Masses and decay constants of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ close to the physical point

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"Scattering Amplitudes and Resonance Properties from Lattice QCD", MITP
Mainz, 27th August 2018

Outline

★ Introduction

- Motivate interest in the $J^P = 0^+$ $D^*_{s0}(2317)$ and $J^P = 1^+$ $D_{s1}(2460)$.
- Lie close to strong decay thresholds and expected to have an interesting internal structure.
- Can lattice say anything about the internal structure?

★ Results for lower lying D_s mass spectrum.

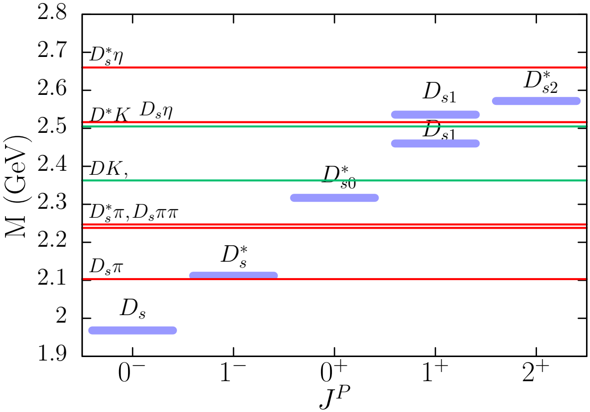
★ Decay constants of the $D^*_{s0}(2317)$ and $D_{s1}(2460)$.

- Compare with decay constants of conventional mesons, 0^- and 1^-.

More details in [Bali,1706.01247].

Lower lying D_s spectrum: mesons with $C = S = \pm 1$

Experimentally observed meson spectra:



Strong decays:

$D_s^* \longrightarrow D_s\pi$

$D_{s0}^* \longrightarrow D_s\pi$
$D_{s0}^* \longrightarrow D_s\pi\pi$

$D_{s1} \longrightarrow D_s^*\pi$
$D_{s1} \longrightarrow D_s\pi\pi$

$D_{s1} \longrightarrow D^*K$
$D_{s1} \longrightarrow DK\pi$
$D_{s1} \longrightarrow D_s\pi\pi$

- Widths: $D_{s0}^* < 3.8$ MeV, $D_{s1}(2460) < 3.5$ MeV, $D_{s1}(2536) = 0.92$ MeV, $D_{s2}^* = 17(4)$ MeV, $D_s^* < 1.9$ MeV, $D_s \sim 10^{-3}$ eV.
- Additional states: $D_{s1}^*(2700)^\pm$, $D_{sJ}(2860)$, $D_{sJ}(3040)^\pm$.
- Radiative + weak decays also observed/possible.

What is the nature of these states?

Quark Model: $c\bar{s}$

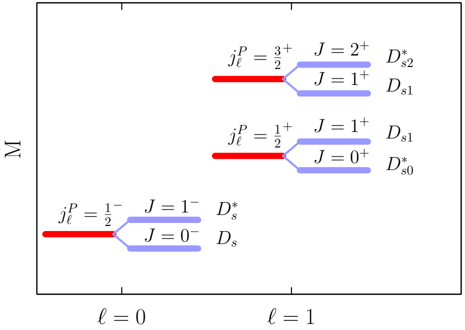
Minimum quark content to satisfy the flavour quantum numbers $S = 1,\ C = 1$.

HQET: $Q\bar{\ell}$ meson, hydrogen-like system, Q acts as a colour source.

Limit $m_Q \to \infty$
QNs: $j_\ell = l + s_\ell = \frac{1}{2}, \frac{3}{2}, \ldots,$

Finite m_Q
QNs: $J = l + S = 0, 1, 2, \ldots,$
$S = s_\ell + s_Q$

$P = -(-1)^l$

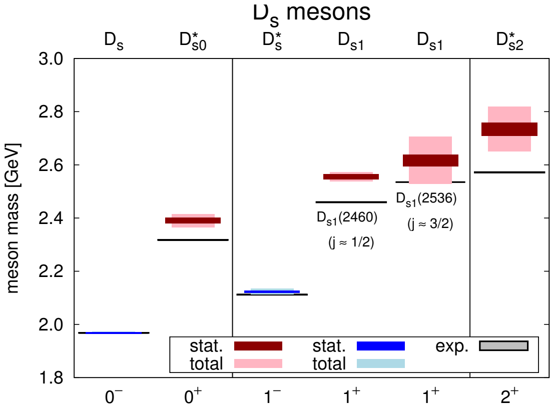


Other possibilities: $c\bar{q}q\bar{s}$
Molecule: weakly bound $(c\bar{q})$ and $(q\bar{s})$. Tetraquark: $(c\bar{q}q\bar{s})$ and more.

Lattice studies: standard approach

- Early theoretical studies and lattice simulations predicted $D_{s0}^*(2317)$ and $D_{s1}(2460)$ to be broad states above threshold.

More recently: [ETMC,1603.06467], $N_f = 2+1+1$, m_π^{phys} and continuum extrapolation.

