Properties of radial and orbital excitations of the heavy-light meson

Benoît Blossier

CNRS/Laboratoire de Physique Théorique d'Orsay

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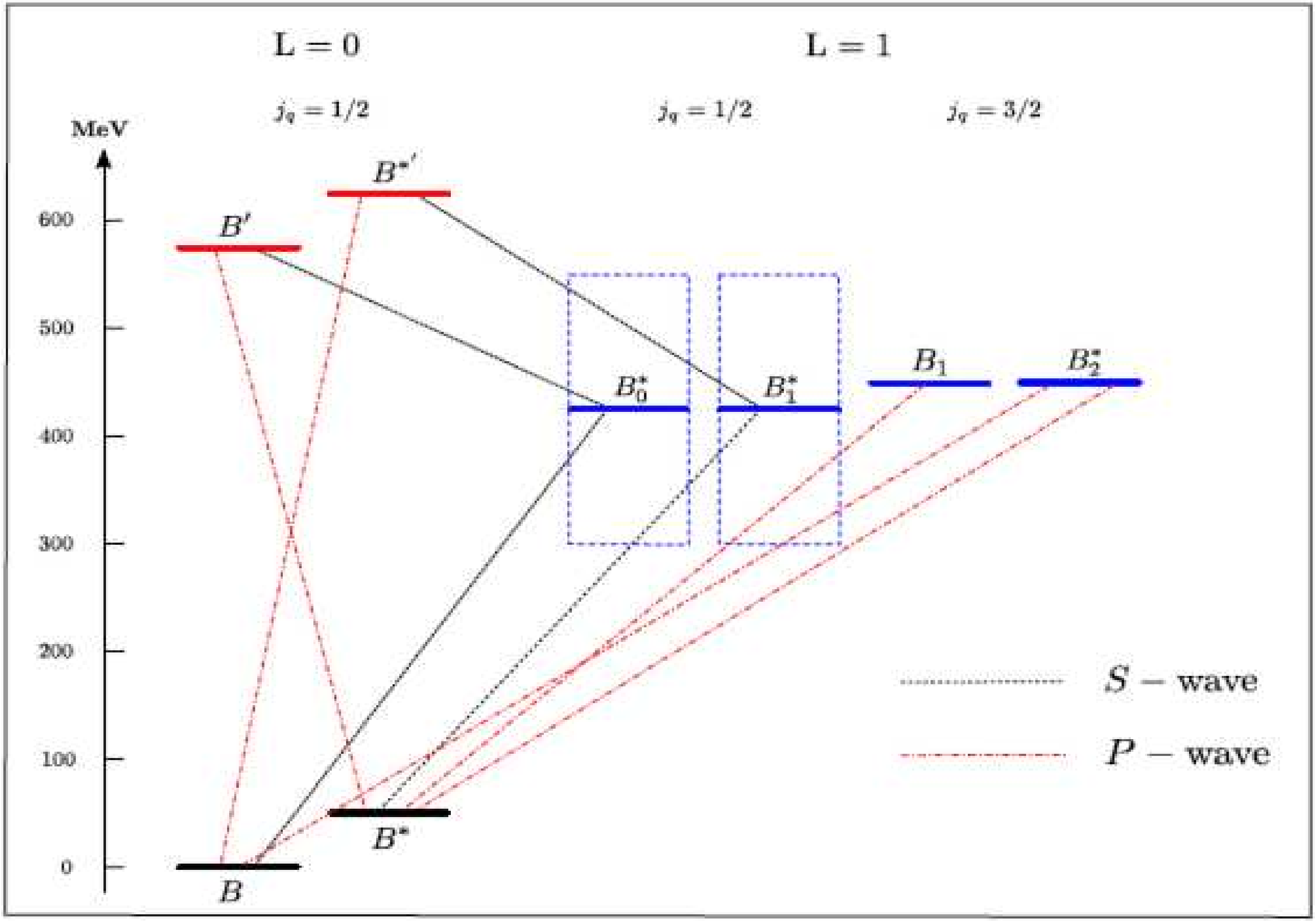
- Radial excitations: molecules or quark-antiquark bound states?
- Decay of states near thresholds

[B. B., J. Bulava, M. Donnellan and A. Gérardin, PRD**87**, 9, 094518 (2013)]
[B. Blossier, N. Garron and A. Gérardin, EPJC **75**, 103 (2015)]
[B. B. and A. Gérardin, PRD**94**, 7, 074504 (2016)]

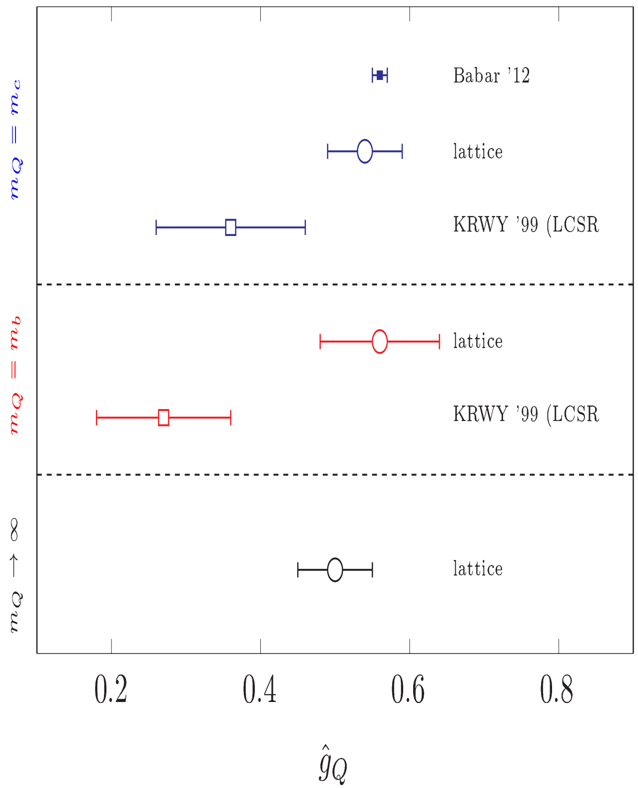
B meson spectroscopy



Radial excitations: molecules or quark-antiquark bound states?

