Charmonium spectroscopy from CLS ensembles

M. Padmanath

with G. Bali, S. Collins, D. Mohler, S. Piemonte, S. Prelovsek, A. Schäfer and S. Weishäupl (RQCD)

Universität Regensburg

Regensburg, Germany

Mainz

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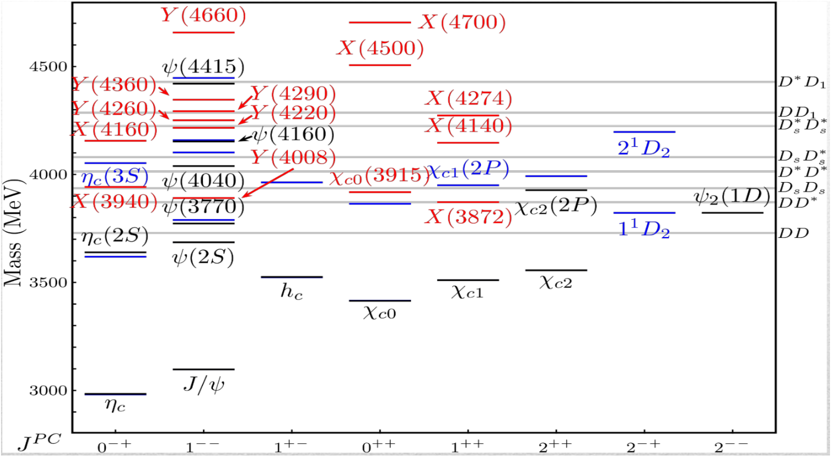
Experimental charmonium spectrum



Esposito, Pilloni, Polosa Phys.Rept. 668

X(3915) Vs X(3860) and $\chi_{c0}(2P)$

$c\bar{c}$ MESONS
(including possibly non- $q\bar{q}$ states)

$X(3915)$ $\quad I^G(J^{PC}) = 0^+(0 or 2^{++})$

was $\chi_{c0}(3915)$

Candidate for $\chi_{c0}(2P)$, but

- expected open-charm decay mode not observed (X(3915) $\not\to$ $\bar{D}D$).
- Spin splitting $m_{\chi_{c2}(2P)} - \chi_{c0}(2P)$ too small.
- observed in OZI suppressed mode $J/\psi\omega$.

Guo and Meissner arXiv:1208.1134; Olsen arXiv:1410.6534

$c\bar{c}$ MESONS
(including possibly non- $q\bar{q}$ states)

$\chi_{c0}(3860)$ $\quad I^G(J^{PC}) = 0^+(0^{++})$

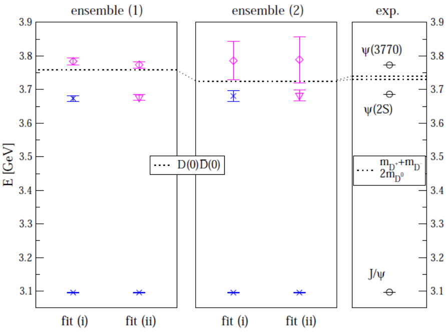
Observation by Belle!

Chilikin *et al*, arXiv:1704.01872

Previous investigation : $(I)J^{PC} = (0)1^{--}$

$\psi(2S)$ and $\psi(3770)$ from $D\bar{D}$ elastic scattering in P-wave.

First resonance determination of a charmonium state.



Leskovec *et al*, JHEP, **1509**, 089, 2015

Ensemble 1 :

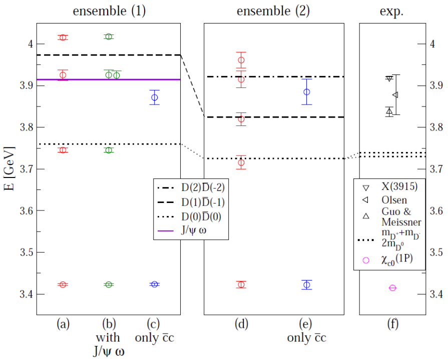
- $N_f = 2$, $m_\pi \sim 266$ MeV, $L \sim 2$ fm
- Wilson clover fermions
- Full distillation.