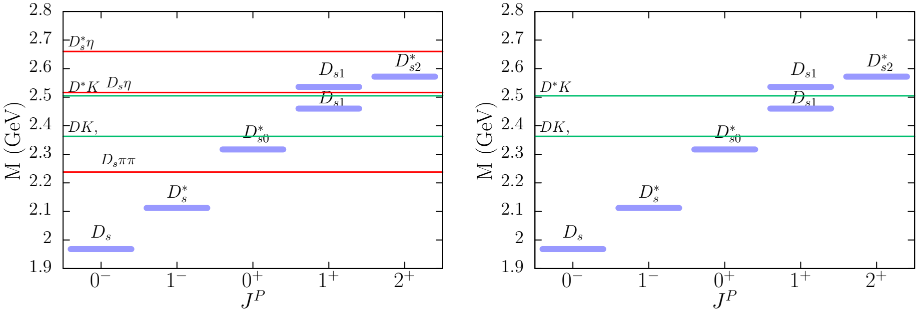


What can the lattice provide?

D_s spectrum:

- Postdiction of states well established experimentally.
  - Demonstration of lattice techniques.
- Investigating internal structure of non-standard candidates.
  - Determine the light quark mass dependence of the spectrum.
  - Calculate the decay constants and compare with those of conventional mesons.

Isospin limit, electrically neutral

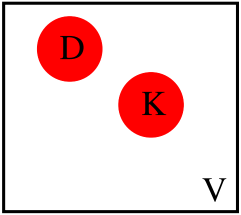


- D_{s0}^* is stable. D_{s1} can decay to $D_s\pi\pi$.
- Ignore $D_s\pi\pi$ and $D_s\eta$ (0^+), $D_s^*\eta$ (1^+).

Only consider (s-wave) DK and D^*K thresholds.

Finite volume mass spectrum

Interested in states close to $D + K$ and $D^* + K$ thresholds.

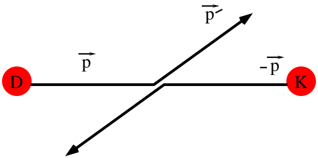


Infinite volume: $E = m_{D^{(*)}} + m_K$

Finite volume: $E = m_{D^{(*)}} + m_K + \Delta E$

$\Delta E > 0$ scattering/resonance

$\Delta E < 0$ bound state



Center of momentum frame: $\vec{p}_{D^{(*)}} = -\vec{p}_K$

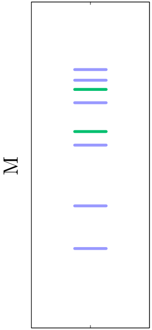
Elastic scattering: $|\vec{p}\,'_{D^{(*)}}| = |\vec{p}_{D^{(*)}}| = p$

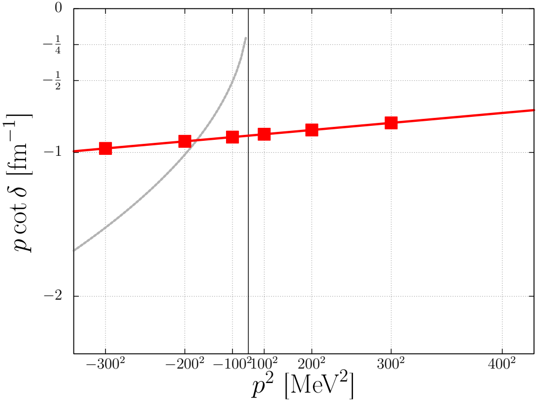
$E = \sqrt{m^2_{D^{(*)}} + p^2} + \sqrt{m^2_K + p^2}$

Lüscher's relation (s-wave):

$p \cot \delta(p) = \frac{1}{\pi L} \sqrt{4\pi}\, \mathcal{Z}_{00}\left(1; \frac{L^2}{4\pi^2} p^2\right)$

Finite volume mass spectrum





Lüscher's relation: $p \cot \delta\,(p) = \frac{1}{\pi L}\sqrt{4\pi}\mathcal{Z}_{00}\left(1; \frac{L^2}{4\pi^2}p^2\right)$

$T(s) = -8\pi\sqrt{s}/(p \cot \delta(p) - ip)$, $E = \sqrt{s}$

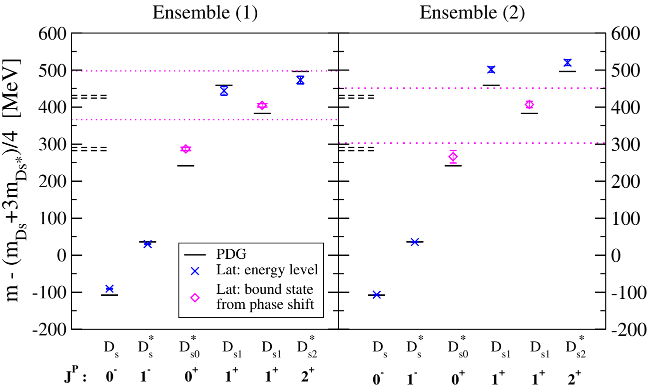
∞ volume bound state pole condition: $p \cot \delta\,(p) = ip$

Effective range approximation: $p \cot \delta\,(p) \approx a_0^{-1} + \frac{1}{2}r_0 p^2$

Can map out the phase shift by varying the volume, moving frames..