$D^* \to D\pi$: an ideal process to test analytical computations based on the soft pion theorem

$$\langle D(p')\pi(q)|D^*(p,\epsilon_\lambda)\rangle = g_{D^*D\pi} q\cdot\epsilon_\lambda, \quad g_{H^*H\pi} \equiv \frac{2\sqrt{m_H m_{H^*}}\hat{g}_Q}{f_\pi}$$



Claim: a negative radial excitation contribution to the hadronic side of LCSR might explain the discrepancy between $g^{\exp}_{D^*D\pi}$ and $g^{\rm LCSR}_{D^*D\pi}$ [D. Becirevic *et al*, '03].

Check on the lattice that statement in the heavy quark limit

Transition amplitude under interest, with $q = p' - p$, $\mathcal{A}^\mu = \bar{d}\gamma^\mu\gamma_5 u$,
$T^{mn\,\mu} = \langle B_m(p)|\mathcal{A}^\mu|B_n^*(p',\lambda)\rangle$ and $\epsilon_\perp^\mu(p',\lambda) = \epsilon(p',\lambda)^\mu - \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu$:

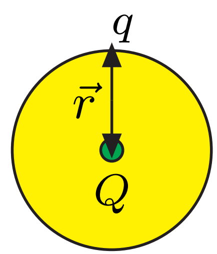
$$\begin{aligned} T^{mn\,\mu} &= 2m_{B_n^*} A_0^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu + (m_{B_m} + m_{B_n^*}) A_1^{mn}(q^2) \epsilon_\perp^\mu(p',\lambda) \\ &+ A_2^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{m_{B_m} + m_{B_n^*}} \left[(p+p')^\mu + \frac{m_{B_m}^2 - m_{B_n^*}^2}{q^2} q^\mu \right] \end{aligned}$$

With $\langle B_m(p)|q_\mu \mathcal{A}^\mu|B_n^*(p',\lambda)\rangle = 2\, m_{B_n^*}\, A_0^{mn}(q^2)\; q\cdot\epsilon(p',\lambda)$, PCAC relation, LSZ reduction formula and $\sum_\lambda \epsilon_\mu(k,\lambda)\,\epsilon_\nu^*(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$:

$$g_{H_n^* H_m \pi} = \frac{2\, m_{H_n^*} A_0^{mn}(0)}{f_\pi}\;,\; A_0^{mn}(q^2) = -\sum_\lambda \frac{\langle H_m(p)|q_\mu \mathcal{A}^\mu|H_n^*(p',\lambda)\rangle}{2m_{H_n^*}\, q_i}\, \epsilon_i^*(p',\lambda)$$

Back to the x space: $A_0^{mn}(q^2=0) = -\frac{q_0}{q_i}\int d^3r\, f_{\gamma_0\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} + \int d^3r\, f_{\gamma_i\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$

Axial density distributions $f_{\gamma_\mu\gamma_5}^{mn}(r)$ defined
in terms of 2-pt and 3-pt HQET correlation functions



Density distributions

The concept is not new:
[C. Alexandrou, Ph. de Forcrand and A. Tsapalis, 03; J. Green and J. Negele, ’10]

Application to $B(L=0)$ states

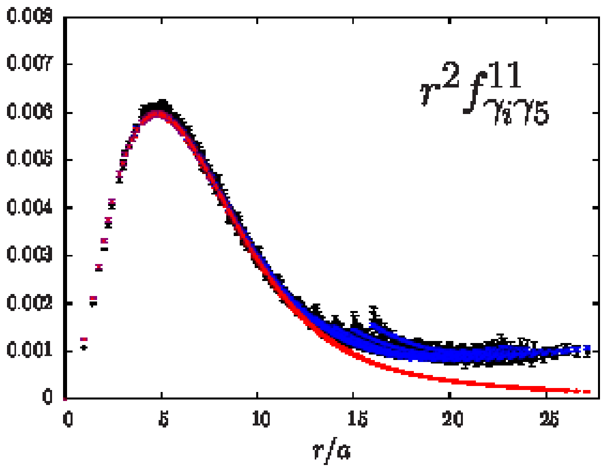
Lattice set-up: $\mathcal{O}(a)$ improved Wilson-Clover (light quark), HYP2 (static quark)

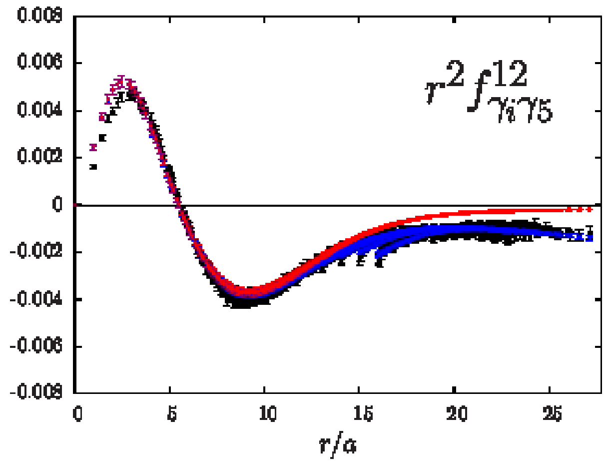
CLS based

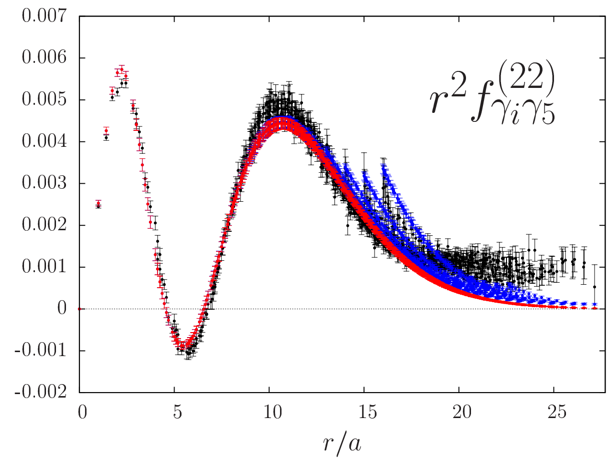
lattice	β	$L^3 \times T$	a[fm]	m_π[MeV]	Lm_π
A5	5.2	$32^3 \times 64$	0.075	330	4
B6		$48^3 \times 96$		280	5.2
D5	5.3	$24^3 \times 48$	0.065	450	3.6
E5		$32^3 \times 64$		440	4.7
F6		$48^3 \times 96$		310	5
N6	5.5	$48^3 \times 96$	0.048	340	4
Q1	6.2885	$24^3 \times 48$	0.06	-	-
Q2	6.2885	$32^3 \times 64$	0.06	-	-

Basis of interpolating fields (4×4 matrix of correlators, Gaussian smearing) large enough to well isolate the ground state and the first excited state *via* GEVP.

