Electromagnetic radii of the proton from atomic spectroscopy

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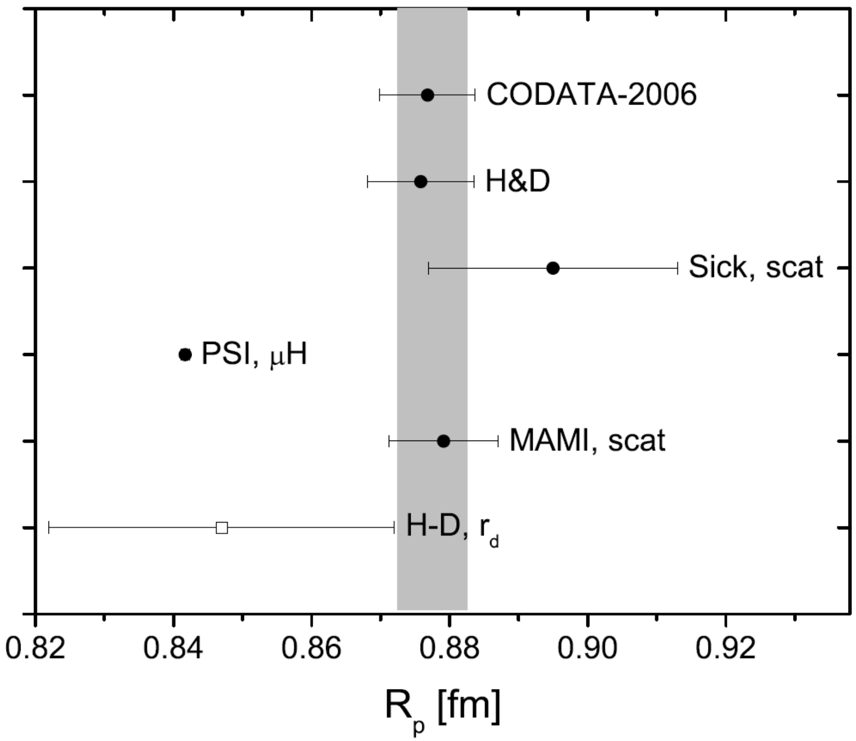
Outline

- Different determinations of the electric and magnetic radii
- Proton-finite-size contributions to the energy level
  - *Leading and next-to-leading contributions*
  - *Spin-independent and spin-dependent terms*
  - *Former evaluations*
  - *Consistency problem*
  - *Fits and fits*
- Strategy of the self-consistent consideration
  - *Low momentum area*
  - *High momentum area*
- The proton charge radius
  - *Results*
  - *Tests*
- Hyperfine interval in H and μH and proton magnetic radius

Different methods to determine the proton charge radius

- *Spectroscopy of hydrogen (and deuterium)*
- *The Lamb shift in muonic hydrogen*
- *Electron-proton scattering*

DIFFERENT METHODS TO DETERMINE THE PROTON CHARGE RADIUS



- *Electron-proton scattering*

Different methods to determine the proton charge radius

- *Spectroscopy of hydrogen (and deuterium)*
  - *spectroscopic data are fitted with R_∞ and R_p (H) or R_∞ and R_d (D)*
- *The Lamb shift in muonic hydrogen*
  - *higher-order finite-proton-size corrections are involved*
- *Electron-proton scattering*
  - *QED corrections*
  - *two-photon exchange (polarizability)*
  - *limited accuracy of data points*
  - *extrapolation to $q^2=0$ and differentiation*
- *overall model dependence*

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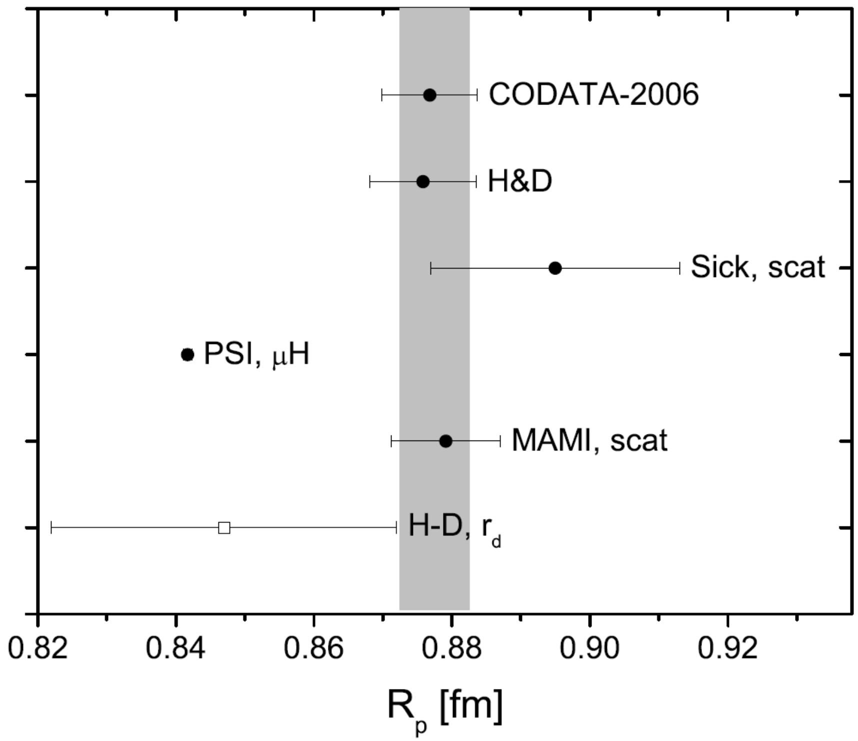
  - *extrapolation to $q^2=0$ and differentiation*
- *overall model dependence*
- *Results on the magnetic radius are controversial*

DIFFERENT METHODS TO DETERMINE THE PROTON CHARGE RADIUS



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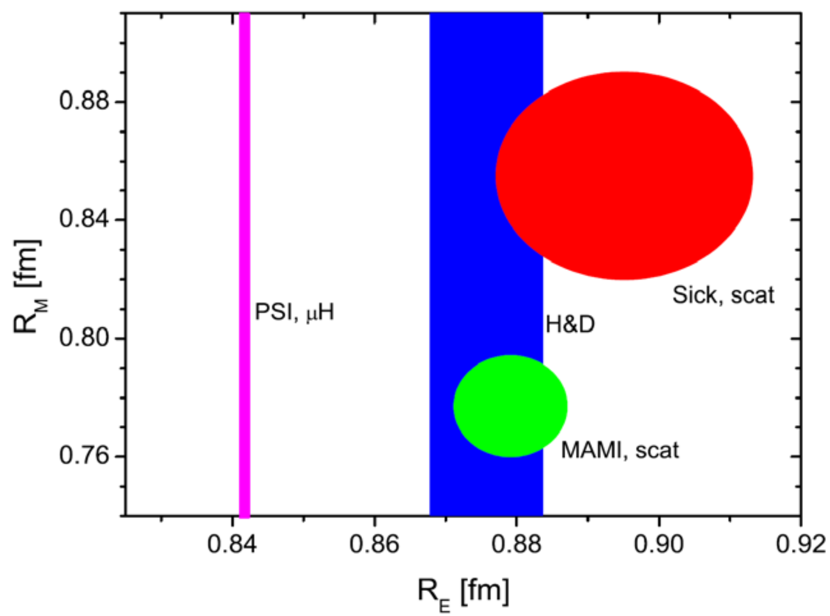


- *No results on magnetic radius*

Contradiction in the determination: two questions

- How to resolve it?
- I partly address this question producing an independent value of the magnetic radius.

