Experimental motivations for studying few-hadron systems on the lattice

Alessandro Pilloni

Scattering Amplitudes and Resonances properties from Lattice QCD
MITP, Mainz, August 27th, 2018
Outline

• Introduction

• The light sector: the $3\pi$ system
  • $\eta', \omega$ and $\phi$
  • The $a_1(1260)$
  • The hybrid $\pi_1$
  • The $a_1(1420)$

• The heavy sector: XYZ
  • The $X(3872)$ and the $Y$ states
  • Two-body subchannels: $Z_c$'s and $Z_b$'s
  • Complicated Dalitz plots
Interpretations on the spectrum leads to understanding fundamental laws of nature.
Experiment vs. Lattice QCD

- Higher and higher statistics ✓ ✗
- Lots of multiparticles decay channels available ✓
- Scattering information entangled to production mechanisms ✗
- Experiments happen at the physical point only ✗
- Orthogonal systematics ✓
- Scattering information separated from production; unaccessible channels ✓
- Although QCD is rigid, one can vary the input parameters (quark masses, $N_c$ and $n_f$) and study the effect on amplitudes ✓
Experiment vs. Lattice QCD

Intermediate step through a 2-body isobar (partial wave truncation)
Experiment vs. Lattice QCD

Intermediate step through a 2-body isobar (partial wave truncation)
Light spectrum (1-particle correlators)

The higher the mass, the more channels open

\[ a_1(1260) \]

\[ a_1(1420) ? \]

\[ \pi_1(1600) \]

**A. Pilloni – Experimental motivation for multihadrons on the lattice**
3-body stuff

Unitarity constraints on the Isobar-Spectator amplitude

\[ 2 \text{Im} = \sum_n + \sum_{n,r \neq r} + \sum_{n \neq j} + \sum_{r \neq k} + \sum_{r \neq k} (1 - \delta_{jk}) \]

\[ \tilde{A}_{k,j} = B_{k,j} + \sum_n \int B_{kn} \tau_n \tilde{A}_{n,j} \]

- B-Matrix composed of OPE and Contact

\[ \text{Contact (Real Function)} \quad \text{OPE (required by unitarity)} \]

M. Mai, B. Hu, M. Doring, AP, A. Szczepaniak EPJA53, 9, 177

A. Jackura, et al., to appear

D. Sadasivan, et al., in progress

→ See Michael’s talk on Friday
The $a_1(1260)$

\[ I^G(J^{P C}) = 1^-(1^{++}) \]

Mass $m = 1230 \pm 40$ MeV
Full width $\Gamma = 250$ to 600 MeV

<table>
<thead>
<tr>
<th>$a_1(1260)$ DECAY MODES</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>$\rho$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\rho \pi)_S$-wave</td>
<td>seen</td>
<td>353</td>
</tr>
<tr>
<td>$(\rho \pi)_D$-wave</td>
<td>seen</td>
<td>353</td>
</tr>
<tr>
<td>$(\rho(1450)\pi)_S$-wave</td>
<td>seen</td>
<td>↑</td>
</tr>
<tr>
<td>$(\rho(1450)\pi)_D$-wave</td>
<td>seen</td>
<td>↑</td>
</tr>
<tr>
<td>$\sigma \pi$</td>
<td>seen</td>
<td></td>
</tr>
<tr>
<td>$f_0(980)\pi$</td>
<td>not seen</td>
<td>179</td>
</tr>
<tr>
<td>$f_0(1370)\pi$</td>
<td>seen</td>
<td>↑</td>
</tr>
<tr>
<td>$f_0(1270)\pi$</td>
<td>seen</td>
<td>↑</td>
</tr>
<tr>
<td>$K^0 K^*(892)^+ c.c.$</td>
<td>seen</td>
<td>↑</td>
</tr>
<tr>
<td>$\pi \gamma$</td>
<td>seen</td>
<td>608</td>
</tr>
</tbody>
</table>

Despite it has been known since forever, the resonance parameters of the $a_1(1260)$ are poorly determined.
The production (and model) dependence is affecting their extraction.
The $a_1(1260)$

The extraction of the resonance in the $\tau$ decay should be the cleanest, but the determination of the pole is still unstable.

