The I=1 pion-pion scattering amplitude and timelike form factor: finite volume and cutoff effects

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CP3-Origins

Based on arXiv:1808.05007
with Christian Andersen (SDU), Ben Hörz (Mainz), Colin Morningstar (CMU)

Scattering Amplitudes and Resonance Properties from Lattice QCD
MITP, U. of Mainz
Aug. 28th, 2018

Isovector pion-pion scattering

- Needed to understand electromagnetic processes for e.g. HVP, HlBl:

$$\gamma^* \to \pi\pi \qquad \gamma^{(*)}\gamma^{(*)} \to \pi\pi \qquad \pi\gamma \to \pi\pi, \quad \ldots$$

Briceno et al. `16;
Alexandrou et al. '18

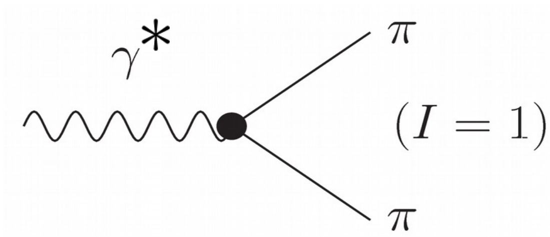
- Needed for weak form factors with rho in final state. See also K*.

$$B \to \rho\ell\bar{\ell} \qquad B_{\mathrm{s}} \to K^*\ell\bar{\ell}$$

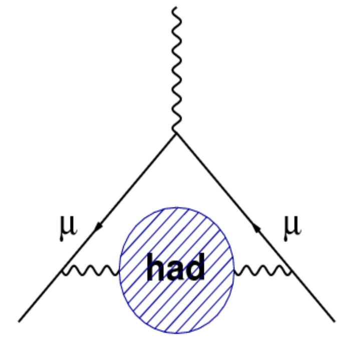
Horgan et al. `14

- Statistically precise, experimentally well-understood playground to study lattice spacing, finite volume effects.

Timelike pion form factor



$\Longrightarrow$



- HVP from time-momentum rep.:

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0\, \tilde{K}(x_0) G(x_0),$$

Jegerlehner and Nyffeler '09;
Bernecker and Meyer `11;
Mainz group `17

$$G(x_0) = -a^3 \sum_{\boldsymbol{x}} \langle \hat{V}_j(x)\, \hat{V}_j(0)\rangle = G^{\rho\rho}(x_0) + G^{I=0}(x_0)$$

- Isovector (dominant) part:

$$G^{\rho\rho}(x_0) = \frac{1}{48\pi^2} \int_{2m_\pi}^\infty d\omega\, \omega^2 \left(1 - \frac{4m_\pi^2}{\omega}\right)^{3/2} |F_\pi(\omega)|^2\, \mathrm{e}^{-\omega|x_0|}$$

Scattering amplitudes in lattice QCD

- In Euclidean time, asymptotic limit of $\langle 0|T\left\{\hat{O}'(x')\ldots\hat{O}^\dagger(x)\right\}|0\rangle$ contains no info about on-shell amplitudes.

L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585

- Finite volume method: below $n \geq 3$ hadron thresholds:

$$\det[K^{-1}(E_{\rm cm}) - B(L\mathbf{q}_{\rm cm})] + {\rm O}({\rm e}^{-ML}) = 0$$

$$S = (1-iK)^{-1}(1+iK)$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

- Determinant over total angular momentum, channel, and total spin
- Block-diagonal in finite-volume irreps.

Scattering amplitudes in lattice QCD (II)

To calculate $|F_\pi(E_{\mathrm{cm}})|$:

Lellouch and Luscher `01
Meyer `11
Feng, Aoki, Hashimoto, Kaneko `15

- Ignore partial waves $\ell \geq 3.$ (Small effect in the amplitude.)
- Determine finite volume matrix element
- Calculate and parametrize $\delta_1(E_{\mathrm{cm}})$
- Combine matrix element and LLM factor:

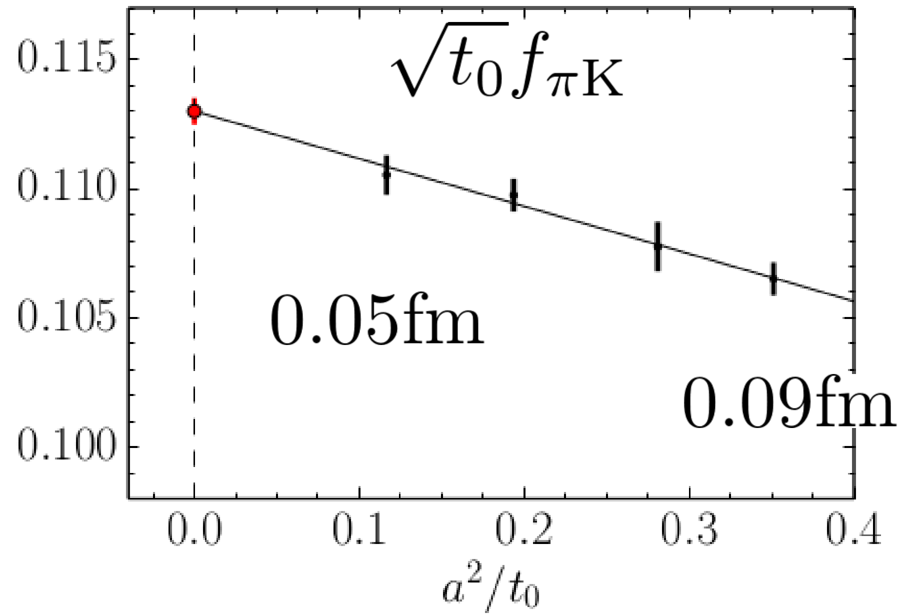
$$|F_\pi(E_{\mathrm{cm}})|^2 = \frac{2\pi E_{\mathrm{cm}}}{2L^3 p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial \delta_1}{\partial p} \right) |\langle 0|\hat{j}_{\mathrm{em}}|\pi(\vec{p}_1)\pi(\vec{p}_2)\rangle|^2$$

Systematic errors in lattice QCD

In order to provide QCD results, systematics must be assessed:

- Lattice Spacing:

$$F^{\rm lat} = F^{\rm QCD} + {\rm O}(a^2)$$
$$(+{\rm O}(g_0^n a))$$



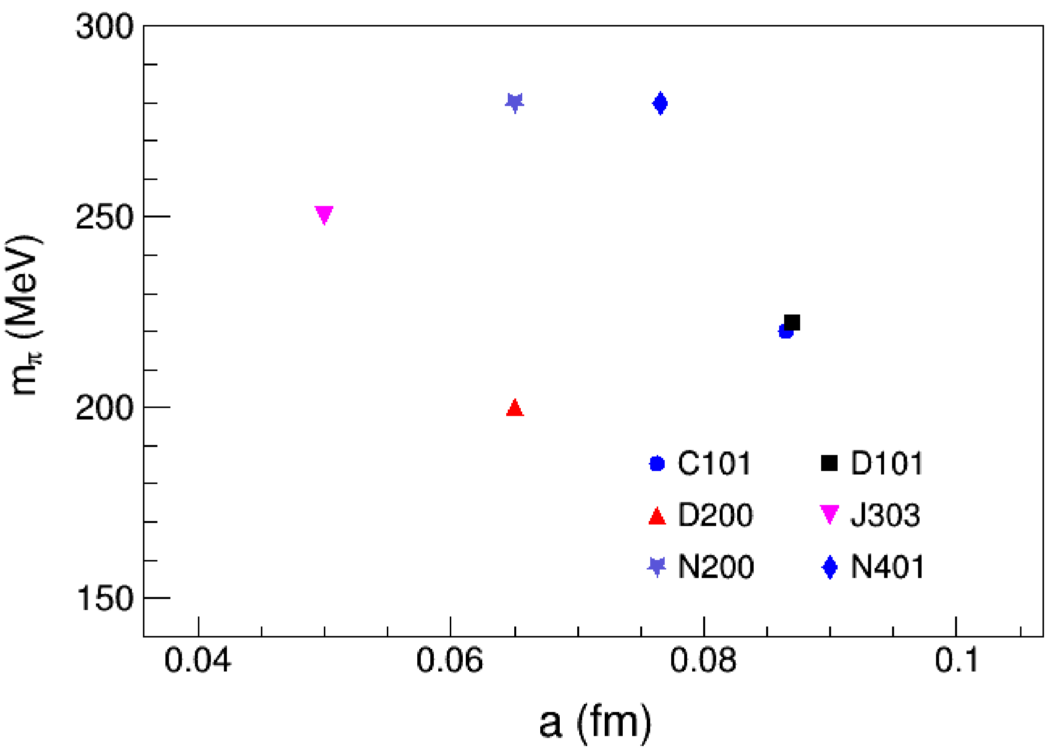
M. Bruno, T. Korzec, S. Schaefer, *Phys. Rev.* **D95** 074504 (2017)

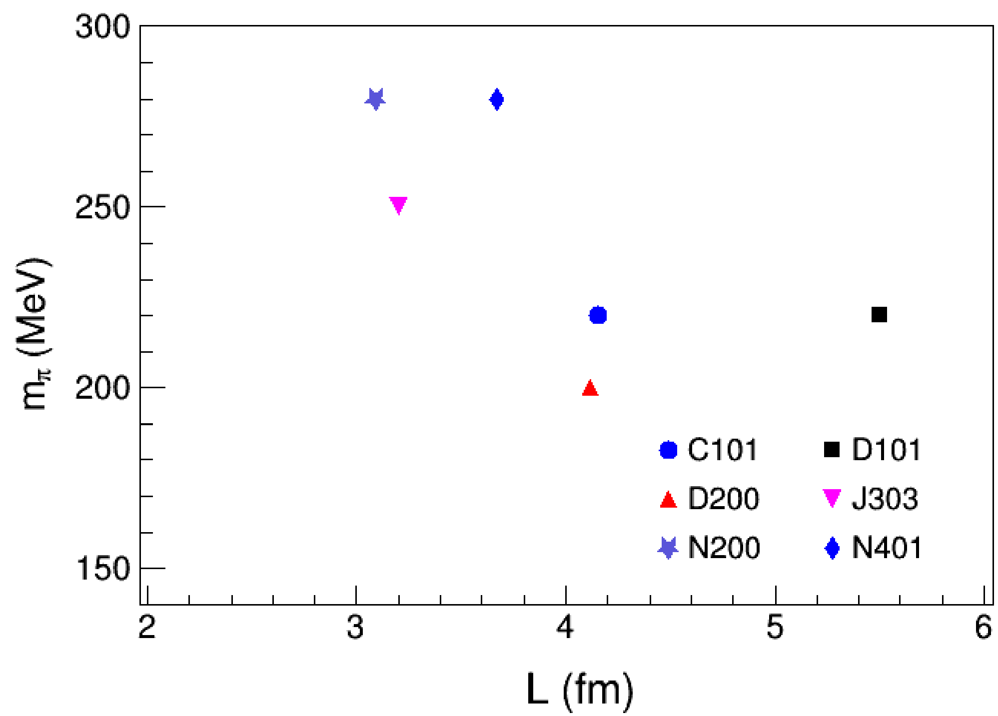
- (Residual) Finite volume effects
- Unphysical quark masses (dependence on $m_{\rm u,d}$, $m_{\rm s}$ also interesting)
- Determination from asymptotic-time limit in temporal correlators

CLS ensembles

M. Bruno, D. Djukanovic, G. Engel, A. Francis, G. Herdoiza, H. Horch, P. Korcyl, T. Korzec, M. Papinutto, S. Schaefer, E. Scholz, J. Simeth, H. Simma, W. Söldner, JHEP **1502** (2015) 043

- 4 lattice spacings $a \geq 0.05\text{fm}$, pion masses $m_\pi \gtrsim 200\text{MeV}$
- $N_\mathrm{f} = 2+1$ chiral limit: $\mathrm{Tr}\, M = 2m_{\mathrm{u,d}} + m_\mathrm{s} = const.$
- Finite volume check: $m_\pi L = 4.6,\ 6.1$





Analysis and data

- All total momenta $d^2 \leq 4$ => 711 correlators.
- Result: 133 energies and 68 O(a)-improved matrix elements

$$(V_\mathrm{R})_\mu = Z_\mathrm{V}\,(1 + ab_\mathrm{V} m_\mathrm{l} + a\bar{b}_\mathrm{V}\,\mathrm{tr}\,M_\mathrm{q})(V_\mathrm{I})_\mu$$

$$(V_\mathrm{I})_\mu = V_\mu + ac_\mathrm{V}\tilde{\partial}_\nu T_{\mu\nu}$$

Renormalization and improvement coeffs. from A. Gerardin, T. Harris, H. Meyer (in prep.)

- How to get the data:
  - Tables in paper
  - Python analysis suite (jupan) on github, hdf5 data files on zenodo
- Not going to talk about:
  - Efficient alg. for correlators
  - Excited state energies and matrix elements
  - Finite-T effects w/ open b.c.'s
  - Cuts chosen so systematic error smaller than statistical

Analysis notebook (jupan)

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ebatz / **jupan**

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Jupyter Analysis Notebook for Lattice QCD Spectroscopy

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README.md

Jupyter Analysis Notebook for Lattice QCD Spectroscopy

Introduction

This package contains utilities and a frontend for the determination of spectra and scattering amplitudes from Lattice QCD correlation functions. A Jupyter notebook is used to illustrate the analysis choices leading to the spectrum.

Setup

Prerequisites

The following analysis libraries need to be accessible for all features of this code to work properly:

- Eigen3
- Minuit2
- Pybind11

Data (zenodo)

zenodo Search Upload Communities

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bulava@cp3.sdu.dk

August 7, 2018

Dataset Open Access

Correlator data for determination of the I=1 pion-pion scattering amplitude and timelike pion form factor from Nf=2+1 lattice QCD

Andersen, Christian; Bulava, John; Hörz, Ben; Morningstar, Colin

Bootstrap samples of all correlation functions involved in the analysis of pion-pion scattering data and the timelike pion form factor described in "The I =1 pion-pion scattering amplitude and timelike pion form factor from N f = 2 + 1 lattice QCD". Additionally, an analysis file is provided for each ensemble which stored the analysis choices made in that work. These data are intended for use with the Jupyter notebook located in https://github.com/ebatz/jupan, which provides an interface. This notebook performs the entire analysis chain discussed in the above paper.