- How to live with it?
- The contradiction means that the overall picture is not self-consistent and certain calculations may involve inconsistencies.

Higher-order proton-finite-size contributions: new approach

- The Lamb in muonic hydrogen
  - consistency (between form factors from scattering and extracted radius)
  - a model-independent self-consistent treatment
- HFS in ordinary and muonic hydrogen
  - sensitive to the destribution of the magnetic moment
  - no model independent result up to date
  - a self-consistent treatment and a model-independent constraint for the first time

Proton finite-size contributions

- Leading term:

$$\Delta E^{\text{lead}}_{\text{pfs}}(ns) = \frac{2}{3}\frac{(Z\alpha)^4}{n^3} m_r^3 R_E^2$$

- Next-to-leading (spin-independent): Friar term

$$I_3^{\text{E}} \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$

- Next-to-leading (spin-dependent): Zemach term

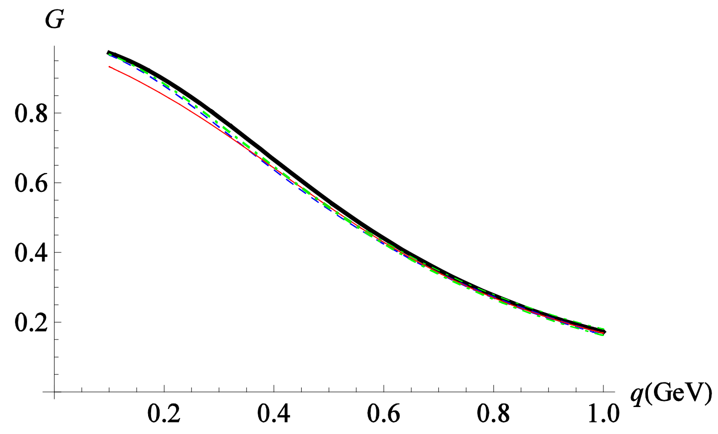
$$I_1^{\text{EM}} \equiv \int_0^\infty \frac{dq}{q^2}\left[\frac{G_E(q^2)G_M(q^2)}{\mu_p} - 1\right]$$

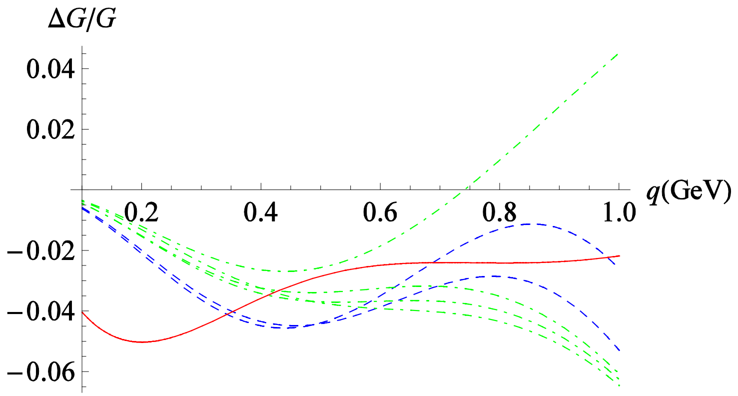
Standard dipole approximation and empiric fits

$$G_{\mathrm{dip}}(q^2) = \left(\frac{\Lambda^2}{q^2 + \Lambda^2}\right)^2$$

$$\Lambda^2 = 0.71\,\mathrm{GeV}^2$$

$R_p = 0.81$ fm



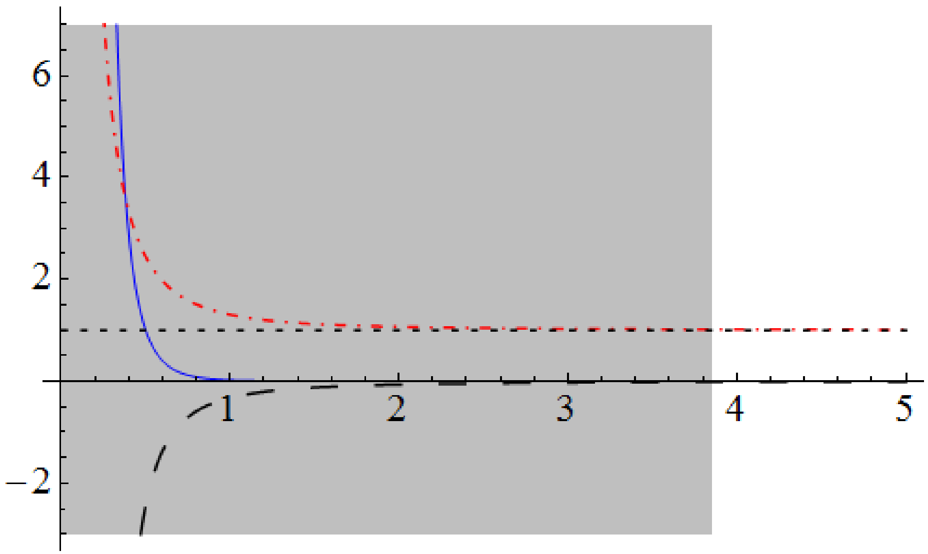


The Lamb shift in muonic hydrogen: Consistency problem !!!

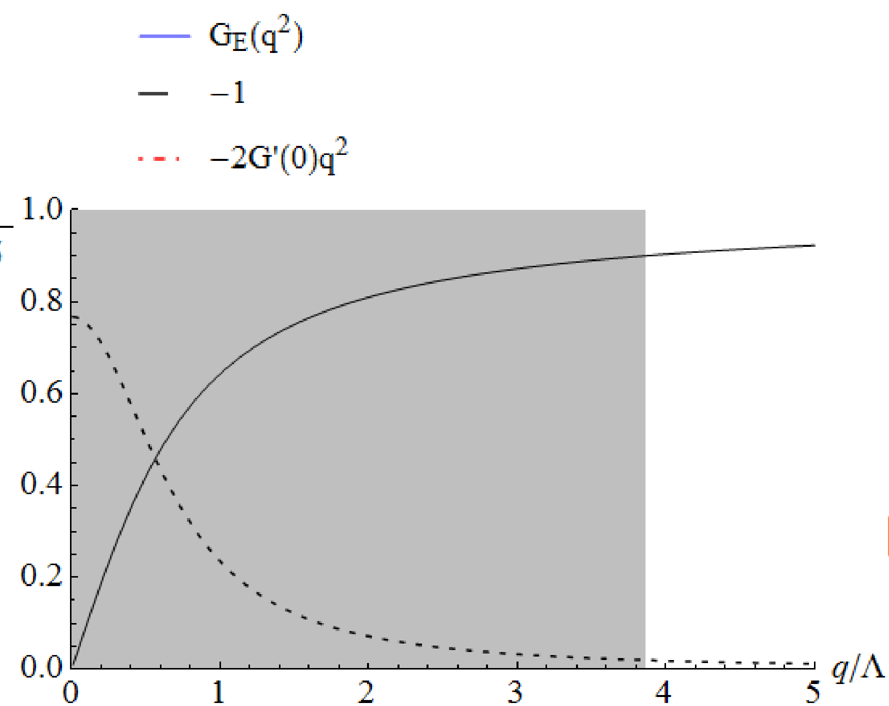
- The integrand includes

  Three terms:

$$I_3^{\mathrm{E}} \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$



90% of the integral



THE LAMB SHIFT IN MUONIC HYDROGEN: CONSISTENCY PROBLEM

- The integrand includes
  Three terms:

$$I_3^E \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\,q^2\right]$$

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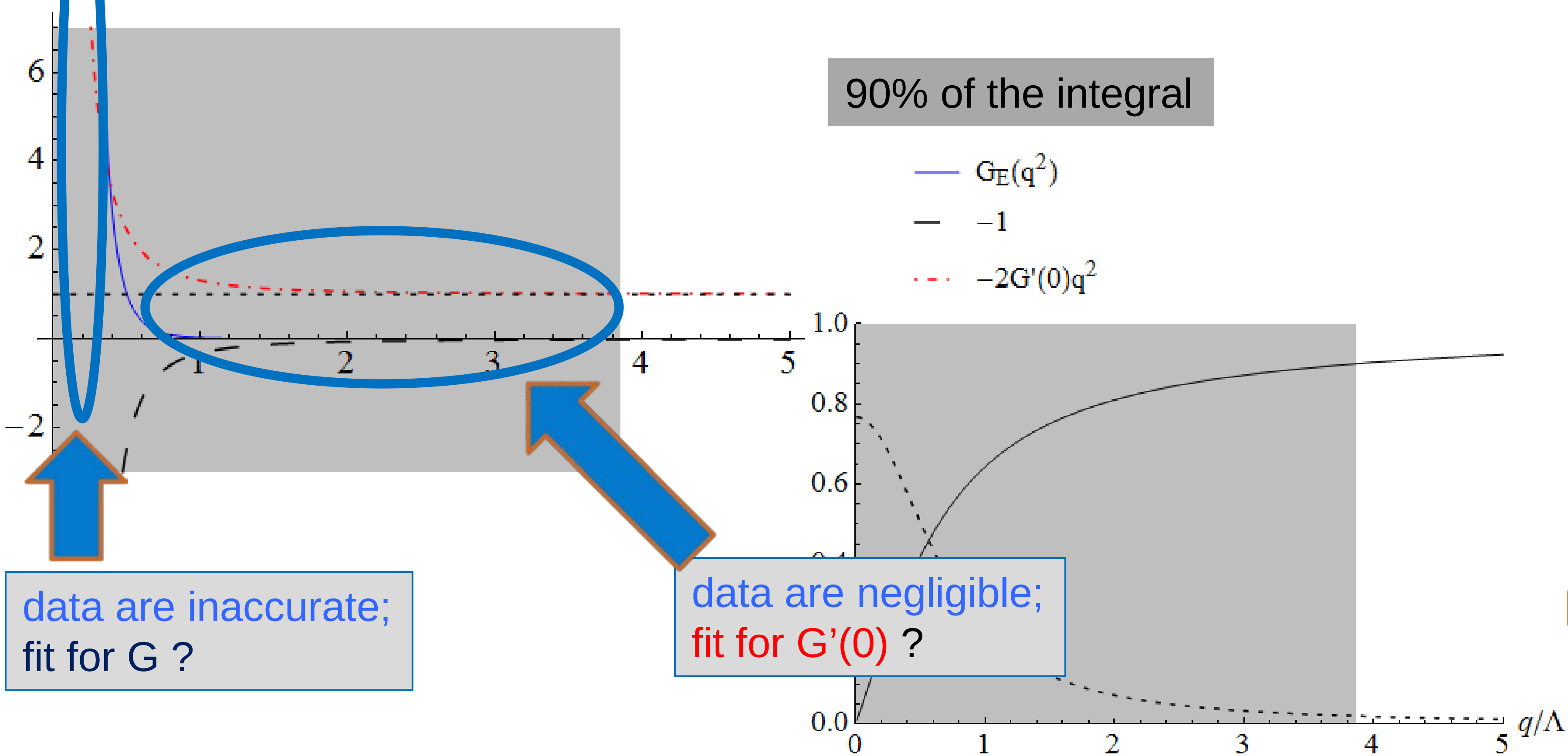

data are inaccurate;
fit for G ?

data are negligible;
fit for G'(0) ?

The Lamb shift in muonic hydrogen: Consistency problem

- The integrand includes
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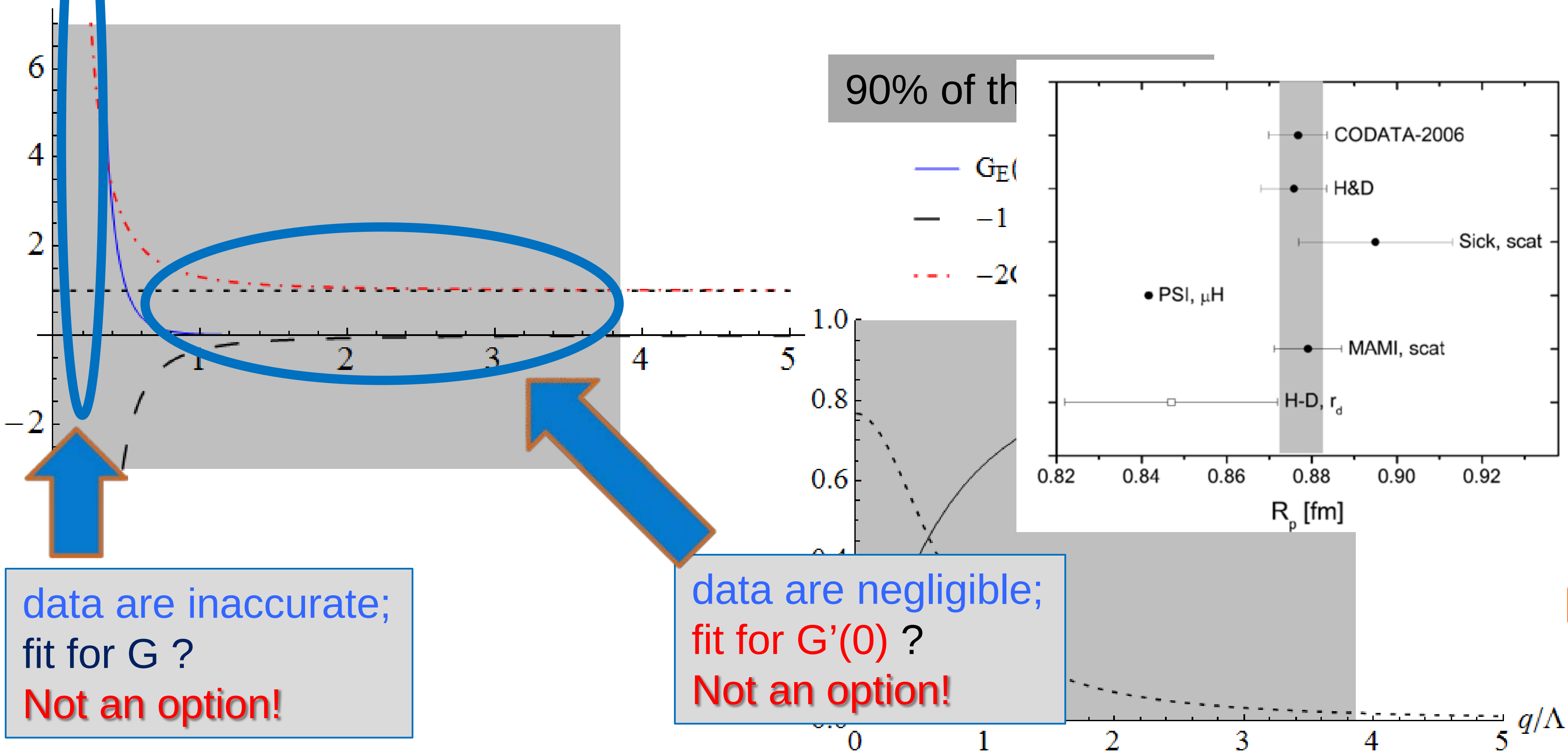

