Scalar resonances in coupled-channel scattering from lattice QCD

David Wilson

based on work with the Hadron Spectrum Collaboration

hadspec

hadspec.org

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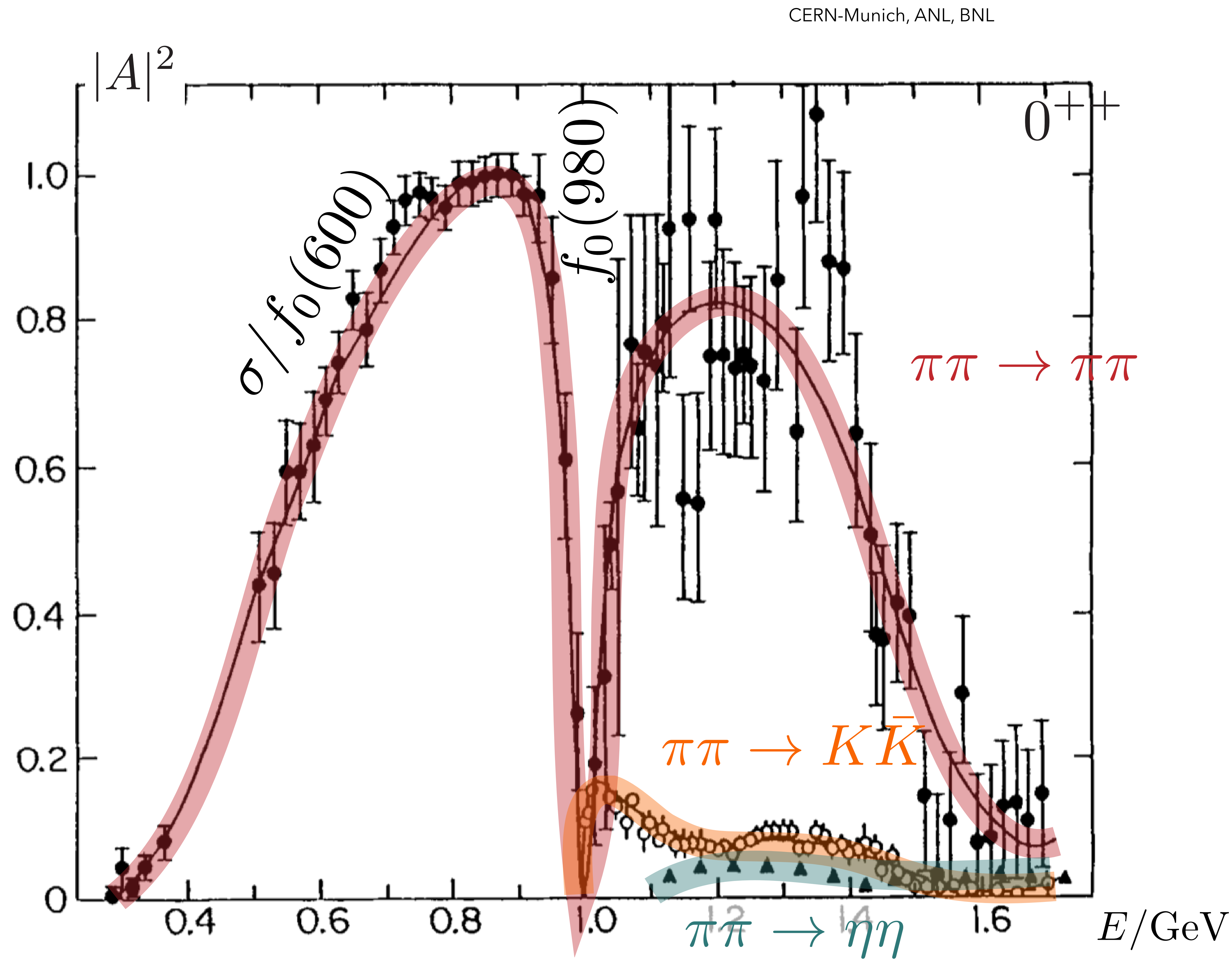
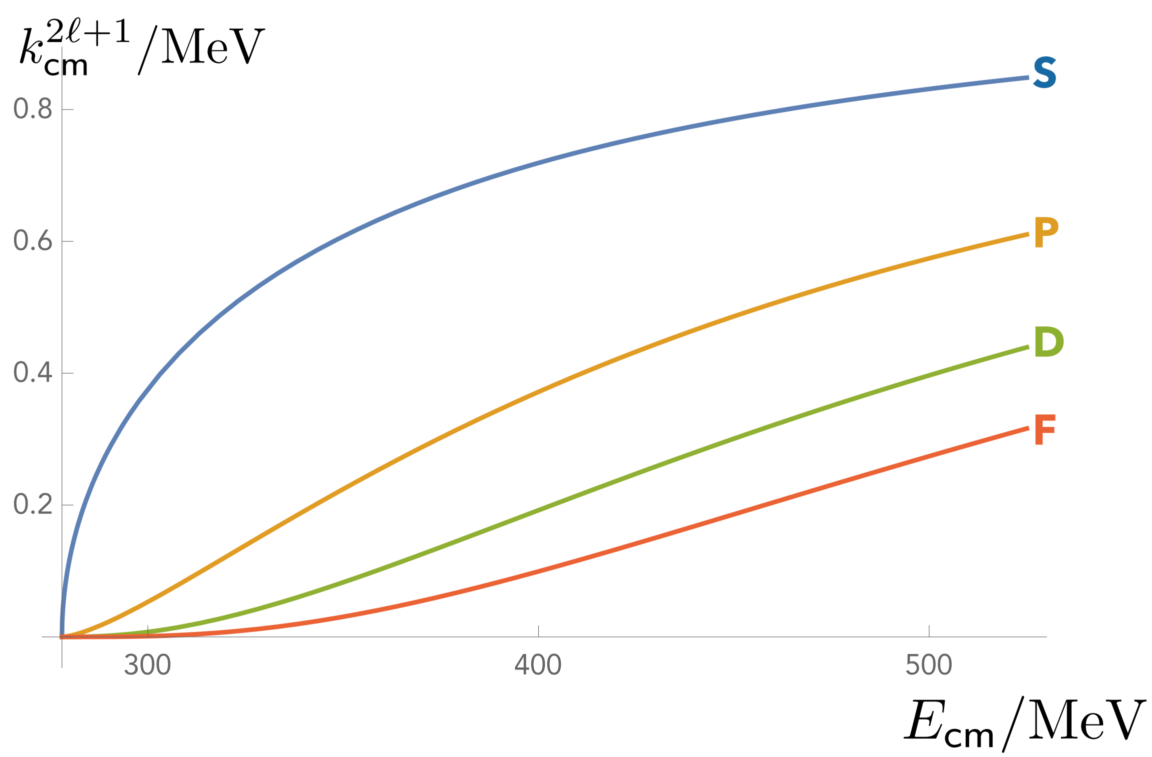
Science Foundation Ireland
For what's next

THE ROYAL SOCIETY


CERN-Munich, ANL, BNL
$|A|^2$
0^{++}
1.0
0.8
0.6
0.4
0.2
0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
E/GeV
$\sigma/f_0(600)$
$f_0(980)$
$\pi\pi \to \pi\pi$
$\pi\pi \to K\bar{K}$
$\pi\pi \to \eta\eta$

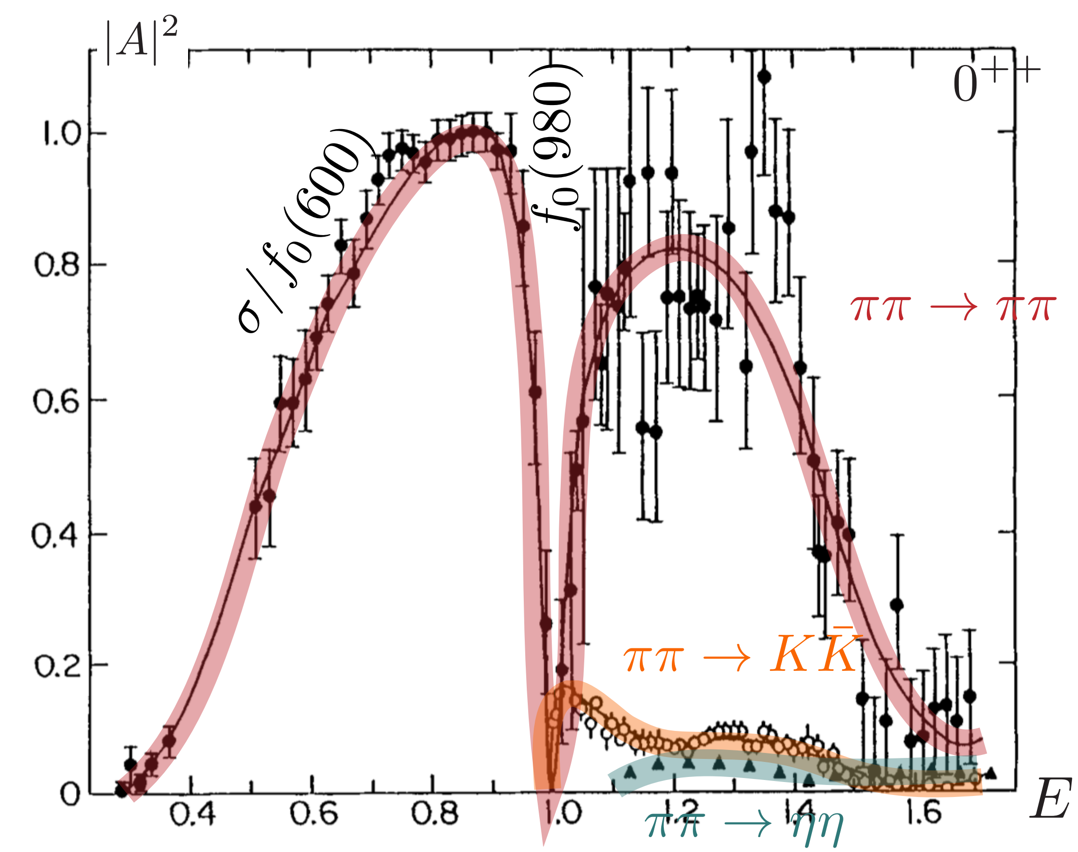
In the scalar sector, amplitudes grow rapidly from threshold:

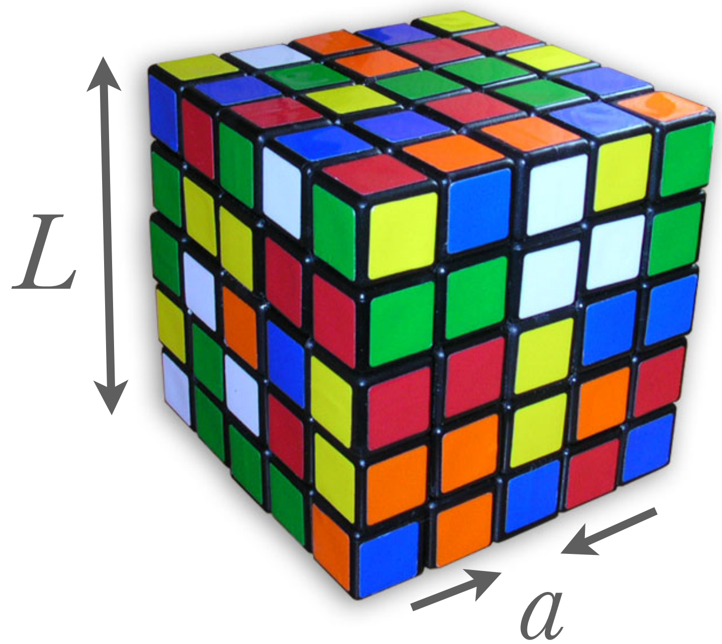


σ and κ are broad (width ~ mass)

$f_0(980)$ and $a_0(980)$ lie very close to KK threshold

CERN-Munich, ANL, BNL





3 volumes
L=16, 20, 24
anisotropic (3.5 finer spacing in time)
Wilson-Clover

m_π=391MeV

m_K=549MeV

$$\begin{bmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{bmatrix}$$

operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$ local qq-like constructions

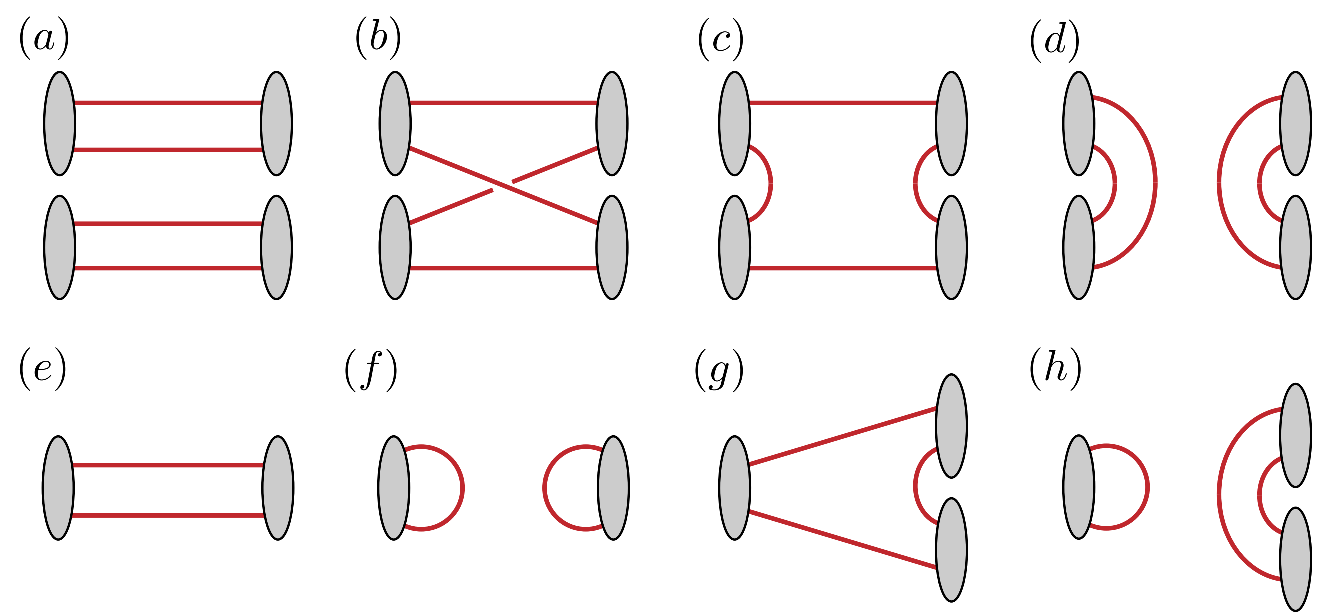
$$\sum_{\vec{p}_1+\vec{p}_2 \in \vec{p}} C(\vec{p}_1, \vec{p}_2; \vec{p}) \Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$

two-hadron constructions

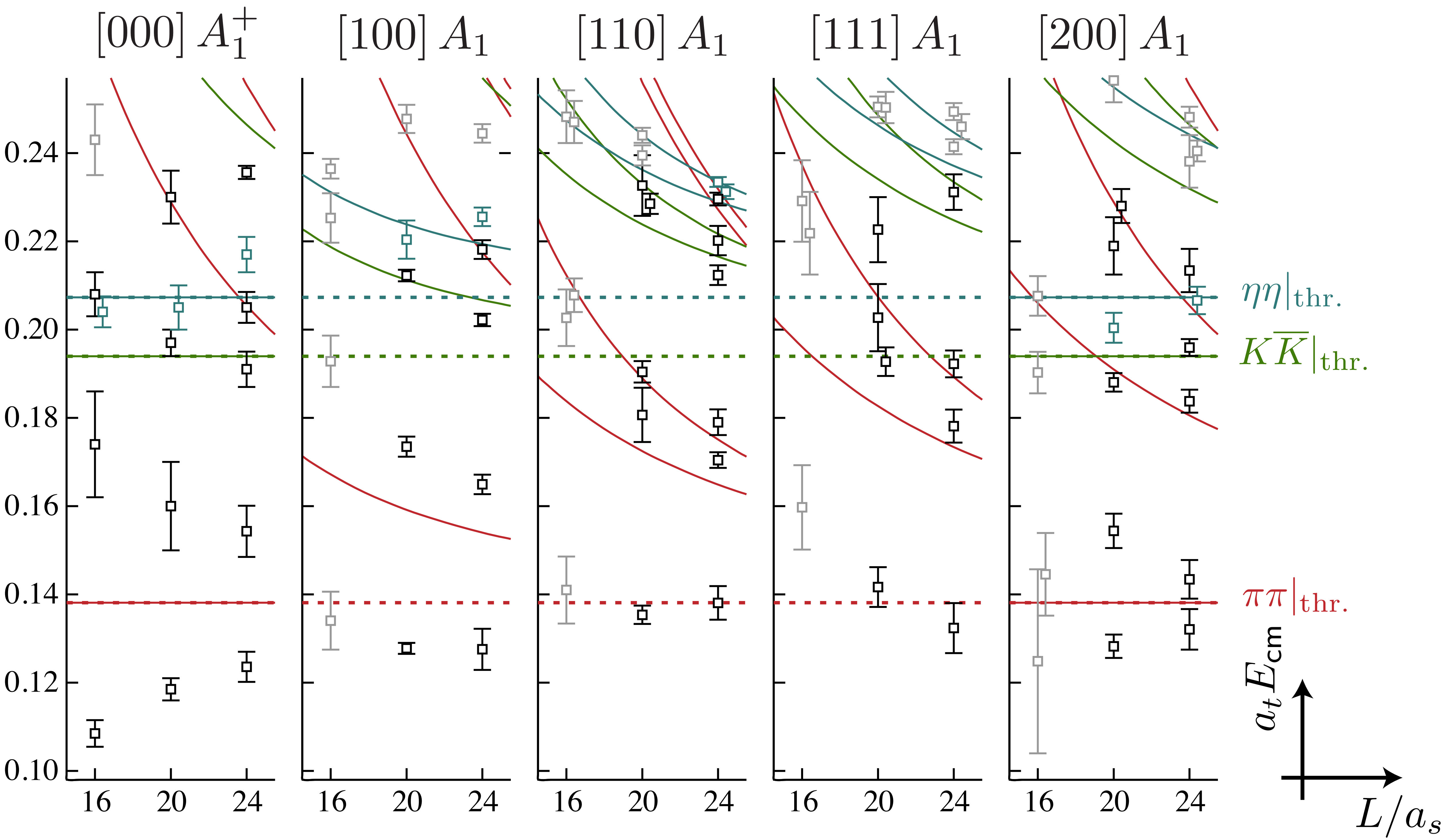
$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

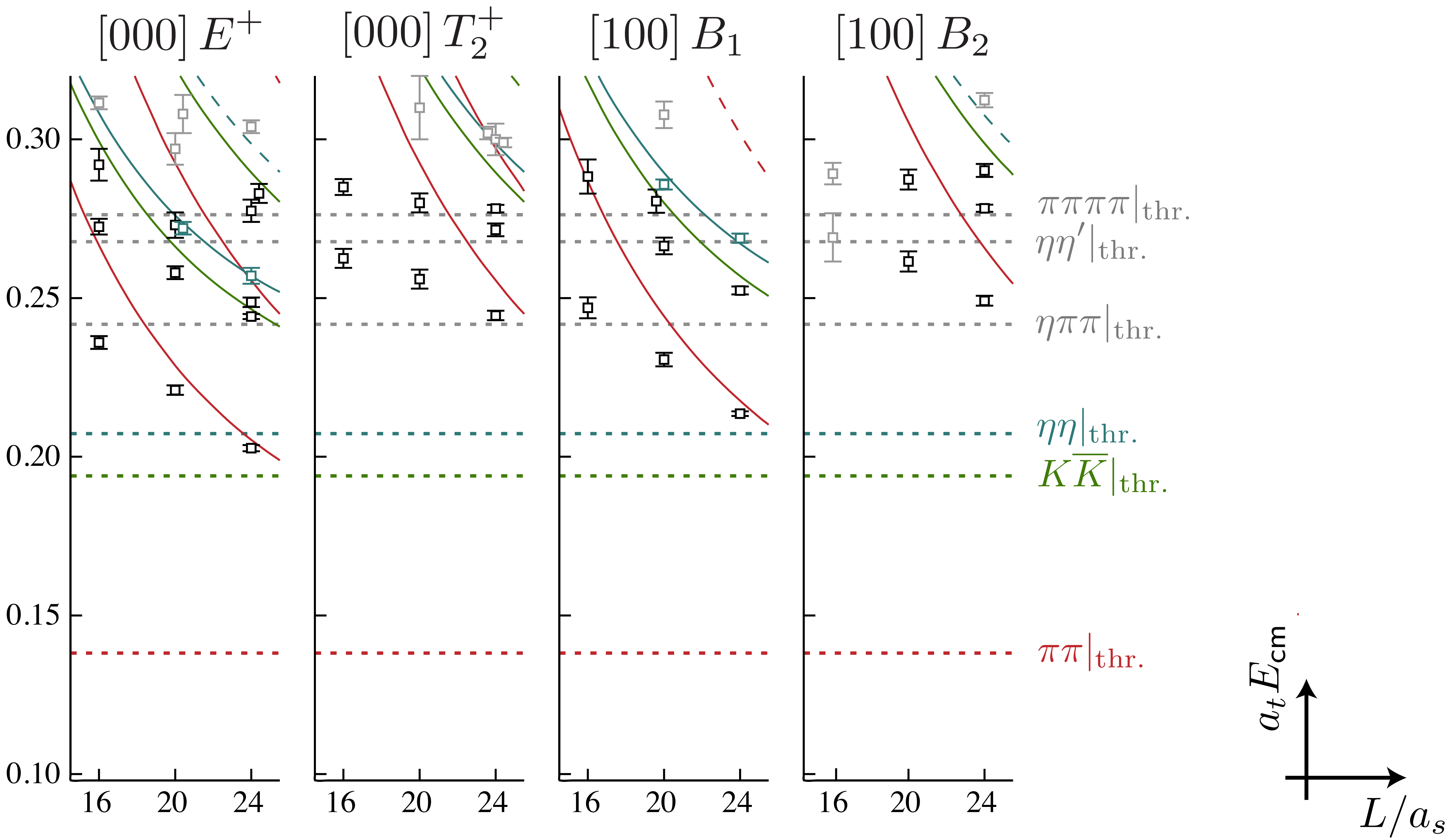
using *distillation* (Peardon *et al* 2009)
many wick contractions, eg just pi-pi & qq operators:



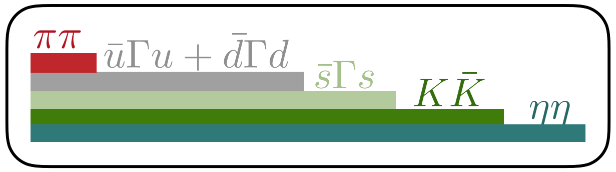
- we compute a large correlation matrix
- then use GEVP to extract energies



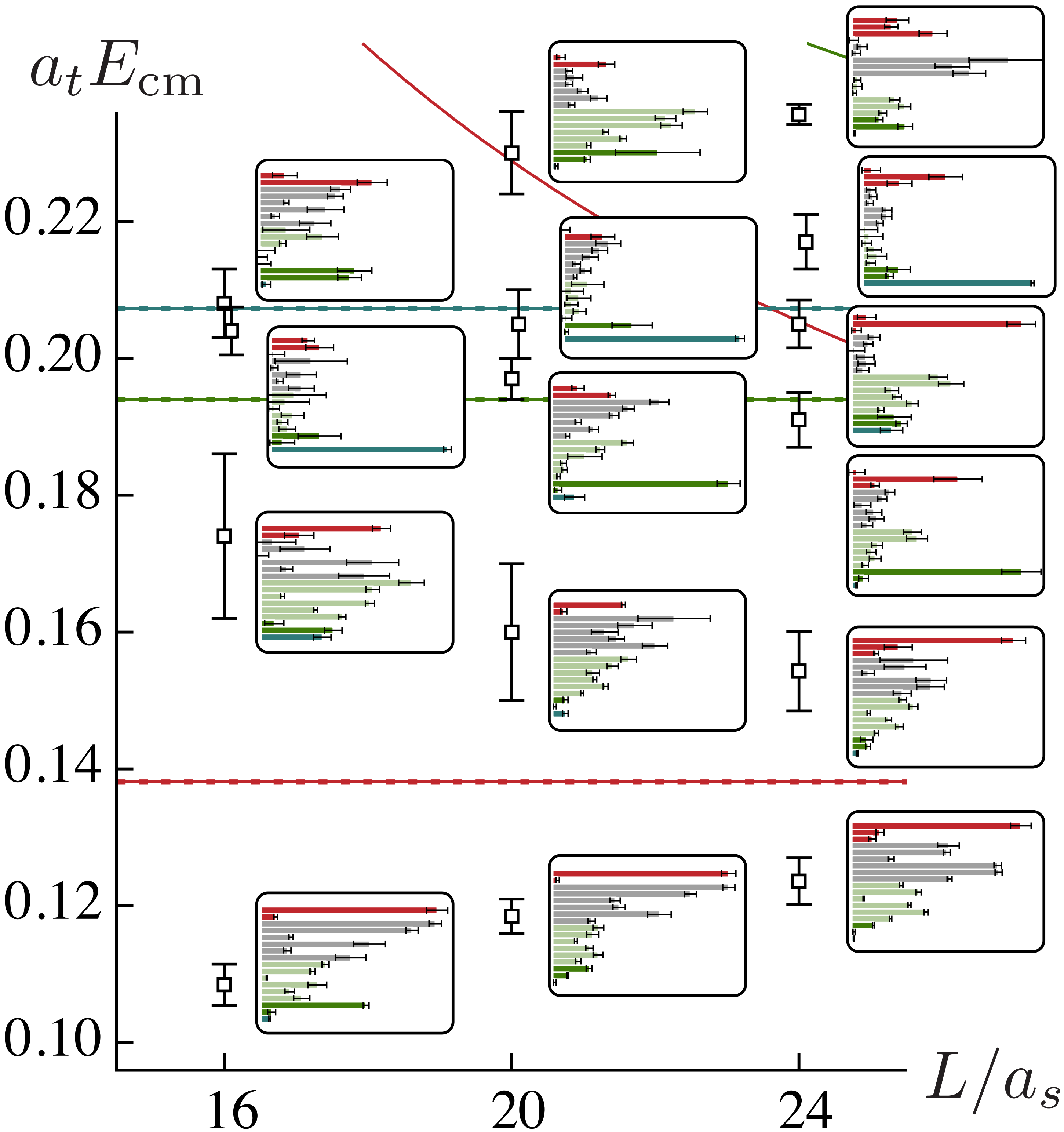
~local $q\bar{q}$ & 2-hadron operators
conservatively **57 energy levels**
dominated by S-wave interactions



conservatively **34 energy levels**
dominated by D-wave interactions



[000]A_1^+

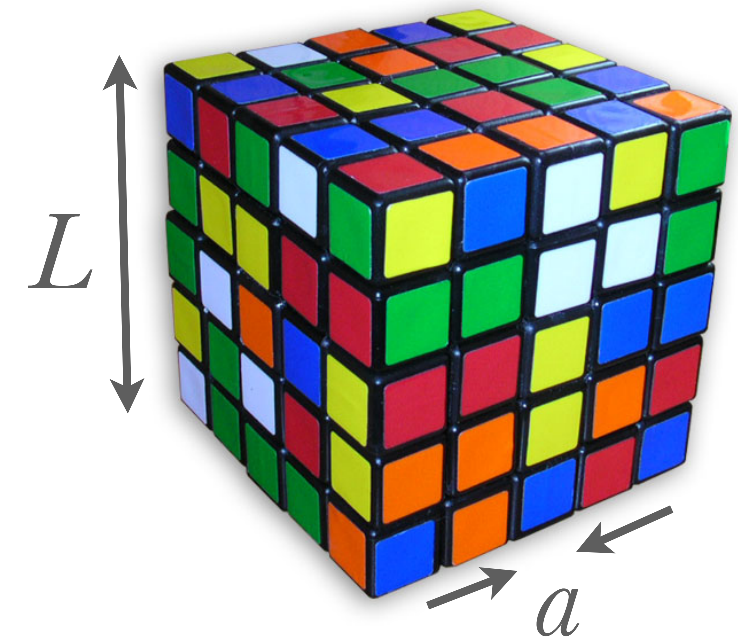


operator overlaps give some intuition

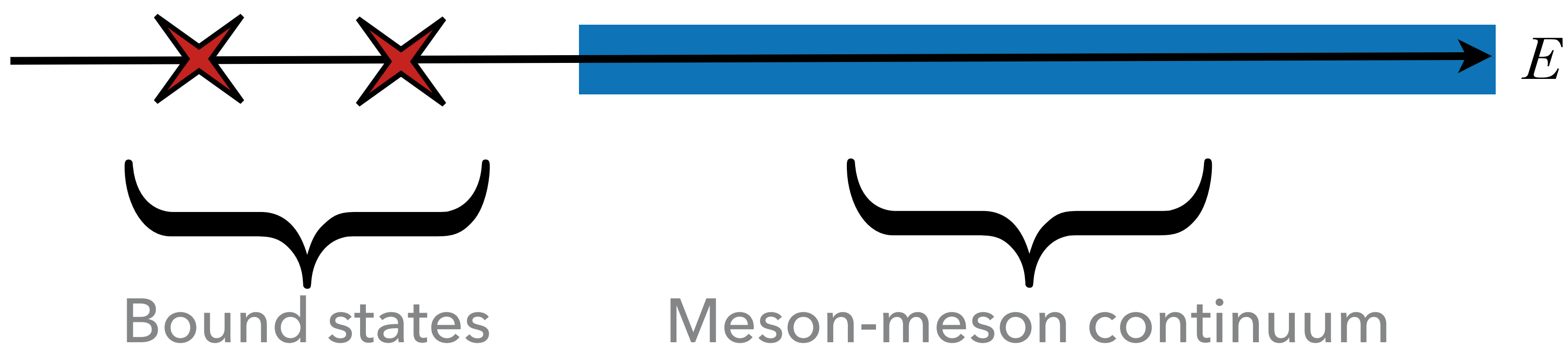
lots of mixing in the scalar sector

- essential to have meson-meson ops even below threshold
- can't always 'read-off' resonance content

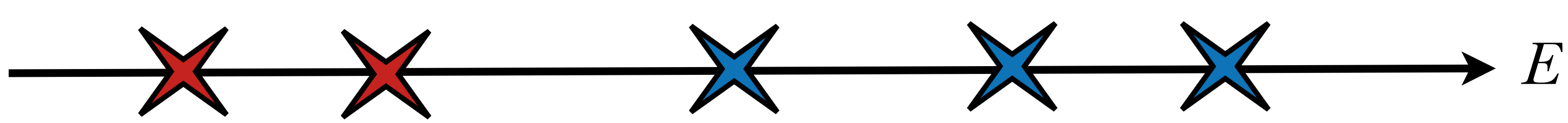
recent review by Briceno, Dudek, Young: arXiv:1706.06223



Infinite volume



Finite volume

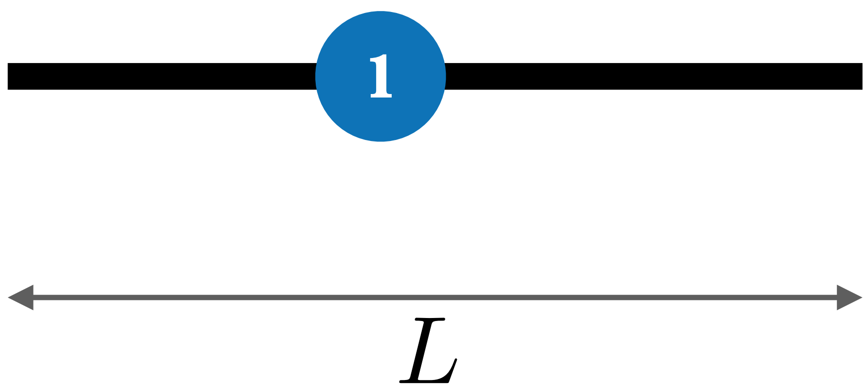


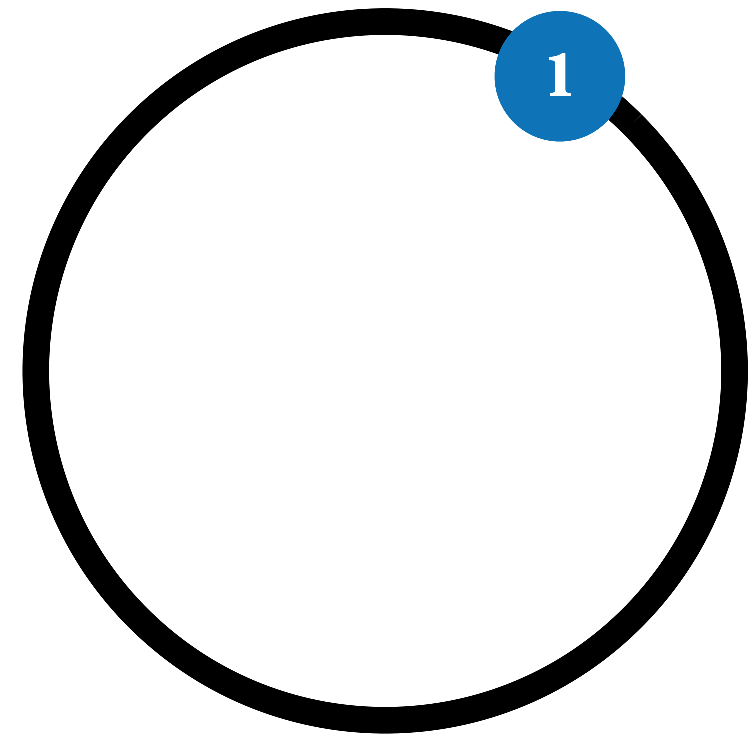
Momentum is quantised - no continuum

$$\vec{p} = \frac{2\pi}{L}\vec{n}$$

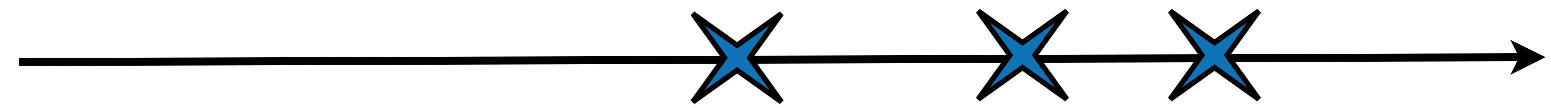
Lüscher et al

1-dimensional QM, periodic BC, single particle:

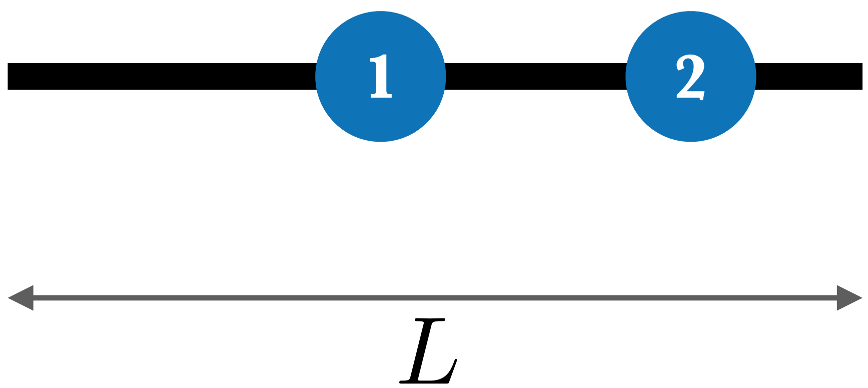


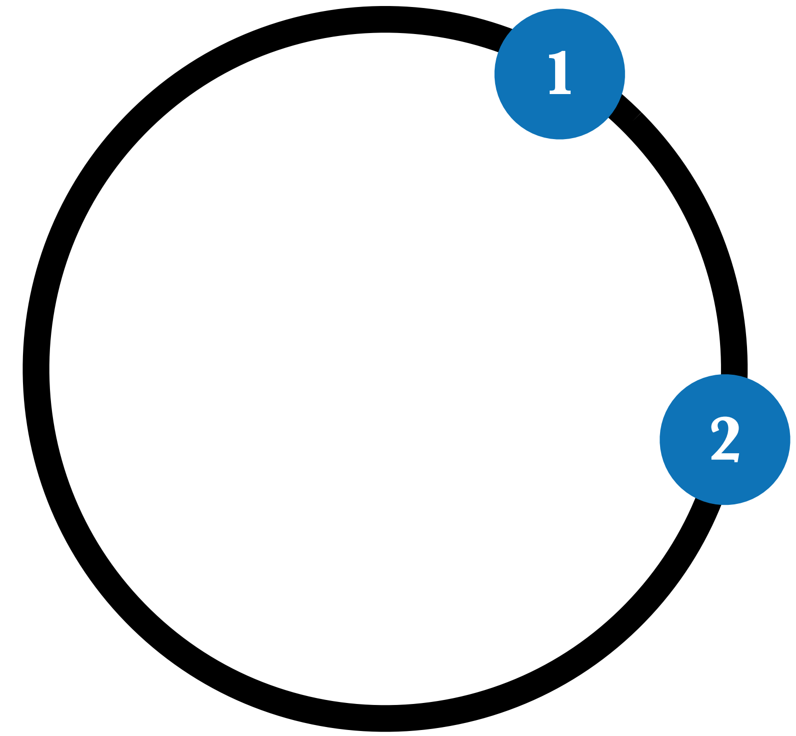


momentum is quantised: $p = \frac{2\pi n}{L}$



1-dimensional QM, periodic BC, two particles, no interactions

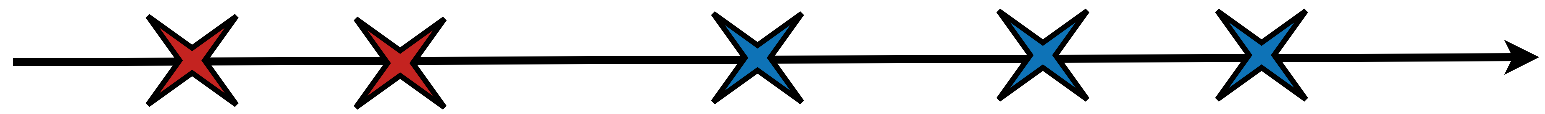




momentum is quantised: $p_i = \frac{2\pi n_i}{L}$

two particle energies are discrete:

$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$

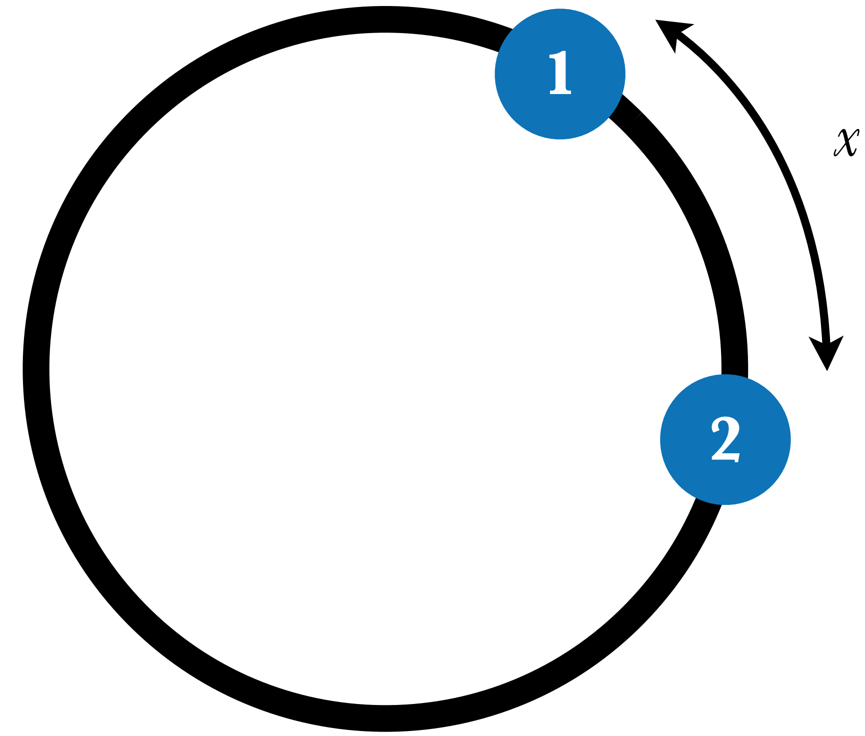


1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \frac{\partial\psi}{\partial x}\Big|_{x=0} = \frac{\partial\psi}{\partial x}\Big|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

Extracting the t-matrix

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space → $\boldsymbol{\rho}$

infinite volume scattering *t*-matrix → $\boldsymbol{t}$

known finite-volume functions → $\boldsymbol{\mathcal{M}}$

Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

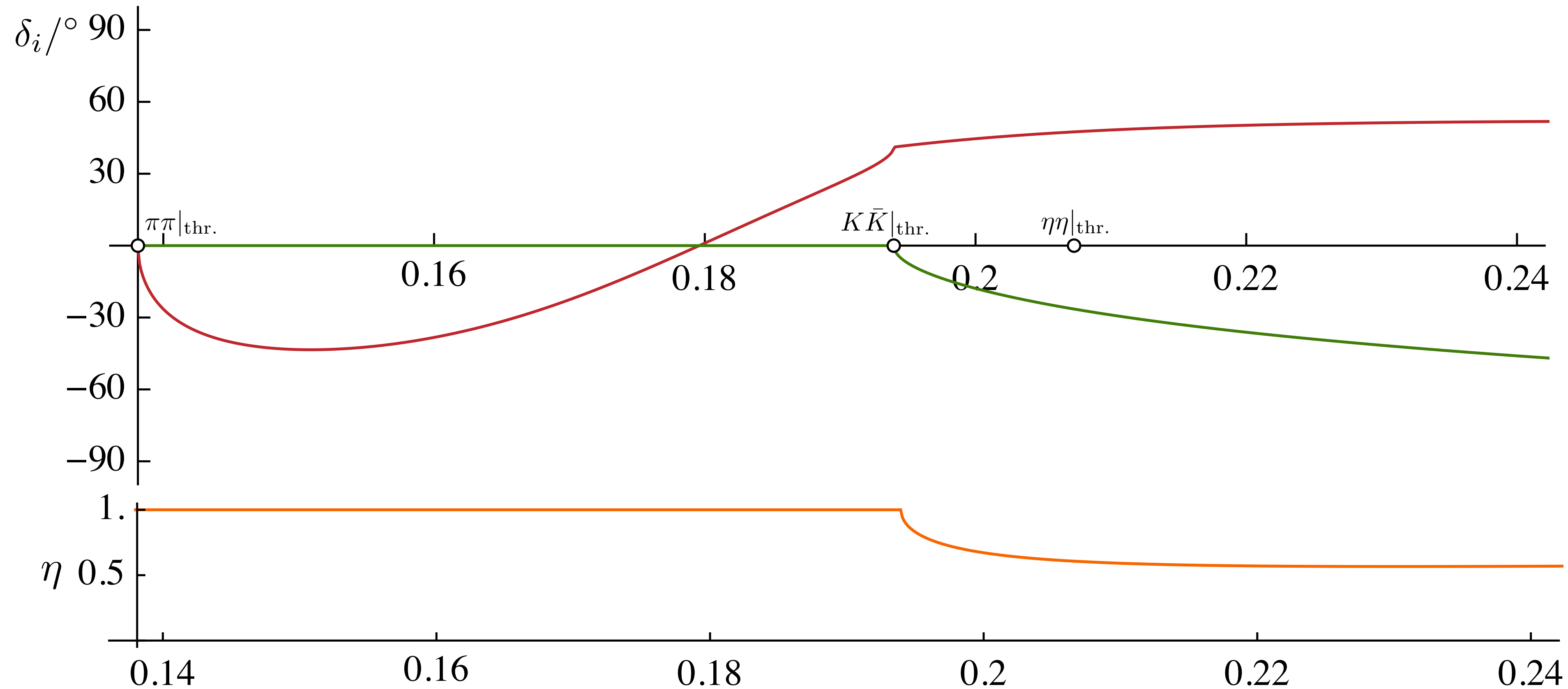
Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