Spatial component of the axial density distributions: systematics from excited states, finite-volume effects and cut-off effects taken into account

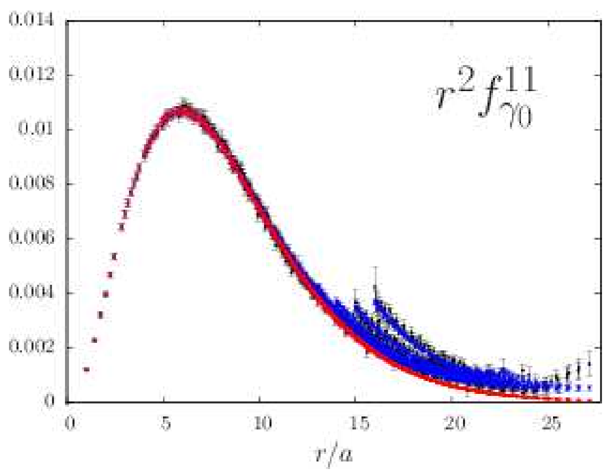


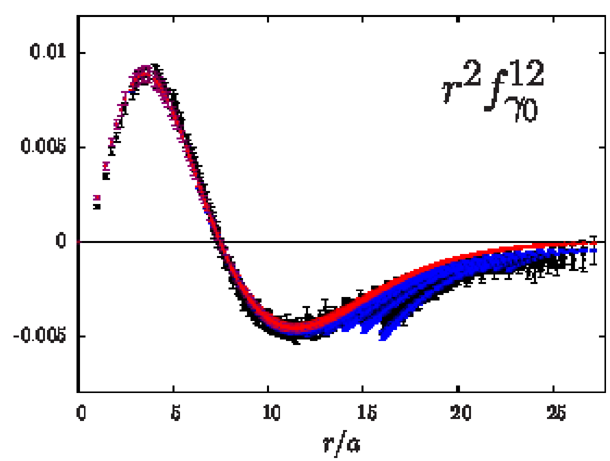




$f^{11}_{\gamma_i\gamma_5}(r)$: positive everywhere; $f^{12}_{\gamma_i\gamma_5}(r)$: there is a node; $f^{22}_{\gamma_i\gamma_5}(r)$: almost positive, negative part interpreted by relativistic effects

Techniques employed also for the charge density distribution $f^{mn}_{\gamma_0}(r)$





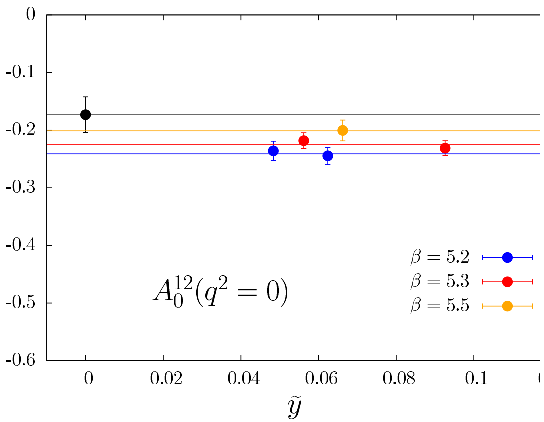
Including Z_V, $\int dr\, r^2\, f^{11}_{\gamma_0}(r)$ compatible with 1. $\int dr\, r^2 f^{12}_{\gamma_0}(r)$ compatible with 0.

There are also time components of density distributions.

Matrix elements obtained at q after a Fourier transform of the distributions to get $g_{B^{*\prime} B\pi}$

$$\begin{aligned}\mathcal{M}_i(q^2_{\max}-\vec{q}^{\,2}) &= 4\pi\int_0^\infty \mathrm{d}r\, r^2\, \frac{\sin(|\vec{q}|r)}{|\vec{q}|r} f^{(12)}_{\gamma_i\gamma_5}(\vec{r}) \\ \frac{q_0}{q_i}\mathcal{M}_0(q^2_{\max}-\vec{q}^{\,2}) &= -q_0 4i\pi\int_0^\infty \mathrm{d}r_\parallel \int_0^\infty \mathrm{d}r_\perp\, r_\perp\, f^{(12)}_{\gamma_0\gamma_5}(r_\parallel, r_\perp)\, \frac{\sin(|\vec{q}|\, r_\parallel)}{|\vec{q}|}\end{aligned}$$

$$A_0^{12}(q^2) = -\frac{q_0}{q_i}\mathcal{M}_0(q^2_{\max}-\vec{q}^{\,2}) + \mathcal{M}_i(q^2_{\max}-\vec{q}^{\,2})$$



Extrapolation of $A_0^{12}(q^2=0)$ to the physical point:
$A_0^{12}(0, m_\pi^2) = D_0 + D_1 a^2 + D_2 m_\pi^2/(8\pi f_\pi^2)$

Qualitative agreement between lattice and quark models: $q_0\mathcal{M}_0/q_i$ dominates in $A_0^{12}(q^2)$ and explains why $A_0^{12}(q^2=0) < 0$.

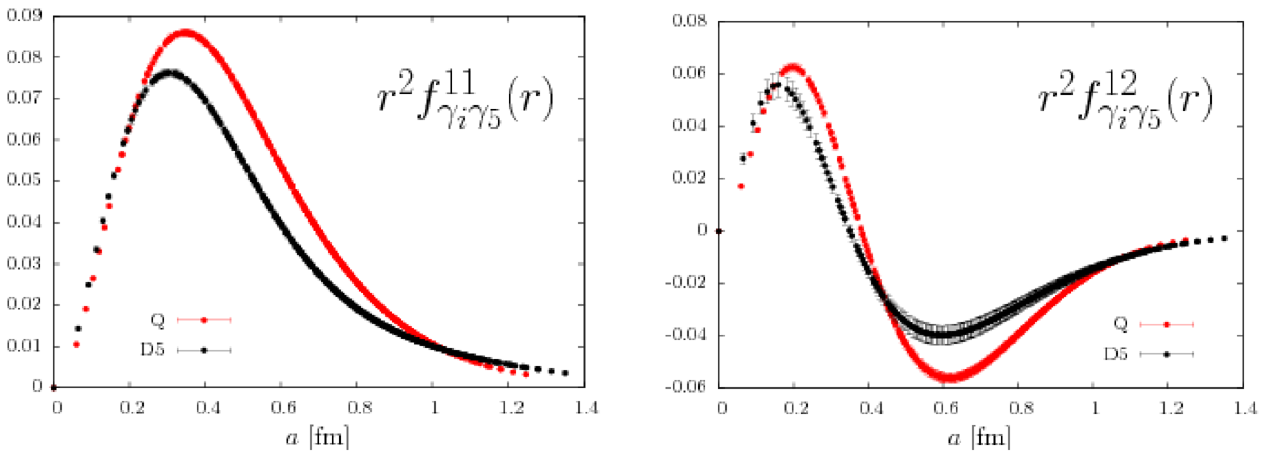
Issue with multihadron states?