The fits & the fits

- The fits theoretically motivated
  - the shape is determined independently of the fitting
  - fitting serves to find the related parameters
  - suitable for any operations on the fit
- There is no a priori model for the proton form factors

- The fits produced empirically
  - the shape is arbitrary chosen
  - literally it has no sense
  - just a good smooth function
  - suitable for rough application
- Suitable with reservations for extrapolations, differentiations, large subtractions

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reservations for extrapolations, differentiations, large subtractions

Empiric fits of the proton form factors

- Literally incorrect:
  - wrong analytic behavior
  - questionable asymptotic behavior
- Based on certain rough theoretical constraints

EVALUATIONS OF I_3^{EM} WITHIN THE FORMER EXTRACTIONS

- *Natural* extraction
  - following Borie, Eides et al, and others
  - dipole shape with an adjustable parameter
- A posteriori: the parameter follows the radius from muonic hydrogen
  - does not well agree with the expt data!

The size of the proton

Randolf Pohl[1], Aldo Antognini[1], François Nez[2], Fernando D. Amaro[3], François Biraben[2], João M. R. Cardoso[3], Daniel S. Covita[3,4], Andreas Dax[5], Satish Dhawan[5], Luis M. P. Fernandes[3], Adolf Giesen[6]†, Thomas Graf[6], Theodor W. Hänsch[1], Paul Indelicato[2], Lucile Julien[2], Cheng-Yang Kao[7], Paul Knowles[8], Eric-Olivier Le Bigot[2], Yi-Wei Liu[7], José A. M. Lopes[3], Livia Ludhova[8], Cristina M. B. Monteiro[3], Françoise Mulhauser[8]†, Tobias Nebel[1], Paul Rabinowitz[9], Joaquim M. F. dos Santos[3], Lukas A. Schaller[8], Karsten Schuhmann[10], Catherine Schwob[2], David Taqqu[11], João F. C. A. Veloso[4] & Franz Kottmann[12]

- *Scientific* extraction
  - Crema cited Birse and McGovern
  - B&M cited Carlson and Vanderhaeghen
  - C&V calculated TPE
- TPE includes recoil, but the Friar term dominates
  - empiric fits (with a bad radius!)

Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen

Aldo Antognini,[1,2]* François Nez,[3] Karsten Schuhmann,[2,4] Fernando D. Amaro,[5] François Biraben,[3] João M. R. Cardoso,[5] Daniel S. Covita,[5,6] Andreas Dax,[7] Satish Dhawan,[7] Marc Diepold,[1] Luis M. P. Fernandes,[5] Adolf Giesen,[4,8] Andrea L. Gouvea,[5] Thomas Graf,[8] Theodor W. Hänsch,[1,9] Paul Indelicato,[3] Lucile Julien,[3] Cheng-Yang Kao,[10] Paul Knowles,[11] Franz Kottmann,[2] Eric-Olivier Le Bigot,[3] Yi-Wei Liu,[10] José A. M. Lopes,[5] Livia Ludhova,[11] Cristina M. B. Monteiro,[5] Françoise Mulhauser,[11] Tobias Nebel,[1] Paul Rabinowitz,[12] Joaquim M. F. dos Santos,[5] Lukas A. Schaller,[11] Catherine Schwob,[3] David Taqqu,[13] João F. C. A. Veloso,[6] Jan Vogelsang,[1] Randolf Pohl[1]

Strategy of the evaluation

- Split the integral

$$I = \int_0^\infty dq ... \equiv I_< + I_> \equiv \int_0^{q_0} dq ... + \int_{q_0}^\infty dq ...$$

- Low momentum

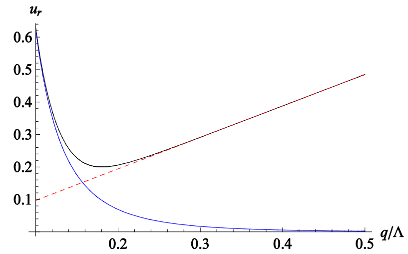
$$\left(G_E(q^2)\right)^2 \simeq 1 - \frac{R_E^2}{3} q^2 + C^{\rm dip}(1 \pm 1)\, q^4$$

- High momentum

$$I_{3>}^{\rm E} = \int_{q_0}^\infty \frac{dq}{q^4} \left(G_E(q^2)\right)^2 - \frac{1}{3q_0^3} + \frac{1}{3}\frac{R_E^2}{q_0}$$

$$\delta I_{3>}^{\rm E} = \frac{1}{3q_0^3} \frac{2\delta G_E(q_0^2)}{G_E(q_0^2)} \left(G_{\rm dip}(q_0^2)\right)^2$$

- Minimization of the uncertainty



Low momentum contribution

- The integrand involves a substantial cancelation

$$I_3^{\mathrm{E}} \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\, q^2\right]$$

- The low-momentum approximation

$$\left(G_E(q^2)\right)^2 \quad \simeq \quad 1 - \frac{R_E^2}{3} q^2 + C^{\mathrm{dip}}(1 \pm 1)\, q^4$$

- The result:

$$I_{3<}^{\mathrm{E}} = 10\,(1 \pm 1)\,\frac{q_0}{\Lambda^4}$$

Low momentum contribution

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High momentum contribution

- The integral

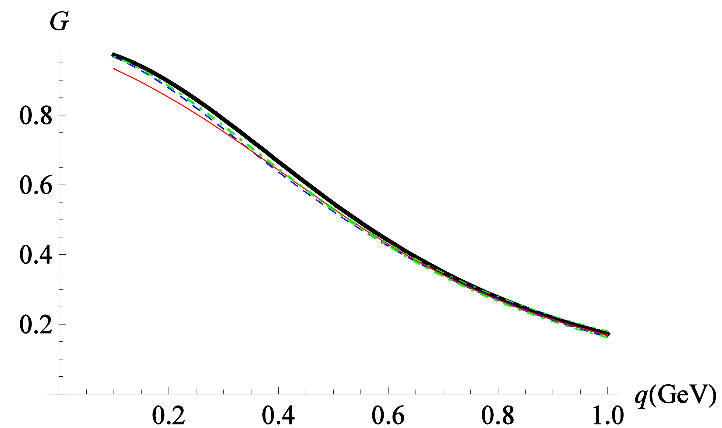
$$I^{\mathrm{E}}_{3>} = \int_{q_0}^{\infty} \frac{dq}{q^4}\left(G_E(q^2)\right)^2 - \frac{1}{3q_0^3} + \frac{1}{3}\frac{R_E^2}{q_0}$$

