Testing Exotic Explanations of Proton Radius Puzzle

Ben Rislow

4 June, 2014

Carl Carlson and BR: PRD 86, 035013 (2012);
PRD 89, 035003 (2014)

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

- Pohl et al. measured muonic Lamb shift between $2S_{1/2}^{F=1}$ and $2P_{3/2}^{F=2}$ levels.

- Extracted charge radius

$$\boxed{R_E = 0.84184(67)\ \text{fm}}$$

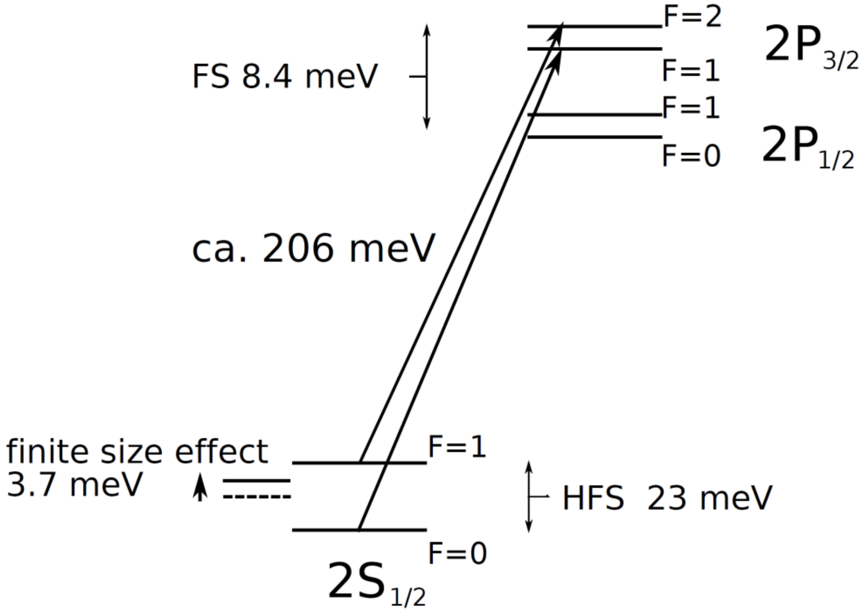
- 2013 Update (Science 339, 417-420)

$$\boxed{R_E = 0.84087(39)\ \text{fm}}$$

- Both are 7σ smaller than CODATA value of 0.8775(51) fm.

Could discrepancy be due to new muonic physics?

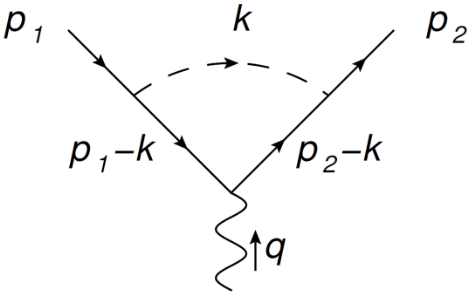
- Perhaps new physics is producing an energy shift presently misinterpreted as a contribution to the proton size :



$$\Delta E_{\mu H,\,\text{size effects}} = \Delta E_{eH,\,\text{size effects}} + \Delta E_{\text{New Physics}}$$

$$3.7\,\text{meV} = \quad 4\,\text{meV} \qquad -0.3\,\text{meV}$$

- New physics must also respect muon's anomalous magnetic moment discrepancy:



$\delta a_{\mu} = (249 \pm 87) \times 10^{-11} \quad [2.1\ \text{ppm} \pm 0.7\ \text{ppm}]$ [PRL 109, (2012) 111808]

- It is a challenge to find new physics models that simultaneously account for a 2 ppm and 10^4 ppm discrepancy.
- Meson decays will also set constraints.

Lepton Universality-Violating Models

- Tucker-Smith and Yavin [PRD 83, (2011) 101702]
  - New particle with either scalar or vector coupling.

- Batell, McKeen, and Pospelov [PRL 107, (2011) 011803]
  - New gauge boson that kinetically mixes with $F^{\mu\nu}$. The model also requires a new scalar particle to account for muon's magnetic moment.

All leptons Muons only

$$-ie\gamma^{\mu} \rightarrow -i\kappa e\gamma^{\mu} - i\frac{g_R}{2}\gamma^{\mu}(1+\gamma^5)$$

- Carlson and Rislow [PRD 86, (2012) 035013]
  - New particles with fine-tuned scalar and pseudoscalar (or polar and axial vector) couplings to the muon.