Significant steps towards a general 3-body quantization condition are being made

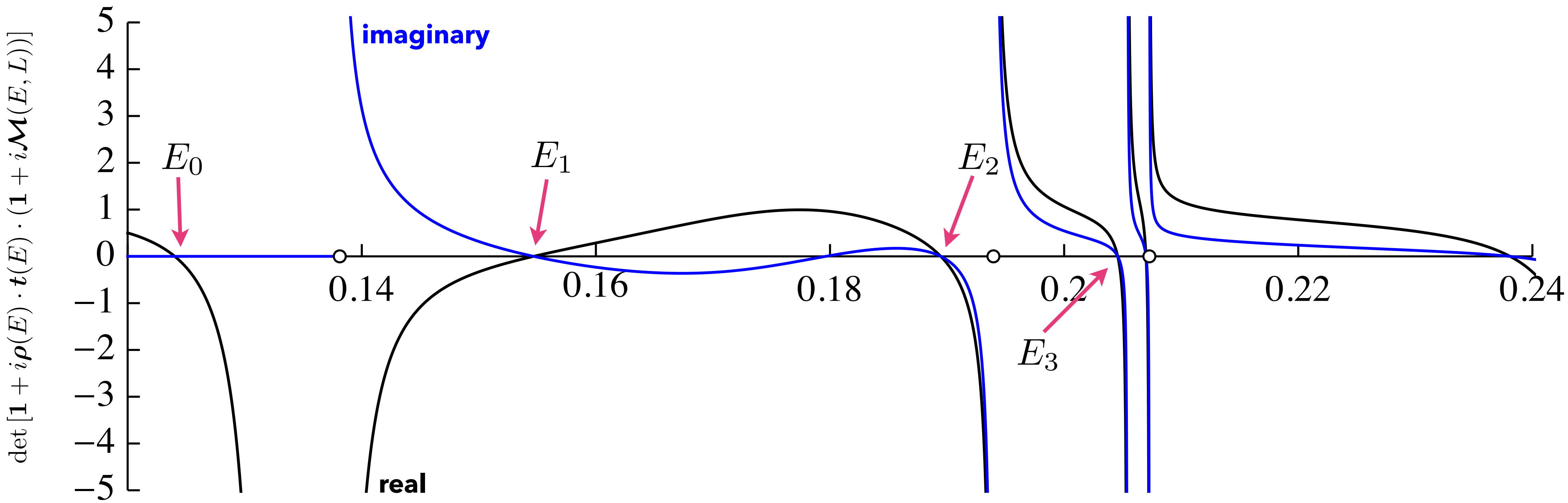
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

$$S_{ii} = \eta e^{2i\delta_i}$$



identify the solutions



$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

determinant condition:
- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations
- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad\qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

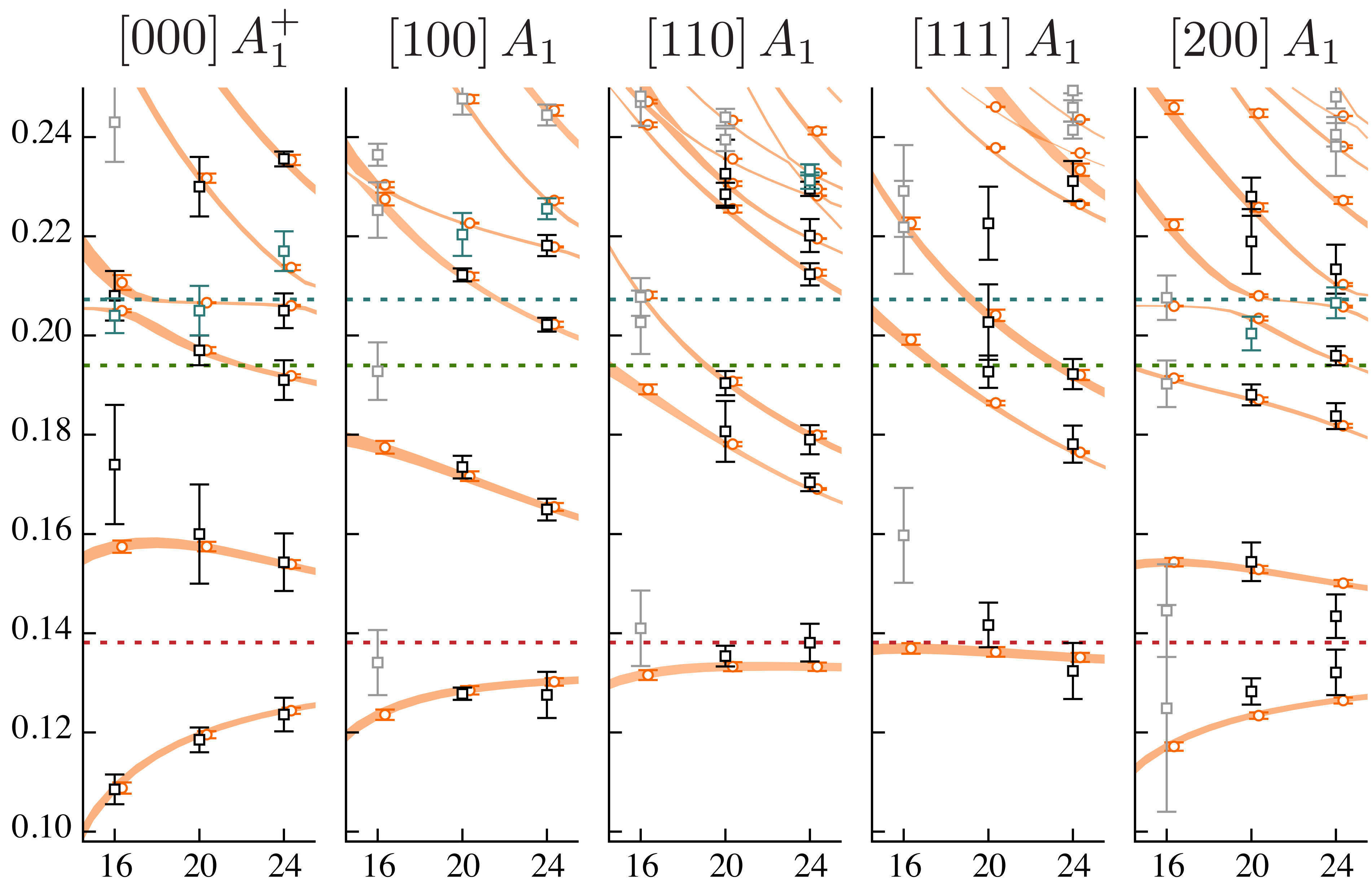
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

use a dispersion relation to generate a real part from iρ

- any form real for real energies is valid
- we use a broad selection of K-matrices
- neglects left-hand cut

An example S-wave spectrum fit

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$



$$\chi^2/N_{\rm dof} = \frac{44.0}{57-8} = 0.90$$

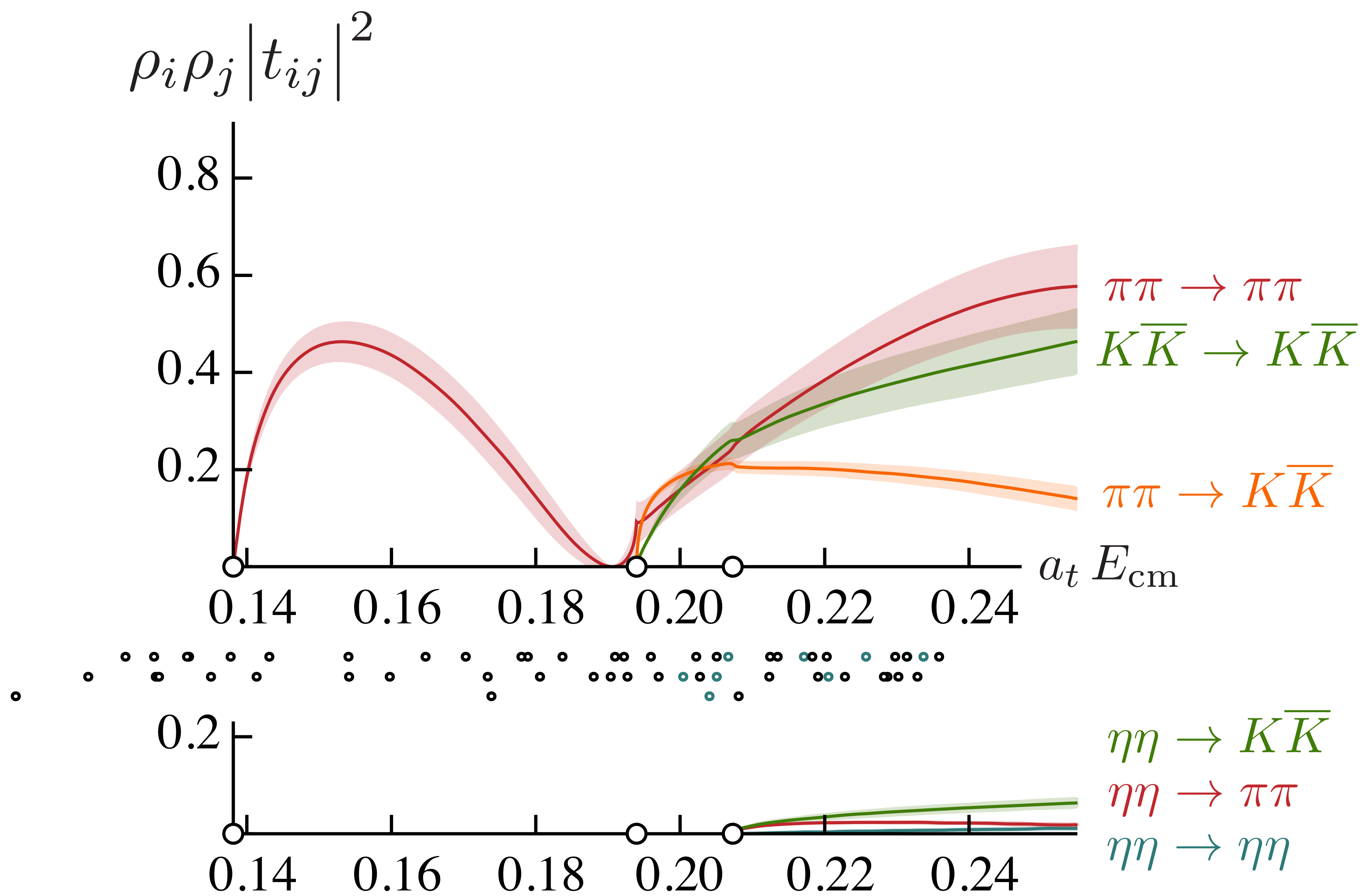
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels



An example S-wave spectrum fit

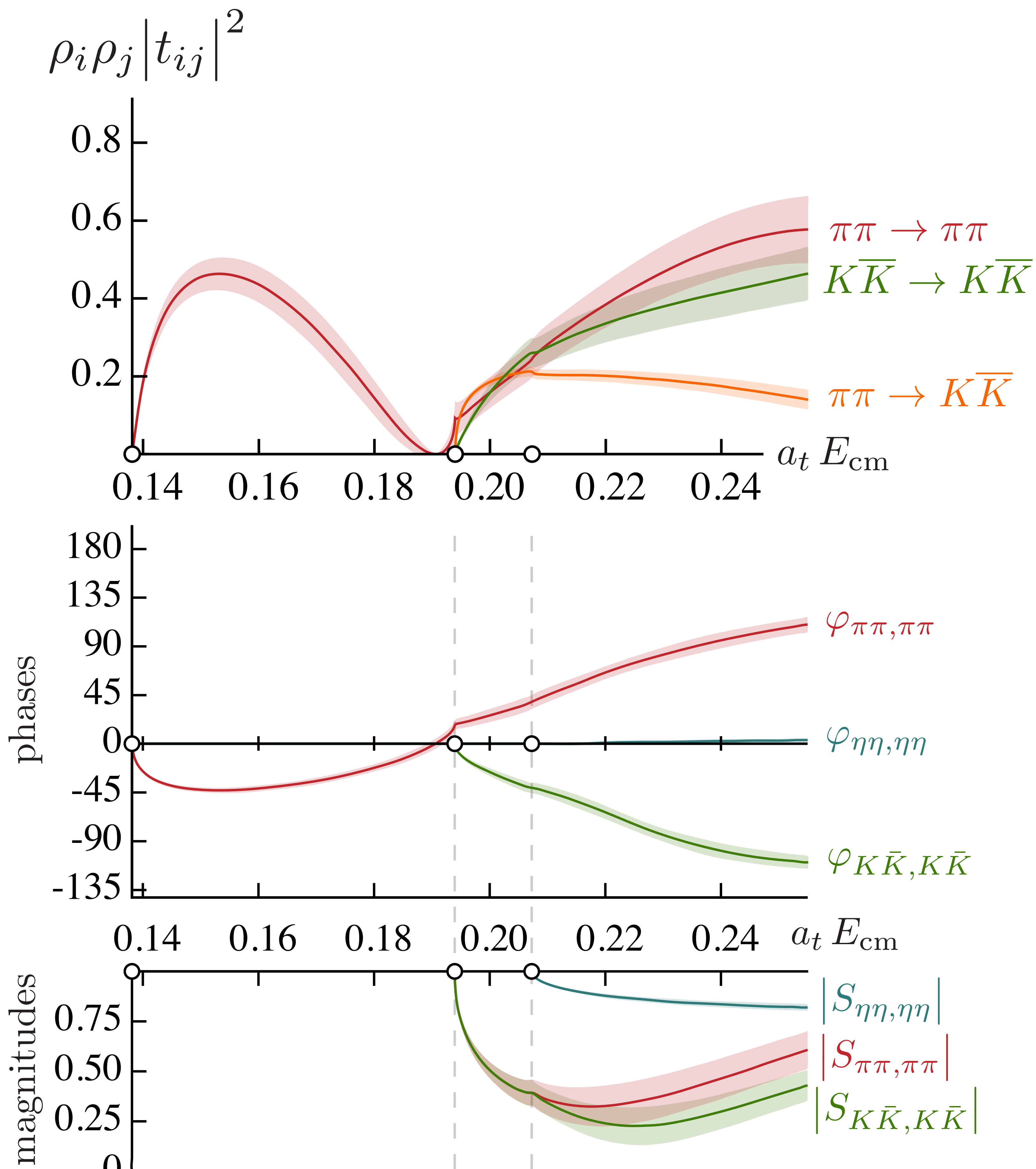
$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

57 energy levels

$$S_{ii}(E_{\mathsf{cm}}) = |S_{ii}(E_{\mathsf{cm}})|\, e^{2i\phi_{ii}(E_{\mathsf{cm}})}$$



An example S-wave spectrum fit

$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

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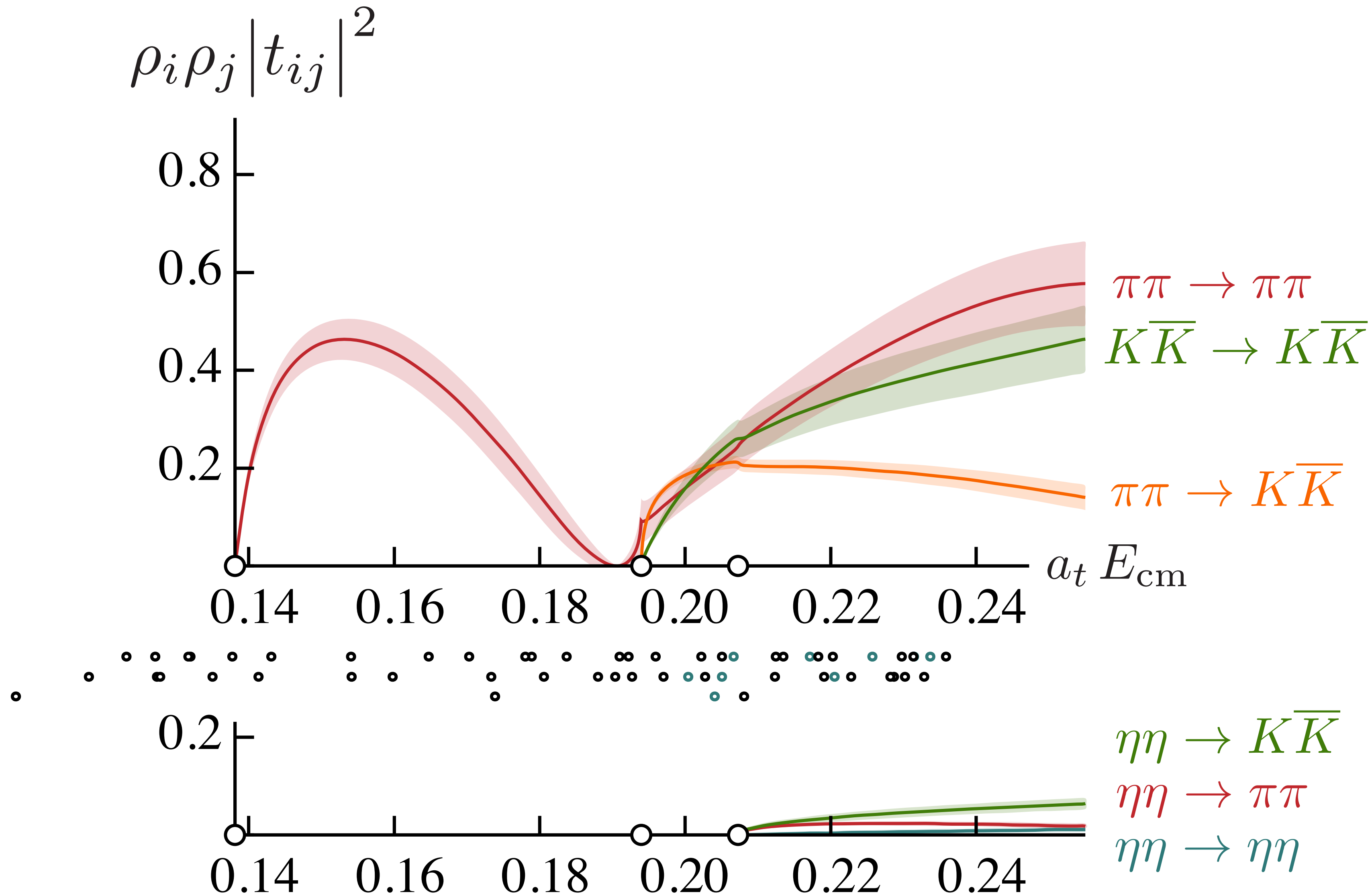
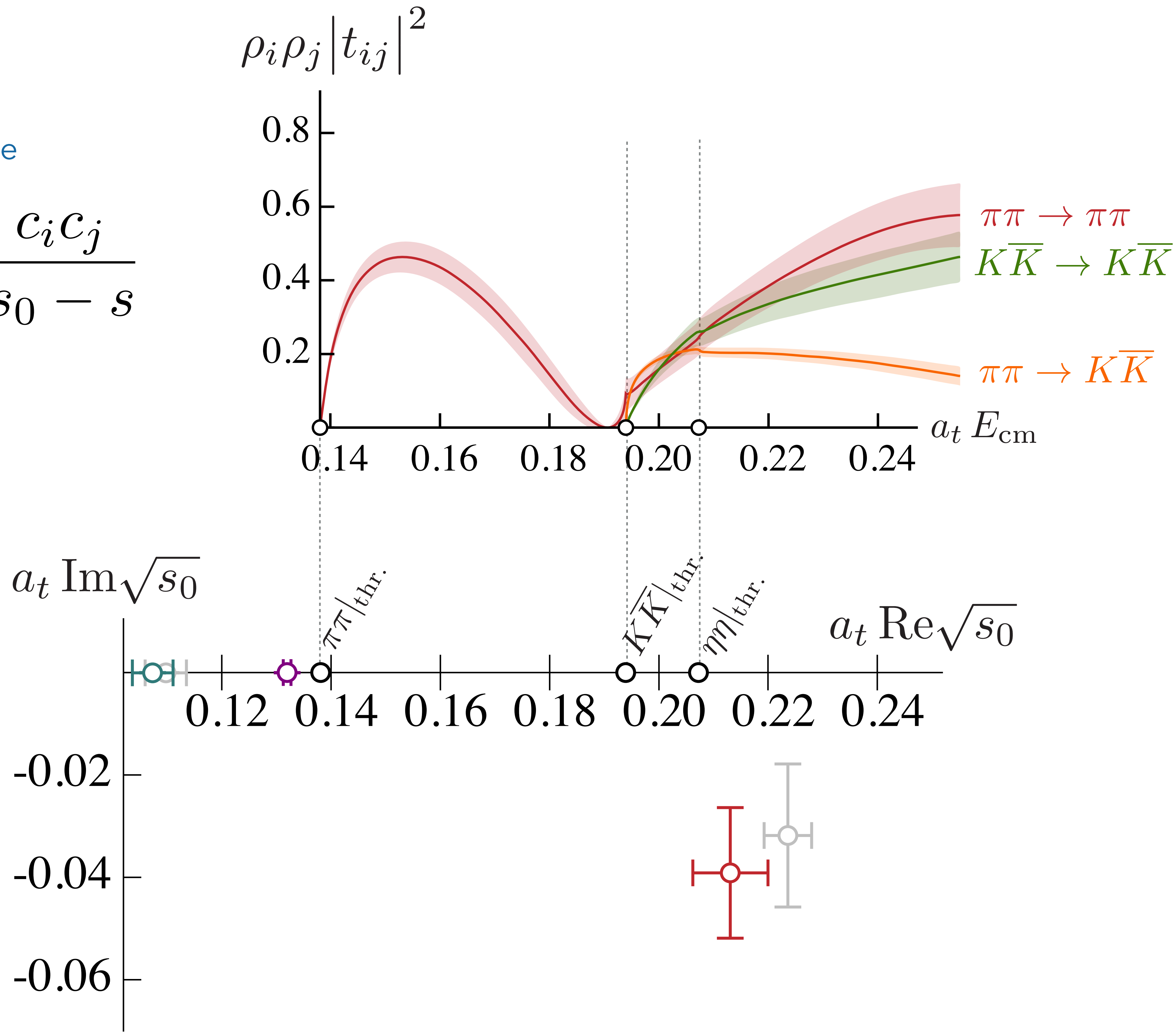
57 energy levels



Near a t-matrix pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$



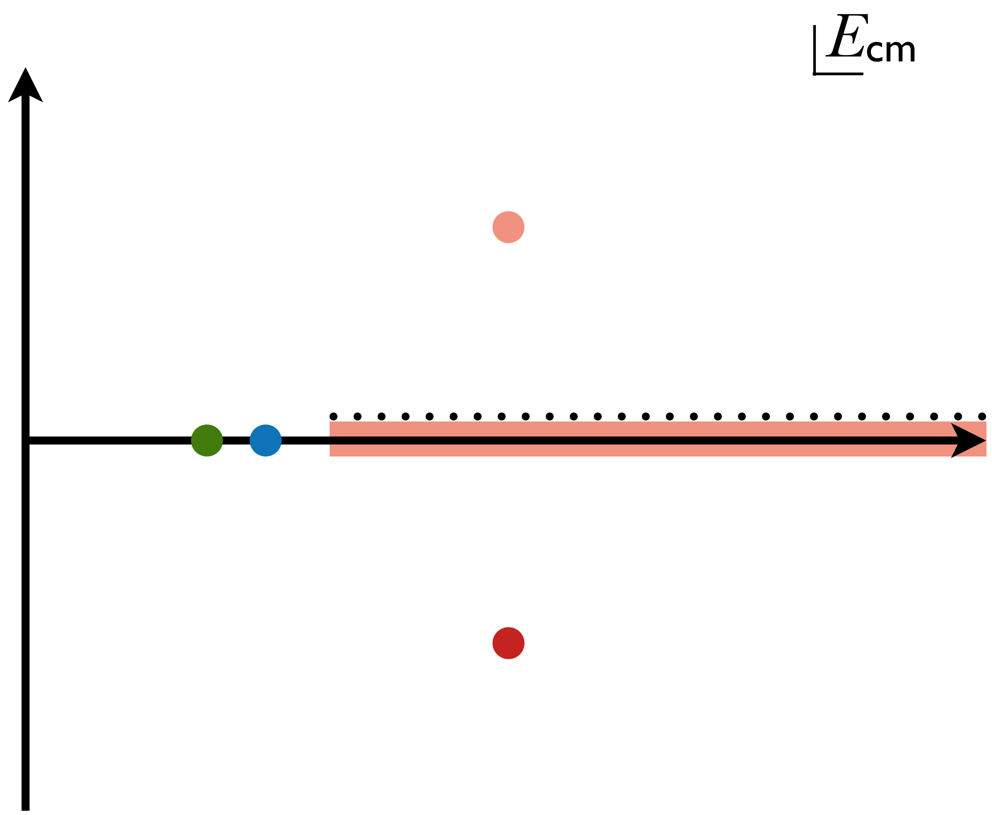
sheet I: bound state

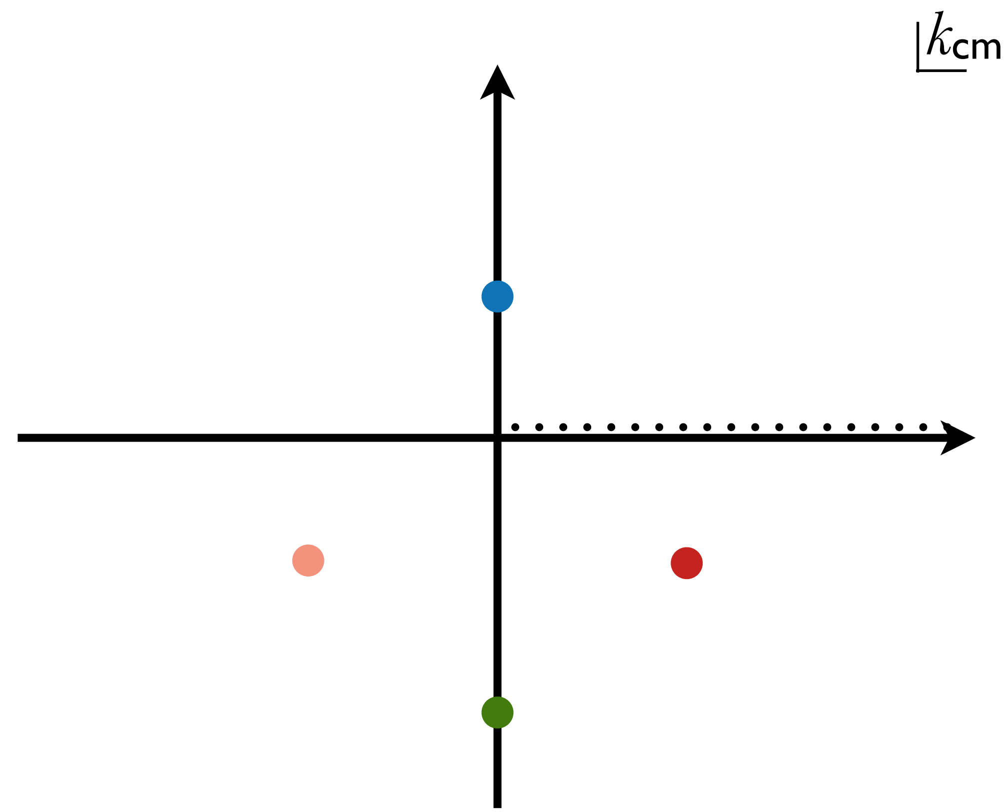
sheet II: resonance

Multi-sheeted complex plane due to square-root branch cuts at each threshold, in single channel case for now:

$$k_{\text{cm}} = \pm\frac{1}{2}\left(E_{\text{cm}}^2 - 4m^2\right)^{\frac{1}{2}}$$

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$





Bound state

Resonance

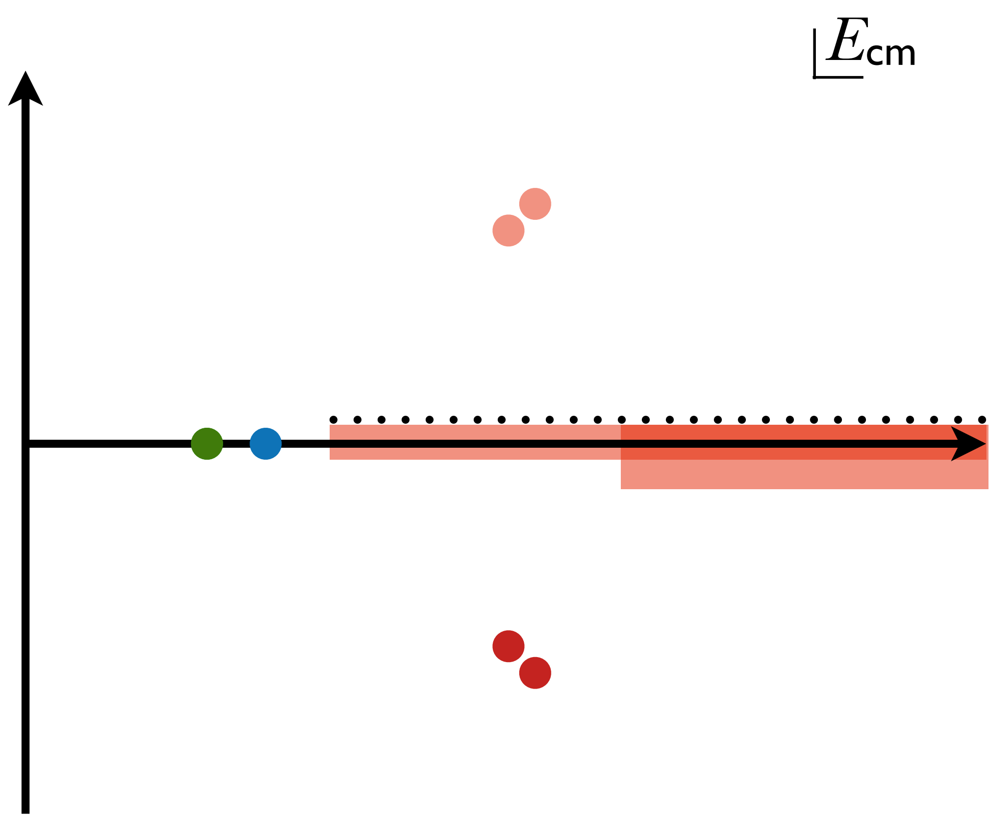
Virtual Bound state

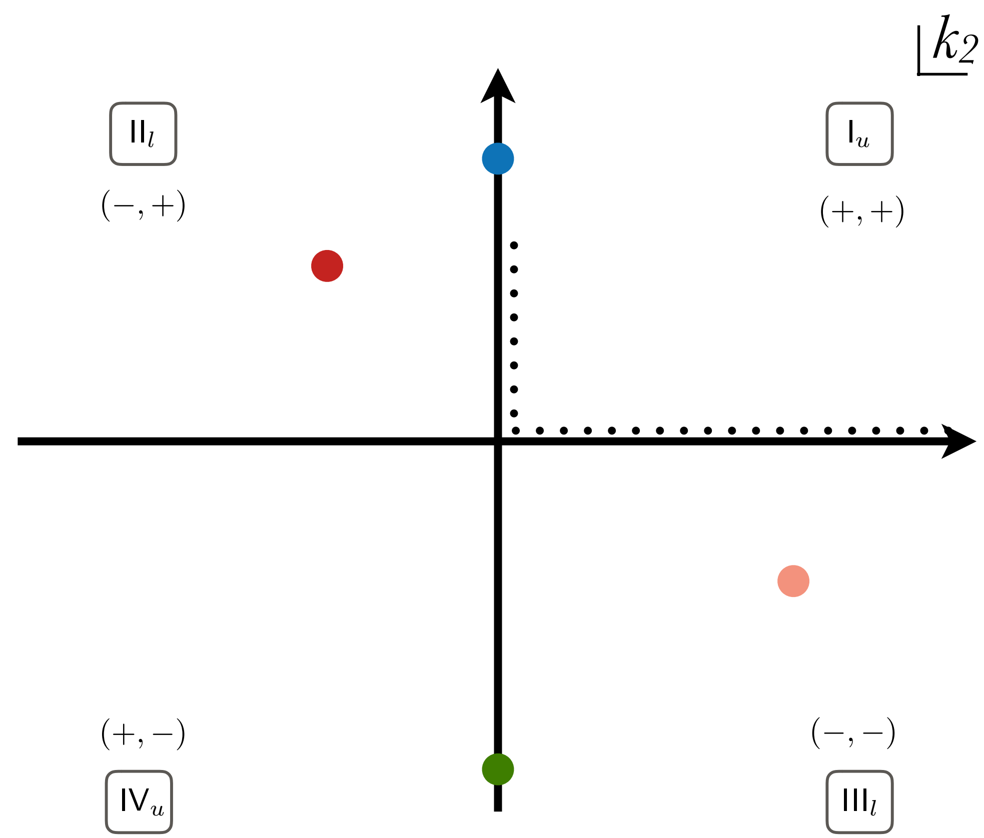
Poles in amplitudes

for n-channels, there are 2^n sheets

$$k_{\mathrm{cm}} = \pm\frac{1}{2}\left(E_{\mathrm{cm}}^2 - 4m^2\right)^{\frac{1}{2}}$$

$$t_{ij}(s \sim s_0) \sim \frac{c_i c_j}{s_0 - s}$$





label sheets by signs of Im(k)

many distributions of pole positions possible

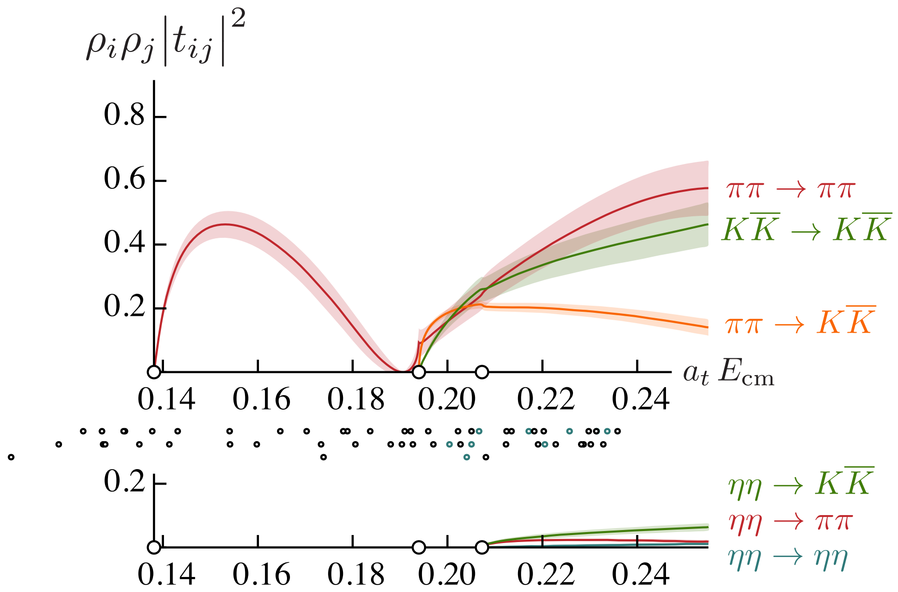
in some cases they can tell us about the composition the state

