Experimental motivations for studying few-hadron systems on the lattice

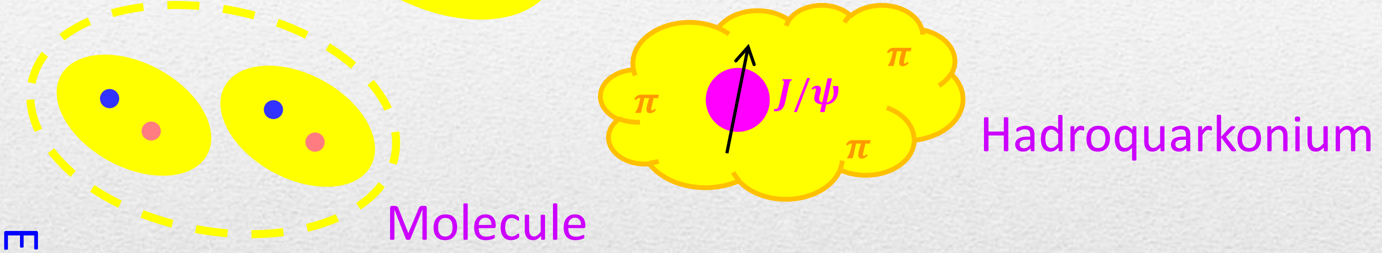
Alessandro Pilloni

Scattering Amplitudes and Resonances properties from Latticd QCD
MITP, Mainz, August 27th, 2018

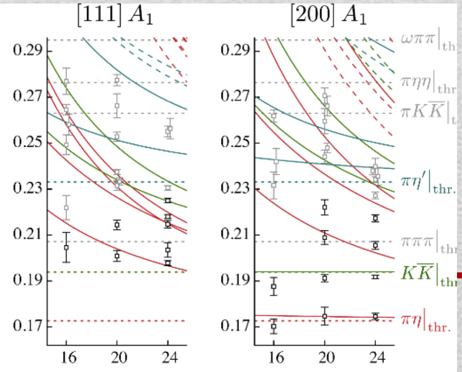
Outline

- Introduction
- The light sector: the 3π system
  - ~~η', ω and ϕ~~
  - The $a_1(1260)$
  - The hybrid π_1
  - The $a_1(1420)$
- The heavy sector: XYZ
  - The $X(3872)$ and the Y states
  - Two-body subchannels: Z_cs and Z_bs
  - Complicated Dalitz plots

Hadron Spectroscopy



Experiment

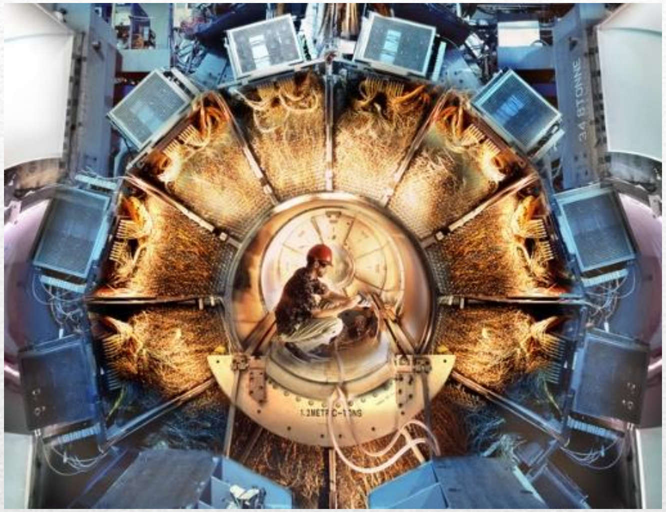


Lattice QCD

Data → Amplitude analysis → Properties, Model building

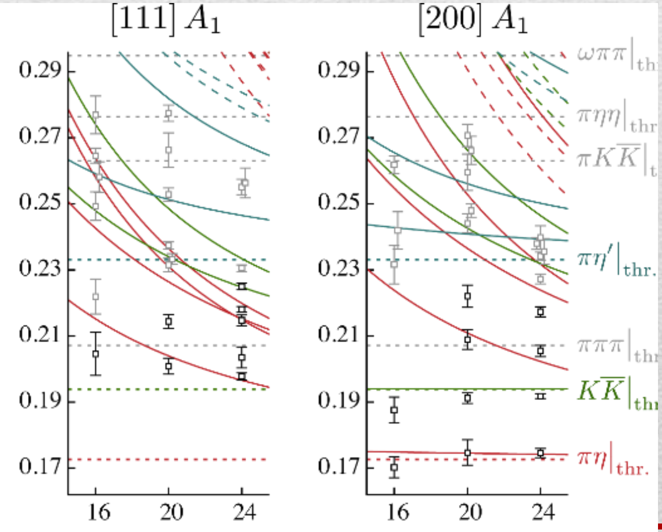
Interpretations on the spectrum leads to understanding fundamental laws of nature

Experiment vs. Lattice QCD



Experiment

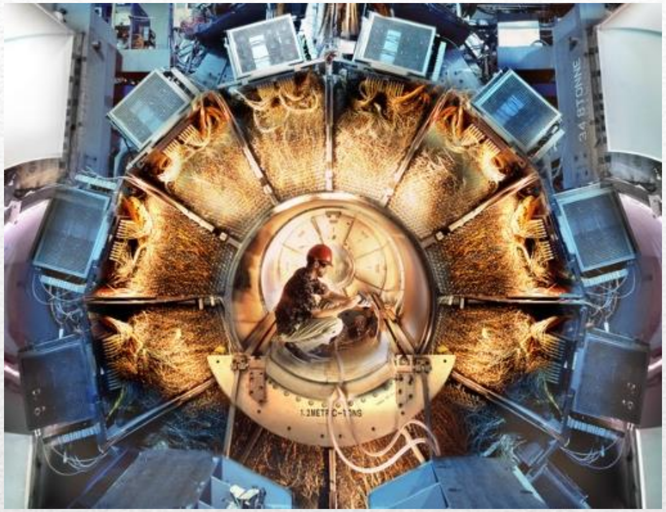
- Higher and higher statistics ✓✗
- Lots of multiparticles decay channels available ✓
- Scattering information entangled to production mechanisms ✗
- Experiments happen at the physical point only ✗



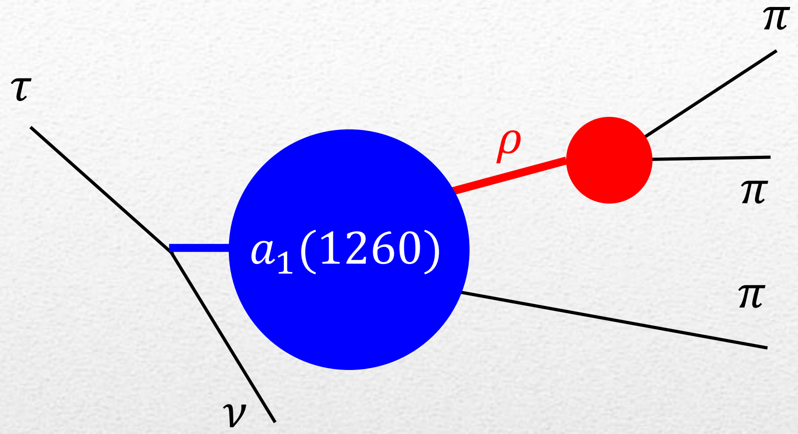
Lattice QCD

- Orthogonal systematics ✓
- Scattering information separated from production; unaccessible channels ✓
- Although QCD is rigid, one can vary the input parameters (quark masses, N_c and n_f) and study the effect on amplitudes ✓

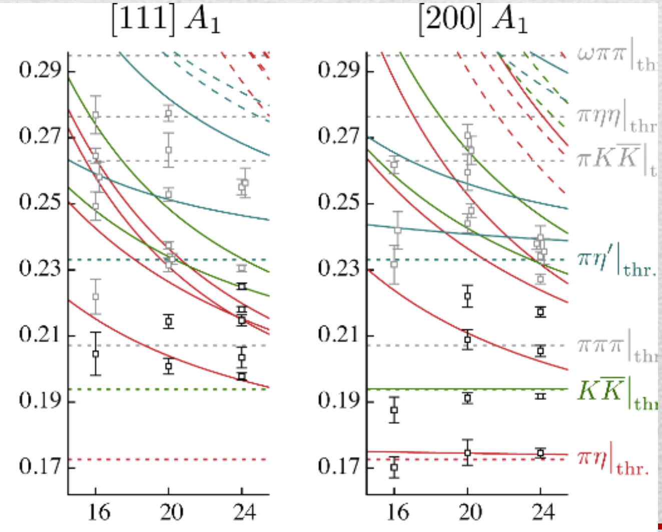
Experiment vs. Lattice QCD



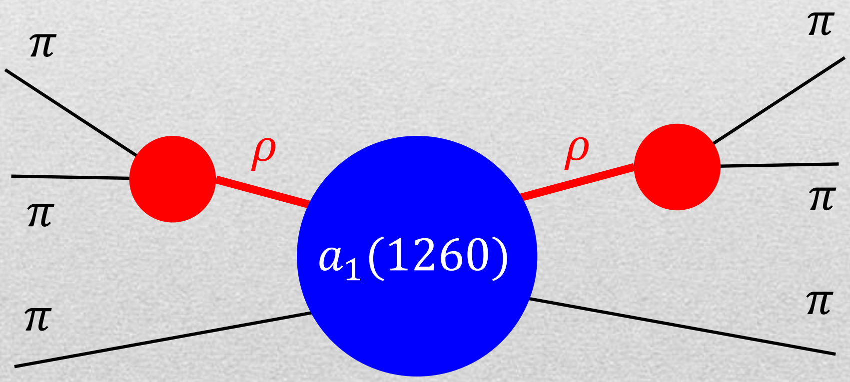
Experiment



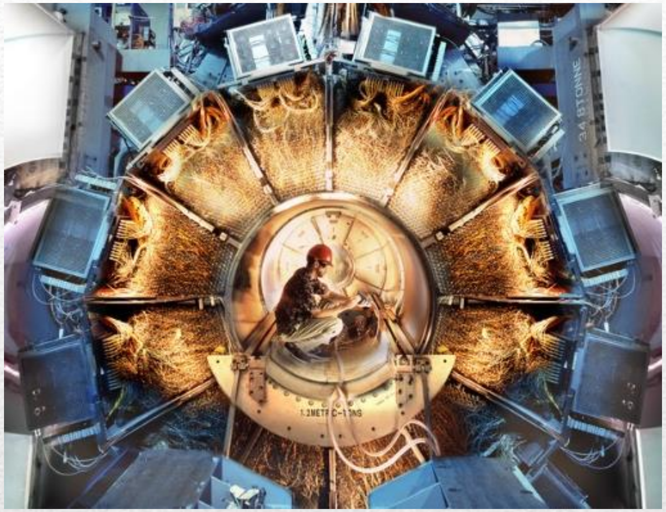
Intermediate step through a 2-body isobar (partial wave truncation)



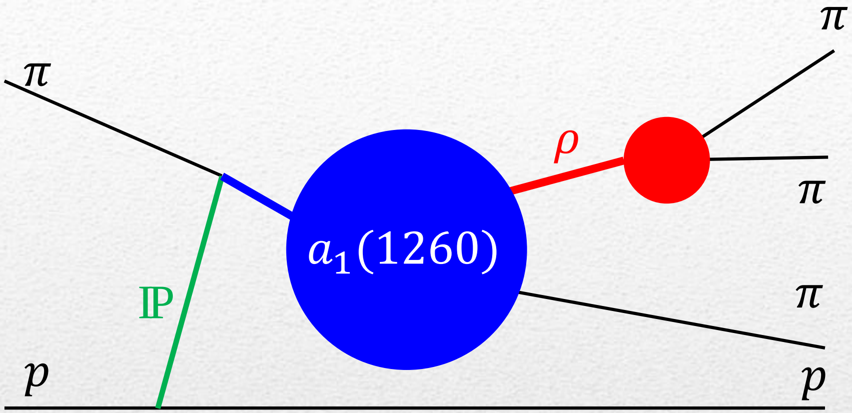
Lattice QCD



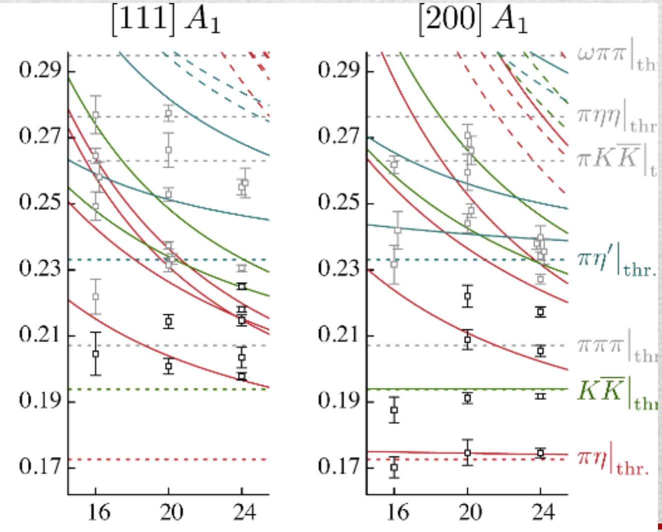
Experiment vs. Lattice QCD



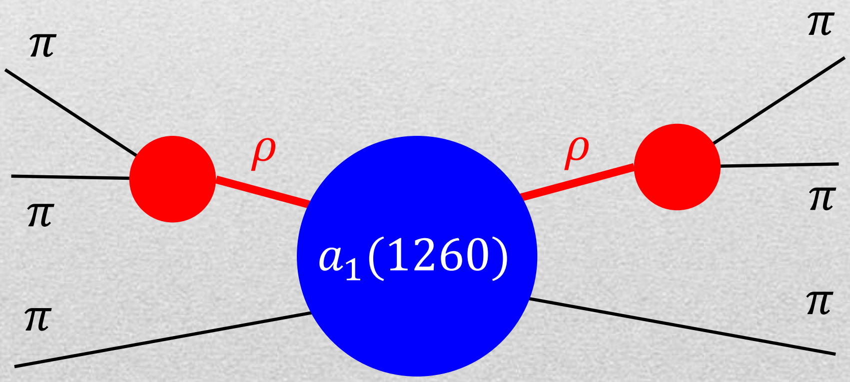
Experiment



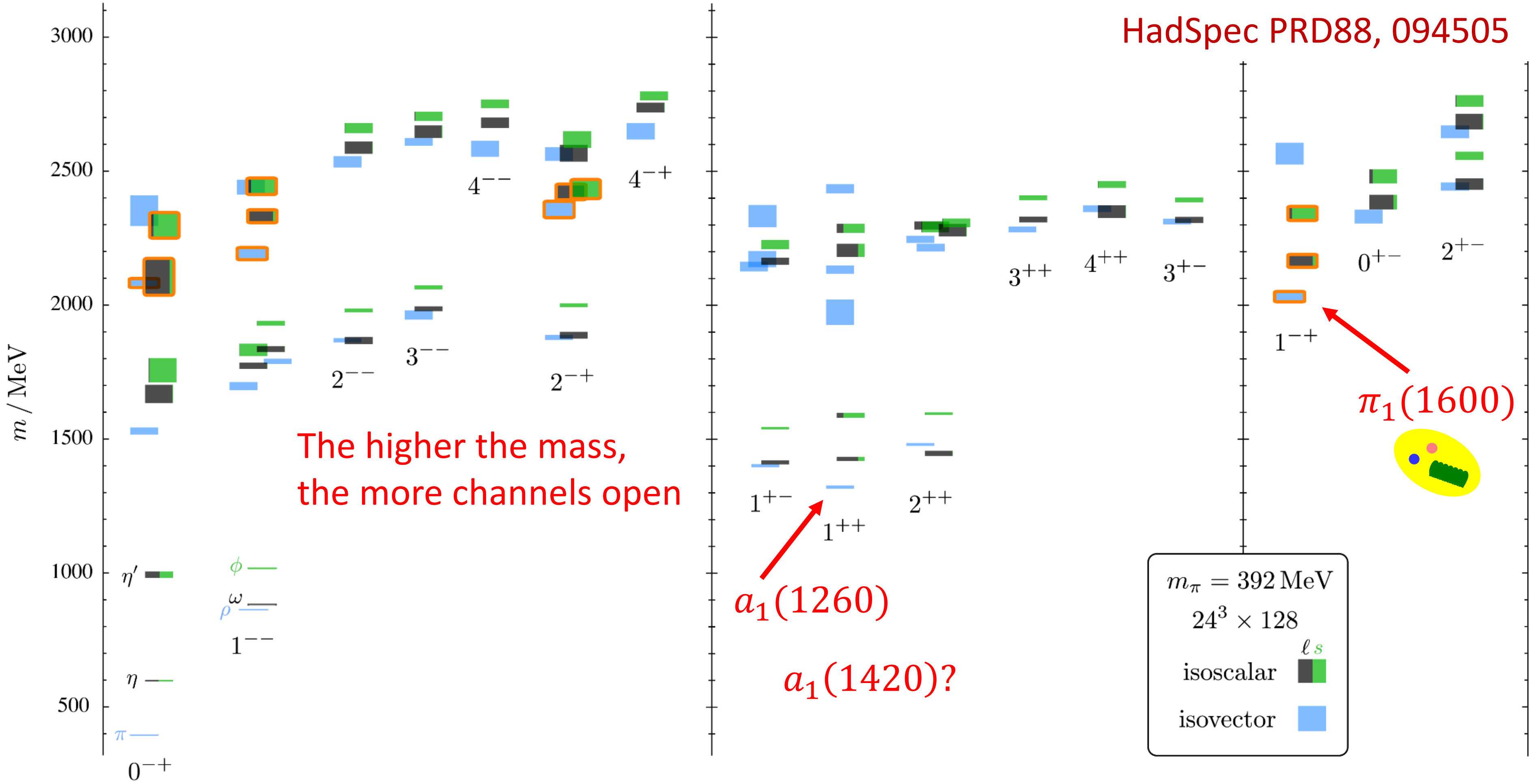
Intermediate step through a 2-body isobar (partial wave truncation)