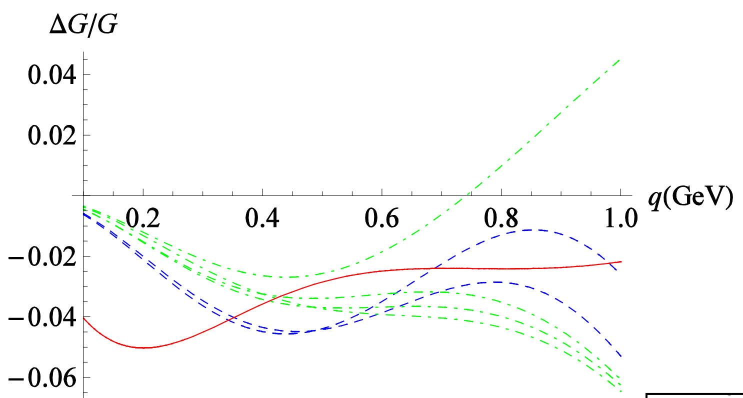
- The central value is the median of a few empiric fits
- The uncertainty

$$\delta I^{\mathrm{E}}_{3>} = \frac{1}{3q_0^3}\frac{2\delta G_E(q_0^2)}{G_E(q_0^2)}\left(G_{\mathrm{dip}}(q_0^2)\right)^2$$

$$\delta G_E(q_0^2)/G_E(q_0^2) \simeq 1\%$$

The empiric fits: shape





- Chain fraction

$$G_E(q^2) = \cfrac{1}{1 + \cfrac{3.44Q^2}{1 - \cfrac{0.178Q^2}{1 - \cfrac{1.212Q^2}{1 + \cfrac{1.176Q^2}{1 - 0.284Q^2}}}}}$$

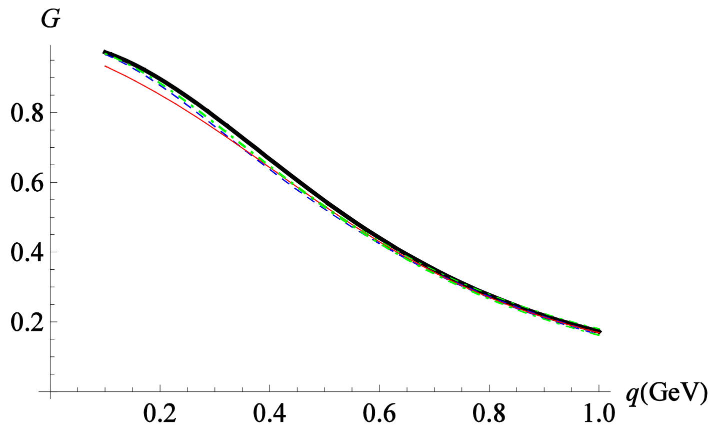
- Padé approximation

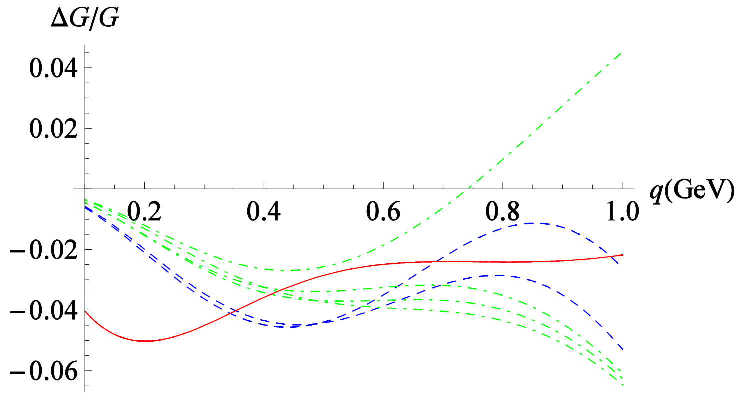
$$G_E = \frac{1 - 0.24\tau}{1 + 10.98\tau + 12.82\tau^2 + 0.863\tau^3}$$

$$\tau = q^2/4m_p^2$$

[9] J.J. Kelly, Phys. Rev. C **70**, 068202 (2004).
[10] J. Arrington and I. Sick, Phys. Rev. C**76**, 035201 (2007).
[11] J. Arrington, W. Melnitchouk, and J.A. Tjon, Phys. Rev. C**76**, 035205 (2007).
[12] W.M. Alberico, S.M. Bilenky, C. Guinti, and K.M. Graczyk, Phys. Rev. C**79**, 065204 (2009).
[13] S. Venkat, J. Arrington, G.A. Miller and X. Zhan, Phys. Rev. C**83**, 015203 (2011).

The empiric fits: parameters





fit	ref.	type	R_E [fm]	C [GeV^{-4}]
(A1)	[10]	chain fraction	0.90	34.3
(A2)	[10]	chain fraction	0.90	35.3
(A3)	[9]	Padé approximation (q^2)	0.86	28.0
(A4)	[11]	Padé approximation (q^2)	0.88	31.1
(A5)	[12]	Padé approximation (q^2)	0.87	28.2
(A6)	[13]	Padé approximation (q^2)	0.88	31.3

$$\boxed{C = 19.8\ \mathrm{GeV}^{-4}}$$

Scatter, uncertainty and the Bosted's fit (1995)

q_0/Λ	q_0	$\delta I^{\mathrm{E}}_{3<}/I^{\mathrm{dip}}_3$	$\delta(I^{\mathrm{E}}_{3>} - I^{\mathrm{R}}_3)/I^{\mathrm{dip}}_3$	total	*scatter*
0.10	0.084 GeV	9.7%	62%	63%	*11%*
0.15	0.126 GeV	14.6%	17.5%	22.3%	*5.5%*
0.20	0.169 GeV	19.4%	6.9%	20.6%	*3.0%*
0.25	0.211 GeV	24.2%	3.2%	24.4%	*1.7%*
0.30	0.253 GeV	29.1%	1.7%	29.2%	*0.9%*
0.40	0.337 GeV	38.8%	0.6%	38.8%	*0.3%*

Scatter, uncertainty and the Bosted's fit (1995)

q_0/Λ	$I_3^{\mathrm{E}} - I_3^{\mathrm{R}}$	scatter of $I_{3>}^{\mathrm{E}}$	scatter* of $I_{3>}^{\mathrm{E}}$
0.10	$-57(11)$ GeV^{-3}	2.0 GeV^{-3}	18 GeV^{-3}
0.15	$-30.8(3.9)$ GeV^{-3}	1.0 GeV^{-3}	5.0 GeV^{-3}
0.20	$-18.0(3.5)$ GeV^{-3}	0.5 GeV^{-3}	1.7 GeV^{-3}
0.25	$-10.4(4.2)$ GeV^{-3}	0.3 GeV^{-3}	0.6 GeV^{-3}
0.30	$-5.4(5.0)$ GeV^{-3}	0.2 GeV^{-3}	0.2 GeV^{-3}
0.40	$+0.9(6.7)$ GeV^{-3}	0.06 GeV^{-3}	0.06 GeV^{-3}