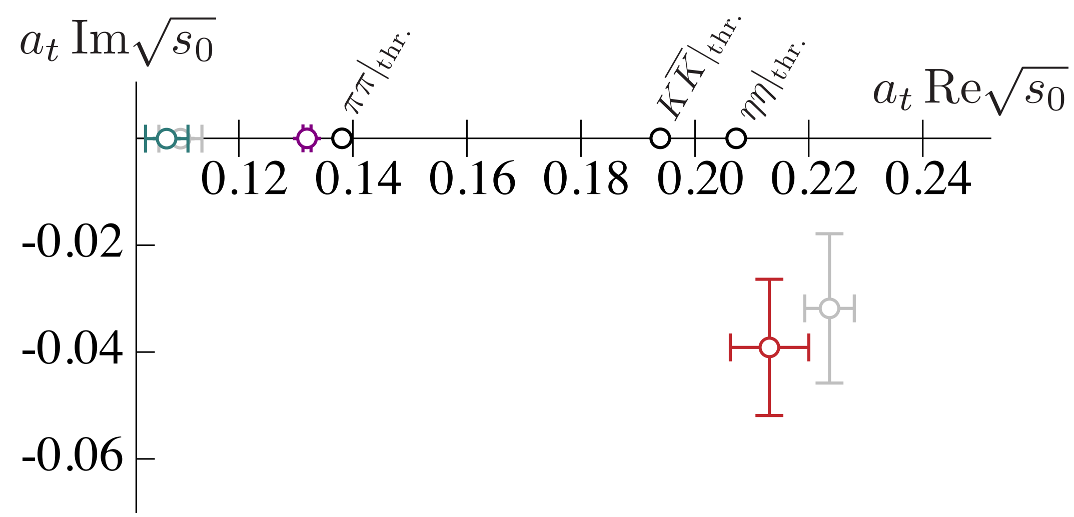
Bound state

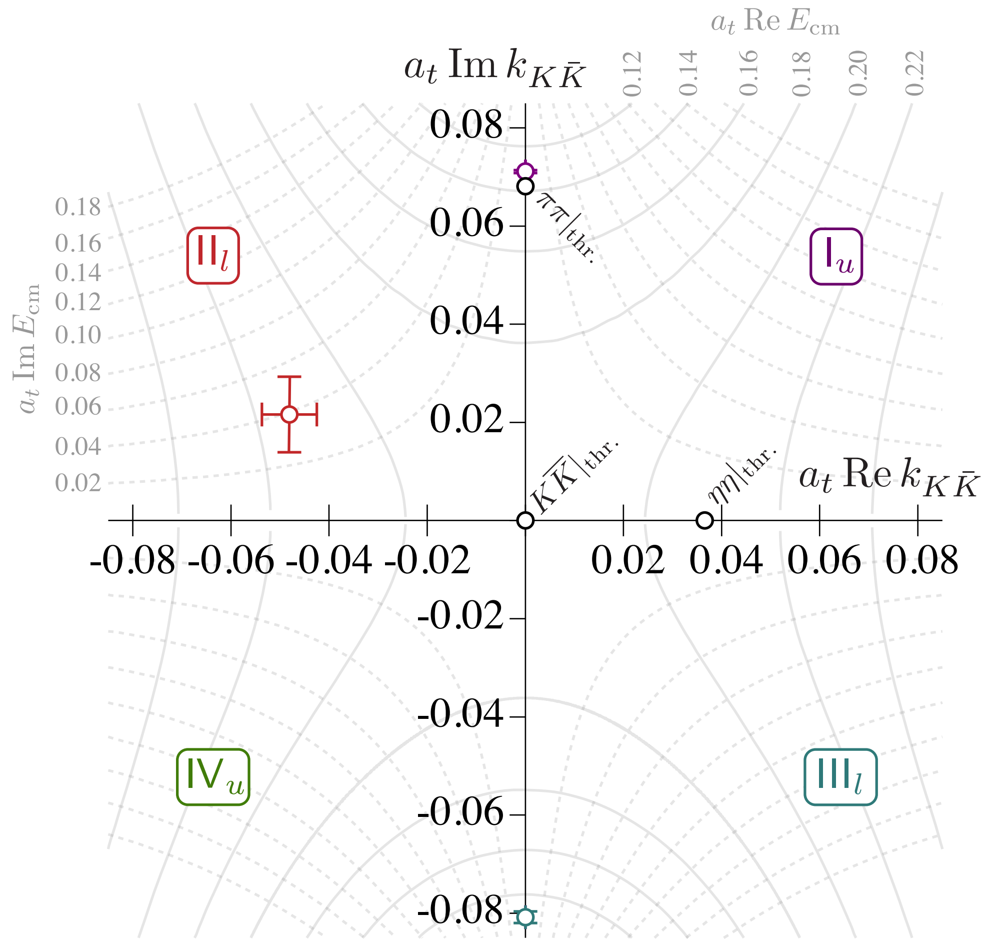
Resonance

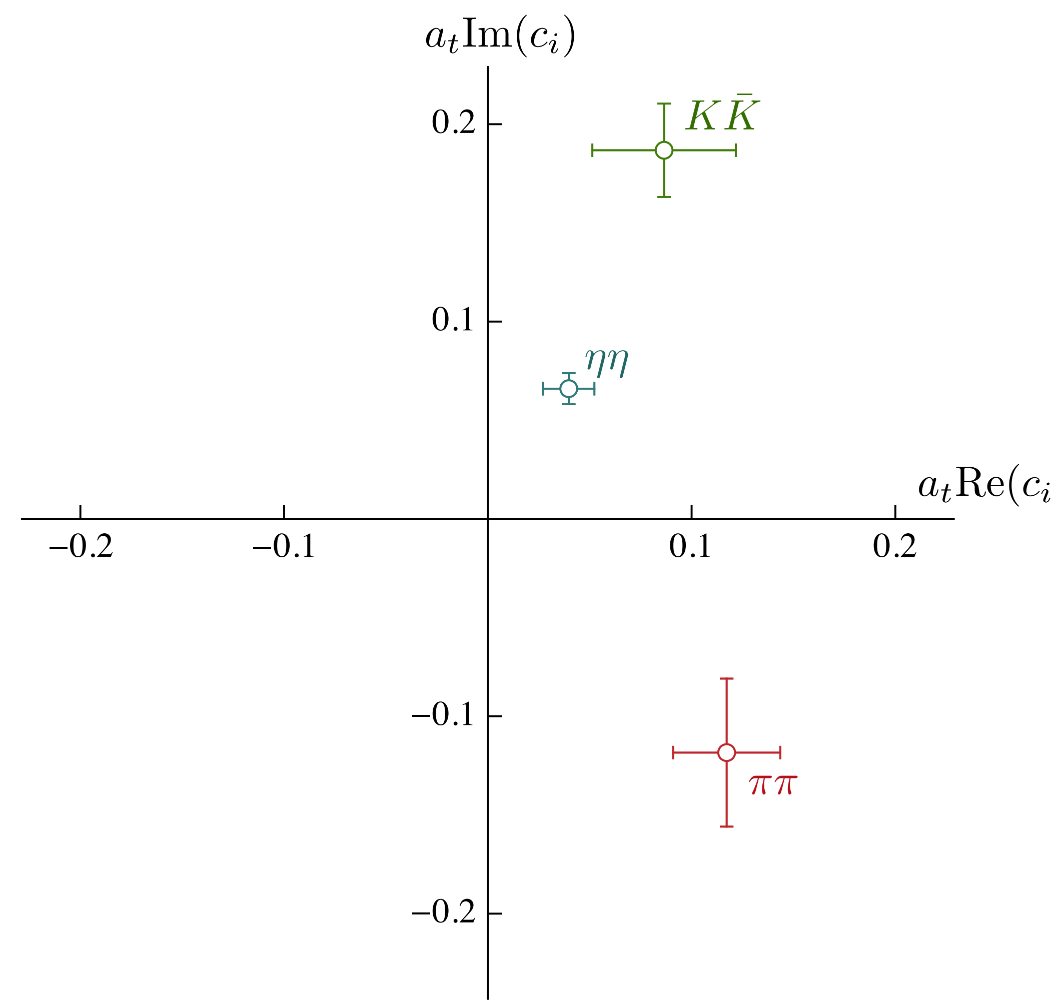
Virtual Bound state

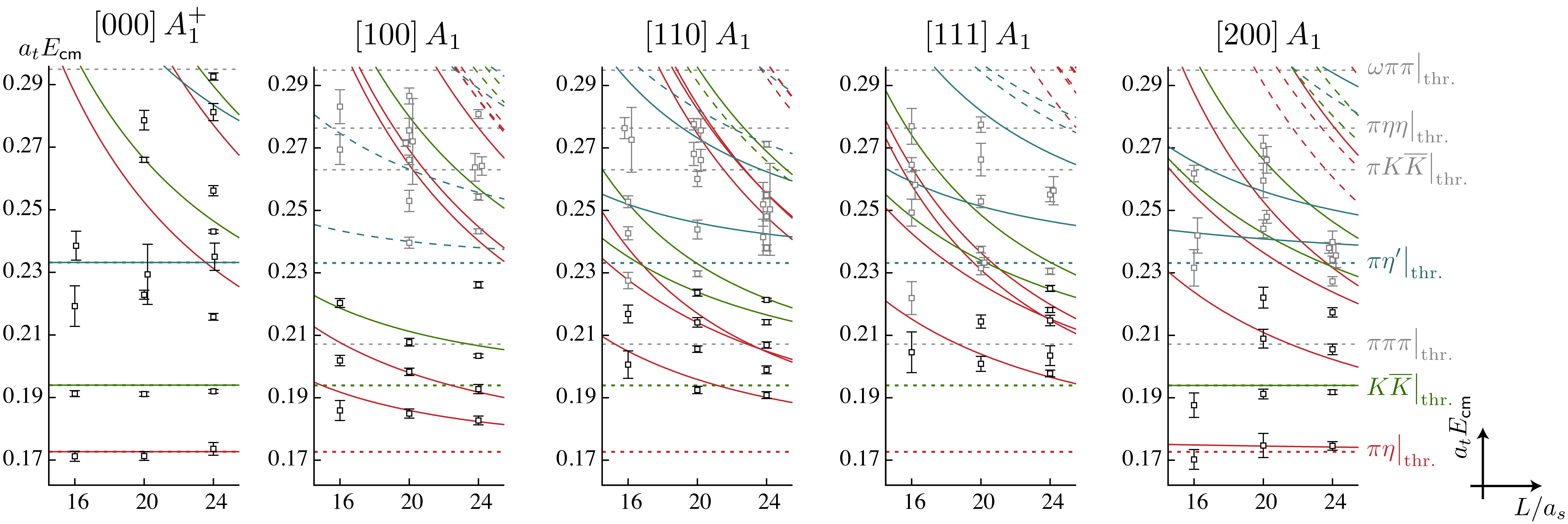
Scalar resonance poles



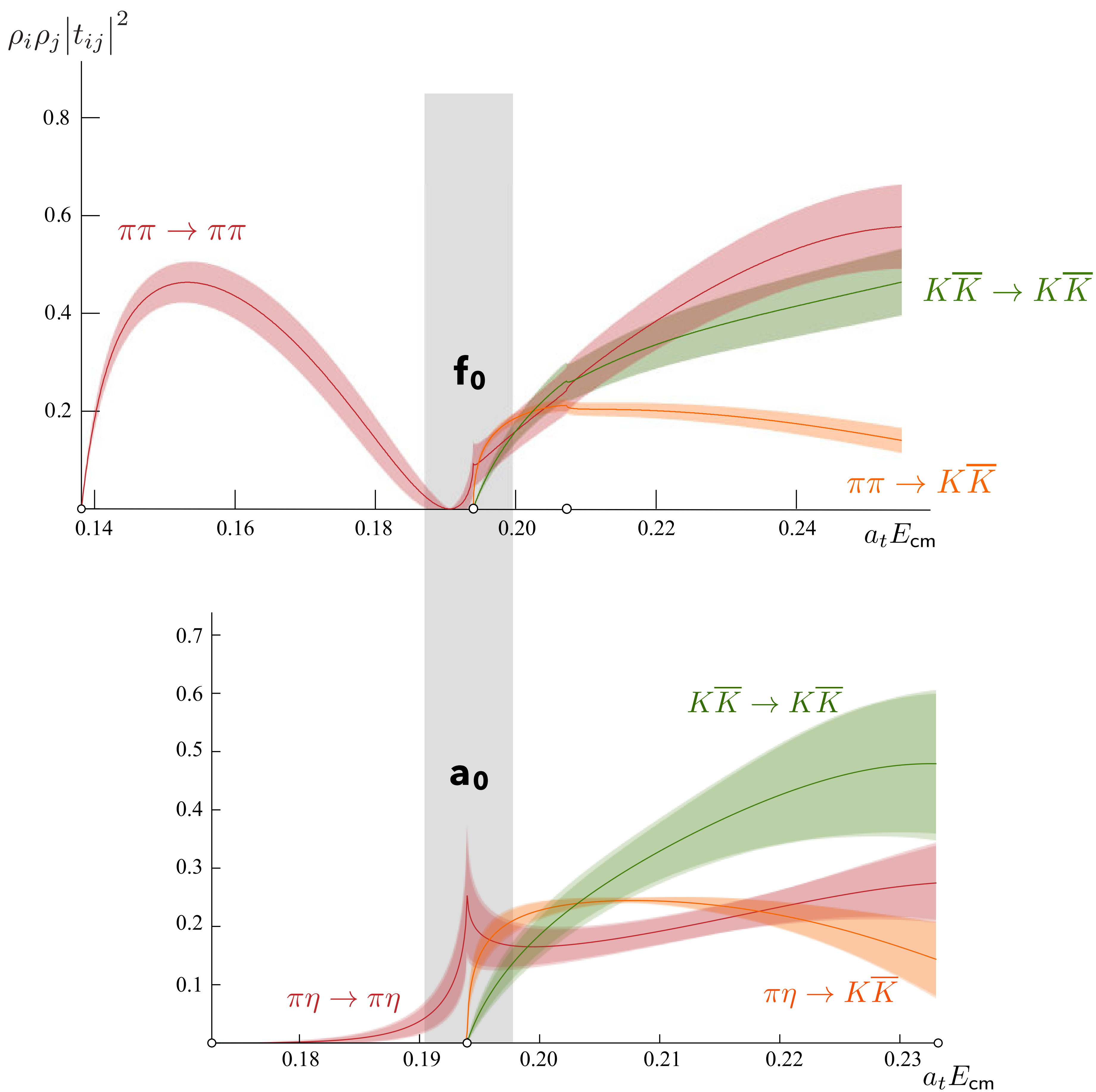


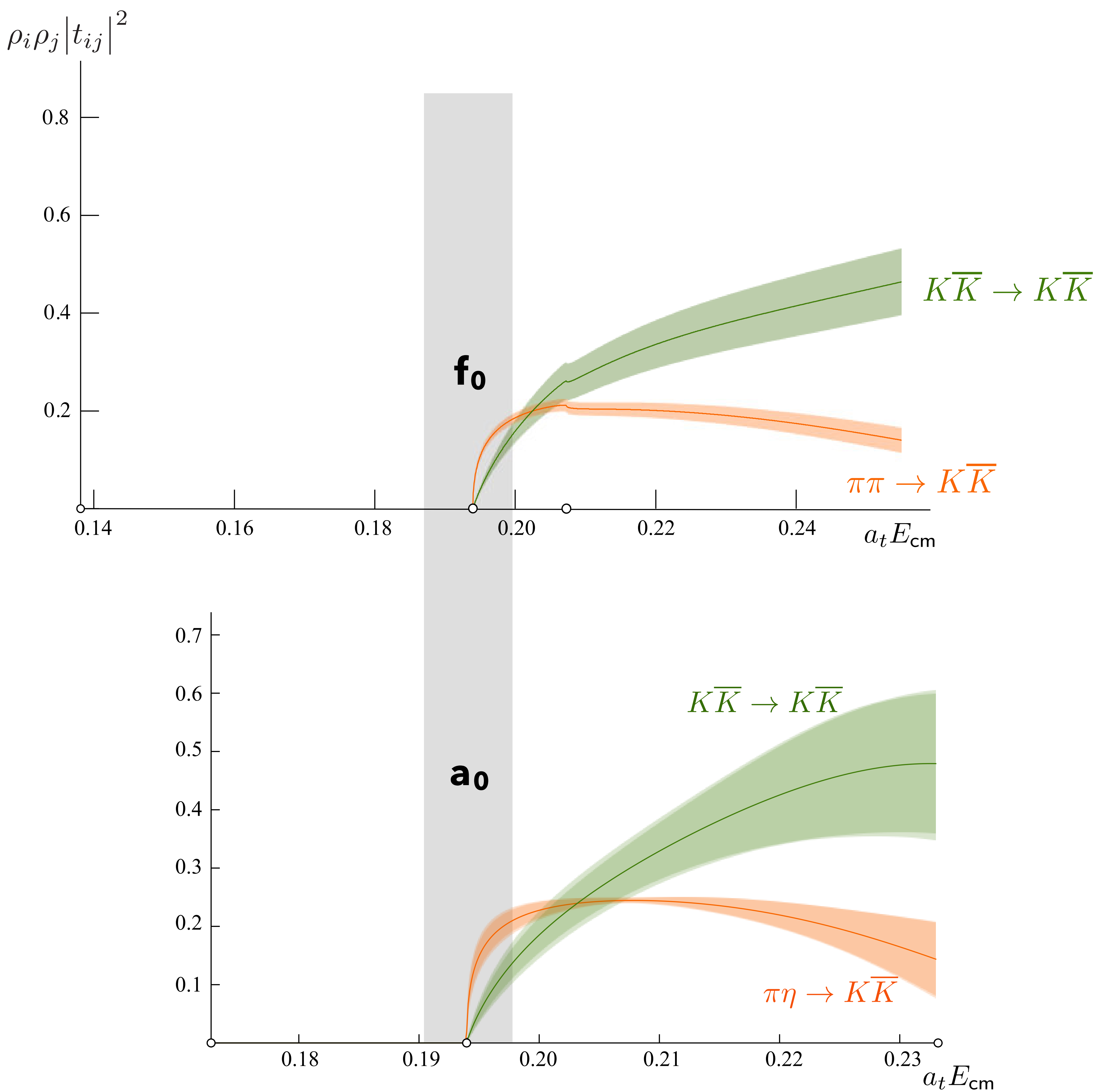


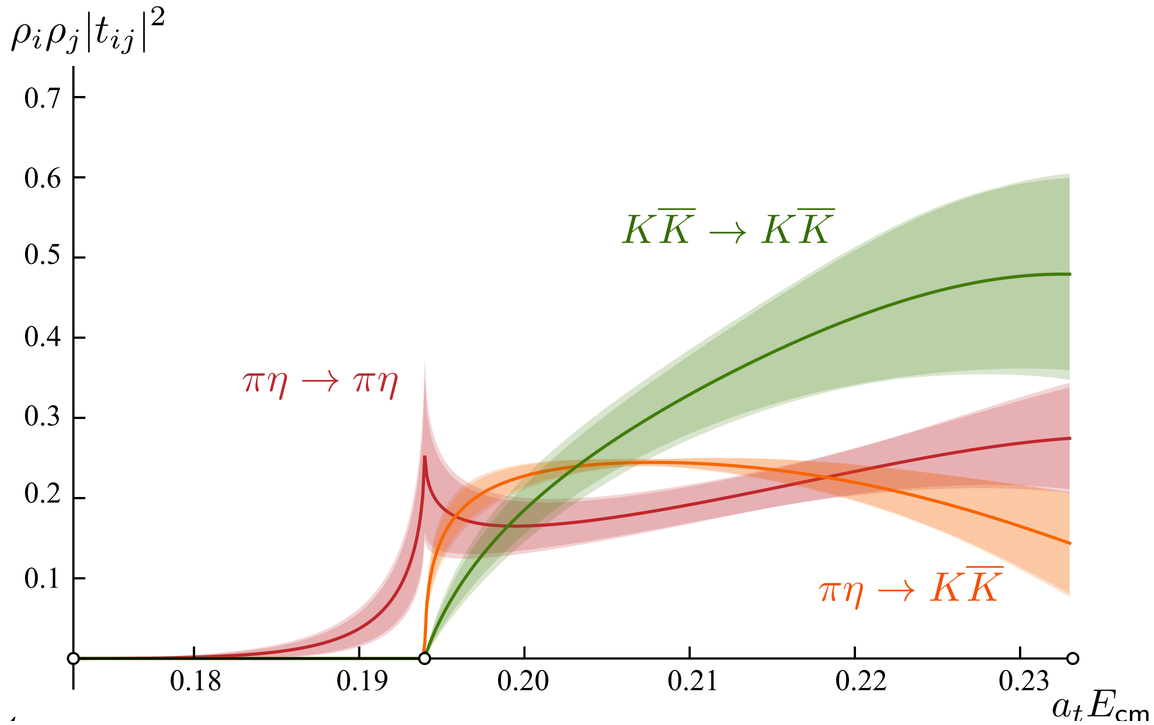


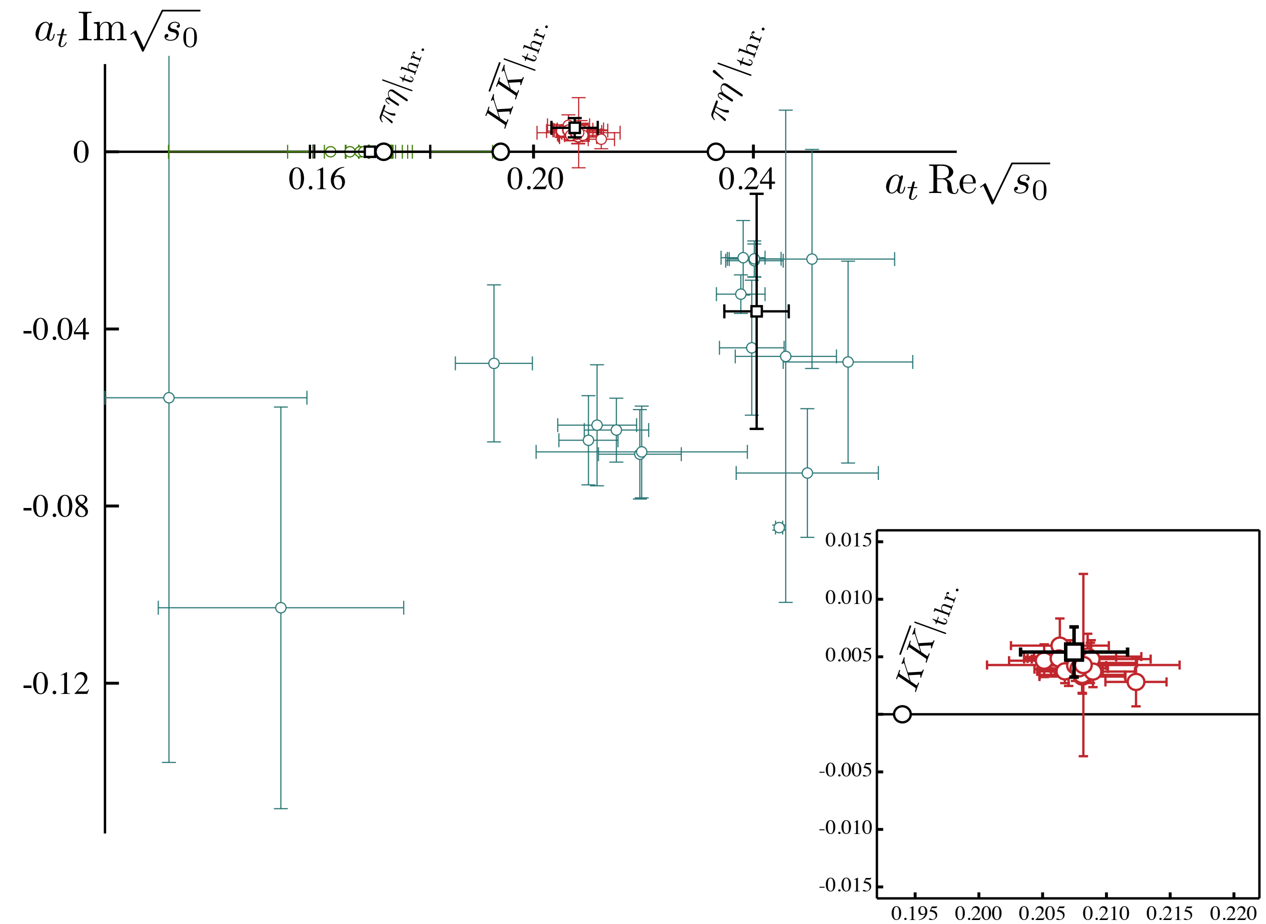

[000] A_1^+
[100] A_1
[110] A_1
[111] A_1
[200] A_1
$a_t E_{\mathsf{cm}}$
0.29
0.27
0.25
0.23
0.21
0.19
0.17
16
20
24
$\omega\pi\pi|_{\text{thr.}}$
$\pi\eta\eta|_{\text{thr.}}$
$\pi K\overline{K}|_{\text{thr.}}$
$\pi\eta'|_{\text{thr.}}$
$\pi\pi\pi|_{\text{thr.}}$
$K\overline{K}|_{\text{thr.}}$
$\pi\eta|_{\text{thr.}}$
L/a_s

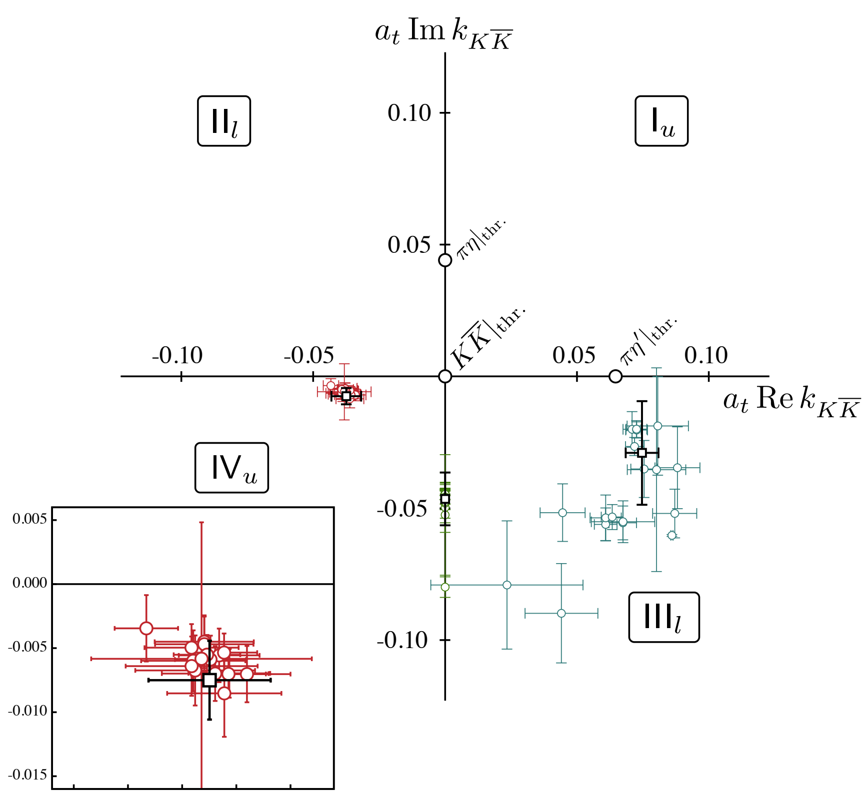
f_0 vs a_0

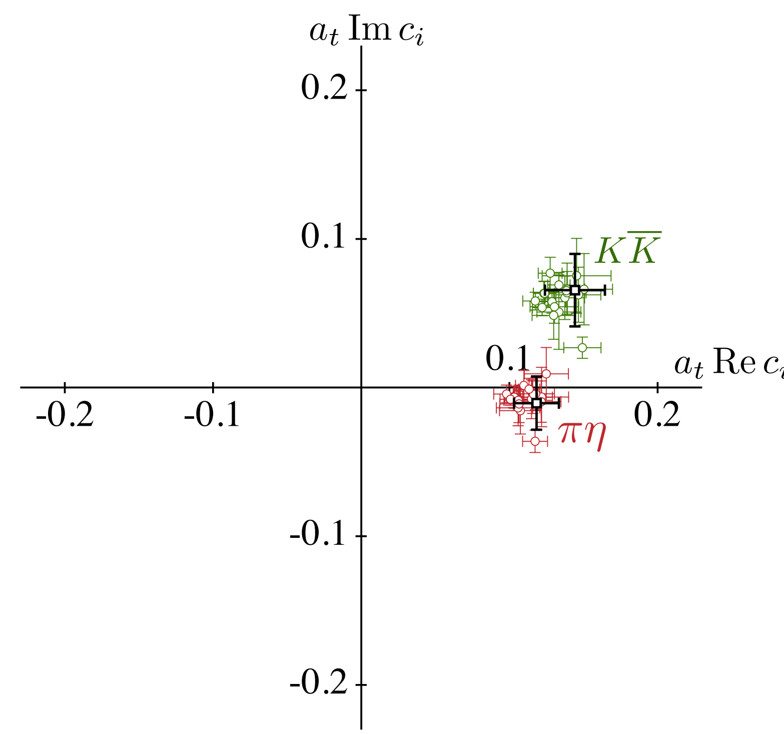

$\rho_i \rho_j |t_{ij}|^2$
0.8
0.6
0.4
0.2
$\pi\pi \to \pi\pi$
f_0
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
0.14
0.16
0.18
0.20
0.22
0.24
$a_t E_{\mathrm{cm}}$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$K\overline{K} \to K\overline{K}$
a_0
$\pi\eta \to \pi\eta$
$\pi\eta \to K\overline{K}$
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{\mathrm{cm}}$


$\rho_i \rho_j |t_{ij}|^2$
0.8
0.6
0.4
0.2
0.14
0.16
0.18
0.20
0.22
0.24
$a_t E_{cm}$
f0
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{cm}$
a0
$K\overline{K} \to K\overline{K}$
$\pi\eta \to K\overline{K}$


$\rho_i \rho_j |t_{ij}|^2$
$K\overline{K} \to K\overline{K}$
$\pi\eta \to \pi\eta$
$\pi\eta \to K\overline{K}$
$a_t E_{\mathsf{cm}}$


$a_t\, \mathrm{Im}\sqrt{s_0}$
$a_t\, \mathrm{Re}\sqrt{s_0}$
$\pi\eta|_{\mathrm{thr.}}$
$K\overline{K}|_{\mathrm{thr.}}$
$\pi\eta'|_{\mathrm{thr.}}$

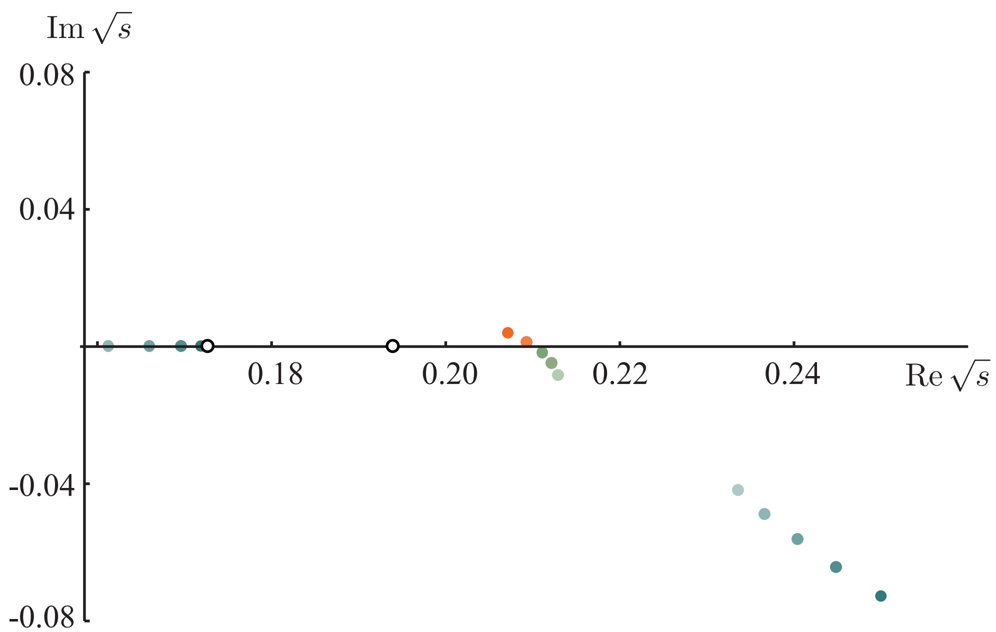

$a_t\, \mathrm{Im}\, k_{K\overline{K}}$
$a_t\, \mathrm{Re}\, k_{K\overline{K}}$
II_l
I_u
III_l
IV_u
$\pi\eta|_{\mathrm{thr.}}$
$K\overline{K}|_{\mathrm{thr.}}$
$\pi\eta'|_{\mathrm{thr.}}$

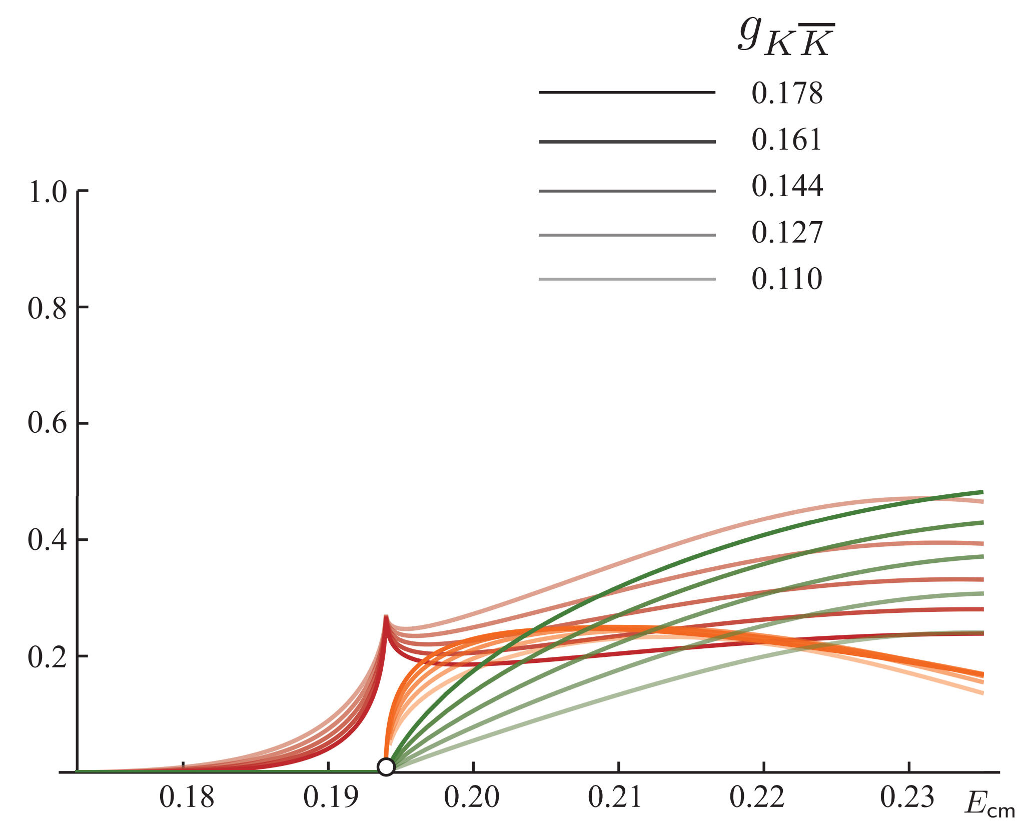

$a_t\, \mathrm{Im}\, c_i$
$a_t\, \mathrm{Re}\, c_i$
$K\overline{K}$
$\pi\eta$

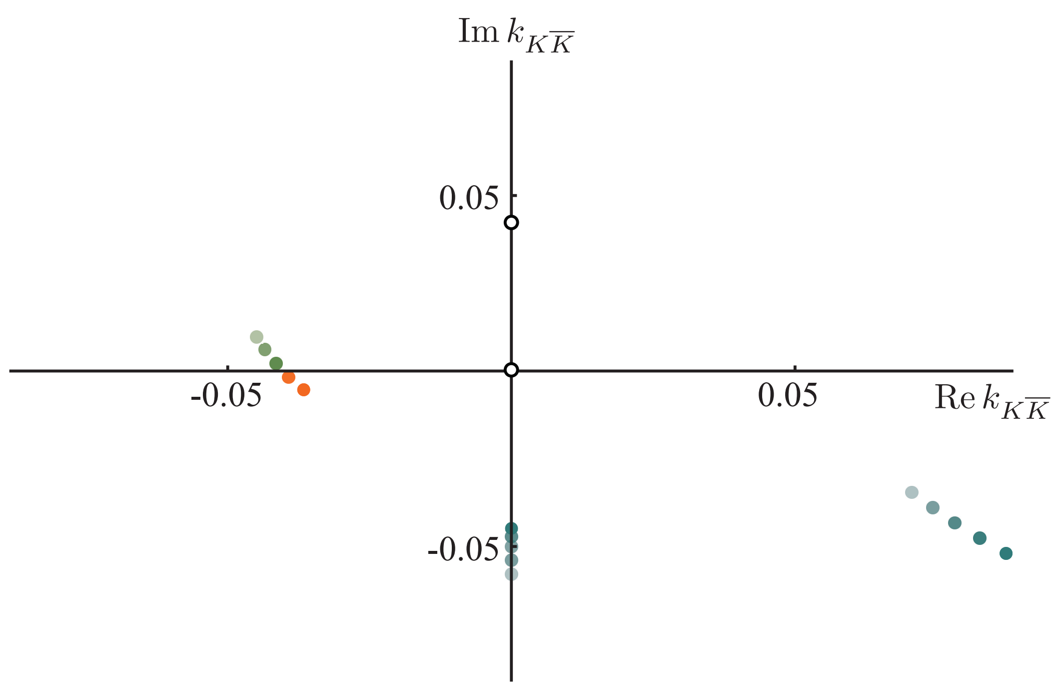
simplest parameterisation that describes the spectra

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

$$\mathbf{K} = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi\eta}^2 & g_{\pi\eta}\, g_{K\bar{K}} \\ g_{\pi\eta}\, g_{K\bar{K}} & g_{K\bar{K}}^2 \end{bmatrix} + \begin{bmatrix} 0 & \gamma \\ \gamma & 0 \end{bmatrix}$$







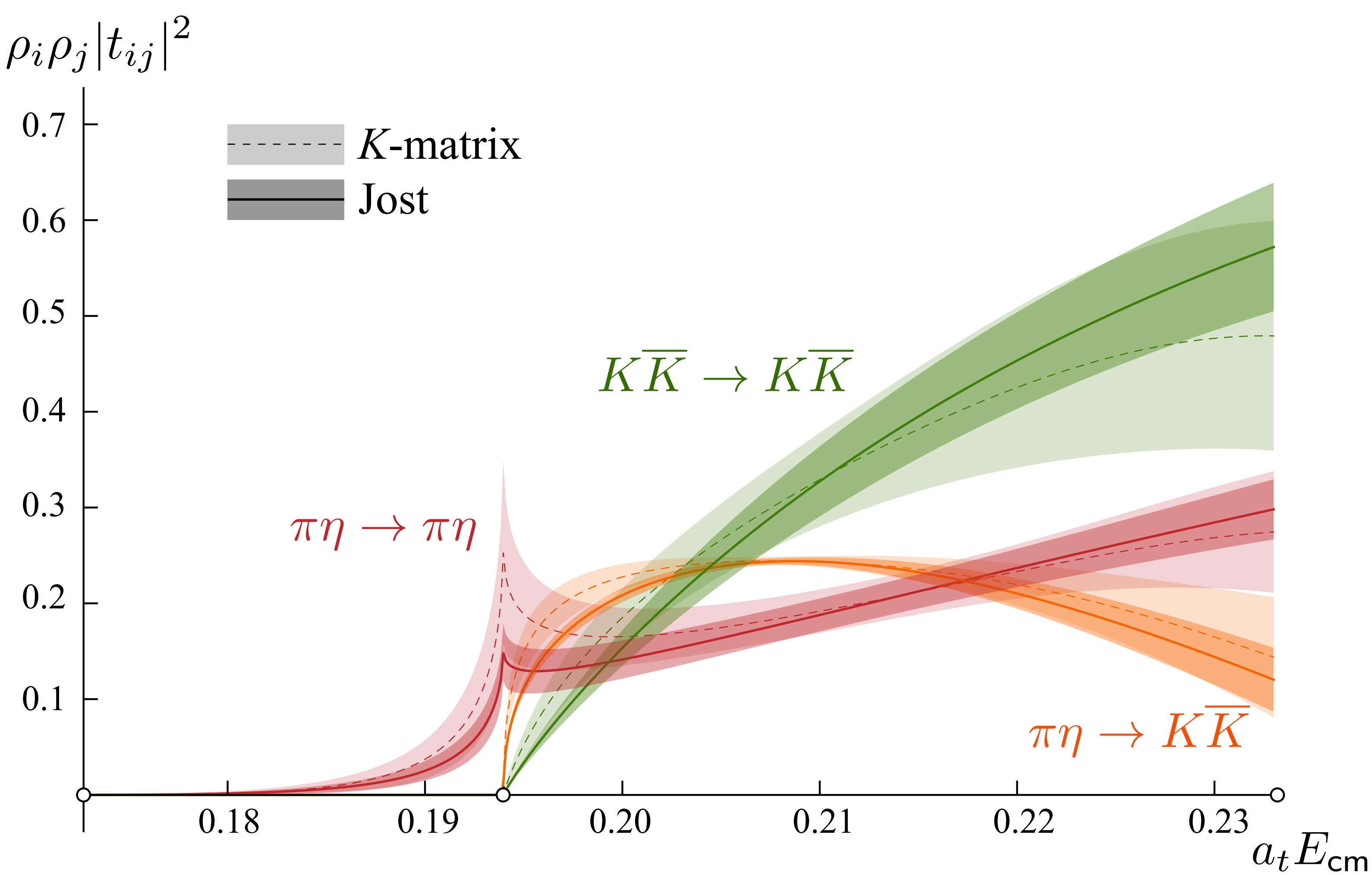
simple functions of scattering momentum

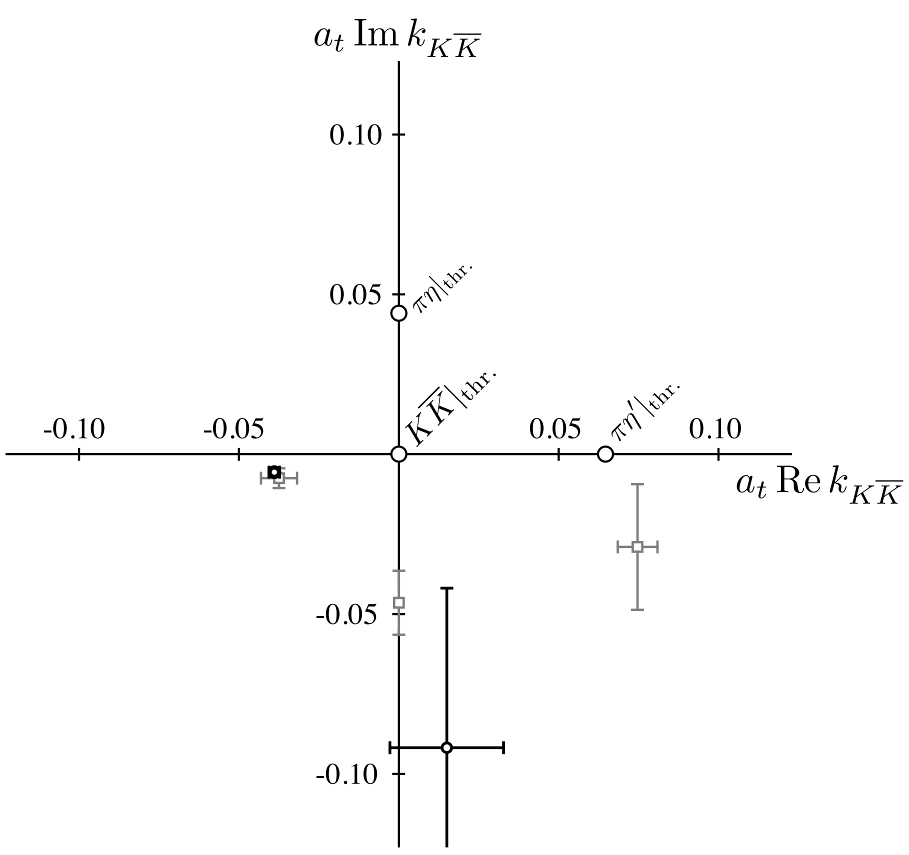
- control of pole positions as a free parameter
- unitarity is not guaranteed (test for a given set of parameters)

$$S_{11} = \frac{\mathfrak{J}(-k_1, k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$S_{22} = \frac{\mathfrak{J}(k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}$$

$$\det \mathbf{S} = \frac{\mathfrak{J}(-k_1, -k_2)}{\mathfrak{J}(k_1, k_2)}.$$





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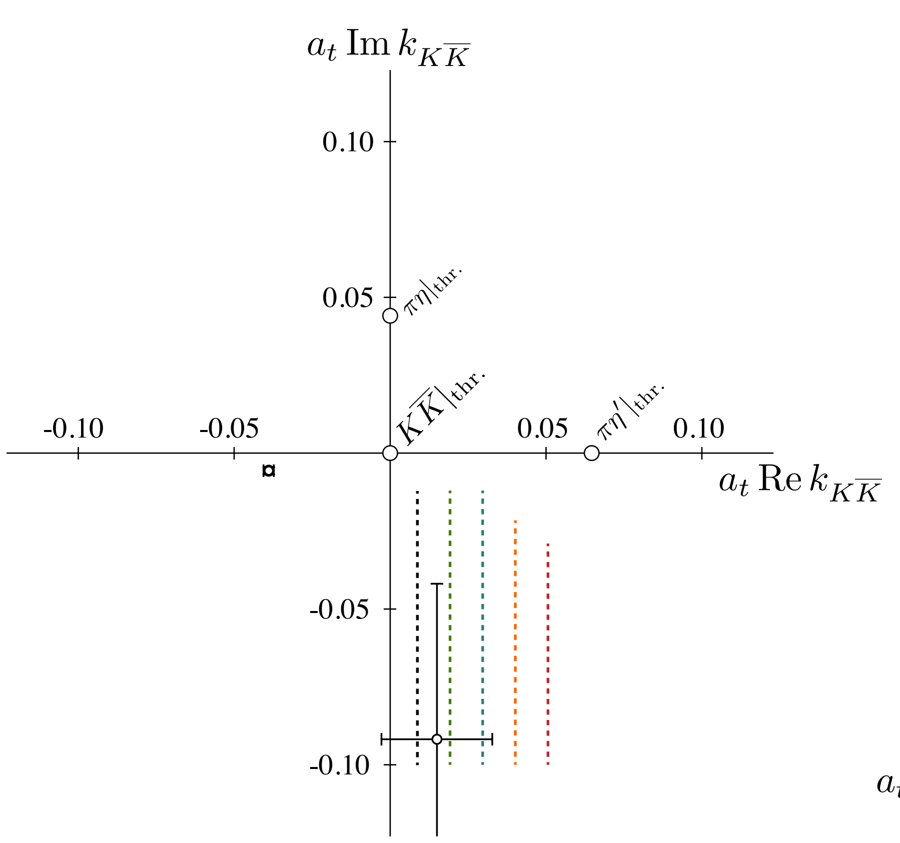
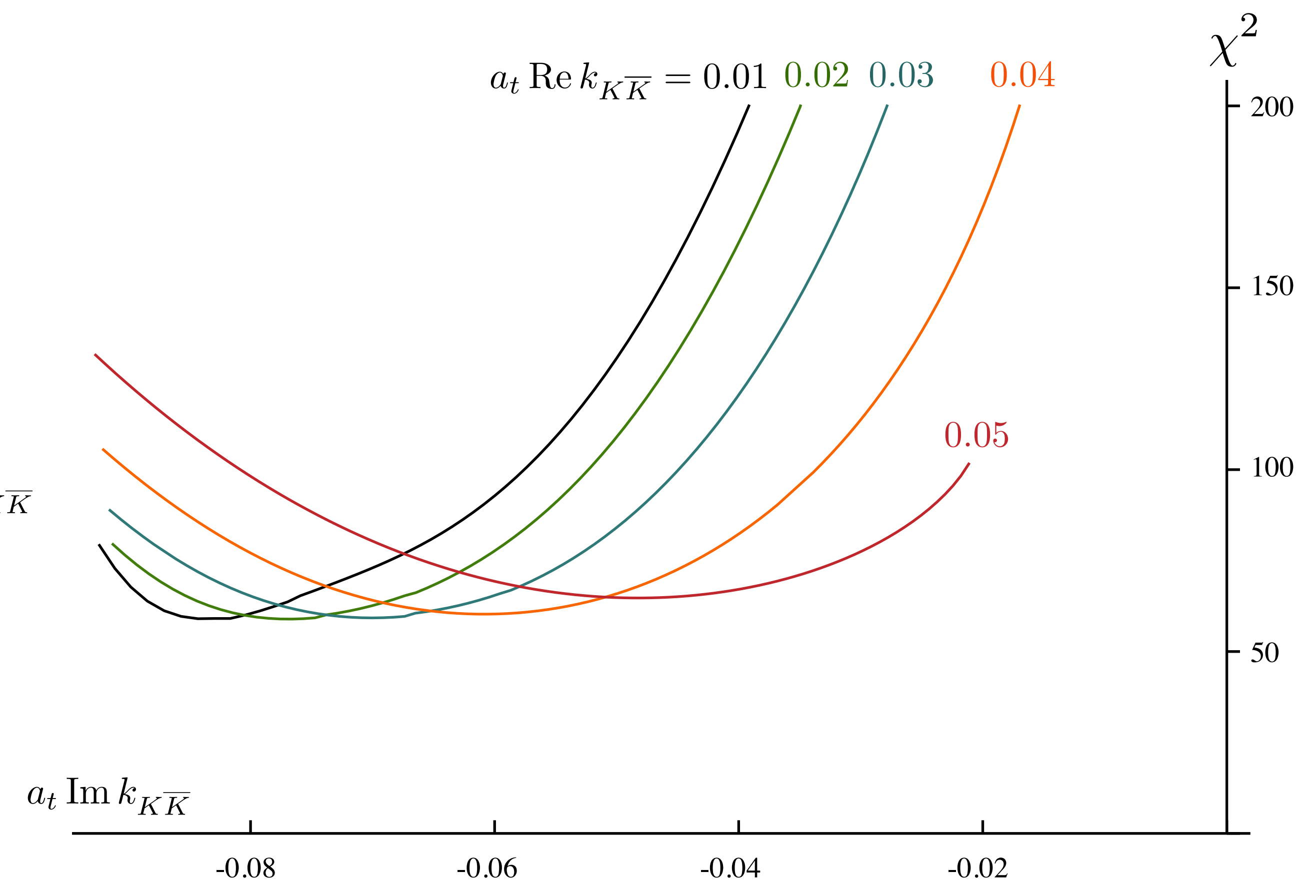
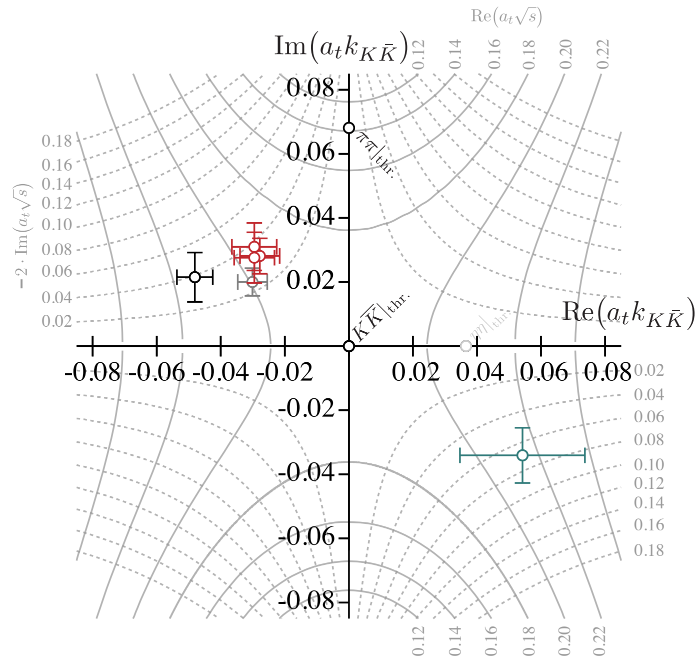
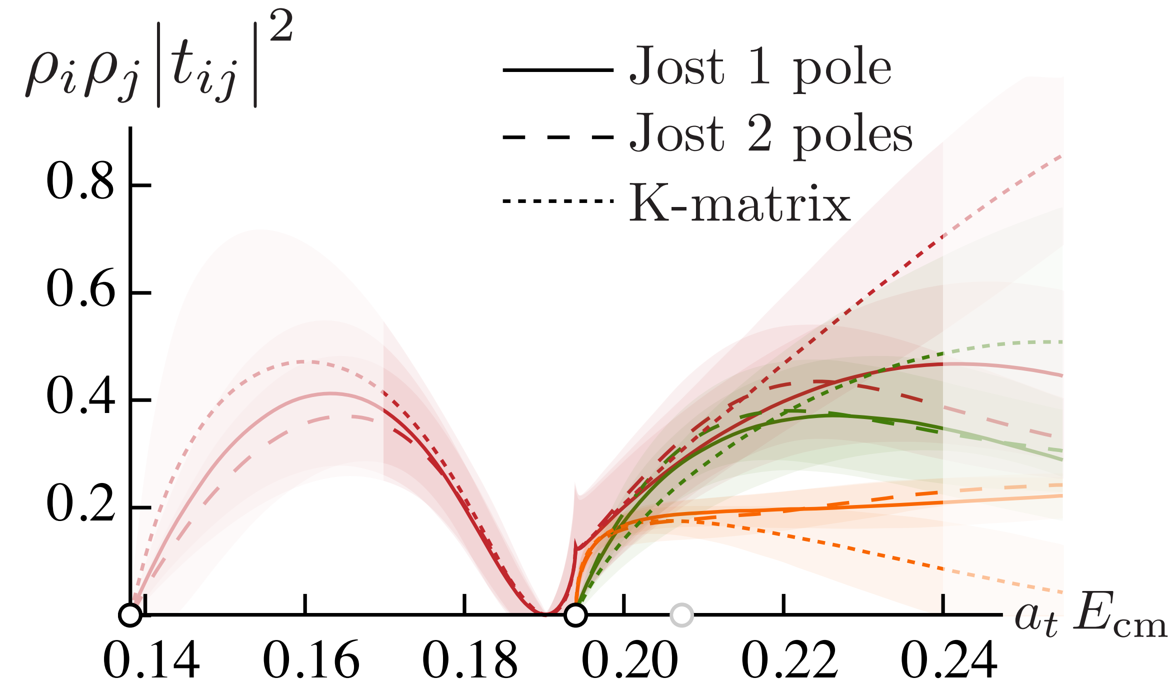




