Dynamically-coupled partial-waves in ρπ scattering and meson-meson operators

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UNIVERSITY OF CAMBRIDGE

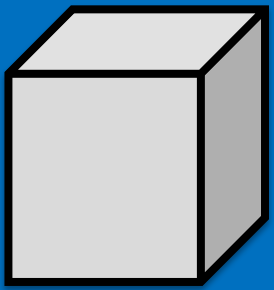
hadspec

Outline

- Introduction
- Multi-hadron operators
- Isospin-2 ρπ scattering with dynamically-coupled partial waves
- Summary

Excited lattice QCD spectroscopy

Finite-volume energy eigenstates from:



$$C_{ij}(t) = \langle 0 | \underline{\mathcal{O}_i(t)} \underline{\mathcal{O}_j^\dagger(0)} | 0 \rangle$$

Large bases of interpolating operators and variational method (GEVP)

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

$$\lambda^{(n)}(t) \to e^{-E_n(t-t_0)} \qquad (t >> t_0)$$

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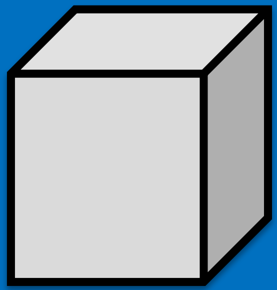

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Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**.

Talk by Dave Wilson earlier

Recent review: Briceño, Dudek, Young [arXiv:1706.06223]

Reduced symmetry

Finite cubic lattice, $\vec{p} = \vec{0}$

Broken sym: 3D rotation group → octahedral (cubic) group $\mathrm{O}_h^{\mathrm{D}}$

Finite number of *irreps* Λ:
A_1, A_2, T_1, T_2, E
(+ others for
half-integer spin)

Λ^P	J^P
$A_1^\pm$	$0^\pm,\ 4^\pm,\ \ldots$
$T_1^\pm$	$1^\pm,\ 3^\pm,\ 4^\pm,\ \ldots$
$T_2^\pm$	$2^\pm,\ 3^\pm,\ 4^\pm,\ \ldots$
$E^\pm$	$2^\pm,\ 4^\pm,\ \ldots$
$A_2^\pm$	$3^\pm,\ \ldots$

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$\vec{p} \neq \vec{0}$

Little group: $\mathrm{LG}(\vec{p}) \subset \mathrm{O}_h^{\mathrm{D}}$ $\quad \left\{ R \in \mathrm{O}_h^{\mathrm{D}} \mid R\vec{p} = \vec{p} \right\}$

Single-meson operators

[PRL 103, 262001; PR D82, 034508; D84, 074508; similarly for baryons PR D85, 014507]

Fermion-bilinear 'single-meson' operators:

$$\mathcal{O}^{J^P,M}(\vec{p},t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\,\bar{\psi}(x)\left[\Gamma \times \overleftrightarrow{D} \times \overleftrightarrow{D} \ldots\right]\psi(x)$$

Circular basis for *D* and Γ, couple using SU(2) Clebsch-Gordans → J^P, M

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'Subduce' operators into lattice irrep (Λ), row (μ):

$$\left[\mathcal{O}^{[J]}_{\Lambda^P,\mu}(\vec{0})\right]^\dagger = \sum_M \mathcal{S}^{J,M}_{\Lambda,\mu}\left[\mathcal{O}^{J^P,M}(\vec{0})\right]^\dagger$$

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For $\vec{p} \neq \vec{0}$ 'helicity ops':

$$\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_M \mathcal{D}^{(J)}_{M\lambda}(R) \left[\mathcal{O}^{J^P,M}(\vec{p})\right]^\dagger \qquad R: \ (0,0,|\vec{p}|) \to \vec{p}$$

Subduce → little group irreps

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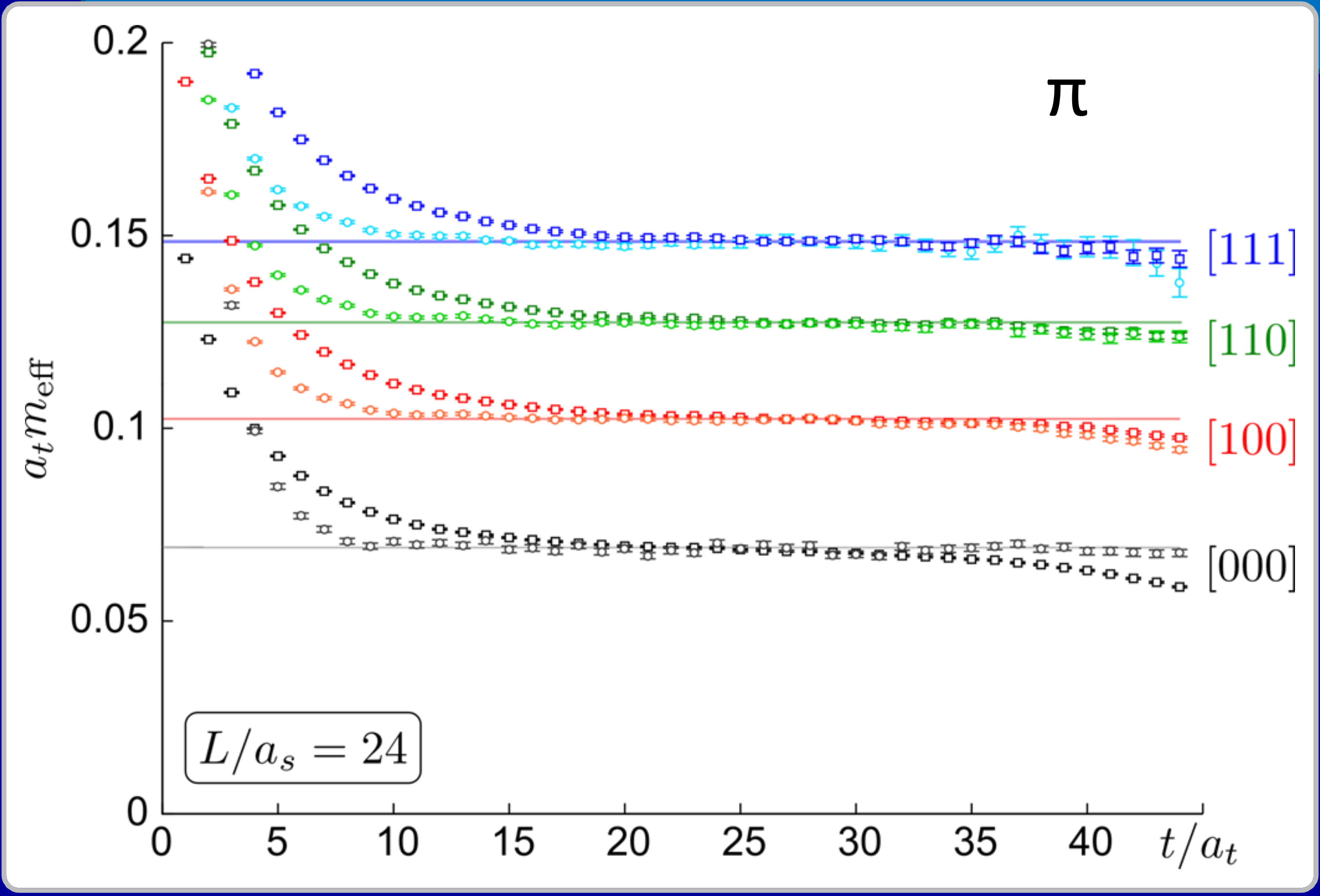
Subduce → little group irreps