First study: $D_{s0}^*(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}^*(2573)$

Lang, Leskovec, Mohler, Prelovsek, Woloshyn: 1308.3175, 1403.8103

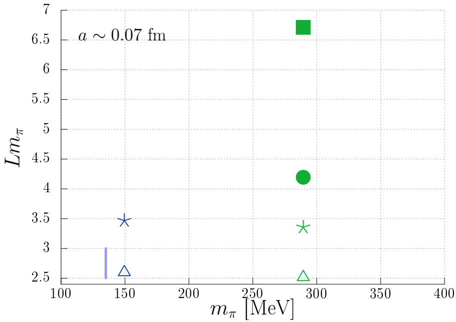


Ensemble 1: $N_f = 2$, $m_\pi = 280$ MeV, $a = 0.12$ fm, and $L = 2.0$ fm ($Lm_\pi = 2.7$)
Ensemble 2: $N_f = 2 + 1$, $m_\pi = 156$ MeV, $a = 0.09$ fm and $L = 2.9$ fm ($Lm_\pi = 2.3$)

Earlier: [Liu,1208.4535], $D\bar{K}$ channel (instead of DK) and SU(3) flavour symmetry.

Lattice details

RQCD+QCDSF: $N_f = 2$, assume valence strange makes the dominant contribution.
Gauge+quark action: $O(\Lambda^2 a^2)$, $O(m_q^2 a^2)$ discretisation effects. $am_c \sim 0.5$.



Near physical pion mass important to reproduce the physical threshold.

Volume varies 1.7-4.5 fm ($m_\pi = 150$ MeV) and 3.4-4.5 fm ($m_\pi = 290$ MeV).

High statistics: 800-2000 configurations.

Extracting the mass spectrum on the lattice

Construct matrix of correlators from operators with relevant QNs.

$$\sum_{\vec{x}} \langle O_j(t,\vec{x}) O_i^\dagger(0,\vec{0})\rangle = \sum_n \langle 0|O_j|n\rangle\langle n|O_i^\dagger|0\rangle e^{-E_m t} \sim A e^{-E_1 t}(1 + B e^{-(E_2 - E_1)t} + \ldots)$$

Operators respect lattice cubic symmetry, for bosons:

- $\mathbf{A_1} \to \mathbf{J} = \mathbf{0}, 4, \ldots$, $\mathbf{T_1} \to \mathbf{J} = \mathbf{1}, 3, 4, \ldots$

Irreducible representations: continuum O(3) symmetry has $J = 0, 1, 2, \ldots$, lattice cubic symmetry A_1, A_2, E, T_1, T_2.

Expect: A_1 channel, g.s. + DK level + ...
T_1 channel, g.s. + D^*K level + third level + ...

Extracting the mass spectrum on the lattice

For $J^P = 0^+$ use operators:

$c\bar{s}$: $O_{c\bar{s}} = c\bar{s}$, $O'_{c\bar{s}} = c\gamma_4\bar{s}$, $\quad D(\vec{0})K(\vec{0})$: $O_{c\bar{\ell}\ell\bar{s}} = c\gamma_5\bar{\ell}(\vec{0})\ell\gamma_5\bar{s}(\vec{0})$

Construct a matrix of correlators, $C(t)$, and solve for the eigenvalues:

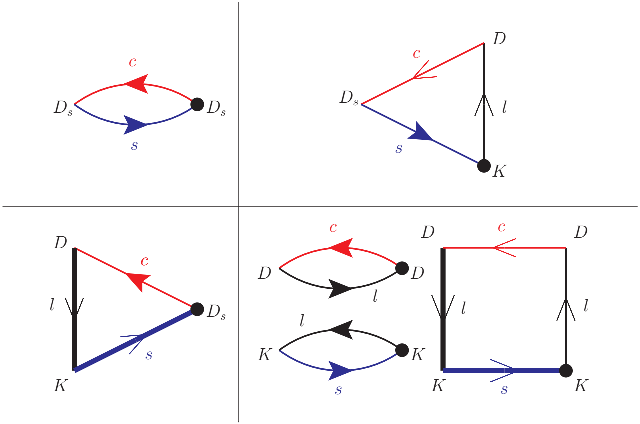
$$\begin{bmatrix} C_{c\bar{s}\to c\bar{s}}(t) & C_{c\bar{s}\to c\bar{s}'}(t) & C_{c\bar{s}\to c\bar{\ell}\ell\bar{s}}(t) \\ C_{c\bar{s}'\to c\bar{s}}(t) & C_{c\bar{s}'\to c\bar{s}'}(t) & C_{c\bar{s}'\to c\bar{\ell}\ell\bar{s}}(t) \\ C_{c\bar{\ell}\ell\bar{s}\to c\bar{s}}(t) & C_{c\bar{\ell}\ell\bar{s}\to c\bar{s}'}(t) & C_{c\bar{\ell}\ell\bar{s}\to c\bar{\ell}\ell\bar{s}}(t) \end{bmatrix}, \quad \lambda_n \sim De^{-E_n t}(1 + O(e^{-\Delta E_m t}))$$

For stability actually solve generalised eigenvalue problem: $t > t_0$

$$C(t)\, v_n(t, t_0) = \lambda_n(t, t_0)\, C(t_0)\, v_n(t, t_0) \qquad \lambda_n(t, t_0) = e^{-E_n(t-t_0)}\left(1 + \mathcal{O}\left(e^{-\Delta E_m t}\right)\right)$$

Actual matrix larger: 6×6, three of type $O_{c\bar{s}}$, two of type $O'_{c\bar{s}}$ and $O_{c\bar{\ell}\ell\bar{s}}$.

