Computing tetraquark resonances with two static quarks and two dynamical quarks: II - Born-Oppenheimer approximation and emergent wave method

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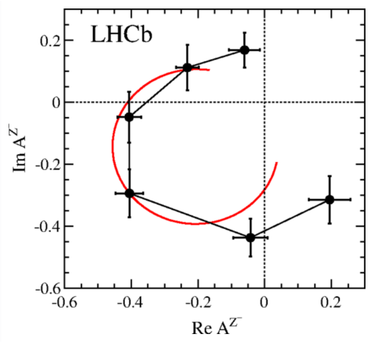
Outline

1. Introduction
   - Experimental observation of double heavy exotics
   - Applying the Born-Oppenheimer approximation
   - Our case study: potential of static $\bar{Q}\bar{Q}$ with light qq
   - Boundstates with double-heavy $\bar{Q}\bar{Q}$ and with light qq
2. The emergent wave method
   - Emergent and incident wavefunctions
   - Partial wave decomposition
   - Solving the differential equations for the emergent wave
   - Phase shifts and **S** and **T** matrix poles
3. Results for phase shifts, **S** matrix and **T** matrix poles and resonances
   - Phase shifts
   - Resonances as poles of the **S** and **T** matrices
   - Summary and outlook

Outline

Introduction

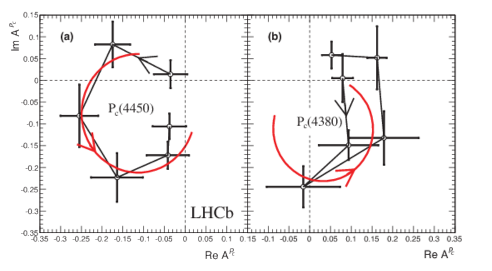
Confirmation of $Z_c(4430)^-$ by LHCb at CERN: $\pi\ J/\psi$ Argand plot



- Exotic hadrons have been a *holy grail* of modern physics since the onset of QCD.
- In the 2010's, we have finally confirmed experimental double-heavy exotic hadrons, see Alessandro Pilloni's talk.
- There are two $Z_b^\pm$ observed by BELLE, slightly below $B\,B^*$ and B^*B^* thresholds, the $Z_b(10610)^+$ and $Z_b(10650)^+$.
- The two $Z_c(3940)^\pm$and $Z_c(4430)^\pm$ are clearly well above *DD* threshold, and have several confirmations, LHCb at CERN recently confirmed $Z_c(4430)^-$ with a resonance mass of 4475 MeV and width of 172 MeV.

Introduction

LHCb $c\bar{c}$ pentaquark



- LHCb has also observed two pentaquarks candidates with again an extremely large significance > 9.
- The two $p_c(4450)^{\pm}$and $P_c(4380)^{\pm}$ are clearly seen in the decay to a $J/\psi p$.
- The recent experimental success of resides in a very high luminosity and a the good resolution.

- However these states are extremely hard to model because they may decay to many many channels (order of 30 for Zc'), being impractical for instance to apply techniques such as the Lüscher 's phase shift method.

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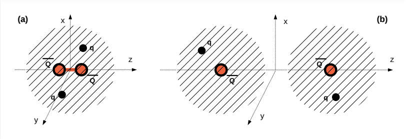
Outline

Introduction

Physics of heavy-heavy hadrons



(a) At small separations the static quarks (for instance in the figure $\bar{Q}\bar{Q}$) interact by perturbative one-gluon exchange.
(b) At large separations the light quarks screen the interaction and the four quarks form two rather weakly interacting heay-light mesons (or barions).

- Nevertheless, we separate the problem in two scales: the heavy quarks can be approximated as static sources, and with Wilson lines we compute lattice QCD potentials.
- The light quarks are also incorporated in lattice QCD with dynamical configurations and propagators.
- Applying the Born-Oppenheimer approximation, we include the quantum kinetic energy of the heavy quarks, and then we should be able to solve the full problem.

Introduction

- Several sorts of hadrons, not just the Zc, Zb and Pc are amenable by the Born-Oppenheimer approximation, including states so far difficult to observe.
- For instance the $b - \bar{b}$ hybrid wave functions and spectra have been studied with lattice QCD and BO.
- Higher excitations, or states with cc or cb or bb can also be studied.
- Moreover the spin-dependent potentials can also be studied with heavy quark effective theories of lattice QCD.
- As a case study we address systems with two heavy antiquarks, anticipated by Ader, Richard, Taxil in 1981, where a $ud\bar{b}\bar{b}$ tetraquark bound state with quantum numbers $I(J^P) = 0(1^+)$ has recently been predicted with lattice QCD potentials.