Ensemble 2 :

- $N_f = 2 + 1$, $m_\pi \sim 196$ MeV, $L \sim 2.9$ fm
- Wilson clover fermions
- Stochastic distillation.

$\chi_{c0}(2P)$ from $D\bar{D}$ elastic scattering in S-wave.

No solid conclusions. Call for more systematic studies.



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- Stochastic distillation.

What we intend

- Resonances around open charm threshold can be studied using lattice QCD.

- Focus on scalar and vector charmonium
  Study multiple inertial frames, coupled channel scenarios, different lattice volumes, ...

- Progressively increase the rigor in the investigation.
  - Multiple inertial frames within single hadron approximation
  - Elastic pseudoscalar-pseudoscalar scattering
  - Coupled channel studies
  - ...

- Assumptions :
  - Effects from charm annihilation to be small
  - Three hadron scattering not to be important

How we do

- Ensemble : CLS
  - U101 $N_f = 2+1$, $L \sim 2$ fm
  - H105 $N_f = 2+1$, $L \sim 2.7$ fm
  - $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV
  - Wilson clover fermions with full distillation ($N_{ev} = 90$)

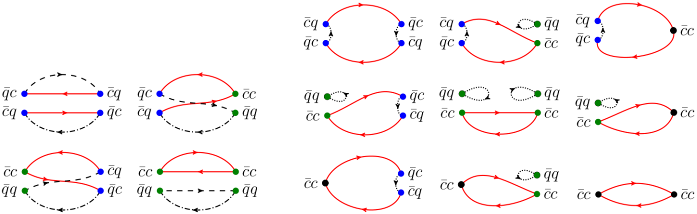
- Multiple excited state extraction
  Correlation matrices using a large basis of interpolating operators
  $$C_{ji}(t_f - t_i) = \langle 0|O_j(t_f)\bar{O}_i(t_i)|0\rangle = \sum_n \frac{Z_n^{i*} Z_n^j}{2E_n} e^{-E_n(t_f - t_i)}$$
  Operator state overlap factors : $Z_n^j = \langle 0|O_j|n\rangle$.

- A good analysis procedure for extraction of energy of physical states.
  Variational fitting method or GEVP.

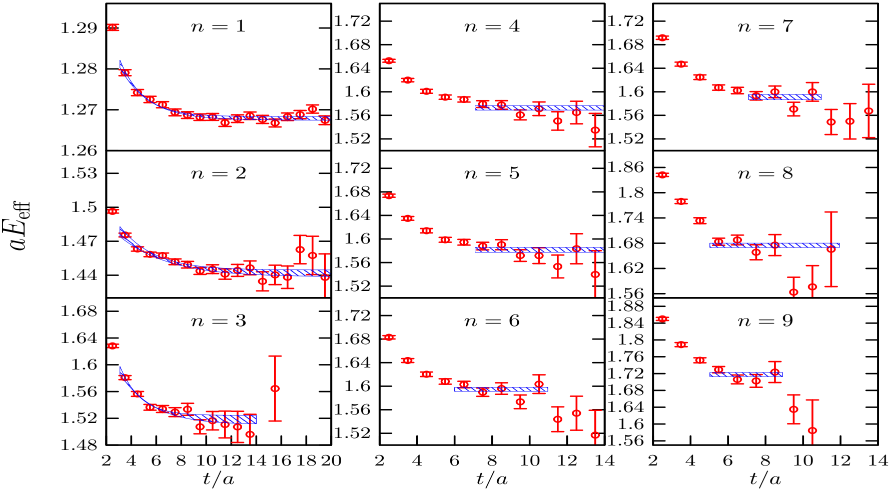
- Utilize "TwoHadronsInBox" toolbox to obtain K-matrix parametrization for our lattice energy levels.
  Morningstar *et al.* Nucl. Phys. B924, 477-507 (2017)