A possible unpleasant systematics of our results is an uncontrolled mixing between radial excitations ($B^{*'}$) and multihadron states ($B_1^*\pi$ in S wave) close to threshold.

$$\Sigma_{12} = m_{B^{*\prime}} - m_B,\ \delta = m_{B_1^*} - m_B$$

lattice	$a\Sigma_{12}$	$a\delta + am_\pi$
A5	$0.253(7)$	$0.281(4)$
B6	$0.235(8)$	$0.248(4)$
E5	$0.225(10)$	$0.278(6)$
F6	$0.213(11)$	$0.233(3)$
N6	$0.166(9)$	$0.176(3)$

Comparison with quenched data: behaviour of $f^{11}_{\gamma_i\gamma_5}$ and $f^{12}_{\gamma_i\gamma_5}$ similar

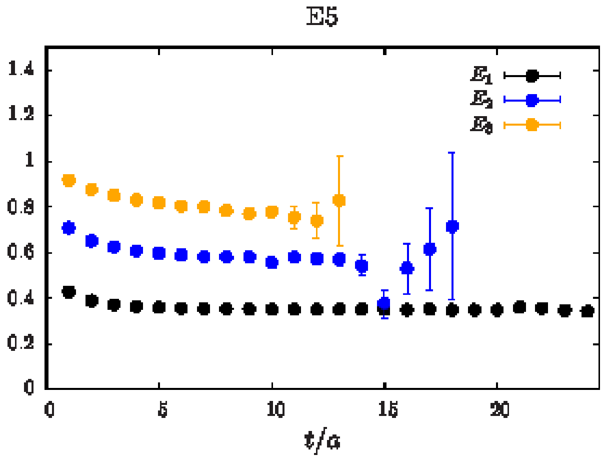


At $\mathrm{N_f} = 2$, position of the node of $f^{12}_{\gamma_i\gamma_5}$ weakly dependent of m_π in the range we have considered

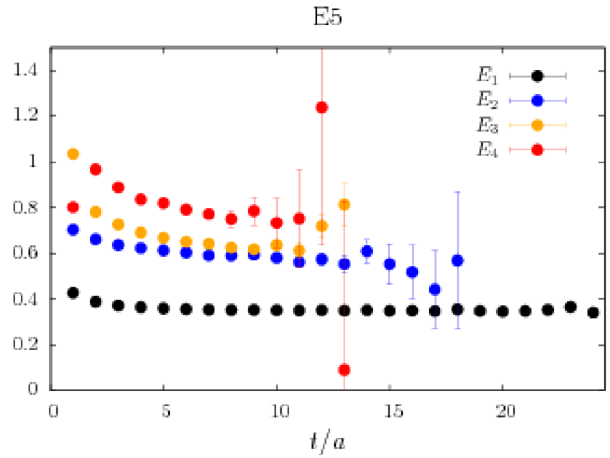
lattice	m_π [MeV]	r_n^{12} [fm]
A5	330	$0.369(13)$
B6	280	$0.374(12)$
E5	440	$0.369(11)$
F6	310	$0.379(20)$
N6	340	$0.365(12)$

Change observed when $\bar{q}\nabla_k h$ is included in addition to $\bar{q}\gamma_k h$ to couple to $B^{*'}$

$\bar{q}\gamma_k h$ only



with $\bar{q}\nabla_k h$



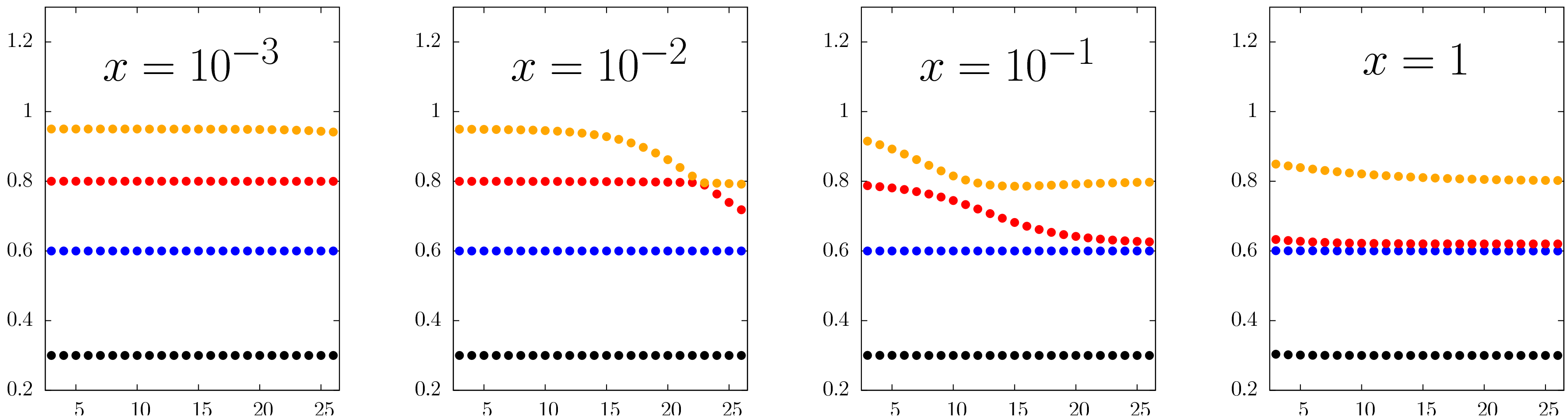
A new state, not seen before, is present in the spectrum close to the first excited state.