(Lattice simulations with stable $\rho$, Lang, Leskovec, Mohler, Prelovsek, JHEP 1404, 162)
The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, et al., to appear

We can use these models to fit $\tau^- \rightarrow 2\pi^- \pi^+ \nu$ and describe the $a_1(1260)$

The dispersed improved model describes better the data at threshold
$\pi p \rightarrow 3\pi p$ diffractive production

COMPASS, PRD95, 032004 (2017)
Slide by B. Ketzer
This production mechanism allows for a nonresonant contribution (Deck effect). Because of the light mass of the pion, the singularity is close to the physical region and generates a peaking background.
$\pi_1(1600) \to \rho \pi \to \pi \pi \pi$

The strength of the Deck effect depends on the momentum transferred $t$, but the precise estimates rely on the model for the Deck amplitude.
Coupled channel $\pi_1(1600) \rightarrow \eta^{(')}\pi$

A strong signal is also observed in $\eta^{(')}\pi$, consistent with the naive expectation for a hybrid meson.

Having the $3\pi \rightarrow 3\pi$ scattering data from Lattice will allow for a coupled channel analysis unaffected by the Deck effect.
Coupled channel $\pi_1(1600) \rightarrow \eta^{(')}\pi$

- Coupled channel analysis of $\eta\pi$ and $\eta'\pi$ almost completed

A. Rodas, AP et al. (JPAC), to appear
Coupled channel $\pi_1(1600) \rightarrow \eta^{(')}\pi$

Production amplitude

$\pi^- \rightarrow \eta \rightarrow \pi^- \pi^- \eta$

$\text{Im} \gamma \rho = \sum_{n} n(s) \text{Im} \gamma D(s)$

Scattering amplitude

$\eta \rightarrow \pi^- \pi^- \pi^-$

$\text{Im} \gamma = \sum_{n} N(s) \text{Im} \gamma D(s)$

$t(s) = \frac{N(s)}{D(s)}$

The $D(s)$ has only right hand cuts; it contains all the Final State Interactions constrained by unitarity $\rightarrow$ universal

$D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \rho_{i}(s') \frac{N_{ij}(s')}{s' (s' - s')} ds'$
Coupled channel $\pi_1(1600) \to \eta^{(')}\pi$

**Production amplitude**

$\text{Im} \frac{\pi^-}{\pi^-} = \sum_n \frac{n(s)}{D(s)}$

**Scattering amplitude**

$\text{Im} \frac{\eta}{\pi^-} = \sum_n \frac{N(s)}{D(s)}$

$t(s) = \frac{N(s)}{D(s)}$

The $n(s), N(s)$ have left hand cuts only, process-dependent, smooth

Having access to scattering directly can help reducing systematics

\[
D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \frac{\rho_i(s')N_{ij}(s')}{s'(s' - s)} ds'
\]
$a_1(1420) \to f_0(980)\pi \to \pi\pi\pi$

COMPASS claimed the observation of another $a_1$ at a slightly higher mass:

- Narrower than the $a_1(1260)$
- Unexpected in quark model or lattice spectra
- Only seen in $f_0(980)\pi$
\[ a_1(1420) \rightarrow f_0(980)\pi \rightarrow \pi\pi\pi \]

It has been proposed that the peak is due to a triangle singularity i.e. a dynamical enhancement generated by rescattering.

Mikhasenko, Ketzer, Sarantsev, PRD91, 094015

If that is the case, the strength of the signal would dramatically depend on the mass of the exchanges: studying the amplitude at different pion/kaon masses will confirm whether this is true.
A host of unexpected resonances have appeared decaying mostly into charmonium + light. Hardly reconciled with usual charmonium interpretation.