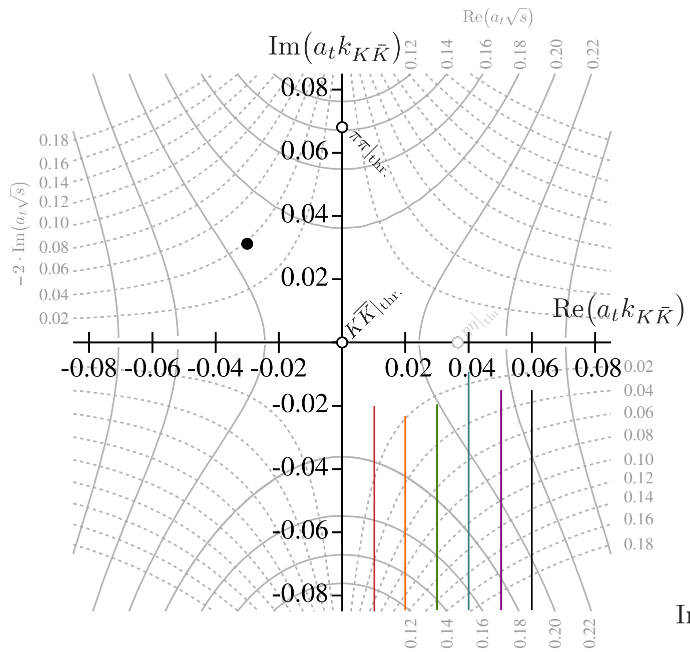
Jost amplitudes - f_0

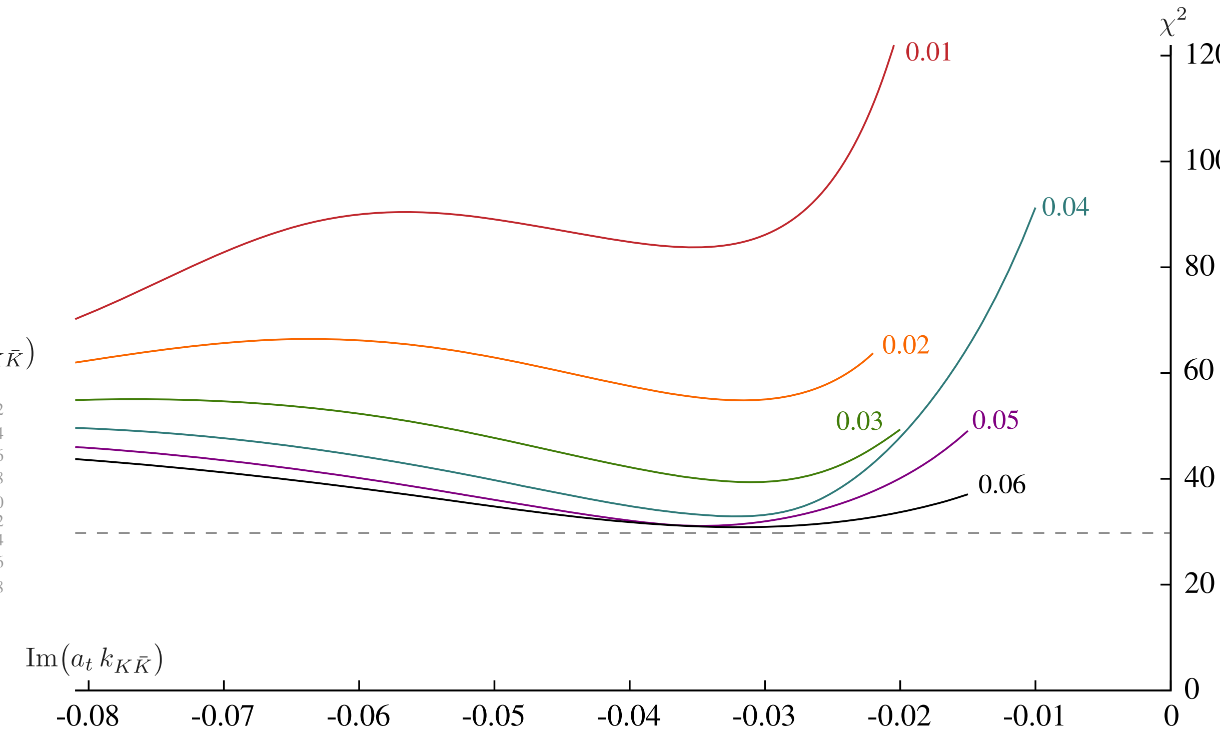


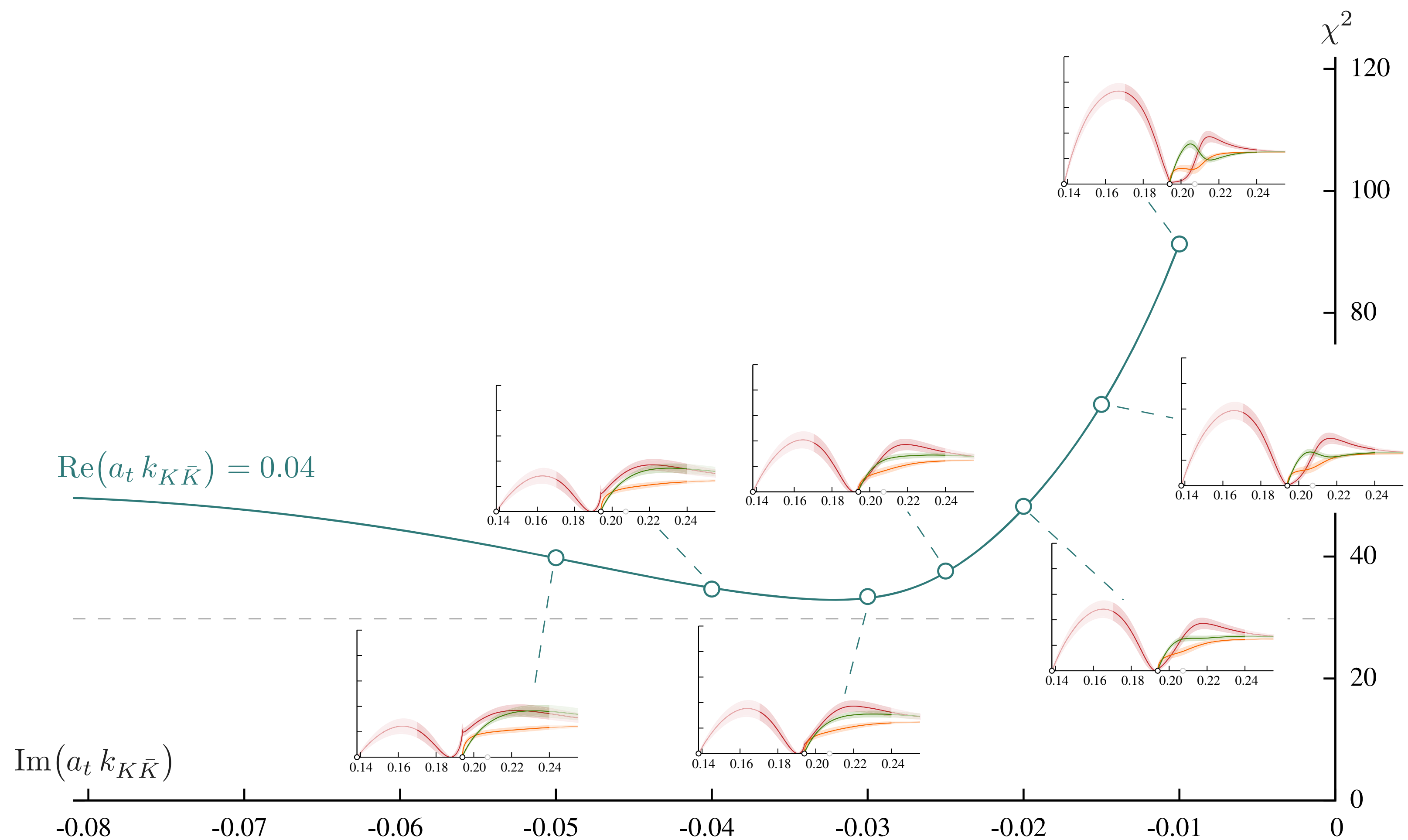


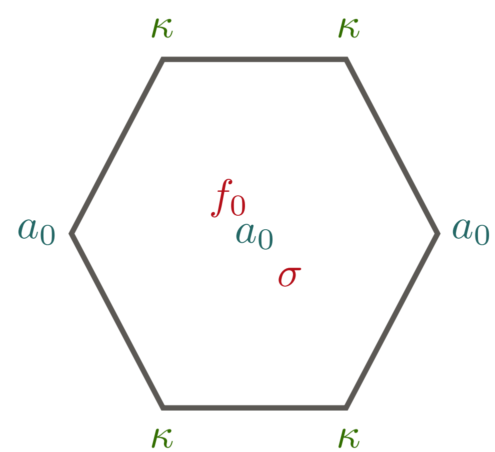
- best used for just a narrow energy region
  à la Morgan & Pennington 1993
- using Jost amps for 2-channel scattering only
- KK threshold region only
- dropping eta-eta
- 1 pole form + smooth background
- 2 pole form

- two pole form,
investigating second pole



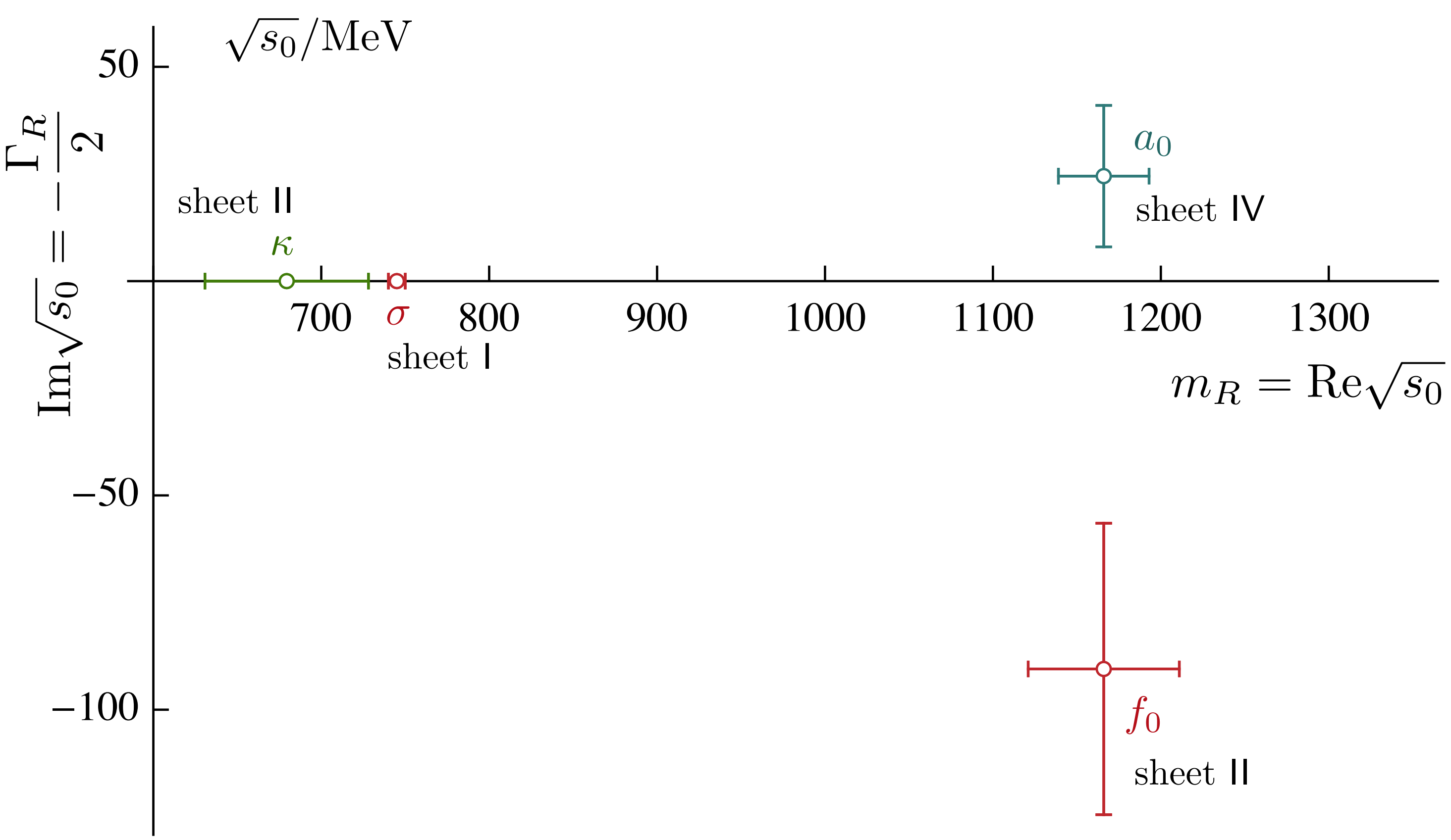


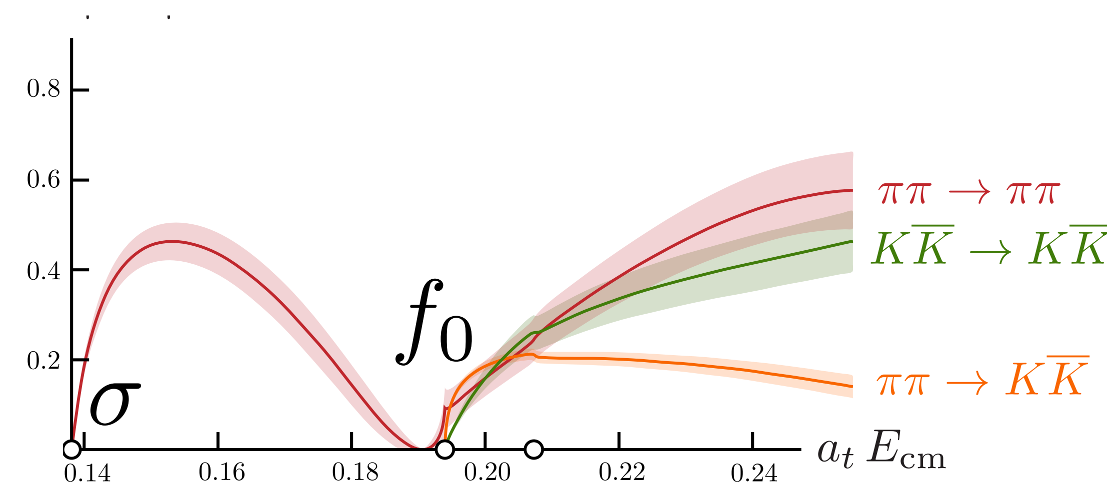


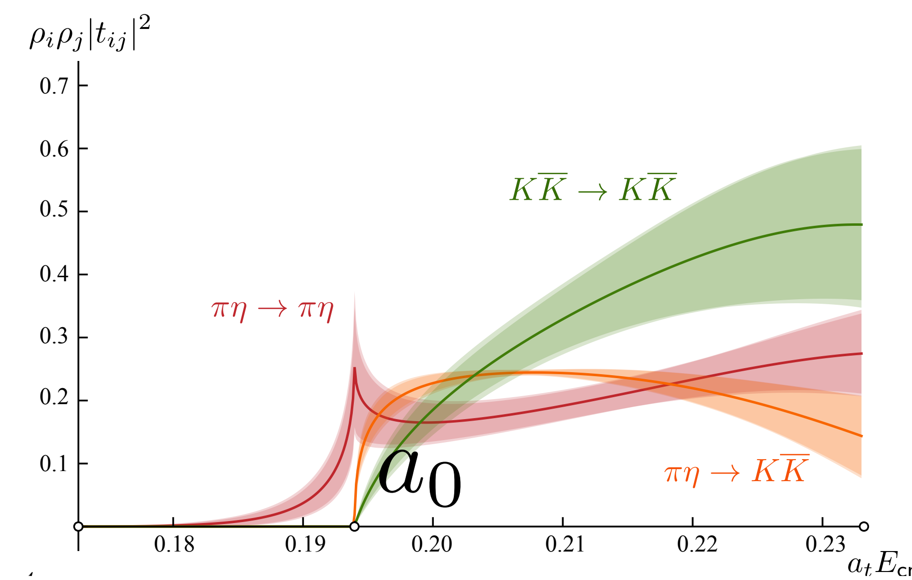


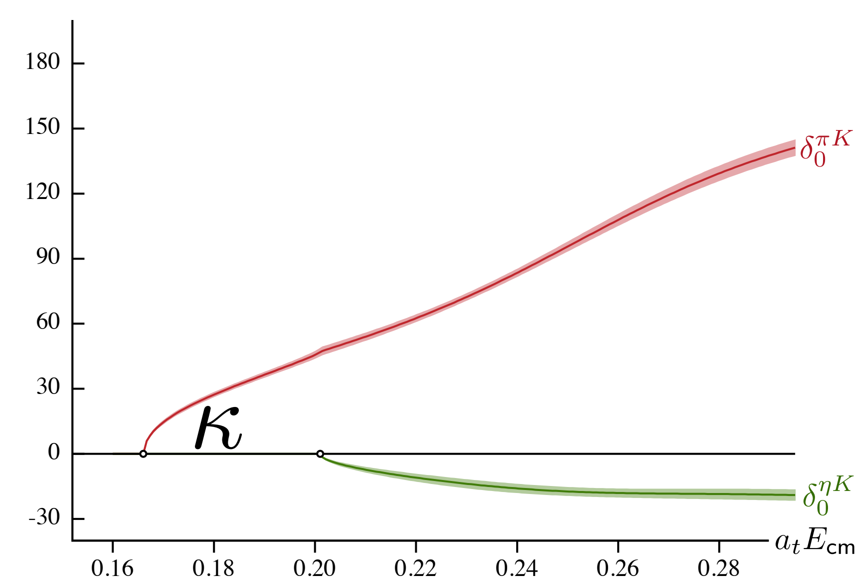
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

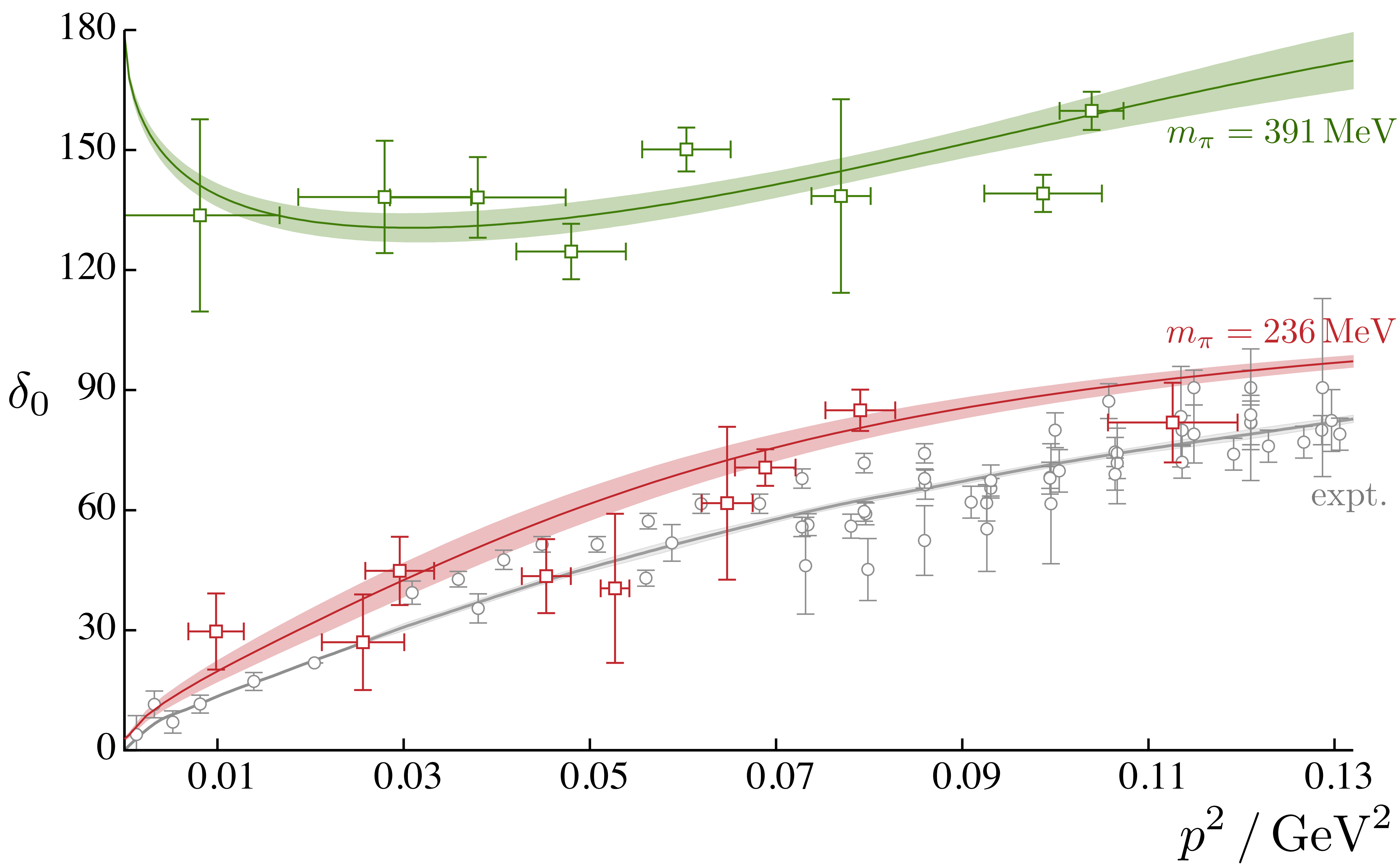
$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$



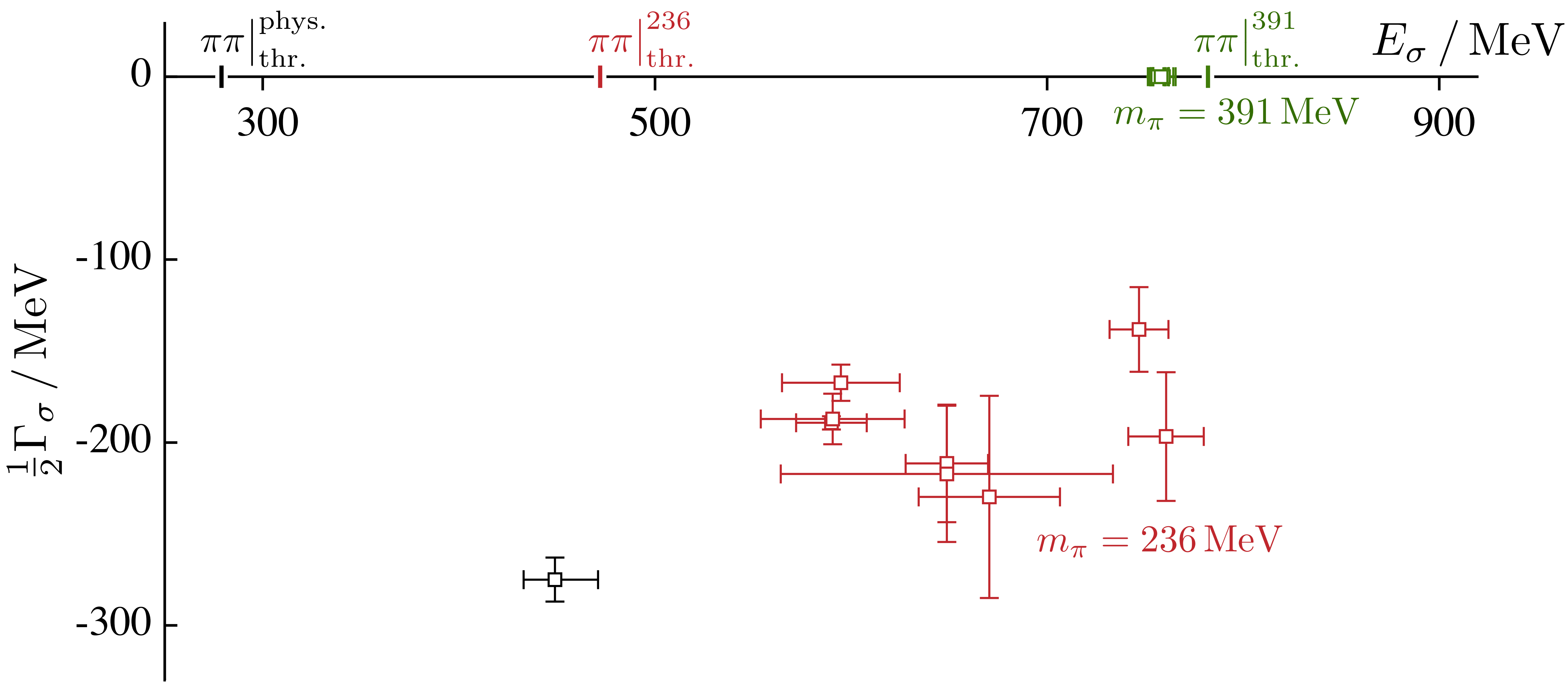







180
150
120
90
60
30
0
δ_0
m_π = 391 MeV
m_π = 236 MeV
expt.
0.01
0.03
0.05
0.07
0.09
0.11
0.13
p^2 / GeV^2

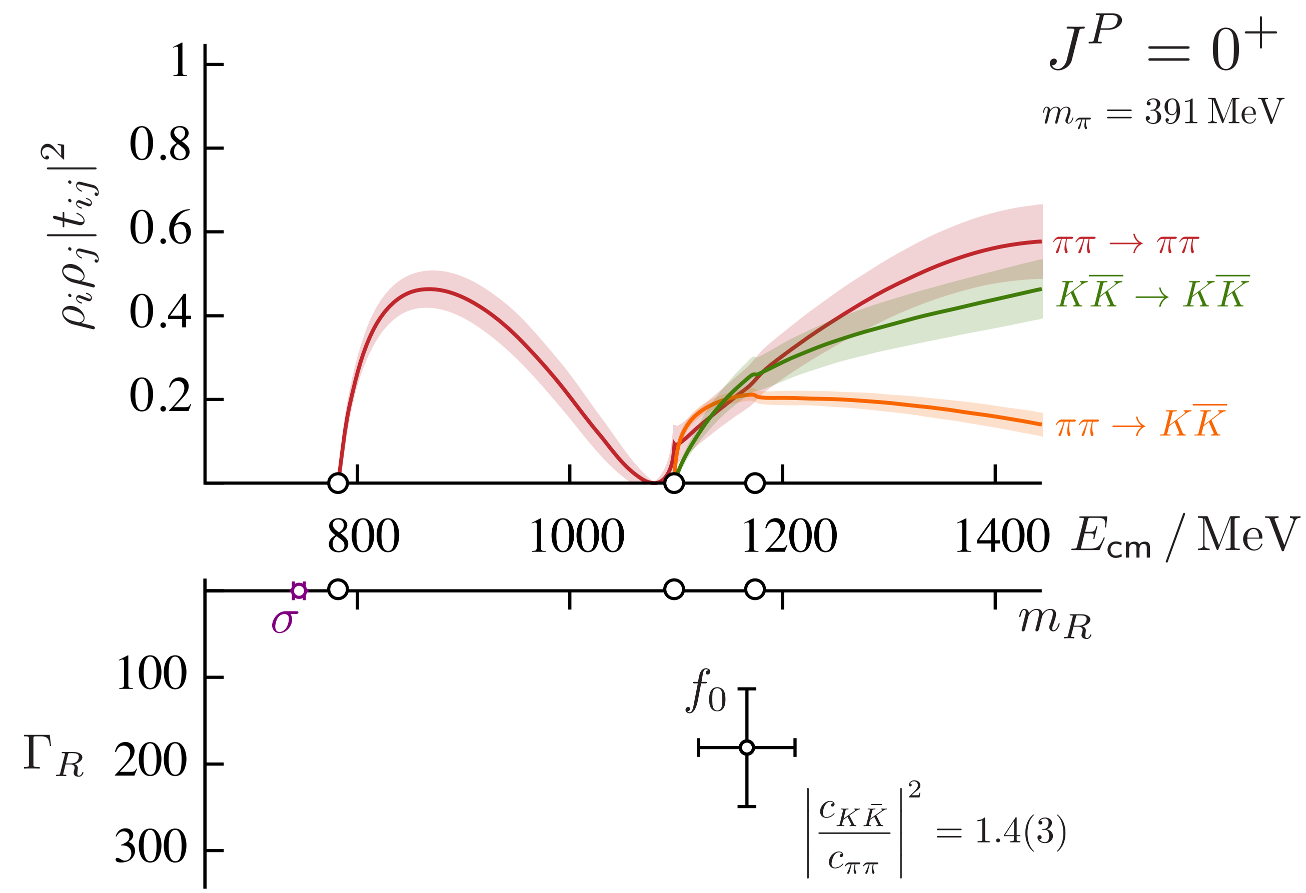
arXiv:1607.05900,
PRL 118, 022002 (2017)



Black point: dispersive + exp.
J.R. Pelaez, Phys. Rep. 658, 1 (2016).

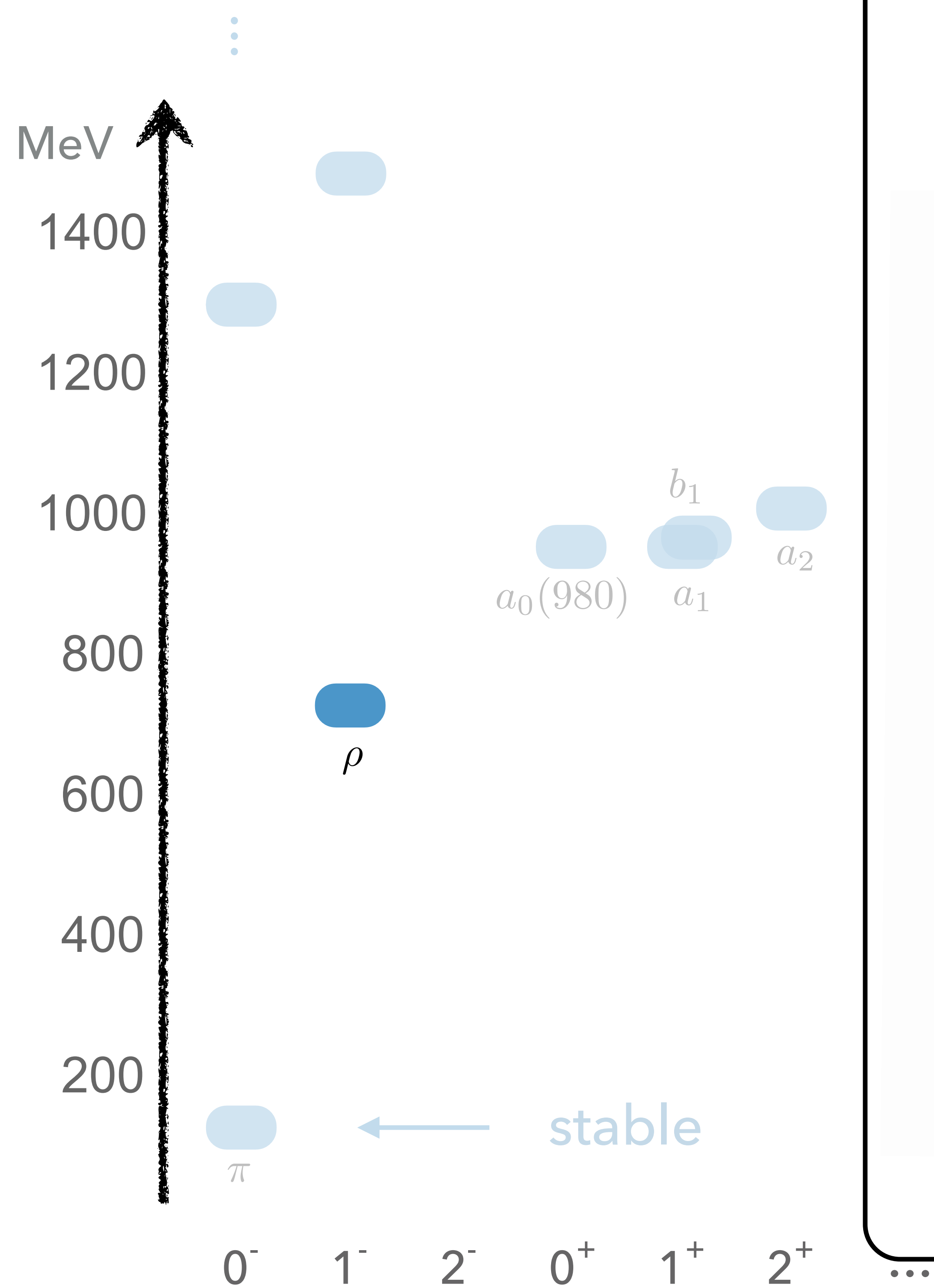
Lattice QCD provides a first-principles tool to do hadron spectroscopy

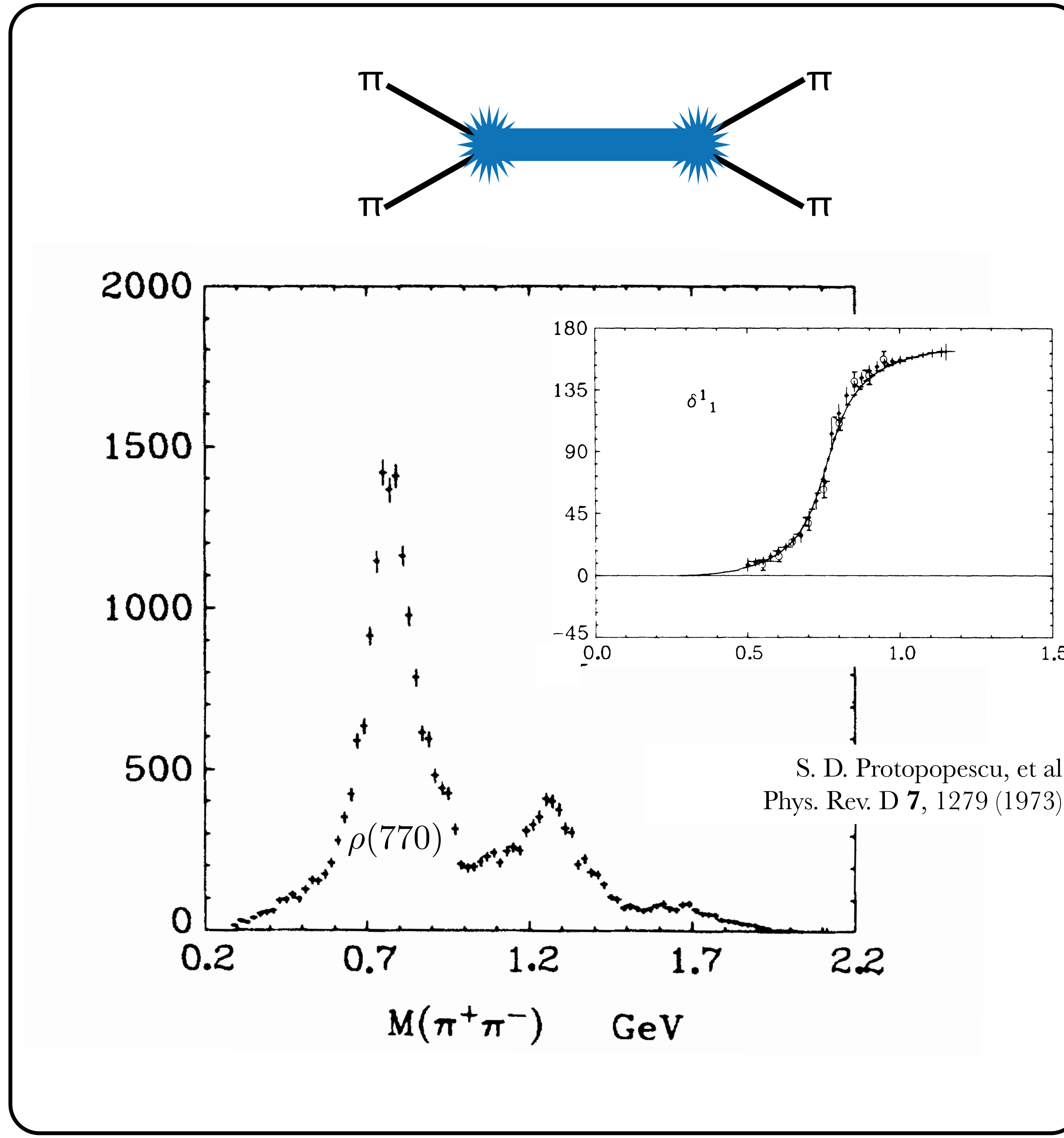
Most excited hadrons decay in multiple channels and these can be computed from lattice QCD

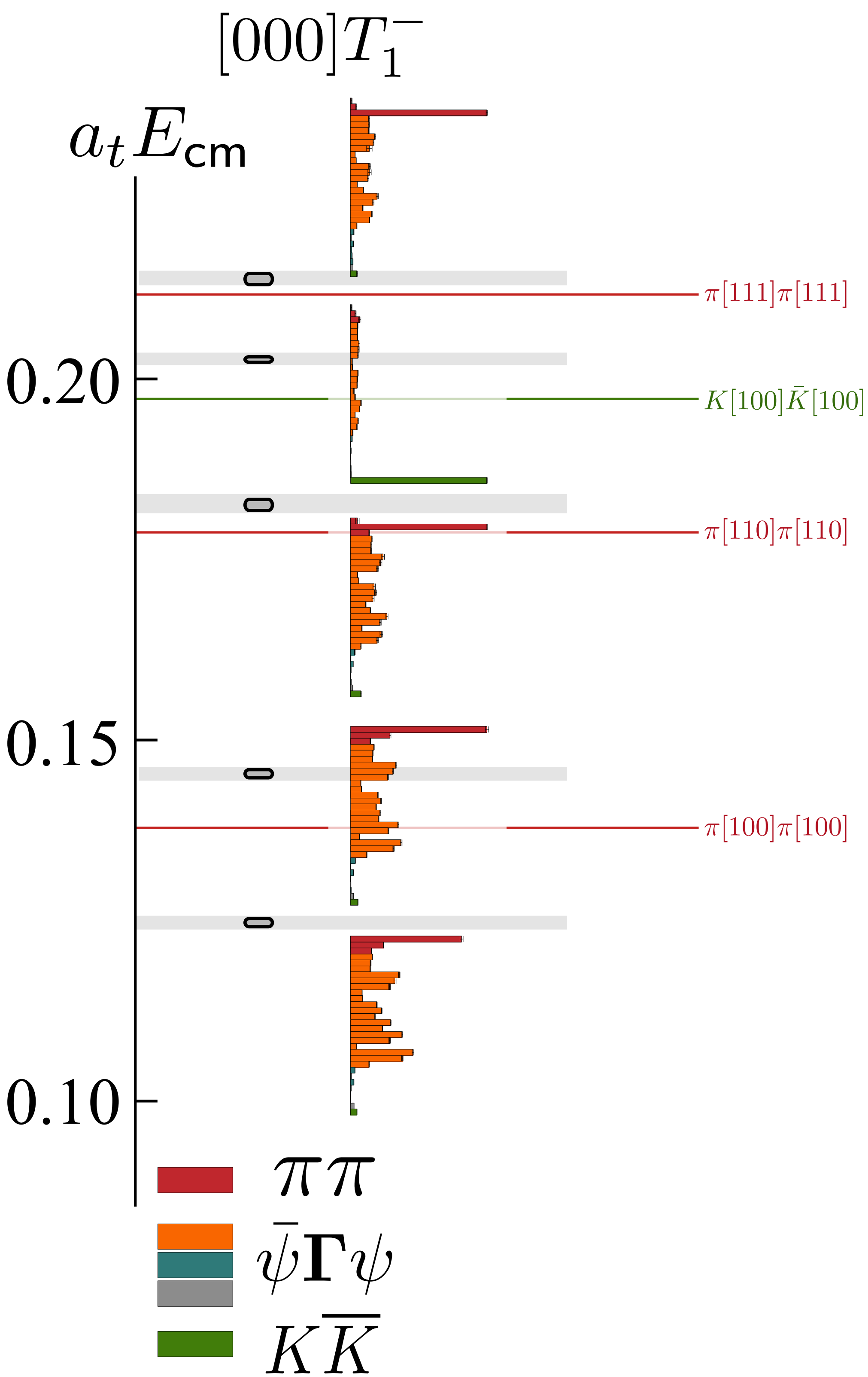


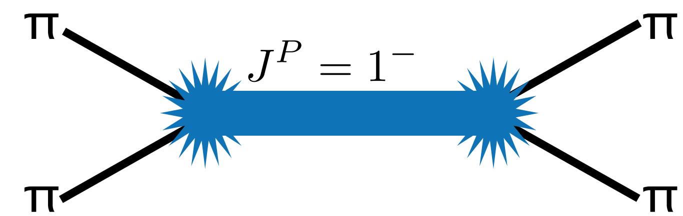
These methods are widely applicable

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances


MeV
1400
1200
1000
800
600
400
200
b_1
$a_0(980)$
a_1
a_2
ρ
π
stable
0-
1-
2-
0+
1+
2+
...