Lattice QCD



Light spectrum (1-particle correlators)



HadSpec PRD88, 094505

The higher the mass, the more channels open

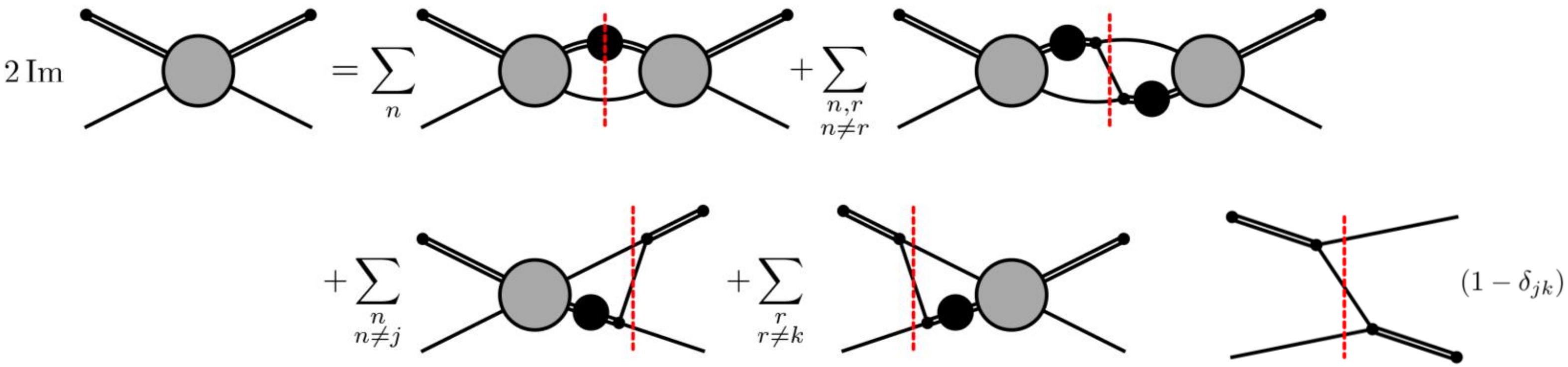
$a_1(1260)$

$a_1(1420)$?

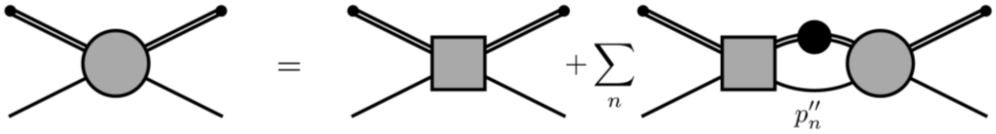
$\pi_1(1600)$

3-body stuff

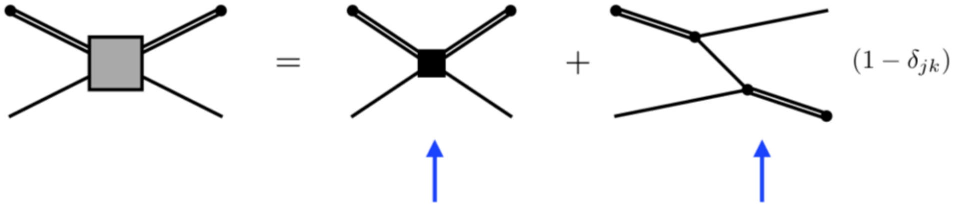
Unitarity constraints on the Isobar-Spectator amplitude



$$\widetilde{\mathcal{A}}_{kj} = \mathcal{B}_{kj} + \sum_n \int \mathcal{B}_{kn} \tau_n \widetilde{\mathcal{A}}_{nj}$$



- B-Matrix composed of OPE and Contact



Contact (Real Function)

OPE (required by unitarity)

A. Jackura

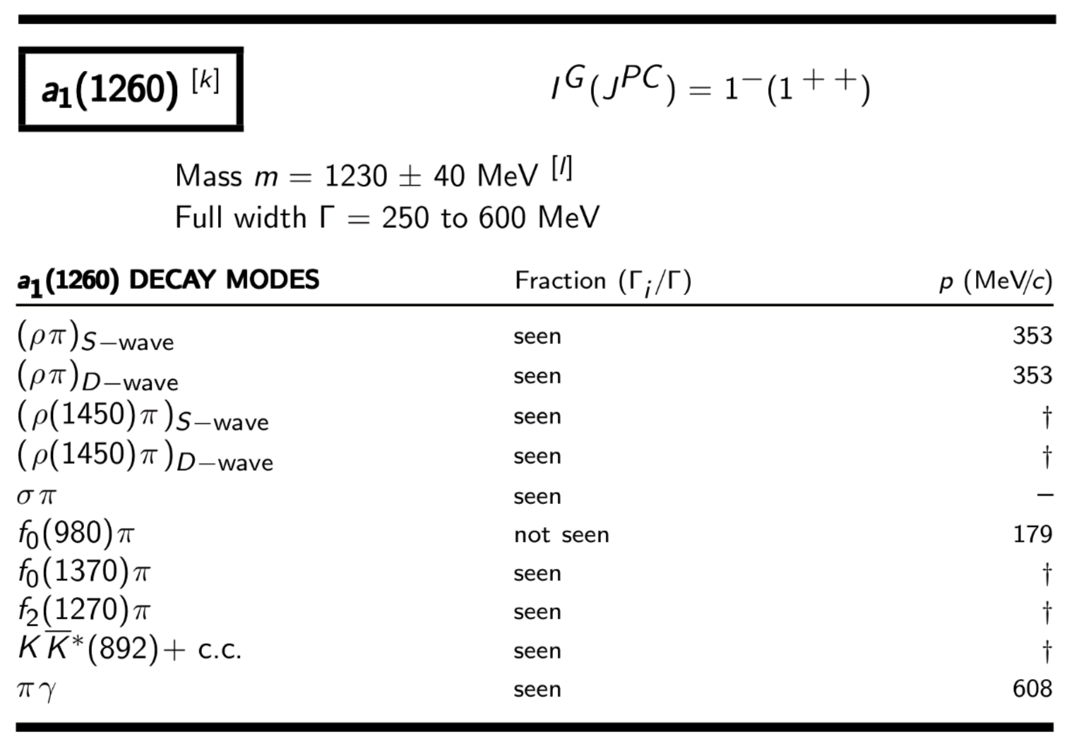
M. Mai, B. Hu, M. Doring,
AP, A. Szczepaniak EPJA53, 9, 177

A. Jackura, *et al.*, to appear

D. Sadasivan, *et al.*, in progress

→ See Michael's talk on Friday

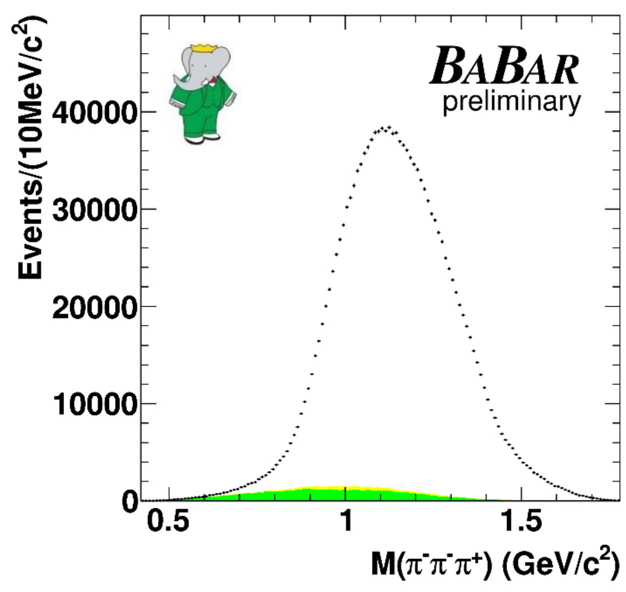
The $a_1(1260)$



$a_1(1260)$ [k] $I^G(J^{PC}) = 1^-(1^{++})$

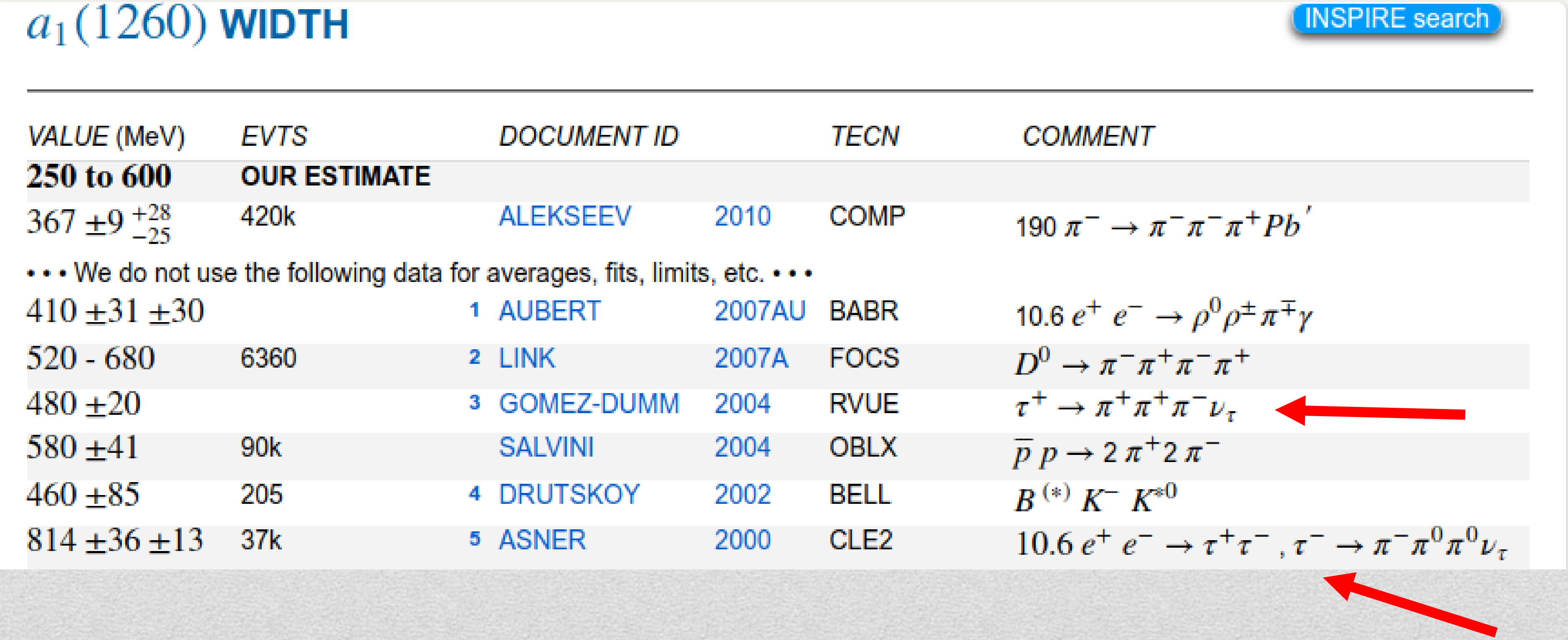
Mass $m = 1230 \pm 40$ MeV [l]
Full width $\Gamma = 250$ to 600 MeV

$a_1(1260)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$(\rho\pi)_{S-\text{wave}}$	seen	353
$(\rho\pi)_{D-\text{wave}}$	seen	353
$(\rho(1450)\pi)_{S-\text{wave}}$	seen	†
$(\rho(1450)\pi)_{D-\text{wave}}$	seen	†
$\sigma\pi$	seen	–
$f_0(980)\pi$	not seen	179
$f_0(1370)\pi$	seen	†
$f_2(1270)\pi$	seen	†
$K\overline{K}^*(892)+$ c.c.	seen	†
$\pi\gamma$	seen	608



Despite it has been known since forever, the resonance parameters of the $a_1(1260)$ are poorly determined
The production (and model) dependence is affecting their extraction

The $a_1(1260)$



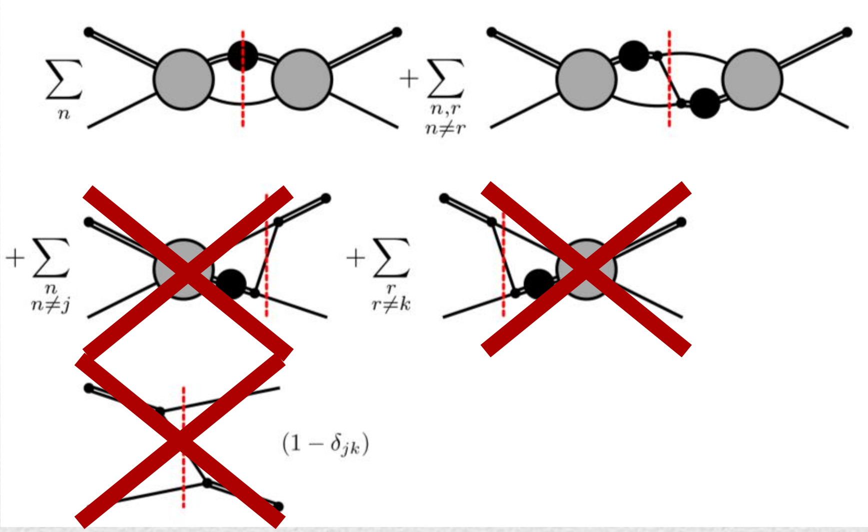
$a_1(1260)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
$367 \pm 9 ^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	$190\ \pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		[1] AUBERT	2007AU	BABR	$10.6\ e^+ e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520 - 680	6360	[2] LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[3] GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\bar{p}\,p \to 2\,\pi^+ 2\,\pi^-$
460 ± 85	205	[4] DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	[5] ASNER	2000	CLE2	$10.6\ e^+ e^- \to \tau^+\tau^-, \tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