Files (1.5 GB)

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New version

Publication date:
August 7, 2018

DOI:
DOI 10.5281/zenodo.1341269

Keyword(s):
lattice QCD | pion-pion scattering | timelike pion form factor

Versions

Version 1.0.1-beta — Aug 7, 2018
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Higher partial waves

	Λ	ℓ
$(0,0,0)$	T_{1u}^+	$1,\,3,\,5^2,\,\ldots$
$(0,0,n)$	A_1^+	$1,\,3,\,5^2,\ldots$
	E^+	$1,\,3^2,\,5^3,\,\ldots$
$(0,n,n)$	A_1^+	$1,\,3^2,\,5^3,\,\ldots$
	B_1^+	$1,\,3^2,\,5^3\,\ldots$
	B_2^+	$1,\,3^2,\,5^3,\,\ldots$
(n,n,n)	A_1^+	$1,\,3^2,\,5^2,\,\ldots$
	E^+	$1,\,3^2,\,5^4,\,\ldots$

- Determinant block-diagonal in finite-volume irreps.
- Each block infinite-dimensional. Elements depend on:

$$R_{\ell m} = (\gamma \pi^{3/2} u^{\ell+1})^{-1}\,\mathrm{Re}\,\mathcal{Z}_{\ell m}(\boldsymbol{s}, \gamma, u^2)$$

- Ok to truncate to leading partial wave in each block?

Higher partial waves

- Automated determination of B-matrix elements

C. Morningstar, JB, B. Singh, R. Brett, J. Fallica, A. Hanlon, B. Hörz, Nucl. Phys. **B924** (2017) 477

- For all partial waves $\ell \leq 6$, all total spin $s \leq 7/2$, all irreps, (non-)identical particles.
- Publicly available C++ code for evaluation. (github)
- Example box matrix element:

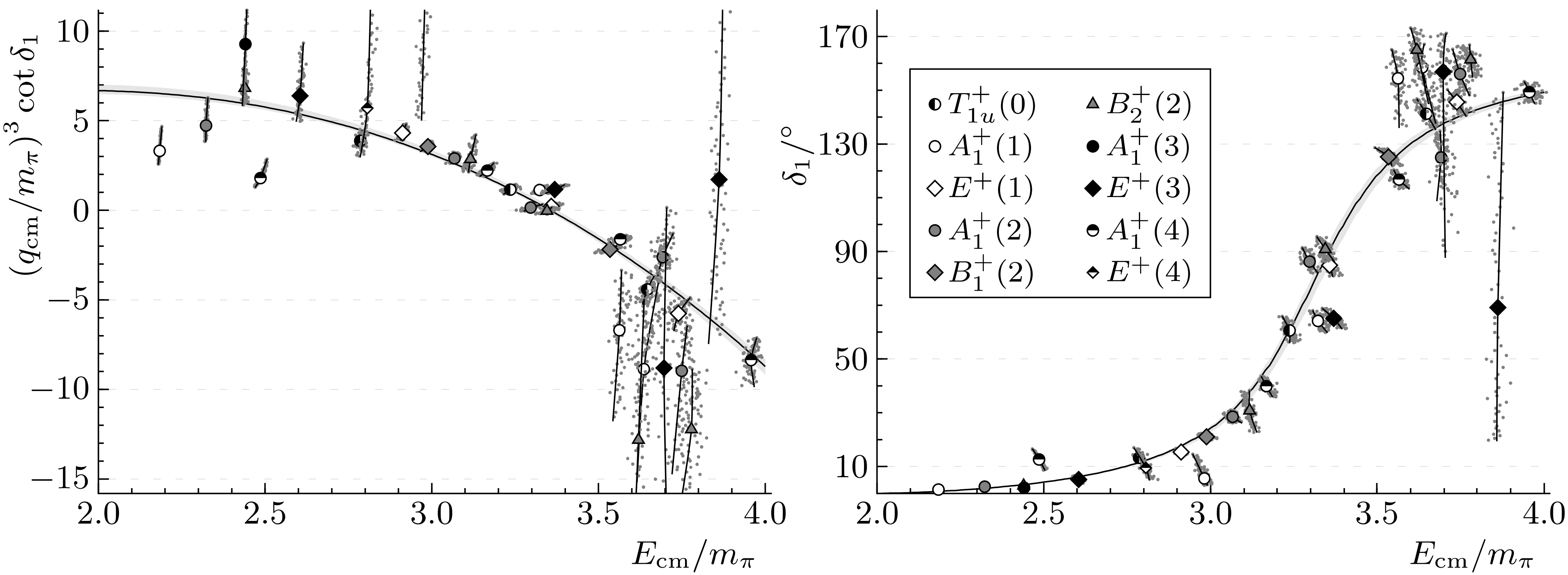
$$B^{A_1,\mathrm{oa}}(\ell_1 = \ell_2 = 6, n_1 = n_2 = 1) = R_{00} - \frac{2\sqrt{5}}{55} R_{20} - \frac{96}{187} R_{40} - \frac{80\sqrt{13}}{3553} R_{60} + \frac{445\sqrt{17}}{3553} R_{80} + \frac{15\sqrt{24310}}{3553} R_{88} - \frac{498\sqrt{21}}{7429} R_{10,0} + \frac{6\sqrt{510510}}{7429} R_{10,8} + \frac{2178}{37145} R_{12,0} + \frac{66\sqrt{277134}}{37145} R_{12,8}$$

Higher partial waves

- *P*-wave: $$(\tilde{K}^{-1})_{11} = \left(\frac{m_\rho^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)\frac{6\pi}{g_{\rho\pi\pi}^2}\frac{E_{\rm cm}}{m_\pi}$$
- *F-wave:* $$(\tilde{K}^{-1})_{33} = -m_\pi^7\, a_3$$