A toy model with 5 states in the spectrum to understand this fact:

spectrum
0.3
0.6
0.63
0.8
0.95

Matrix of couplings

$$\begin{bmatrix} 0.6 & 0.25 & x \times 0.4 & 0.1 & 0.5 \\ 0.61 & 0.27 & x \times 0.39 & 0.11 & 0.51 \\ 0.58 & 0.24 & x \times 0.42 & 0.12 & 0.52 \\ 0.57 & 0.25 & x \times 0.41 & 0.1 & 0.49 \\ 0.56 & 0.26 & x \times 0.36 & 0.08 & 0.48 \end{bmatrix}$$



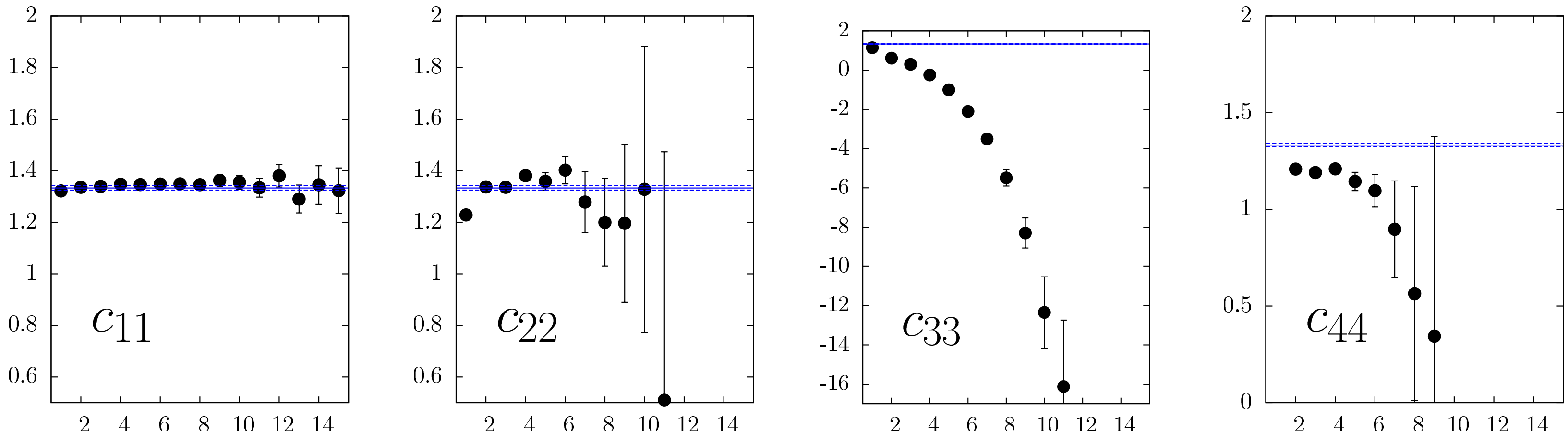
$x \ll 1$: GEVP isolates states 1, 2, 4 and 5; $x \to 1$, GEVP isolates states 1, 2, 3 and 4

A GEVP can "miss" an intermediate state of the spectrum if, by accident, the coupling of the interpolating fields to that state is suppressed.

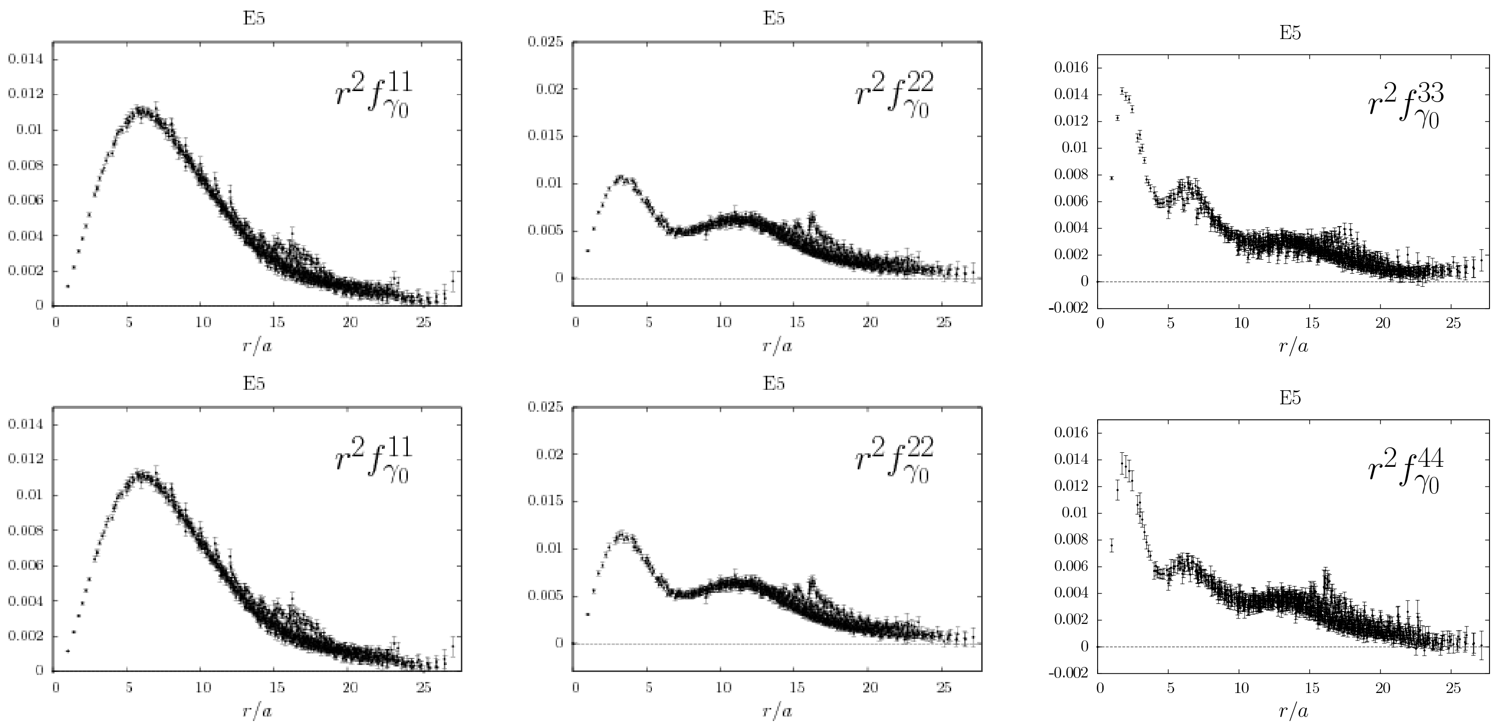
Our claim: using interpolating fields $\bar{q}\gamma_k h$, no chance to couple to multi-hadron states while inserting an operator $\bar{q}\nabla_k h$ may isolate the $B_1^*\pi$ two-particle state.