The extraction of the resonance in the τ decay should be the cleanest, but the determination of the pole is still unstable

(Lattice simulations with stable ρ, Lang, Leskovec, Mohler, Prelovsek, JHEP 1404, 162)

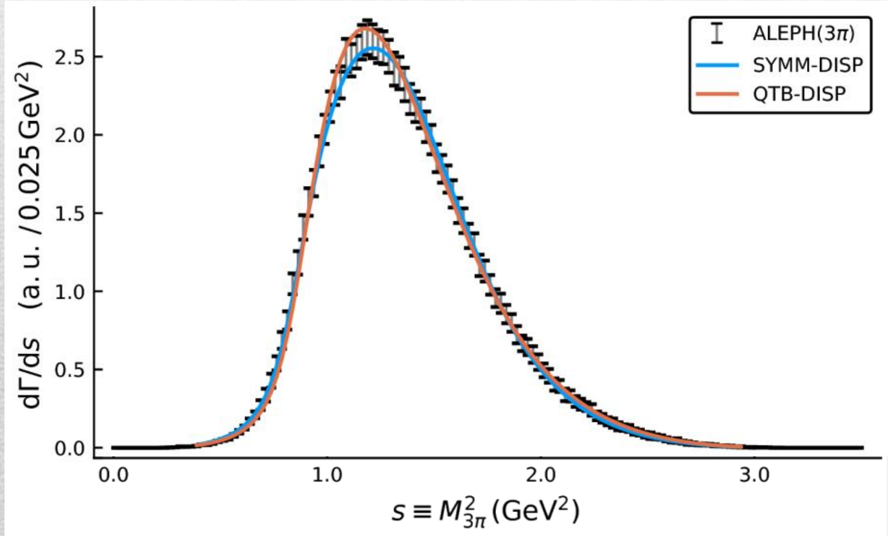
The $a_1(1260)$

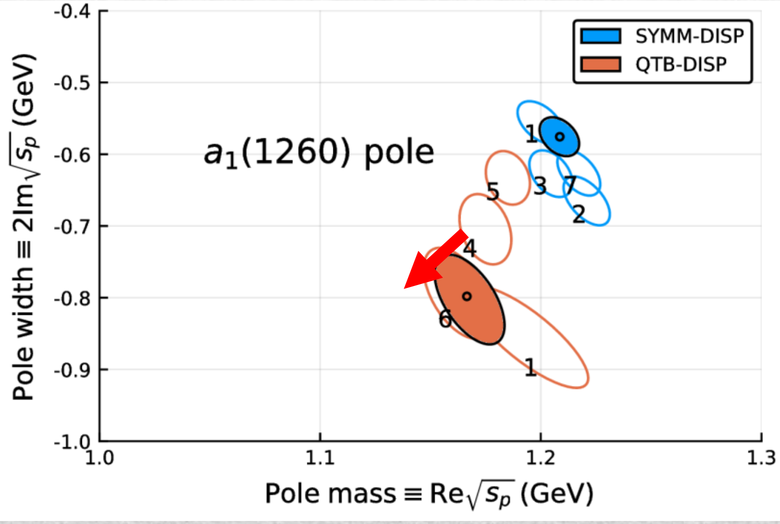


M. Mikhasenko, A. Jackura, AP, *et al.*, to appear

We can use these models to fit $\tau^- \to 2\pi^-\pi^+\nu$ and describe the $a_1(1260)$

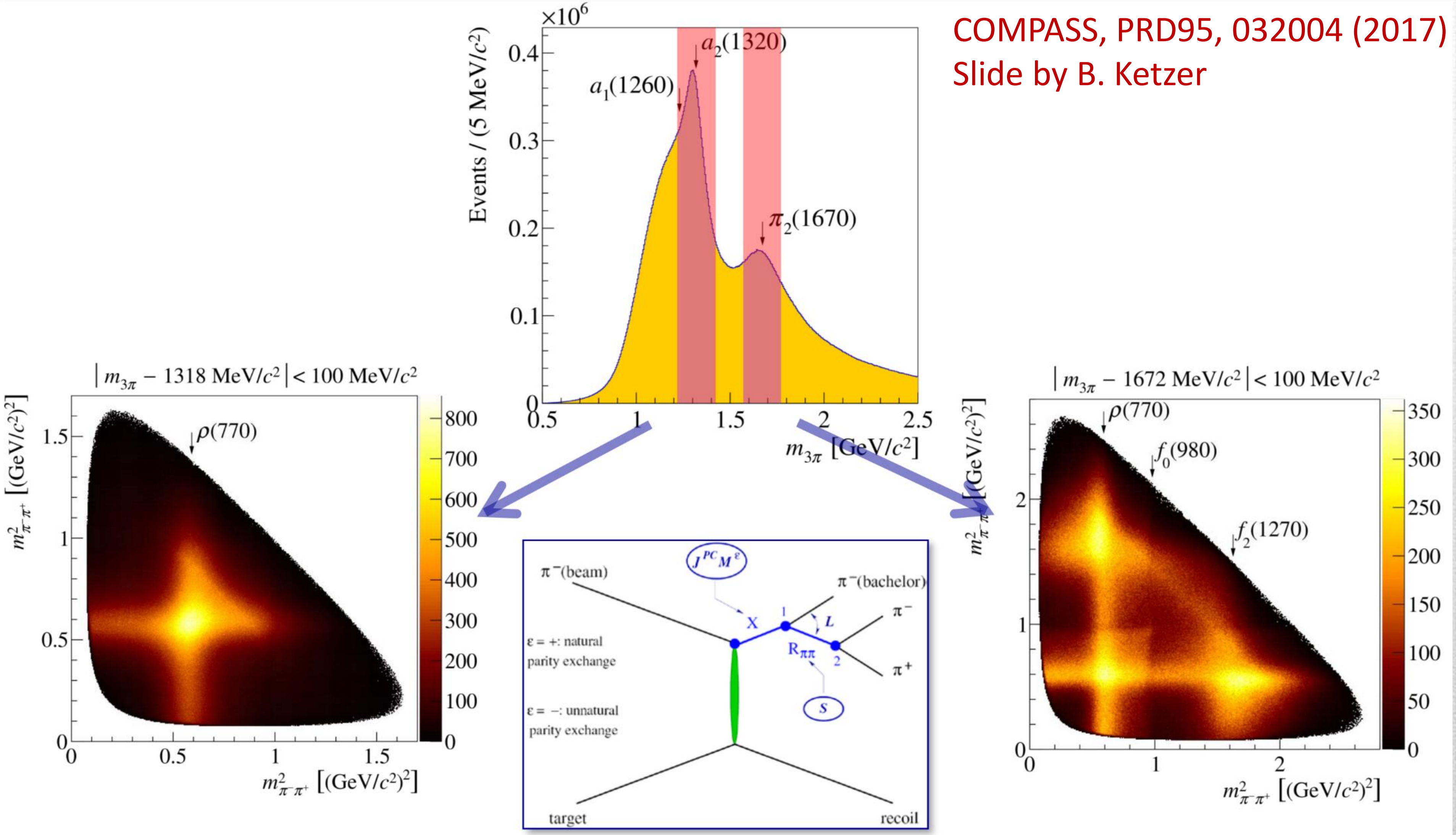
The dispersed improved model describes better the data at threshold



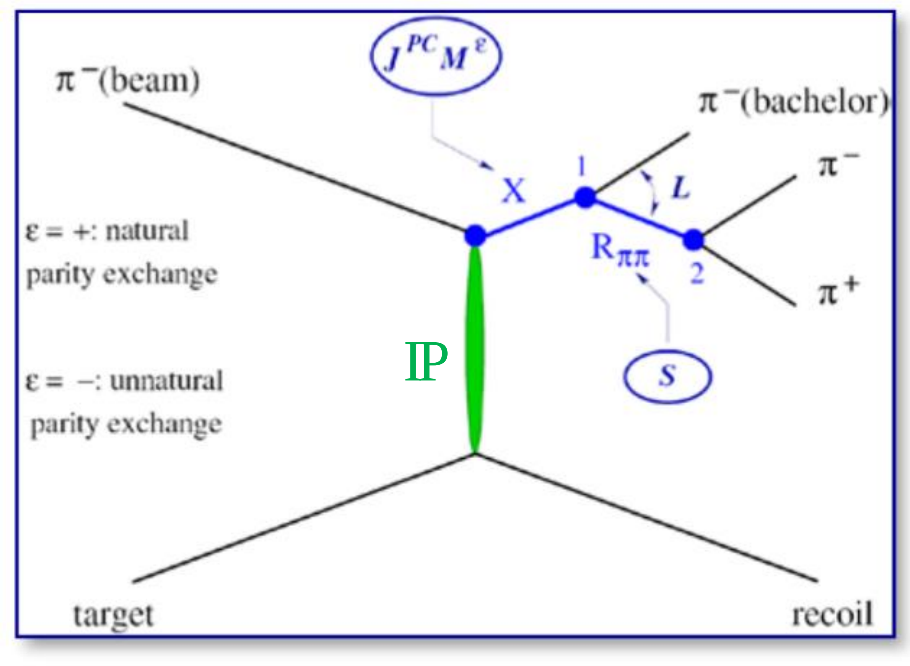


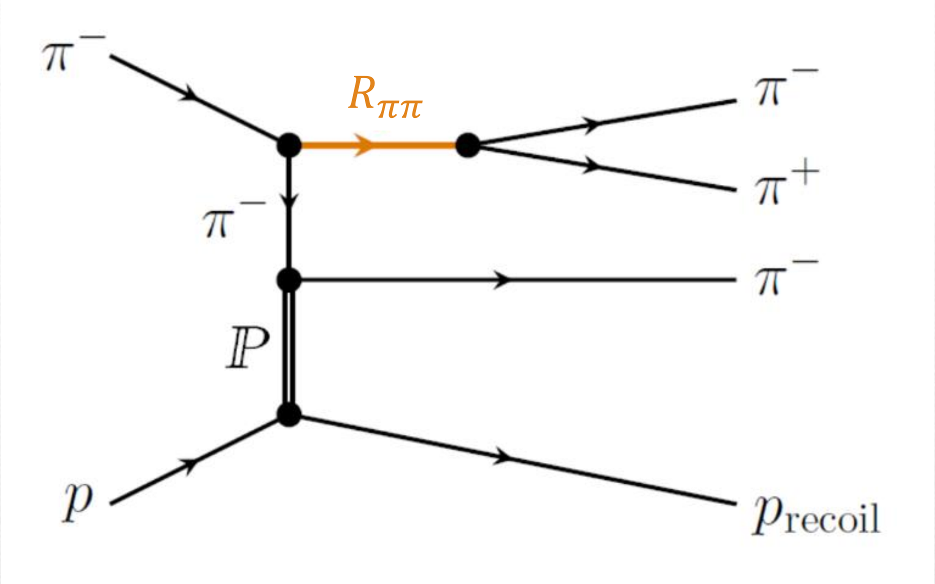
$\pi p \to 3\pi\, p$ diffractive production

COMPASS, PRD95, 032004 (2017)
Slide by B. Ketzer



Deck amplitude

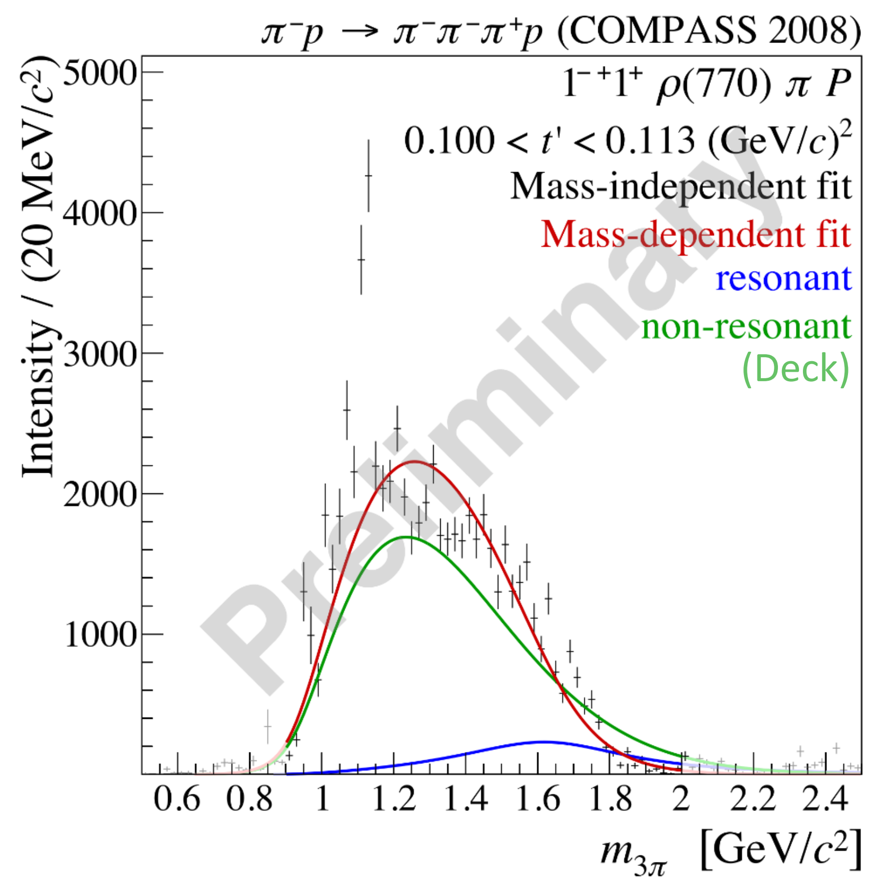


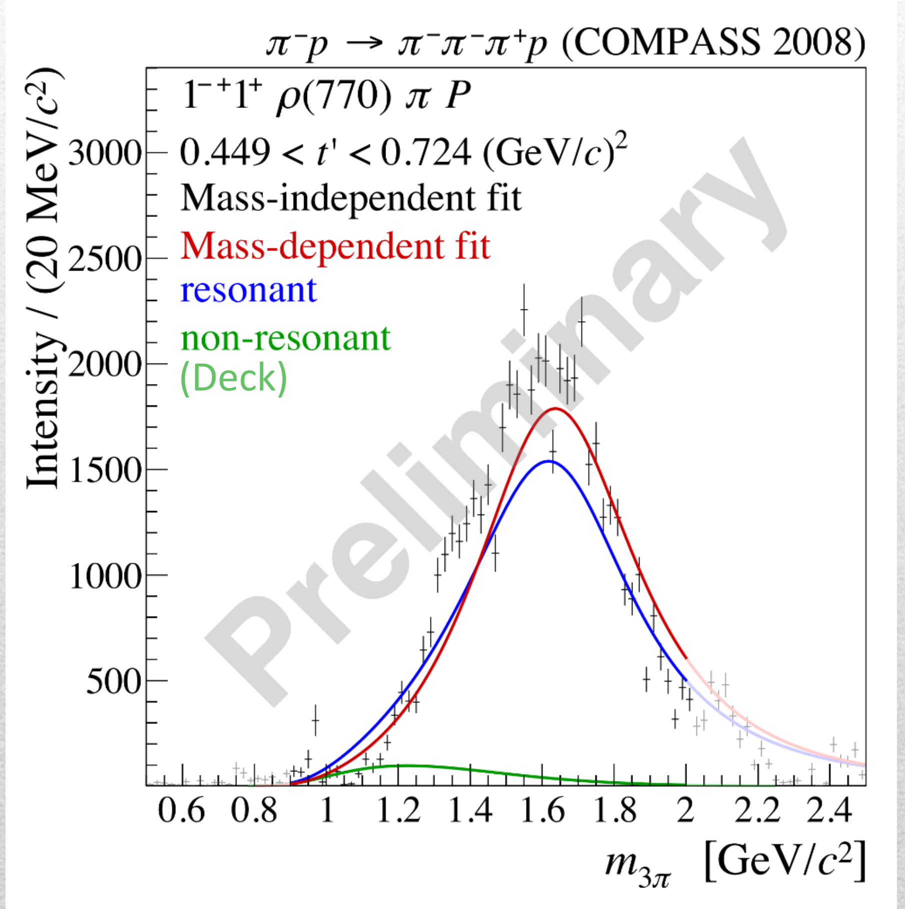


This production mechanism allows for a nonresonant contribution (Deck effect)
Because of the light mass of the pion, the singularity is close to the physical region and generates a peaking background