Many ops in each channel (up to 3 derivs at rest, 2 at non-zero mom.)

Variationally optimised meson ops

For each $\vec{p}$, Λ: variational analysis with large basis of fermion-bilinear ops
n'th eigenvector → optimal operator for *n*'th state

$$\Omega^{(n)\dagger} \sim \sum_i v_i^{(n)} O_i^\dagger$$



$m_\pi \approx 400$ MeV

[PR D86, 34031 (2012)]

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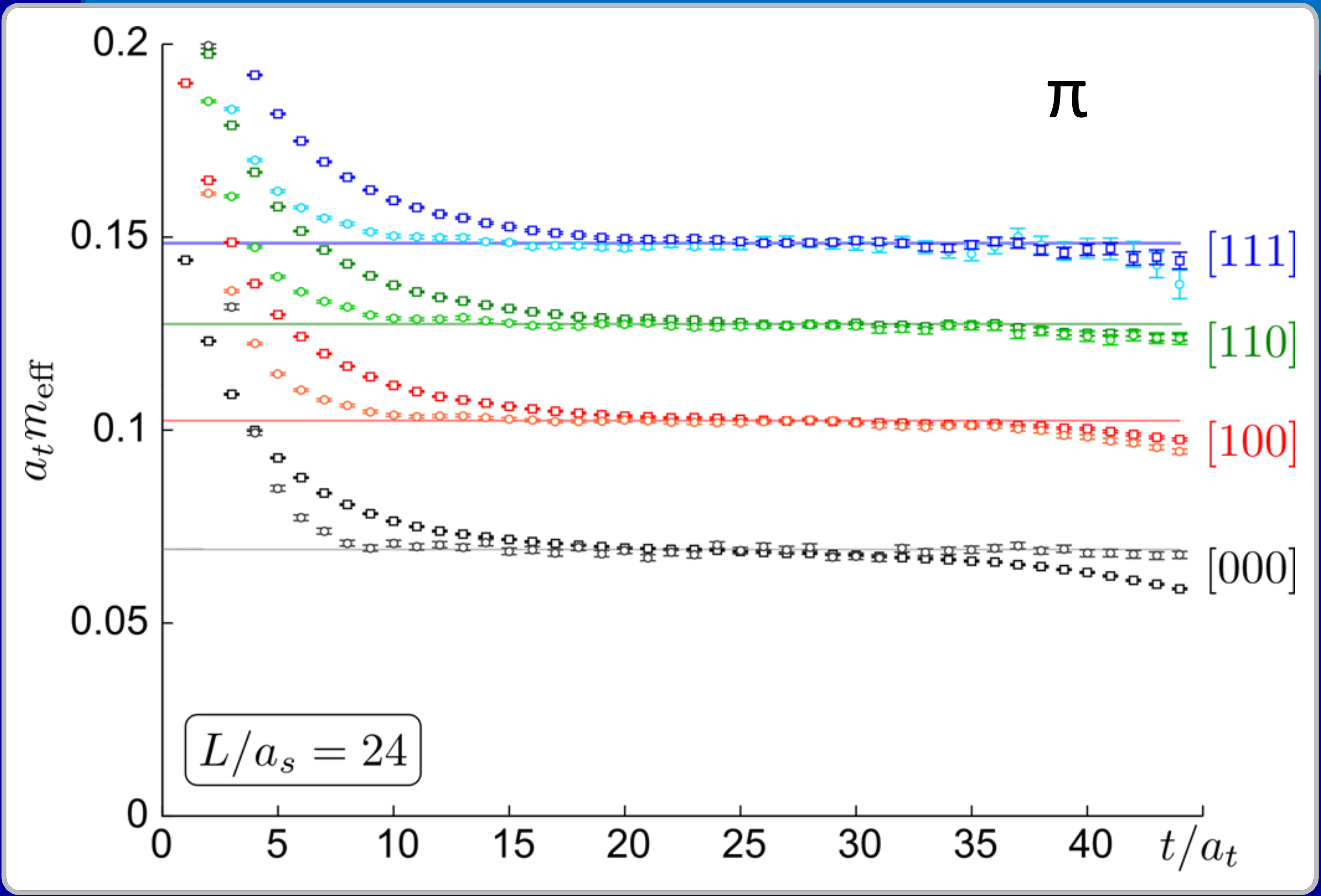