The heavy sector: XYZ states

A. Pilloni – Experimental motivation for multihadrons on the lattice
X(3872)

- Discovered in $B \to K X \to K J/\psi \pi\pi$
- Quantum numbers $1^{++}$
- Very close to $DD^*$ threshold
- Too narrow for an above-threshold charmonium
- Isospin violation too big $\frac{\Gamma(X \to J/\psi \omega)}{\Gamma(X \to J/\psi \rho)} \sim 0.8 \pm 0.3$
- Mass prediction not compatible with $\chi_{c1}(2P)$
  
  $M = 3871.68 \pm 0.17$ MeV
  $M_X - M_{DD^*} = -3 \pm 192$ keV
  $\Gamma < 1.2$ MeV @90%

A. Pilloni – Experimental motivation for multihadrons on the lattice
$X(3872)$

Large prompt production at hadron colliders

$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$

$\sigma_{PR} \times B(X \rightarrow J/\psi \pi \pi \pi)$

$=(1.06 \pm 0.11 \pm 0.15)\text{ nb}$

CMS, JHEP 1304, 154
**$X(3872)$ on the lattice**

Prelovsek, Leskovec, PRL111, 192001

- **Three body dynamics $D\bar{D}\pi$** may play a role. Playing with lighter charm mass?
- A full amplitude analysis is missing, and is now mandatory
Vector $Y$ states

Lots of unexpected $J^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!)

Seen in few final states, mostly $J/\psi \pi\pi$ and $\psi(2S) \pi\pi$

Not seen decaying into open charm pairs
Large HQSS violation

A. Pilloni – Experimental motivation for multihadrons on the lattice
New BESIII data show a peculiar lineshape for the $Y(4260)$, and suggest a state narrower and lighter than in the past.

The state is mature for a coupled channel analysis (on the lattice?)

A. Pilloni – Experimental motivation for multihadrons on the lattice
Charged Z states: $Z_c(3900), Z'_c(4020)$

In the Dalitz plot projections, two states appear slightly above $D^{(*)}D^*$ thresholds

$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi \pi^+\pi^- \text{ and } \to (DD^*)^+\pi^-$

$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$

$e^+e^- \to Z'_c(4020)^+\pi^- \to h_c \pi^+\pi^- \text{ and } \to \bar{D}^*0 D^{*+}\pi^-$

$M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$
Charged Z states: $Z_b(10610), Z'_b(10650)$

Anomalous dipion width in $\Upsilon(5S)$, 2 orders of magnitude larger than $\Upsilon(nS)$

Moreover, observed $\Upsilon(5S) \to h_b(nP)\pi\pi$ which violates HQSS

$\Upsilon(5S) \to Z_b(10610)^+\pi^- \to \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$ and $\to (B B^*)^+\pi^-$

$M = 10607.2 \pm 2.0$ MeV, $\Gamma = 18.4 \pm 2.4$ MeV

$\Upsilon(5S) \to Z'_b(10650)^+\pi^- \to \Upsilon(nS)\pi^+\pi^-, h_b(nP)\pi^+\pi^-$ and $\to \bar{B}^*0 B^+\pi^-$

$M = 10652.2 \pm 1.5$ MeV, $\Gamma = 11.5 \pm 2.2$ MeV
The number of energy levels we find is equal to the number of expected non-interacting meson-mesons.

Finite-volume spectrum lies close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.

There is no strong indication for a bound state or narrow resonance in this channel, $Z_C(3900)$?

Tetraquark operators do not have a significant effect on calculating the spectrum.

No calculations have found evidence for a resonance

Prelovsek, Leskovec, PLB727, 172-176
HALQCD, PRL117, 242001
HadSpec, JHEP 1711, 033
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities $\rightarrow$ different natures

- Triangle rescattering, logarithmic branching point
  - Szczepaniak, PLB747, 410

- (anti)bound state, II/IV sheet pole
  - Szczepaniak, PLB747, 410

- Resonance, III sheet pole
  - "$\sigma, f_0(980)$"

AP et al. (JPAC), PLB772, 200

D1(2420)

$u: D_0(2400)$

$D_1(2420)$

$Z_c(3900)$?