π
π
π
π
2000
1500
1000
500
0
0.2
0.7
1.2
1.7
2.2
$M(\pi^+\pi^-)$ GeV
$\rho(770)$
180
135
90
45
0
−45
0.0
0.5
1.0
1.5
δ^1_1
S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)





excited state spectra from a variational method

$$C_{ij}(t) = \langle 0|\, \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\, |0\rangle$$

$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t,t_0)C_{ij}(t_0)v_j^{\mathfrak{n}}$$

operators used:

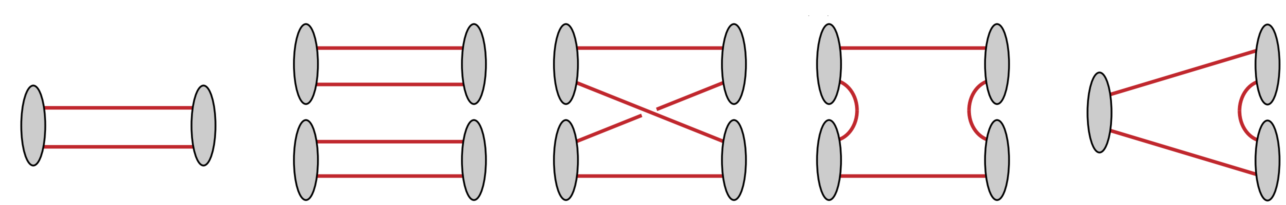
$\bar{\psi}\Gamma \overleftrightarrow{D}\ldots\overleftrightarrow{D}\,\psi$ local qq-like constructions

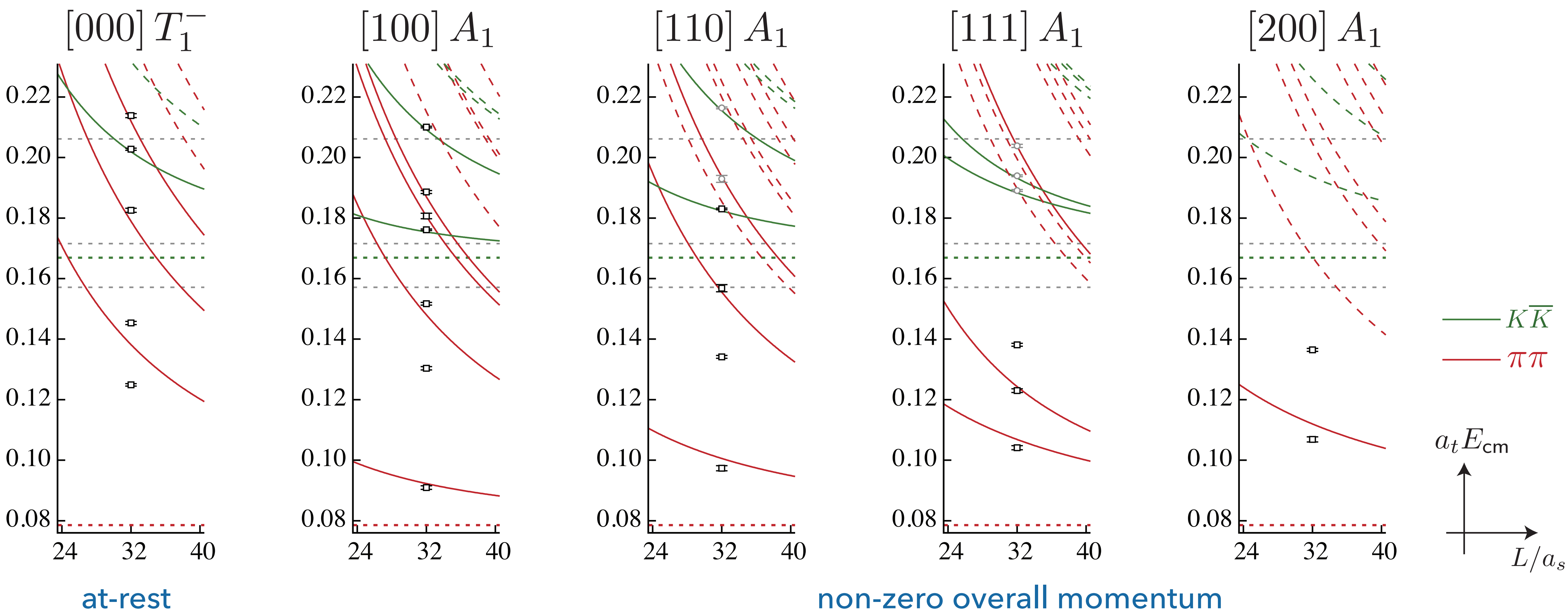
$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$ two-hadron constructions

$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$ uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)

many wick contractions:

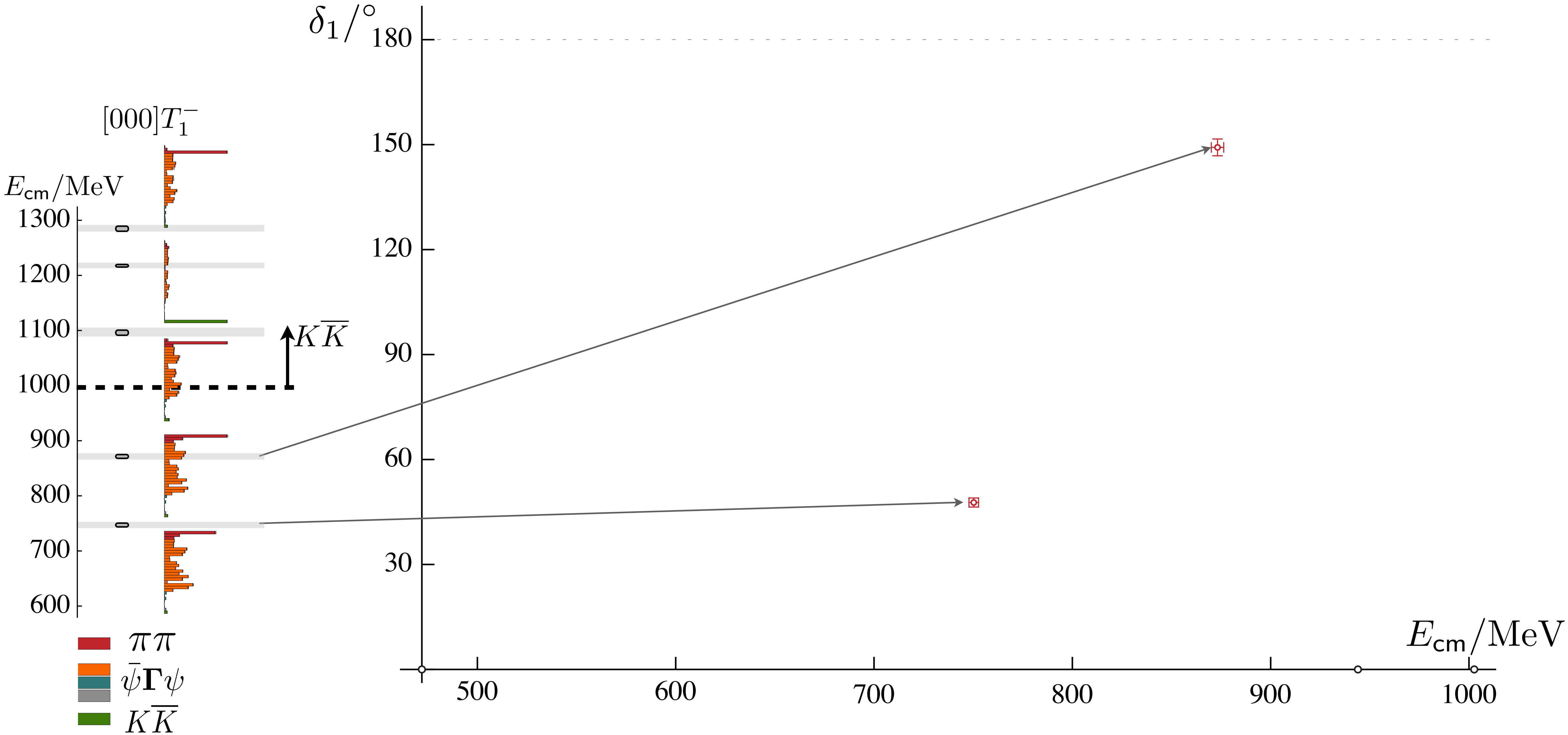



[000] T_1^-
[100] A_1
[110] A_1
[111] A_1
[200] A_1
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
24
32
40
$K\overline{K}$
$\pi\pi$
$a_t E_{\mathsf{cm}}$
L/a_s
at-rest
non-zero overall momentum

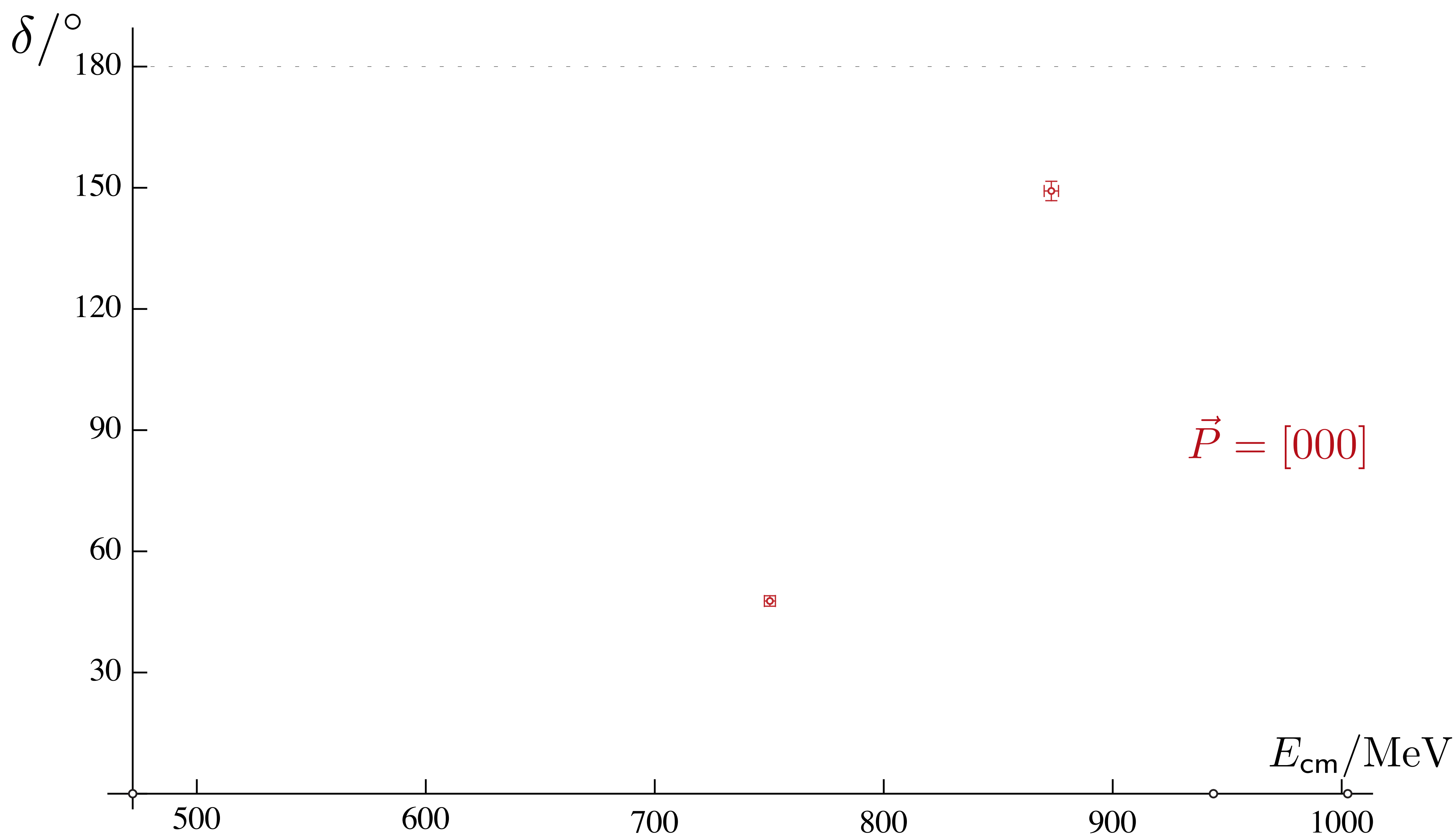
$m_\pi = 236$ MeV

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

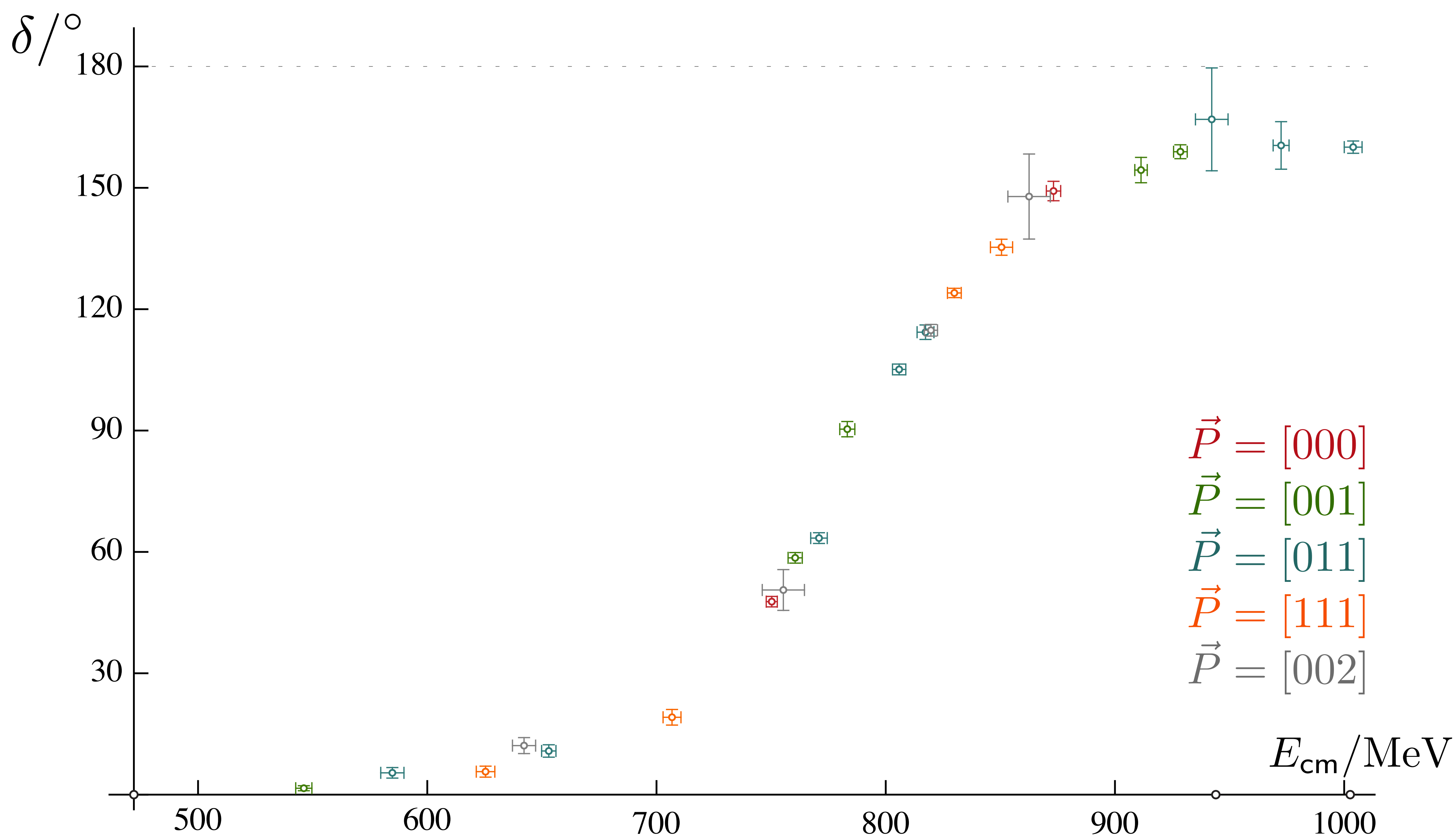
$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$



$m_\pi = 236$ MeV



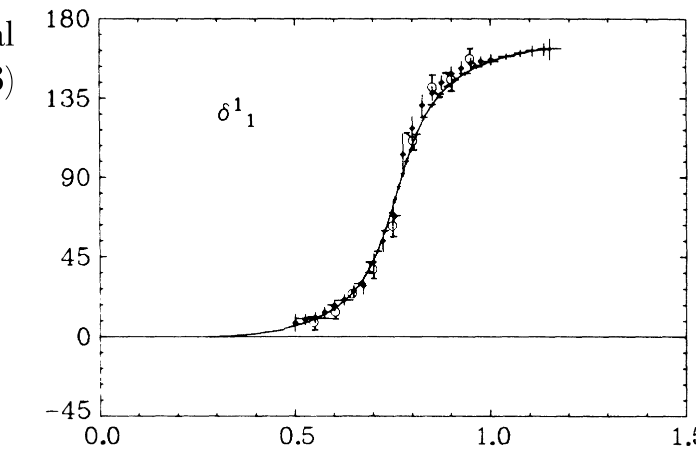
$m_\pi = 236$ MeV

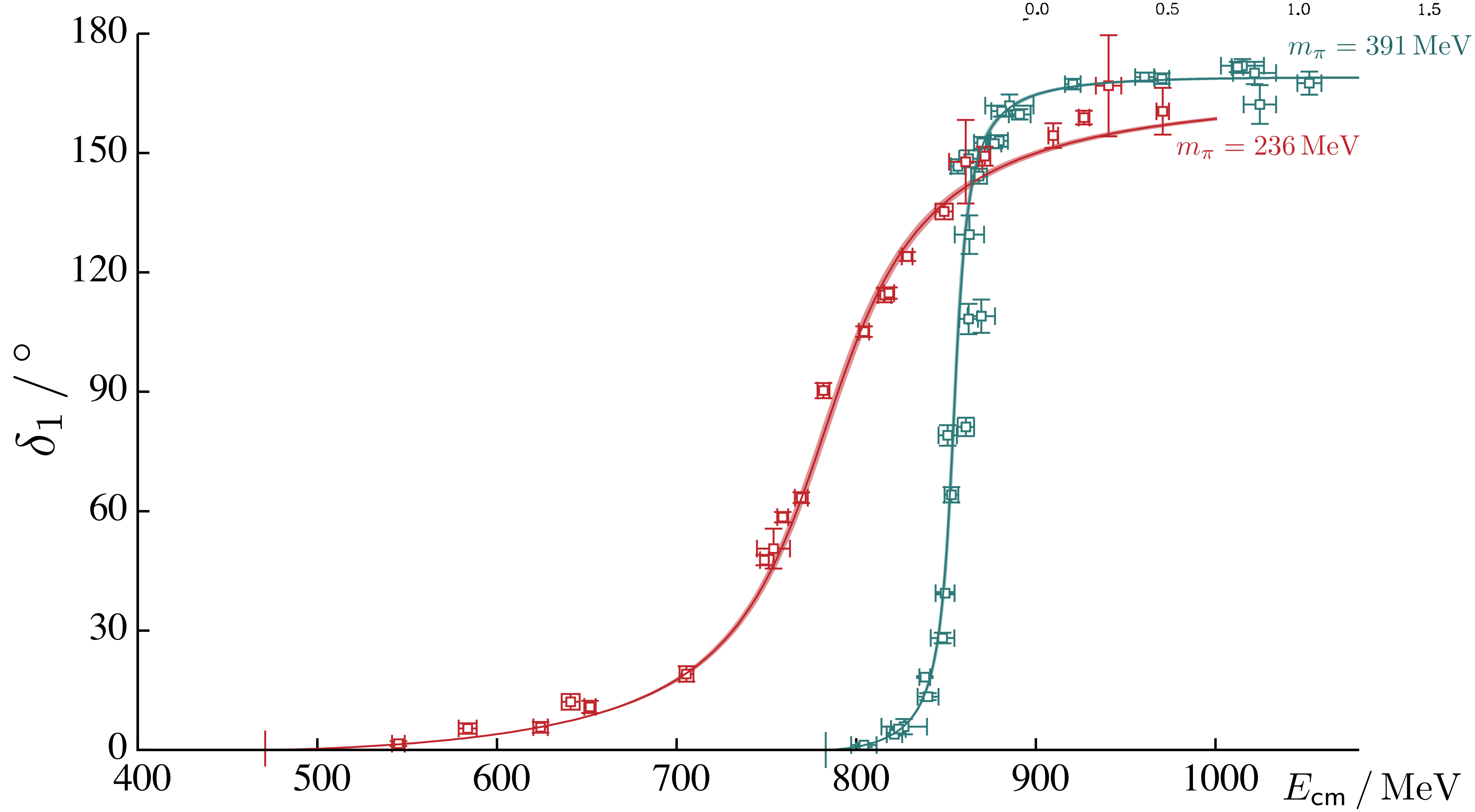


$\delta/^\circ$
180
150
120
90
60
30
500
600
700
800
900
1000
$E_{\mathsf{cm}}/\mathrm{MeV}$
$\vec{P} = [000]$
$\vec{P} = [001]$
$\vec{P} = [011]$
$\vec{P} = [111]$
$\vec{P} = [002]$

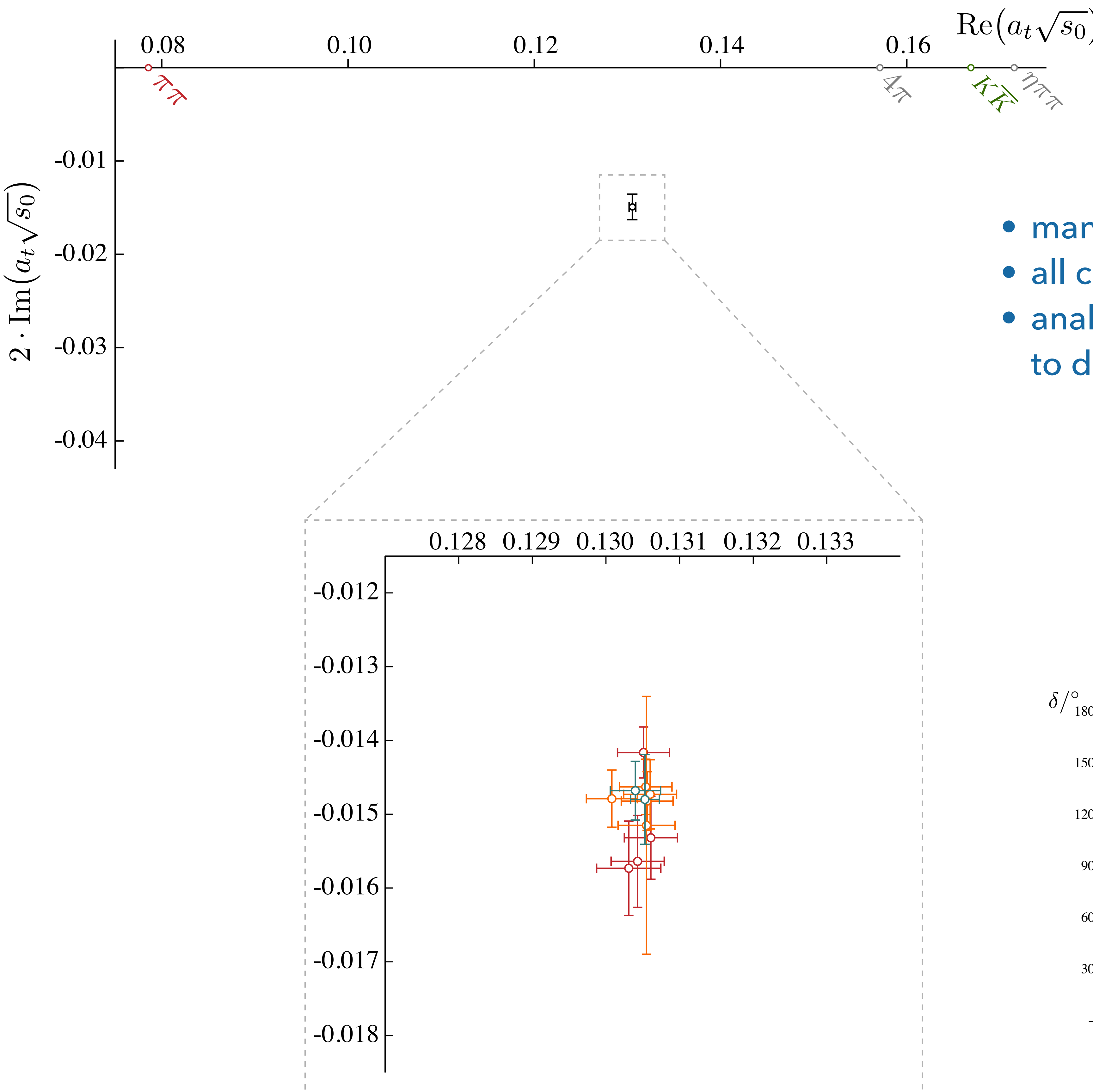
$m_\pi = 236$ MeV

S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)





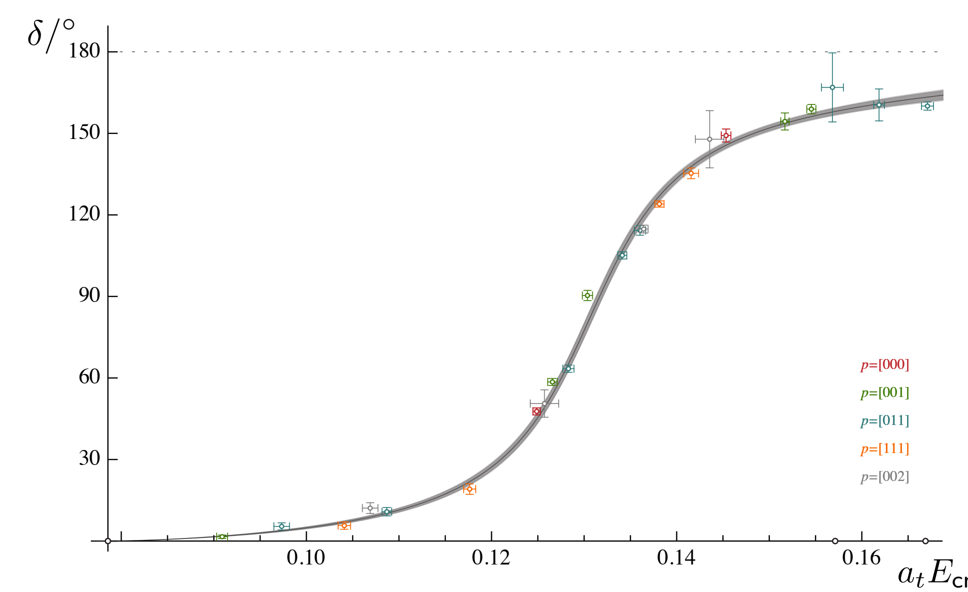
rho resonance pole

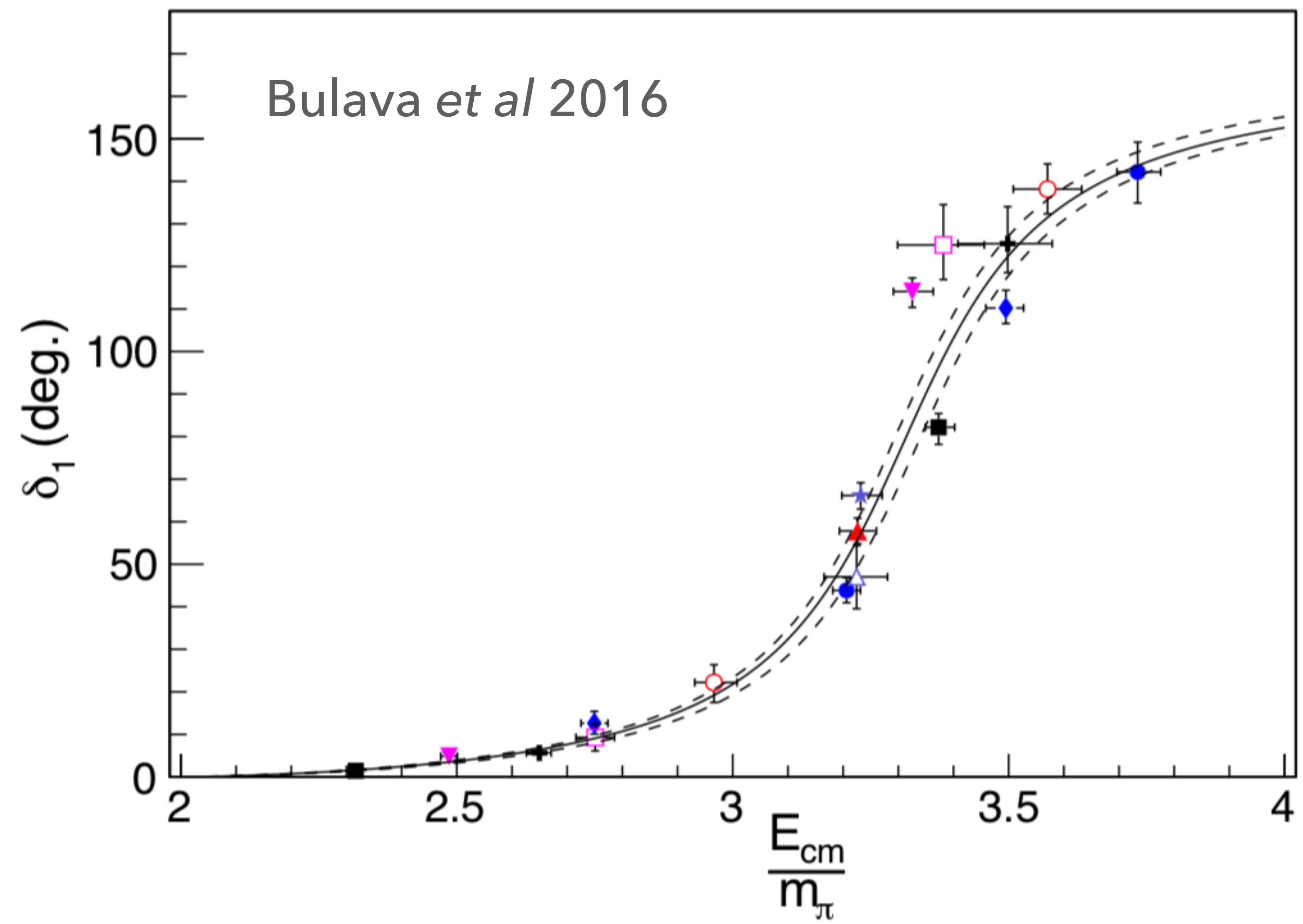


near a pole:

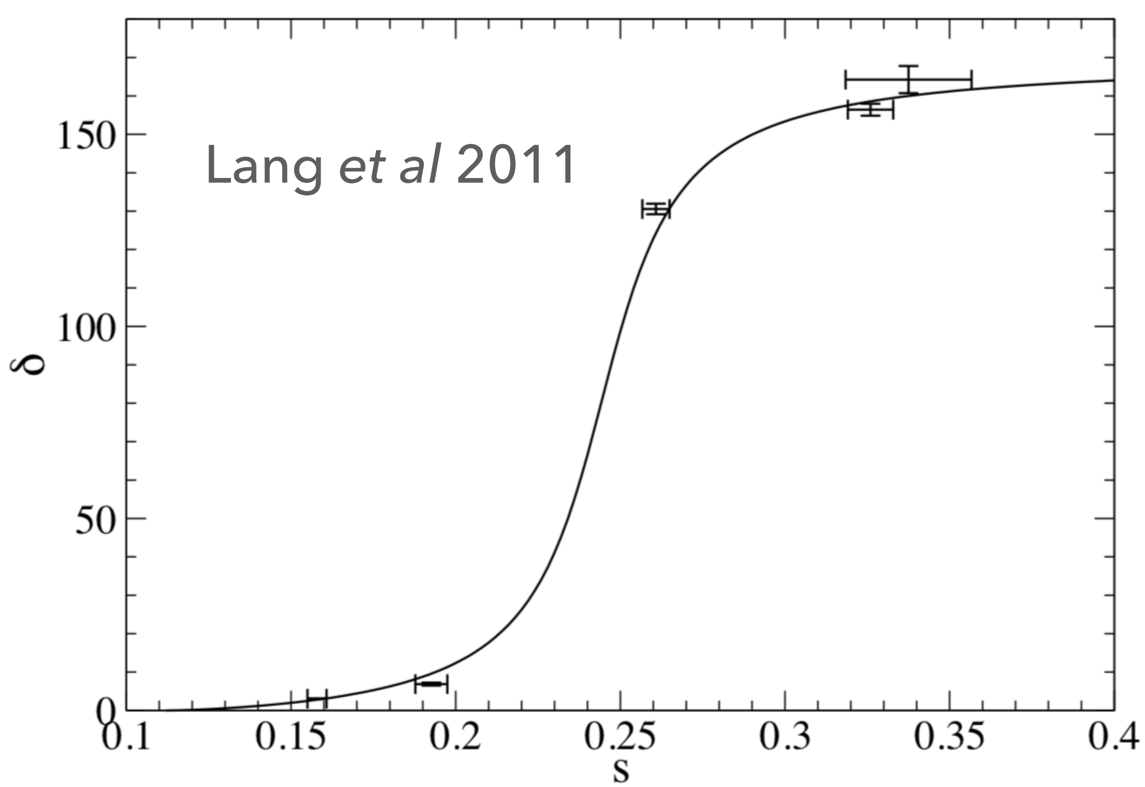
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

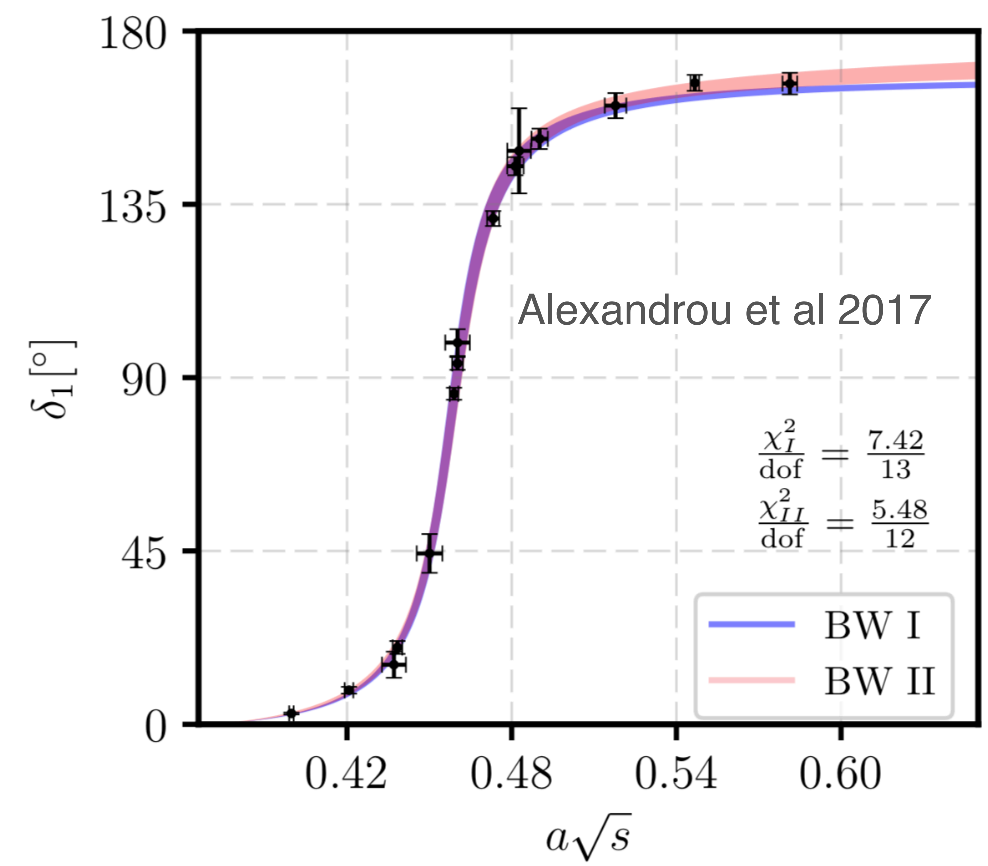
- many parameterisations of *t*
- all consistent are for real energies
- analytically continue to complex energies to determine t-matrix poles





- just a selection for the ρ
- … many more in the literature
- nucleons, K, D, B, charmonium

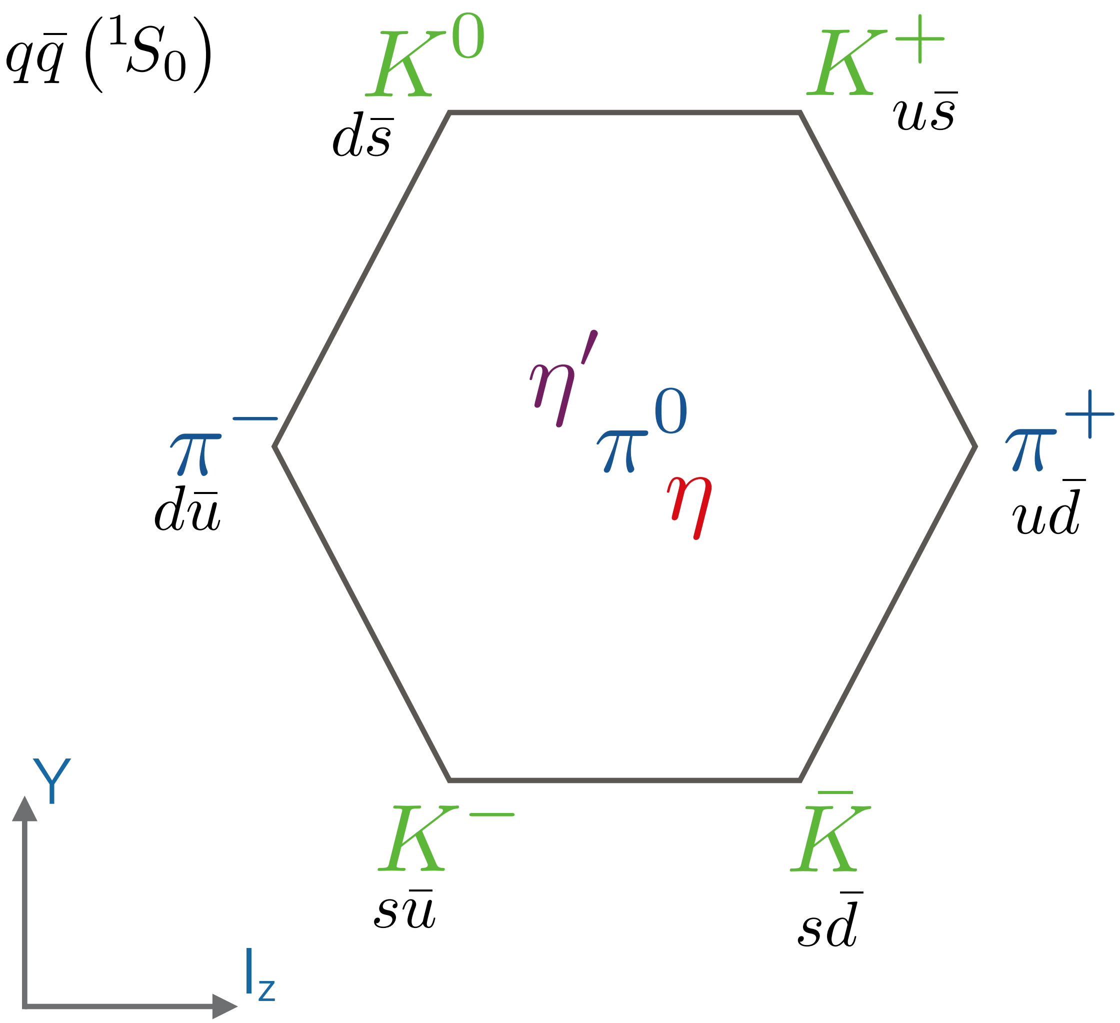




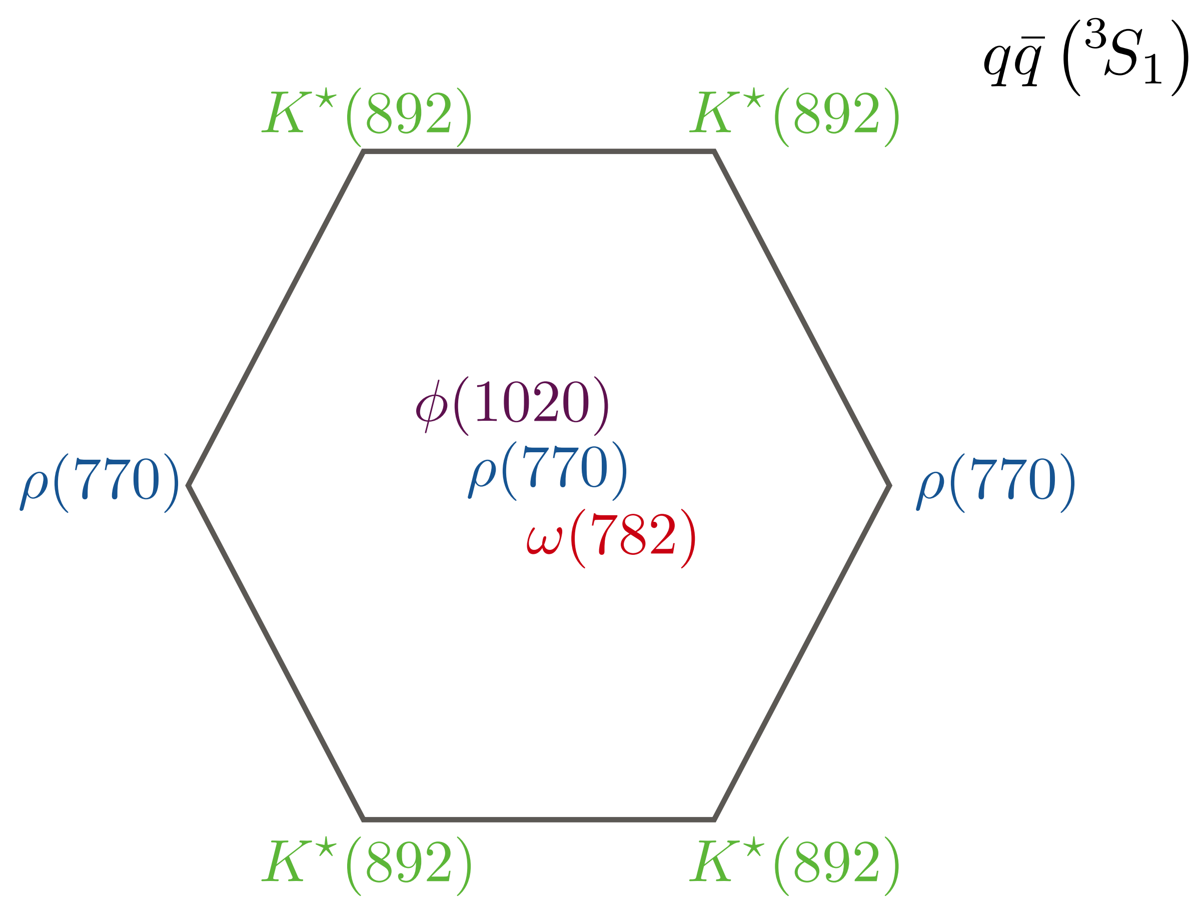
Quark model

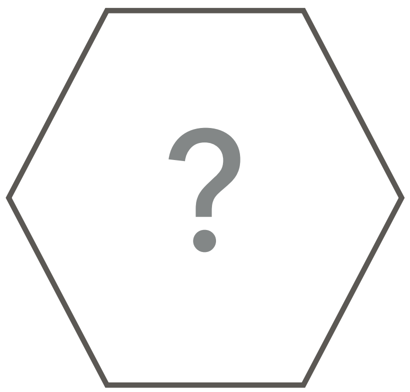
Considering *u,d,s* quarks: $q\bar{q}\,({}^{2S+1}L_J)$