Could have multi-hadron ops

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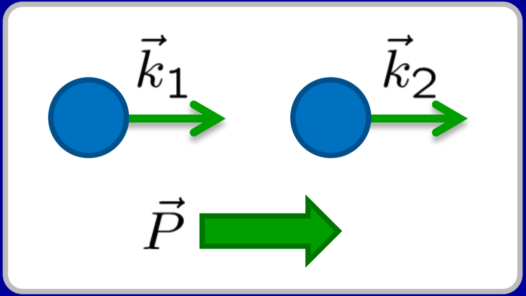
Hadron-hadron (or multi-hadron) operators

$$\mathbb{O}^{\dagger}_{\Lambda,\mu}(\vec{P}) = \sum_{\mu_1,\mu_2} \sum_{\vec{k}_1,\vec{k}_2} C(\vec{P},\Lambda,\mu;\vec{k}_1,\Lambda_1,\mu_1;\vec{k}_2,\Lambda_2,\mu_2)\; \mathbb{O}^{\dagger}_{\Lambda_1\mu_1}(\vec{k}_1)\; \mathbb{O}^{\dagger}_{\Lambda_2\mu_2}(\vec{k}_2)$$



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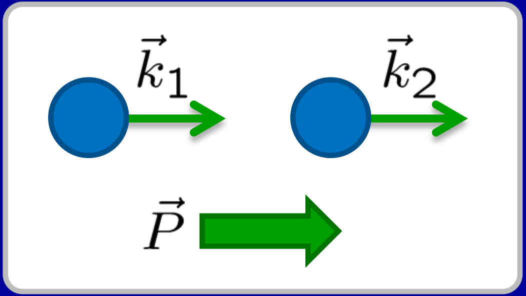
Could be: simple fermion-bilinears, optimised ops, multi-hadron ops, ...



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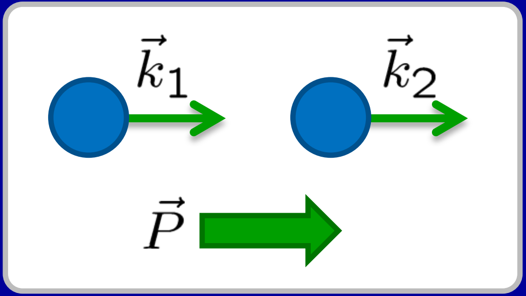
'Generalised Clebsch-Gordans' $\Lambda_1 \otimes \Lambda_2 \to \Lambda$

$\Lambda_1 \in \mathsf{LG}(\vec{k}_1),\ \Lambda_2 \in \mathsf{LG}(\vec{k}_2),\ \Lambda \in \mathsf{LG}(\vec{P})$

(calculate using induced representation)



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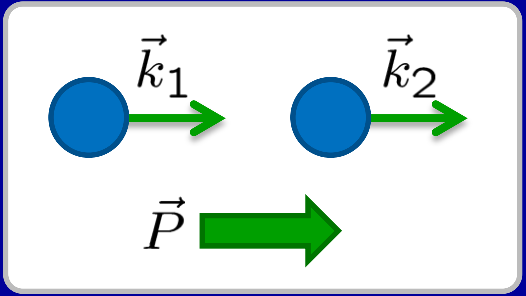

Sum over all $\vec{k}_1,\ \vec{k}_2$ related by allowed lattice rot. such that $\vec{P} = \vec{k}_1 + \vec{k}_2$
$R\,\vec{k}_{1,2}\ \forall\, R \in \mathsf{LG}(\vec{P})$

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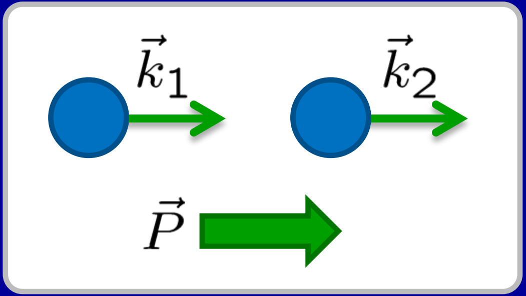

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Why this approach?

- Don't mix different Λ_i
- Can use optimised single-hadron ops
- Can iteratively construct >2 hadron ops

[PR D86, 034031 (2012)]

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Compare continuum formulation:

$$\mathcal{O}^{\dagger[S,\ell]}_{J,M} \sim \sum_{\lambda_1\lambda_2} \int d\hat{p}\; C(J,\ell,S,M;\vec{p},S_1,\lambda_1;-\vec{p},S_2,\lambda_2)\; \mathcal{O}^{\dagger S_1\lambda_1}(\vec{p})\; \mathcal{O}^{\dagger S_2\lambda_2}(-\vec{p})$$

$$C = \langle S_1,\lambda_1;S_2,-\lambda_2|S,\lambda\rangle\langle \ell,0;S,\lambda|J,\lambda\rangle\; \mathcal{D}^{(J)*}_{M\lambda}(\hat{p})$$