Maiani et al., PRD71, 014028
Faccini et al., PRD87, 111102
Esposito et al., Phys.Rept. 668

A. Pilloni – Experimental motivation for multihadrons on the lattice
Amplitude model

\[ f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\text{max}}} (2l + 1) \left( a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u) \right) \]

Khuri-Treiman

\[ f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s) \]

\[ f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s P_l(z_s) \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' - s}, \]

\[ f_i(s, t, u) = 16\pi \left[ a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' (s' - s)} \right) \right], \]
A. Pilloni – Experimental motivation for multihadrons on the lattice
Fit: III+tr.
Fit: IV+tr.

\[ E_{CM} = 4.26 \text{ GeV} \]

\[ E_{CM} = 4.23 \text{ GeV} \]

\[ E_{CM} = 4.26 \text{ GeV} \]

\[ E_{CM} = 4.23 \text{ GeV} \]

\[ E_{CM} = 4.26 \text{ GeV} \]

\[ E_{CM} = 4.23 \text{ GeV} \]

A. Pilloni – Experimental motivation for multihadrons on the lattice
Fit: tr.
Data can hardly distinguish these scenarios. Lattice QCD can actually provide the scattering matrix as an input to this analysis.
More complicated Dalitz plots

In the reaction $e^+e^- \rightarrow \psi'\pi^+\pi^-$, the situation looks even more obscure.

Data refused to be fitted with any simple model.

BESIII, PRD96, 032004
More complicated Dalitz plots

LHCb, $B \rightarrow K J/\psi \phi$

Belle, $B \rightarrow K \chi_{c1} \pi$

LHCb, $\Lambda_b \rightarrow K J/\psi p$

Very complicated Dalitz plots

They can all benefit of the knowledge of the underlying $2 \rightarrow 2$ scattering amplitude
Outlook

• The light sector: the $3\pi$ system
  • The $a_1(1260)$
  • The hybrid $\pi_1$
  • The $a_1(1420)$

• The heavy sector: XYZ
  • The $X(3872)$ and the $Y$ states
  • Two-body subchannels: $Z_c$s and $Z_b$s
  • Complicated Dalitz plots

Lattice can disentangle the scattering from the production mechanism
Three body dynamics AND coupled channels

Lattice can provide the $2 \rightarrow 2$ scattering amplitude that can be used as input in the phenomenological models
BACKUP
Pole extraction

Scenario | III+tr. | IV+tr. | tr. |
---|---|---|---|
III | 1.5σ (1.5σ) | 1.5σ (2.7σ) | “2.4σ” (“1.4σ”) |
III+tr. | – | 1.5σ (3.1σ) | “2.6σ” (“1.3σ”) |
IV+tr. | – | – | “2.1σ” (“0.9σ”) |

Not conclusive at this stage
Pentaquarks!

LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

Two states seen in $\Lambda_b \rightarrow (J/\psi p) K^-$, evidence in $\Lambda_b \rightarrow (J/\psi p) \pi^-$

$M_1 = 4380 \pm 8 \pm 29$ MeV
$\Gamma_1 = 205 \pm 18 \pm 86$ MeV
$M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV
$\Gamma_2 = 39 \pm 5 \pm 19$ MeV

Quantum numbers

$$J^P = \left( \frac{3^-}{2}, \frac{5^+}{2} \right) \text{ or } \left( \frac{3^+}{2}, \frac{5^-}{2} \right) \text{ or } \left( \frac{5^+}{2}, \frac{3^-}{2} \right)$$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by $\Lambda^*$ (model dependence?)

No obvious threshold nearby

A. Pilloni – Experimental motivation for multihadrons on the lattice
Pentaquarks!

Two states seen in $\Lambda_b \to (J/\psi p) K^-$, evidence in $\Lambda_b \to (J/\psi p) \pi^-$

- $M_1 = 4380 \pm 8 \pm 29$ MeV
- $\Gamma_1 = 205 \pm 18 \pm 86$ MeV
- $M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV
- $\Gamma_2 = 39 \pm 5 \pm 19$ MeV

Quantum numbers

$$J^P = \left( \frac{3^-}{2} , \frac{5^+}{2} \right) \text{ or } \left( \frac{3^+}{2} , \frac{5^-}{2} \right) \text{ or } \left( \frac{5^+}{2} , \frac{3^-}{2} \right)$$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by $\Lambda^*$ (model dependence?)