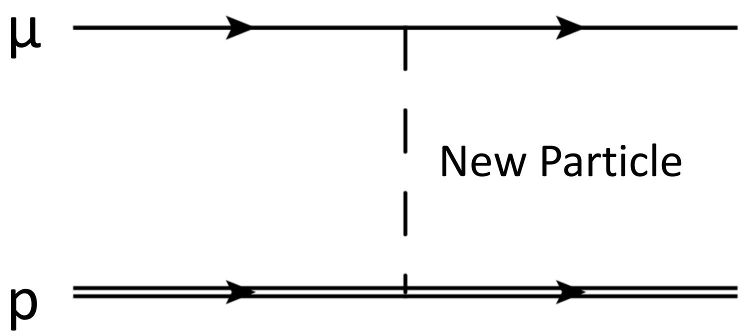
$$-ie\gamma^{\mu} \rightarrow -i\varepsilon(C_S(m_{A'}) + iC_P(m_{A'})\gamma^5)$$

$$-ie\gamma^{\mu} \rightarrow -i\gamma^{\mu}\varepsilon(C_V(m_{A'}) + C_A(m_{A'})\gamma^5)$$

Our Model

- Scalar-Pseudoscalar Langrangian

$$\mathcal{L}_S = -C_S^\mu \phi \bar{\psi}_\mu \psi_\mu - iC_P^\mu \varphi \bar{\psi}_\mu \gamma_5 \psi_\mu \\ -C_S^p \phi \bar{\psi}_p \psi_p - iC_P^p \varphi \bar{\psi}_p \gamma_5 \psi_p,$$



- We consider the case where $C^p = C^\mu$ and the masses of ϕ and φ are the same.
- We also construct a Polar-Axial Vector Lagrangian

Model Construction: Lamb Shift Constraint

- For a new particle with scalar couplings to the proton and muon,

$$V(r) = -C_S^{\mu} C_S^{p} \frac{e^{-Mr}}{4\pi r}$$

- For vector couplings, the sign in the potential is positive.
- The shift in energy is given by

$$\Delta E = \int dr r^2 V(r)(\left|R_{20}\right|^2 - \left|R_{21}\right|^2)$$

which simplifies to

Adjust so that ΔE = -0.3 meV

$$\Delta E = -\frac{C^{\mu} C^{p}}{8\pi a^3 M^2} f(Ma) \; ; \; f(x) = \frac{x^4}{(1+x)^4}$$

where M is the new particle mass and a is the Bohr radius.

Model Construction: δa_μ Constraint

- For scalar and pseudoscalar couplings

$$\delta a_\mu = \frac{m_\mu^2}{8\pi^2} \int_0^1 dz \, \frac{C_S^2 \, z^2(2-z) - C_P^2 \, z^3}{z^2 m_\mu^2 + (1-z) M^2}$$

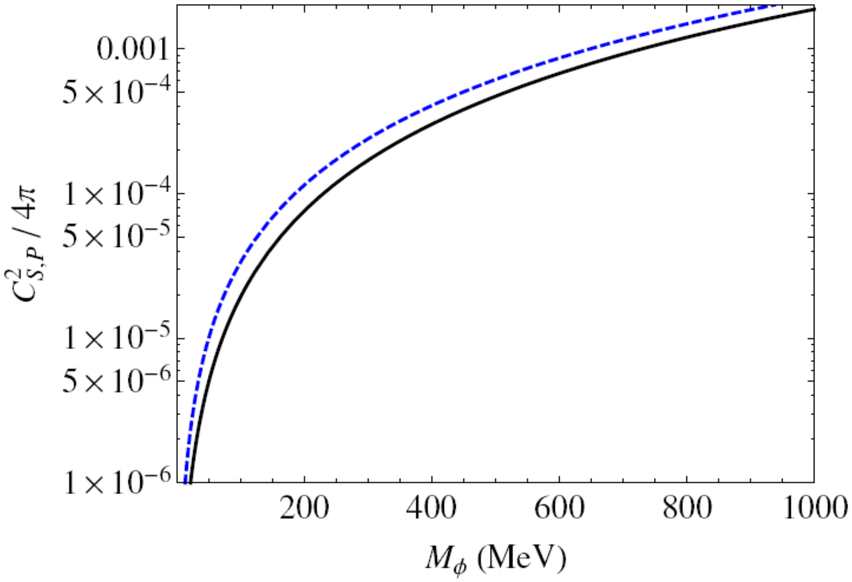
- For polar and axial vector couplings

$$\delta a_\mu = \frac{m_\mu^2}{4\pi^2} \int_0^1 \frac{dz}{z^2 m_\mu^2 + (1-z) M^2} \Bigg\{ C_V^2 z^2 (1-z) \\ - C_A^2 \left[z(1-z)(4-z) + \frac{2 m_\mu^2}{M^2} z^3 \right] \Bigg\}$$

- Sign difference allows fine-tuning.