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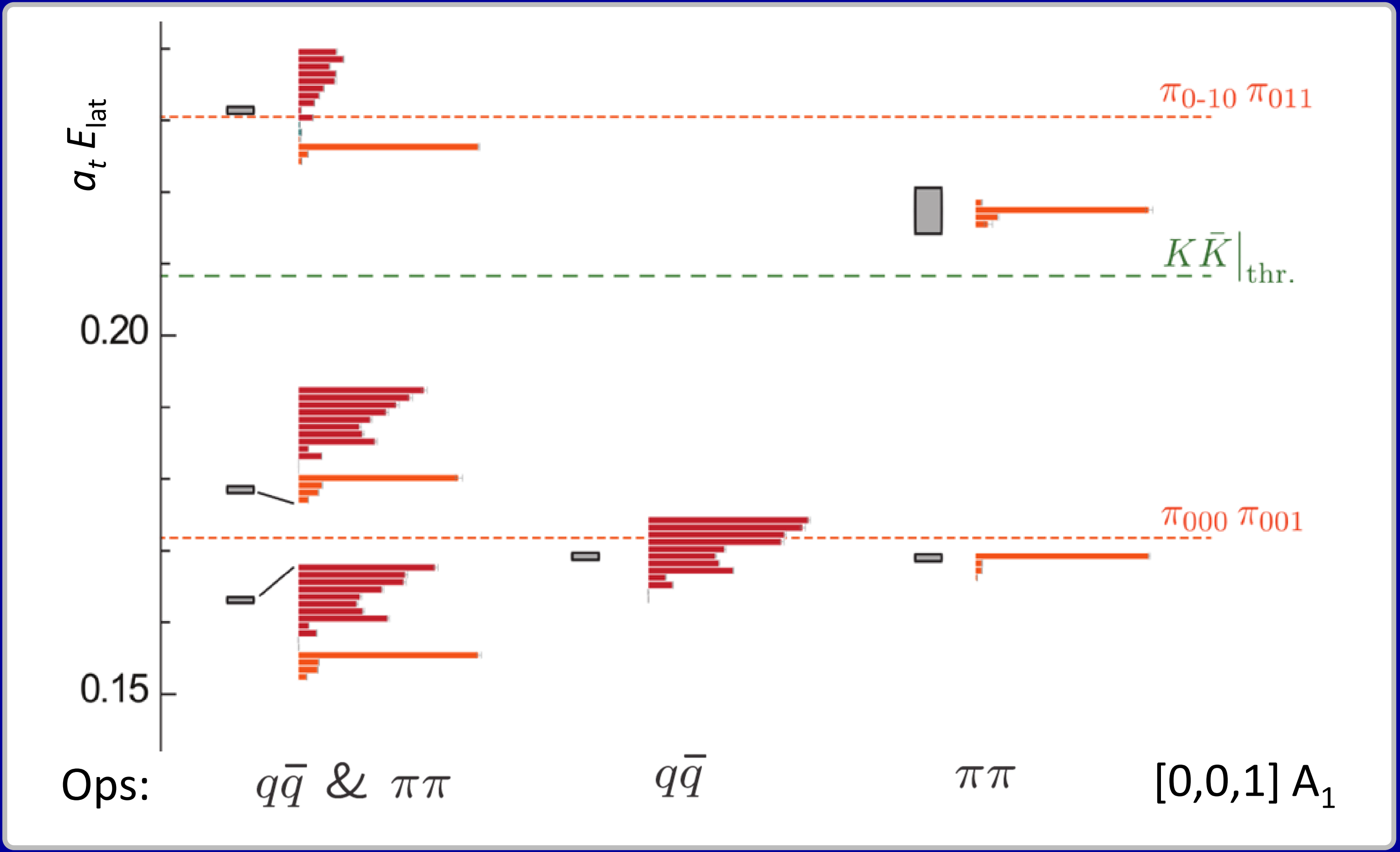
Operators and energy levels

I=1 $\pi\pi$ scattering (ρ resonance)



$m_\pi \approx 400$ MeV [PR D87, 034505 (2013)]

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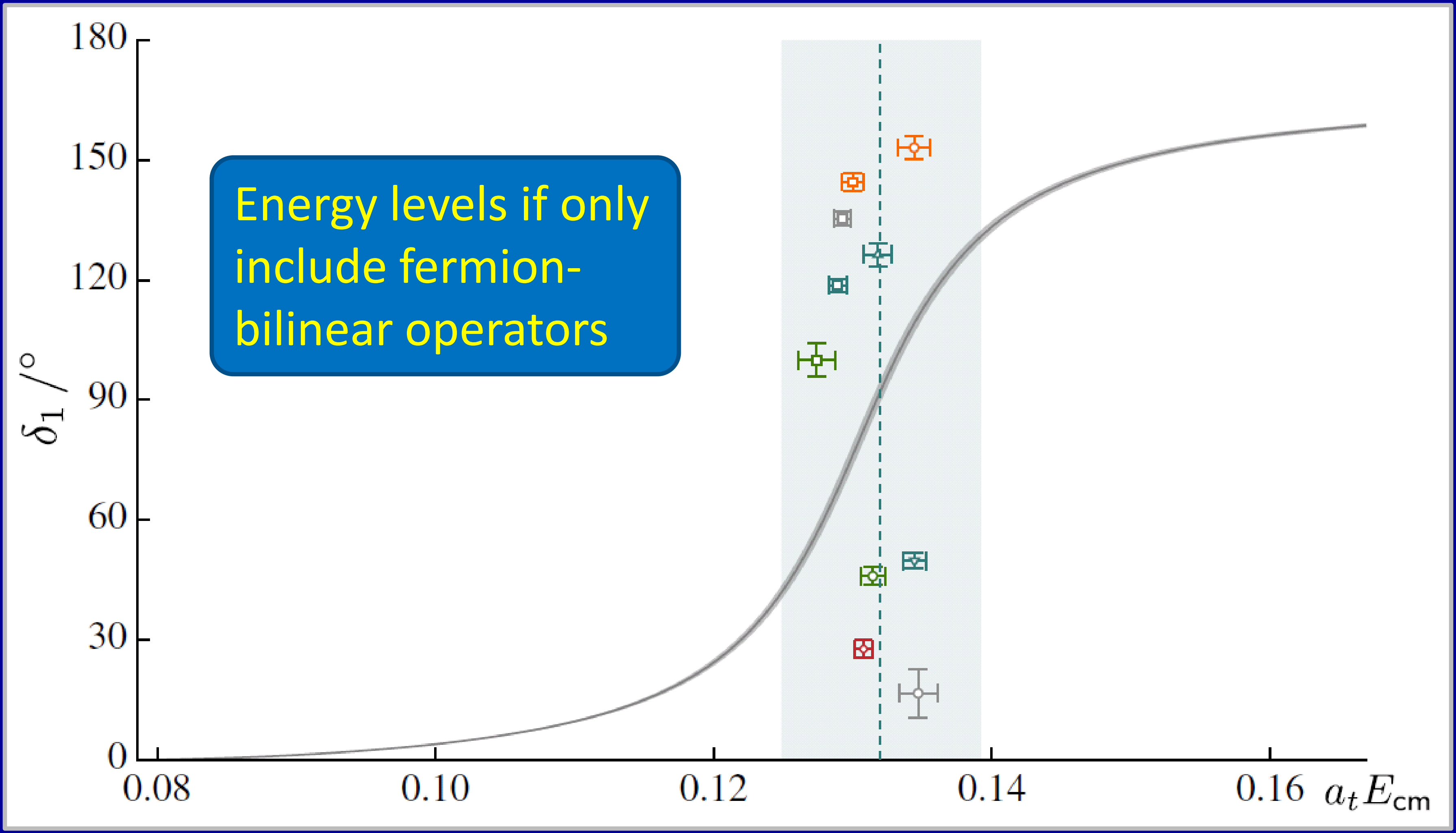