Quark line diagrams that need to be computed:



Use stochastic estimation: one-end trick + sequential propagators following [CP-PACS,0708.3705] ($\rho \to \pi\pi$) and [RQCD,1512.08678] ($\rho \to \pi\pi$).

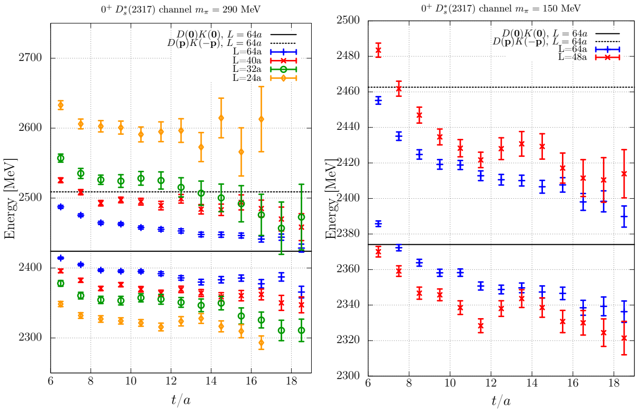
Computational cost restricts t in $C(t)$ to range $5 - 19$, $(N_T = 15)$.

Main overhead compared to standard $c\bar{s}$ analysis: $N_T + 3$ light propagators per configuration.

Effective masses of eigenvalues: $J^P = 0^+$

$$C(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0), \qquad \lambda_n(t,t_0) = e^{-E_n(t-t_0)} + \dots$$

$E_n(t+a/2, t_0) = \log\left(\lambda_n(t,t_0)/\lambda_n(t+a,t_0)\right)$, expect g.s. and DK level.

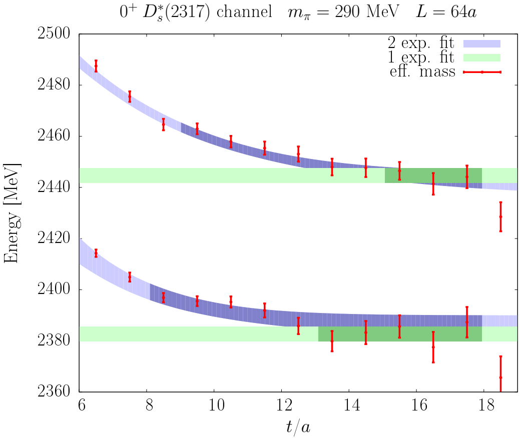


From 4×4 correlator matrix, three $O_{c\bar{s}}$ and one $O_{c\bar{\ell}\ell\bar{s}}$.

Fitting to eigenvalues: $J^P = 0^+$

Fit to
$\lambda_n \sim e^{-E_n t}(1 + ce^{-\Delta E_m t})$
in the range t_{min} to t_{max}.

From 4×4 correlator matrix:
$3 \times O_{c\bar{s}}$ and $1 \times O_{c\bar{\ell}\ell\bar{s}}$.



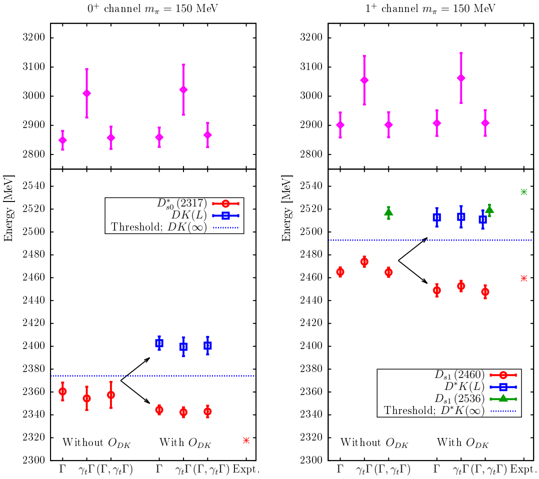
Thermal states: possible contributions for (anti) periodic b.c. of the form

$$\langle D|O_i|K\rangle\langle K|O_j^\dagger|D\rangle e^{-(T-t)m_K} e^{-m_D t}.$$

Estimate: keep $t_{max} < 19a$ ($17a$) for $T/a = 64$ (48) to avoid these contributions.

Finite volume spectrum: $J^P = 0^+$ and 1^+

Vary the operator basis for the correlator matrix.