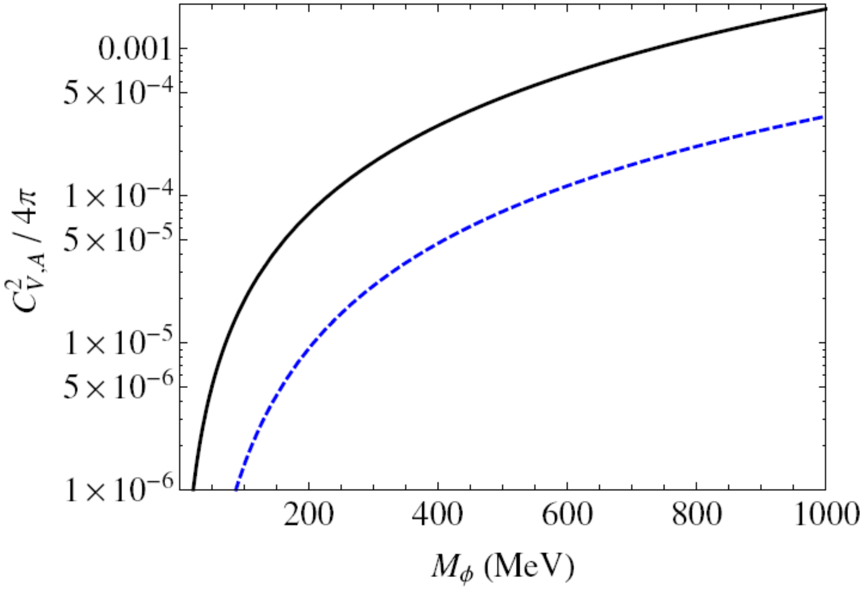
Couplings

Scalar-Pseudoscalar



Black: Scalar
Blue: Pseudoscalar

Polar-Axial Vector



Black: Polar
Blue: Axial

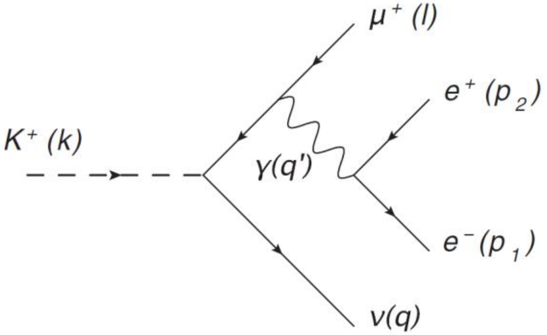
Kaon Decays at JPARC E36

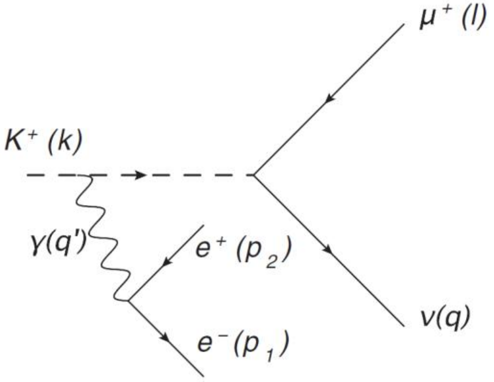
- The decay channel

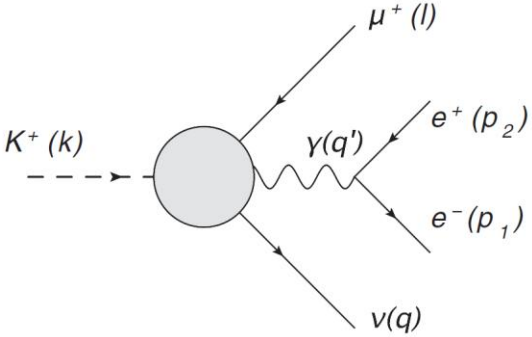
$$K^+ \to \mu^+ + \nu + e^+ + e^-$$

  could distinguish and eliminate models.

- Previously measured for $m_{ee} > 140$ MeV, calculated by Bijnens et al. [Nuc. Phys. B396 (1993) 81-118].
- If Standard Model holds, E36 experiment expects to see 200,000 such decays, 1000 decays per 1 MeV bin.



