# The H-dibaryon from Lattice QCD using Two-baryon Operators with Distillation

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# Outline

- Motivation for studying the H-dibaryon
- Interpolating operators
- Overview of recent results from the Mainz group
    - $N_f = 2$ CLS ensembles with quenched strange quark
    - Distillation vs. smeared point sources
- Preliminary results on $N_f = 2 + 1$ CLS ensembles
    - Larger basis of operators
    - Use of spin-1 baryon-baryon operators
- Future work

# Motivation

- In 1977, Jaffe predicts deeply bound dibaryon with quark content $uuddss$, $J^P = 0^+$, $I = 0$
- Conclusive experimental evidence for such a state is still lacking
- Early quenched calculations disagree on existence of such a bound state
- More recent results with dynamical quarks from NPLQCD and HAL QCD disagree on the binding energy for $m_\pi \approx 800\,\mathrm{MeV}$

# The Mainz Dibaryon Project

- In collaboration with:

  A. Francis, J. Green, P. Junnarkar, H. Wittig

- Recent results on $N_f = 2$ CLS ensembles with quenched strange quark (arXiv:1805.03966)
  - Main focus on two ensembles with $a = 0.0658\,\mathrm{fm}$ and $L = 2.1\,\mathrm{fm}$
    - E1: $m_\pi = 960\,\mathrm{MeV}$, quenched $m_s = m_{u,d}$
    - E5: $m_\pi = 440\,\mathrm{MeV}$, quenched $m_s \approx m_s^{\mathrm{phys}}$
  - Uses smeared point sources and Distillation
  - Finite volume analysis

- Recent extensions to $N_f = 3$

# $SU(3)$ Flavor Structure

- The singlet can be formed from two flavor octets

$$\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27})_S \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})_A$$

- Can rotate to multiplets of $SU(3)$ flavor

$$\begin{bmatrix} BB_{27} \\ BB_{8_S} \\ BB_1 \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{1}{40}} & \sqrt{\frac{12}{40}} \\ -\sqrt{\frac{1}{5}} & -\sqrt{\frac{3}{5}} & \sqrt{\frac{1}{5}} \\ -\sqrt{\frac{1}{8}} & \sqrt{\frac{3}{8}} & \sqrt{\frac{4}{8}} \end{bmatrix} \begin{bmatrix} [\Lambda\Lambda]^{I=0} \\ [\Sigma\Sigma]^{I=0} \\ [N\Xi]_s^{I=0} \end{bmatrix}$$

- **8** and **27** mix with **1** upon $SU(3)$ symmetry breaking

# Interpolating Operators

- Hexaquark operators inspired by Jaffe's bag model prediction:

$$[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}(s^i C\gamma_5 P_+ t^j)(v^l C\gamma_5 P_+ w^m)(r^k C\gamma_5 P_+ u^n)$$

  - Can form singlet $H^1$ and 27-plet $H^{27}$ flavor combinations

- Two-baryon operators:
  - Momentum-projected octet baryon operators

$$B_\alpha(\boldsymbol{p}, t)[rst] = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \epsilon_{abc}(s^a C\gamma_5 P_+ t^b) r^c_\alpha$$

  - Can form spin-zero and spin-one operators

$$[B_1 B_2]_0(\boldsymbol{p}_1, \boldsymbol{p}_2) = B^{(1)}(\boldsymbol{p}_1) C\gamma_5 P_+ B^{(2)}(\boldsymbol{p}_2)$$
$$[B_1 B_2]_i(\boldsymbol{p}_1, \boldsymbol{p}_2) = B^{(1)}(\boldsymbol{p}_1) C\gamma_i P_+ B^{(2)}(\boldsymbol{p}_2)$$

# Rotational Properties of Operators

- Python package using SymPy libary to determine rotation properties
- Can very simply construct needed operators:

```
u = QuarkField.create('u')
a = ColorIdx('a')
i = DiracIdx('i')
...
Delta = Eijk(a,b,c) * u[a,i] * u[b,j] * u[c,k]
```

- Project to definite momentum, and determine Little Group

```
delta_ops = Operator(Delta, P([0,0,1]))
delta_op_rep = OperatorRepresentation(*delta_ops)
delta_op_rep.littleGroupContents()
# output: 6 G1 + 4 G2
```