Energy levels if only include fermion-bilinear operators

$m_\pi \approx 236$ MeV [PR D92, 094502 (2015)]

ρπ isospin-2 scattering

[**Woss**, CT, Dudek, Edwards, Wilson, JHEP 1807, 043 (2018)]

Different partial waves with same J^P can mix dynamically

E.g. $J^P = 1^+$ ($^3S_1$, $^3D_1$), $J^P = 2^-$ ($^3P_2$, $^3F_2$)

$$\mathbf{t} = \begin{bmatrix} t(^3S_1|\,^3S_1) & t(^3S_1|\,^3D_1) \\ t(^3S_1|\,^3D_1) & t(^3D_1|\,^3D_1) \end{bmatrix}$$

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Finite-volume lattice QCD calculations: reduced symmetry →additional 'mixing'

[0,0,0]	T_1^+
$J^+(^3\ell_J)$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$ $3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$ $4^+ \; (^3G_4)$

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$$\mathbf{t} = \begin{bmatrix} t({}^3S_1|{}^3S_1) & t({}^3S_1|{}^3D_1) & 0 \\ t({}^3S_1|{}^3D_1) & t({}^3D_1|{}^3D_1) & 0 \\ 0 & 0 & t({}^3D_3|{}^3D_3) \end{bmatrix}$$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

[00n] Λ	A_1	A_2	E
$J^P(^3\ell_J)$		$0^-\ (^3P_0)$	
		$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$	$1^+ \begin{pmatrix} ^3S_1 \\ ^3D_1 \end{pmatrix}$
	$1^-\ (^3P_1)$		$1^-\ (^3P_1)$
	$2^+\ (^3D_2)$		$2^+\ (^3D_2)$
		$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$	$2^- \begin{pmatrix} ^3P_2 \\ ^3F_2 \end{pmatrix}$
		$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}$	$3^+ \begin{pmatrix} ^3D_3 \\ ^3G_3 \end{pmatrix}_{[2]}$
	$3^-\ (^3F_3)$		$3^-\ (^3F_3)_{[2]}$
	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$	$4^- \begin{pmatrix} ^3F_4 \\ ^3H_4 \end{pmatrix}_{[2]}$

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]

Calc. at $SU(3)_F$ symmetric point (m_u=m_d=m_s)

$m_\pi \approx 700$ MeV

$m_\rho \approx 1020$ MeV (stable)

Large bases of ρπ $SU(3)_F$ ops:

'optimised' ρ and π ops $\sim \bar{\psi}\Gamma D \ldots \psi$

Use 'distillation' [PR D80 054506 (2009)]

One lattice spacing, 2 volumes (L ≈ 2, 3 fm)

$\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

$1\ T_1^+$

$\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

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$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

1 T_1^+

$\rho[001]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow E_2$, $\pi[001] \rightarrow A_2$

$[001]\ A_1 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, A_1^-, E^-$

$[001]\ E_2 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, T_2^+, T_1^-, T_2^-$

2 T_1^+

$\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

1 T_1^+

$\rho[001]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow E_2$, $\pi[001] \rightarrow A_2$

$[001]\ A_1 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, A_1^-, E^-$

$[001]\ E_2 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, T_2^+, T_1^-, T_2^-$

2 T_1^+

$\rho[011]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow B_1, B_2$, $\pi[011] \rightarrow A_2$

$[011]\ A_1 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

$[011]\ B_1 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

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3 T_1^+

$\rho\pi$ isospin-2 scattering

[JHEP 1807, 043 (2018)]

$\rho[000] \rightarrow T_1^-$, $\pi[000] \rightarrow A_1^-$

$[000]\ T_1^- \times [000]\ A_1^- \rightarrow [000]\ \mathbf{T_1^+}$

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$[001]\ A_1 \times [001]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, A_1^-, E^-$

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$2\ T_1^+$

$\rho[011]\ \lambda=0 \rightarrow A_1$, $\lambda=\pm1 \rightarrow B_1, B_2$, $\pi[011] \rightarrow A_2$