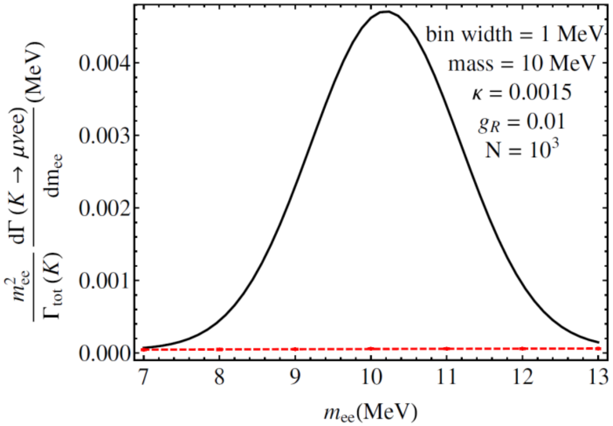


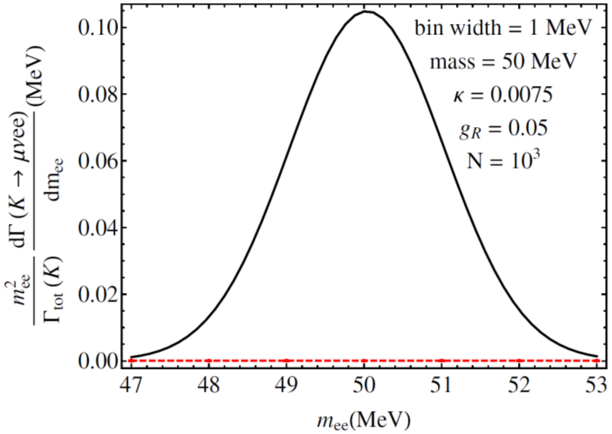


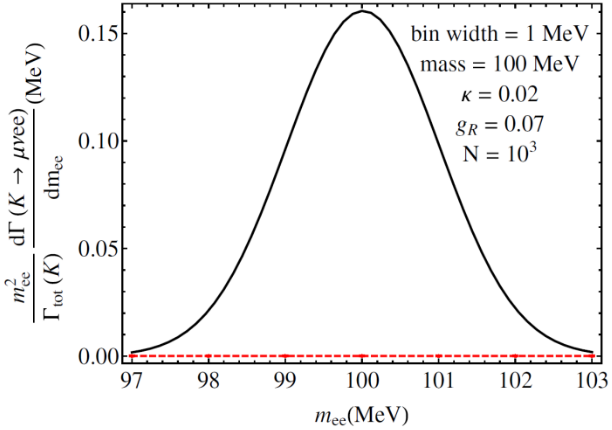
$$i\mathcal{M} = -G_F(-ie)^2 V_{us} \bar{u}(p_1)\gamma_\rho v(p_2) \frac{-i}{q'^2} (f_K m_\mu L^\rho - H^{\rho\nu} j_\nu)$$

- Batell *et al.* model: additional axial couplings, different couplings to electrons and muons.

$$\frac{-i}{q'^2} \rightarrow \frac{-i}{q'^2 - m_{A'}^2 + i m_{A'}\Gamma} \qquad -ie\gamma^\mu \rightarrow -i\kappa e\gamma^\mu - i\frac{g_R}{2}\gamma^\mu(1+\gamma^5)$$







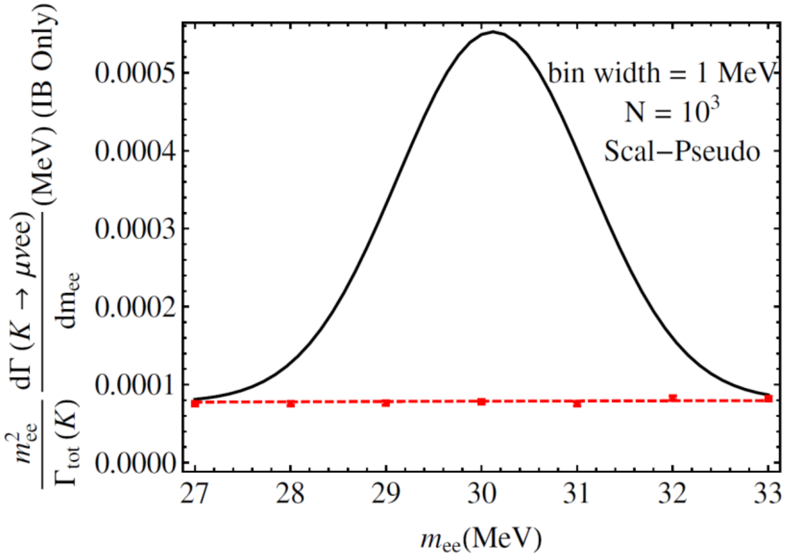
Red Curve: Standard Model

Black Curve: Additional Modified Dark Photon

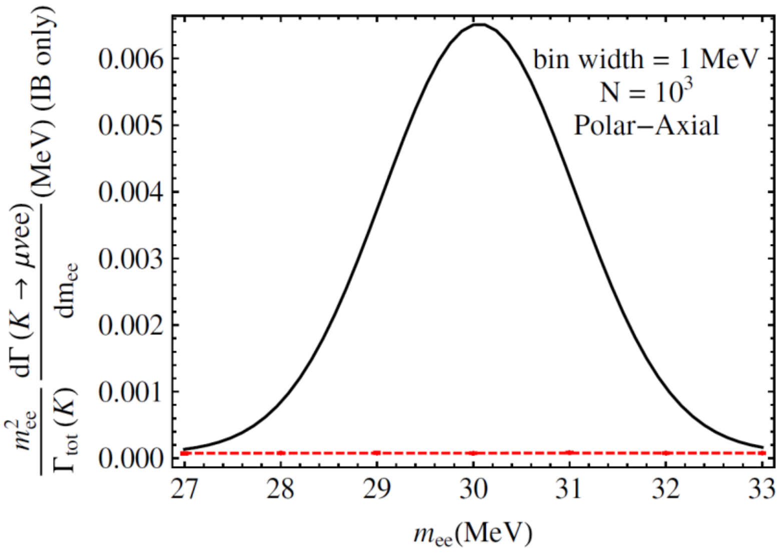
- Carlson and Rislow model: additional fine-tuned couplings, different couplings to electrons and muons.