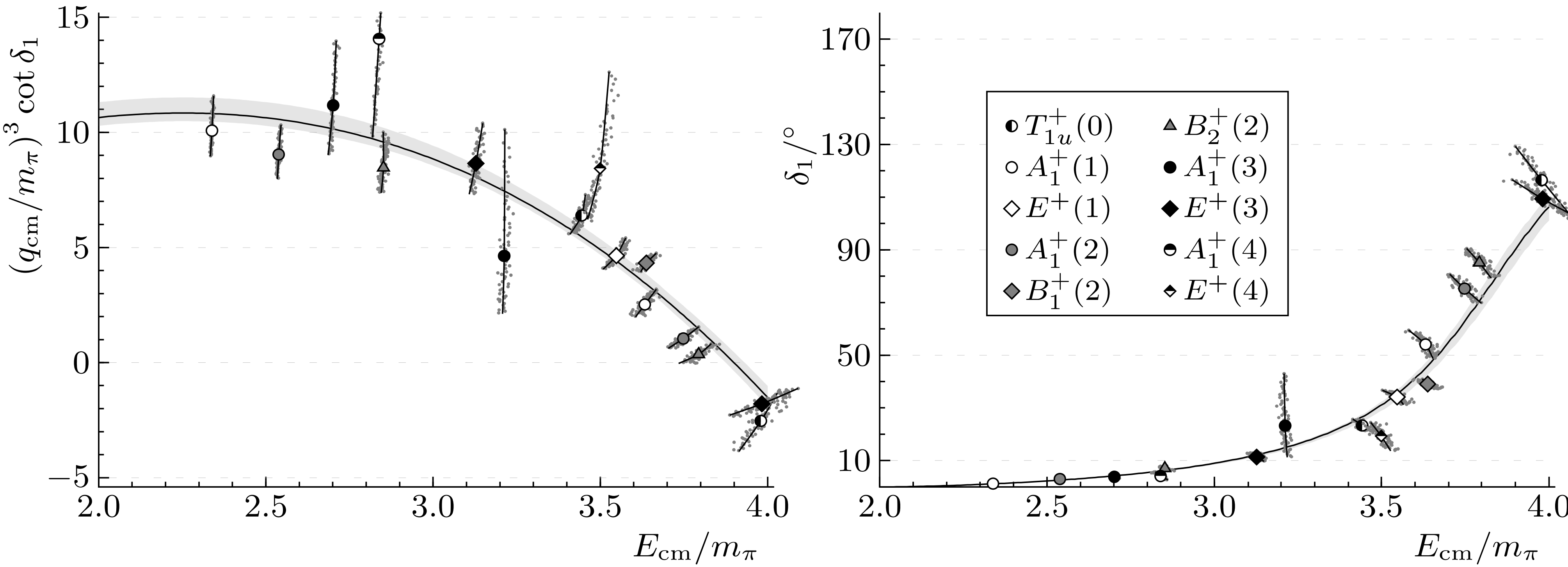
	$\ell = 1$ fits		$\ell = 1, 3$ fits		
ID	m_ρ/m_π	$g_{\rho\pi\pi}$	m_ρ/m_π	$g_{\rho\pi\pi}$	$m_\pi^7 a_3 \times 10^3$
D101	3.366(15)	6.19(10)	3.370(15)	6.23(10)	-0.56(30)
C101	3.395(26)	5.67(17)	3.399(30)	5.72(19)	-0.18(26)
N401	2.717(16)	5.84(12)	2.721(16)	5.88(13)	-2.7(3.0)
N200	2.733(16)	5.94(10)	2.733(16)	5.94(10)	0.0(2.9)
D200	3.877(34)	6.16(19)	3.883(36)	6.15(20)	-0.61(94)
J303	3.089(25)	6.30(17)	3.096(25)	6.32(17)	-4.2(3.6)

Results – Highlights (I)



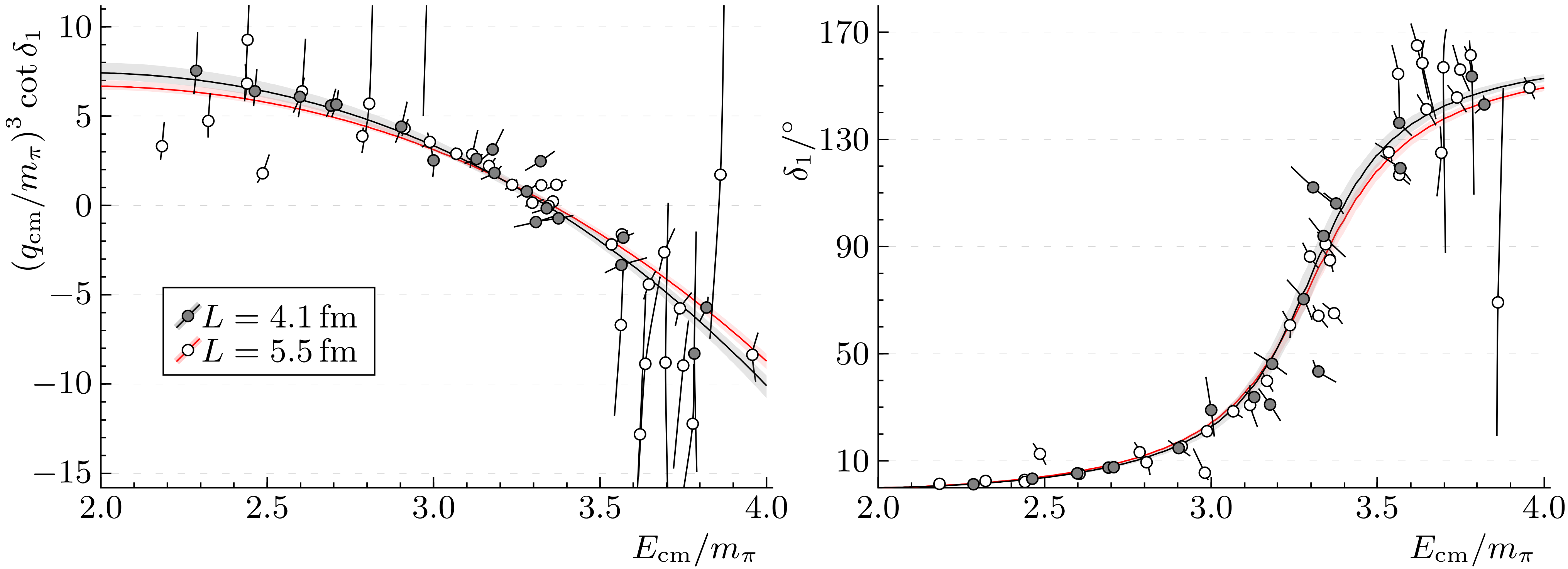
D101: $m_\pi = 220\,\mathrm{MeV}, \quad L = 5.5\,\mathrm{fm}, \quad (N_{\rm ev} = 928)$

Results – Highlights (II)