Interpolators and contractions

- Interpolators create states with correct quantum numbers.
  $\mathcal{O} \sim \bar{c}\Gamma c,\ \bar{c}\overleftrightarrow{D}_i\Gamma c,\ \bar{c}\overleftrightarrow{D}_i\overleftrightarrow{D}_j\Gamma c,\ \ldots$
- All physical states with given J^{PC} can appear in the lattice spectrum. Single meson states, two-meson states, etc.
- In practical calculations, $\bar{c}c$ couple very weakly to two meson states.
- Necessitates the inclusion of multi-hadron operators
  $\mathcal{O} = \bar{Q}\Gamma Q,\ \ (\bar{Q}\Gamma_1 q)_{1_c}(\bar{q}\Gamma_2 Q)_{1_c},\ \ (\bar{Q}\Gamma_1 Q)_{1_c}(\bar{q}\Gamma_2 q)_{1_c}.$
- Wick contractions



Effective masses : quality of fits



E^- irrep spectrum in inertial frame with momentum $\mathbf{P} = (0, 0, 1)$.

Rest frame interpolators : Single hadron approximation

- In the infinite volume continuum

$$O^{J,M,P}(\mathbf{0}) = \sum_{m_i} C_{CG}(m_1, m_2, m_3, M) \times \sum_{\mathbf{x}} \bar{c}(\mathbf{x})\Gamma_{m_1} \overleftrightarrow{D}_{m_2} \overleftrightarrow{D}_{m_3} c(\mathbf{x})$$

- Projection on to lattice irreducible representations

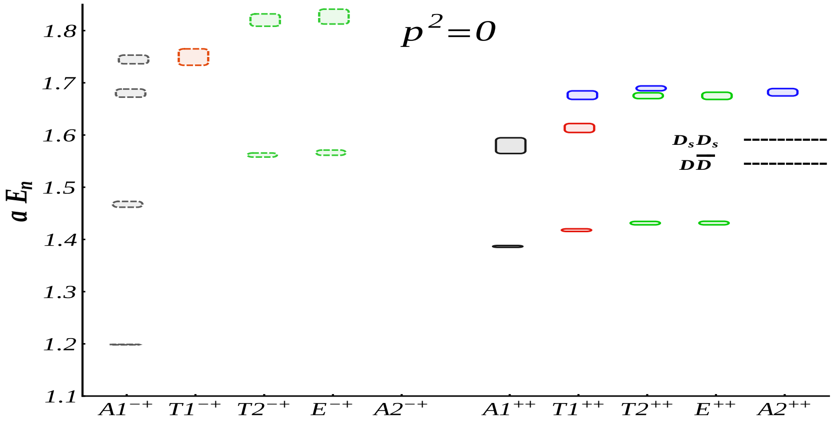
$$O^{[J,P]}_{\Lambda,\mu}(\mathbf{p}=\mathbf{0}) = \sum_{M} S^{J,M}_{\Lambda,\mu} \; O^{J,M,P}(\mathbf{p}=\mathbf{0})$$

Dudek *et al*, PRD 82 034508 (2010)

- Parity and charge conjugation remains good also on the lattice

$\mathbf{p}=0,\ O_h,\ P, C = \pm$	
Λ (dim)	J
A_1 (0)	0, ...
T_1 (3)	1, 3, ...
T_2 (3)	2, 3, ...
E (2)	2, ...
A_2 (1)	3, ...

Charmonium spectrum rest frame, $C = +$

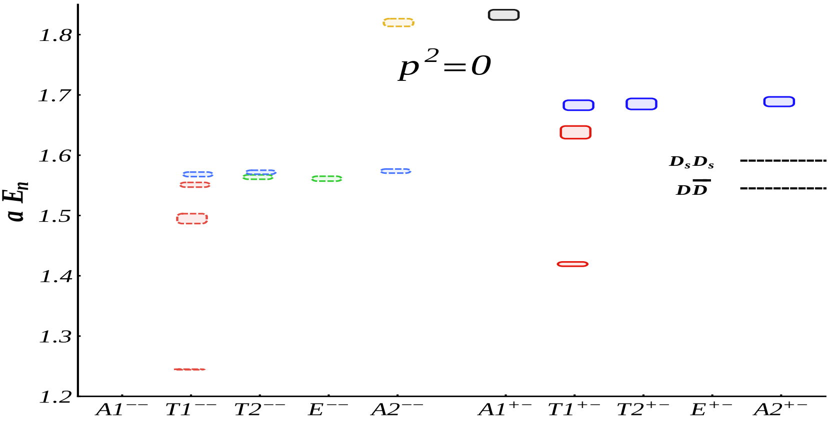


Spin 0, 1, 2, 3.