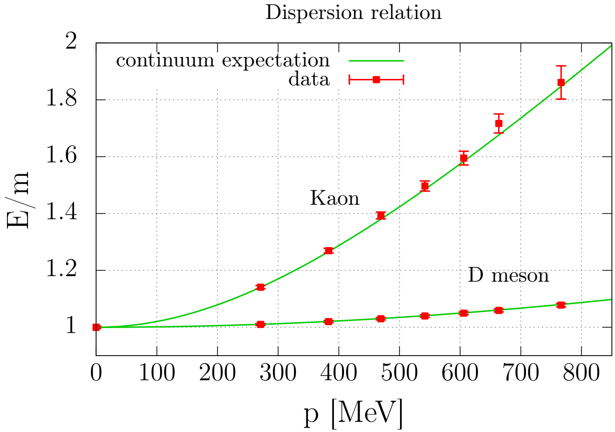
$c\bar{s}$ and $D^{(*)}K$ operators are needed to resolve g.s.+ DK level, depends on set up c.f. ETMC.

Axial-vector: need $O'_{c\bar{s}}$ in order to see third close lying level.

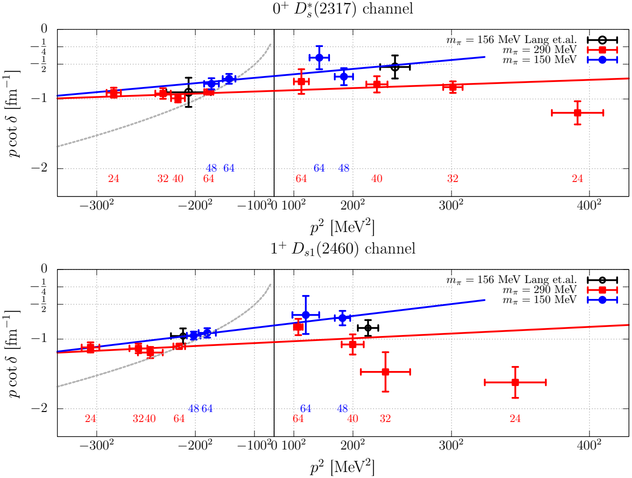
Extraction of the phase shift

$$E_n = \sqrt{m_K^2 + p_n^2} + \sqrt{m_{D^{(*)}}^2 + p_n^2} \quad \Rightarrow \quad p_n \cot\delta(p_n) = \frac{2}{L\sqrt{\pi}} \mathcal{Z}_{00}\left(1; \frac{L^2}{4\pi^2} p_n^2\right)$$

Dispersion relation: interested in p^2 up to 400^2 MeV.



Phase shift



Results for largest volumes close to infinite volume for g.s..

Effective range approximation: $p \cot \delta(p) = 1/a_0 + r_0 p^2/2 + \mathcal{O}(p^4)$

Omit $L = 24a$ for $m_\pi = 290$ MeV, p^2 may be too large or finite volume effects.

$J^P = 1^+$: third level ($D_{s1}(2536)$) not considered .

a_0, r_0 and g

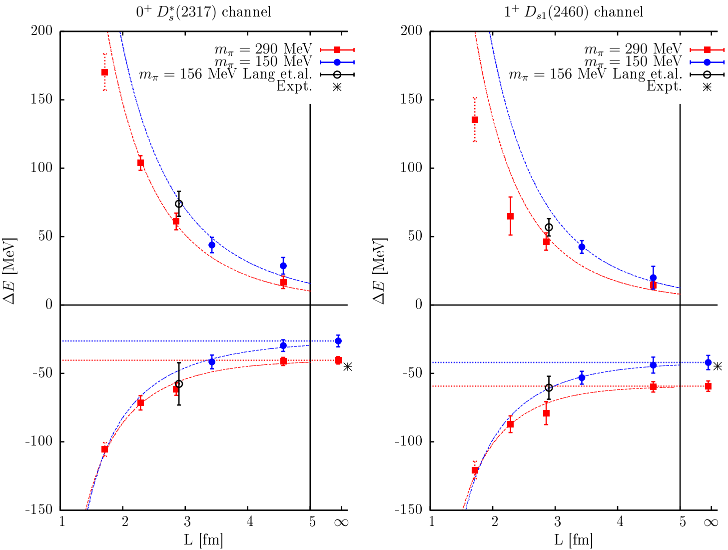
In the vicinity of the pole $T(s) = g^2/(s - s_B)$

Errors: (stat.)(finite V) finite V, drop smallest L.

	a_0 [fm]	r_0 [fm]	g [GeV]
	Scalar		
RQCD	**-1.49(0.13)(-0.30)**	**0.20(0.09)(+0.31)**	**11.0(0.6)(+1.2)**
[a]Lang et al.	-1.33(20)	0.27(17)	12.6(1.5)
[1]HMChPT,LQCD	-1.3(5)(1)	-0.1(3)(1)	11.3
[2]LQCD,HMChPT	-0.86(3)		
[3]HMChPT,Expt			10.203
[4]HMChPT,Expt,LQCD	$-1.04^{+0.06}_{-0.03}$		
[5]HMChPT,Expt,LQCD	$-0.89^{+0.06}_{-0.10}$		
[6]HMChPT,Expt	$-0.95^{+0.15+0.08}_{-0.15-0.13}$		
	Axialvector		
RQCD	**-1.24(0.09)(-0.12)**	**0.27(0.07)(+0.13)**	**13.8(0.7)(+1.1)**
[a]Lang et al.	-1.11(11)	0.10(10)	12.6(7)
[1]HMChPT,LQCD	-1.1(5)(2)	-0.2(3)(1)	14.2