Juge:1999ie, McNeile:2006bz, McNeile:2002az.
(using inpirehep.net code for references)

Outline

Introduction

- The lattice QCD results for the potentials of two static antiquarks $\bar{Q}\bar{Q}$ in the presence of two light quarks qq can be parametrized by a screened Coulomb potential,

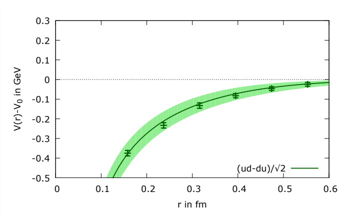
$$V(r) = -\frac{\alpha}{r} e^{-r^2/d^2} . \qquad (1)$$

- inspired by one-gluon exchange at small $\bar{Q}\bar{Q}$ separations r and a screening of the Coulomb potential by the two B mesons at large r.

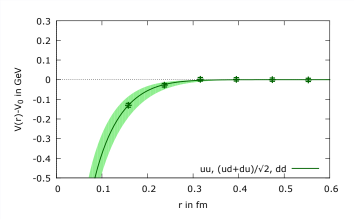
Wagner:2010ad, Wagner:2011ev, Bicudo:2015kna.

Fit with a screened Coulomb potential

scalar:



vector:



Introduction

Fit of the lattice QCD potential

I	j	α	d in fm
0	0	$0.34^{+0.03}_{-0.03}$	$0.45^{+0.12}_{-0.10}$
1	1	$0.29^{+0.05}_{-0.06}$	$0.16^{+0.05}_{-0.02}$

Table: Parameters α and d of the potential of Eq. (1) for two static antiquarks $\bar{Q}\bar{Q}$, in the presence of two light quarks qq with quantum numbers I and j.

- There are both attractive and repulsive channels.
- Most promising with respect to the existence of tetraquark bound states or resonances are light quarks $q \in \{u, d\}$ together with $(I = 0, j = 0)$ or $(I = 1, j = 1)$,
- the corresponding potentials $V(r)$ are not only attractive, but also rather wide and deep

Bicudo:2012qt, Brown:2012tm. Bicudo:2015vta, Bicudo:2016ooe.

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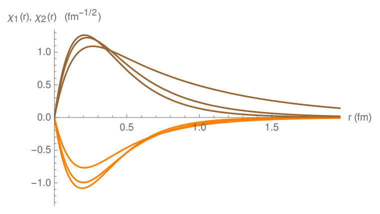
Introduction

- Using the Born-Oppenheimer approximation (very good for $\bar{b}$, fair for $\bar{c}$ quarks), we provide a quantum kinetic energy $p^2/2\mu$ to the heavy quarks.
- Solving the Schrödinger equation, we found evidence for the existence of ONLY ONE double heavy tetraquark boundstate $ud\bar{b}\bar{b}$ with $I = 0$ and $J^p = 1^+$ equivalent to a $B\,B^* \oplus B^*\,B^*$ state.
- We found several non-existence evidences of I=1 $ud\bar{b}\bar{b}$, nor of $u/ds\bar{b}\bar{b}$, $ss\bar{b}\bar{b}$, $u/dc\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$, $cc\bar{b}\bar{b}$, $ud\bar{c}\bar{b}$, $ss\bar{c}\bar{b}$, $u/dc\bar{c}\bar{b}$, $sc\bar{c}\bar{b}$, $cc\bar{c}\bar{b}$, $ud\bar{c}\bar{c}$, $u/ds\bar{c}\bar{c}$, $ss\bar{c}\bar{c}$, $u/dc\bar{c}\bar{c}$, $sc\bar{c}\bar{c}$, $cc\bar{c}\bar{c}$ tetraquarks.

Wavefunctions of the $ud\bar{b}\bar{b}$
$I = 0$, $J^p = 1^+$, $E_b = 59^{+30}_{-38}$ MeV



The two components $B\,B^*$ and $B^*\,B^*$ of the wave function, including the error bars.

Bicudo:2015vta, Bicudo:2015kna, Bicudo:2016ooe.

Outline

Emergent wave method

- Our goal now is to study resonances, a 1st technical step to address the exotics such as Z_b, Z_c and P_c observed at BELLE, BESIII, LHCb... and predict the future resonances observed at PANDA.
- Notice systematic error bars come from the ansatz to fit (or interpolate) the potentials, and in the heavy quark $1/m_Q$ expansion.
- We tried several techniques, first with a toy model. Typically momentum space techniques are used in effective theories, but a position space technique is more convenient for lattice QCD potentials.
- It turns out the best approach is to get back to fundamental quantum mechanics. We adopt a simple technique, we call it the emergent wave method.
- As a first case study, here we explore the resonances produced by the $ud\bar{b}\bar{b}$ potential detailed in the previous talk of Marc Wagner.