$$G_E(q^2) = \frac{1}{1 + 0.62Q + 0.68Q^2 + 2.8Q^3 + 0.83Q^4}$$

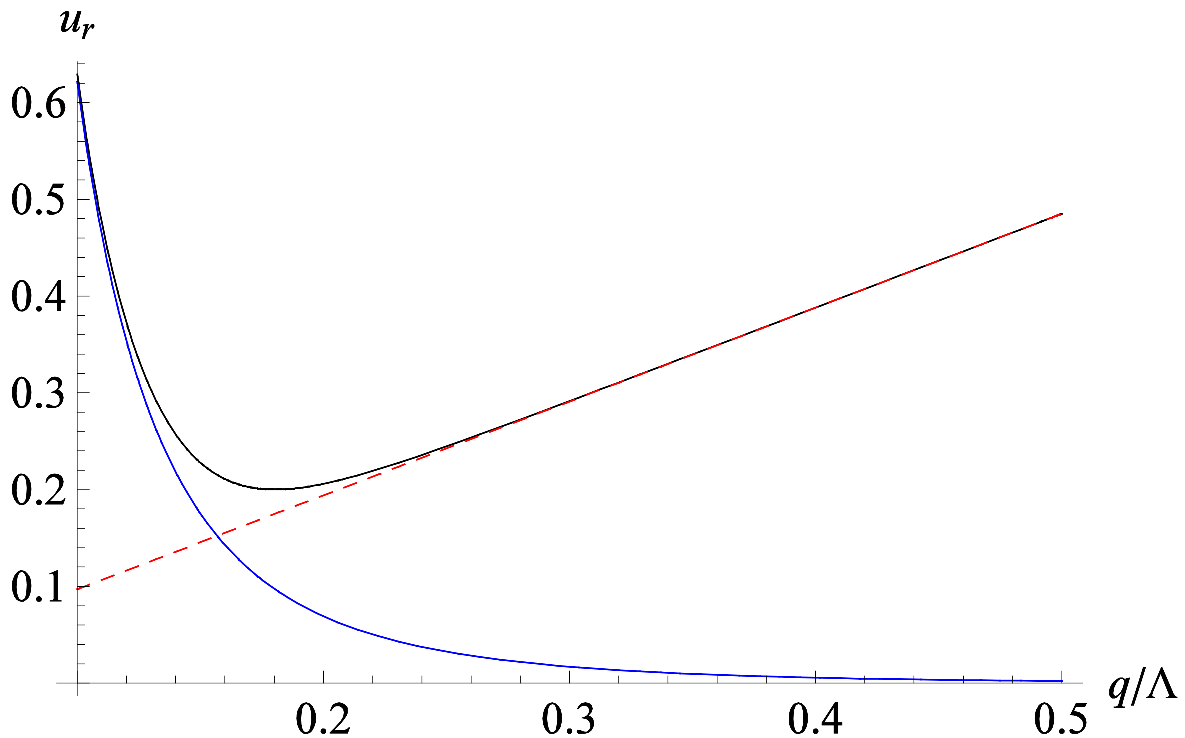
G'(0) = ∞ (the fit was designed only for medium q)

STRATEGY: THE CENTRAL VALUE AND THE UNCERTAINTY

- The central value
  - Low-momentum part
    - Dipole value for G"(0)
    - G'(0) is adjustable
  - High-momentum part
    - empiric fits with somewhat different R_E and G"(0)
    - the median
- The uncertainty
  - Low-momentum part
    - 100% uncertainty
  - High-momentum part
    - 1% uncertainty due to integration around q_0
    - that is larger than the scatter

Minimization of the uncertainty

- The uncertainty is modeled with a smooth simple function and we find its minimum



Determination of the charge radius

- Extraction equation

$$(R_E)^2 - (R_E^0)^2 = \frac{1}{\left[1 - \frac{8(Z\alpha)}{\pi}\frac{m_r}{q_0}\right]}\left[\frac{8(Z\alpha)}{\pi}\frac{m_r}{q_0}(R_E^0)^2 + \frac{24(Z\alpha)m_r}{\pi}\left(I_3^{\mathrm{E-R}} - I_3^0\right)\right]$$

$$I_3^{\mathrm{E-R}} = \int_0^{q_0}\frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\,q^2\right] + \int_{q_0}^{\infty}\frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1\right]$$

- Results

Ref.	R_E^0 [fm]	I_3^0 [GeV^{-3}]	$R_E - R_E^0$ [fm]	R_E [fm]
[1]	0.841 84(67)	19.25	−0.000 19(43)	0.841 65(79)
[2]	0.840 87(39)	22.9(1.2)	−0.000 65(43)(15)	0.840 22(56)

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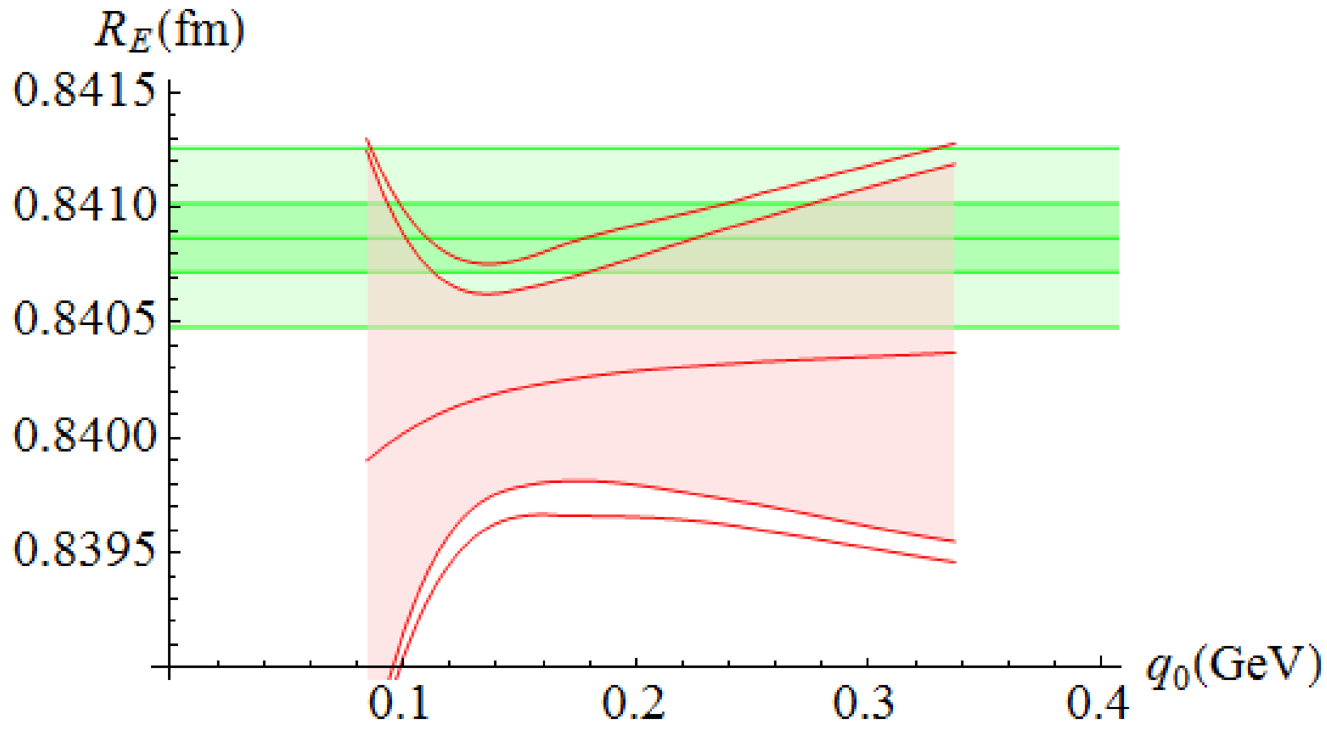
Dependence on Q_0