$[011]\ A_1 \times [011]\ A_2 \rightarrow [000]\ \mathbf{T_1^+}, \ldots$

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$3\ T_1^+$

Overall non-zero momentum

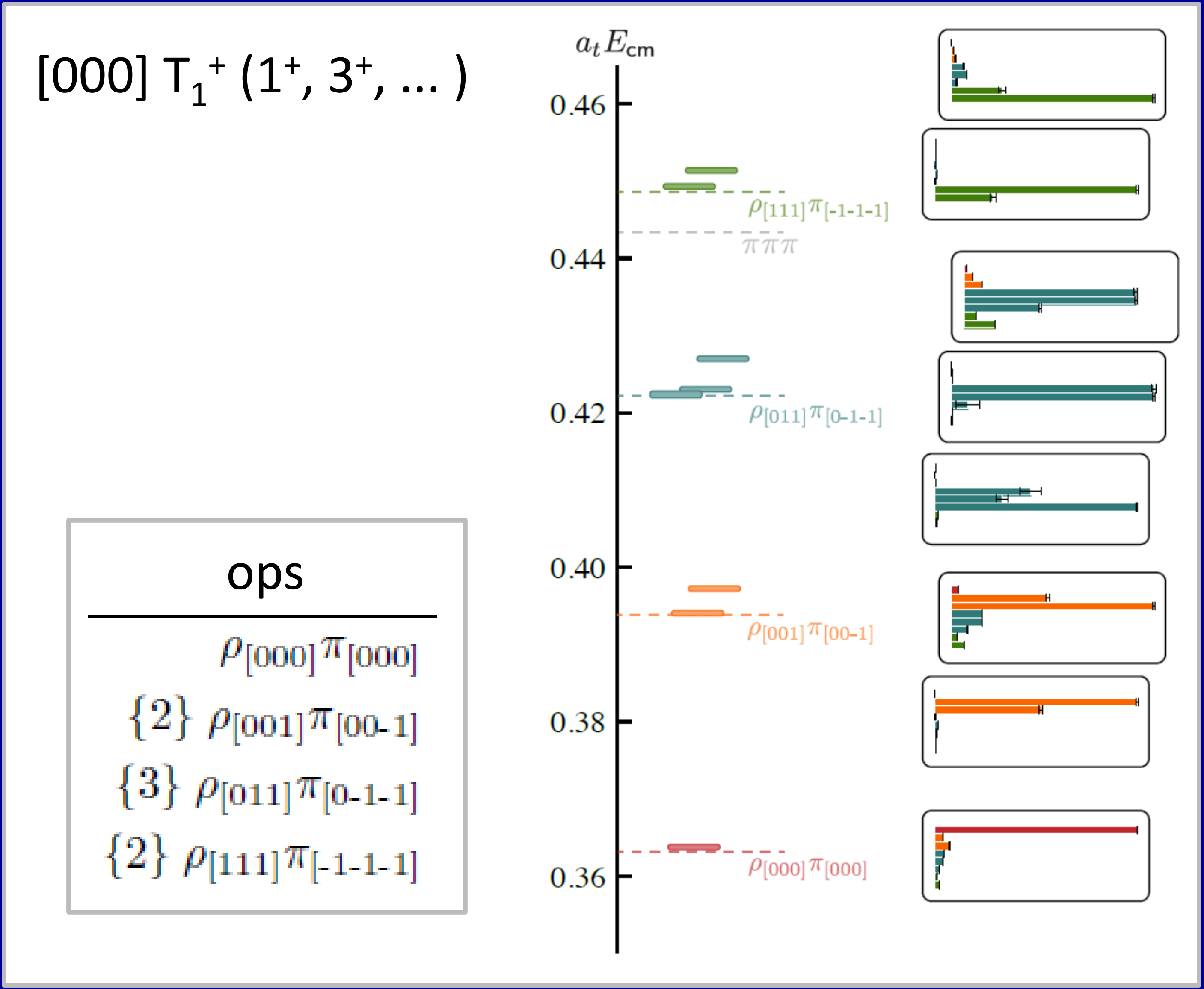
$[001]\ A_1 \times [011]\ A_2 \rightarrow [001]\ \mathbf{E_2}, \ldots$

$[001]\ E_2 \times [011]\ A_2 \rightarrow [001]\ \mathbf{E_2}, \mathbf{E_2}, \ldots$