Bicudo:2015bra

Emergent wave method

The first step in the emergent wave method is to split the wave function of the Schrödinger Eq. $\Big(H_0 + V(r) - E\Big)\psi = 0$, into two parts,

$$\Psi = \Psi_0 + X \; , \tag{2}$$

where Ψ_0 is the incident wave, a solution of the free Schrödinger equation, $H_0\Psi_0 = E\Psi_0$, and X is the emergent wave. We obtain

$$\Big(H_0 + V(r) - E\Big)X = -V(r)\Psi_0 \; . \tag{3}$$

- For any energy E we calculate the emergent wave X by providing the corresponding Ψ_0 and fixing the appropriate boundary conditions.
- From the asymptotic behaviour of the emergent wave X we then determine the phase shifts δ_l , the S matrix and the T matrix.
- Continuing to complex energies $E \in \mathbb{C}$ we find the poles of the S matrix and the T matrix in the complex plane.
- We identify a resonance with a pole of S in the second Riemann sheet at $m - i\Gamma/2$, where m is the mass and Γ is the resonance decay width.

Outline

Emergent wave method

We consider an incident plane wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$, which can be expressed as a sum of spherical waves,

$$\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_l (2l+1) i^l j_l(kr) P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \ , \tag{4}$$

where j_l are spherical Bessel functions, P_l are Legendre polynomials and the relation between energy and momentum is $\hbar k = \sqrt{2\mu E}$. For a spherically symmetric potential $V(r)$ as in Eq. (1) and an incident wave $\Psi_0 = e^{i\mathbf{k}\cdot\mathbf{r}}$ the emergent wave X can also be expanded in terms of Legendre polynomials P_l,

$$X = \sum_l (2l+1) i^l \frac{\chi_l(r)}{kr} P_l(\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}) \ . \tag{5}$$

Inserting Eq. (4) and Eq. (5) into Eq. (3) leads to a set of ordinary differential equations for χ_l,

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r) - E \right) \chi_l(r) = -V(r) kr j_l(kr) \ . \tag{6}$$

Outline

Emergent wave method

The potentials $V(r)$, Eq. (1), are exponentially screened, i.e. $V(r) \approx 0$ for $r \geq R$, where $R \gg d$. For large separations $r \geq R$ the emergent wave is, hence, a superposition of outgoing spherical waves, i.e.

$$\frac{\chi_l(r)}{kr} = i\, t_l h_l^{(1)}(kr), \tag{7}$$

where $h_l^{(1)}$ are the spherical Hankel functions of first kind.
Our aim is now to compute the complex prefactors t_l, which will eventually lead to the phase shifts. To this end we solve the ordinary differential equation (6). The corresponding boundary conditions are the following:

- At $r = 0$: $\chi_l(r) \propto r^{l+1}$.
- For $r \geq R$: Eq. (7).

The boundary condition for $r \geq R$ fixes t_l as a function of E.
We solve it numerically, with two different numerical techniques approaches:
(1) a fine uniform discretization of the interval $[0, R]$, which reduces the differential equation to a large set of linear equations, which can be solved rather efficiently, since the corresponding matrix is tridiagonal;
(2) a standard 4-th order Runge-Kutta shooting method.

Outline

Emergent wave method

The quantity t_l is a T matrix eigenvalue. From t_l we directly calculate the phase shift δ_l and also read off the corresponding S matrix eigenvalue s_l, [1]

$$s_l \equiv 1 + 2it_l = e^{2i\delta_l} \ . \tag{8}$$

Moreover, note that both the S matrix and the T matrix are analytical in the complex plane. They are well-defined for complex energies $E \in \mathbb{C}$.

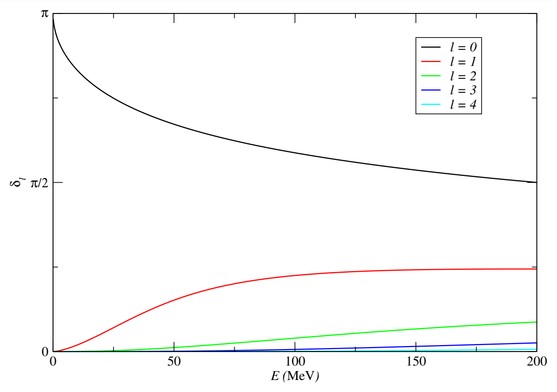
- Thus, our numerical method can as well be applied to solve the differential Eq. (6) for complex $E \in \mathbb{C}$.
- We find the S and T matrix poles by scanning the complex plane $(\mathrm{Re}(E), \mathrm{Im}(E))$ and applying Newton's method to find the roots of $1/t_l(E)$. The poles of the S and the T matrix correspond to complex energies of resonances.
- Note the resonance poles must be in the second Riemann sheet with a negative imaginary part both for the energy E and the momentum k.