$$\frac{-i}{q'^2} \rightarrow \frac{-i}{q'^2 - m_{A'}^2 + i m_{A'} \Gamma}$$

$$-ie\gamma^\mu \rightarrow -i\varepsilon(C_S(m_{A'}) + iC_P(m_{A'})\gamma^5)$$



$$-ie\gamma^\mu \rightarrow -i\gamma^\mu\varepsilon(C_V(m_{A'}) + C_A(m_{A'})\gamma^5)$$



Red Curve: Standard Model

Black Curve: Additional Dark Photon

HFS in Muonic Hydrogen

- The measured HFS in 2S muonic hydrogen is

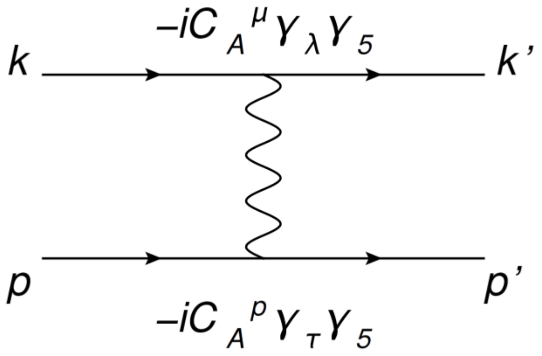
$$\Delta E_{\rm HFS}^{\rm exp} = 22.8089(51)\ \rm meV$$

- The theory value is

$$\Delta E_{\rm HFS}^{\rm thy} = 22.8146(49)\ \rm meV$$ [PRA 83 (2011) 042509]

- Thus, HFS contribution from new physics must not exceed 5.1 μeV.
- Pseudoscalar and axial vector couplings will contribute.

HFS from axial exchange



- Compare to Drell & Sullivan (1965)

$$\mathcal{M}_A = C_A^{\mu} C_A^{p} \bar{u}(k')\gamma_\lambda\gamma_5 u(k) \frac{g^{\lambda\tau} - q^\lambda q^\tau / m_A^2}{q^2 - m_A^2} \bar{u}(p')\gamma_\tau\gamma_5 u(p)$$

- NR,

$$\mathcal{M}_A = 2m_\mu\, 2m_p \frac{C_A^{\mu} C_A^{p}}{\vec{q}^2 + m_A^2}\, \vec{\sigma}_e \cdot \vec{\sigma}_p$$

and

$$\Delta\mathcal{M}_A = \langle \mathcal{M}_A \rangle_{S=1} - \langle \mathcal{M}_A \rangle_{S=0}$$

Axial exchange

- Convert scattering amplitude to energy,

$$\Delta E^A_{\mathrm{HFS}} = -\frac{1}{2m_\mu 2m_p} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \phi^*(k')\, \Delta\mathcal{M}_A(\vec{k}' - \vec{k})\, \phi(k)$$

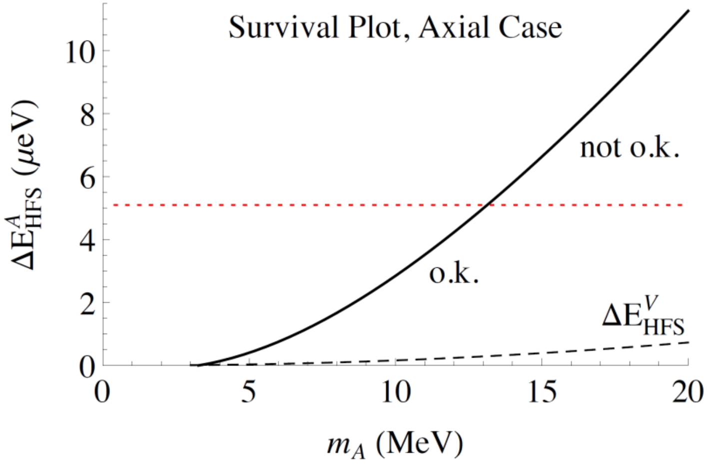
- For the 2S state,

$$\Delta E^A_{\mathrm{HFS}} = -\frac{C^\mu_A C^p_A}{4\pi} \frac{2m_r^3\alpha^3}{m_A^2} \frac{m_A^2(m_A^2 + \frac{1}{2}m_r^2\alpha^2)}{(m_A + m_r\alpha)^4}$$

- Using existing results for $C_A^\mu\ C_A^p/(4\pi)$ as a function of the exchanged mass m_A leads to