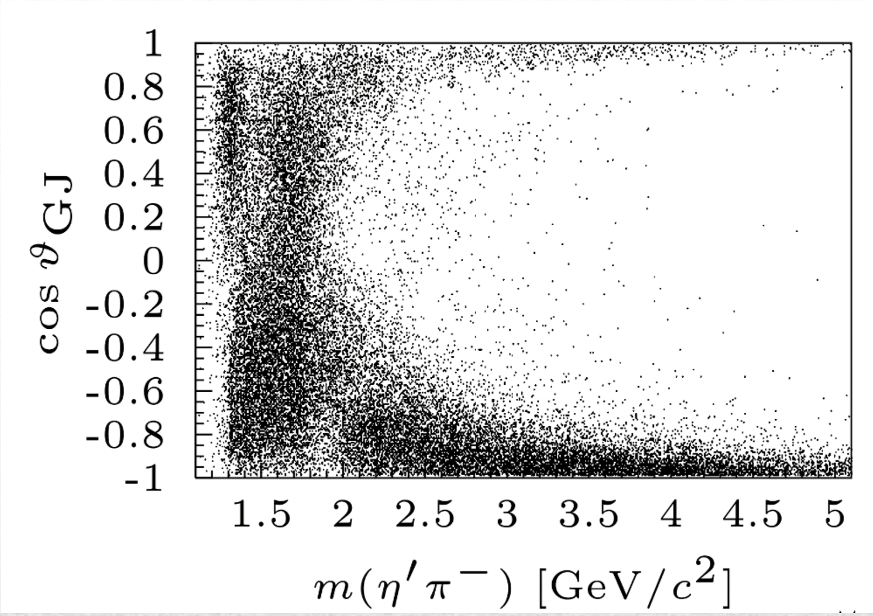
$\pi_1(1600) \to \rho\pi \to \pi\pi\pi$

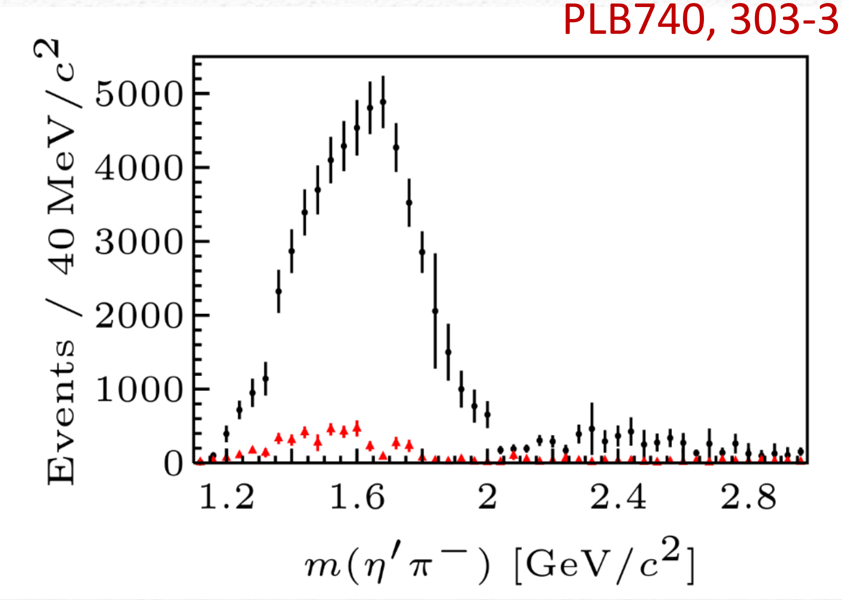
The strength of the Deck effect depends on the momentum transferred t, but the precise estimates rely on the model for the Deck amplitude





Coupled channel $\pi_1(1600) \to \eta^{(\prime)}\pi$





PLB740, 303-311

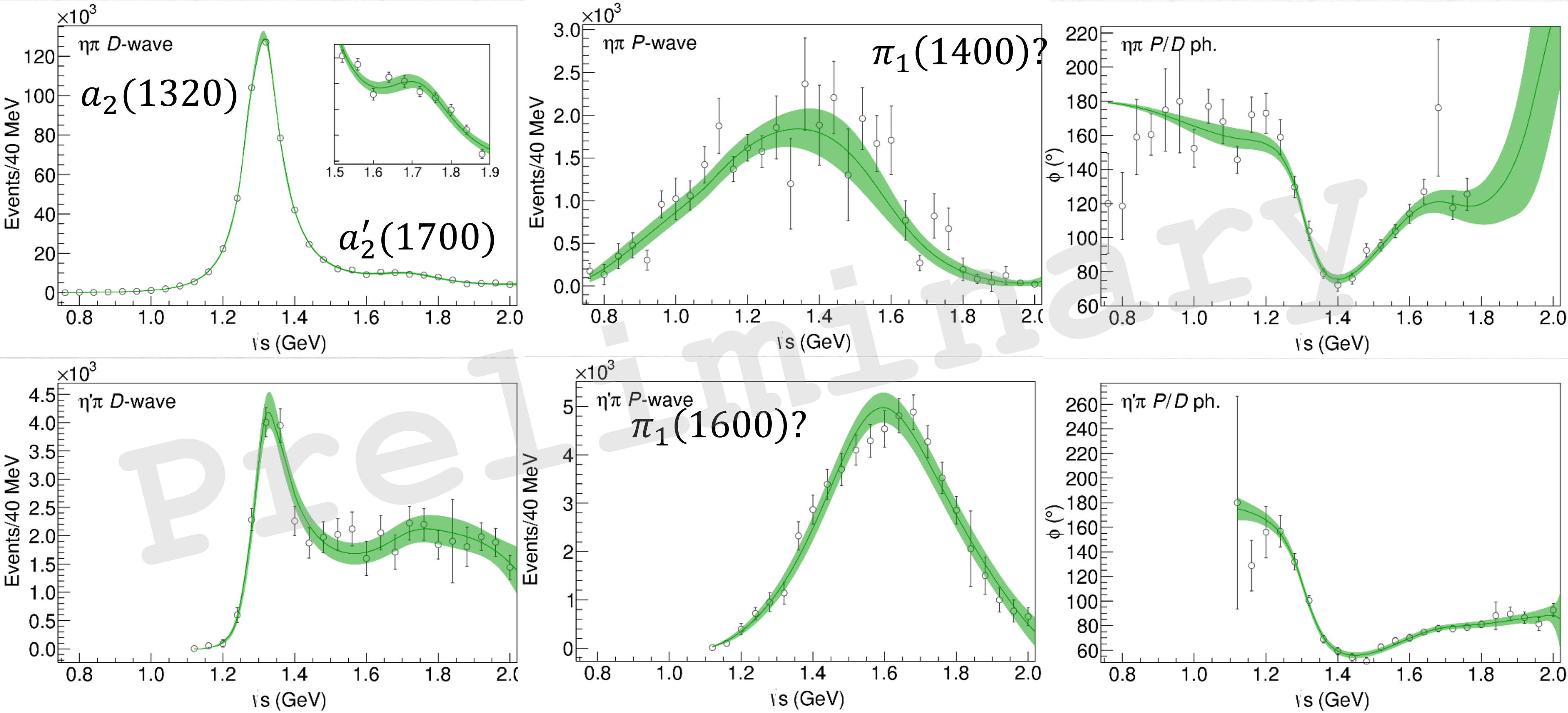
A strong signal is also observed in $\eta^{(\prime)}\pi$, consistent with the naive expectation for a hybrid meson

Having the $3\pi \to 3\pi$ scattering data from Lattice will allow for a coupled channel analysis unaffected by the Deck effect

Coupled channel $\pi_1(1600) \to \eta^{(\prime)}\pi$

- Coupled channel analysis of $\eta\pi$ and $\eta'\pi$ almost completed

A. Rodas, AP *et al.* (JPAC), to appear



$a_2(1320)$

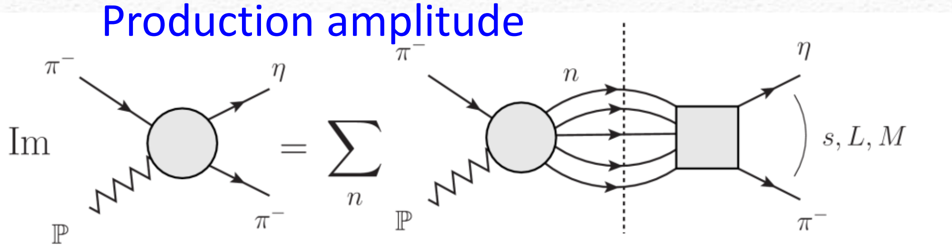
$a_2'(1700)$

$\pi_1(1400)$?

$\pi_1(1600)$?

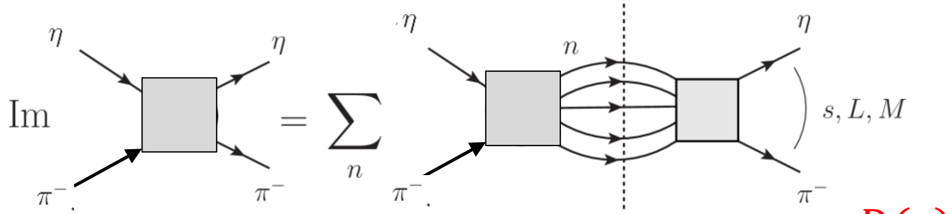
Coupled channel $\pi_1(1600) \to \eta^{(\prime)}\pi$

Production amplitude



$n(s)$ $D(s)$

Scattering amplitude

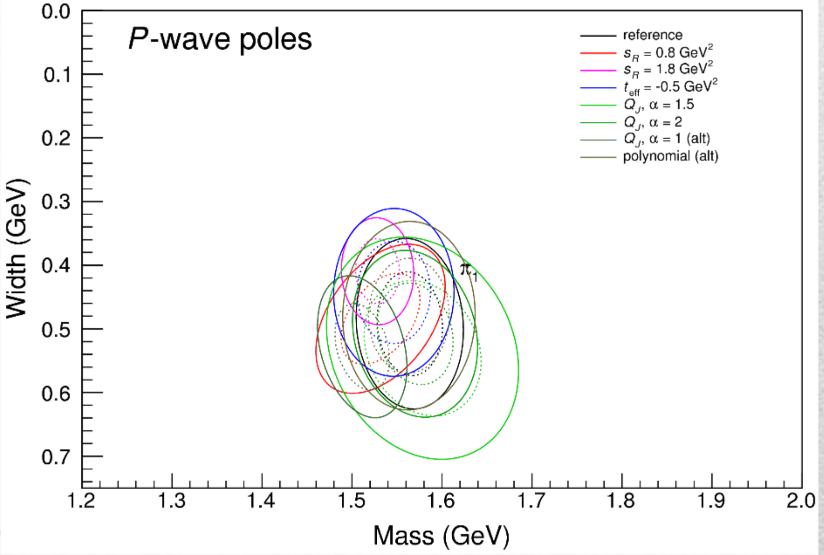


$N(s)$ $D(s)$

$$D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho_i(s')N_{ij}(s')}{s'\,(s'-s)}ds'$$

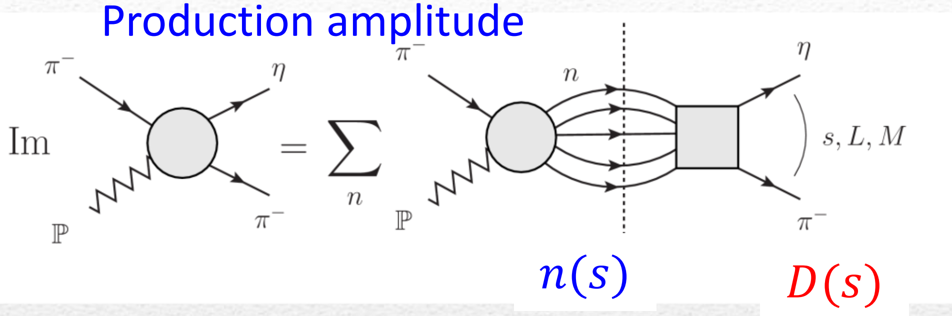
$$t(s) = \frac{N(s)}{D(s)}$$

The $D(s)$ has only right hand cuts;
it contains all the Final State Interactions
constrained by unitarity → universal

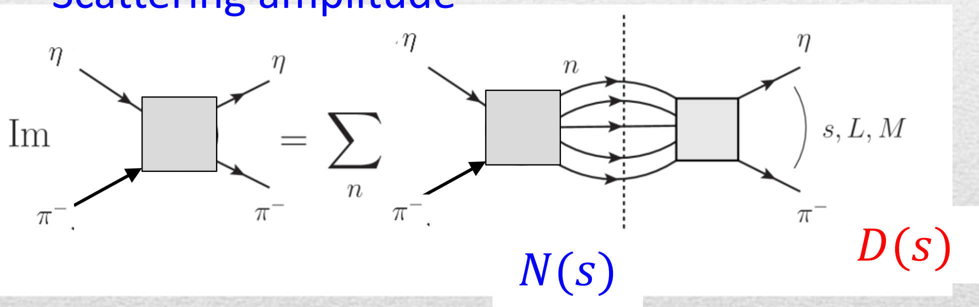


Coupled channel $\pi_1(1600) \to \eta^{(\prime)}\pi$

Production amplitude



Scattering amplitude

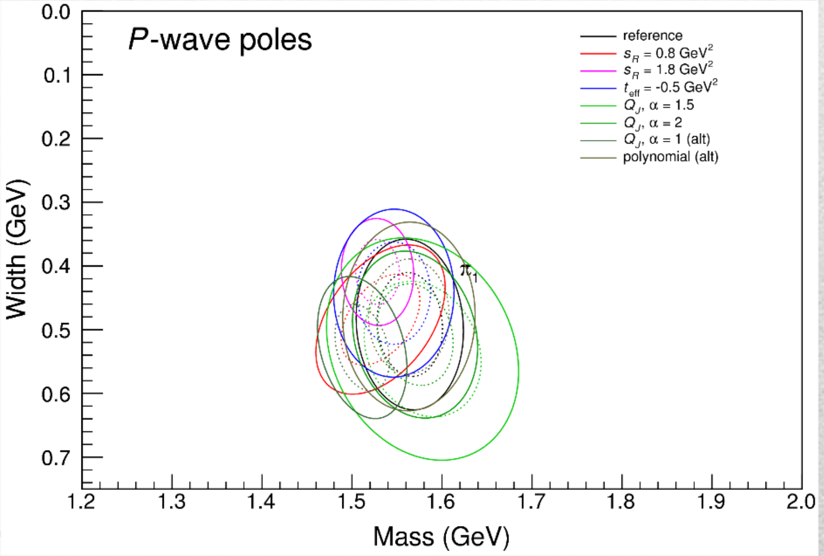


$$D(s)_{ij} = (K^{-1})_{ij}(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho_i(s')N_{ij}(s')}{s'(s'-s)}ds'$$