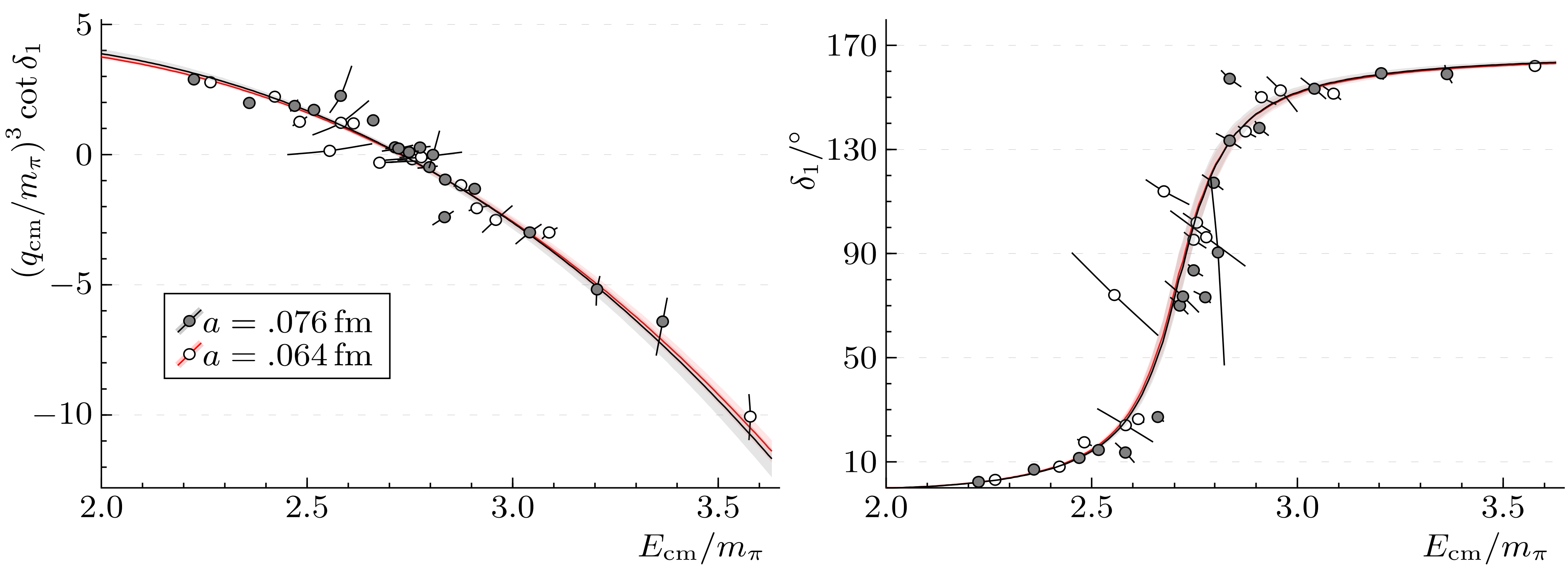
D200: $m_\pi = 200\,\mathrm{MeV}, \quad m_\pi L = 4.2, \quad a = 0.065\,\mathrm{fm}$

Results – finite volume check



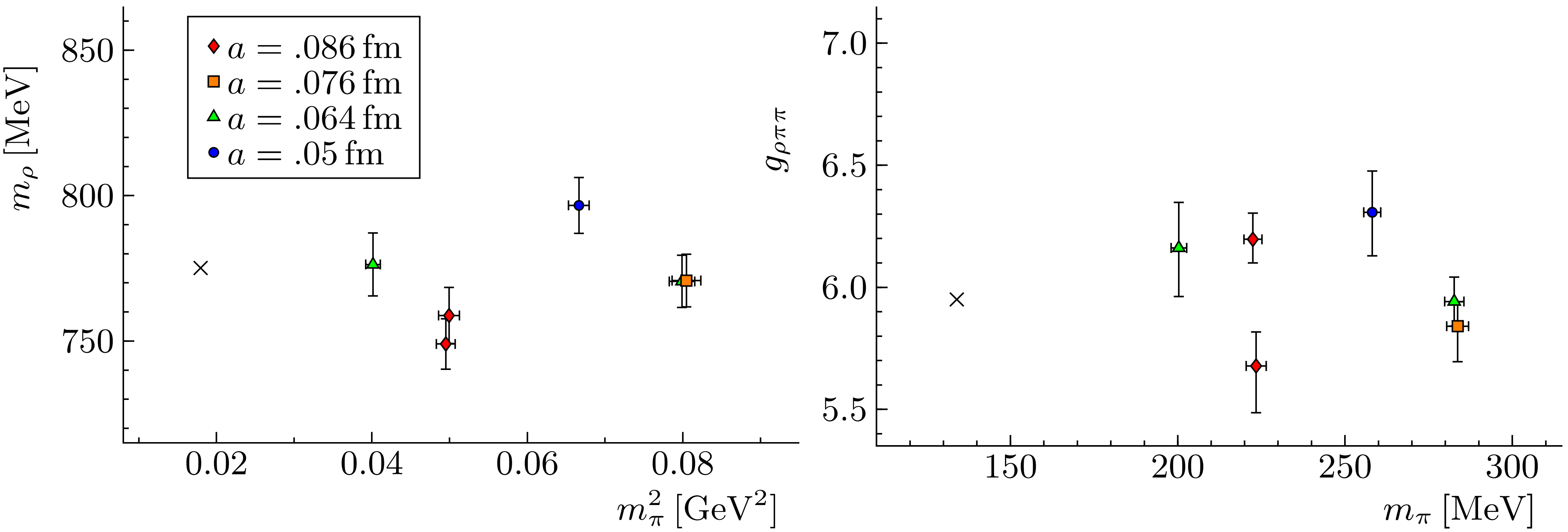
C101/D101: $m_\pi = 220\,\mathrm{MeV}, \quad a = 0.086\,\mathrm{fm}$

Results – lattice spacing check



N401/N200: $m_\pi = 280\,\mathrm{MeV}, \quad L^3 \times T = 48^3 \times 128$

Results – amplitude summary



- Chiral behavior of mass somewhat flat
- Chiral/continuum extrapolation should be done properly

B. Hu, R. Molina, M. Döring, M. Mai, A. Alexandru, `17;
D. Bolton, R. Briceno, D. Wilson, `16; …..

Fits to Form Factor (I)

- Gonaris-Sakurai parametrization (not a fit):

$$F_\pi^{\rm GS}(\sqrt{s}) = \frac{f_0}{q_{\rm cm}^2 h(\sqrt{s}) - q_\rho^2 h(m_\rho) + b(q_{\rm cm}^2 - q_\rho^2) - \frac{q_{\rm cm}^3}{\sqrt{s}} i},$$

$$b = -h(m_\rho) - \frac{24\pi}{g_{\rho\pi\pi}^2} - \frac{2q_\rho^2}{m_\rho} h'(m_\rho), \qquad f_0 = -\frac{m_\pi^2}{\pi} - q_\rho^2 h(m_\rho) - b\frac{m_\rho^2}{4},$$

$$h(\sqrt{s}) = \frac{2}{\pi}\frac{q_{\rm cm}}{\sqrt{s}} \ln\left(\frac{\sqrt{s} + 2q_{\rm cm}}{2m_\pi}\right),$$

- 'Commonly' used:

Feng et al. `15; Giusti et al. `18; Meyer and Wittig '18; …

Fits to Form Factor (II)

- nth-subtracted dispersion relation:

$$F_\pi(s) = \sum_{k=0}^{n-1} \frac{s^k}{k!} \frac{d^k}{ds^k} F_\pi(0) + \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dz}{z^n} \frac{\tan\delta_1(z) \mathrm{Re}\, F_\pi(z)}{z - s - i\epsilon}$$