[1] At large distances $r \geq R$, the radial wavefunction is $kr[j_l(kr) + i\, t_l h_l^{(1)}(kr)] = (kr/2)[h_l^{(2)}(kr) + e^{2i\delta_l} h_l^{(1)}(kr)]$.

Outline

Results for the phase shifts and resonances

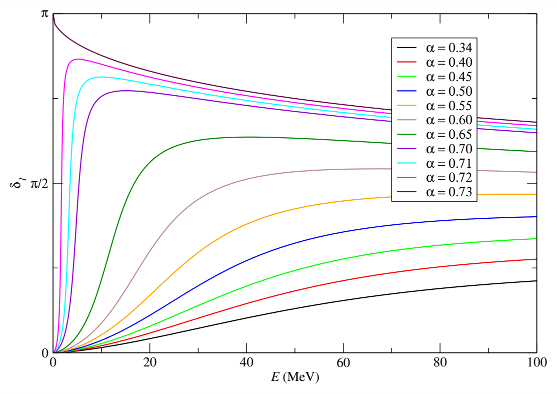
Phase shifts



Phase shift δ_l as a function of the energy E for different angular momenta $l = 0, 1, 2, 3, 4$ for the ($I = 0, j = 0$) potential ($\alpha = 0.34$, $d = 0.45\,\mathrm{fm}$).

Results for the phase shifts and resonances

δ_1 for different α parameters

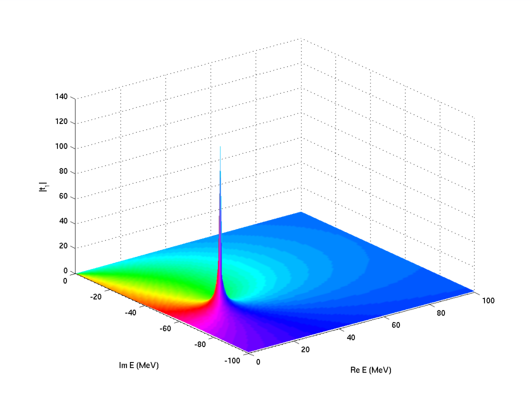


Phase shift δ_1 as a function of the energy E for different parameters α for the $(I = 0, j = 0)$ potential $(d = 0.45\,\mathrm{fm})$.

Outline

Results for the phase shifts and resonances

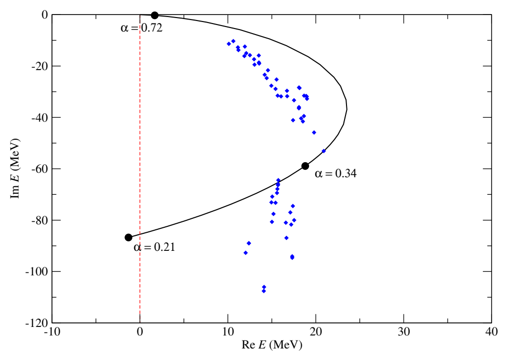
Pole in the complex plane of $E \in \mathbb{C}$



T matrix eigenvalue t_1 as a function of the complex energy E . The vertical axis shows the norm $|t_1|$, the colours represent the phase $\arg(t_l)$.

Results for the phase shifts and resonances

Pole trajectory in the complex space of $E \in \mathbb{C}$ as a function of α



Trajectory of the pole of the eigenvalue t_1 of the T matrix in the complex plane $(\mathrm{Re}(E), \mathrm{Im}(E))$, corresponding to a variation of parameter α. We also illustrate with a cloud of diamond points the systematic error Bicudo:2015vta .

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Summary and outlook

- For more details, please see the recent Phys.Rev. D96, 054510 (2017), Pedro Bicudo, Marco Cardoso (CeFEMA, IST, Lisbon Univ.) , Antje Peters, Martin Pflaumer, Marc Wagner (Frankfurt Univ.).
- Searching for $ud\bar{b}\bar{b}$ resonances, we utilized lattice QCD potentials computed for two static antiquarks in the presence of two light quarks, the Born-Oppenheimer approximation and the emergent wave method.
- First we computed scattering phase shifts of a BB meson pair.
- Then we performed the analytic continuation of the S matrix and the T matrix to the second Riemann sheet and have searched for poles $\in \mathbb{C}$.
- From these results we have predicted a novel $ud\bar{b}\bar{b}$ resonance, with quantum numbers $I(J^P) = 0(1^-)$. Performing a careful statistical and systematic error analysis has led to a resonance mass $m = 10\,576^{+4}_{-4}\,\mathrm{MeV}$ and a decay width $\Gamma = 112^{+90}_{-103}\,\mathrm{MeV}$.
- As and outlook we plan to address the experimentally observed quarkonia exotics Z_c, Z_b, P_c (including $b\bar{b}$ or $c\bar{c}$), and other resonances.

Bicudo:2017szl