- Plot for scientific extraction



$$R_E = 0.840\,22(56)\ \text{fm}$$

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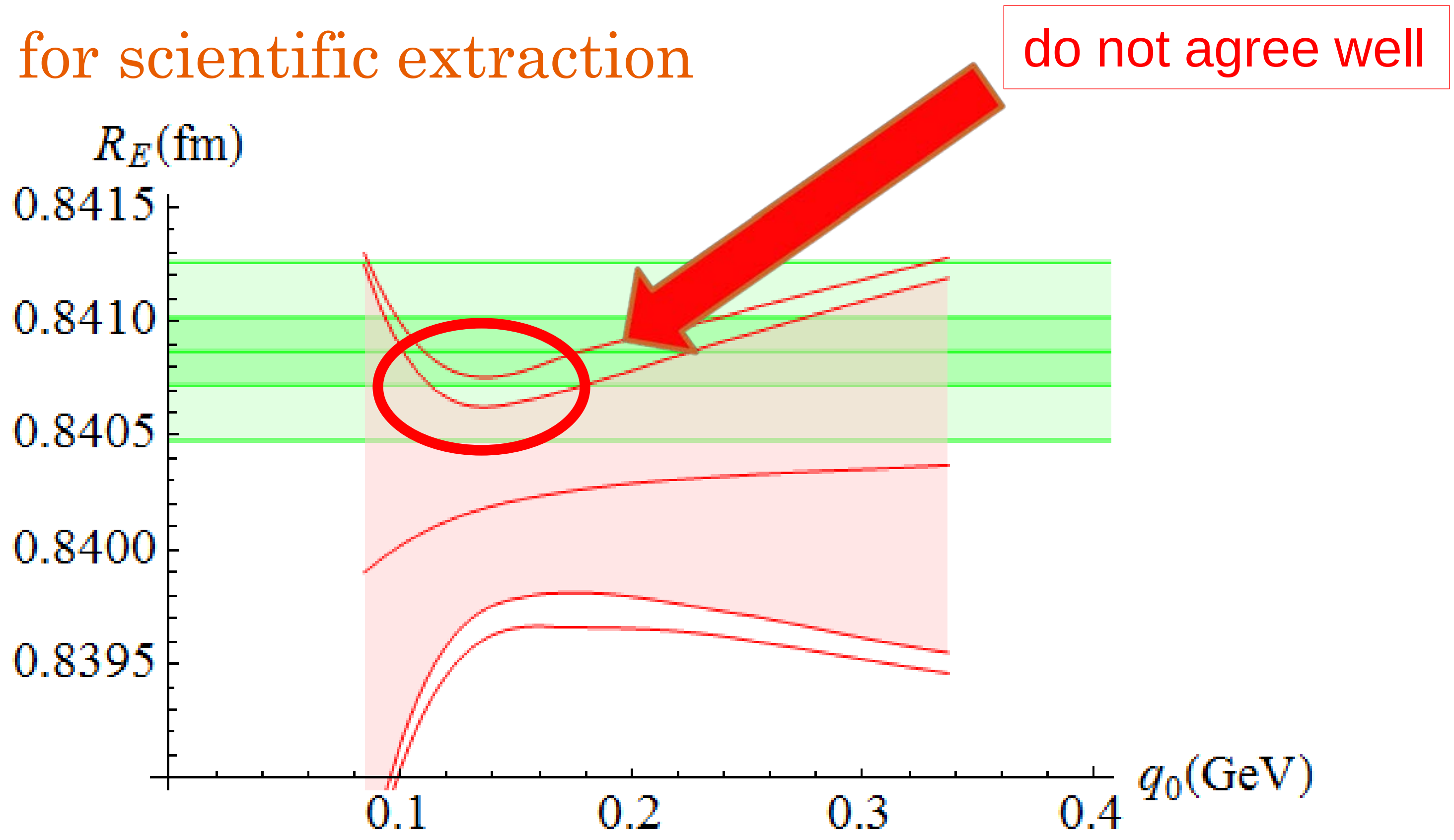


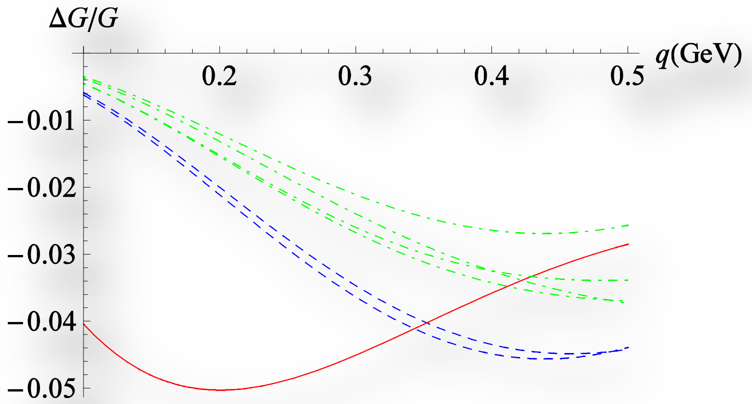
$$R_E = 0.840\,22(56)\ \text{fm}$$

Continuity of the form factor:

- Low momentum (R_E is extracted from μH)

$$G_E^2(q^2) = 1 - \frac{R_E^2 q^2}{3} + Cq^4$$

- High momentum (the median)

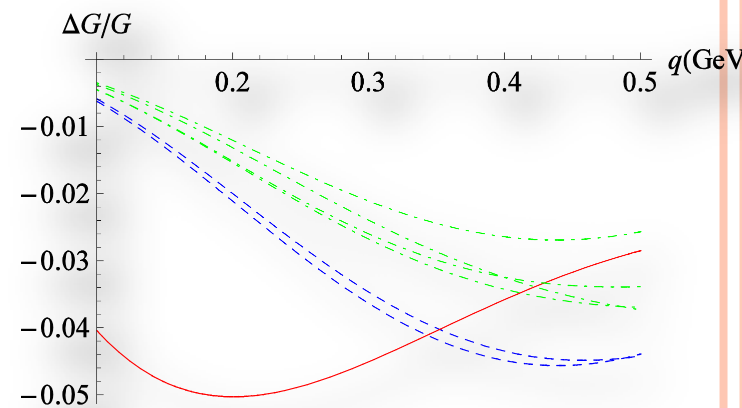


Continuity of the form factor: *A posteriori*

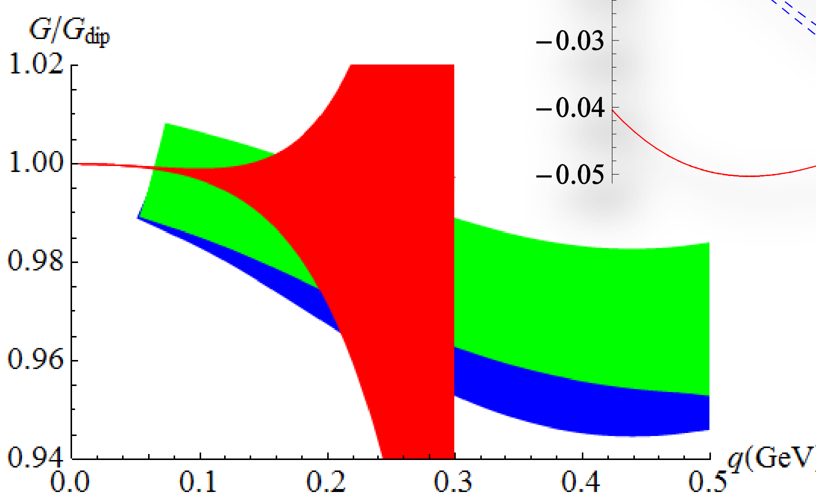
- Low momentum (R_E is extracted from μH)

$$G_E^2(q^2) = 1 - \frac{R_E^2 q^2}{3} + C q^4$$

- High momentum (the median)