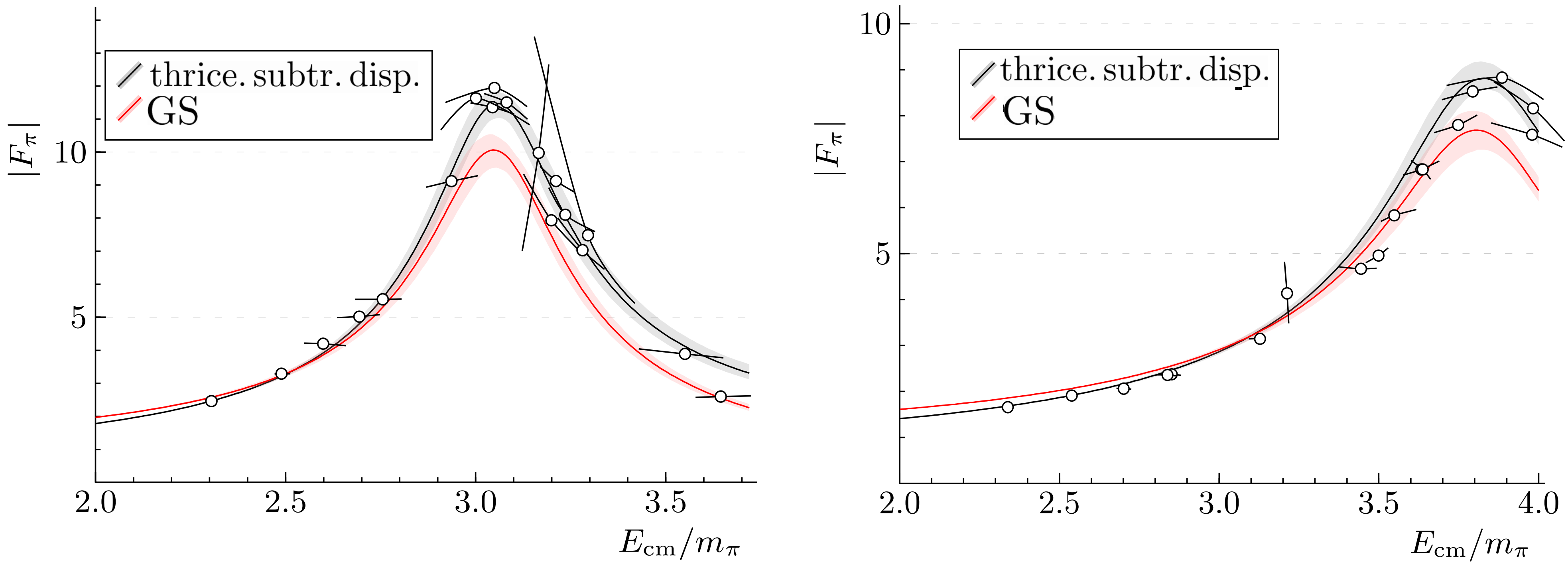
- Omnès-Muskhelishvili solution:

$$F_\pi(s) = Q_n(s)\ \exp\left\{ \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dz}{z^n} \frac{\delta_1(z)}{z - s - i\epsilon} \right\}$$

$$= Q_n(s)\, \Omega_n[\delta_1](s), \qquad \ln\, Q_n(s) = \sum_{k=1}^{n} p_k\, s^k,$$

- n = 2 has one parameter, n = 3 has two parameters.

GS vs. 3-subtracted disp. rel.

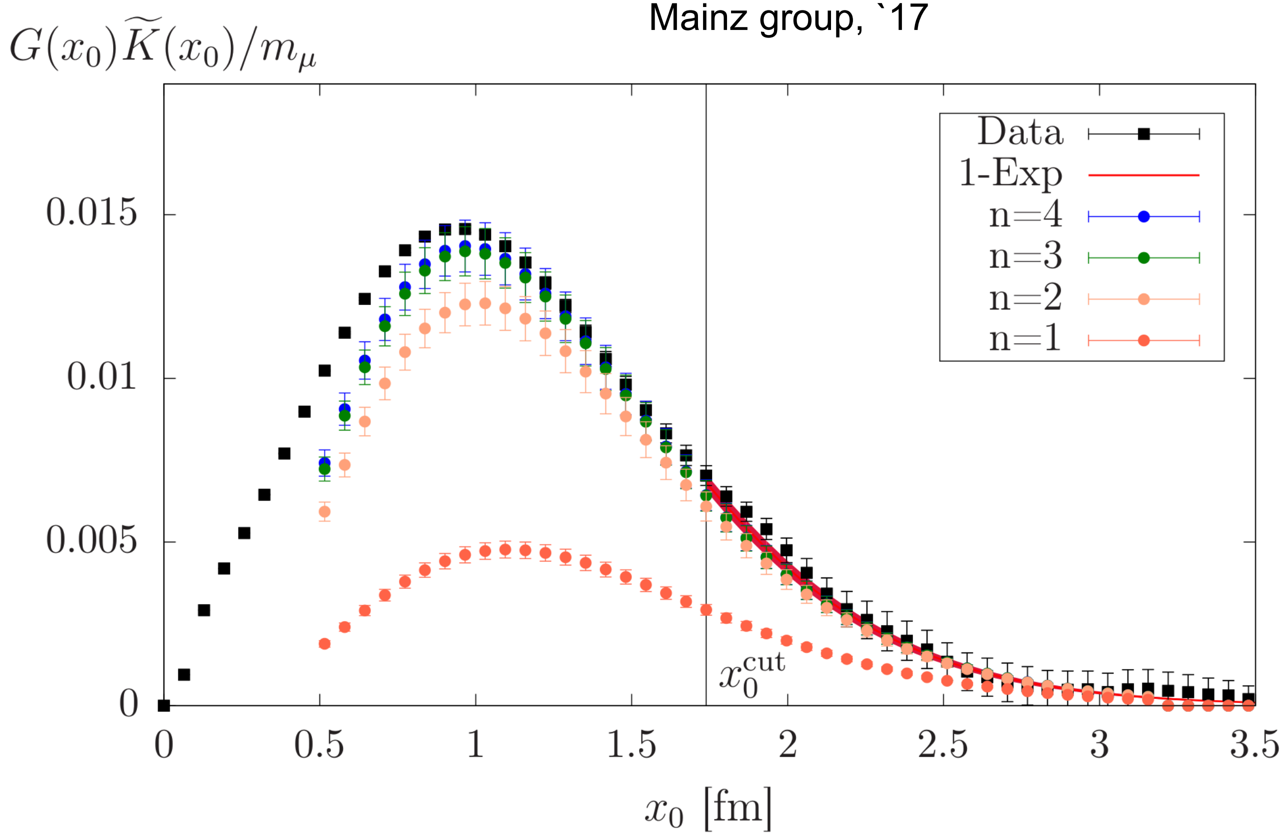


J303 (left): $m_\pi = 260\,\mathrm{MeV}, \quad a = 0.050\,\mathrm{fm},\ L = 3.2\,\mathrm{fm}$

D200 (right): $m_\pi = 200\,\mathrm{MeV}, \quad a = 0.065\,\mathrm{fm},\ L = 4.2\,\mathrm{fm}$

Look ahead: application to HVP

Mainz group, `17



Vector-vector correlator in finite volume (D200):

$$\lim_{x_0\to\infty} G(x_0, L) = \sum_{j=1}^{n} \left| \langle 0|\hat{V}|\pi\pi, j\rangle \right|^2 \mathrm{e}^{-E_j x_0}$$

Look ahead: K*(892)