No obvious threshold nearby

A. Pilloni – Experimental motivation for multihadrons on the lattice
Tetraquark: the $c\bar{c}s\bar{s}$ states

Much narrower than LHCb! Look for prompt!

Good description of the spectrum but one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with $0^{++}$ and $2^{++}$)

Maiani, Polosa and Riquer, PRD 94, 054026
Other beasts

\(X(3915)\), seen in \(B \to X K \to J/\psi \omega\) and \(\gamma\gamma \to X \to J/\psi \omega\)
\(J^{PC} = 0^{++}\), candidate for \(\chi_{c0}(2P)\)
But \(X(3915) \not\to D\bar{D}\) as expected, and the hyperfine splitting
\(M(2^{++}) - M(0^{++})\) too small

One/two peaks seen in \(B \to XK \to J/\psi \phi K\), close to threshold
To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.

**Hadronic vertex**

$$\langle \lambda_{\psi} \lambda_{p'} | T_{\text{dec}} | \lambda_{\gamma} \lambda_p \rangle = \frac{\langle \lambda_{\psi} \lambda_{p'} | T_{\text{em}} | \lambda_{\gamma} \lambda_p \rangle}{M_r^2 - W^2 - i\Gamma_r M_r}$$

**EM vertex**

**Hadronic part**

- 3 independent helicity couplings, → approx. equal, $g_{\lambda_{\psi}, \lambda_{p'}} \sim g$
- $g$ extracted from total width and (unknown) branching ratio

**Vector meson dominance** relates the radiative width to the hadronic width

$$\Gamma_\gamma = 4\pi\alpha \Gamma_{\psi p} \left( \frac{f_\psi}{M_\psi} \right)^2 \left( \frac{p_i}{p_f} \right)^{2\ell+1} \times \frac{4}{6}$$

Hiller Blin, AP et al. (JPAC), PRD94, 034002

A. Pilloni – Experimental motivation for multihadrons on the lattice
Dictionary - Quark model

\[ L = \text{orbital angular momentum} \]
\[ S = \text{spin} \ q + \bar{q} \]
\[ J = \text{total angular momentum} \]
\[ = \text{exp. measured spin} \]

\[ I = \text{isospin} = 0 \text{ for quarkonia} \]

<table>
<thead>
<tr>
<th>( J^{PC} )</th>
<th>( L )</th>
<th>( S )</th>
<th>Charmonium ((c\bar{c}))</th>
<th>Bottomonium ((b\bar{b}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0++</td>
<td>0 ((S\text{-wave}))</td>
<td>0</td>
<td>( \eta_c(nS) )</td>
<td>( \eta_b(nS) )</td>
</tr>
<tr>
<td>1--</td>
<td>0 ((S\text{-wave}))</td>
<td>1</td>
<td>( \psi(nS') )</td>
<td>( \Upsilon(nS) )</td>
</tr>
<tr>
<td>1+-</td>
<td>1 ((P\text{-wave}))</td>
<td>0</td>
<td>( h_c(nP) )</td>
<td>( h_b(nP) )</td>
</tr>
<tr>
<td>0++</td>
<td>1 ((P\text{-wave}))</td>
<td>1</td>
<td>( \chi_{c0}(nP) )</td>
<td>( \chi_{b0}(nP) )</td>
</tr>
<tr>
<td>1++</td>
<td>1 ((P\text{-wave}))</td>
<td>1</td>
<td>( \chi_{c1}(nP) )</td>
<td>( \chi_{b1}(nP) )</td>
</tr>
<tr>
<td>2++</td>
<td>1 ((P\text{-wave}))</td>
<td>1</td>
<td>( \chi_{c2}(nP) )</td>
<td>( \chi_{b2}(nP) )</td>
</tr>
</tbody>
</table>

