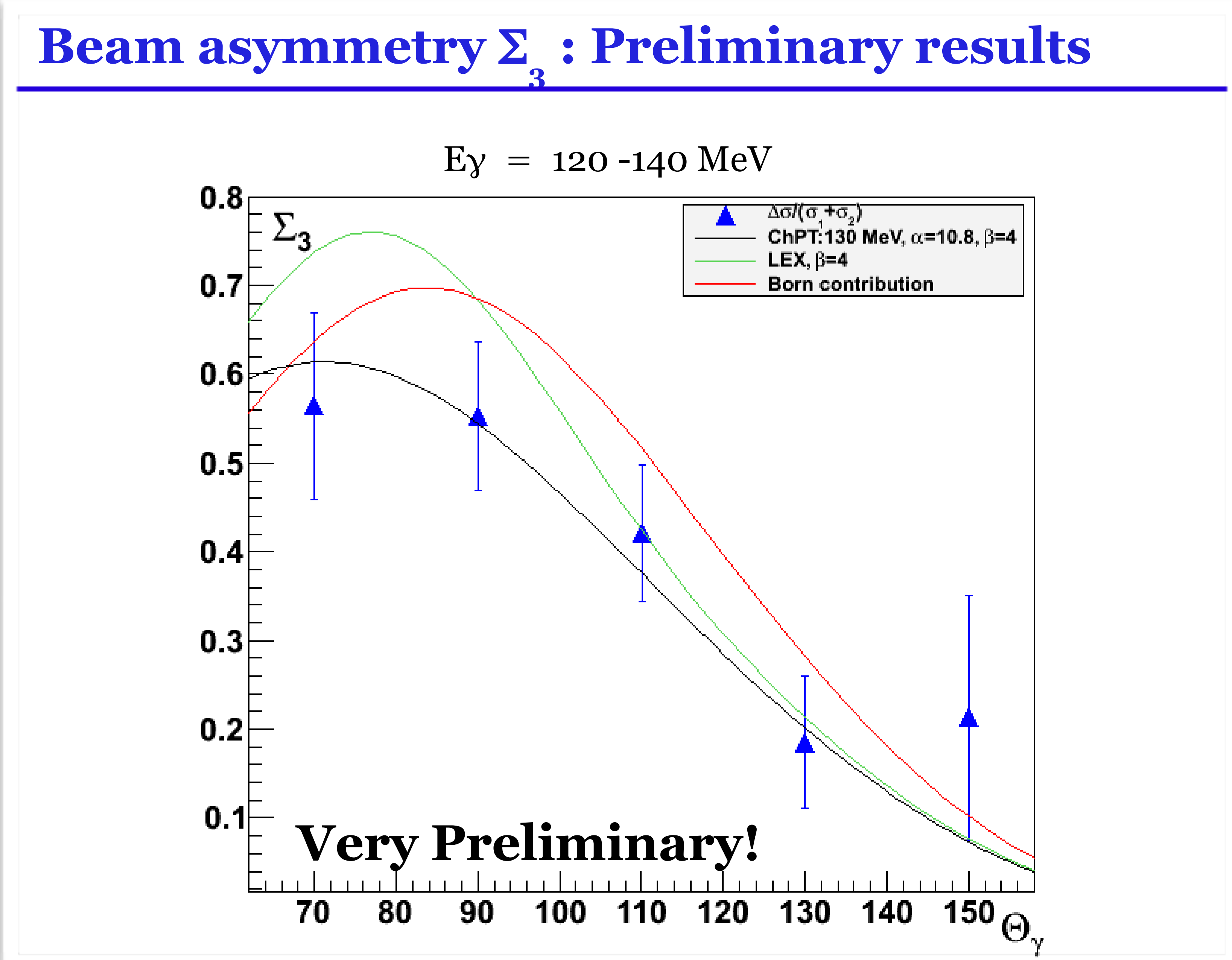
$$\Sigma_3 \equiv \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}} \overset{\text{LEX}}{=} \Sigma_3^{(\text{Born})} - \frac{4\beta_{M1}}{Z^2\alpha_{em}} \frac{\cos\theta \sin^2\theta}{(1+\cos^2\theta)^2}\,\omega^2 + O(\omega^4)$$



New Mainz data for Compton beam asymmetry

Data taken: 28.05. – 17.06.2013, 327 h

V. Sokhoyan, E. Downie et al. [A2 Coll.]



first data on this observable below pion production threshold!

better precision needed!!