- First test on anisotropic Nf = 2+1 lattice: $a_{\rm s}/a_{\rm t} = 3.5$

$$32^3 \times 256,\ m_\pi = 240\text{MeV},\ a_{\rm s} = 0.12\text{fm},\ L = 3.8\text{fm}$$

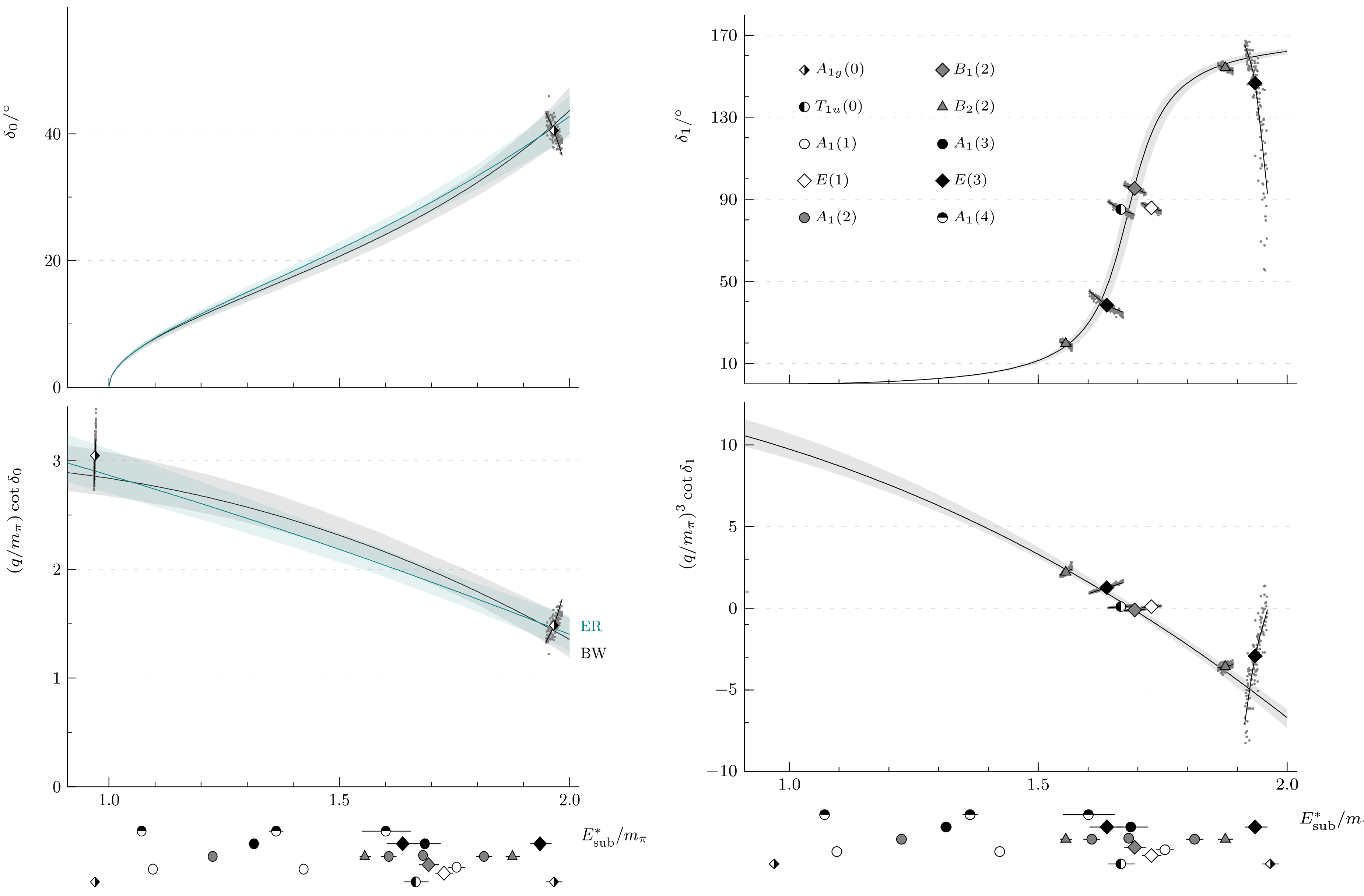
R. Brett, JB, J. Fallica, A. Hanlon, B. Hörz, C. Morningstar, Nucl. Phys. **B932** (2018) 29-51

- Non-identical particles: even-odd partial wave mixing

- Both s- and p-wave are fit simultaneously

mom.	irrep	ℓ
0	A_{1g}	$0,\ 4,\ \ldots$
	T_{1u}	$1,\ 3,\ \ldots$
1	A_1	$0,\ 1,\ 2,\ \ldots$
	E	$1,\ 2,\ 3,\ \ldots$
2	A_1	$0,\ 1,\ 2,\ \ldots$
	B_1	$1,\ 2,\ 3,\ \ldots$
	B_2	$1,\ 2,\ 3,\ \ldots$
3	A_1	$0,\ 1,\ 2,\ \ldots$
	E	$1,\ 2,\ 3,\ \ldots$
4	A_1	$0,\ 1,\ 2,\ \ldots$

Look ahead: K*(892)