(a) [Lang,1403.8103] (1) [Torres,1412.1706], (2) [Liu,1208.4535],
(3) [Guo,hep-ph/0603072], (4) [Yao,1502.05981], (5) [Guo,1507.03123],
(6) [Albaladejo,1604.01193]

Splitting with the threshold: $E_n = m_D + m_K + \Delta E_n$



Spectrum: $m_\pi = 150$ MeV

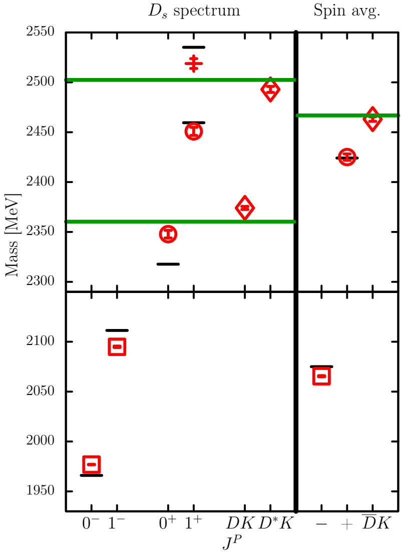
D_s, D_s^*, $D_{s1}(2536)$ from $L = 64a$.

Deviation from expt.

Likely discretisation effects:
$O(a^2)$, $O((ma)^2)$, $am_c \sim 0.5$.

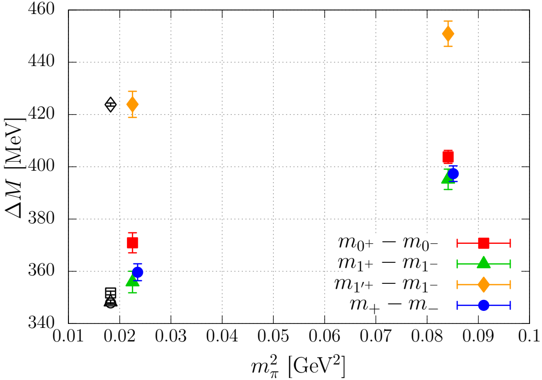
HQET: fine structure splittings $\rightarrow$
momentum scales close to $m_c \not\ll a^{-1}$.

Spin-average splittings $\rightarrow$
$O(\Lambda) \ll a^{-1} = 2.8$ GeV.



Spectrum: splittings

Separate out the light quark dependence.



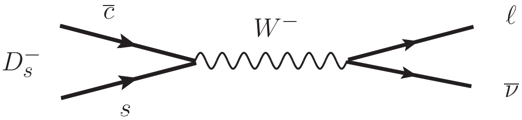
Short (crude) extrapolation: $m_+ - m_- = 356(3)$ MeV, c.f. 349 MeV from expt..

m_π dependence of splittings significant: due to mass shifts for D_{s0}^*, D_{s1}.

D_s and D_s^* masses only shift by $3 - 7$ MeV.

Decay constants

Leptonic decay: pseudoscalar D_s meson, $J^P = 0^-$,



Decay constant: $\langle 0|\bar{s}\gamma_\mu (1-\gamma_5)\, c|D_s(\boldsymbol{p})\rangle \longrightarrow \langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s(\boldsymbol{p})\rangle = f_A\, p_\mu$

Decay width:

$$\Gamma = \frac{G_F^2}{8\pi} f_{D_s}^2 m_l^2 M_{D_s} \left(1 - \frac{m_l^2}{M_{D_s}^2}\right)^2 |V_{cs}|^2$$

Lattice: FLAG review [Aoki,1607.00299]:

$$N_f = 2 \qquad f_{D_s} = 250(7) \text{ MeV} \qquad N_f = 2+1 \qquad f_{D_s} = 249.8(2.3) \text{ MeV}$$
$$N_f = 2+1+1 \qquad f_{D_s} = 248.83(1.27) \text{ MeV}$$

Decay constants

Vector meson, D_s^*, $J^P = 1^-$

[Becirevic,1201.4039], $N_f = 2$ twisted mass fermions, $f_{D_s^*}/f_{D_s} = 1.26(3)$,
[ETMC,1610.09671] $N_f = 2+1+1$ twisted mass fermions $f_{D_s^*}/f_{D_s} = 1.09(2)$.
[HPQCD,1312.5264] $N_f = 2+1+1$ HISQ fermions $f_{D_s^*}/f_{D_s} = 1.10(2)$,

Higher positive parity states: D_{s0}^* and D_{s1}

$J^P = 0^+$ **Vector** $\langle 0|\bar{s}\gamma_\mu c|D_{s0}^*(\boldsymbol{p})\rangle = \mathbf{f_V^{0^+}} p_\mu$

$J^P = 1^+$ **Axial-vector** $\langle 0|\bar{s}\gamma_\nu\gamma_5 c|D_{s1}(\boldsymbol{p},\epsilon)\rangle = \mathbf{f_A^{1^+}} m_{D_{s1}}\epsilon_\nu$

Compare the magnitude of $f_V^{0^+}$ and $f_A^{1^+}$ with those of conventional D_s and D_s^*.