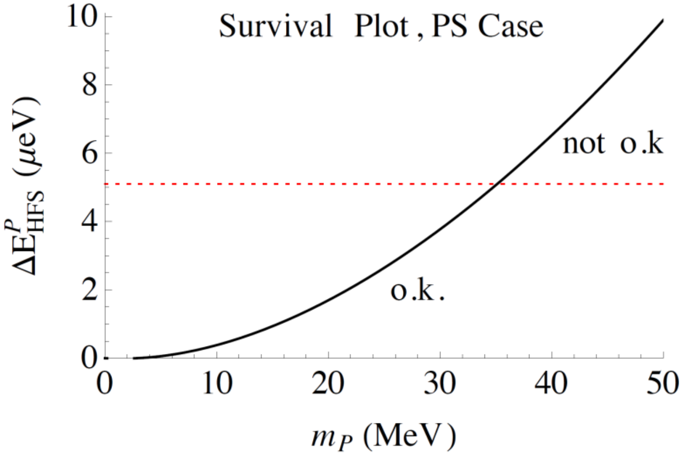
Carlson PSAS 2014

Axial exchange allowed range



- O.k. if mass below about 13 MeV
- Polar vector also shown

- Similar result for PS case
- Mass o.k. below about 35 MeV



Carlson PSAS 2014

Comment on muonic ^{4}He

- Yesterday we performed a back of envelope prediction for Lamb shift contribution of our model.
- Reminder:

$$\Delta E = -\frac{C^\mu C^p}{8\pi a^3 M^2} f(Ma) \; ; \; f(x) = \frac{x^4}{(1+x)^4}$$

$$\Delta E \propto \delta R_p^2$$

- After some algebra:

$$\frac{\delta R_h}{R_h} = \frac{\delta R_p}{R_p} \frac{R_p^2}{R_h^2} \frac{f(x_h)}{f(x_p)}$$

$$\frac{\delta R_h}{R_h} = 0.25\% = 4\% \frac{R_p^2}{R_h^2} \frac{f(x_h)}{f(x_p)} \quad ; \quad x_h = \frac{M}{Z_h m_r \alpha}$$

- Particle mass < 1 MeV.

Conclusion

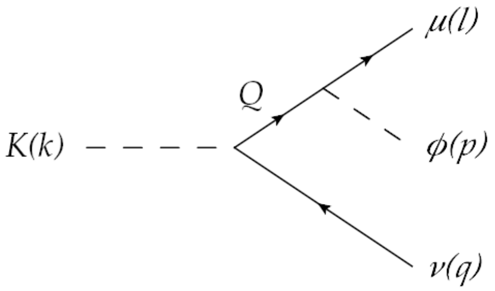
- There are several new physics models constrained by the Lamb shift, magnetic moment, and HFS of the muon.
- A new particle, if it exists, will have a mass of a few MeV.
- Kaon decay measurements at JPARC could help further constrain or eliminate models.

Kaon Constraint

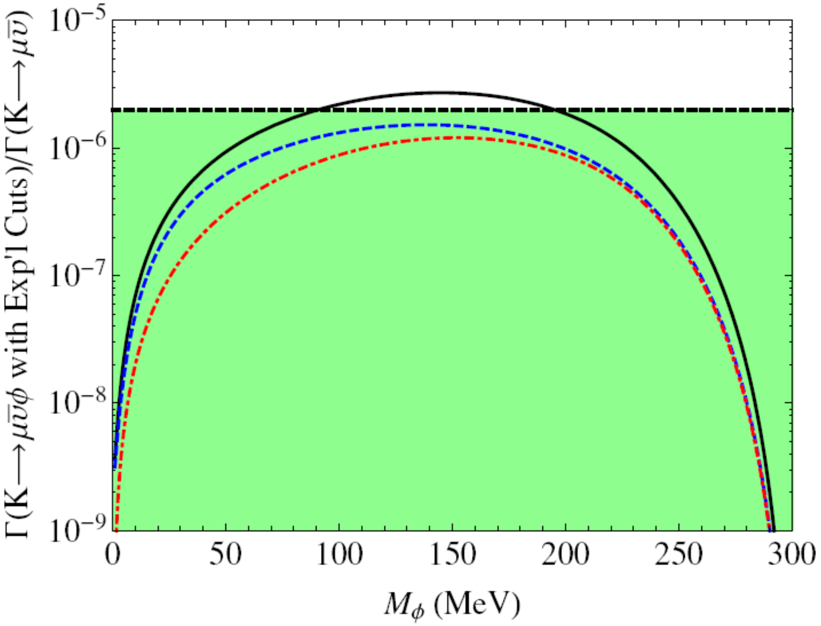
- Pang et al. (PRD 8, 1989 (1973)) looked for the multibody decay   K -> μ X(invisible). They found the experimental limit

$$\frac{1}{\Gamma(K \to \mu\nu)} \int \frac{d\Gamma(K \to \mu X)}{dE_\mu} D(E_\mu)\, dE_\mu < 2 \times 10^{-6}$$