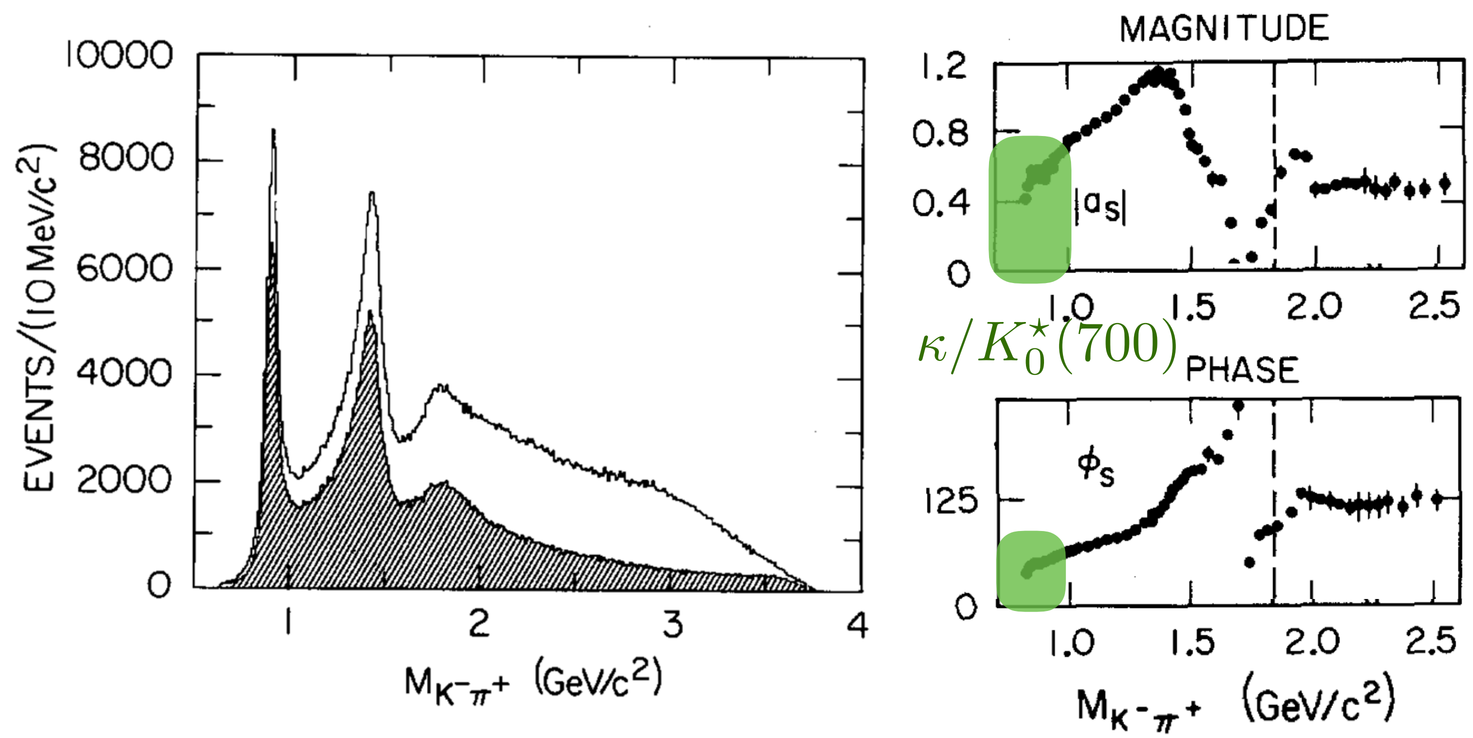
pseudoscalar nonet $J^P=0^-$



vector nonet $J^P=1^-$

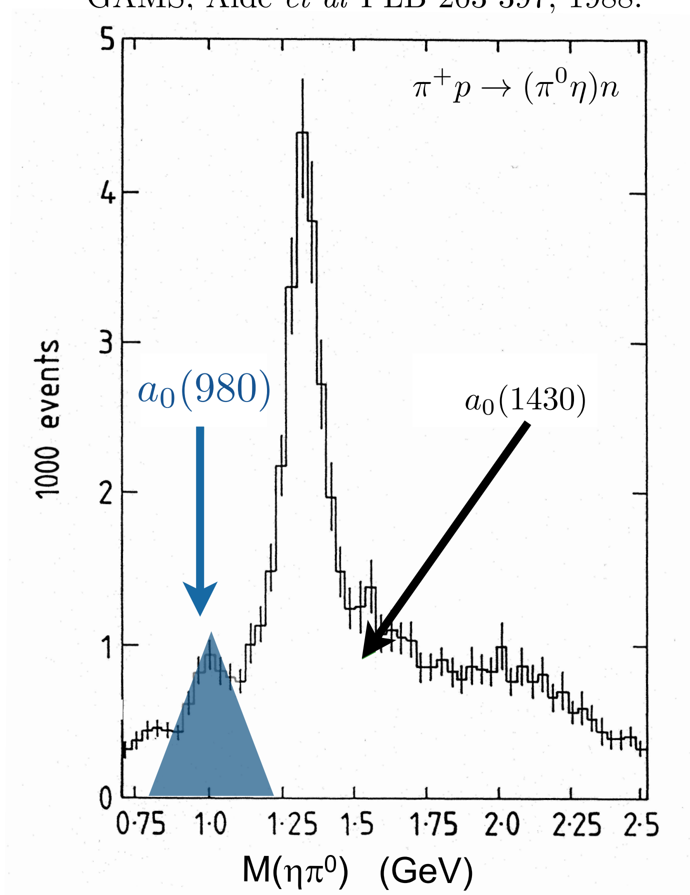


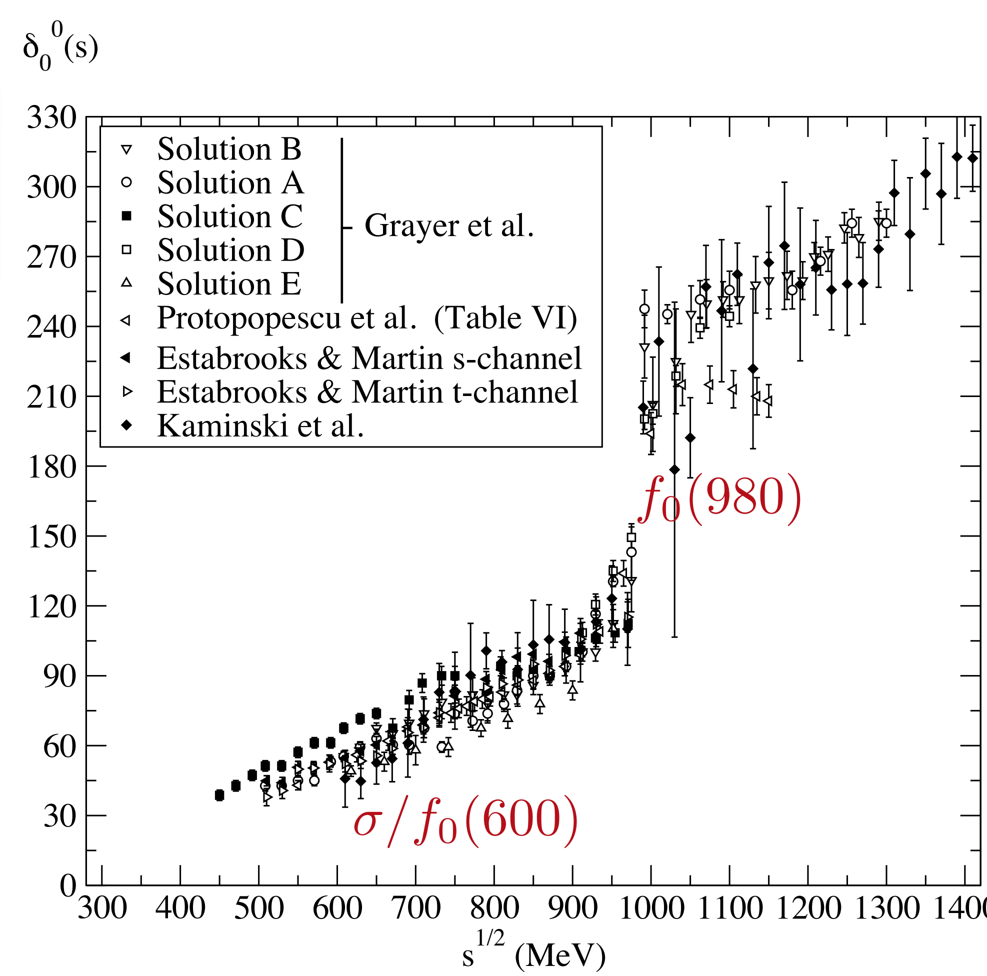




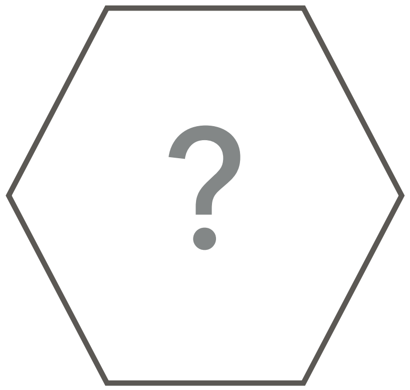
LASS experiment at SLAC E_K = 11 GeV

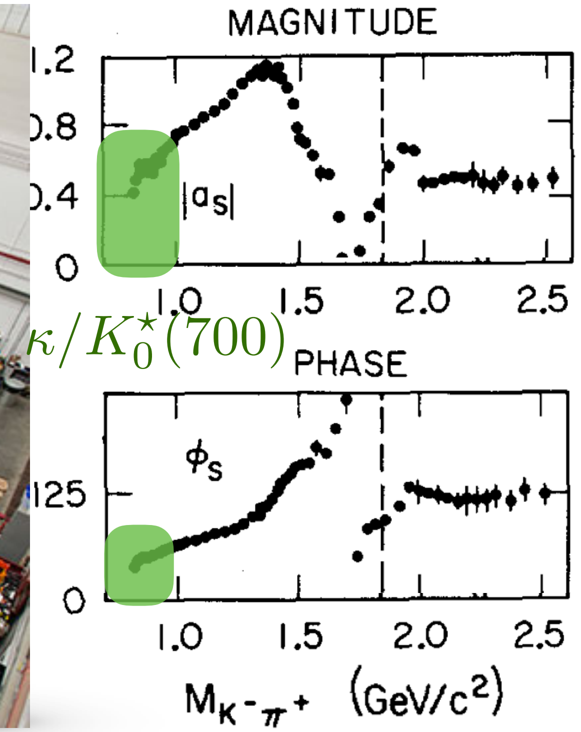
GAMS, Alde *et al* PLB 203 397, 1988.



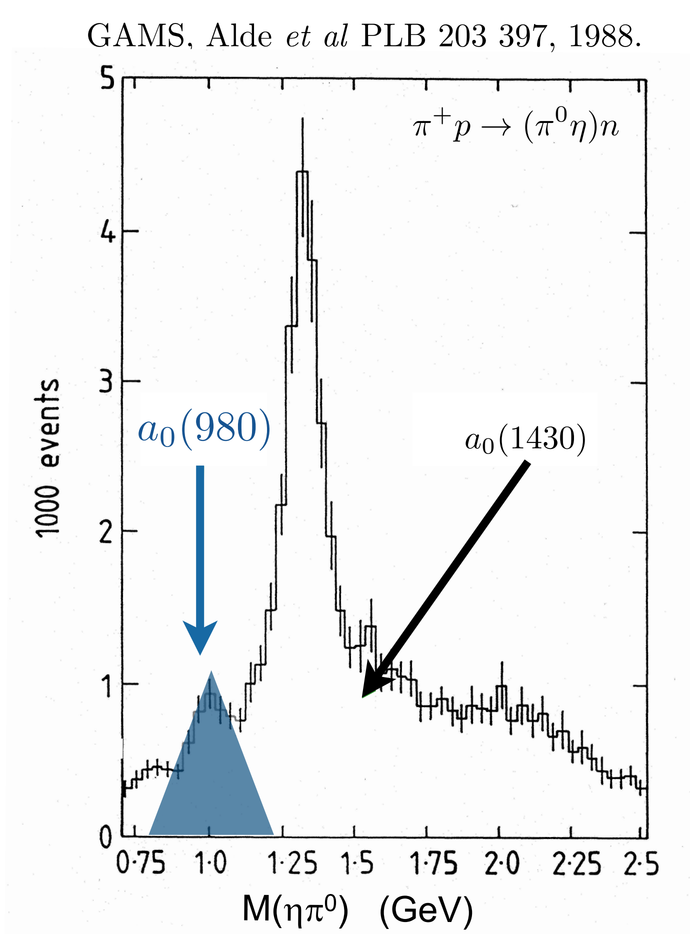


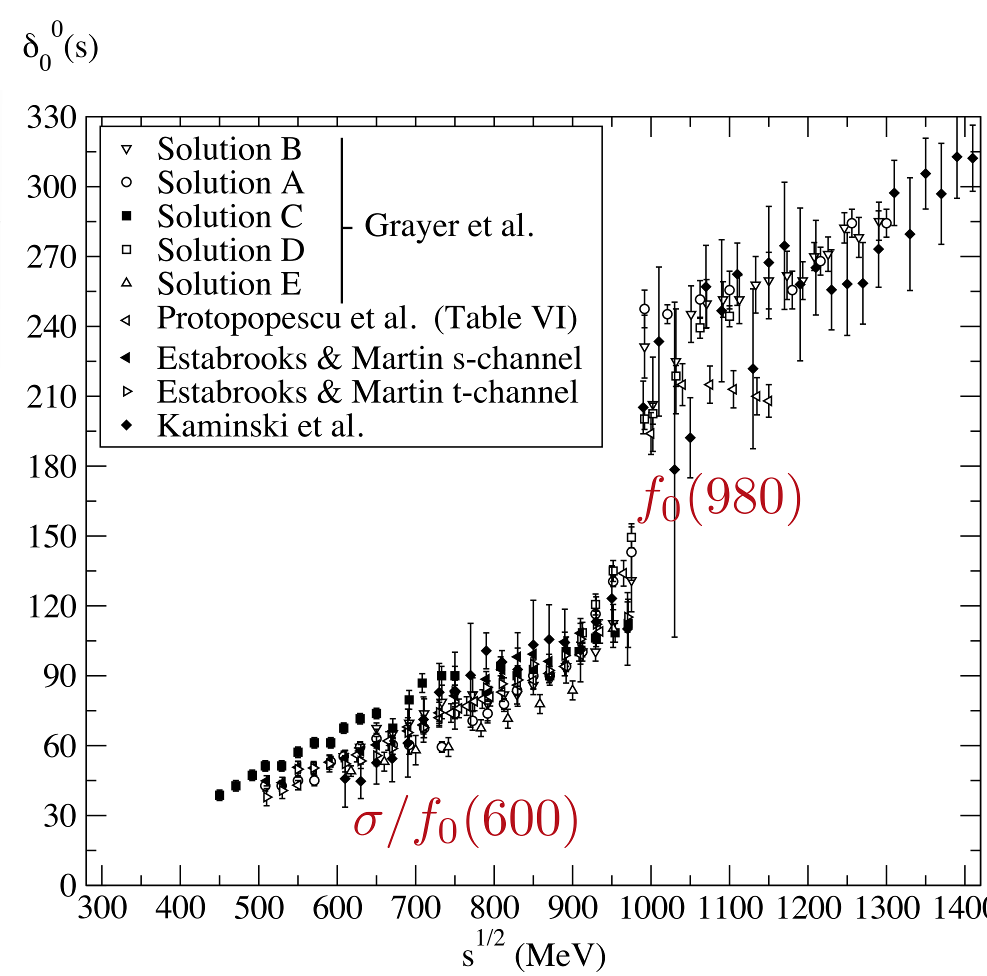
How do we understand the scalars?

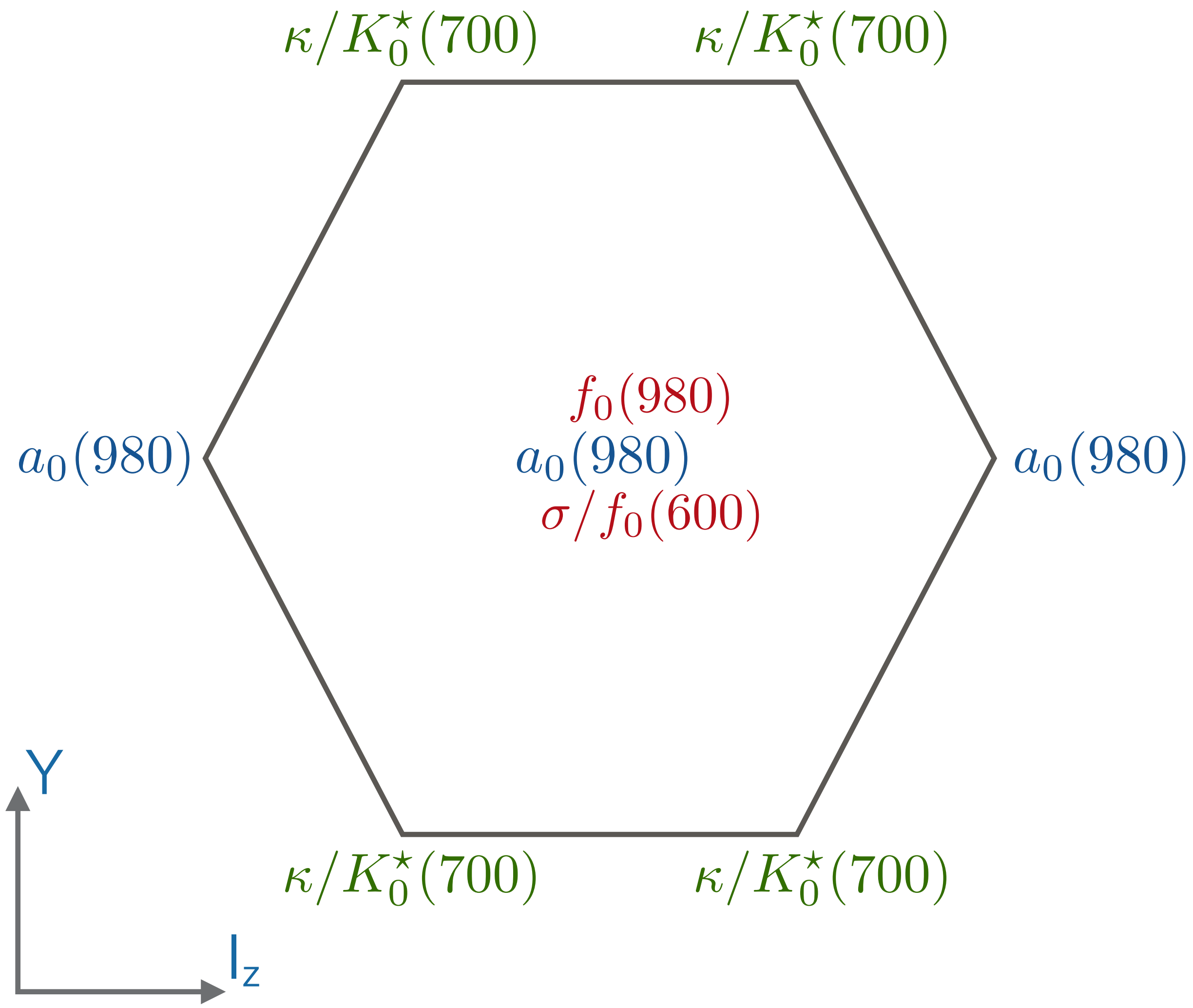




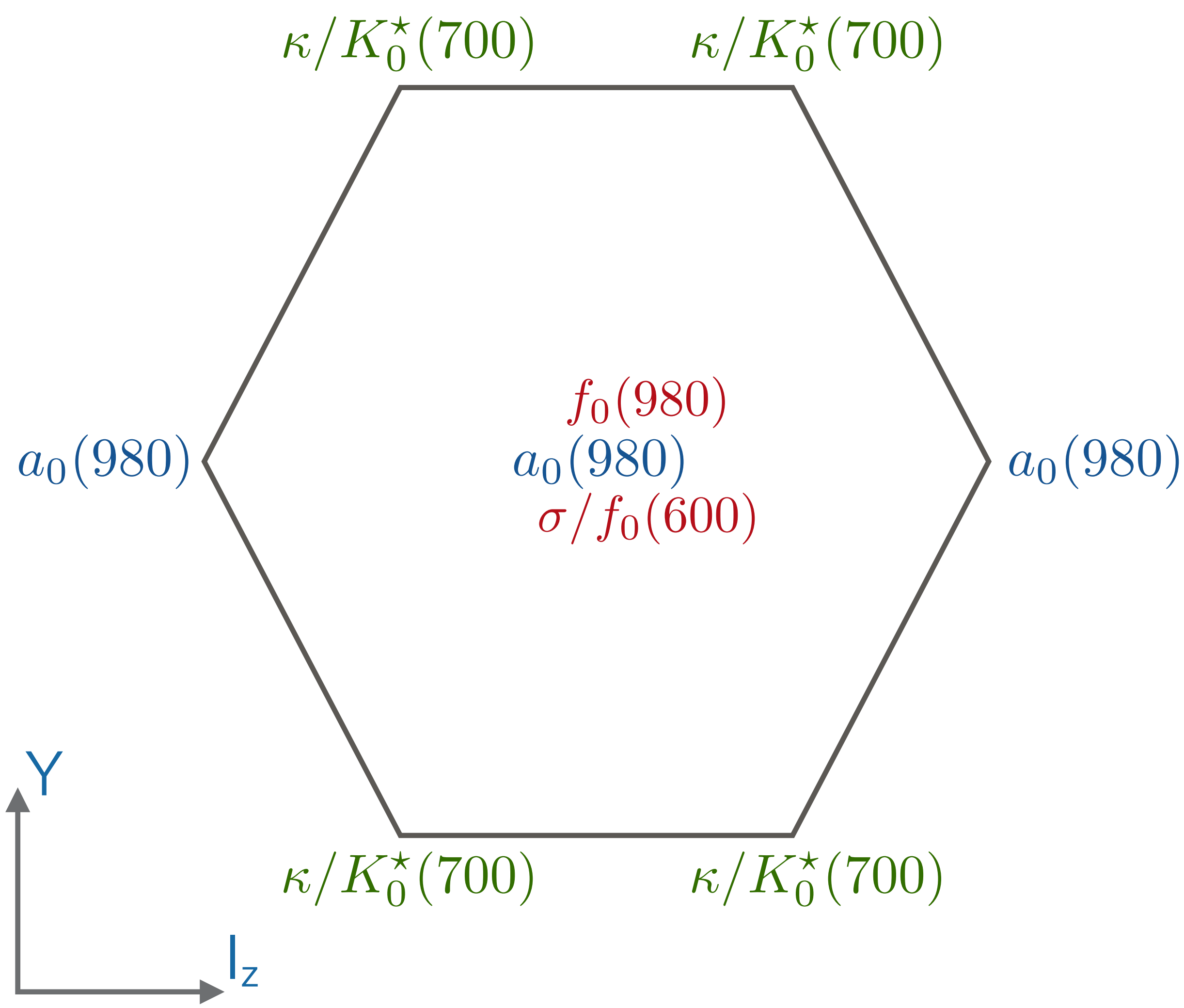
LASS experiment at SLAC $E_K = 11$ GeV

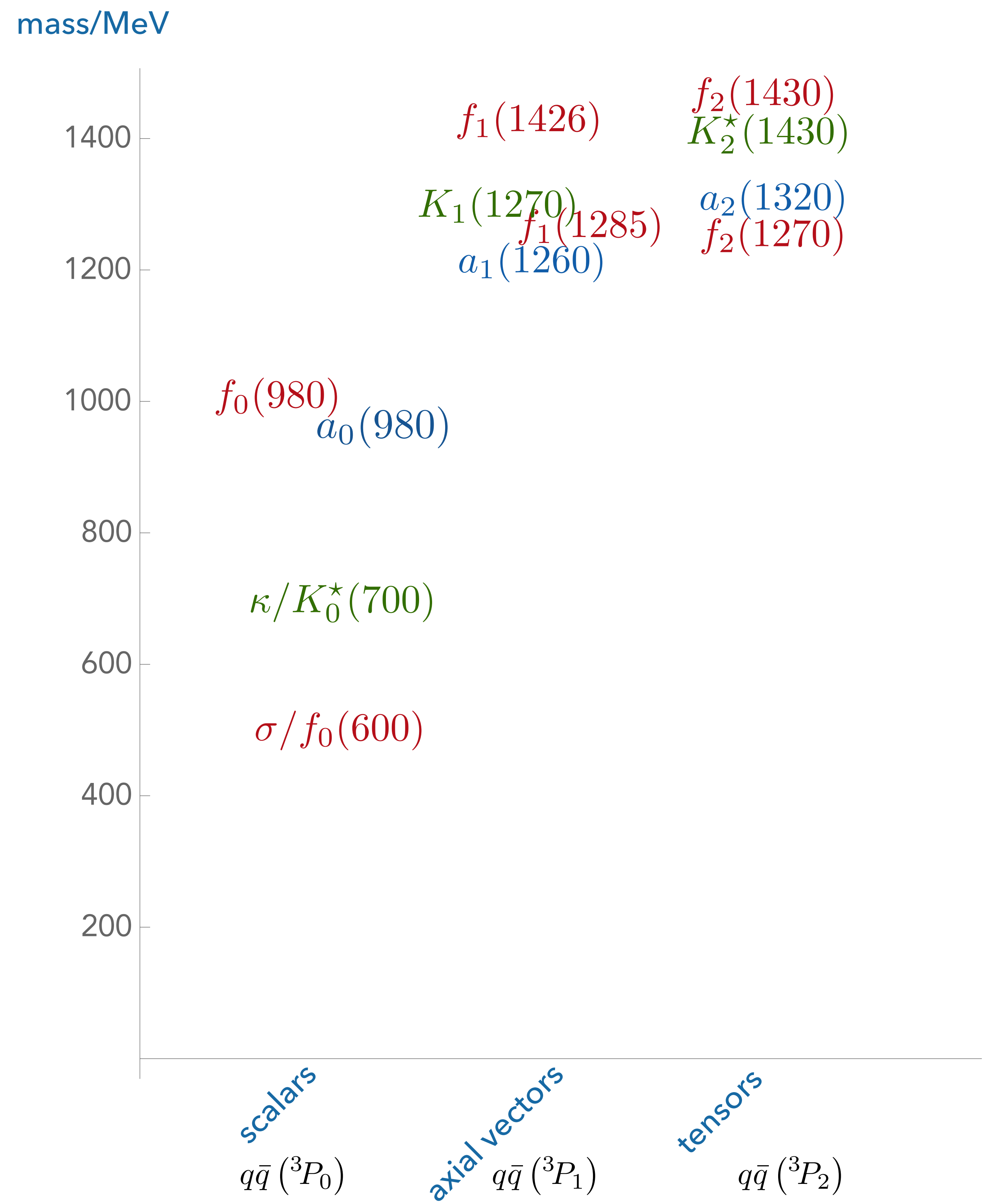






$q\bar{q}\left({}^3P_0\right)$


$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
$a_0(980)$
$f_0(980)$
$a_0(980)$
$\sigma/f_0(600)$
$a_0(980)$
$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
Y
I_z


mass/MeV
1400
1200
1000
800
600
400
200
$f_1(1426)$
$f_2(1430)$
$K_2^\star(1430)$
$K_1(1270)$
$f_1(1285)$
$a_1(1260)$
$a_2(1320)$
$f_2(1270)$
$f_0(980)$
$a_0(980)$
$\kappa/K_0^\star(700)$
$\sigma/f_0(600)$
scalars
axial vectors
tensors
$q\bar{q}\ (^3P_0)$
$q\bar{q}\ (^3P_1)$
$q\bar{q}\ (^3P_2)$

An example D-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

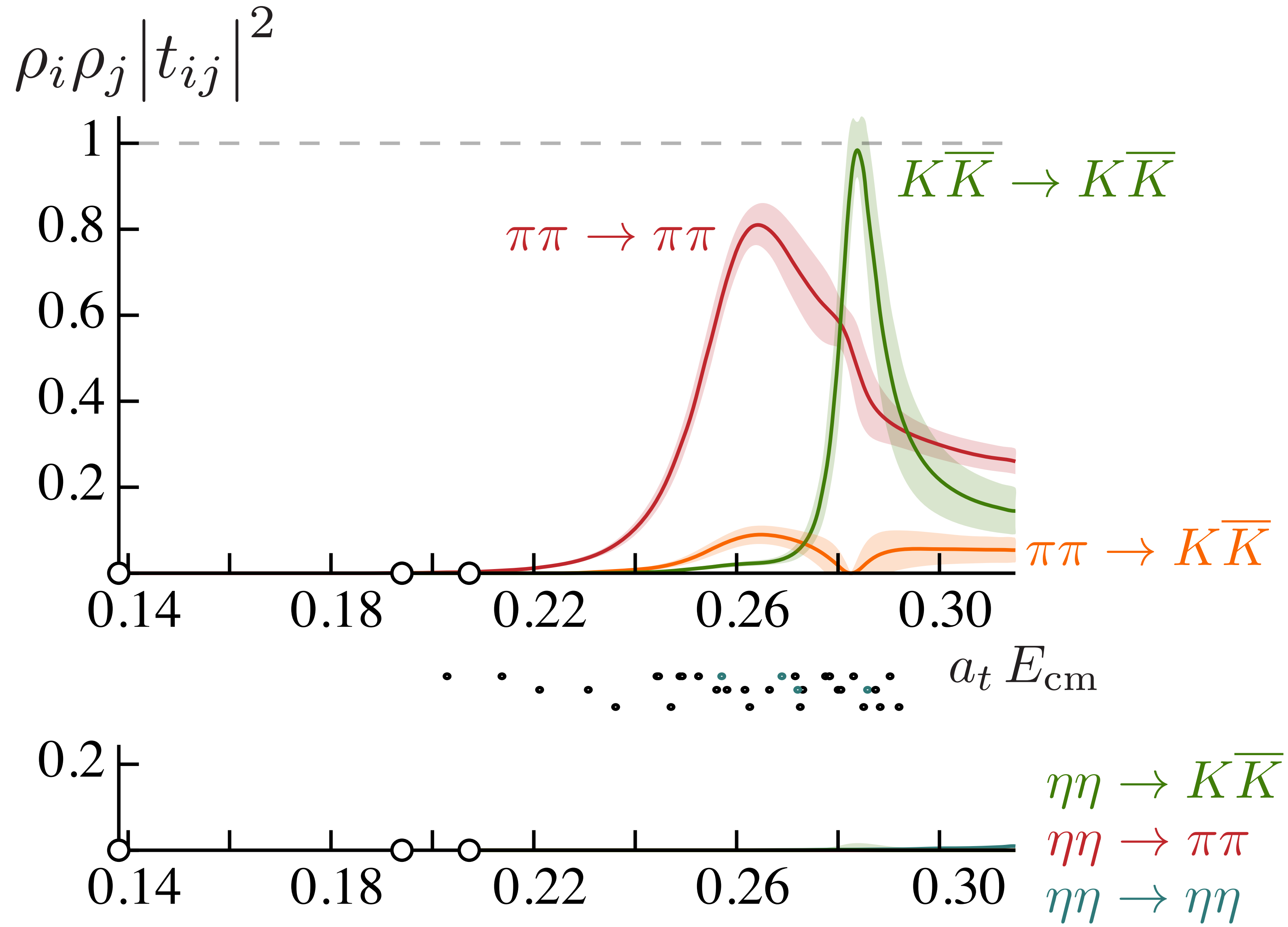
$$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$

$\gamma_{\eta\eta} \neq 0$

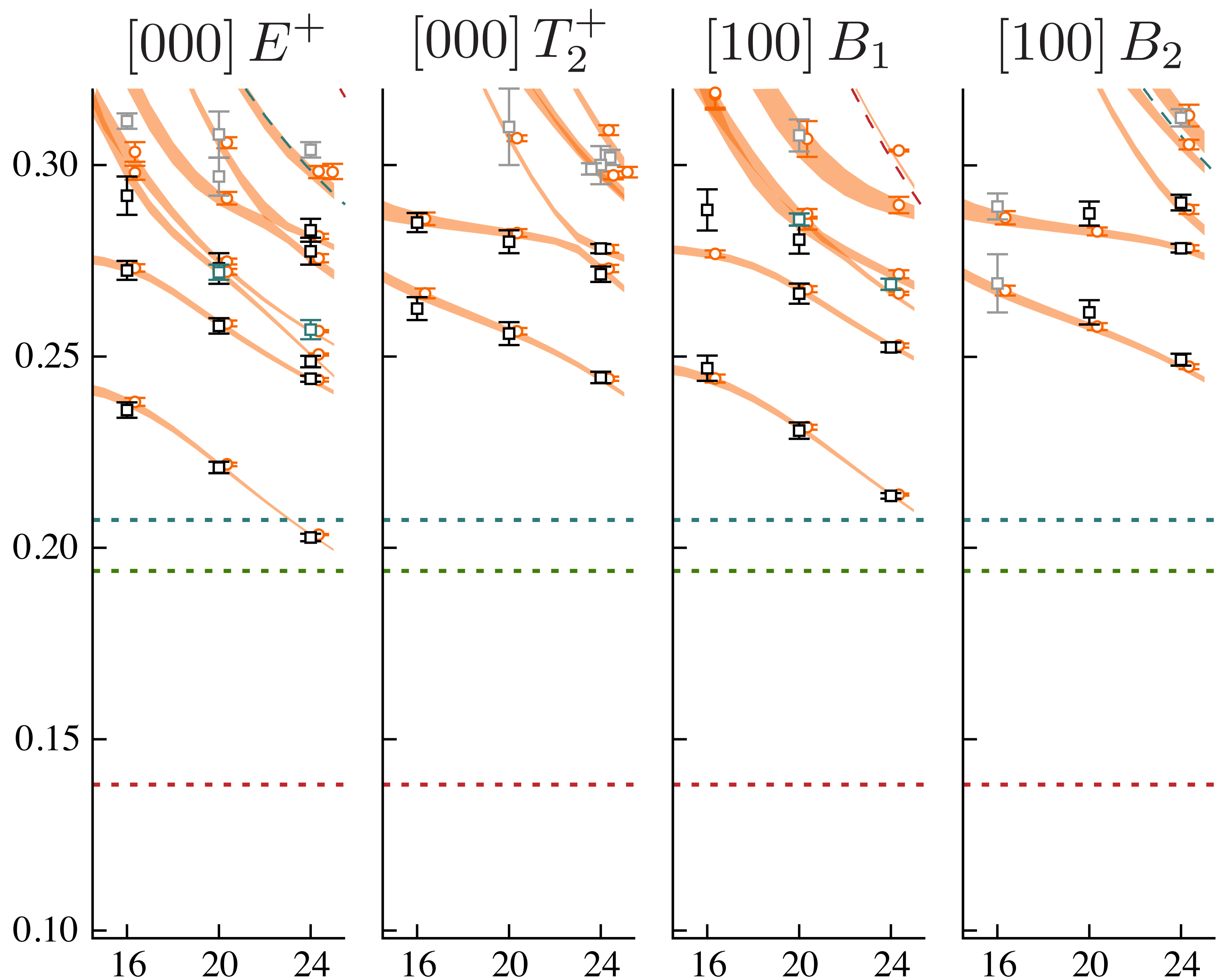
$\gamma_{ij} = 0$ otherwise

$$\chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15$$

34 energy levels



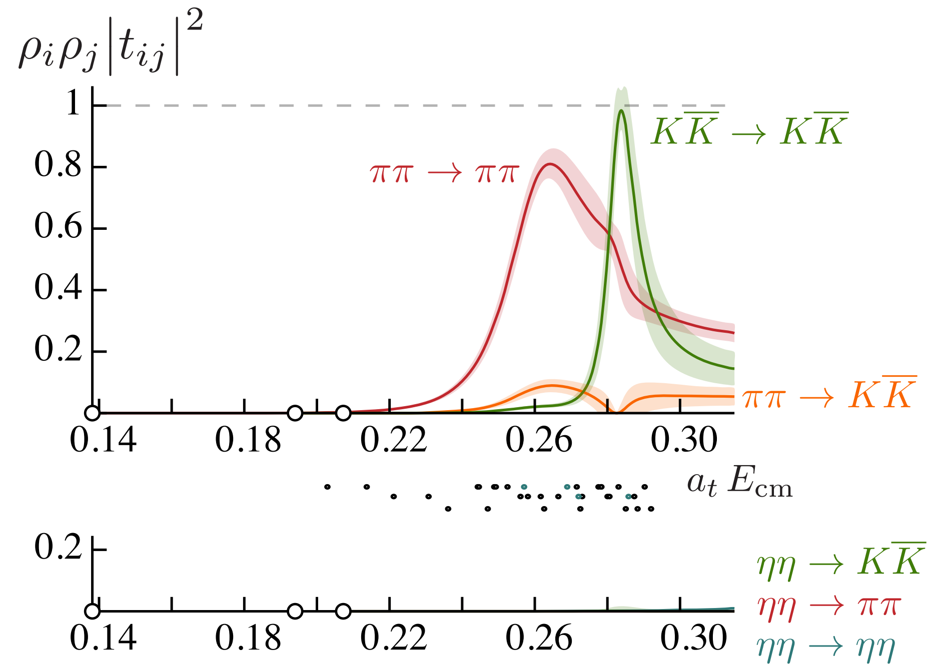
An example D-wave spectrum fit

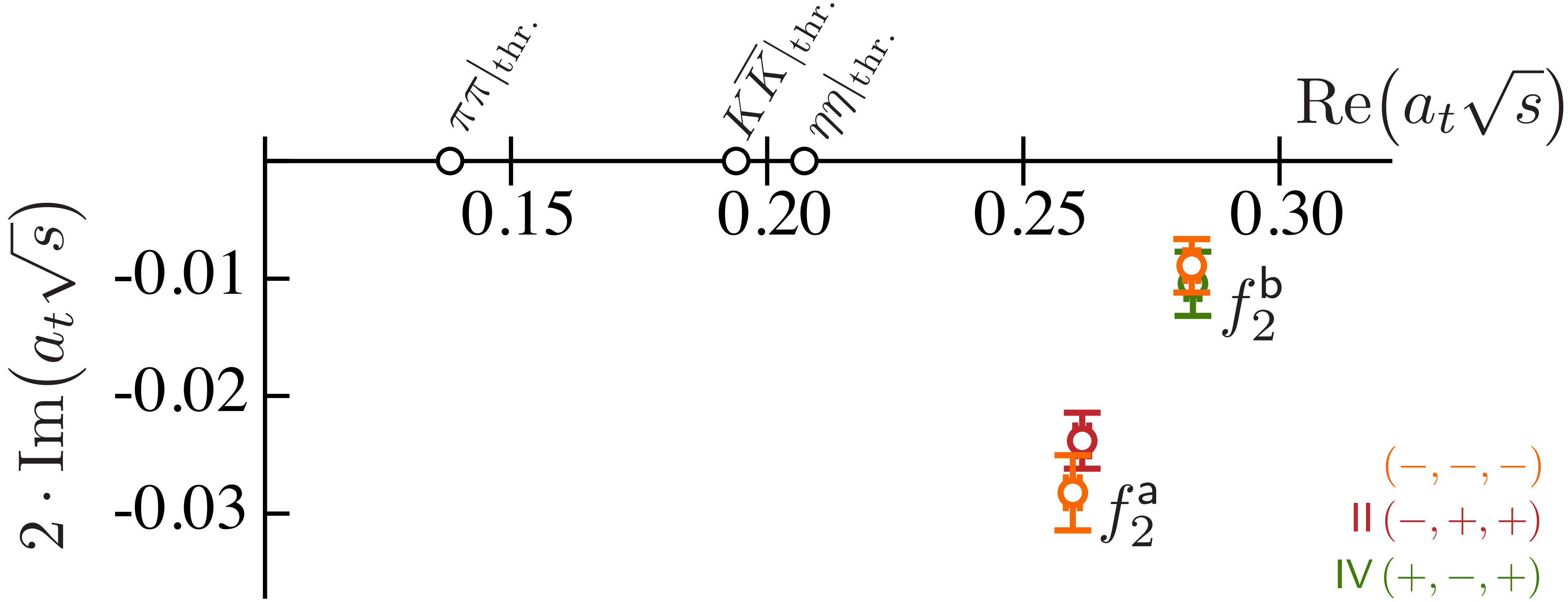


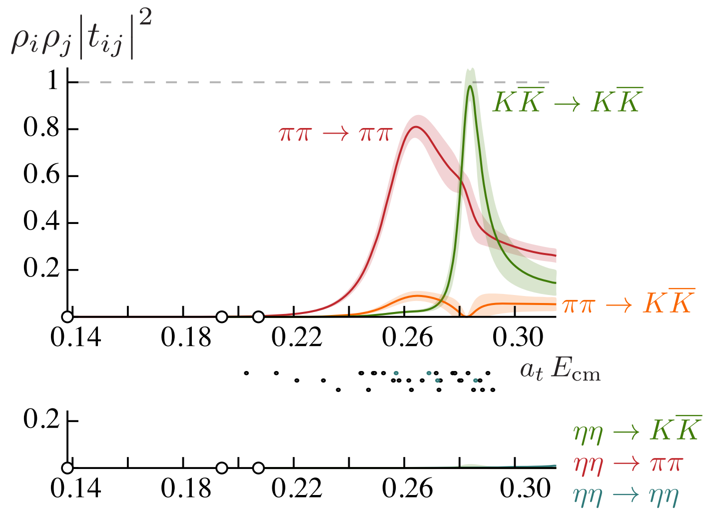
$$\chi^2/N_{\mathrm{dof}} = \frac{28.9}{34-9} = 1.15$$