Clues come from density distributions obtained with that interpolating field.

Conservation of vector charge: not verified in the case of second excited state if the basis of interpolating fields incorporates $\bar{q}\nabla_k h$.



Including or not $\bar{q}\nabla_k h$ does not change the profile of $f^{11}_{\gamma_0}$ nor $f^{22}_{\gamma_0}$: it does in the case of $f^{33}_{\gamma_0}$.



Question addressed in the framework of the workshop

Can density distributions provide any relevant information about the nature of exotic charmonia or $\bar{b}b\bar{q}q$ hadrons (bound states, molecules)?

Broad “diquark" density distributions within $\bar{b}b\bar{q}q$ state: tetraquark bound state

Peaked “diquark" density distributions within $\bar{b}b\bar{q}q$ state: molecular state

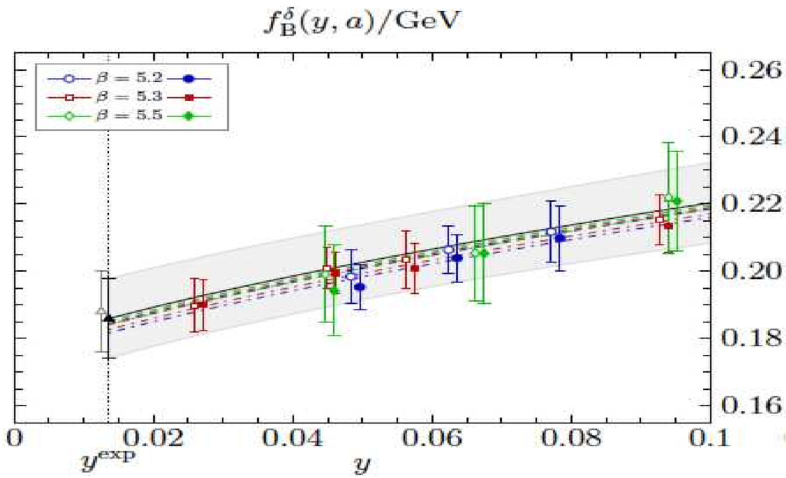
Decay of states near thresholds

Heavy Meson Chiral Perturbation Theory is often used to extrapolate lattice data in the heavy-light sector.

Example on f_B:

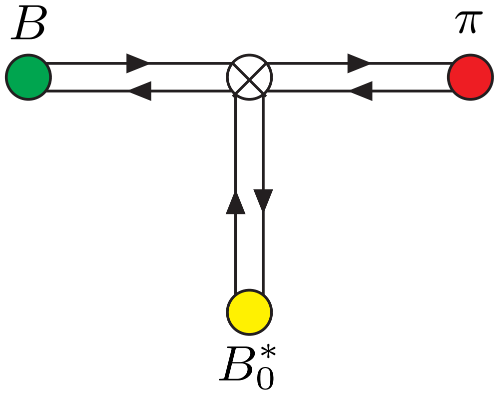
$$f_B\sqrt{\frac{m_B}{2}}(y,a,\delta) = A\left[1-\frac{3}{4}\frac{1+3\hat{g}^2}{2}(y\ln y - y^{\exp}\ln y^{\exp})\right] + C(y-y^{\exp}) + D^{\delta}a^2$$

[F. Bernardoni *et al*, '14]



$$\begin{aligned}\mathcal{L}_{\mathrm{HM}\chi\mathrm{PT}} &= \frac{f_\pi^2}{8}\mathrm{Tr}(\partial^\mu\Sigma\partial_\mu\Sigma^\dagger) + i\mathrm{Tr}(Hv\cdot\mathcal{D}\bar{H}) + i\mathrm{Tr}(Sv\cdot\mathcal{D}\bar{S}) \\ &+ i\hat{g}\mathrm{Tr}(H\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{H}) + i\tilde{g}\mathrm{Tr}(S\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{S}) + ih\mathrm{Tr}(S\gamma_\mu\gamma_5\mathcal{A}^\mu\bar{H})\end{aligned}$$

$H : j^P = \frac{1}{2}^-$ heavy-light meson doublet $\quad S : j^P = \frac{1}{2}^+$ heavy-light meson doublet

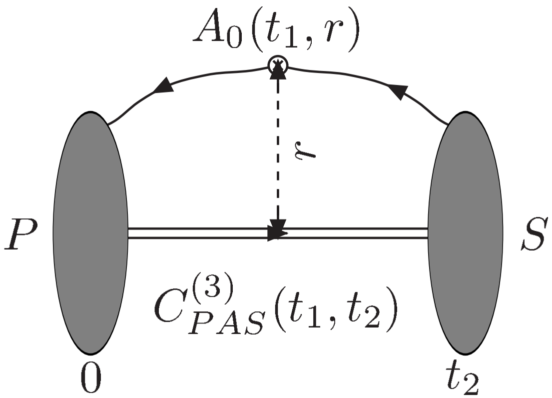


$$\Gamma(B_0^{*0} \to B^+\pi^-) = \frac{1}{8\pi}\, g^2_{B_0^* B\pi}\, \frac{|\vec{q}_\pi|}{m^2_{B_0^*}}$$

$$|\vec{q}_\pi| = \frac{\sqrt{[m^2_{B_0^*} - (m_B + m_\pi)^2][m^2_{B_0^*} - (m_B - m_\pi)^2]}}{2 m_{B_0^*}}$$

HMχ PT: $\Gamma(B_0^* \to B^+\pi^-) = \frac{h^2}{8\pi f_\pi^2} \frac{m_B}{m^3_{B_0^*}} \left(m^2_{B_0^*} - m^2_B\right)^2 |\vec{q}_\pi|$

$$g_{B_0^* B\pi} = \sqrt{\frac{m_B}{m_{B_0^*}}} \left(m^2_{B_0^*} - m^2_B\right) \frac{h}{f_\pi}$$



Extract h from the density distribution [D. Becirevic *et al*, '12]

$$A_+(\delta^2 - q_\pi^2) = 4\pi \int_0^\infty r^2 dr \frac{\sin(q_\pi r)}{q_\pi r} f_{PAS}(r)$$

$$\delta = m_{B_0^*} - m_B \quad f_{PAS}(r) = \langle B | [\bar{q}\gamma_0\gamma_5 q](r) | B_0^* \rangle$$

$$\vec{q}_\pi = (0, 0, \delta)$$

We have followed another strategy, valid near thresholds [C. McNeile *et al*, '01; '03; '04]