In addition:

Scalar $\langle 0|\bar{s}c|D_{s0}^*(\boldsymbol{p})\rangle = \mathbf{f_S^{0^+}} m_{0^+}$,

Tensor $\langle 0|\bar{s}\gamma_5\sigma_{\mu\nu}c|D_{s1}(\boldsymbol{p},\epsilon)\rangle = \mathbf{f_T^{1^+}}(p_\mu\epsilon_\nu - p_\nu\epsilon_\mu)$

For D_{s0}^*, scalar and vector decay constants are related:

Conserved vector current relation $f_V = f_S(m_c - m_s)/m_{D_{s0}^*}$

Non-leptonic $B \to D^{(*)} D_{sJ}^{(*)}$ decays

Decay constants not yet directly determined in expt..

Instead: non-leptonic $B \to D^{(*)} D_{sJ}^{(*)}$ decays

In low energy limit (effective Hamiltonian) and factorisation (heavy quark limit).

Amplitude approx $\propto \langle \mathbf{D_{s0}^*} | \bar{\mathbf{s}} \gamma_\mu (\mathbf{1} - \gamma^{\mathbf{5}}) \mathbf{c} | \mathbf{0} \rangle \langle \mathbf{D} | \bar{\mathbf{c}} \gamma^\mu (\mathbf{1} - \gamma^{\mathbf{5}}) \mathbf{b} | \mathbf{B} \rangle$

$$\mathbf{R_{D0}} = \frac{\mathcal{B}(B \to D D_{s0}^*(2317))}{\mathcal{B}(B \to D D_s)} \approx \mathbf{R_{D^*0}} = \frac{\mathcal{B}(B \to D^* D_{s0}^*(2317))}{\mathcal{B}(B \to D^* D_s)} \approx \left| \frac{\mathbf{f_{D_{s0}^*}}}{\mathbf{f_{D_s}}} \right|^{\mathbf{2}}$$

$$\mathbf{R_{D1}} = \frac{\mathcal{B}(B \to D D_{s1}(2460))}{\mathcal{B}(B \to D^* D_s^*)} \approx \mathbf{R_{D^*1}} = \frac{\mathcal{B}(B \to D^* D_{s1}(2460))}{\mathcal{B}(B \to D^* D_s^*)} \approx \left| \frac{\mathbf{f_{D_{s1}}}}{\mathbf{f_{D_s^*}}} \right|^{\mathbf{2}}$$

Expt: [Belle,1102.0935]

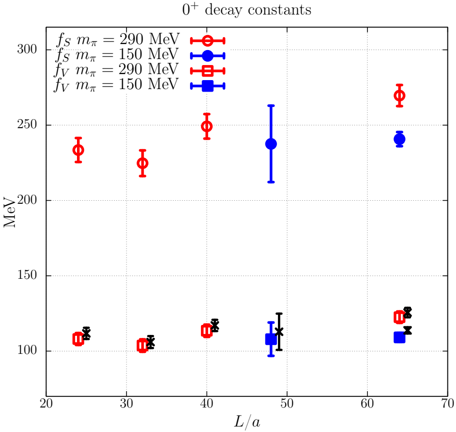
$\mathbf{R_{D0}} = 0.10(3)$, $\mathbf{R_{D^*0}} = 0.15(6)$ are similar as are

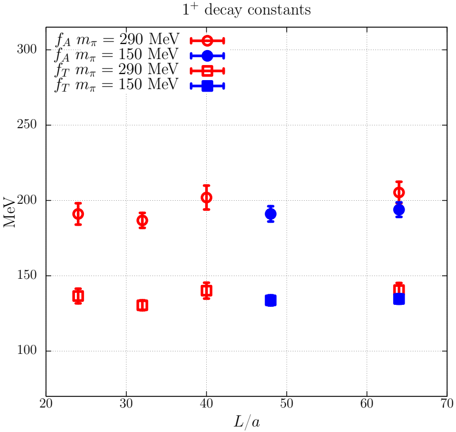
$\mathbf{R_{D1}} = 0.44(11)$, $\mathbf{R_{D^*0}} = 0.58(12)$

Decay constants: results

$$C^X_{LS}(t) = \langle 0|J_X(t)\,O^\dagger(0)|0\rangle \approx \sqrt{\frac{mL^3}{2}e^{mt_0}}\,\mathbf{f}_\mathbf{X}^{\mathrm{latt}}\,e^{-mt} \qquad \mathbf{f}_\mathbf{X}^{\mathrm{ren}} = Z_X\,(1 + a\overline{m}b_X)\,f_X^{\mathrm{latt}}$$

$X \in \{S, V, A, T\}$ and $m \in \{m_{0^+}, m_{1^+}\}$





D_{s1} is a narrow resonance (p wave decay to $D_s\pi\pi$): [Briceño and Hansen,1502.04314]
$0 \to 2$ but no $0 \to 3$.

Decay constants: results $m_\pi = 150$ MeV

Errors: (stat.)(renorm.)(finite V)(disc.)

Finite V from extrap. with $f + ge^{-Lm_\pi}/(Lm_\pi)^{3/2}$ from LO ChPT for $m_\pi = 290$ MeV.