- For our case

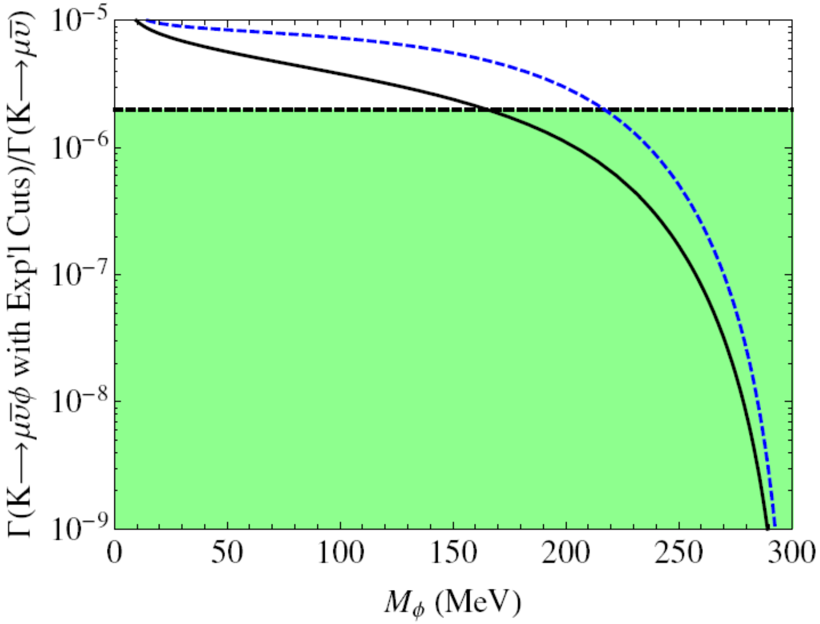


Mass Limits (Shaded region is allowed.)



Scalar-Pseudoscalar

Black: Full Result
Blue: Pseudoscalar Part
Red: Scalar Part



Polar-Axial Vector

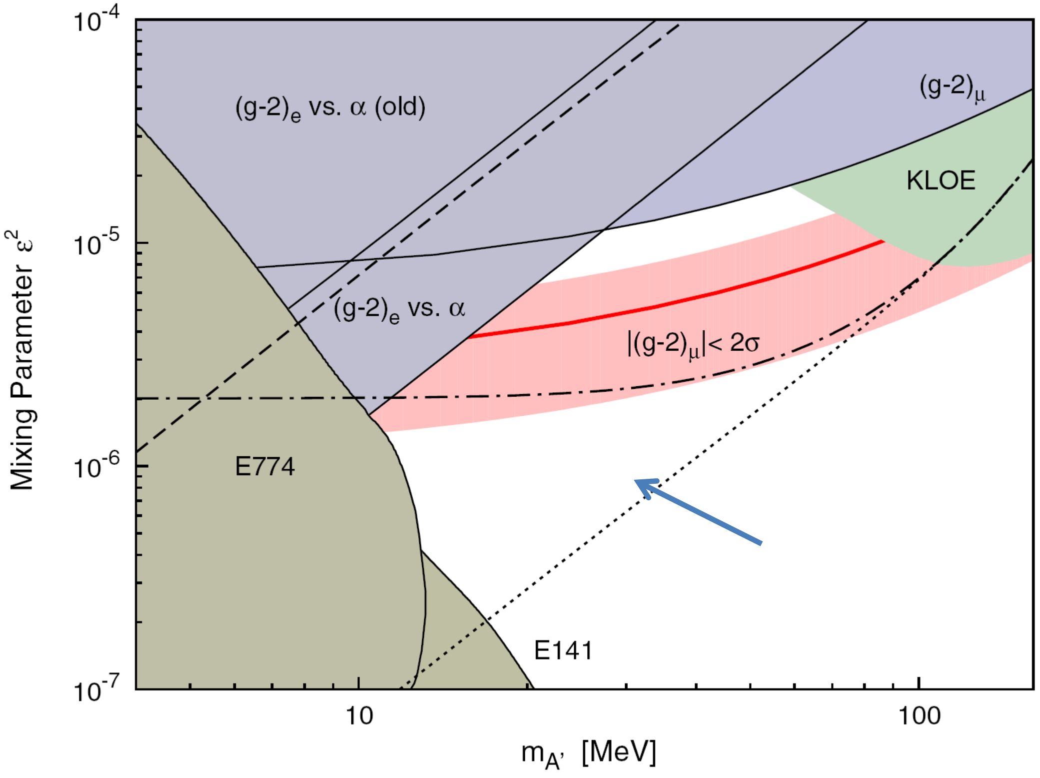
Black: Single Particle
w/polar and axial couplings
Blue: Two Particles
w/equal masses

- Batell et al.'s model evades this constraint because their new particle decays into e^+e^- pair.