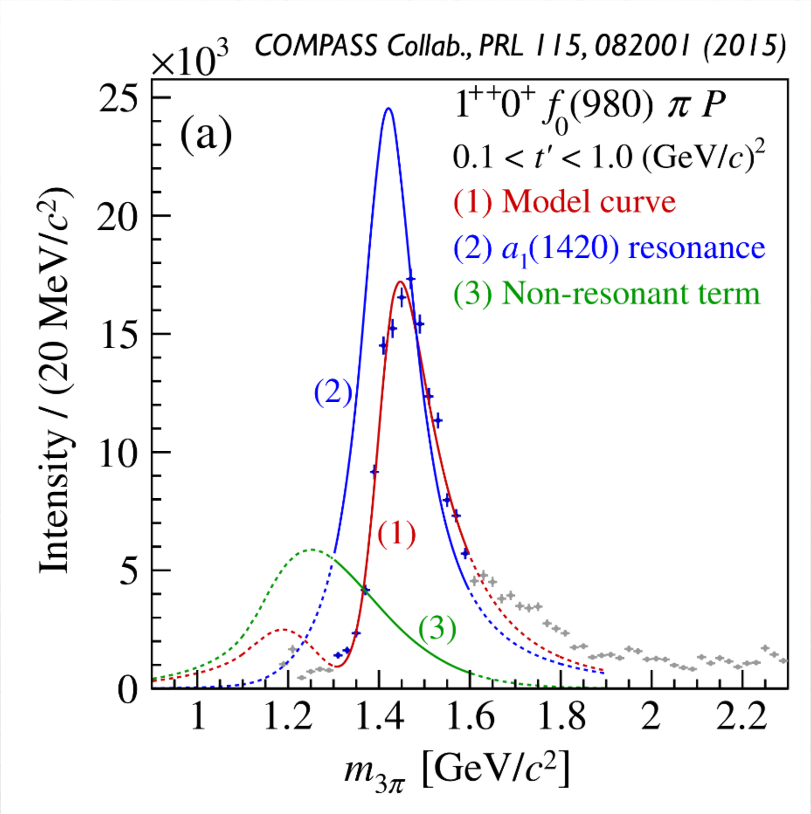
$$t(s) = \frac{N(s)}{D(s)}$$

The $n(s), N(s)$ have left hand cuts only, process-dependent, smooth

Having access to scattering directly can help reducing systematics

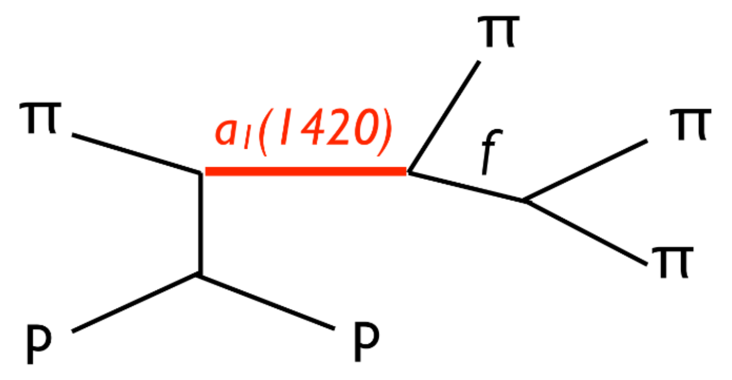


$a_1(1420) \to f_0(980)\pi \to \pi\pi\pi$

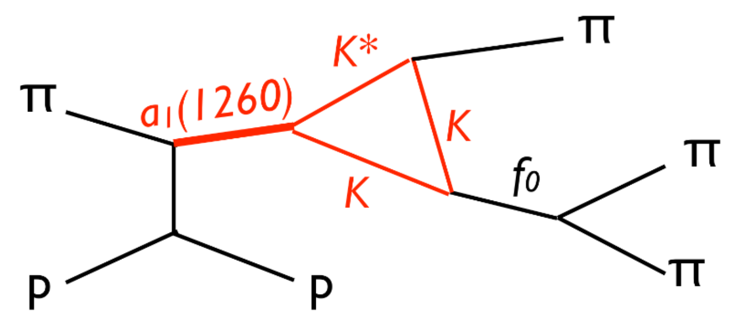


COMPASS claimed the observation of another a_1 at a slightly higher mass

- Narrower than the $a_1(1260)$
- Unexpected in quark model or lattice spectra
- Only seen in $f_0(980)\pi$



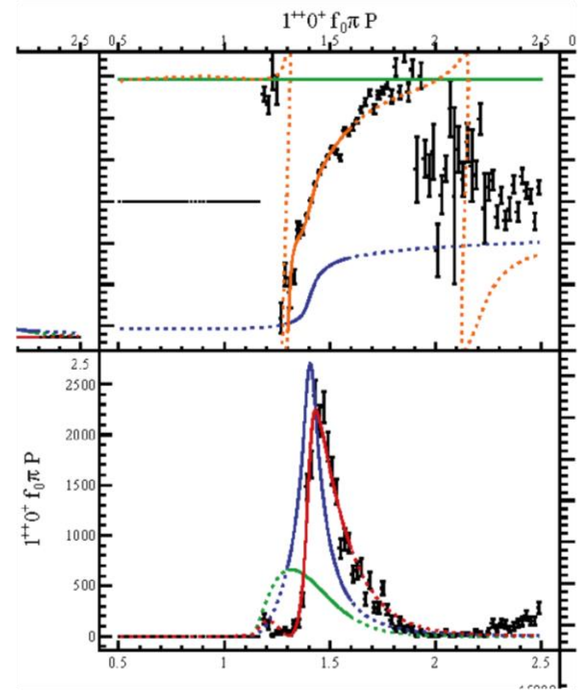
$a_1(1420) \to f_0(980)\pi \to \pi\pi\pi$



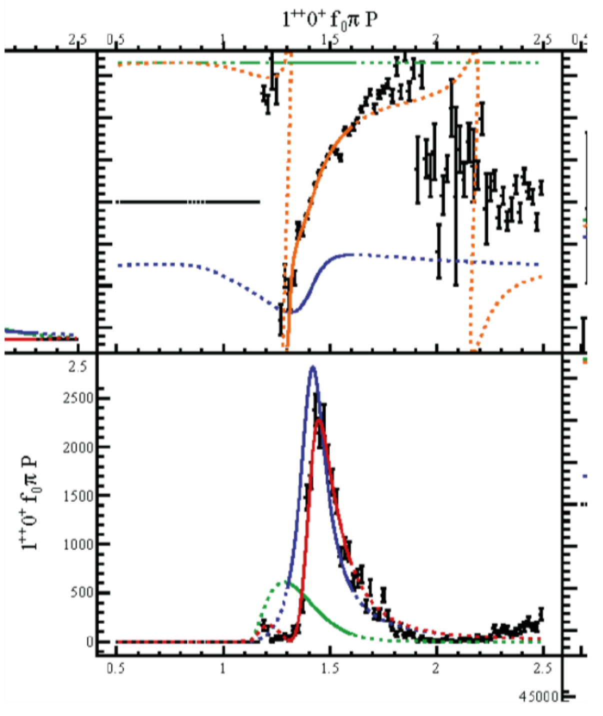
It has been proposed that the peak is due to a triangle singularity i.e. a dynamical enhancement generated by rescattering

Mikhasenko, Ketzer, Sarantsev, PRD91, 094015

Triangle



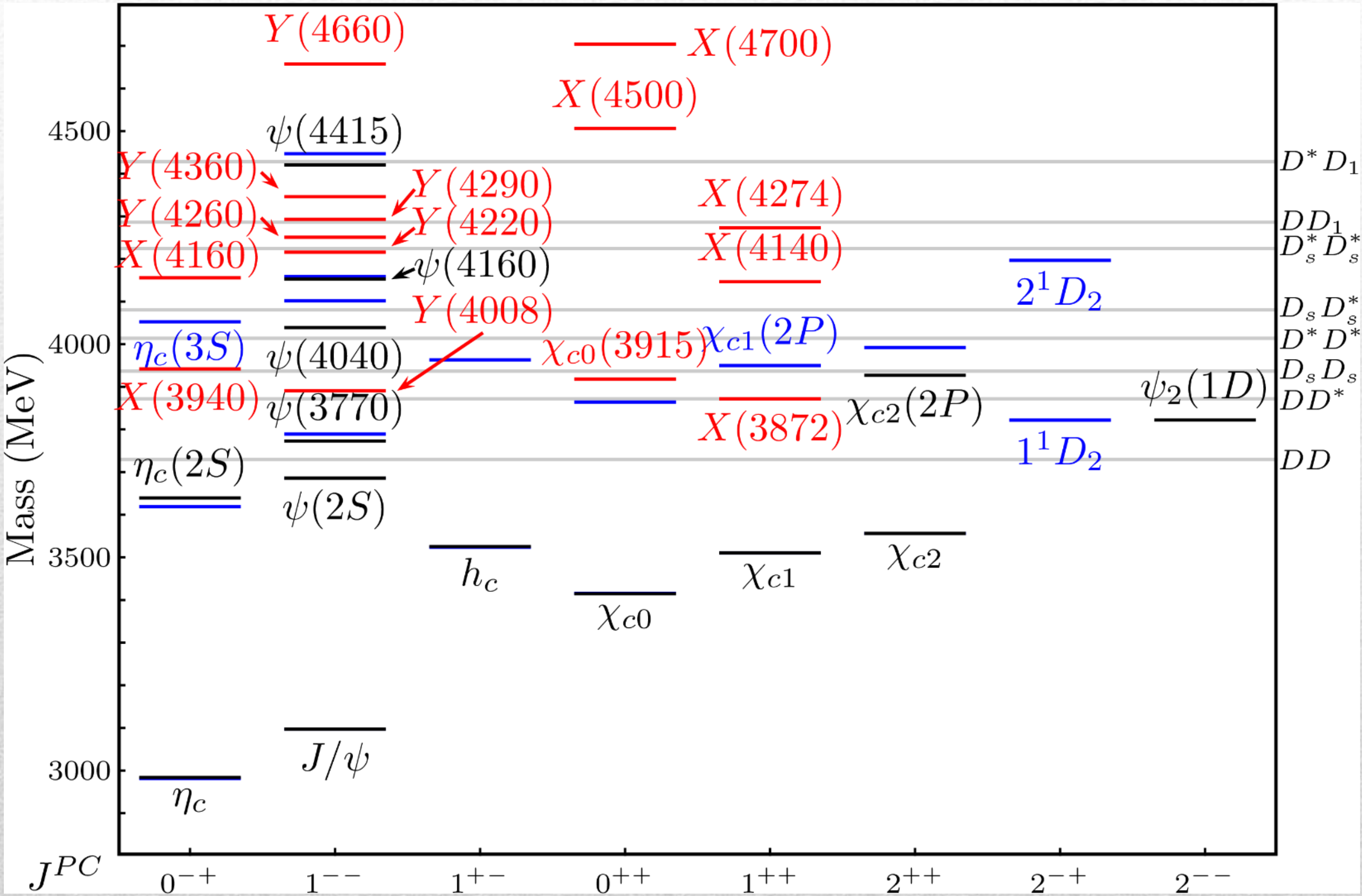
Breit-Wigner



If that is the case, the strength of the signal would dramatically depend on the mass of the exchanges: studying the amplitude at different pion/kaon masses will confirm whether this is true

The heavy sector: XYZ states

Esposito, AP, Polosa, Phys.Rept. 668

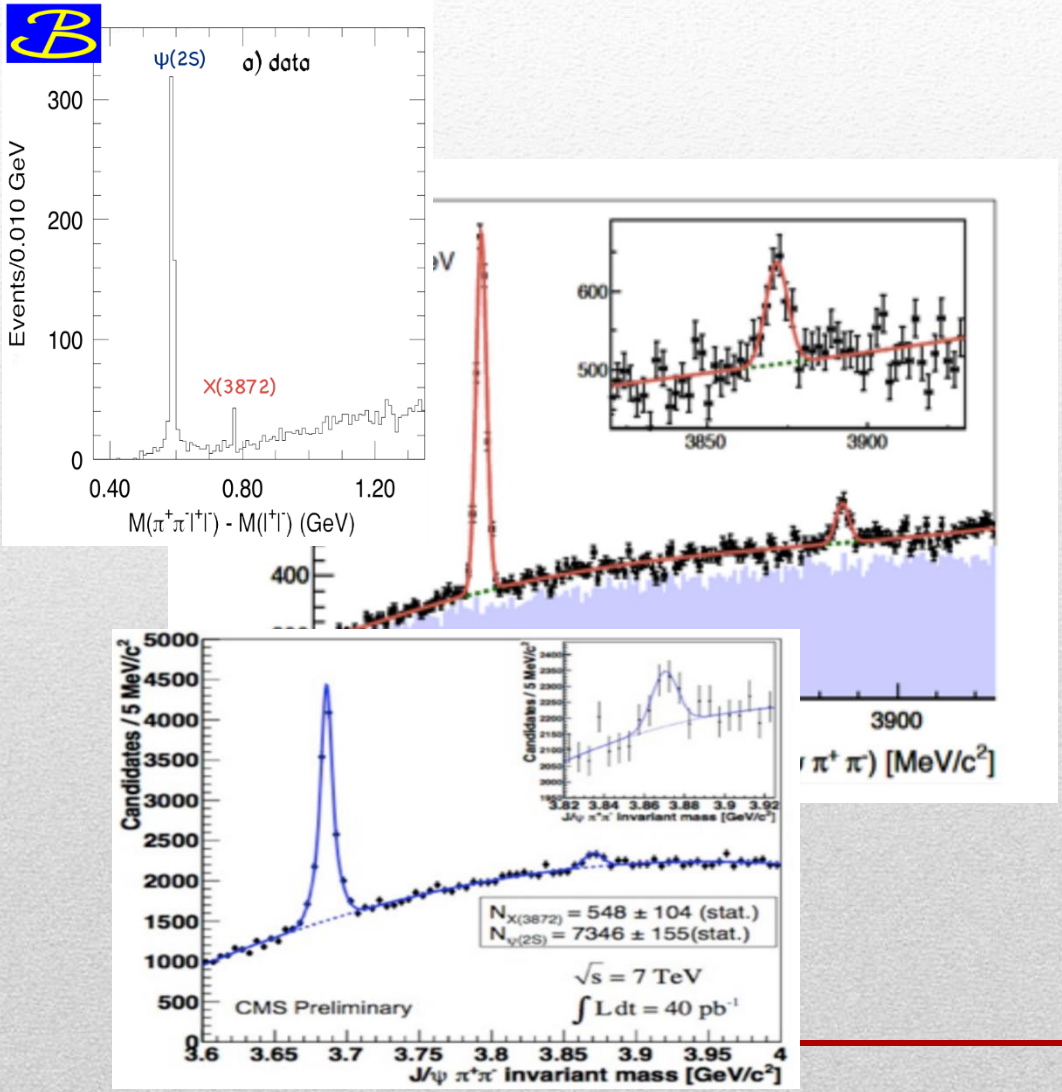


A host of unexpected resonances have appeared

decaying mostly into charmonium + light

Hardly reconciled with usual charmonium interpretation

$X(3872)$



- Discovered in
  $B \to K\, X \to K\, J/\psi\, \pi\pi$
- Quantum numbers 1^{++}
- Very close to DD^* threshold
- Too narrow for an above-treshold charmonium
- Isospin violation too big
  $$\frac{\Gamma(X\to J/\psi\ \ \omega)}{\Gamma(X\to J/\psi\ \ \rho)} \sim 0.8 \pm 0.3$$
- Mass prediction not compatible with $\chi_{c1}(2P)$

$M = 3871.68 \pm 0.17 \text{ MeV}$
$M_X - M_{DD^*} = -3 \pm 192 \text{ keV}$
$\Gamma < 1.2 \text{ MeV @}90\%$