But \( J/\psi = \psi(1S), \ \psi' = \psi(2S) \)
Charged $Z$ states: $Z(4430)$

$Z(4430)^+ \rightarrow \psi(2S) \pi^+$

$I^G J^{PC} = 1^+ 1^{+-}$

$M = 4475 \pm 7_{-25}^{+15} \text{ MeV}$

$\Gamma = 172 \pm 13_{-34}^{+37} \text{ MeV}$

Far from open charm thresholds

If the amplitude is a free complex number, in each bin of $m_{\psi',\pi^-}^2$, the resonant behaviour appears as well.
\[ Y(4260) \rightarrow \bar{D}D_1? \]

\[ e^+e^- \rightarrow Y(4260) \rightarrow \pi \bar{D}^0D^{**} \]

BESIII PRL 112, 022001

\[ Z_c(3900)^- \rightarrow \bar{D}^0D^{**} \]

\[ A = \frac{N_{|\cos\theta|>0.5} - N_{|\cos\theta|<0.5}}{N_{|\cos\theta|>0.5} + N_{|\cos\theta|<0.5}} \]

<table>
<thead>
<tr>
<th>DD_1 MC</th>
<th>Z_c+ps MC</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43±0.04</td>
<td>0.02±0.02</td>
<td>0.12±0.06</td>
</tr>
</tbody>
</table>

Not a lot of room for \( \bar{D}D_1(2410) \)
Vector $Y$ states in BESIII

$e^+ e^- \rightarrow \gamma/\psi \pi\pi$

$e^+ e^- \rightarrow h_c \pi\pi$

A. Pilloni – Modeling exotic XYZ states
Flavored $X(5568)$

A flavored state seen in $B_s^0 \pi$ invariant mass by D0 (both $B_s^0 \rightarrow J/\psi \phi$ and $\rightarrow D_s \mu \nu$),

- not confirmed by LHCb or CMS
- (different kinematics? Compare differential distributions)

Controversy to be solved
Interpretations on the spectrum leads to understanding fundamental laws of nature

Data

Amplitude analysis

Properties, Model building

Experiment

Lattice QCD

Interpretations on the spectrum leads to understanding fundamental laws of nature

Hadron Spectroscopy

Meson

Baryon

Glueball

Hybrids

Tetraquark

Hadroquarkonium

Molecule
S-Matrix principles

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.

A. Pilloni – Experimental motivation for multihadrons on the lattice
Pole hunting

I sheet

Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet

II sheet

\[ V(r) \]

\[^3S\text{ (bound state)}\]
\[^1S\text{ (virtual state)}\]

\[^3S\text{ (bound state)}\]
\[^1S\text{ (virtual state)}\]

A. Pilloni – Experimental motivation for multihadrons on the lattice
Pole hunting

More complicated structure when more thresholds arise: two sheets for each new threshold

- III sheet: usual resonances
- IV sheet: cusps (virtual states)
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.

$$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^- \text{ and } \rightarrow (D\bar{D}^*)^+\pi^-$$

$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$

Such a state would require a minimal 4q content and would be manifestly exotic.
Amplitude model

\[ f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\text{max}}} (2l + 1) \left( a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right) \]

Khuri-Treiman

\[ f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s \ f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)}(s) + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s) \]

\[ f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s \ P_l(z_s) \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for} \ l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s}, \]

\[ f_i(s, t, u) = 16\pi \left[ a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right], \]
Triangle singularity

Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438). However, this effect cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363).

\[ f_{0,i}(s) = b_{0,i}(s) + \frac{t_{i,j}}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' - s} \]

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo, Meissner, Wang, Yang PRD92, 071502
Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters.

\[
f_i(s, t, u) = 16\pi \left[ a_{0,i}(t) + a_{0,i}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right],
\]

The scattering matrix is parametrized as \((t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}\)

Four different scenarios considered:

- **III**: the K matrix is \(\frac{g_i g_j}{M^2 - s}\), this generates a pole in the closest unphysical sheet. The rescattering integral is set to zero.
- **III+tr.**: same, but with the correct value of the rescattering integral.
- **IV+tr.**: the K matrix is constant, this generates a pole in the IV sheet.
- **tr.**: same, but the pole is pushed far away by adding a penalty in the \(\chi^2\) A. Pilloni – Experimental motivation for multihadrons on the lattice
Singularities and lineshapes

Different lineshapes according to different singularities

- Triangle
- IV sheet pole
- III sheet pole
- III+tr.
- IV+tr.
- tr.
- no pole

A. Pilloni – Experimental motivation for multihadrons on the lattice
### Table: Experimental motivation for multihadrons on the lattice

<table>
<thead>
<tr>
<th>State</th>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$J^{PC}$</th>
<th>Process (mode)</th>
<th>Experiment (#$\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3823)$</td>
<td>3823.1 ± 1.9</td>
<td>&lt; 24</td>
<td>$?^-$</td>
<td>$B \to K(\chi_c17)$</td>
<td>BES II (5.0)</td>
</tr>
<tr>
<td>$X(3872)$</td>
<td>3871.68 ± 0.17</td>
<td>&lt; 1.2</td>
<td>$1^{++}$</td>
<td>$B \to K(\pi^+\pi^-J/\psi)$</td>
<td>BABar (8.6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p\bar{p} \to (\pi^+\pi^-J/\psi)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p\bar{p} \to (\pi^+\pi^-J/\psi)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\pi^+\pi^-\pi^0J/\psi)$</td>
<td>BES II (5.5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\gamma J/\psi)$</td>
<td>BABar (8.6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(\gamma\psi(2S))$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \to K(D\bar{D}^*)$</td>
<td></td>
</tr>
<tr>
<td>$Z_c(3900)^+$</td>
<td>3888.7 ± 3.4</td>
<td>35 ± 7</td>
<td>$1^{--}$</td>
<td>$Y(4260) \to \pi^-(D\bar{D}^*)^+$</td>
<td>BES III (8.9)</td>
</tr>
<tr>
<td>$Z_c(4020)^+$</td>
<td>4023.9 ± 2.4</td>
<td>10 ± 6</td>
<td>$1^{--}$</td>
<td>$Y(4260) \to \pi^-(\pi^+\pi^0\pi^0)$</td>
<td>BES III (10)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y(4260) \to \pi^-(\pi^+\pi^0\pi^0)$</td>
<td></td>
</tr>
<tr>
<td>$X(4140)$</td>
<td>3918.4 ± 1.9</td>
<td>20 ± 5</td>
<td>$0^{++}$</td>
<td>$B \to K(\omega J/\psi)$</td>
<td>BABar (8.6)</td>
</tr>
<tr>
<td>$Z(3930)$</td>
<td>3927.2 ± 2.6</td>
<td>24 ± 6</td>
<td>$2^{++}$</td>
<td>$e^+e^- \to e^+e^- (\omega J/\psi)$</td>
<td>BABar (5.3)</td>
</tr>
<tr>
<td>$X(3940)$</td>
<td>3942 ± 5</td>
<td>37 ± 17</td>
<td>$?^+$</td>
<td>$e^+e^- \to J/\psi (D\bar{D}^*)$</td>
<td>BES III (6.9)</td>
</tr>
<tr>
<td>$Y(4008)$</td>
<td>3891 ± 42</td>
<td>255 ± 42</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+\pi^-J/\psi)$</td>
<td>BES III (7.4)</td>
</tr>
<tr>
<td>Z(10650)$^+$</td>
<td>4051 ± 24</td>
<td>82 ± 55</td>
<td>$?^+$</td>
<td>$B \to K(\pi^+\pi^0\chi_c1)$</td>
<td>BABar (5.0)</td>
</tr>
<tr>
<td>$Y(4140)$</td>