- Supports multi-particle operators

# Some Details of the Python Package

- The representation matrix $W_{ij}(R)$ $(R \in \mathcal{G})$ for a given basis of operators $\mathcal{O}_i$ can be found via $U_R \mathcal{O}_i U_R^\dagger = \mathcal{O}_j W_{ji}(R)$
- Much can be uncovered from $W_{ij}(R)$
  - Is W irreducible?

$$\sum_{R \in \mathcal{G}} |\chi(W(R))|^2 = g_{\mathcal{G}} \iff W \text{ is irreducible}$$

  - How many times does the irrep $\Gamma$ occur in $W$?

$$n_\Gamma^W = \frac{1}{g_{\mathcal{G}}} \sum_{R \in \mathcal{G}} \chi(\Gamma(R))^* \chi(W(R))$$

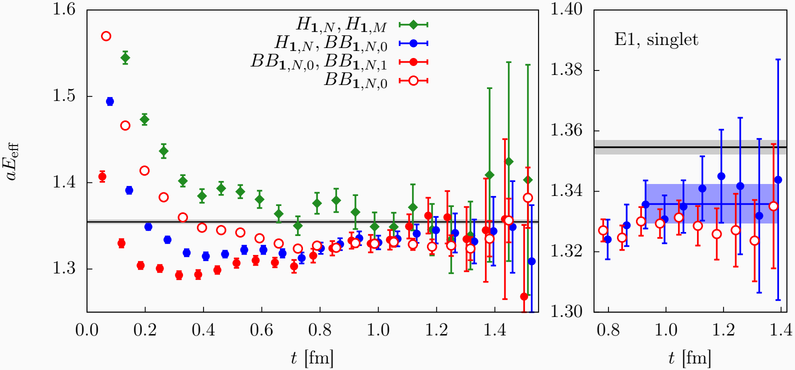
  - Apply group-theoretical projections

$$P_{ij}^{\Lambda\lambda} = \frac{d_\Lambda}{g_{\mathcal{G}}} \sum_{R \in \mathcal{G}} \Gamma_{\lambda\lambda}^{(\Lambda)}(R) W_{ji}(R)$$

- Perform tests for rotations between equivalent momentum frames

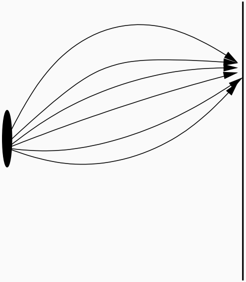
# Ground State for Singlet Channel on $E1$ ($SU(3)$ Symmetric)

- Legend indicates sink operators
- Hexaquark operators noisier and slower ground-state saturation

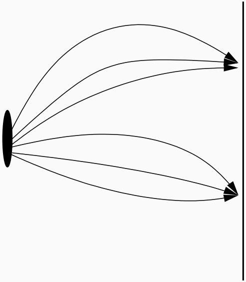


# Adding Distillation to the Mix

- Use of point sources requires local operators at the source
- Leads to non-Hermitian correlator matrices



$\langle H(t)H^{\dagger}(0)\rangle$

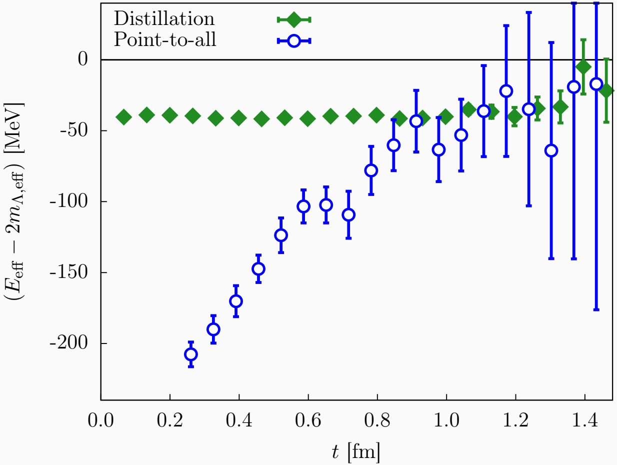


$\langle BB(t)H^{\dagger}(0)\rangle$

- Add use of timeslice-to-all method

# Distillation vs. Smeared Point Sources

- Ensemble E1, ground state in singlet channel
- Better quality data with less inversions