+(-) parity with solid (dashed) boundaries.

States with ambiguous identities in orange color.

Charmonium spectrum rest frame, $C = -$



Spin 0, 1, 2, 3.

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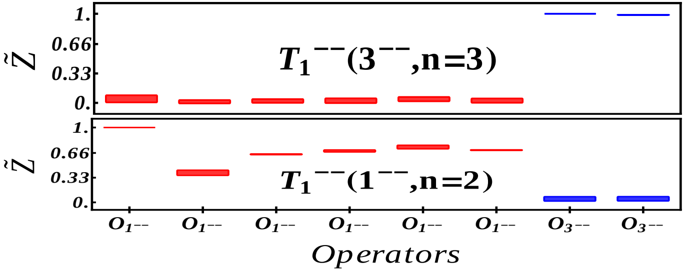
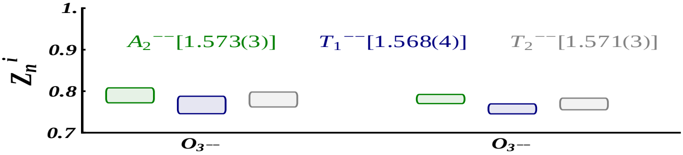
Spin assignment using operator state overlaps





$$\tilde{Z}_n^i = Z_n^i / max(Z_m^i)$$

Moving frame interpolators : Single hadron approximation

- In the infinite volume continuum, J^P no more good quantum no.s! Irreps labelled by the helicity, λ (and $\tilde{\eta} = P(-1)^J$ for $\lambda = 0$.)
- Infinite volume continuum interpolators with good helicity.

$$O^{J,P,\lambda}(\mathbf{p}) = \sum_M \mathcal{D}^{(J)*}_{M,\lambda}(R) O^{J,M,P}(\mathbf{p})$$

- Projection on to lattice irreducible representations

$$O^{[J,P,|\lambda|]}_{\Lambda,\mu}(\mathbf{p}) = \sum_{\hat{\lambda}=\pm|\lambda|} S^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu} \, O^{J,P,\lambda}(\mathbf{p})$$

Thomas *et al*, PRD 85 014507 (2012)

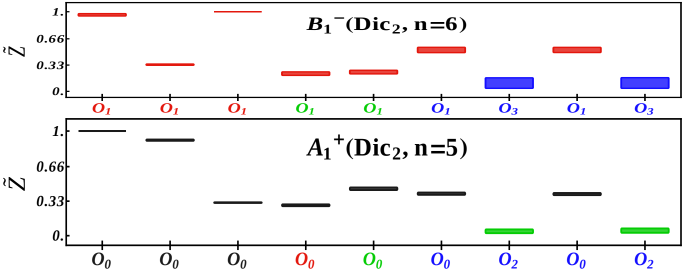
- Charge conjugation remains good also on the lattice.

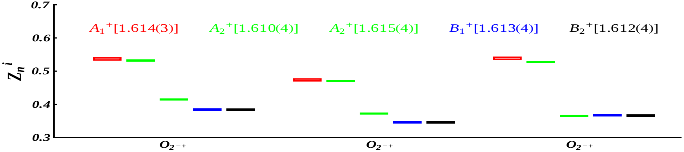
Moving frames : continuum to lattice

$\mathbf{p} = (0,0,1)$, Dic_4		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+,\ 1^-,\ 2^+,\ 3^-$
A_2 (1)	0^-	$0^-,\ 1^+,\ 2^-,\ 3^+$
E (2)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$
B_1 (1)	2	$2^\pm,\ 3^\pm$
B_2 (1)	2	$2^\pm,\ 3^\pm$