- The overall fit



MAGNETIC RADIUS OF THE PROTON: THE STRATEGY

- Zemach correction

$$I_1^{\mathrm{EM}} \equiv \int_0^\infty \frac{dq}{q^2}\left[\frac{G_E(q^2)G_M(q^2)}{\mu_p} - 1\right]$$

- Low-momentum part

$$\frac{G_E(q^2)G_M(q^2)}{\mu_p} \simeq 1 - \frac{R_E^2 + R_M^2}{6}q^2 + C^{\mathrm{dip}}(1 \pm 1)\, q^4$$

$$I_{1<} = -\frac{R_E^2 + R_M^2}{6}q_0 + \frac{10}{3}(1 \pm 1)\frac{q_0^3}{\Lambda^4}$$

- High-momentum part

$$I_{1>} = \int_{q_0}^\infty \frac{dq}{q^2}\frac{G_E(q^2)G_M(q^2)}{\mu_p} - \frac{1}{q_0}$$

$$\delta I_{1>}^{\mathrm{EM}} = \left[\frac{\delta G_E(q_0^2)}{G_E(q_0^2)} + \frac{\delta G_M(q_0^2)}{G_M(q_0^2)}\right]\frac{\left(G_{\mathrm{dip}}(q_0^2)\right)^2}{q_0}$$

Magnetic radius of the proton: the constraints

- Extraction equation

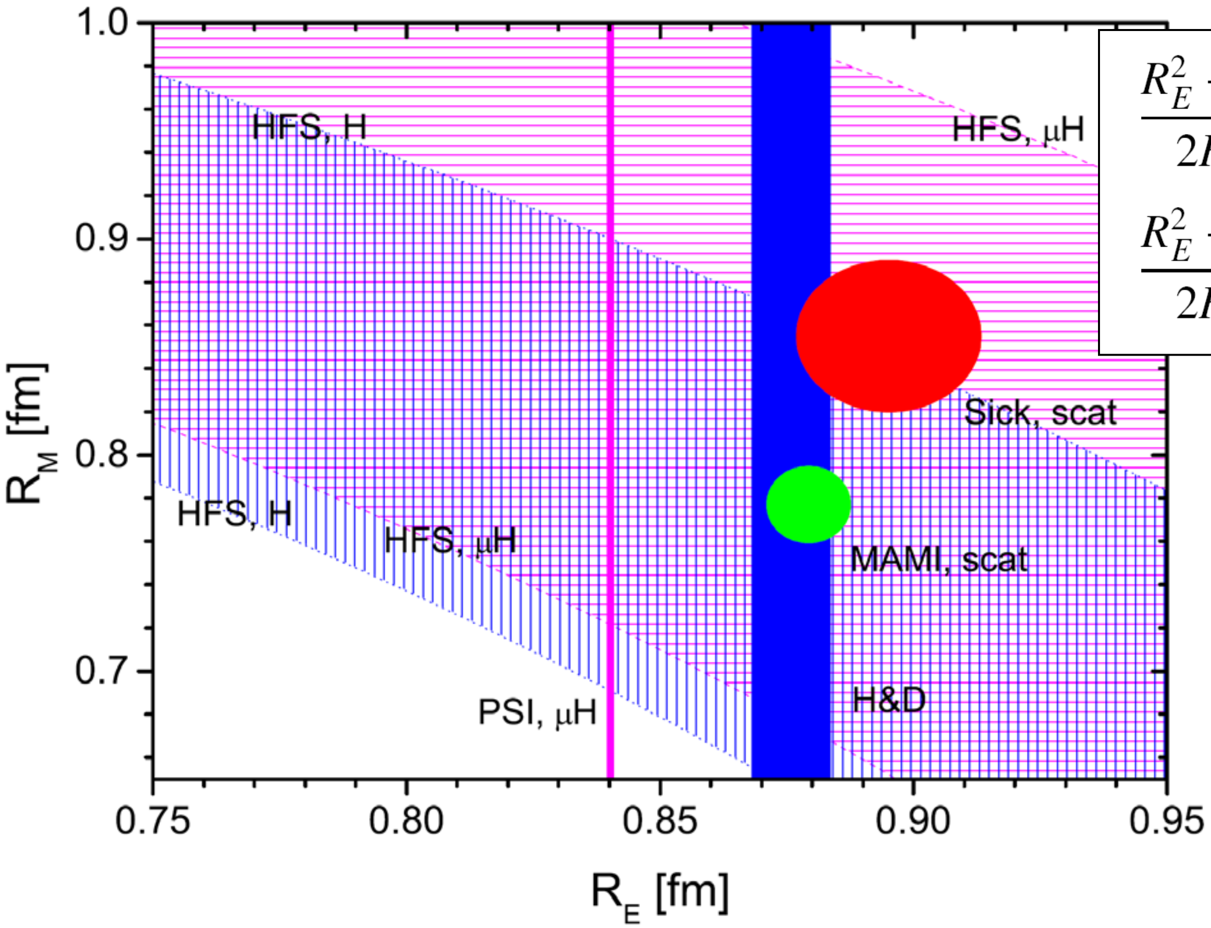
$$R_E^2 + R_M^2 = -\frac{6}{q_0}\left(I_1^{\text{exp}} - I_1^{\text{EM-R}}\right)$$

$$I_1^{\text{EM-R}} = \int_0^{q_0} \frac{dq}{q^2}\left[\frac{G_E(q^2)G_M(q^2)}{\mu_p} - 1 - \left(G'_E(0) + \frac{G'_M(0)}{\mu_p}\right)q^2\right] + \int_{q_0}^{\infty} \frac{dq}{q^2}\left[\frac{G_E(q^2)G_M(q^2)}{\mu_p} - 1\right]$$

- Data

Atom	State	I_1^{exp}	$\delta I_1 / I_1$
H	$1s$	$-4.17(6)\ \text{GeV}^{-1}$	1.5%
μH	$2s$	$-4.31(15)\ \text{GeV}^{-1}$	3.4%

Magnetic radius of the proton: the results



$$\frac{R_E^2 + R_M^2}{2R_{\rm dip}^2} = 1.025(94)\,, \qquad \text{from H}\,,$$

$$\frac{R_E^2 + R_M^2}{2R_{\rm dip}^2} = 1.13(13)\,, \qquad \text{from } \mu\text{H}$$

$$R_{\rm dip}^2 = 0.658\ \text{fm}^2$$