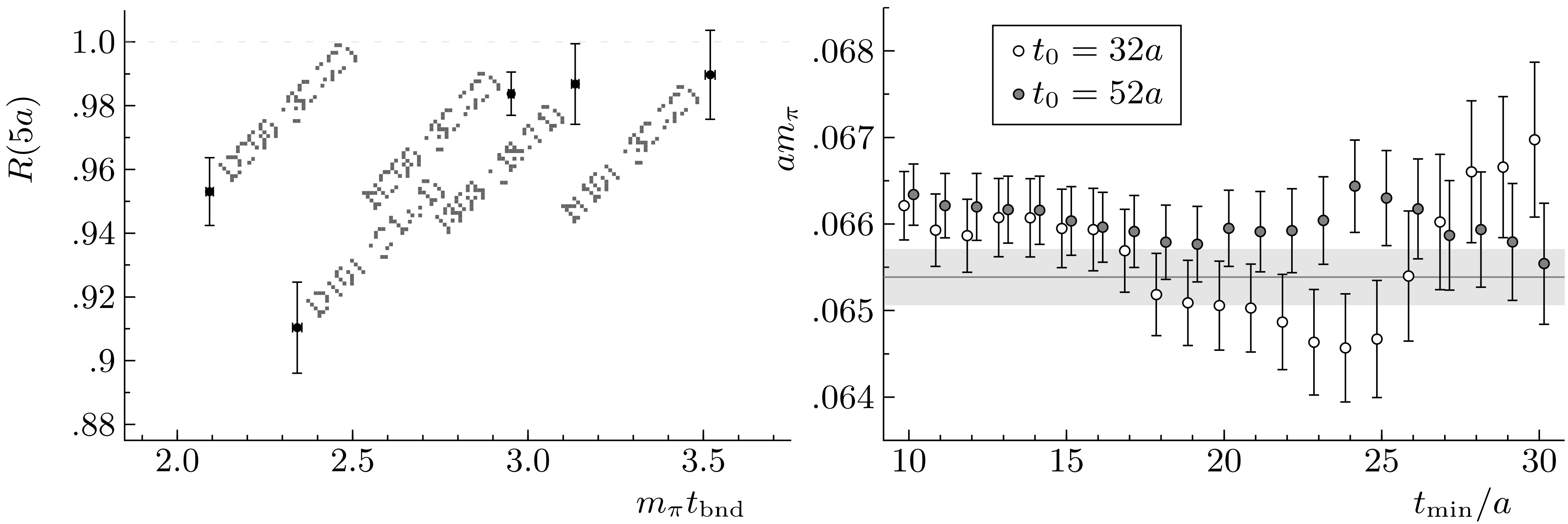


Conclusions

- Completed lattice calculation of $\delta_1(E_{\rm cm})$ and $|F_\pi(E_{\rm cm})|$testing finite volume and cutoff effects.
- At $m_\pi = 280\,{\rm MeV}$, consistency between $a = 0.075\,{\rm fm}$ and $a = 0.065\,{\rm fm}$ (ff. also)
- At $m_\pi = 220\,{\rm MeV}$, consistency between $m_\pi L = 4.6,\ 6.1$
- Chiral behavior of m_ρ a bit flatter than $m_{\rm s} = {\rm const.}$
- (Publicly available) data can be used with EFT's to perform chiral (and continuum) limit.
- Timelike pion form factor data can be used to extend vector-vector correlator, improving HVP determination in TMR.
- Challenges:
    - Disconnected e.m. current insertions
    - Other scattering channels and matrix elements (progress on $K^*(892)$, D(1232))

C. Andersen, J. Bulava, B. Hörz, C. Morningstar, Phys. Rev. D97 (2018) 014506

Finite-T effects (open b.c.'s)



$$\lim_{\substack{T\to\infty \\ t_0,(T-t_{\rm f})\to\infty}} C_T(t_0,t_{\rm f}) = C(t_{\rm f}-t_0)\times\left\{1+{\rm O}({\rm e}^{-E_0 t_{\rm bnd}})\right\},$$

$$R_{t_0,t_0'}(t) = \frac{C_T(t_0,t+t_0)}{C_T(t_0',t+t_0')}$$

Delta(1232) setup

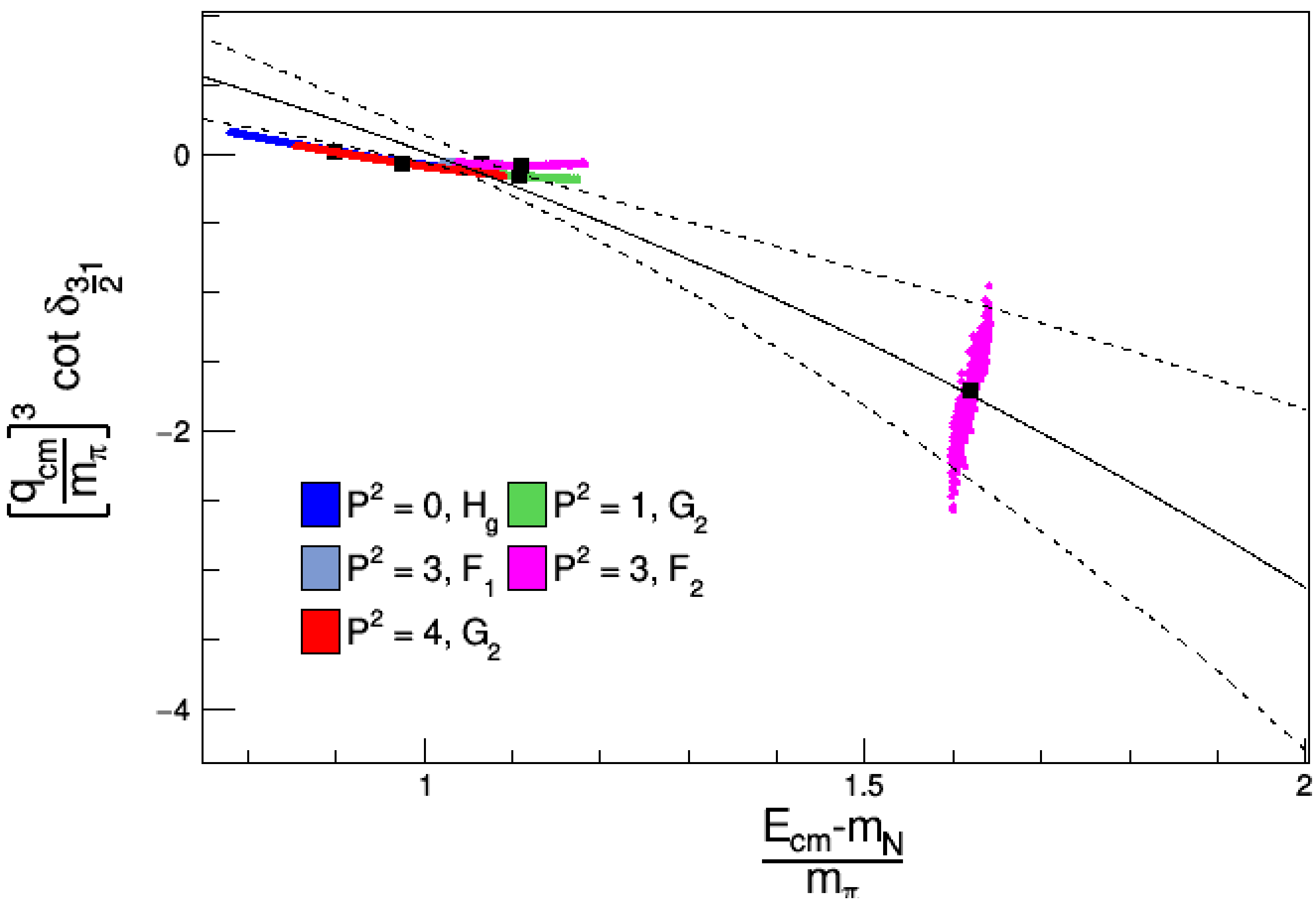
- Choose $I = 3/2$ irreps where $\ell(J^P) = 1(3/2^+)$ is the lowest partial wave

mom.	irrep	$\ell(J^p)$
$(0,0,0)$	H_g	$1(3/2^+), 3(5/2^+), \ldots$
	H_u	$2(3/2^-), 2(5/2^-), \ldots$
$(0,0,n)$	G_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
(n,n,n)	F_1	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$
	F_2	$1(3/2^+), 2(3/2^-), 2(5/2^-), \ldots$

- Neglecting d-wave Delta(1700), relying on orbital angular momentum threshold suppression of d-wave.

Delta(1232) first results

$$(L = 3.6\text{fm}, a = 0.075\text{fm},\ m_\pi = 280\text{MeV})$$



$$\frac{m_\Delta}{m_\pi} = 4.738(47), \qquad g_{\Delta N\pi} = 19.0(4.7), \qquad \frac{\chi^2}{d.o.f} = 1.11$$

C. W. Andersen, JB, B. Hoerz, C. Morningstar, Phys. Rev. D97 (2018) 014506