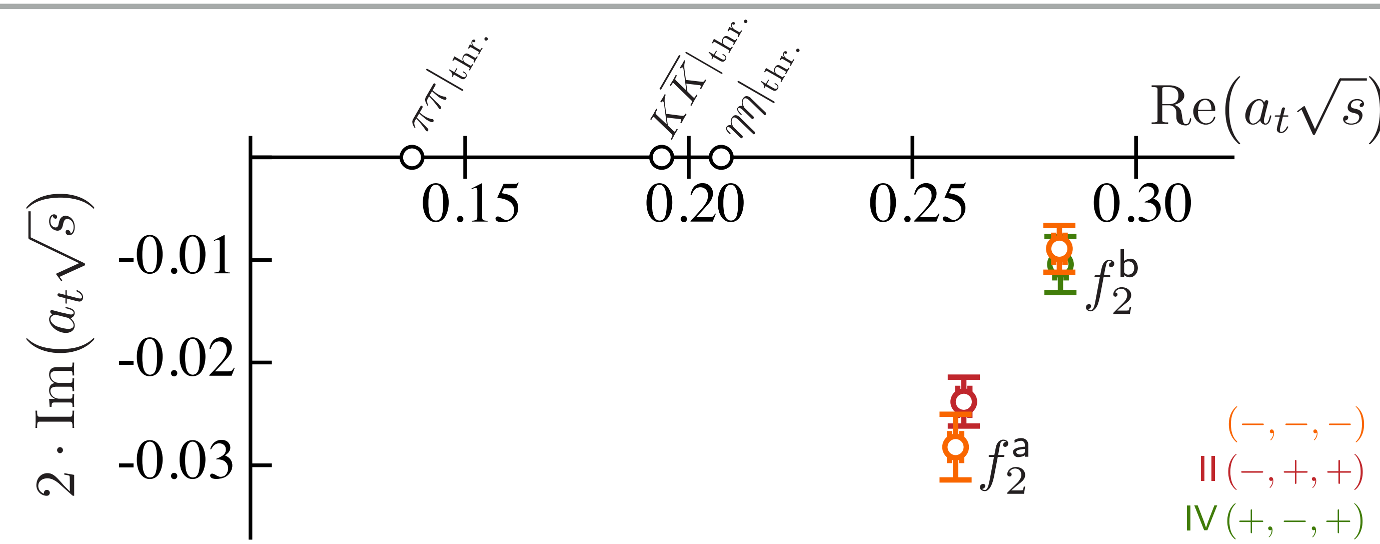
Near a t-matrix pole

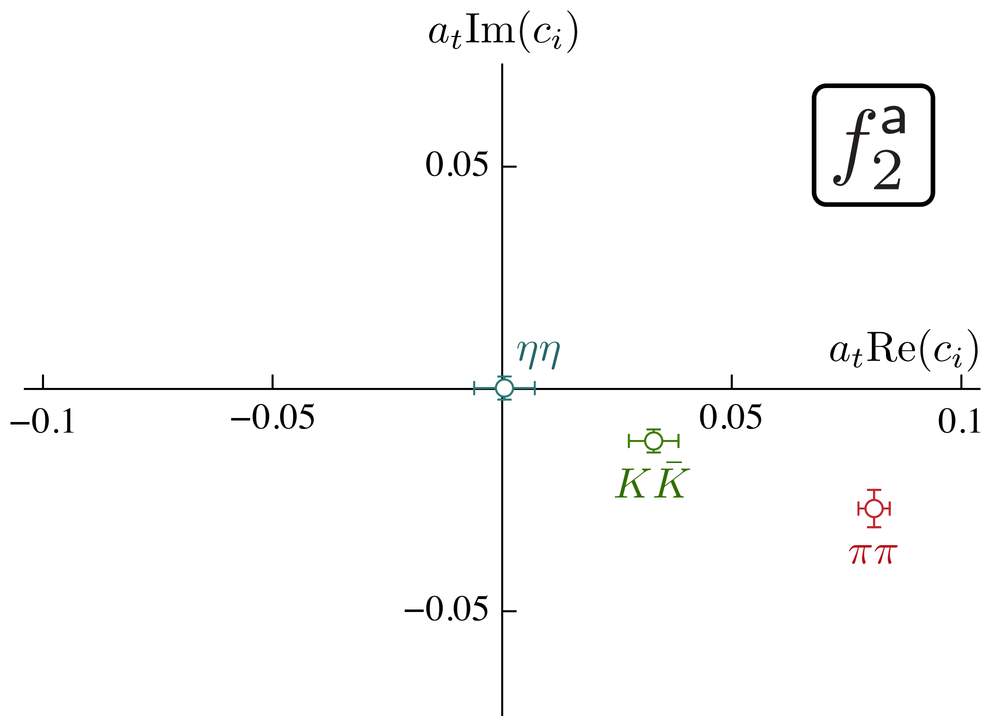
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

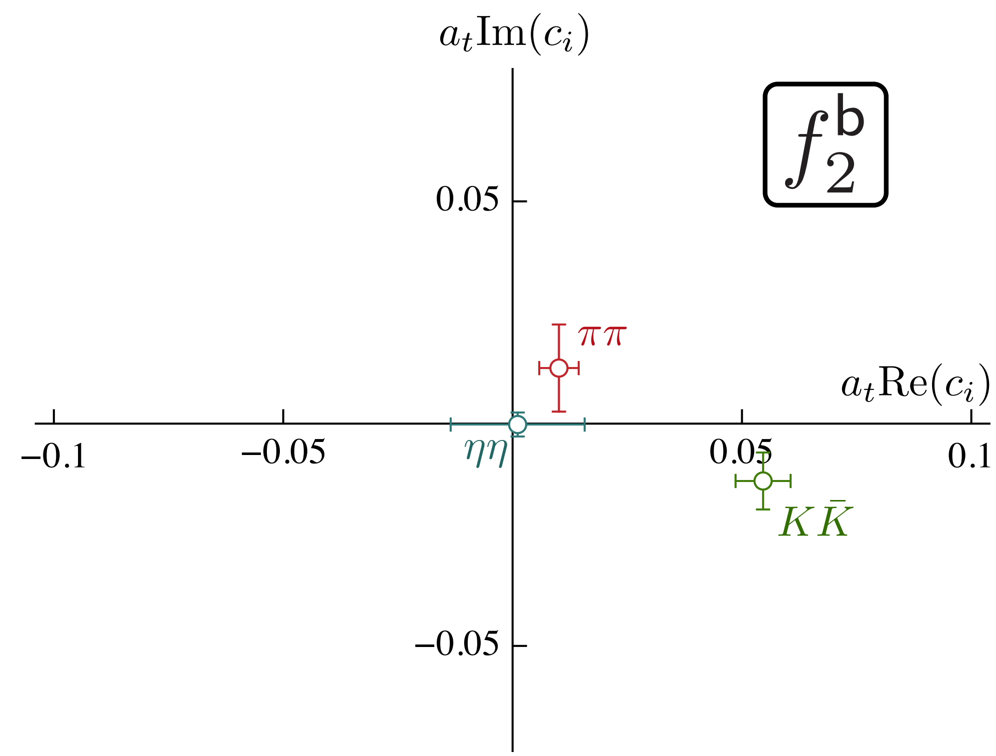












$f_2^{\mathsf{a}}: \quad \sqrt{s_0} = 1470(15) - \tfrac{i}{2}\,160(18)$ MeV

$\mathrm{Br}(f_2^{\mathsf{a}} \to \pi\pi) \sim 85\%, \quad \mathrm{Br}(f_2^{\mathsf{a}} \to K\overline{K}) \sim 12\%$

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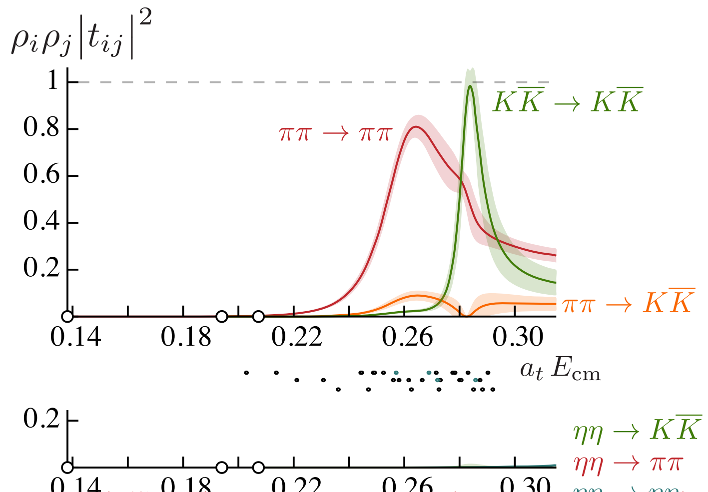
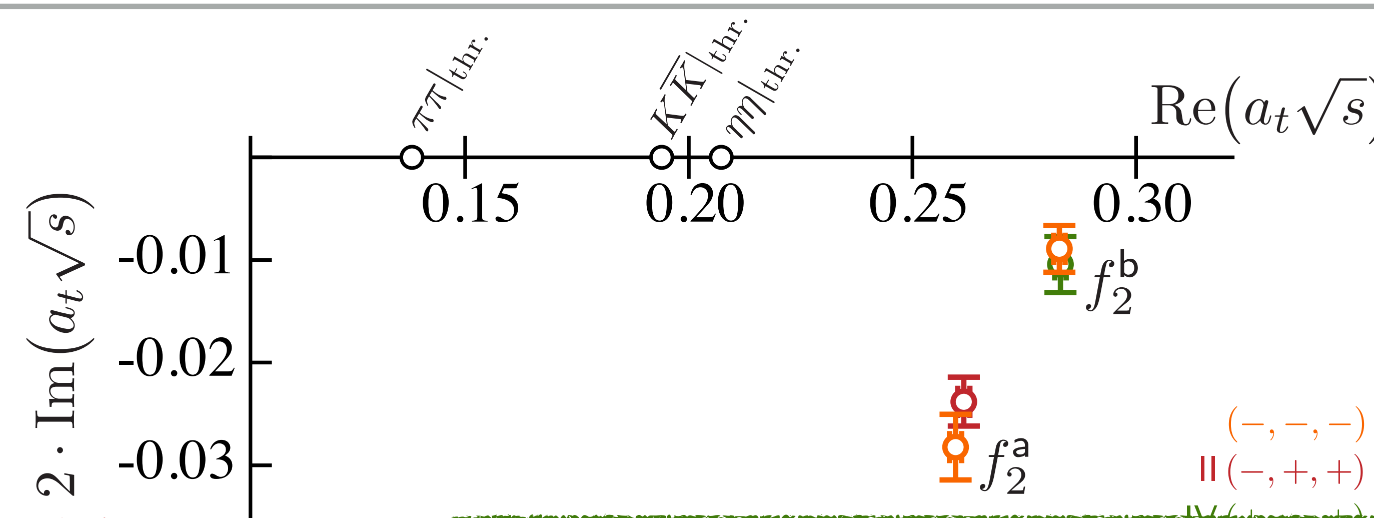
$\mathrm{Br}(f_2^{\mathsf{b}} \to \pi\pi) \sim 8\%, \quad \mathrm{Br}(f_2^{\mathsf{b}} \to K\overline{K}) \sim 92\%$





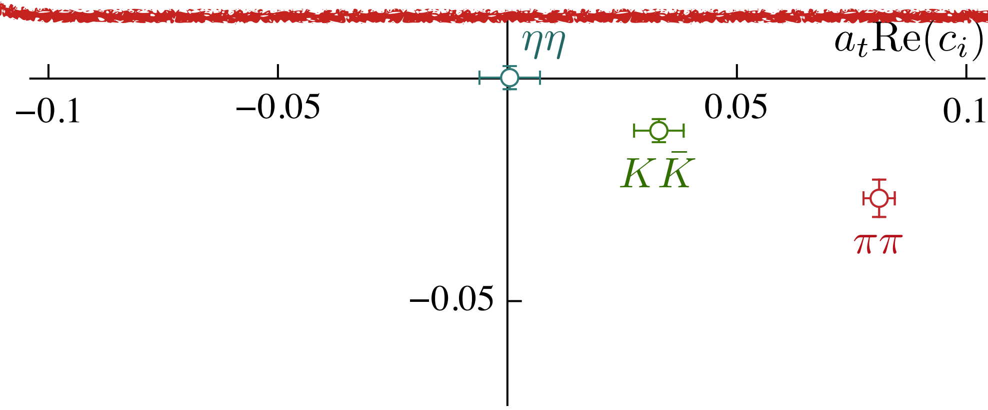
$f_2(1270)$ DECAY MODES

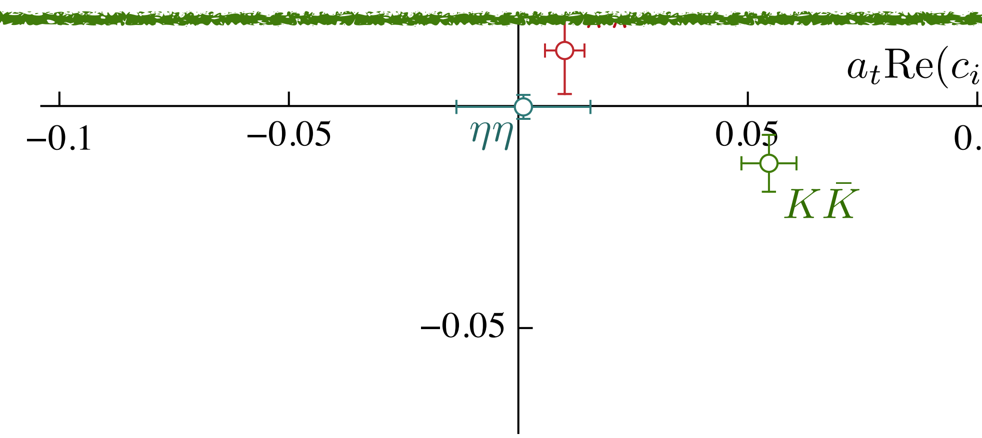
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	$(84.2\ ^{+2.9}_{-0.9})$ %
Γ_2	$\pi^+\pi^-2\pi^0$	$(7.7\ ^{+1.1}_{-3.2})$ %
Γ_3	$K\bar{K}$	$(4.6\ ^{+0.5}_{-0.4})$ %
Γ_4	$2\pi^+2\pi^-$	$(2.8\ \pm 0.4)$ %

PDG2017

$f_2'(1525)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\bar{K}$	$(88.7\ \pm 2.2)$ %
Γ_2	$\eta\eta$	$(10.4\ \pm 2.2)$ %
Γ_3	$\pi\pi$	$(8.2\ \pm 1.5) \times 10^{-3}$
Γ_4	$K\bar{K}^*(892)+$ c.c.	





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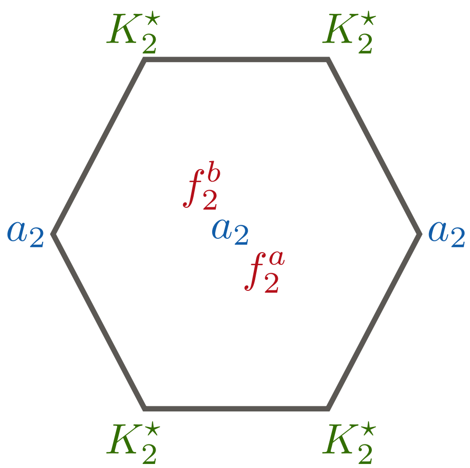
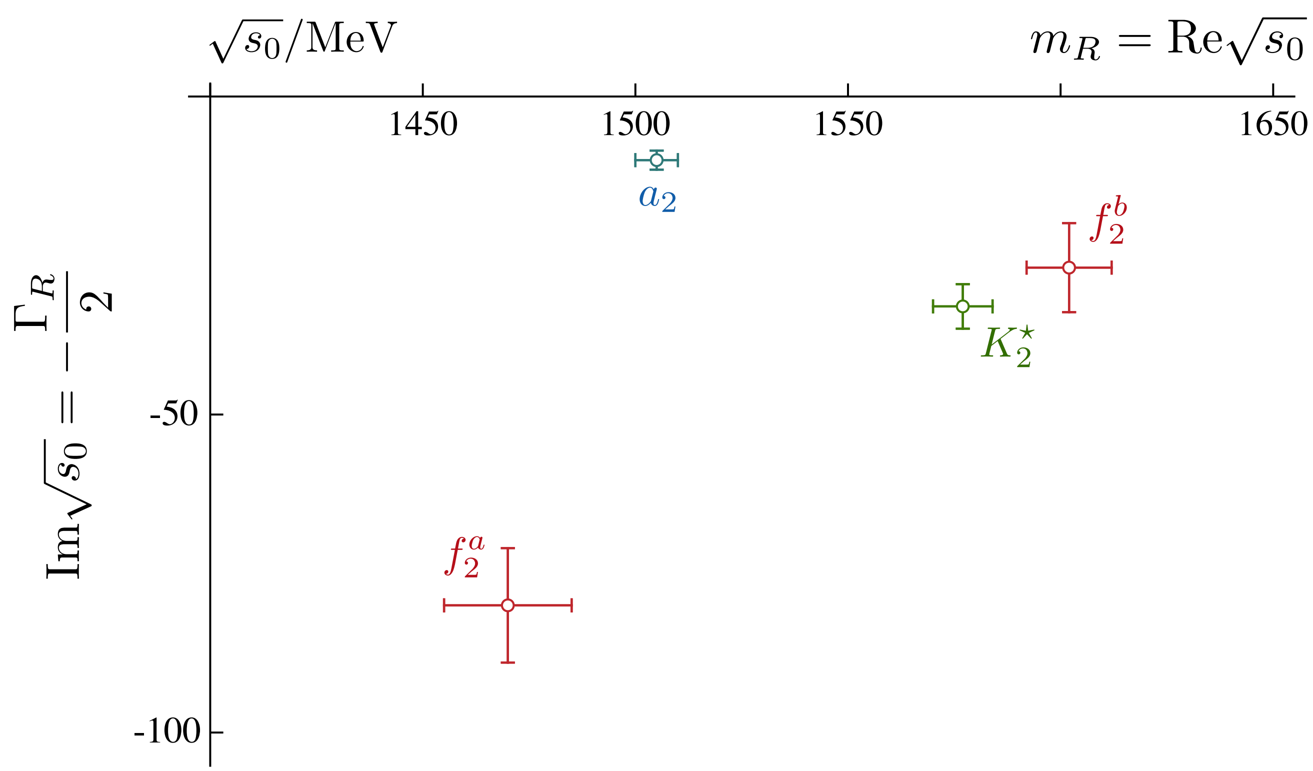
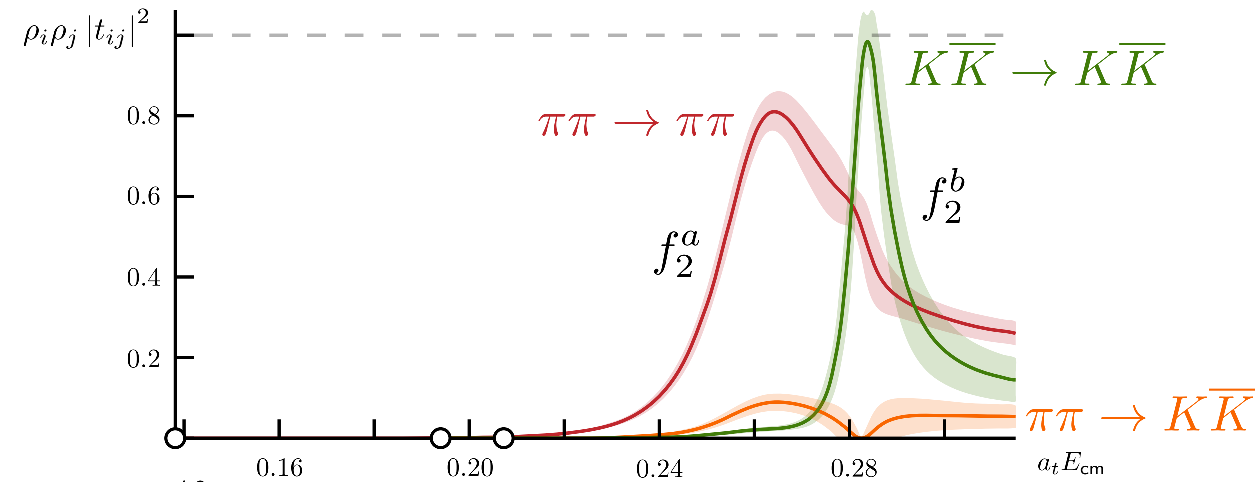
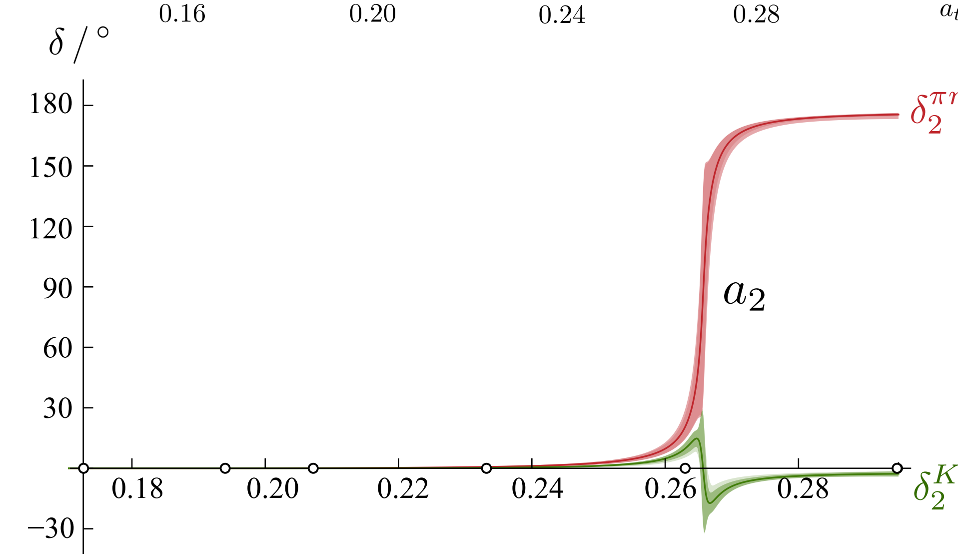


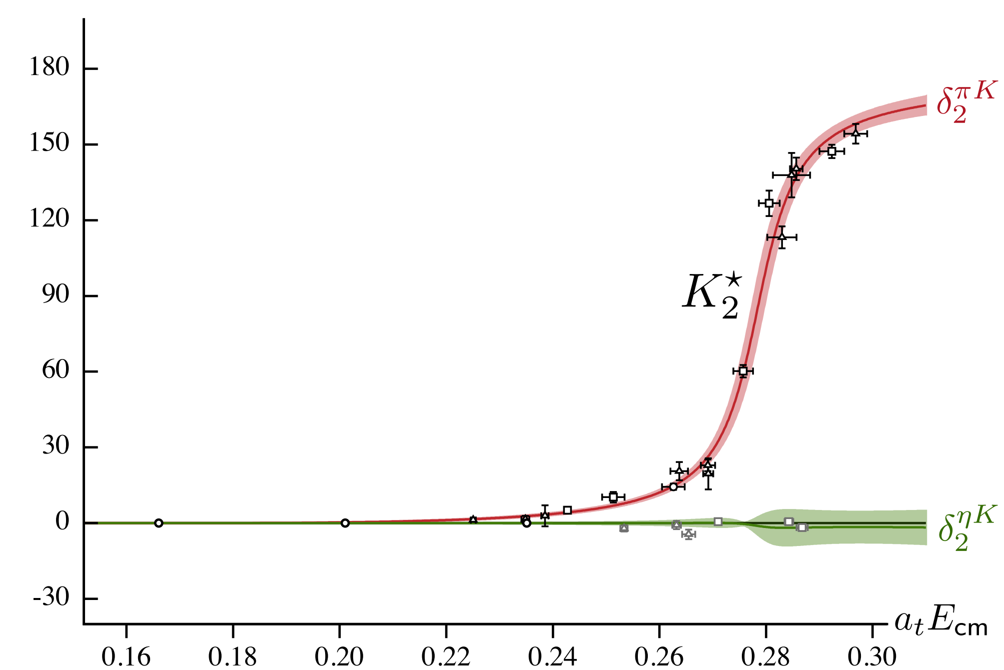
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

$$\sqrt{s_0} = m_R - i\frac{\Gamma_R}{2}$$





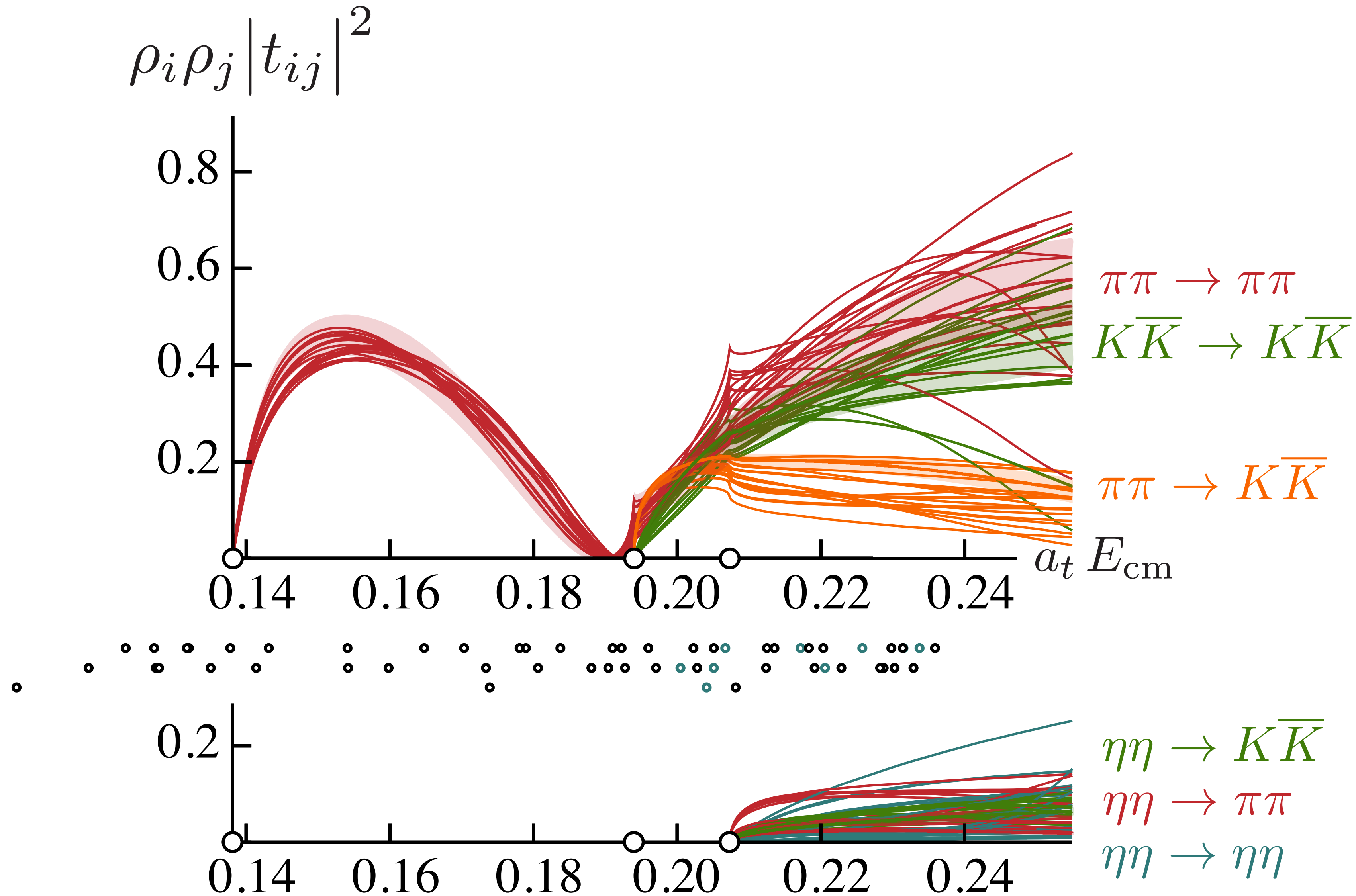




20 amplitudes

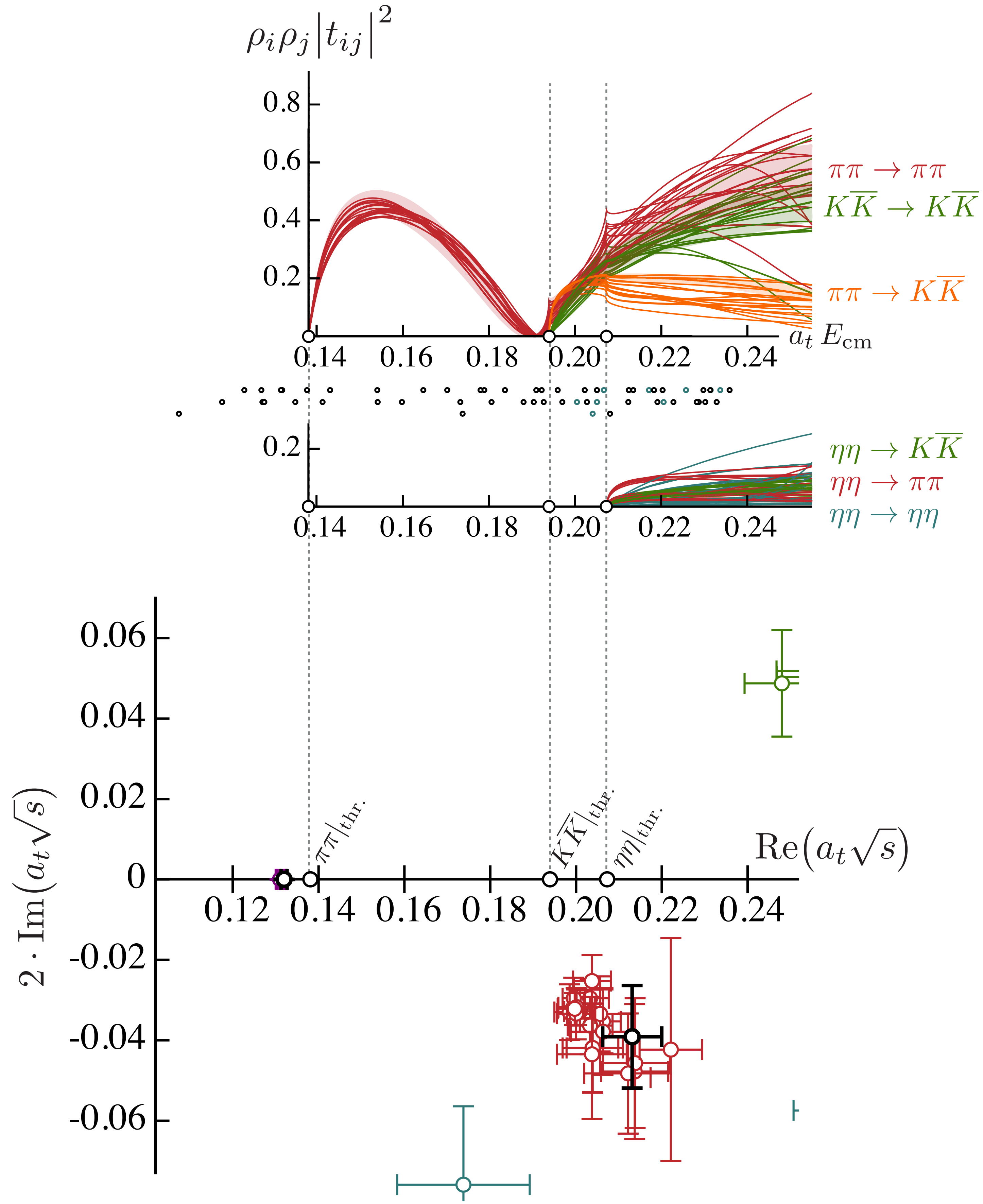
$\chi^2/N_{\mathrm{dof}} < 1.05$

57 energy levels

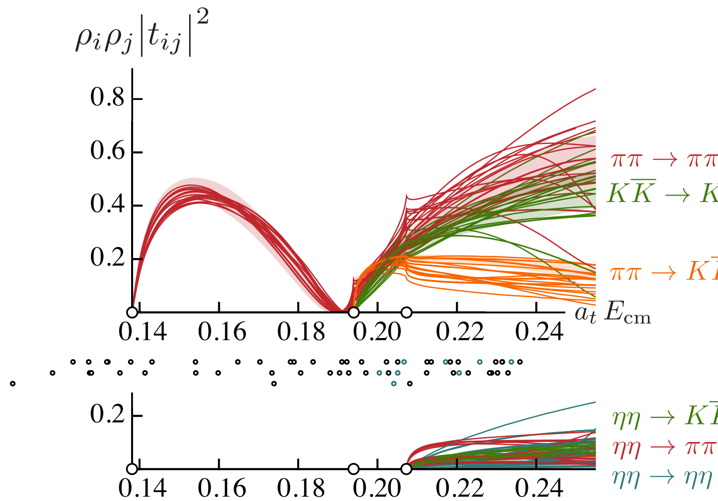


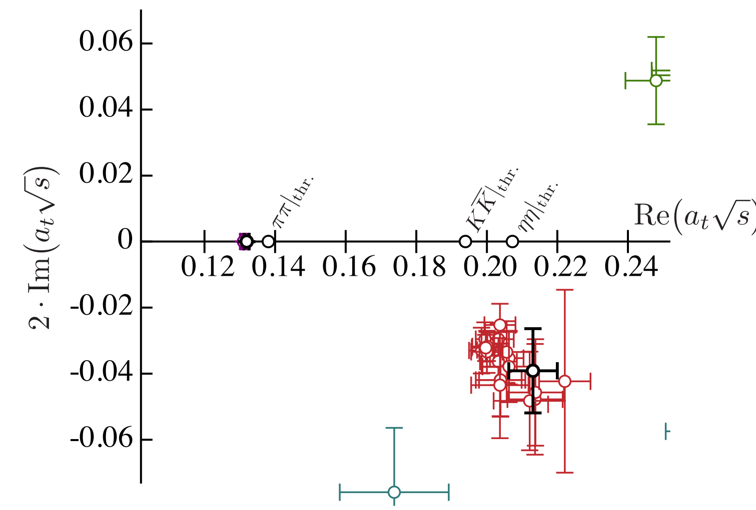
Near a t-matrix pole

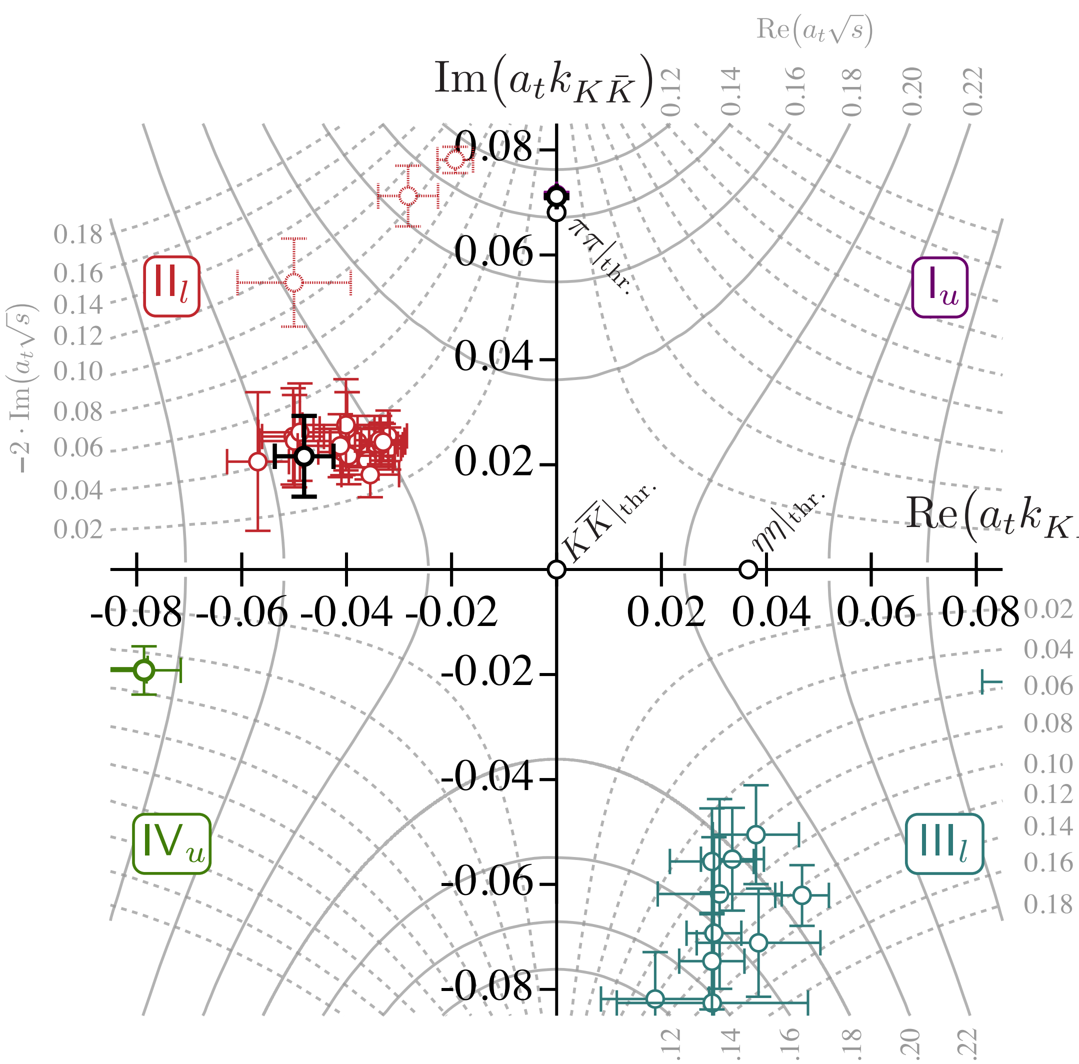
$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