Also: $f_T^{1^+} = 135(2)(2)(+3)(10)$ **MeV**.

MeV	$f_S^{0^+}$	$f_V^{0^+}$	$f_A^{1^+}$
RQCD	**241(4)(2)(+12)(10)**	**114(2)(0)(+5)(10)**	**194(3)(4)(+5)(10)**
[1]Herdoiza et al.	340(110)	200(50)	
[2]B-decays,HQS		74(11)	166(20)
[3]B-decays,HQS		67(13)	
[4]B-decays,HQS		58-86	130-200
[5]QM		440	410
[6]QM		122-154	
[7]LF QM		71	117
[8]LC QCDSR	225(25)		225(25)
[9]DK-molecule		67.1(4.5)	144.5(11.1)
[10]LF QM		$74.4^{+10.4}_{-10.6}$	159^{-36}_{+32}
[11]QM		119	165
[12]QCDSR	333(20)		245(17)

(1) [Herdoiza,hep-lat/0604001] (2) [Hwang,hep-ph/0410301] (3) [Cheng,hep-ph/0305038] (4) [Cheng,hep-ph/0605073] (5) [LeYaounac,hep-ph/0107047] (6) [Hsieh,hep-ph/0312232] (7) [Cheng,hep-ph/0310359] (8) [Colangelo,hep-ph/0505195] (9) [Fässler,0705.0892](10) [Verma,1103.2973] (11) [Segovia,1203.4362] (12) [Wang,1506.01993]

Decay constants: results $m_\pi = 150$ MeV

Unfortunately, f_{D_s} and $f_{D_s^*}$ not computed.

Scalar: use[ALPHA,1312.7693] results with same action and $N_f = 2$:

$f_{D_s} \sim 257$ MeV at $m_\pi = 190$ MeV, $a = 0.065$ fm, $f_{D_s} = 247$ MeV at m_π^{phys}, $a = 0$.

Gives: $f_{D_{s0}^*}/f_{D_s} \approx 0.45, \qquad |f_{D_{s0}^*}/f_{D_s}|^2 \approx 0.20$

Vector: use m_π^{phys}, $a = 0$ results

[Becirevic,1201.4039], $N_f = 2$ twisted mass fermions, $f_{D_s^*}/f_{D_s} = 1.26(3)$,
[HPQCD,1312.5264] HISQ fermions $f_{D_s^*}/f_{D_s} = 1.10(2)$,
[ETMC,1610.09671] $N_f = 2+1+1$ twisted mass fermions $f_{D_s^*}/f_{D_s} = 1.09(2)$.

Gives: $f_{D_{s1}}/f_{D_s^*} \approx 0.6 - 0.7, \qquad |f_{D_{s1}}/f_{D_s^*}|^2 \approx 0.36 - 0.49$

Expt: [Belle,1102.0935]

$$\mathbf{R_{D0}} = 0.10(3) \quad \mathbf{R_{D^*0}} = 0.15(6) \quad \mathbf{R_{D1}} = 0.44(11) \quad \mathbf{R_{D^*0}} = 0.58(12)$$

Decay constants

Heavy quark $m_Q \to \infty$ limit: (D_s, D_s^*), (D_{s0}^*, D_{s1}) form degenerate pairs.

m_c $\quad f_{D_s^*}/f_{D_s} = 1.10 - 1.26, \quad f_{D_{s1}}/f_{D_{s0}^*} \sim 1.7,$

Nature of states: $P = +$ decay constants suppressed relative to $P = -$.

$$f_{D_{s0}^*}/f_{D_s} \approx 0.45, \qquad f_{D_{s1}}/f_{D_s^*} \approx 0.6 - 0.7$$

The states more spatially extended (in a non-relativistic $\bar{q}q$ picture $f \propto |\psi(0)|^2$).

Conventional mesons: charmonium sector

So far lattice results for decay constants of η_c (0^-), J/ψ (1^-) and h_c (1^{+-}).
[Becirevic,1312.2858]: $f_{h_c}/f_{J/\psi} = 0.56$.

However, roughly: $\Gamma(\bar{c}c \to \gamma\gamma) \propto f_{\bar{c}c}^2/m_{\bar{c}c}$

From the expt. results

$$f_{\chi_0}/f_{\eta_c} = f_{0^{++}}/f_{0^{-+}} \sim 0.7$$

D_s: summary and outlook

★ High statistics study with $m_\pi = 290$ MeV and 150 MeV and $L = 1.7 - 4.5$ fm.

★ DK and D^*K thresholds reproduced to within 14 MeV.

★ $D^{(*)}K$ operators essential for reliably extracting the g.s. and DK state.

★ Phase shift, $p \cot \delta(p)$ linear with p^2 for $|p^2| \leq 300$ GeV^2 consistent with the effective range approximation.

★ Discrepancies seen with experimental mass spectrum. Likely due to discretisation effects.

★ Spin-average masses and splittings reasonably consistent with expt.. Significant dependence on light quark mass observed.

★ $f_V^{0^+}$ and $f_A^{1^+}$ roughly compatible with $B \to D^{(*)} D_{sJ}^{(*)}$ branching fractions.

★ $f_S^{0^+}$ and $f_T^{1^+}$ can be compared to model predictions.