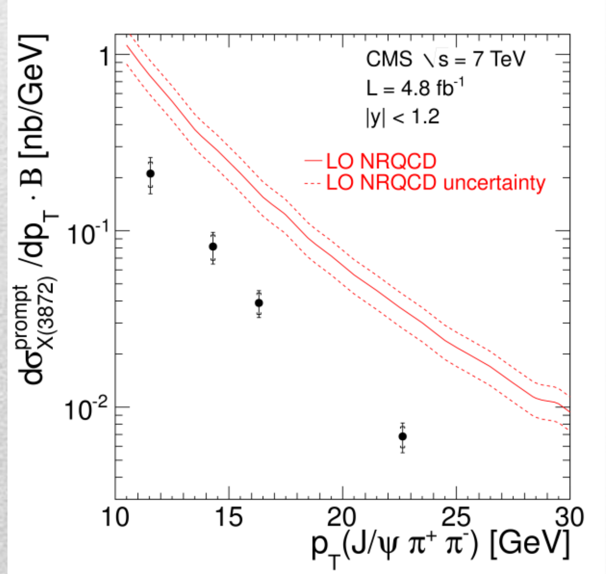
$X(3872)$

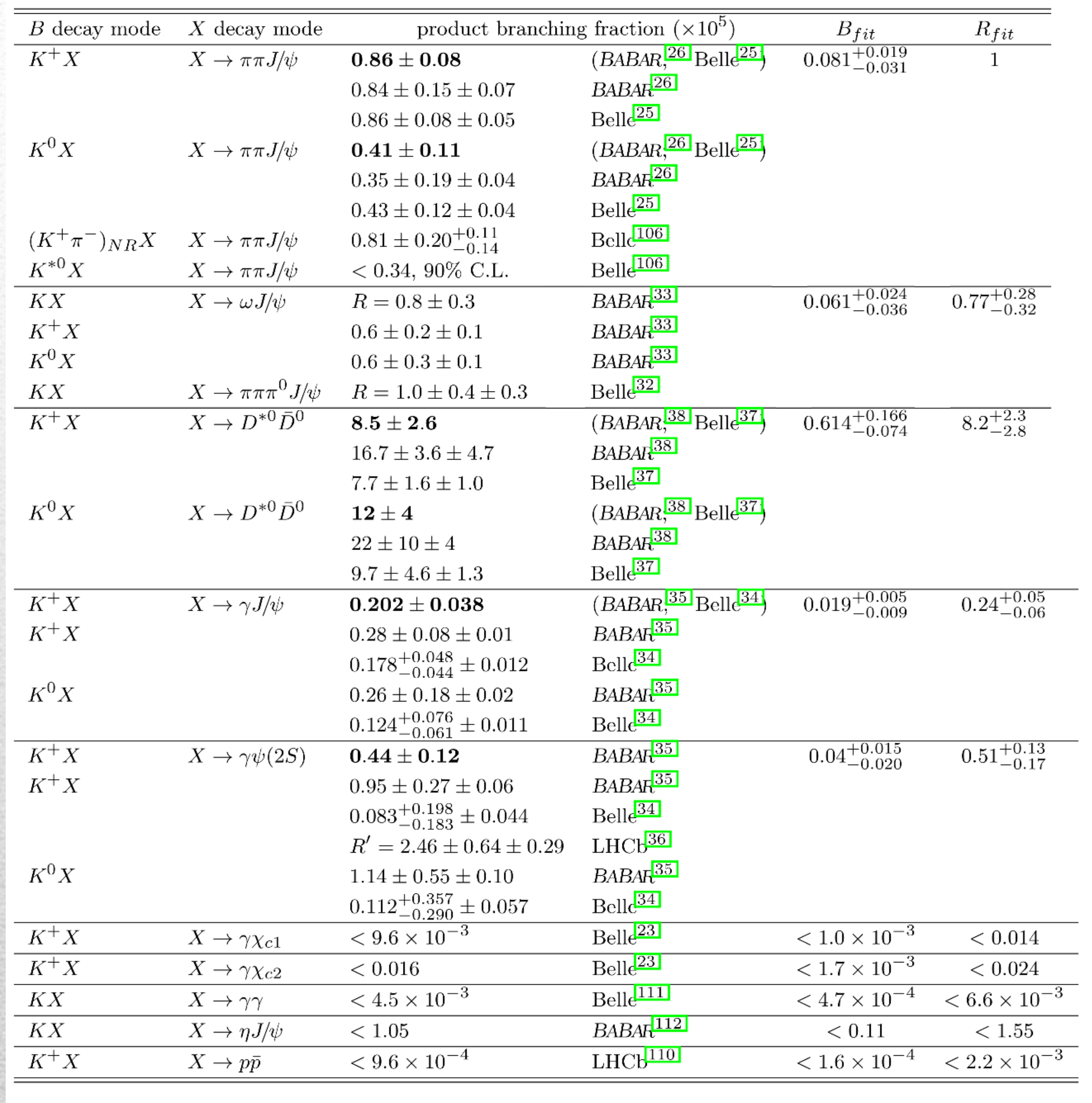
Large prompt production
at hadron colliders

$$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$$

$$\sigma_{PR} \times B(X \to J/\psi\pi\pi) = (1.06 \pm 0.11 \pm 0.15)\ \text{nb}$$

CMS, JHEP 1304, 154

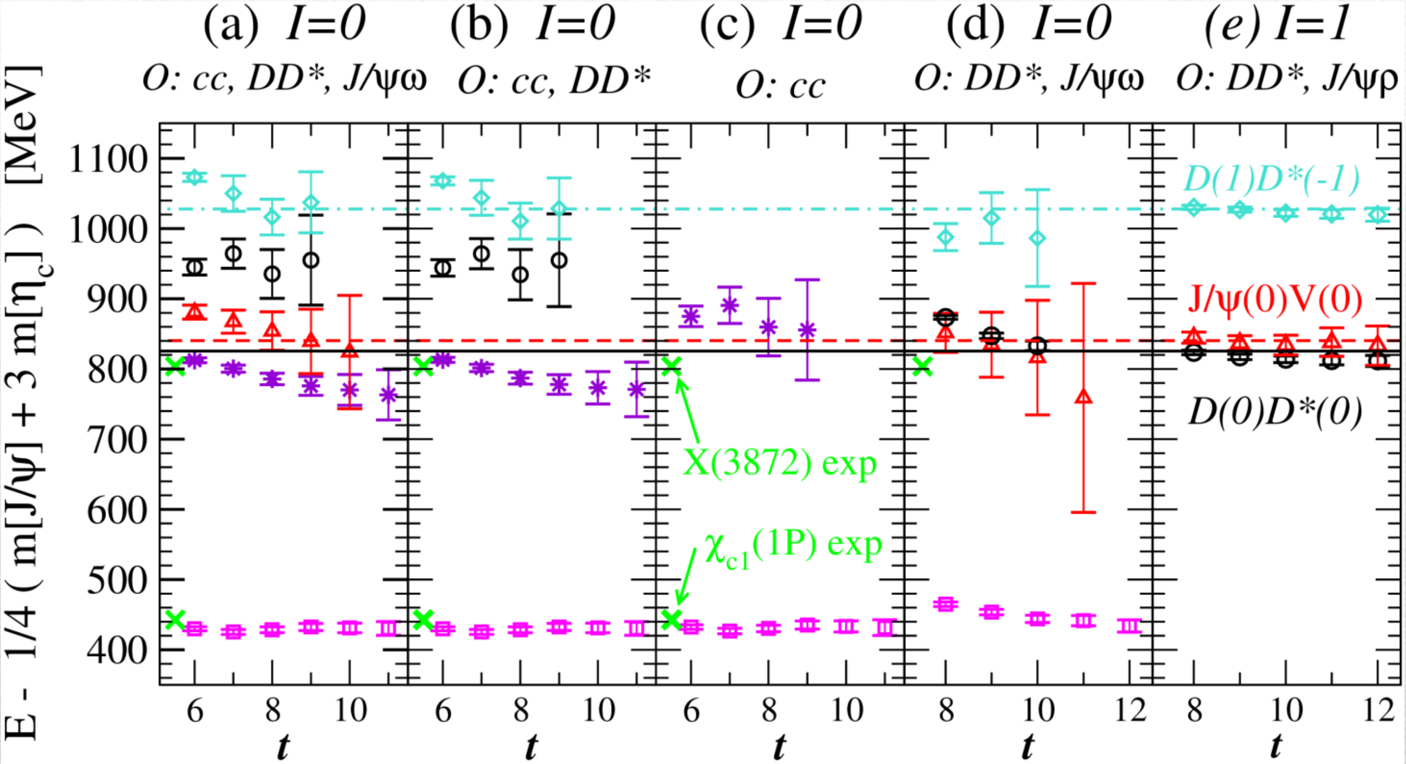




B decay mode	X decay mode	product branching fraction ($\times 10^5$)		B_{fit}	R_{fit}
K^+X	$X \to \pi\pi J/\psi$	$\mathbf{0.86 \pm 0.08}$	(*BABAR*,[26] Belle[25])	$0.081^{+0.019}_{-0.031}$	1
		$0.84 \pm 0.15 \pm 0.07$	*BABAR*[26]		
		$0.86 \pm 0.08 \pm 0.05$	Belle[25]		
K^0X	$X \to \pi\pi J/\psi$	$\mathbf{0.41 \pm 0.11}$	(*BABAR*,[26] Belle[25])		
		$0.35 \pm 0.19 \pm 0.04$	*BABAR*[26]		
		$0.43 \pm 0.12 \pm 0.04$	Belle[25]		
$(K^+\pi^-)_{NR}X$	$X \to \pi\pi J/\psi$	$0.81 \pm 0.20^{+0.11}_{-0.14}$	Belle[106]		
$K^{*0}X$	$X \to \pi\pi J/\psi$	< 0.34, 90% C.L.	Belle[106]		
KX	$X \to \omega J/\psi$	$R = 0.8 \pm 0.3$	*BABAR*[33]	$0.061^{+0.024}_{-0.036}$	$0.77^{+0.28}_{-0.32}$
K^+X		$0.6 \pm 0.2 \pm 0.1$	*BABAR*[33]		
K^0X		$0.6 \pm 0.3 \pm 0.1$	*BABAR*[33]		
KX	$X \to \pi\pi\pi^0 J/\psi$	$R = 1.0 \pm 0.4 \pm 0.3$	Belle[32]		
K^+X	$X \to D^{*0}\bar{D}^0$	$\mathbf{8.5 \pm 2.6}$	(*BABAR*,[38] Belle[37])	$0.614^{+0.166}_{-0.074}$	$8.2^{+2.3}_{-2.8}$
		$16.7 \pm 3.6 \pm 4.7$	*BABAR*[38]		
		$7.7 \pm 1.6 \pm 1.0$	Belle[37]		
K^0X	$X \to D^{*0}\bar{D}^0$	$\mathbf{12 \pm 4}$	(*BABAR*,[38] Belle[37])		
		$22 \pm 10 \pm 4$	*BABAR*[38]		
		$9.7 \pm 4.6 \pm 1.3$	Belle[37]		
K^+X	$X \to \gamma J/\psi$	$\mathbf{0.202 \pm 0.038}$	(*BABAR*,[35] Belle[34])	$0.019^{+0.005}_{-0.009}$	$0.24^{+0.05}_{-0.06}$
K^+X		$0.28 \pm 0.08 \pm 0.01$	*BABAR*[35]		
		$0.178^{+0.048}_{-0.044} \pm 0.012$	Belle[34]		
K^0X		$0.26 \pm 0.18 \pm 0.02$	*BABAR*[35]		
		$0.124^{+0.076}_{-0.061} \pm 0.011$	Belle[34]		
K^+X	$X \to \gamma\psi(2S)$	$\mathbf{0.44 \pm 0.12}$	*BABAR*[35]	$0.04^{+0.015}_{-0.020}$	$0.51^{+0.13}_{-0.17}$
K^+X		$0.95 \pm 0.27 \pm 0.06$	*BABAR*[35]		
		$0.083^{+0.198}_{-0.183} \pm 0.044$	Belle[34]		
		$R' = 2.46 \pm 0.64 \pm 0.29$	LHCb[36]		
K^0X		$1.14 \pm 0.55 \pm 0.10$	*BABAR*[35]		
		$0.112^{+0.357}_{-0.290} \pm 0.057$	Belle[34]		
K^+X	$X \to \gamma\chi_{c1}$	$< 9.6 \times 10^{-3}$	Belle[23]	$< 1.0 \times 10^{-3}$	< 0.014
K^+X	$X \to \gamma\chi_{c2}$	< 0.016	Belle[23]	$< 1.7 \times 10^{-3}$	< 0.024
KX	$X \to \gamma\gamma$	$< 4.5 \times 10^{-3}$	Belle[111]	$< 4.7 \times 10^{-4}$	$< 6.6 \times 10^{-3}$
KX	$X \to \eta J/\psi$	< 1.05	*BABAR*[112]	< 0.11	< 1.55
K^+X	$X \to p\bar{p}$	$< 9.6 \times 10^{-4}$	LHCb[110]	$< 1.6 \times 10^{-4}$	$< 2.2 \times 10^{-3}$

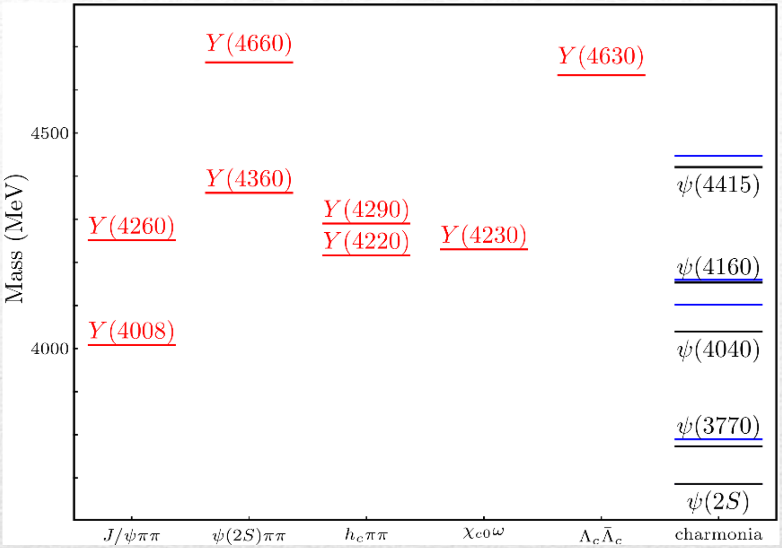
$X(3872)$ on the lattice

Prelovsek, Leskovec, PRL111, 192001



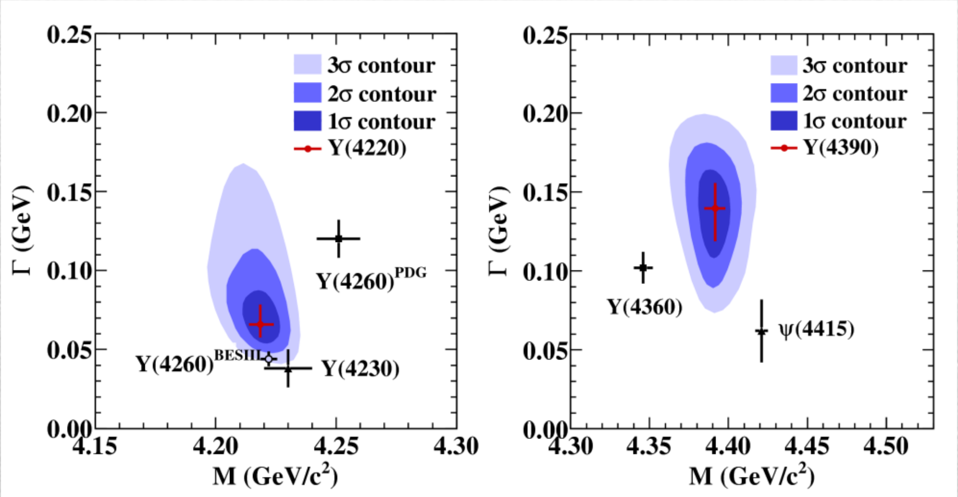
- Three body dynamics $D\bar{D}\pi$ may play a role. Playing with lighter charm mass?
- A full amplitude analysis is missing, and is now mandatory