<td>4145.6 ± 3.6</td>
<td>14.3 ± 5.9</td>
<td>$?^+$</td>
<td>$B^+ \to K^+(\phi J/\psi)$</td>
<td>CDHF (5.0), BABar (1.9),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LHC (1.4), CMS (5.0)&lt;5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Doil (3.1)</td>
</tr>
<tr>
<td>$Z(4200)^+$</td>
<td>4156 ± 25</td>
<td>130 ± 13.13 ± 0.5</td>
<td>$?^+$</td>
<td>$e^+e^- \to J/\psi (D\bar{D}^*)$</td>
<td>BES III (5.5)</td>
</tr>
<tr>
<td>$Z(4200)^+$</td>
<td>4196 ± 35</td>
<td>370 ± 40 ~110</td>
<td>$1^{--}$</td>
<td>$B \to K(\pi^+\pi^-J/\psi)$</td>
<td>BES III (7.2)</td>
</tr>
<tr>
<td>$Y(4220)$</td>
<td>4196 ± 35</td>
<td>39 ± 32</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+\pi^-J/\psi)$</td>
<td>BES III (5.5)</td>
</tr>
<tr>
<td>$Y(4230)$</td>
<td>4230 ± 8</td>
<td>38 ± 12</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\chi_{c0}\omega)$</td>
<td>BES II (5.5)</td>
</tr>
<tr>
<td>$Z(4240)^+$</td>
<td>4248 ± 185</td>
<td>177 ± 121</td>
<td>$?^+$</td>
<td>$B \to K(\pi^-\chi_c1)$</td>
<td>BES III (5.0), BABar (2.0),</td>
</tr>
<tr>
<td>$Y(4260)$</td>
<td>4250 ± 9</td>
<td>108 ± 12</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+J/\psi)$</td>
<td>BES III (8.8)</td>
</tr>
<tr>
<td>$Y(4290)$</td>
<td>4293 ± 9</td>
<td>222 ± 67</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+\pi^-h_c)$</td>
<td>BES III (5.3)</td>
</tr>
<tr>
<td>$X(4350)$</td>
<td>4350 ± 4.6</td>
<td>13 ± 18 ± 10</td>
<td>$0^{2^{++}}$</td>
<td>$e^+e^- \to e^+e^- (\phi J/\psi)$</td>
<td>BES III (3.2)</td>
</tr>
<tr>
<td>$Y(4360)$</td>
<td>4354 ± 11</td>
<td>78 ± 16</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+\pi^-\omega)$</td>
<td>BES III (8.2)</td>
</tr>
<tr>
<td>$Z(4390)^+$</td>
<td>4475 ± 17</td>
<td>180 ± 31</td>
<td>$1^{--}$</td>
<td>$\bar{B} \to K(\pi^-\psi(2S))$</td>
<td>BABar (4.4), BABar (2.4),</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LHC (13.9)</td>
</tr>
<tr>
<td>$Y(4630)$</td>
<td>4634 ± 9</td>
<td>92 ± 41 ± 32</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\Lambda^+_c\Lambda^-_c)$</td>
<td>BES III (4.0)</td>
</tr>
<tr>
<td>$Y(4660)$</td>
<td>4665 ± 10</td>
<td>53 ± 14</td>
<td>$1^{--}$</td>
<td>$e^+e^- \to (\pi^+\pi^-\omega)$</td>
<td>BES III (8.2)</td>
</tr>
<tr>
<td>$Z(6010)^+$</td>
<td>10607 ± 2.0</td>
<td>18 ± 2.4</td>
<td>$1^{--}$</td>
<td>$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$</td>
<td>BES III (10)</td>
</tr>
<tr>
<td>$Z(6050)^+$</td>
<td>10652 ± 2.5</td>
<td>11.5 ± 2.2</td>
<td>$1^{--}$</td>
<td>$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$</td>
<td>BES III (10)</td>
</tr>
<tr>
<td>$Y(5S)$</td>
<td>10607 ± 2.0</td>
<td>18 ± 2.4</td>
<td>$1^{--}$</td>
<td>$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$</td>
<td>BES III (10)</td>
</tr>
</tbody>
</table>

---

**Guerrieri, AP, Piccinini, Polosa, IJMPA 30, 1530002**

---

A. Pilloni – Experimental motivation for multihadrons on the lattice