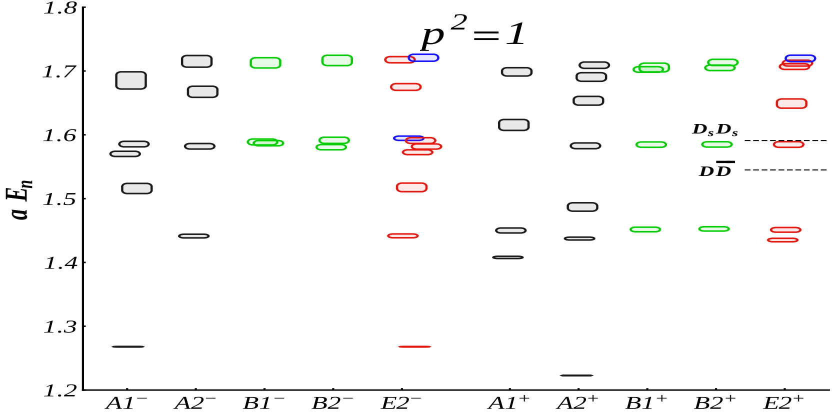
$\mathbf{p} = (1,1,0)$, Dic_2		
Λ (dim)	$\lvert\lambda\rvert^{\tilde{\eta}}$	J^P (at rest)
A_1 (1)	0^+	$0^+,\ 1^-,\ 2^+,\ 3^-$
	2	$2^\pm,\ 3^\pm$
A_2 (1)	0^-	$0^-,\ 1^+,\ 2^-,\ 3^+$
	2	$2^\pm,\ 3^\pm$
B_1 (1)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$
B_2 (1)	1	$1^\pm,\ 2^\pm,\ 3^\pm$
	3	$3^\pm$

Operator state overlaps and spin assignments!



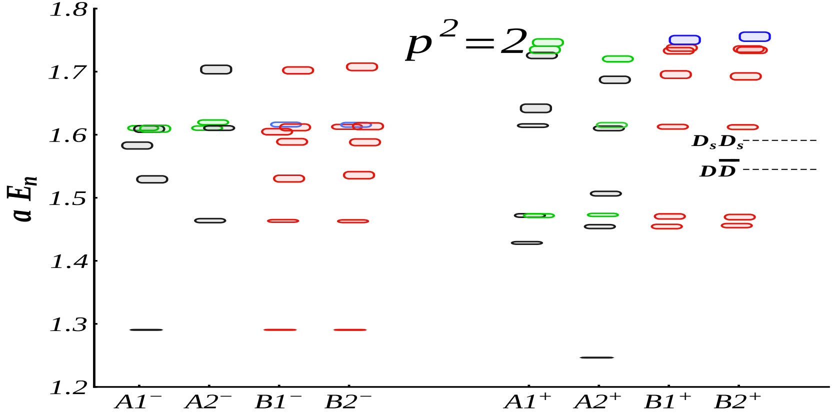


$|\lambda|$ identified spectrum in moving frame $\mathbf{P} = (0, 0, 1)$



Magnitude of helicity 0, 1, 2, 3.

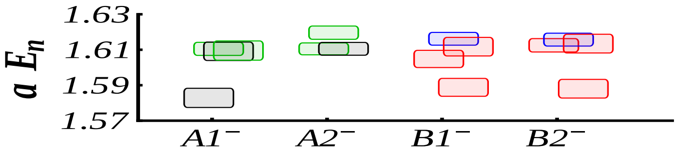
$|\lambda|$ identified spectrum in moving frame $\mathbf{P} = (1, 1, 0)$



Magnitude of helicity 0, 1, 2, 3.

Spin-parity assignments

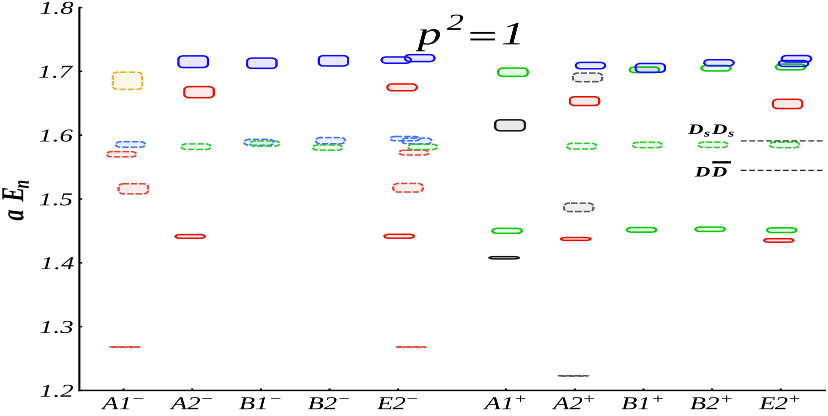
- Inputs from the rest frame spectrum. Dispersion relations.
- Possible quantum numbers based on observed patterns.