We consider the ratio $C^{(2)}_{B_0^*\, B\pi}(t) / \sqrt{C^{(2)}_{B_0^*\, B_0^*}(t) C^{(2)}_{B\pi\, B\pi}(t)}$

$$\langle \pi^+(q_\pi) B^-(p) | B_0^{*0}(p') \rangle = g_{B_0^* B\pi} = \sqrt{m_B m_{B_0^*}} \frac{m^2_{B_0^*} - m^2_B}{m^2_{B_0^*}} \frac{h}{f_\pi}$$

Fermi golden rule: $\Gamma(B_0^* \to B^-\pi^+) = 2\pi \left| \langle \pi^+(q_\pi) B^-(p) | B_0^{*0}(p') \rangle \right|^2 \rho$

$$\rho(E_\pi) = \frac{L^3}{(2\pi)^3} 4\pi \vec{q}_\pi^{\,2} \frac{dq_\pi}{dE_\pi} = \frac{L^3}{2\pi^2} |\vec{q}_\pi| E_\pi$$

$$\frac{\Gamma(B_0^* \to B^-\pi^+)}{q_\pi} = \frac{1}{\pi} \left(\frac{L}{a}\right)^3 (aE_\pi) \left| a \langle \pi^+(q_\pi) B^-(p) | B_0^{*0}(p') \rangle \right|^2$$

$$C^{(2)}_{B_0^*\,B\pi}(t) = \sum_{t_1} \langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle x \langle B\pi|\mathcal{O}^{B\pi}|0\rangle e^{-m_{B_0^*} t_1} e^{-E_{B\pi}(t-t_1)} + \mathcal{O}(x^3) + \text{excited states}$$

Assumption: small overlaps $\langle 0|\mathcal{O}^{B_0^*}|B\pi\rangle$ and $\langle 0|\mathcal{O}^{B\pi}|B_0^*\rangle$
$x = |a\langle \pi^+(q_\pi)B^-(p)|B_0^{*0}(p')\rangle| \quad \langle n|m\rangle = \delta_{mn}$

Close to the threshold $m_{B_0^*} \approx E_{B\pi}$:

$$C^{(2)}_{B_0^*\,B\pi}(t) = \langle 0|\mathcal{O}^{B_0^*}|B_0^*\rangle x \langle B\pi|\mathcal{O}^{B\pi}|0\rangle \times t e^{-m_{B_0^*} t} + \mathcal{O}(x^3) + \text{excited states}$$

$$R(t) = \frac{C^{(2)}_{B_0^*\,B\pi}(t)}{\left(C^{(2)}_{B_0^*\,B_0^*}(t)\, C^{(2)}_{B\pi\,B\pi}(t)\right)^{1/2}} \approx A + xt$$

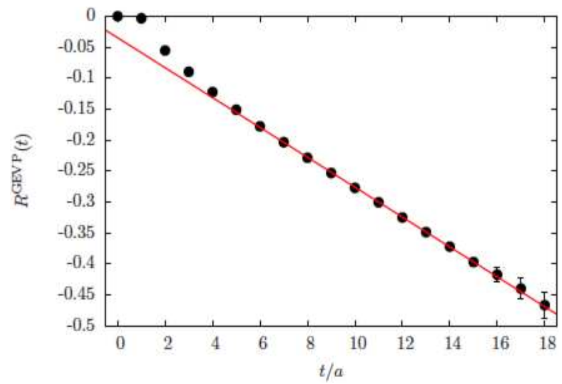
Further away from the threshold, $R(t)$ goes in $t \longrightarrow \frac{2}{\Delta}\sinh\left(\frac{\Delta}{2}t\right) = t + \frac{\Delta^2 t^3}{24} + \mathcal{O}(\Delta^4)$,
$\Delta = m_{B_0^*} - E_{B\pi}$

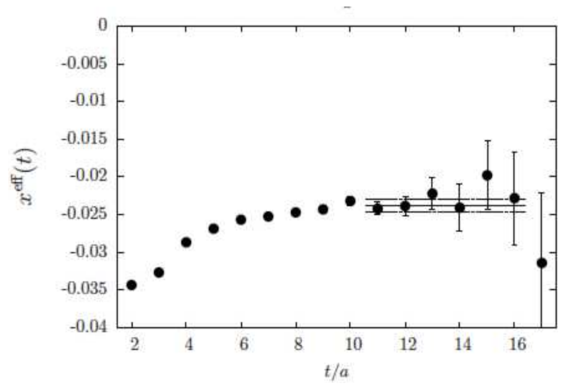
Excited states are suppressed by solving a GEVP:

CLS
based

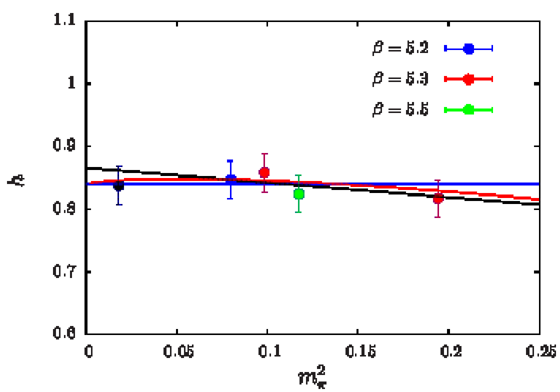
β	a[fm]	L/a	m_π[MeV]
5.2	0.075	48	280
5.3	0.065	32	440
		48	310
5.5	0.048	48	340

[$a = 0.065$ fm, $m_\pi = 440$ MeV]





Several chiral extrapolations to get h, using $m_{B_0^*} - m_B = 399(17)(28)$ MeV