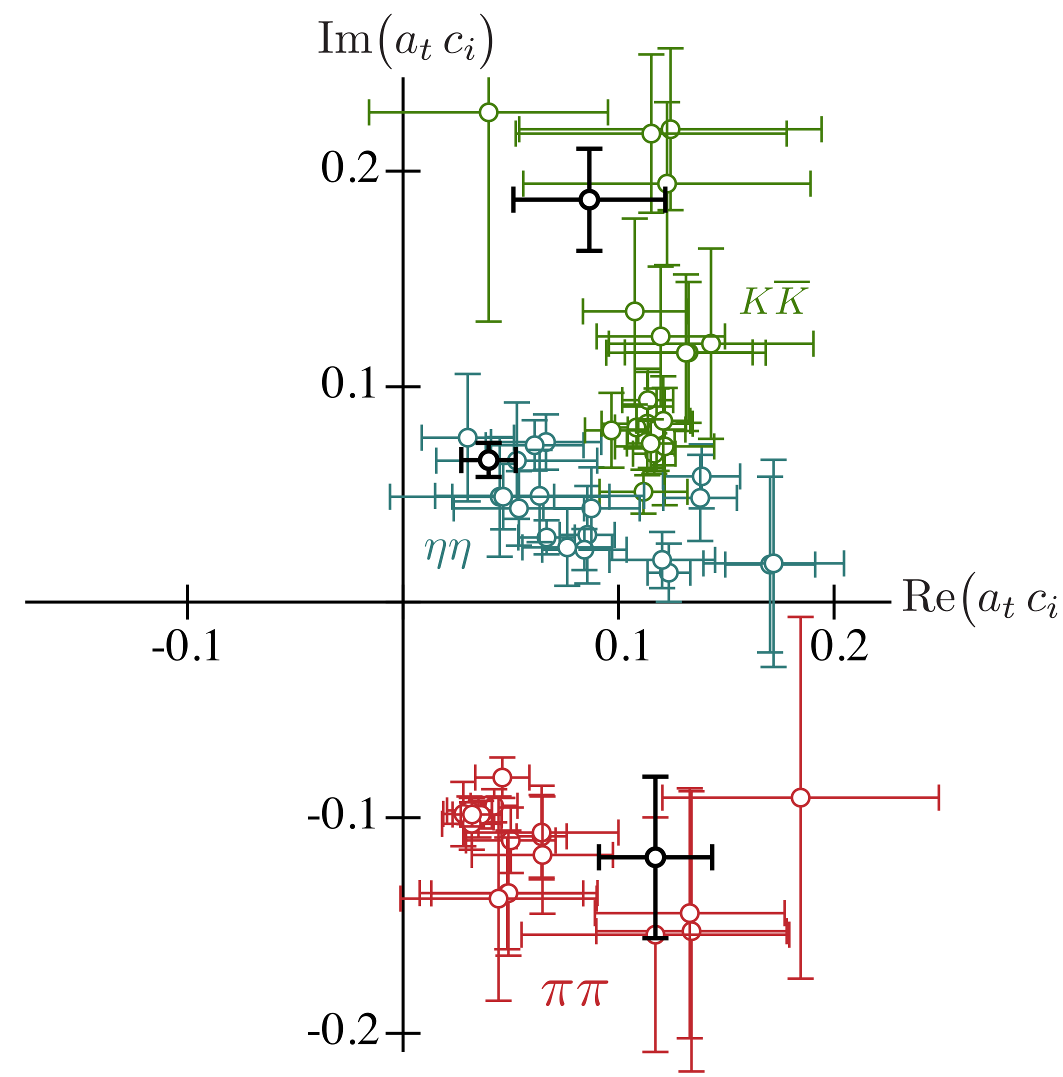


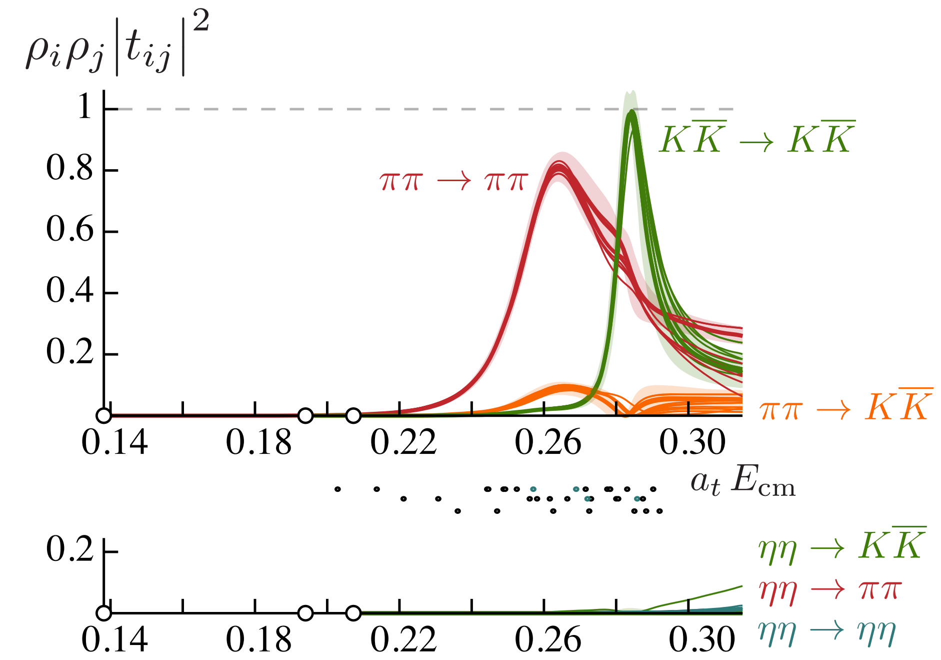
Scalar f0 resonance poles











Near a t-matrix pole

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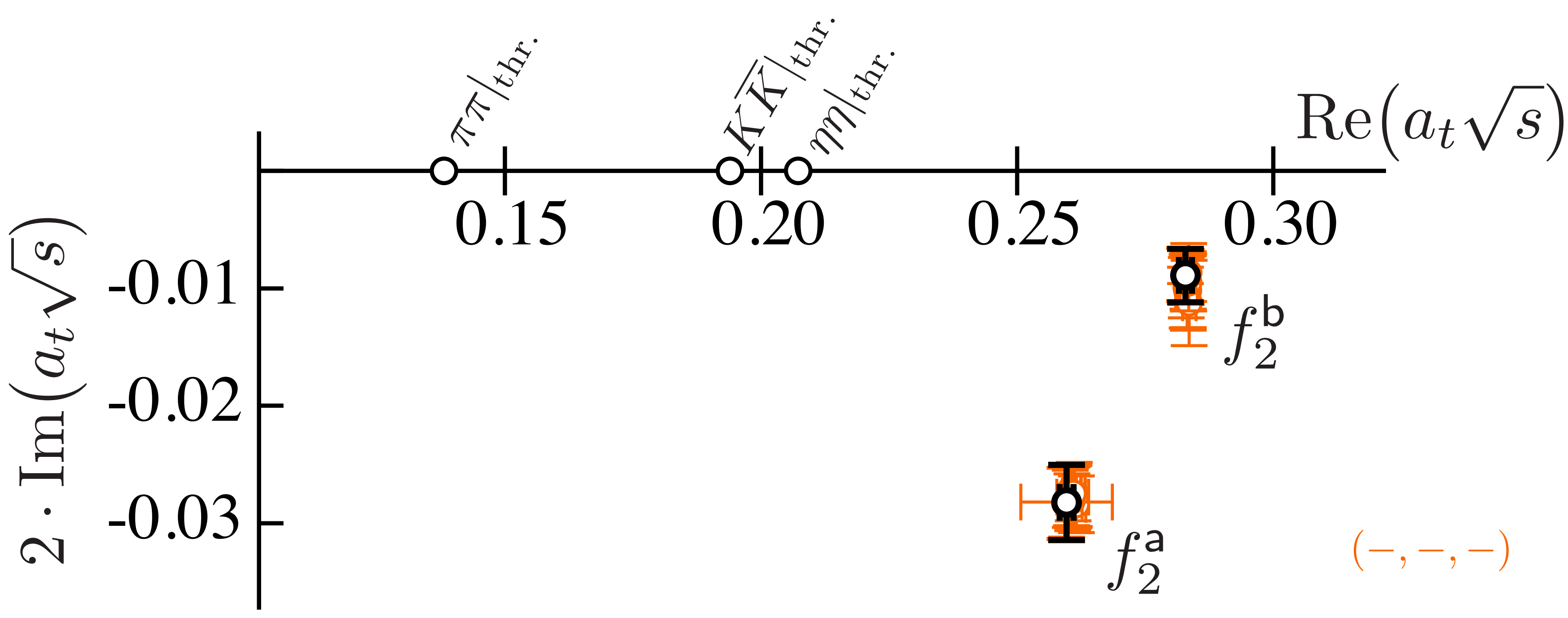
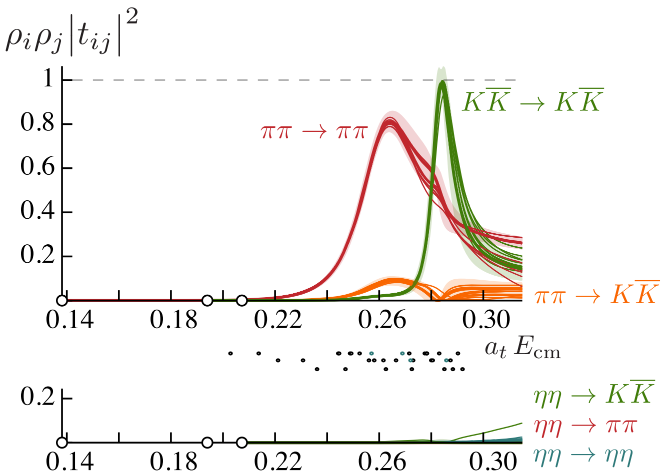
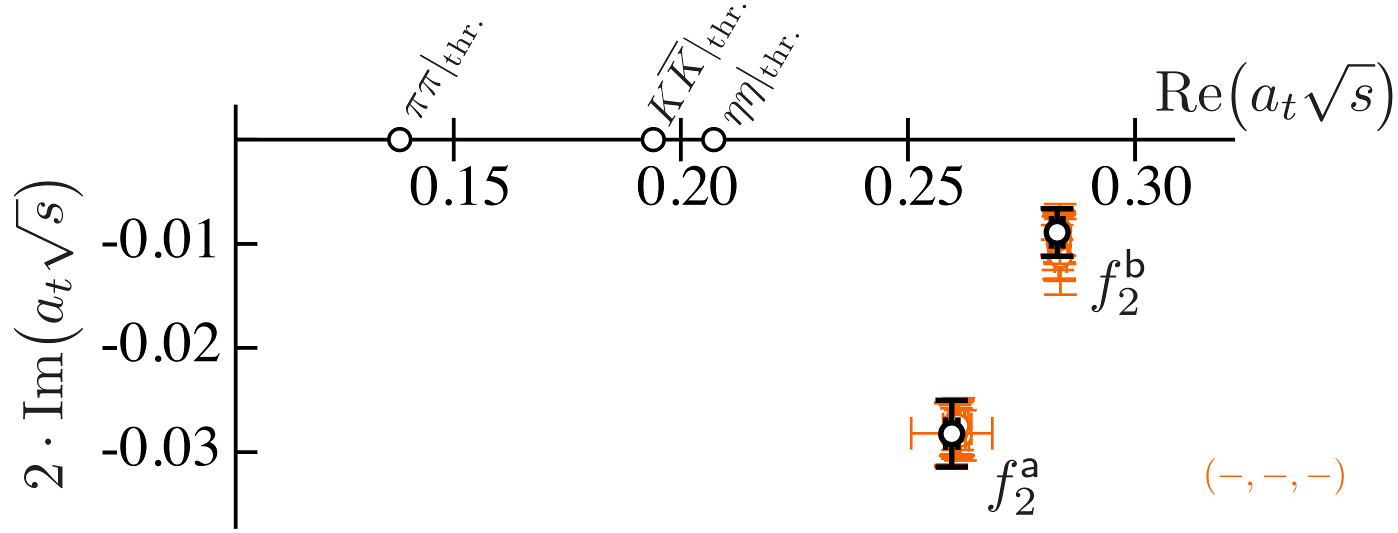
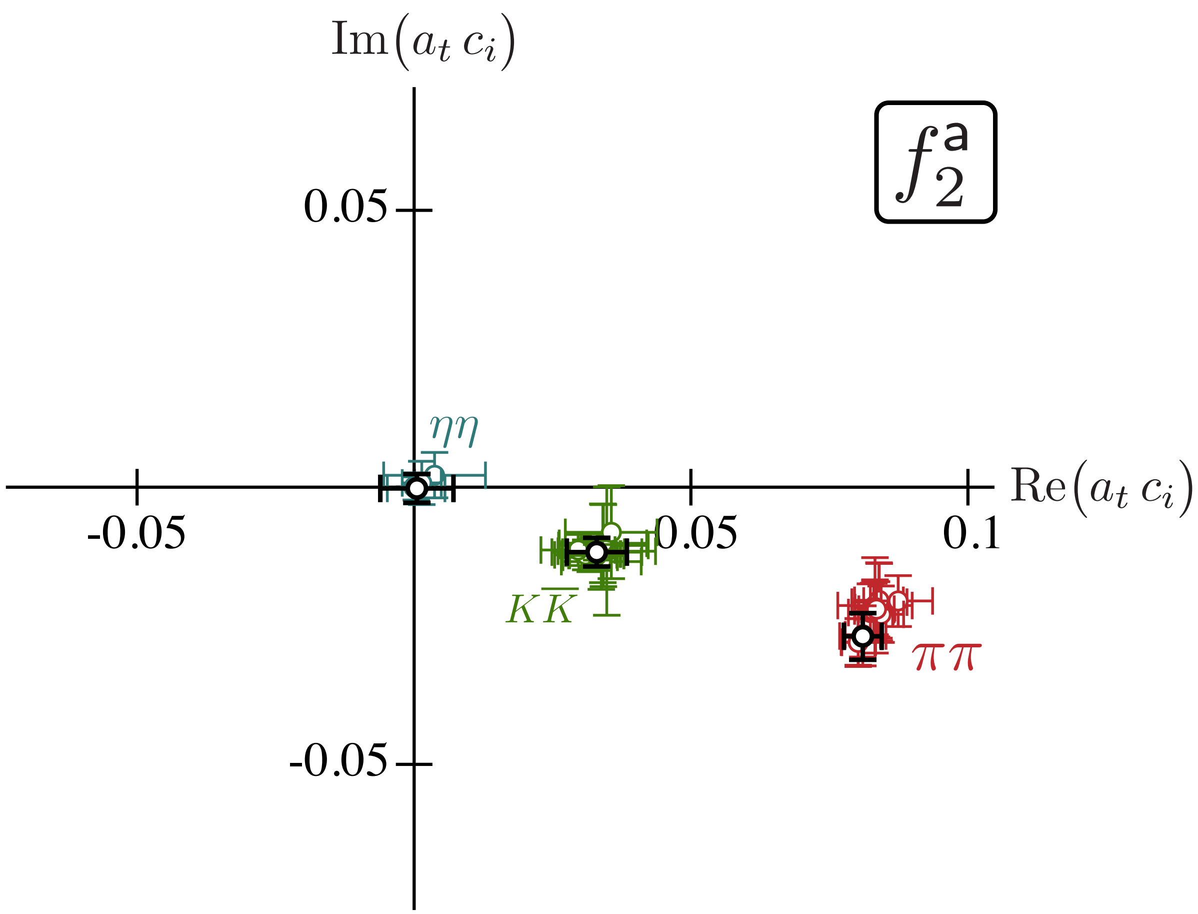
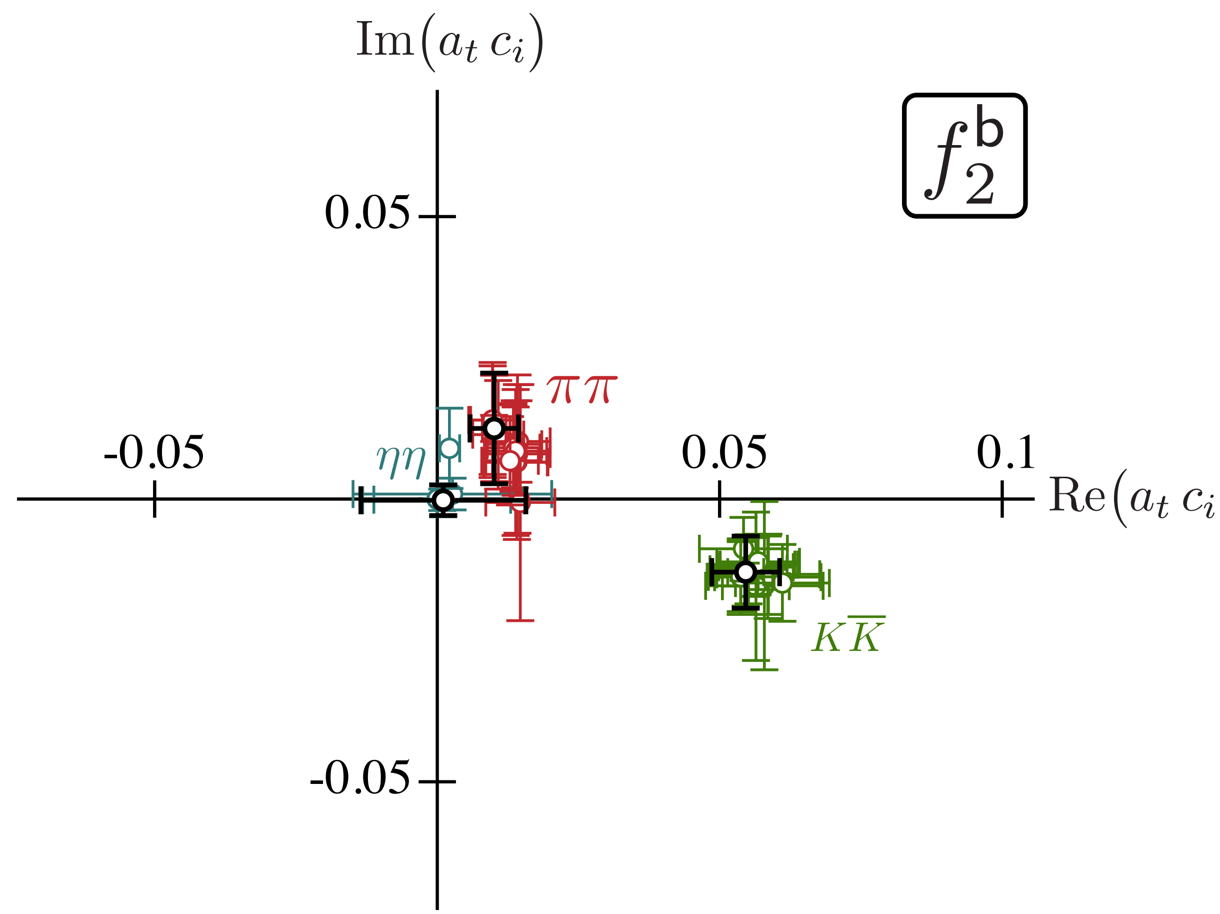


Tensor f2 resonance poles









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Several methods - many challenges similar to experimental analyses

- Weinberg method, uses renormalisation factor Z

  Useful for bound states - distinguishes composite meson-meson vs compact

- Morgan pole counting

  Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact

  e.g. Morgan & Pennington $f_0(980)$

- Photocouplings - determine radial extent

- Decay constants

- N_c dependence

  qq states become stable, meson-meson sink into continuum - Pelaez *et al*

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  How a near-threshold state reacts to changes in the masses can give clues

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  How a near-threshold state reacts to changes in the masses can give clues

requires significant extra computation

- Weinberg method

Useful for bound states - distinguishes composite meson-meson vs compact

$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

$$r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi \epsilon}}$$

$$Z = 1 \sim \text{compact}$$

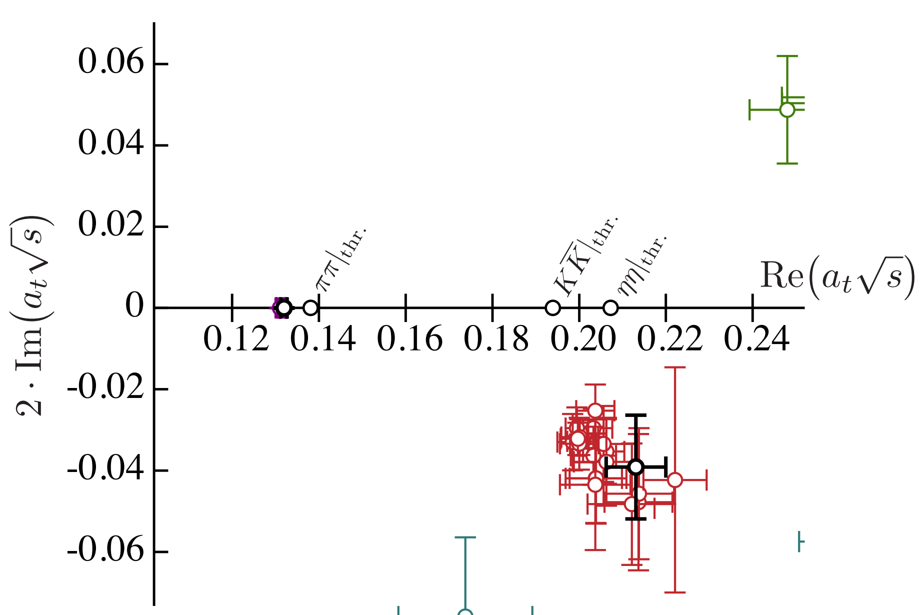
$$Z = 0 \sim \text{molecule}$$

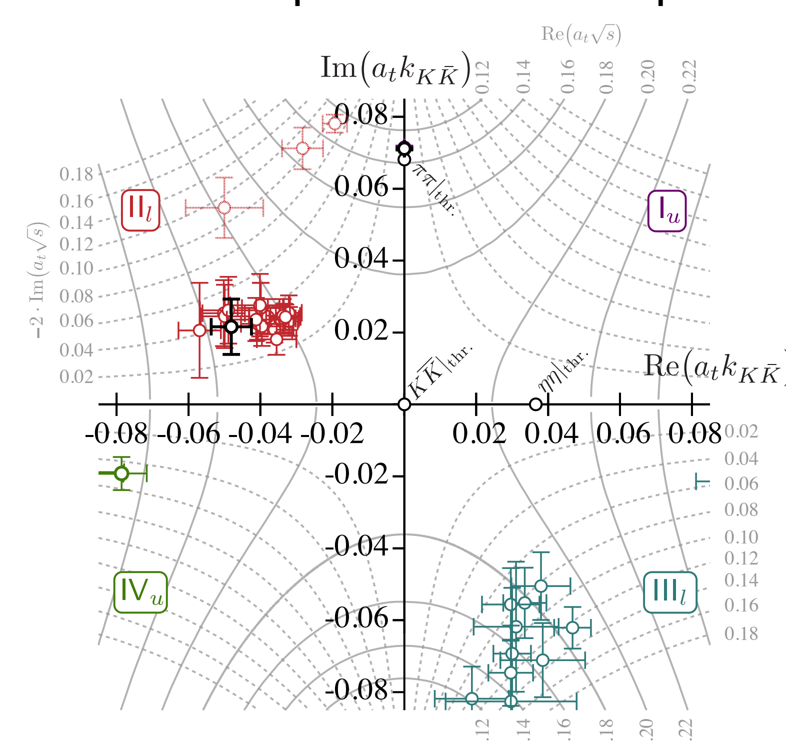
$$Z \sim 0.3(1)$$

(σ bound state)

- Morgan pole counting

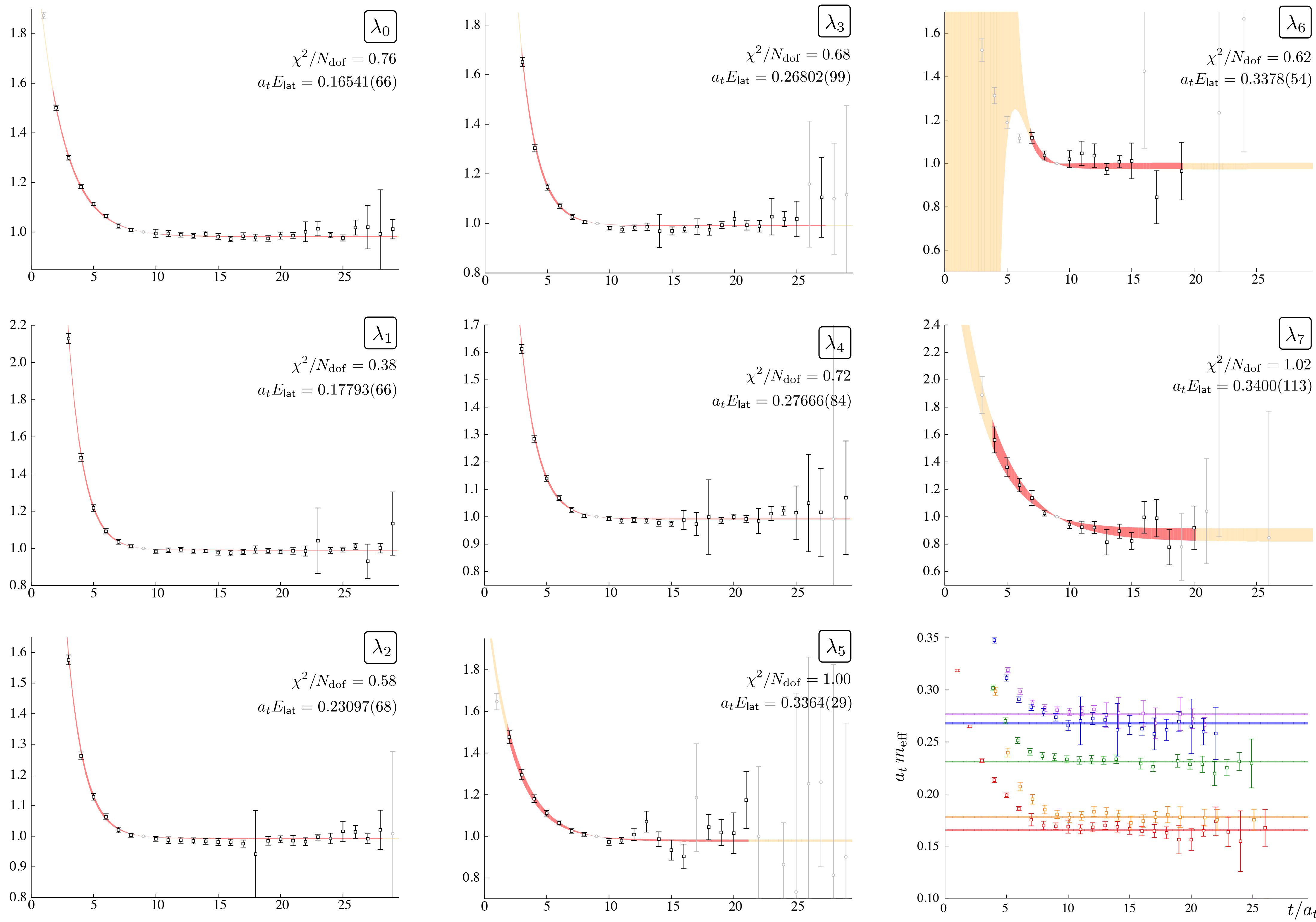
Generalisation of Weinberg - one pole ~ molecular, vs two poles ~ compact



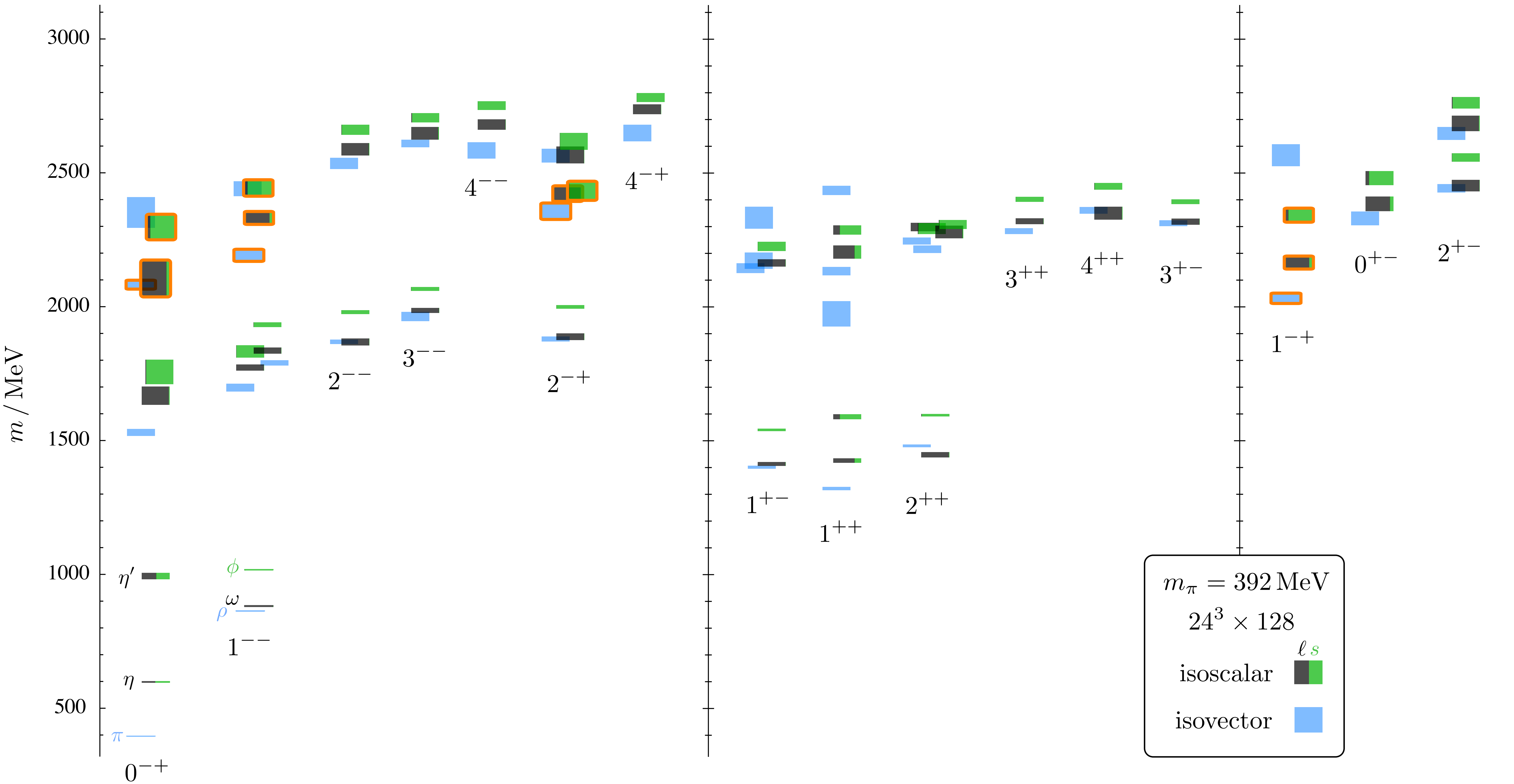


Obtain the finite volume spectrum using a variational method

$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda(t)_{\mathfrak{n}} C_{ij}(t_0)v_j^{\mathfrak{n}} \quad \rightarrow \quad \lambda_{\mathfrak{n}} \sim \exp(-E_{\mathfrak{n}} t)$$



J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.



J. J. Dudek, R. G. Edwards, P. Guo and C. E. Thomas, Phys.Rev. D88 (2013) 9, 094505.

- *K*-matrix contains everything not constrained by unitarity

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) - i\delta_{ij}\rho_i(s)$$

$$K = \frac{1}{m^2 - s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K} \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

- Chew-Mandelstam phase space -- include also *s*-channel cut along with imaginary part.

$$t^{-1}_{ij}(s) = K^{-1}_{ij}(s) + \delta_{ij}\ I_i(s)$$

$$I_i(s) = I_i(s_{\mathrm{thr}_i}) - \frac{s - s_{\mathrm{thr}_i}}{\pi}\int_{s_{\mathrm{thr}_i}}^{\infty} ds'\, \frac{\rho_i(s')}{(s'-s)(s'-s_{\mathrm{thr}_i})}$$

(Subtract at pole so that Re $I(s = m^2) = 0$)

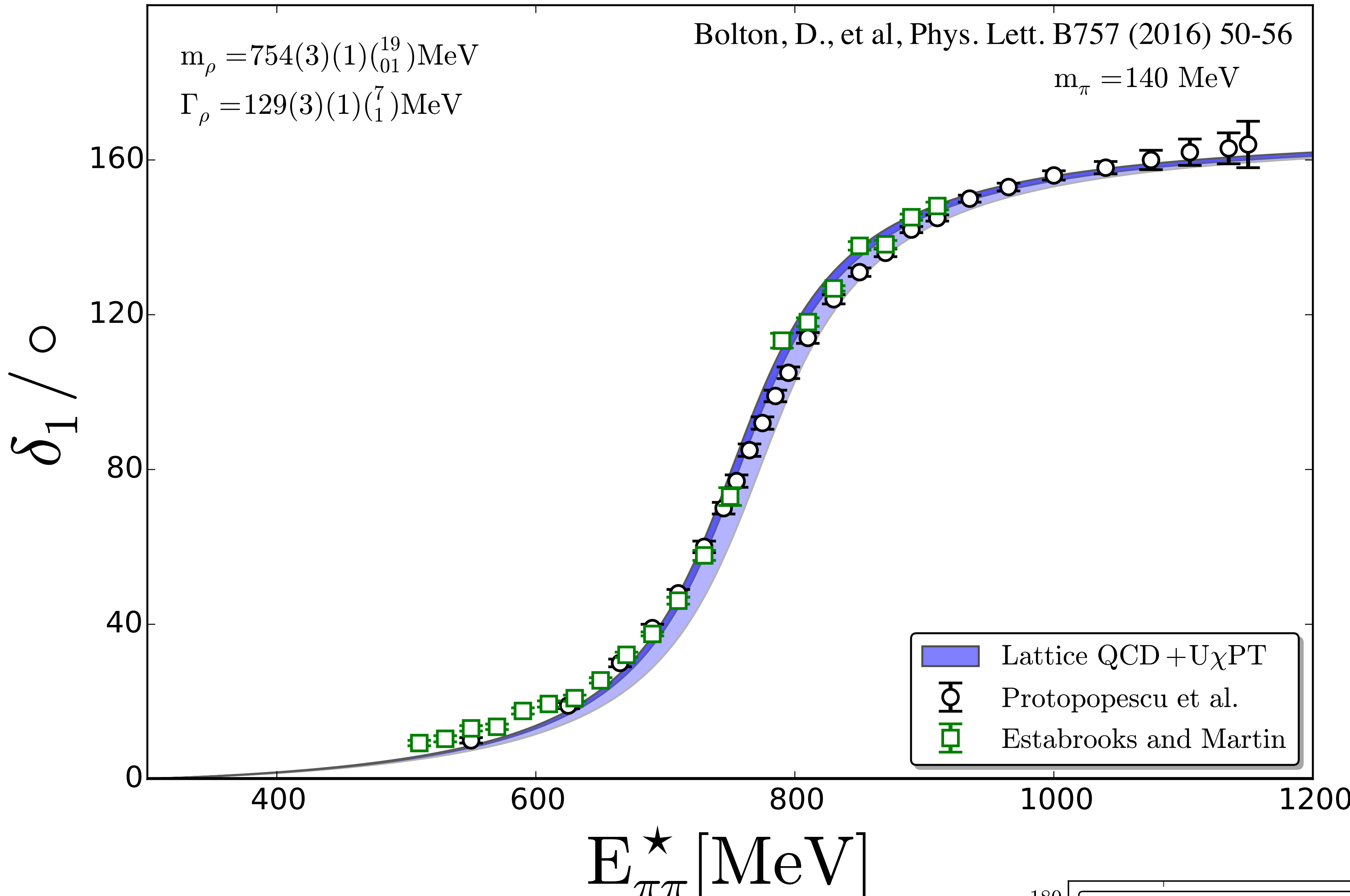
- Threshold factors for $l>0$

$$t^{-1}_{ij}(s) = \frac{1}{(2k_i)^{\ell}} K^{-1}_{ij}(s) \frac{1}{(2k_j)^{\ell}} + \delta_{ij}\ I_i(s)$$

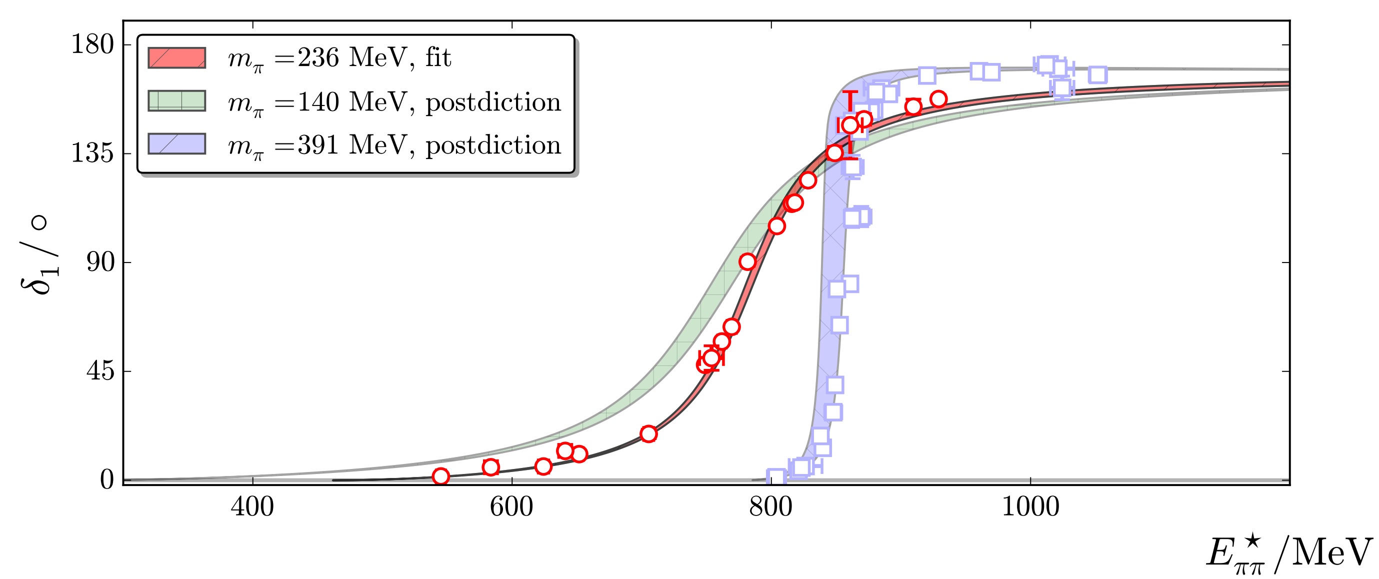
As used in Guo, Mitchell and Szczepaniak Phys.Rev. D82 (2010) 094002

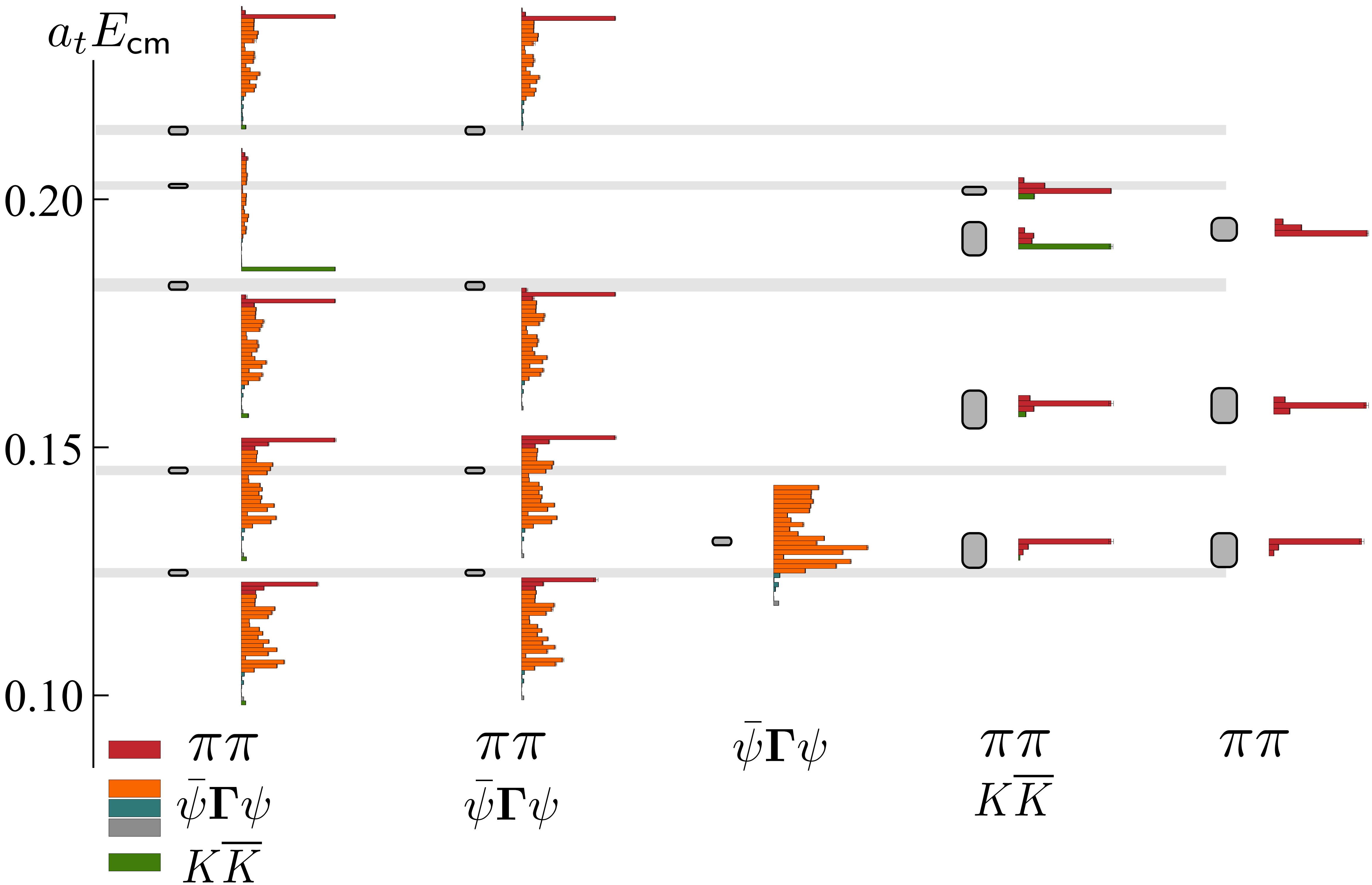
No modifications were used in I(s) for higher waves.

Also tested phase space factors instead of k_i for thresholds.

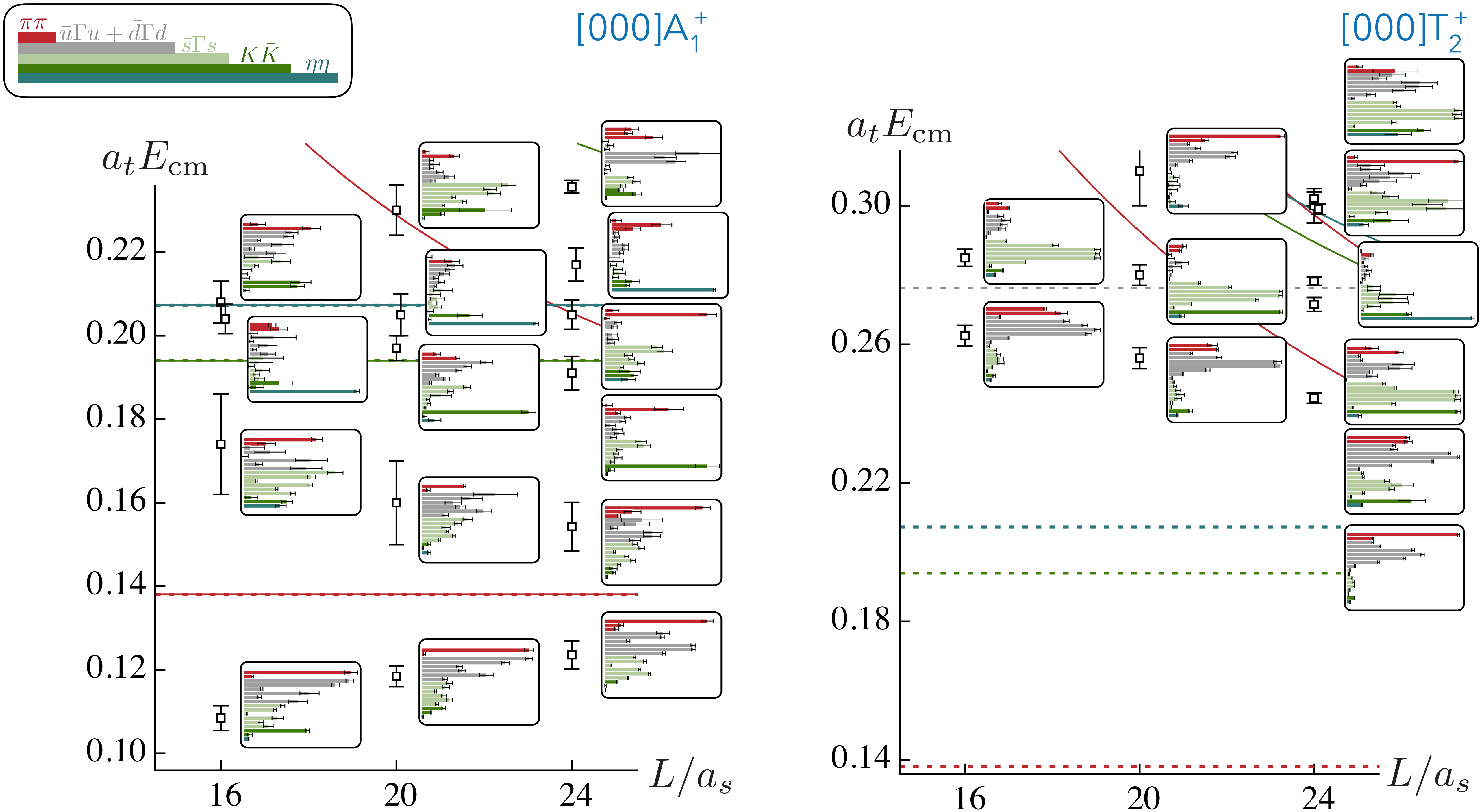


- fitted UChiPT LECS at 236 MeV
- Extrapolated up to 391 MeV
- Extrapolated down to 140 MeV




$a_t E_{\mathrm{cm}}$
0.20
0.15
0.10
$\pi\pi$
$\bar{\psi}\mathbf{\Gamma}\psi$
$K\overline{K}$
$\pi\pi$
$\bar{\psi}\mathbf{\Gamma}\psi$
$\bar{\psi}\mathbf{\Gamma}\psi$
$\pi\pi$
$K\overline{K}$
$\pi\pi$

Spectra and overlaps



operator overlaps give some intuition

lots of mixing in the scalar sector

- essential to have meson-meson ops even below threshold
- can't always 'read-off' resonance content

recent review by Briceno, Dudek, Young: arXiv:1706.06223

Moir et al, JHEP 1610 (2016) 011