States with spin 1, 2 and 3 in this band. Possible $P = -$

- Overlap factors to determine the states dominantly coupled to an interpolator.

J^P identified spectrum in moving frame $\mathbf{P} = (0, 0, 1)$

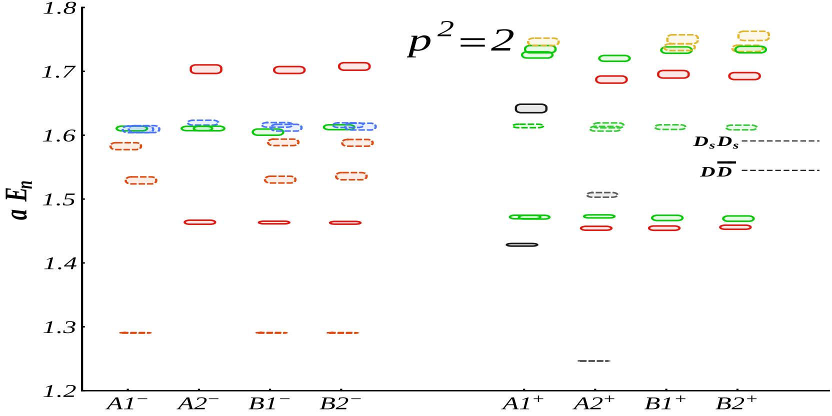


Spin 0, 1, 2, 3.

+(-) parity with solid (dashed) boundaries.

States with ambiguous identities in orange color.

J^P identified spectrum in moving frame $\mathbf{P} = (1, 1, 0)$



Spin 0, 1, 2, 3.

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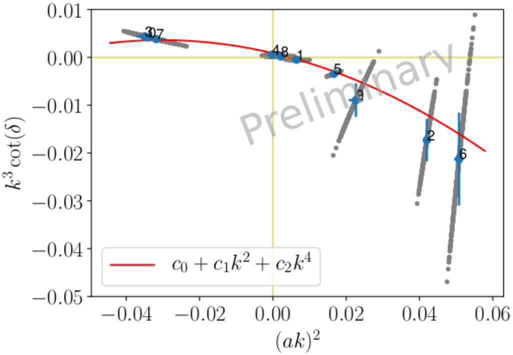
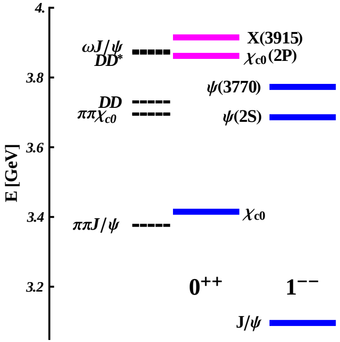
Resonance treatment : 1^{--} charmonia

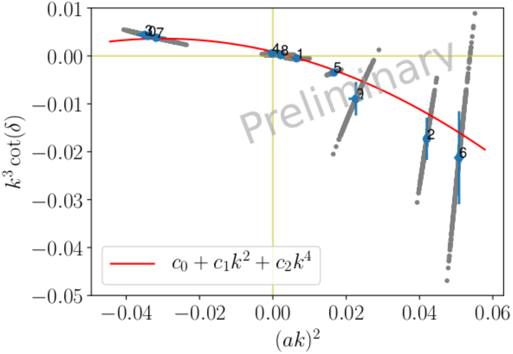


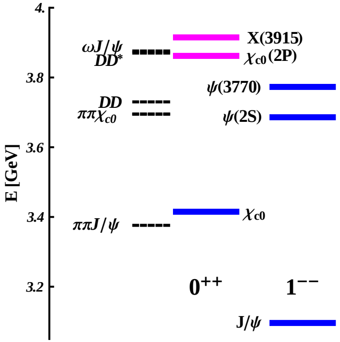


- 1^{--} channel in $\bar{D}D$ scattering in p-wave.
- Neglecting the effects of a spin 3 state.
- Example parametrization for elastic $D\bar{D}$ scattering.