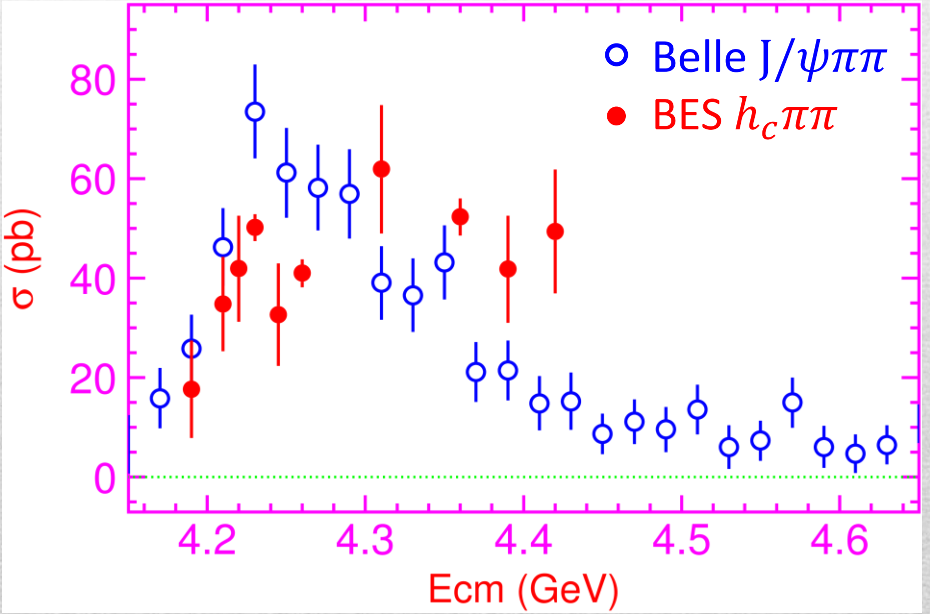
Vector Y states



Lots of unexpected $J^{PC} = 1^{--}$ states
found in ISR/direct production (and nowhere else!)
Seen in few final states,
mostly $J/\psi\,\pi\pi$ and $\psi(2S)\,\pi\pi$

Not seen decaying into open charm pairs
Large HQSS violation

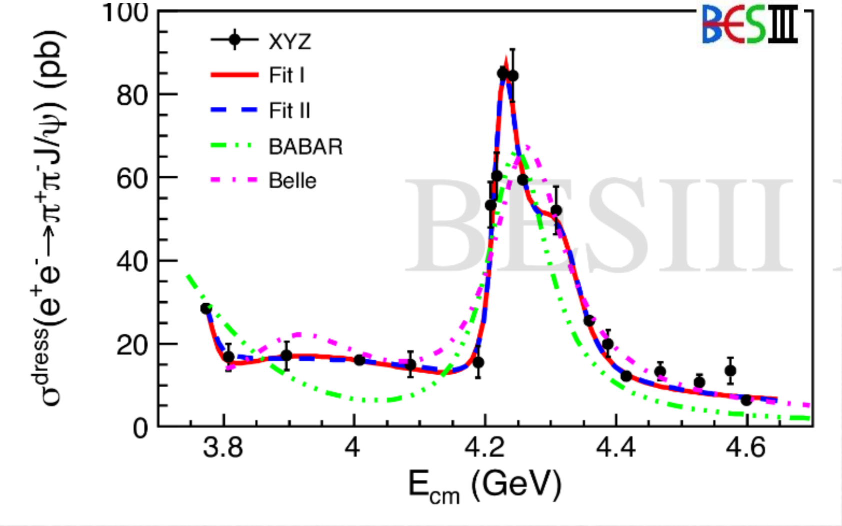




$Y(4260)$

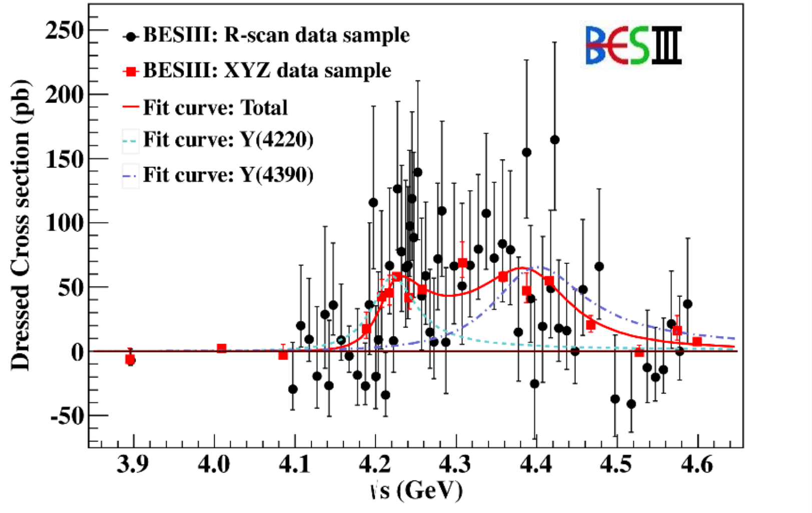
BESIII, PRL118, 092001 (2017)

$e^+e^- \to J/\psi\,\pi\pi$



BESIII, PRL118, 092002 (2017)

$e^+e^- \to h_c\,\pi\pi$

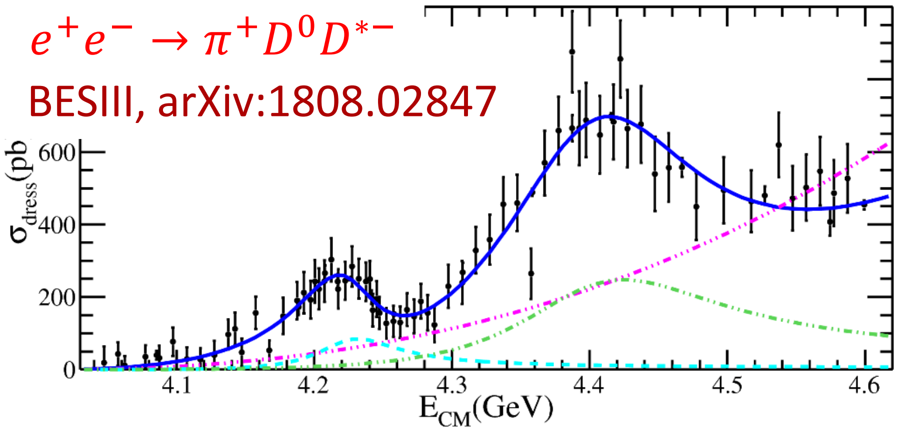


New BESIII data show a peculiar lineshape for the $Y(4260)$, and suggest a state narrower and lighter than in the past

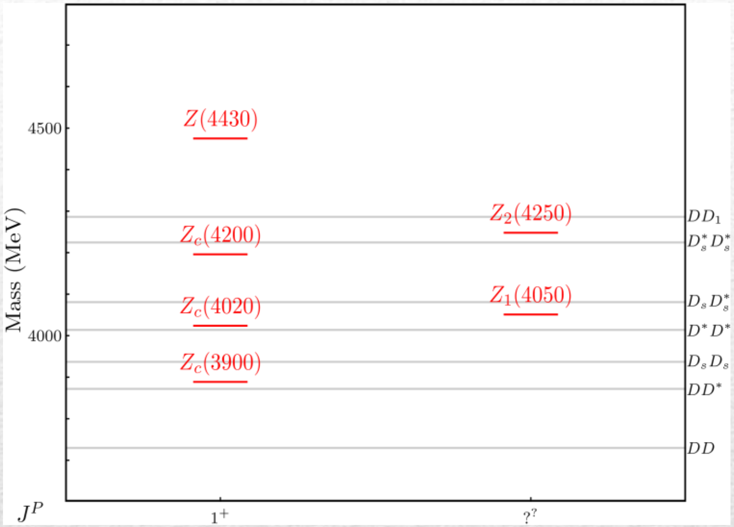
The state is mature for a coupled channel analysis (on the lattice?)

$e^+e^- \to \pi^+ D^0 D^{*-}$

BESIII, arXiv:1808.02847

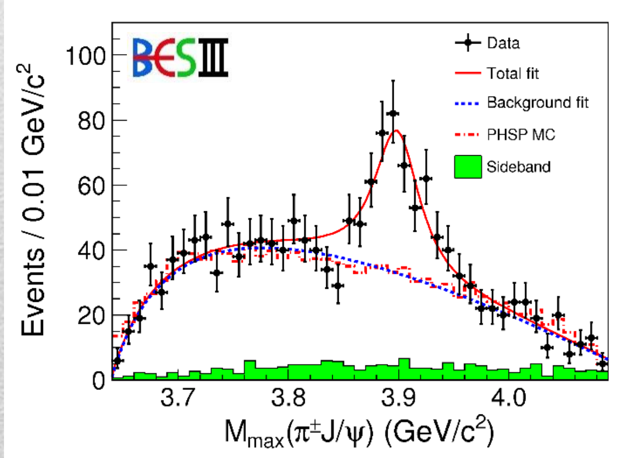


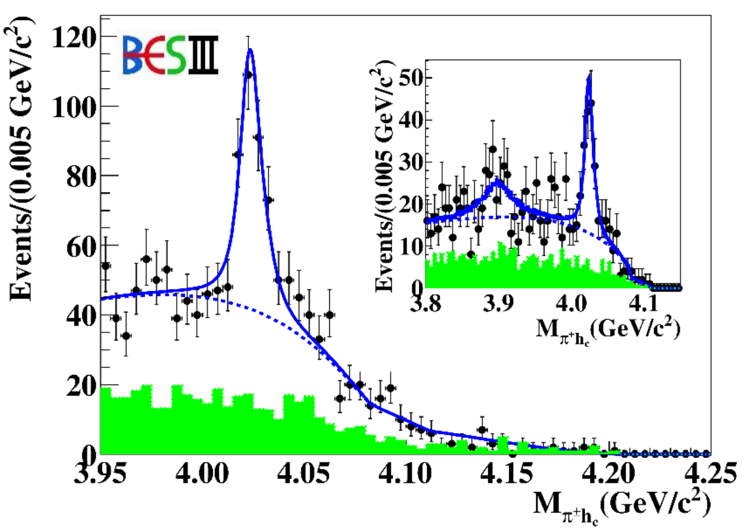
Charged Z states: $Z_c(3900), Z_c'(4020)$



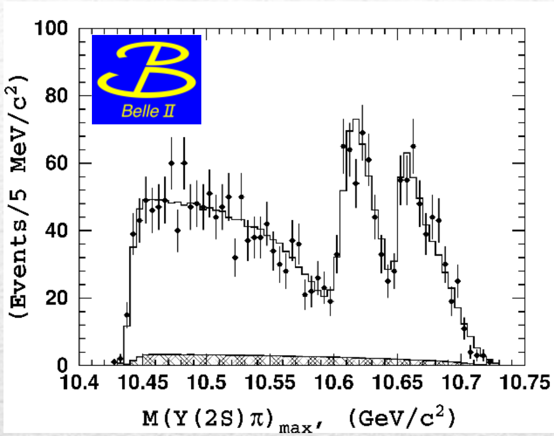
In the Dalitz plot projections, two states appear slightly above $D^{(*)}D^*$ thresholds

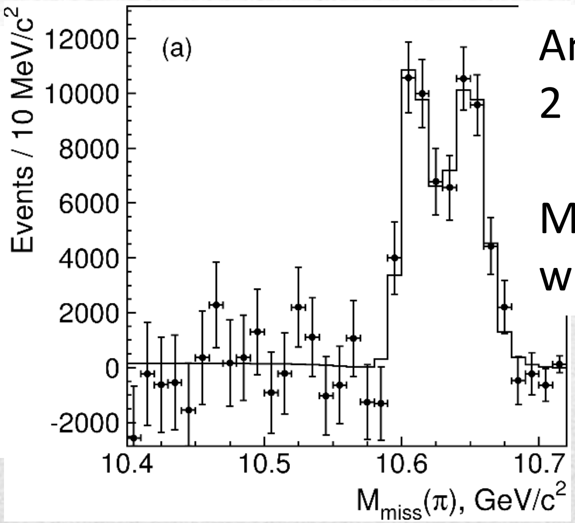
$$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi\,\pi^+\pi^- \text{ and} \to (DD^*)^+\pi^-$$
$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$
$$e^+e^- \to Z_c'(4020)^+\pi^- \to h_c\,\pi^+\pi^- \text{ and} \to \bar{D}^{*0}D^{*+}\pi^-$$
$$M = 4023.9 \pm 2.4 \text{ MeV}, \Gamma = 10 \pm 6 \text{ MeV}$$





Charged Z states: $Z_b(10610), Z_b'(10650)$





Anomalous dipion width in $\Upsilon(5S)$,
2 orders of magnitude larger than $\Upsilon(nS)$

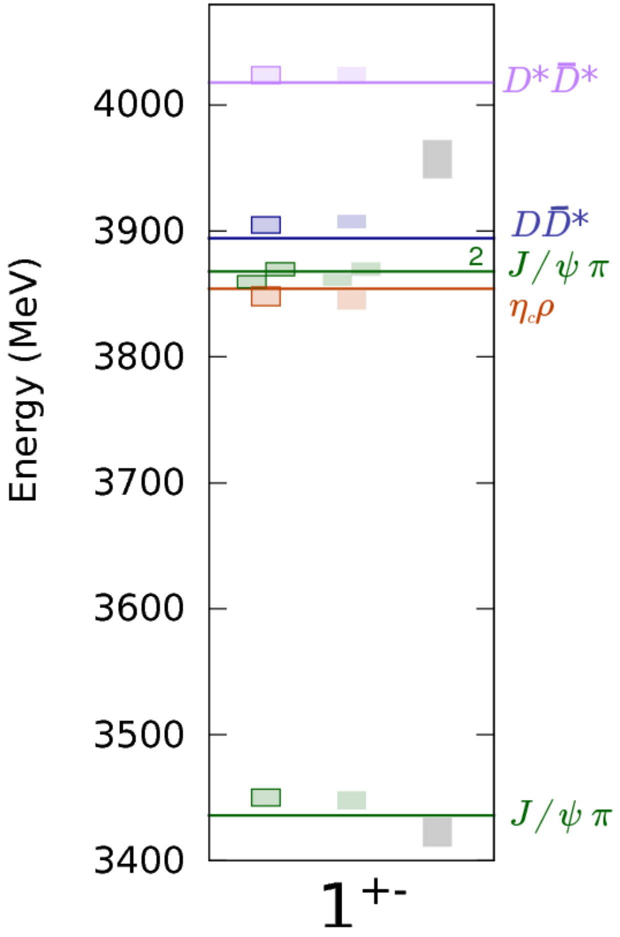
Moreover, observed $\Upsilon(5S) \to h_b(nP)\pi\pi$
which violates HQSS

2 twin peaks

$$\Upsilon(5S) \to Z_b(10610)^+\pi^- \to \Upsilon(nS)\,\pi^+\pi^-, h_b(nP)\,\pi^+\pi^-$$
$$\text{and} \to (BB^*)^+\pi^-$$
$$M = 10607.2 \pm 2.0 \text{ MeV}, \Gamma = 18.4 \pm 2.4 \text{ MeV}$$

$$\Upsilon(5S) \to Z_b'(10650)^+\pi^- \to \Upsilon(nS)\,\pi^+\pi^-, h_b(nP)\,\pi^+\pi^-$$
$$\text{and} \to \bar{B}^{*0}B^{*+}\pi^-$$
$$M = 10652.2 \pm 1.5 \text{ MeV}, \Gamma = 11.5 \pm 2.2 \text{ MeV}$$

Z_cs on the lattice



- The number of energy levels we find is equal to the number of expected non-interacting meson-mesons.
- Finite-volume spectrum lies close to non-interacting meson-meson levels suggesting there are weak meson-meson interactions.
- There is no strong indication for a bound state or narrow resonance in this channel. $Z_C(3900)$?
- Tetraquark operators do not have a significant effect on calculating the spectrum.