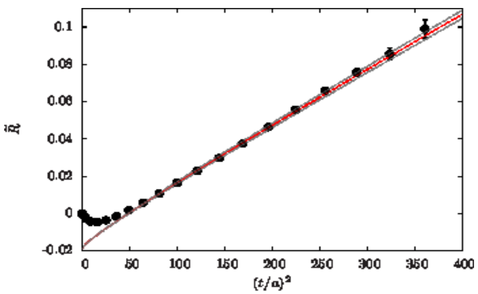
formula	result
$h = \mathrm{C_{ste}}$	0.84(3)
$h = h_0 + \alpha m_\pi^2$	0.86(4)
$h = h_0 \left[1 - \frac{3}{4}\frac{3\hat{g}_0^2+3\tilde{g}_0^2+2\hat{g}_0\tilde{g}_0}{(4\pi f_\pi)^2} m_\pi^2 \ln m_\pi^2\right] + C_h m_\pi^2$	0.84(3)
$h = h_0 \left[1 - \frac{3}{4}\frac{3\hat{g}_0^2+3\tilde{g}_0^2+2\hat{g}_0\tilde{g}_0}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2) - \frac{h_0^2}{(4\pi f_\pi)^2}\frac{m_\pi^2}{2\delta^2} m_\pi^2 \ln(m_\pi^2)\right] + C_h' m_\pi^2$	0.85(3)

$$h = 0.84(3)(2)$$

Check of the analysis using the correlator $C_{B\pi\, B\pi}$, with $C_{\rm conn}(t) = -\frac{3}{2}C_{\rm box}(t) + \frac{1}{2}C_{\rm cross}(t)$:

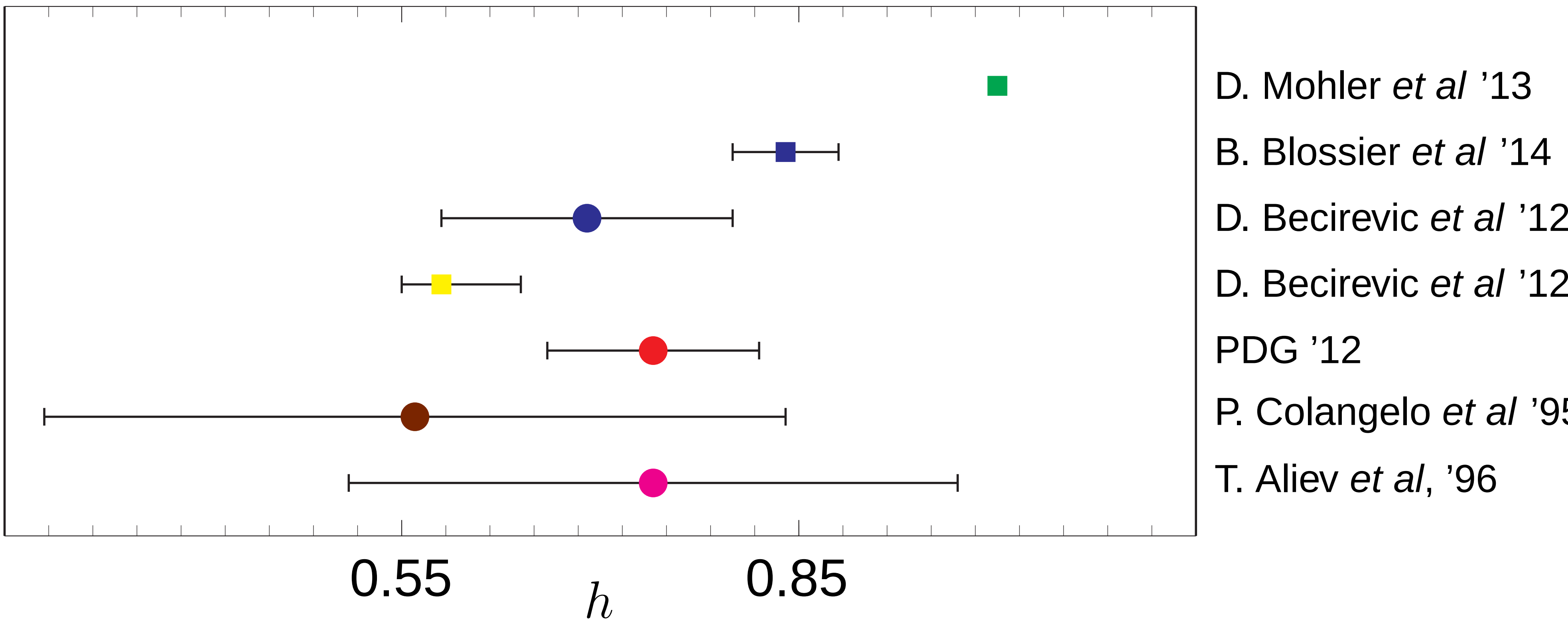
$$\widetilde{R}(t) = \frac{(v_{B\pi}(t), C_{\rm conn}(t) v_{B\pi}(t))}{(v_{B\pi}(t), C_{B\pi\, B\pi}(t) v_{B\pi}(t))} = A' + \frac{1}{2}x^2t^2 + \mathcal{O}(t)$$

[$a = 0.065$ fm, $m_\pi = 440$ MeV]



$|ax| = 0.0237(8)$ in perfect agreement
with $|ax|_{R(t)} = 0.0238(9)$

Collection of results



Different ways to get h: $\Gamma(D_0^*)$, phase shift in $D\pi$ scattering state (small $1/m_c$ corrections), QCD sum rules, density distribution, transition at the threshold $m_{B_0^*} \approx E_{B\pi}$

Adler-Weisberger sum rule: $\sum_\delta |X_{B\delta}|^2 = 1 \quad \Gamma(\mathcal{I} \to \mathcal{F}\pi) = \frac{1}{1\pi f_\pi^2} \frac{|\vec{q}|^3}{2j_{\mathcal{I}}+1} |X_{\mathcal{I}\to\mathcal{F}}|^2$

With $\hat{g} \sim 0.5$, it is almost saturated by B^* and B_0^*.

h is pretty large, some care is required in the application of HMχPT for pion masses close to $m_{B_0^*} - m_B \sim 400$ MeV: B meson orbital excitations degrees of freedom can not be neglected in chiral loops.

Question addressed in the framework of the workshop

Does it make sense to study broad resonant states by “imposing" kinematical configurations very near thresholds?

Example of K^*, $K^* \to K\pi$

$K^*(s\bar{u}') \to K(s\bar{u})\pi'(u\bar{u}')$ where the quark u' is quenched

The threshold condition $E_{K\pi'} \sim E_{K^*}$ is set by imposing twisted BC on the u field in $O_{K\pi'}$.
s and u' quenched $\Longrightarrow$ no issue with isospin breaking effects
[C. Sachrajda and G. Villadoro, ’04]

Numerical advantage of twisted BC compared to moving frames: more efficient scan along the “Breit-Wigner"

Theoretical issue: is the extrapolation $m_{u'} \to m_u$ smooth? (cusp, large finite volume effects because of “shadow" isospin breaking)