$$p^3 cot\delta/\sqrt{s} = c_0 + c_1 s + c_2 s^2$$

Resonance treatment : 1^{--} charmonia



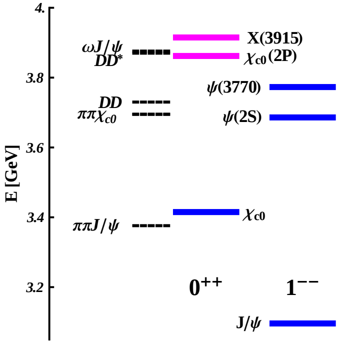


- Including a bound state in the fit ($\psi(2S)$).
- Data from CMF, T_1^{--} from $p^2 = 0$, A_1^- from $p^2 = 1$ and 2. Ensemble H105.
- Non negligible differences between two ensembles (U101 and H105). Exponentially suppressed volume effects?

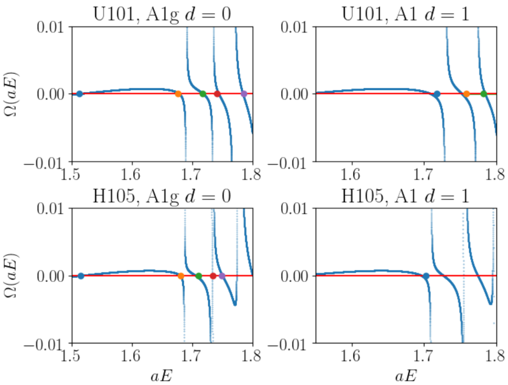
Resonance treatment : 0^{++} and $\chi_{c0}(2P)$

- Include $\bar{c}c$ and meson-meson interpolators of type $D\bar{D}$, D_sD_s, $D^*\bar{D}^*$ and $J/\psi\omega$.
- Do not consider $\eta_c\eta$.
- Currently neglecting presence of states with different quantum numbers.
- Preliminary results : Joint fit to U101 and H105 ensemble data. Not yet including all energy levels, frames.
- Example parameterization :

$$\tilde{K}^{-1} = \begin{bmatrix} c_{11}s + b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}$$



$J^{PC} = 0^{++}$: Determinant residual method

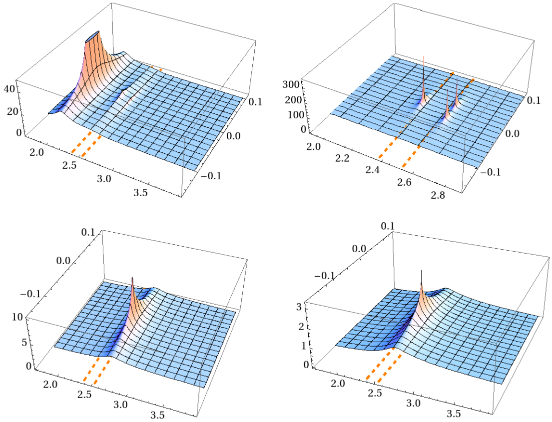


- Minimize a residual built from

$$r_k = \Omega(\mu, A) = \frac{detA}{det[(\mu^2 + AA^\dagger)^{1/2}]}; \quad A = \tilde{K}^{-1}(E^{obs}_{cm,k}) - B(E^{obs}_{cm,k}).$$

- $\Omega(\mu, A)$ crosses zero at our energy levels for fitted parameters.

$J^{PC} = 0^{++}$: Riemann sheets



- Sheets on top : ++, -+; Sheets in the bottom : +-, - -.
- Rich pole structure.
- Strong parameter dependence in some features.

Summary and outlook

- We investigate charmonium bound state and resonances with $J^{PC} = 0^{++}$ and 1^{--}.
- Gradually relaxing the simplifying assumptions.
  - Multiple inertial frames within single hadron approximation
  - Elastic pseudoscalar-pseudoscalar scattering
  - Coupled channel studies
- Future directions :
  - Parameterization dependence.
  - The complex plane structures for various parametrizations.
  - Include effects from other quantum channels in the analysis
  - Light and charm quark mass dependence
  - Discretization effects

Thank you...