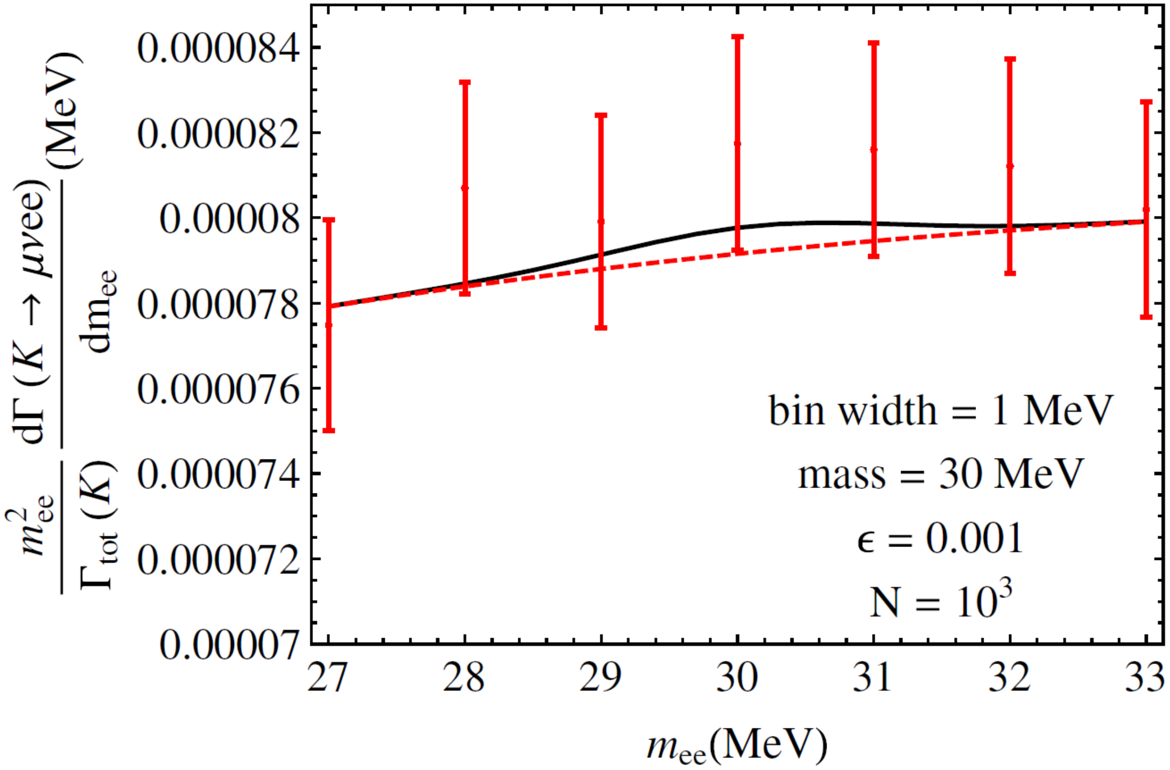
Model Tests

- Dark photon:

$$\frac{-i}{q'^2} \to \frac{-i}{q'^2 - m_{A'}^2 + i m_{A'}\Gamma} \qquad -ie \to -i\varepsilon e$$



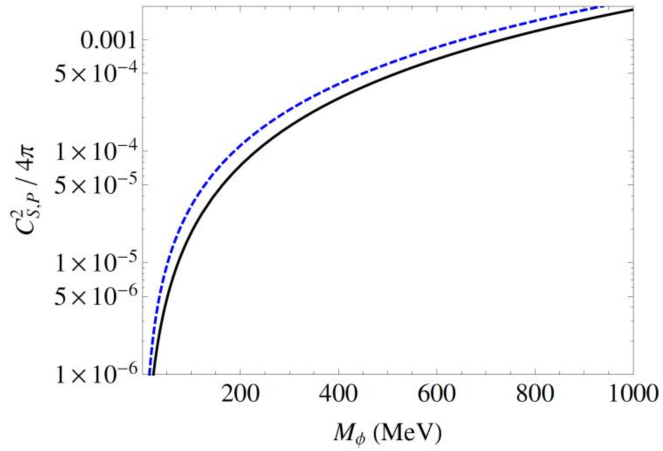
- Dark photon:



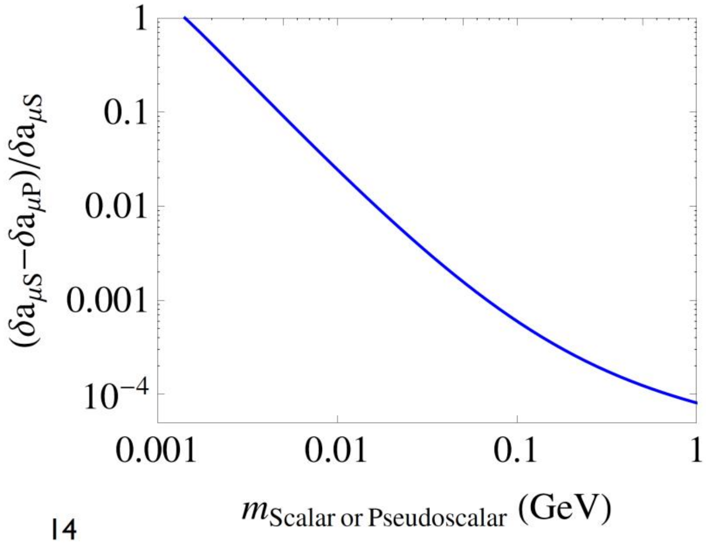
Red Curve: Standard Model
Black Curve: Additional Dark Photon
Error Bars: Simulated Data

Constraint from $(g\text{-}2)_\mu$

- Hence fine-tuning is possible, however unpleasant



Scalar and pseudoscalar couplings needed for cancellation for (g-2)



or, degree of fine tuning required

(There are similar plots for V, A case)