# Finite Volume Analysis - Lüscher Method

- S-wave scattering phase shift:

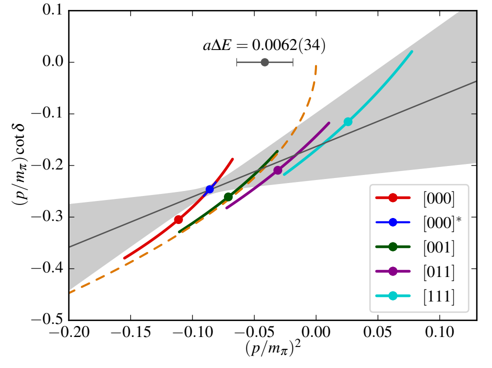
$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi} L \gamma} \mathcal{Z}_{00}^{\boldsymbol{P}}(1, q^2), \qquad q = \frac{pL}{2\pi}, \qquad p^2 = \frac{1}{4}(E^2 - \boldsymbol{P}^2) - m_\Lambda^2$$

- Perform fit to phase shift

- Pole below threshold indicates a bound state

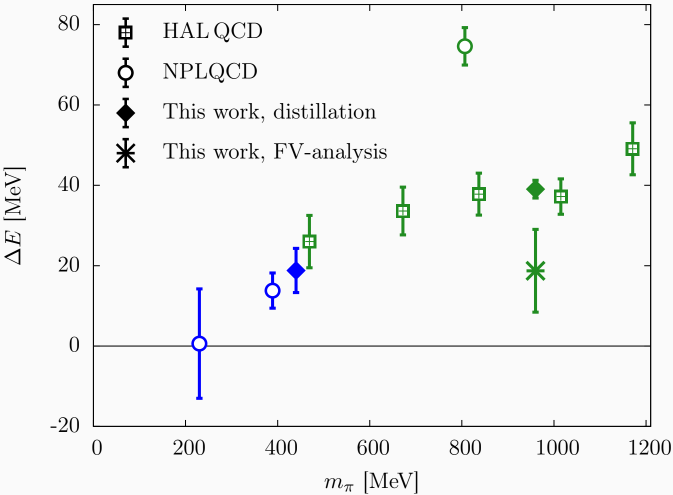
$$\mathcal{A} \propto \frac{1}{p \cot \delta_0(p) - ip}$$

$$\implies p \cot \delta_0(p) = -\sqrt{-p^2}$$



# Comparison to Other Collaborations

- Green are $SU(3)$-symmetric, and blue are $SU(3)$ broken



# $N_f = 2 + 1$ CLS Ensembles

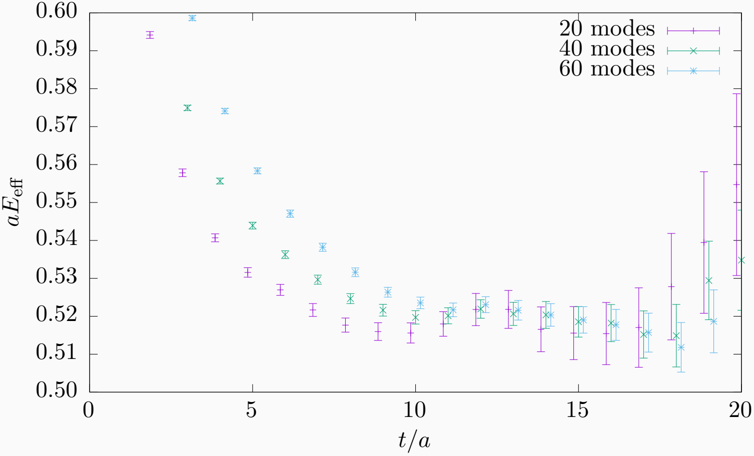
- Beginning extensions to CLS ensembles with $N_f = 2 + 1$ $O(a)$-improved Wilson fermions

- Initial results for the $SU(3)$-symmetric point, $m_\pi = m_K = m_\eta \approx 420\,\mathrm{MeV}$
  - U103 - $\beta = 3.40$, $24^3 \times 128$, $N_{\mathrm{LapH}} = 20$, open BCs
  - H101 - $\beta = 3.40$, $32^3 \times 96$, $N_{\mathrm{LapH}} = 48$, open BCs
  - B450 - $\beta = 3.46$, $32^3 \times 64$, $N_{\mathrm{LapH}} = 32$, periodic BCs

- Very Preliminary!