$3\ E_2$

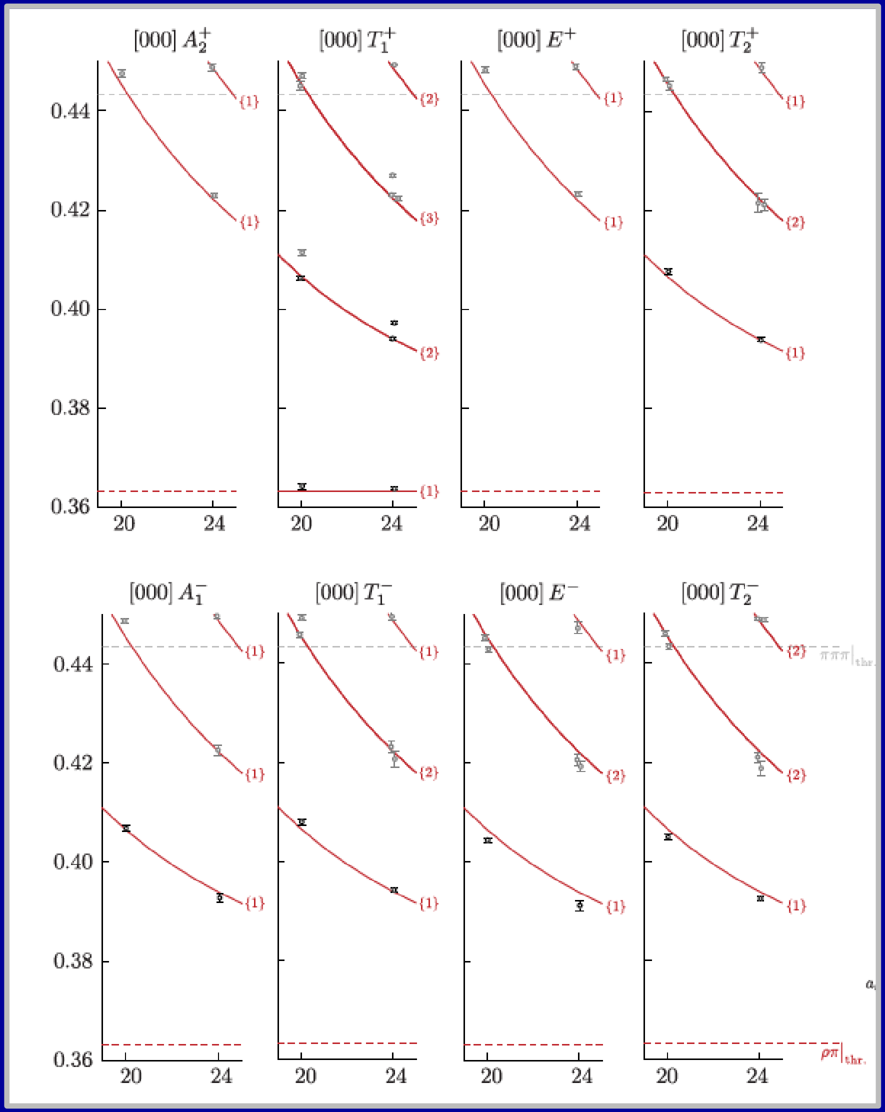
ρπ isospin-2 scattering – spectra

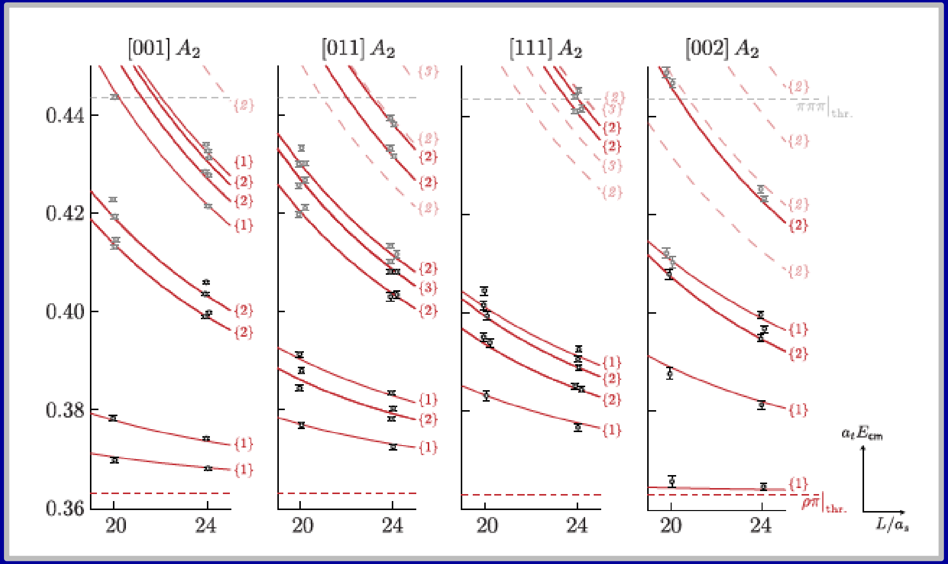
[JHEP 1807, 043 (2018)]



ρπ isospin-2 scattering – spectra

[JHEP 1807, 043 (2018)]