Predictions of HBChPT vs BChPT

HBChPT@LO

Bernard, Keiser, Meissner
Int J Mod Phys(1995)

$$\alpha_p = \alpha_n = \frac{5\pi\alpha}{6m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 12.2\times 10^{-4}\ \text{fm}^3,$$

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$\mu = m_\pi / M_N$

$$\boxed{\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu + \frac{121}{6})\mu^2 + \mathcal{O}(\mu^3)\right]}$$

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diamagnetic

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paramagnetic

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$$\alpha = \underbrace{6.8}_{\mathcal{O}(p^3)} + \underbrace{(-0.1) + 4.1}_{\mathcal{O}(p^4/\Delta)} = 10.8\,,$$

$$\beta = \underbrace{-1.8}_{\mathcal{O}(p^3)} + \underbrace{7.1 - 1.3}_{\mathcal{O}(p^4/\Delta)} = 4.0\,.$$

paramagnetic

diamagnetic

$$\mu = m_\pi/M_N$$

$$\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu + \frac{121}{6})\mu^2 + \mathcal{O}(\mu^3)\right]$$

Bernard, Keiser, Meissner PRL (1991)

Predictions of HBChPT vs BChPT

HBChPT@LO

Bernard, Keiser, Meissner Int J Mod Phys (1995)

$$\alpha_p = \alpha_n = \frac{5\pi\alpha}{6m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 12.2\times 10^{-4}\ \text{fm}^3,$$

$$\beta_p = \beta_n = \frac{\pi\alpha}{12m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 1.2\times 10^{-4}\ \text{fm}^3,$$

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HBChPT@NLO:

Griesshammer & Hemmert (2004)
Griesshammer, McGovern, Phillips (2012)

The Delta contribution is accompanied by “promoted” LECs, hence not predictive

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BChPT@NLO

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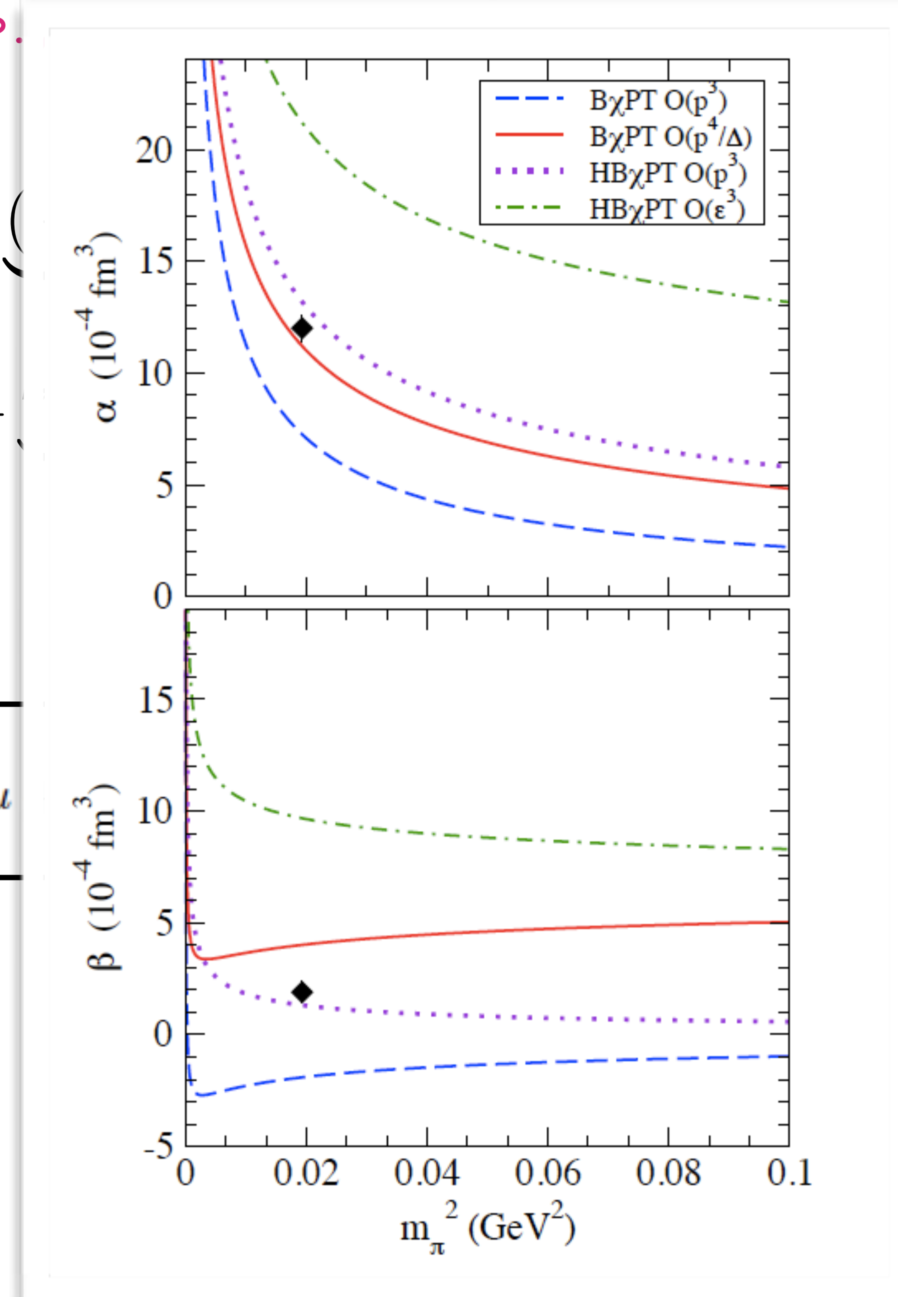

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Lattice QCD data expected soon