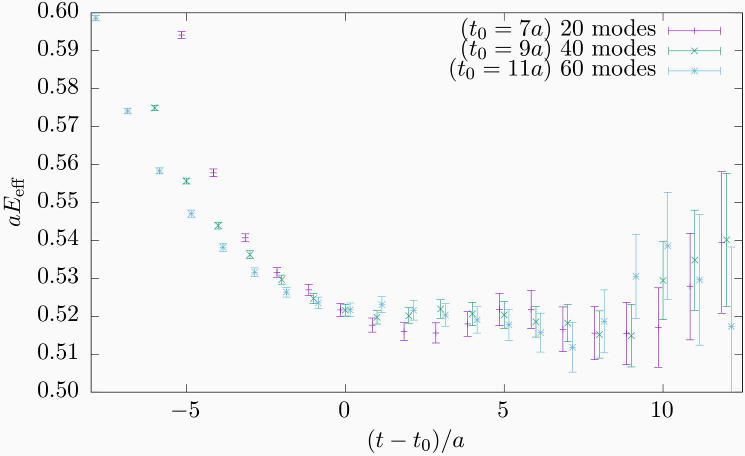
# Choosing $N_{\mathrm{LapH}}$ from Octet Baryon Effective Energy

Statistical error increases for smaller number of modes



# Choosing $N_{\mathrm{LapH}}$ from Octet Baryon Shifted Effective Energy

Plateau is reached earlier for smaller number of modes



# Variational Method to Extract Finite-Volume Spectrum

- Form $N \times N$ correlation matrix, has spectral decomposition

$$C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)*} \; e^{-E_n t}, \quad Z_j^{(n)} = \langle 0 | \, O_j \, | n \rangle$$