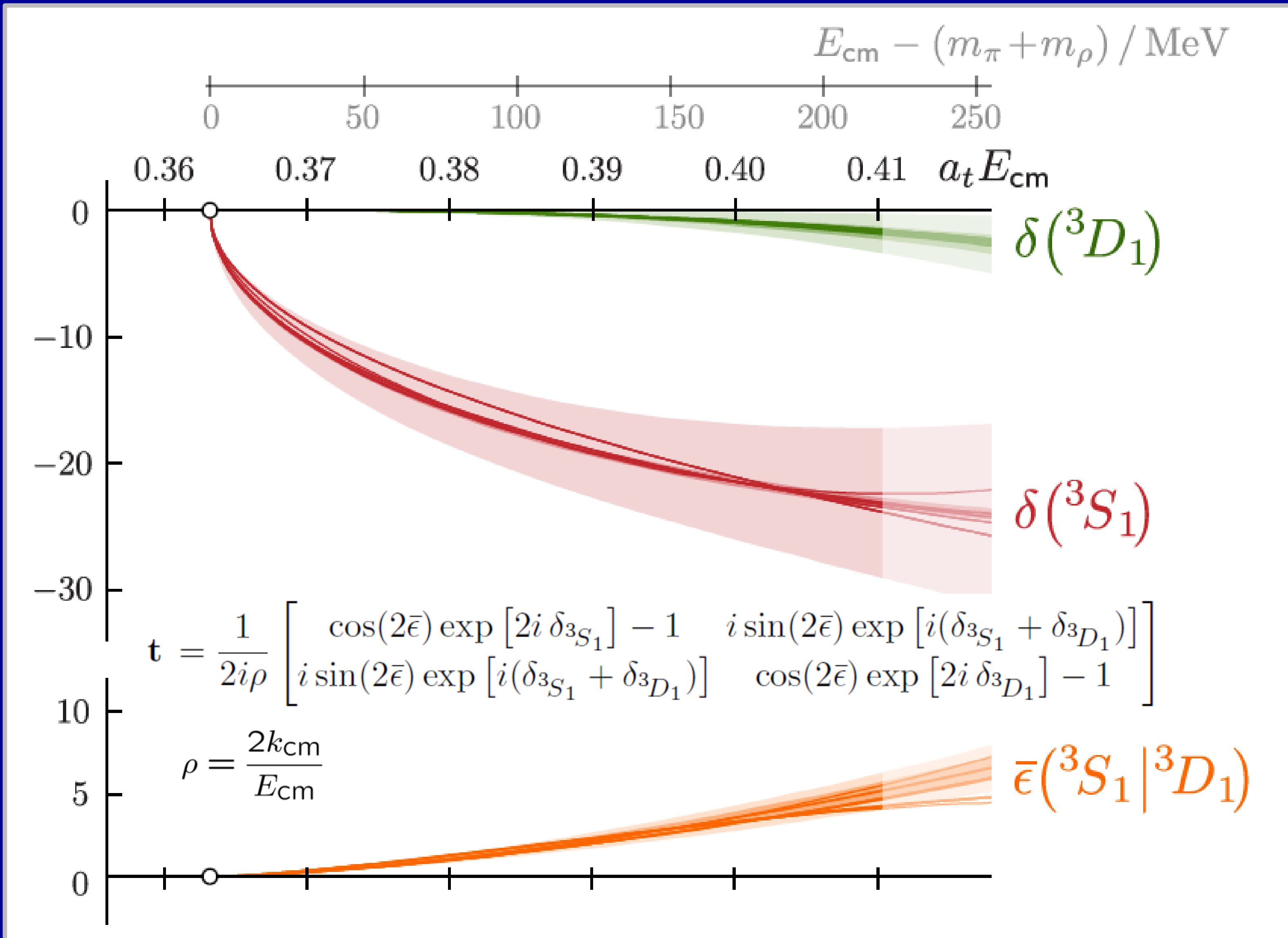


+ others

Used 141 energy levels for ℓ = 0, 1, 2

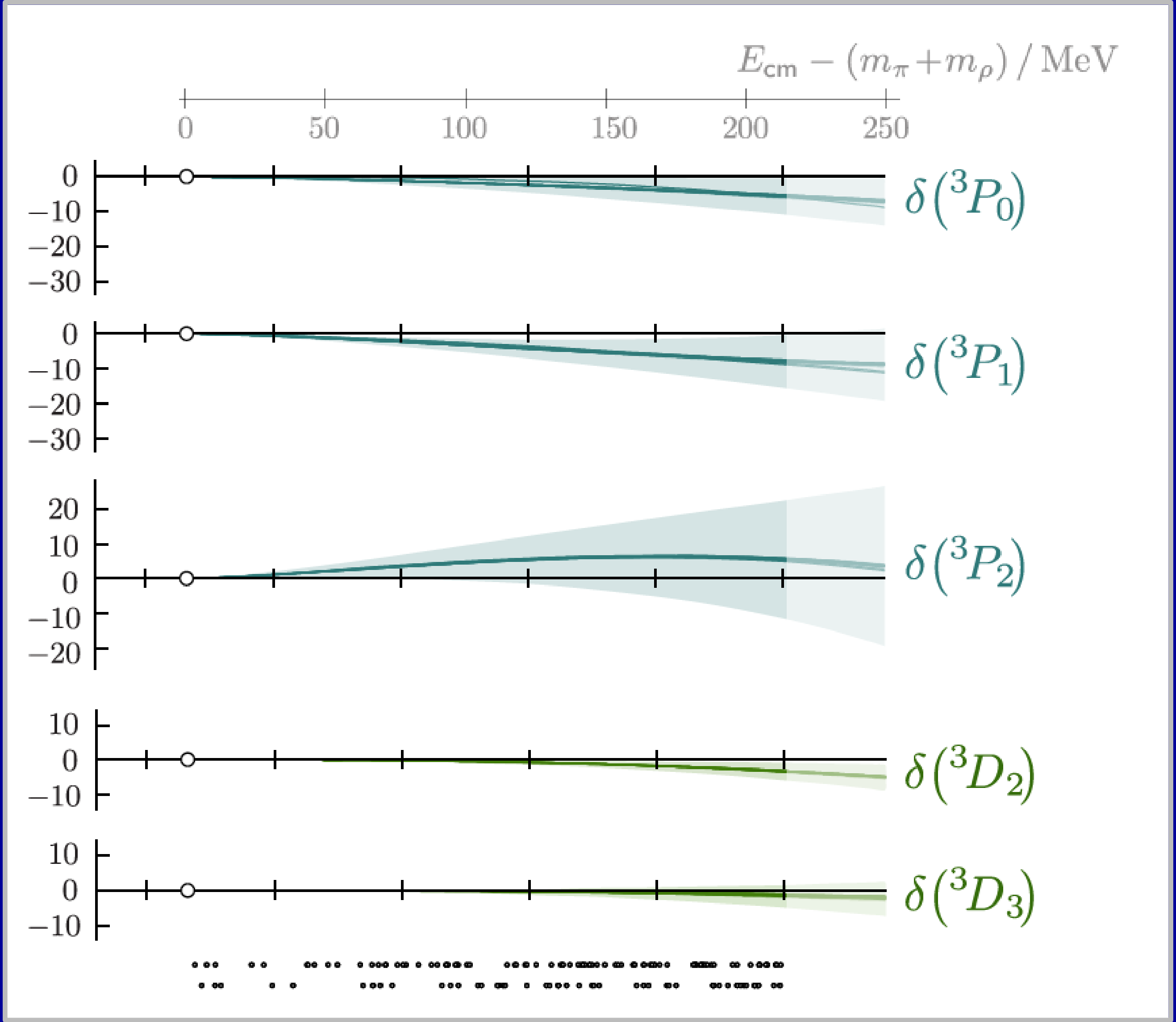
ρπ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]



ρπ isospin-2 scattering – amplitudes

[JHEP 1807, 043 (2018)]



Summary

- General multi-hadron operator constructions
- Example of recent progress:
  - $\rho\pi$ isospin-2 scattering with dynamically-coupled $^3S_1$, $^3D_1$ partial waves
- Work in progress on other channels and different m_π
- Also matrix elements, transitions

UNIVERSITY OF CAMBRIDGE

Science & Technology Facilities Council

Hadron Spectrum Collaboration

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Cian O'Hara, David Tims

University of Cambridge, UK:

CT, *Gavin Cheung, Antoni Woss*

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Nilmani Mathur

www.hadspec.org

Extra slides

ρπ isospin-2 scattering

[JHEP 1807, 043 (2018)]