- Let the columns of $U$ contain the eigenvectors of

$$\hat{C}(\tau_D) = C(\tau_0)^{-1/2} \, C(\tau_D) \, C(\tau_0)^{-1/2}$$

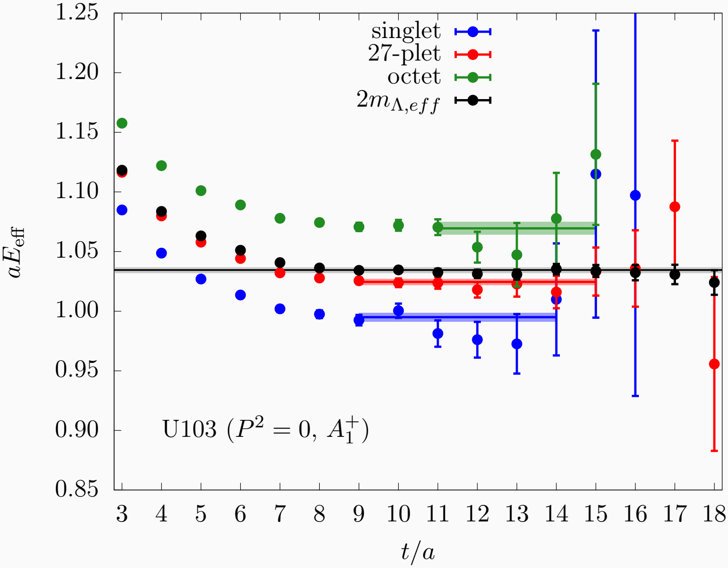
- Use $U$ to rotate at other times

$$\widetilde{C}(t) = U^\dagger \; \hat{C}(t) \; U$$

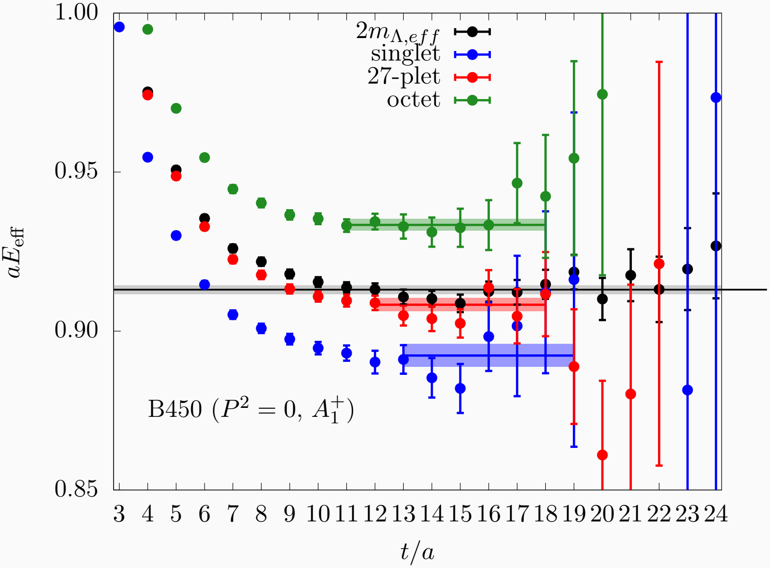
- Must check that $\widetilde{C}(t)$ remains diagonal at $t > \tau_D$.
- If $\tau_0$ is chosen sufficiently, then eigenvalues $\lambda_n(t, \tau_0)$ behave as

$$\lambda_n(t, \tau_0) \propto e^{-E_n t} + O(e^{(E_N - E_n)t})$$

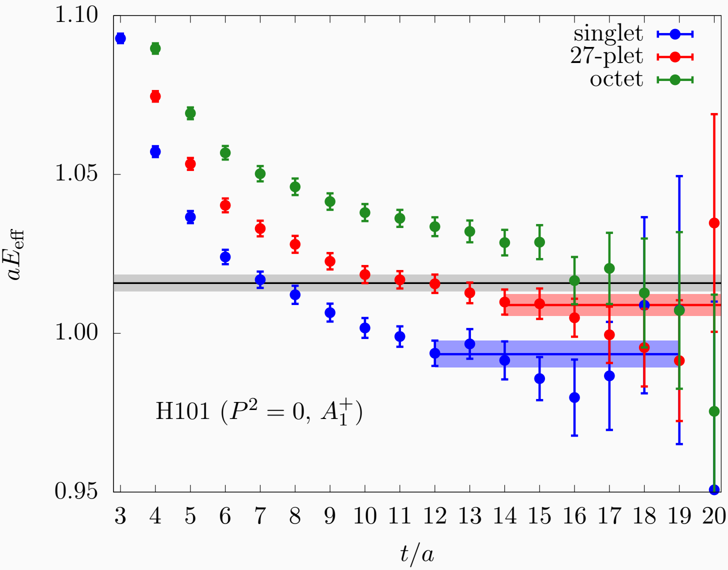
# U103: $P^2 = 0$, $A_1^+$ irrep



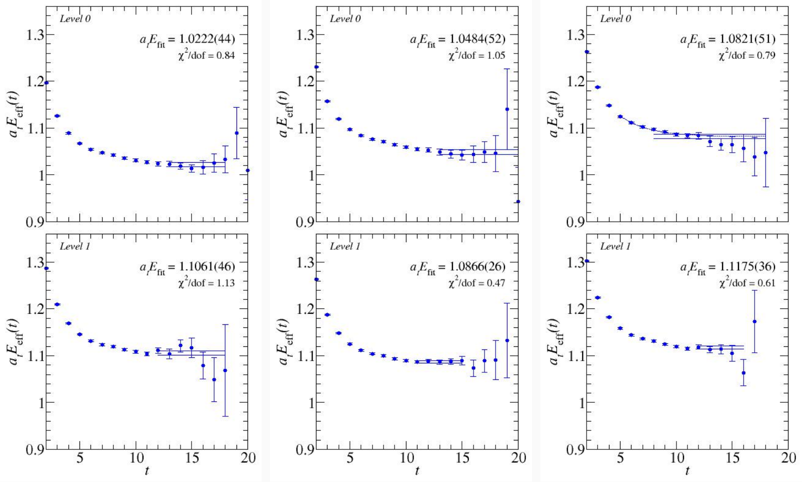
# B450: $P^2 = 0$, $A_1^+$ irrep



# H101: $P^2 = 0$, $A_1^+$ irrep

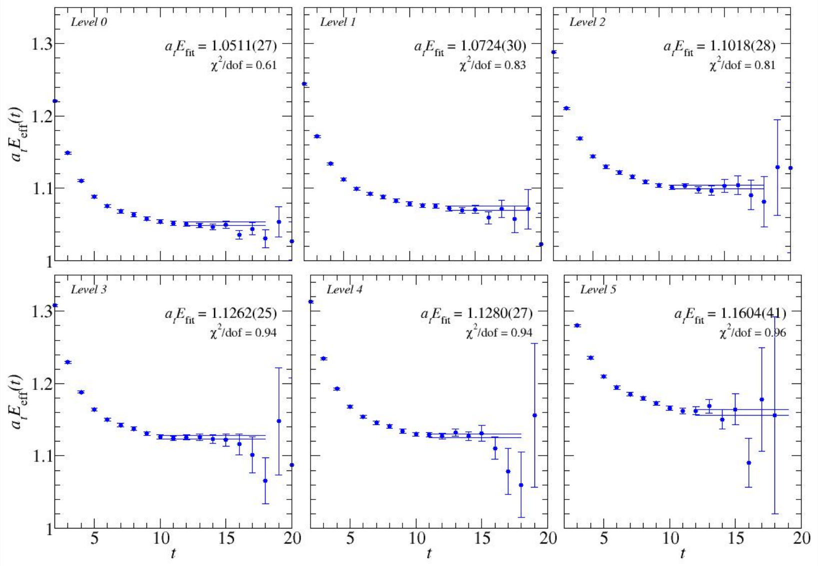


# H101: $P^2 = 1, 2, 3$, $A_1$ irrep - $SU(3)$ singlet




$P^2 = 1$ $P^2 = 2$ $P^2 = 3$

# H101: $SU(3)$ octet, $P^2 = 1$, $A_1$ irrep



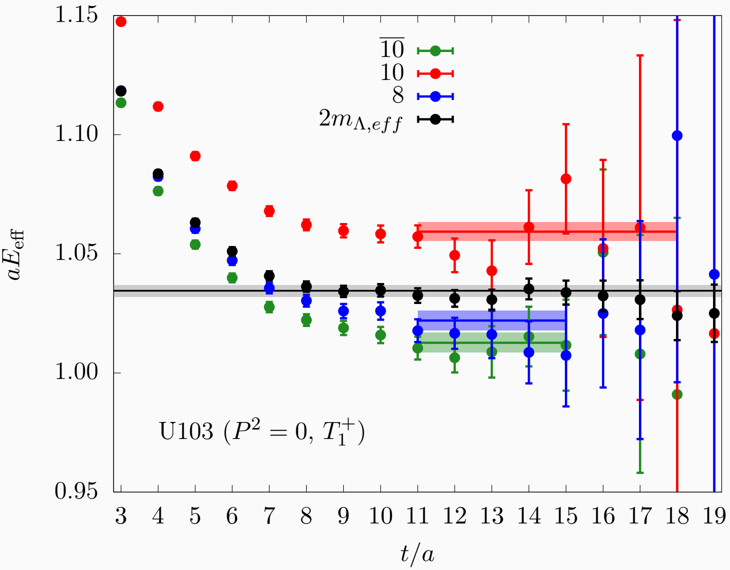
# Extensions to Asymmetric Flavor Combinations

- Can use operators that are flavor asymmetric to access other $SU(3)$ multiplets

$$\begin{bmatrix} BB_{\overline{10}} \\ BB_{10} \\ BB_{8_A} \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} \end{bmatrix} \begin{bmatrix} [N\Xi]_a^{I=1} \\ [\Sigma\Lambda]_a^{I=1} \\ [\Sigma\Sigma]^{I=1} \end{bmatrix}$$

- The deuteron lives in $\overline{\mathbf{10}}$
- To access positive parity states at rest, must include spin-1 operators
  - Mixing between ${}^3S_1$ and ${}^3D_1$

# U103: $P^2 = 0$, $T_1^+$ irrep



# Summary and Outlook

- Results for $N_f = 2$ ensembles shown
- Hexaquark operators not as important
- Distillation substantially improves quality of data
- Preliminary $N_f = 3$ results shown

Future Work

- Finalize $N_f = 3$ results
- Include $SU(3)$ broken ensembles
  - Coupled channels ($\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$)
- Extensions to more ensembles
  - $N_{\mathrm{LapH}}$ scales as $L^3$ for constant smearing radius
  - Large lattices could be very expensive
  - Investigate stochastic LapH

# Questions?