G. Cheung

No calculations have found evidence for a resonance

Prelovsek, Leskovec, PLB727, 172-176

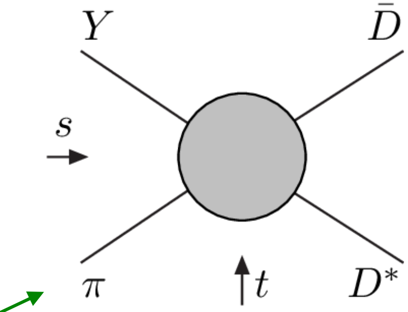
HALQCD, PRL117, 242001

HadSpec, JHEP 1711, 033

Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

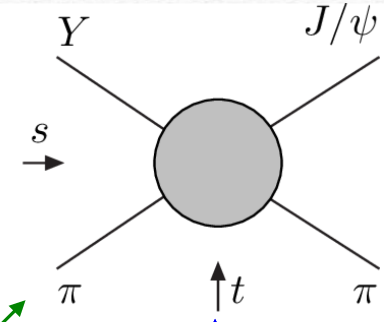
AP *et al.* (JPAC), PLB772, 200



u: $D_0(2400)$

$D_1(2420)$

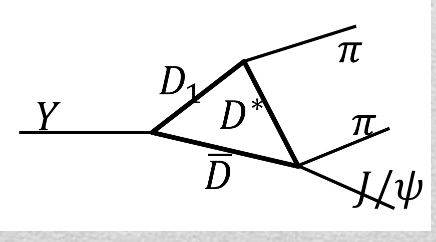
$Z_c(3900)$?



u: $Z_c(3900)$?

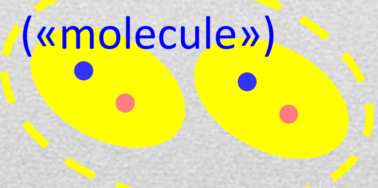
"$\sigma, f_0(980)$"

Triangle rescattering,
logarithmic branching point



Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
(«molecule»)

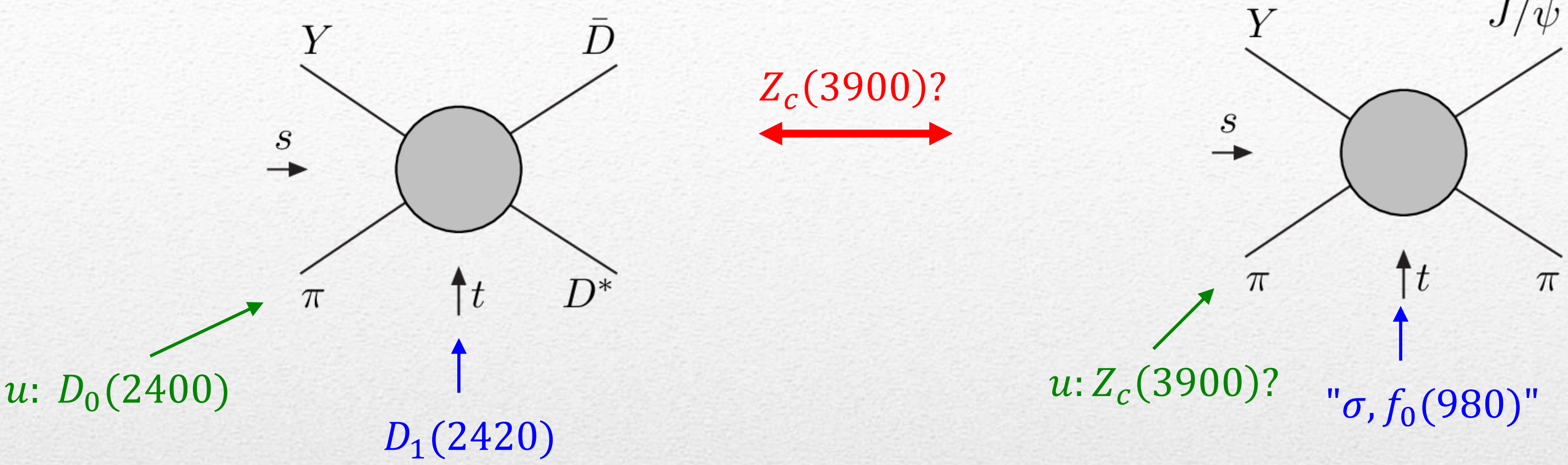


Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)

Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Amplitude model



$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1) \left(a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right)$$

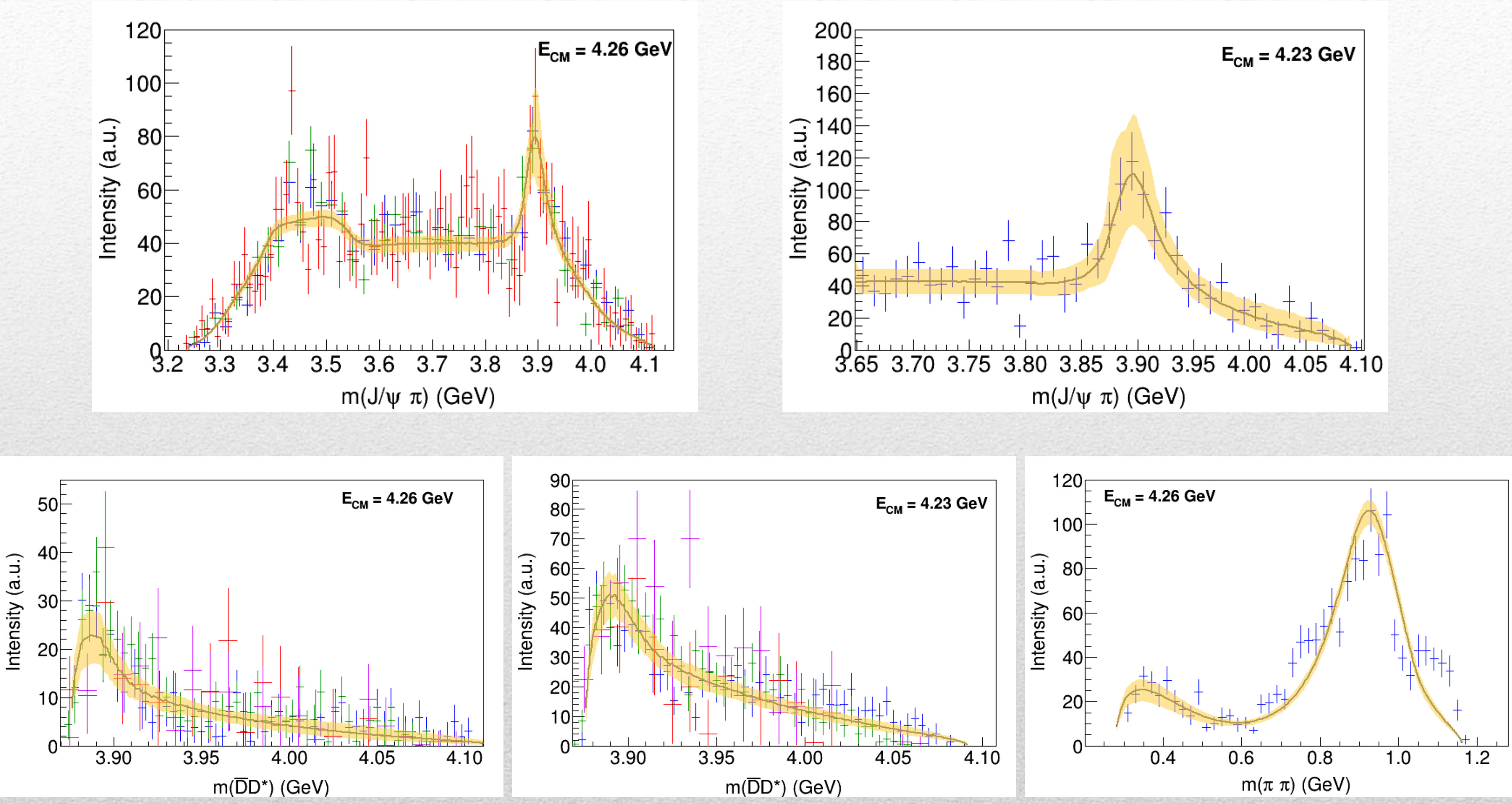
Khuri-Treiman

$$f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

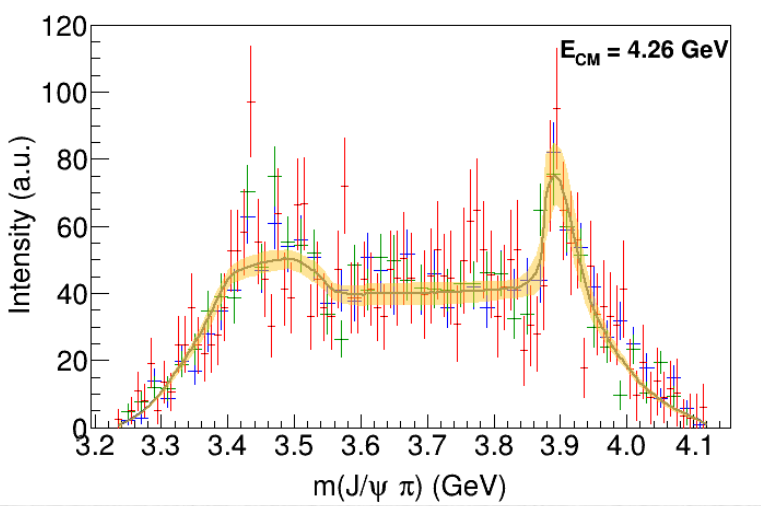
$$f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s\, P_l(z_s) \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s},$$

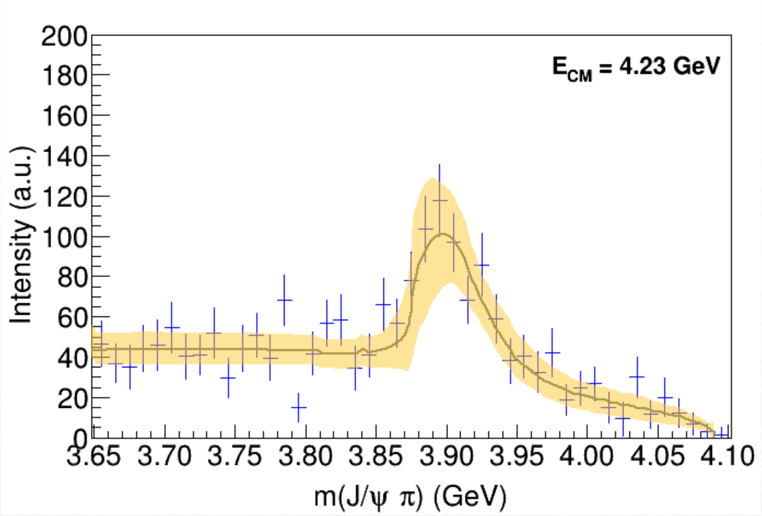
$$\boxed{f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s' - s)} \right) \right],}$$

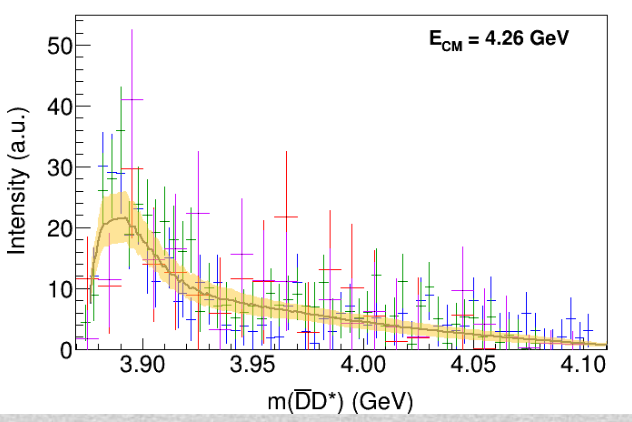
Fit: III

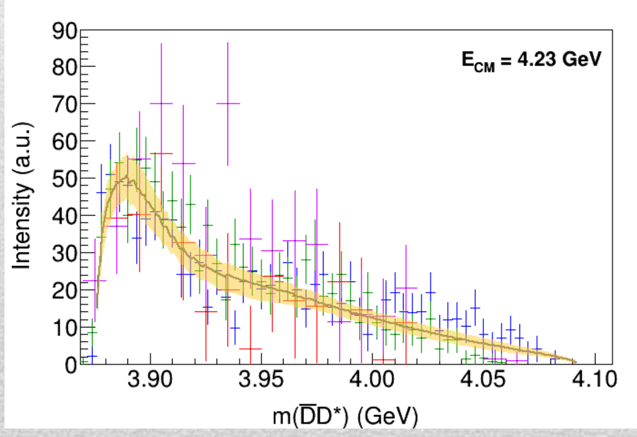

E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
120
100
80
60
40
20
0
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0
4.1
E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
200
180
160
140
120
100
80
60
40
20
0
3.65
3.70
3.75
3.80
3.85
3.90
3.95
4.00
4.05
4.10
E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m($\bar{D}$D*) (GeV)
50
40
30
20
10
0
3.90
3.95
4.00
4.05
4.10
E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
m($\bar{D}$D*) (GeV)
90
80
70
60
50
40
30
20
10
0
3.90
3.95
4.00
4.05
4.10
E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)
120
100
80
60
40
20
0
0.4
0.6
0.8
1.0
1.2

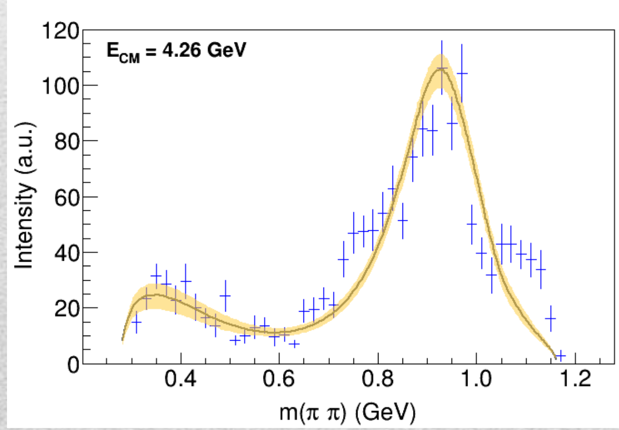
Fit: III+tr.


E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
0
20
40
60
80
100
120
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0
4.1
m(J/ψ π) (GeV)

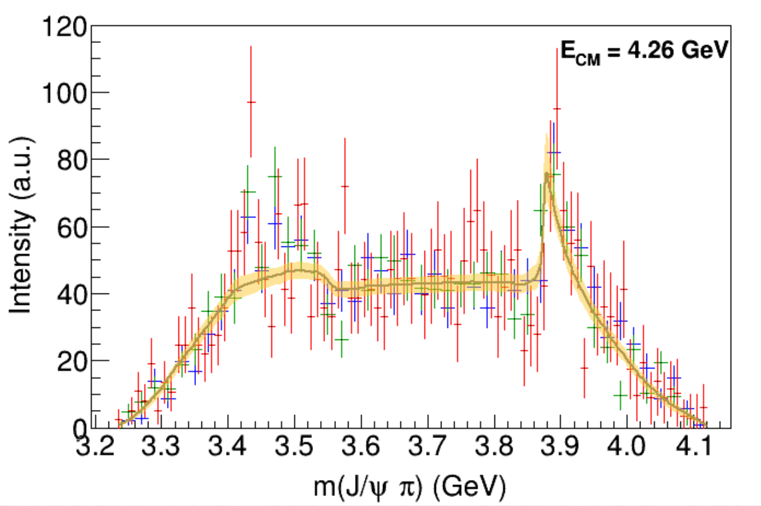

E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
0
20
40
60
80
100
120
140
160
180
200
3.65
3.70
3.75
3.80
3.85
3.90
3.95
4.00
4.05
4.10
m(J/ψ π) (GeV)

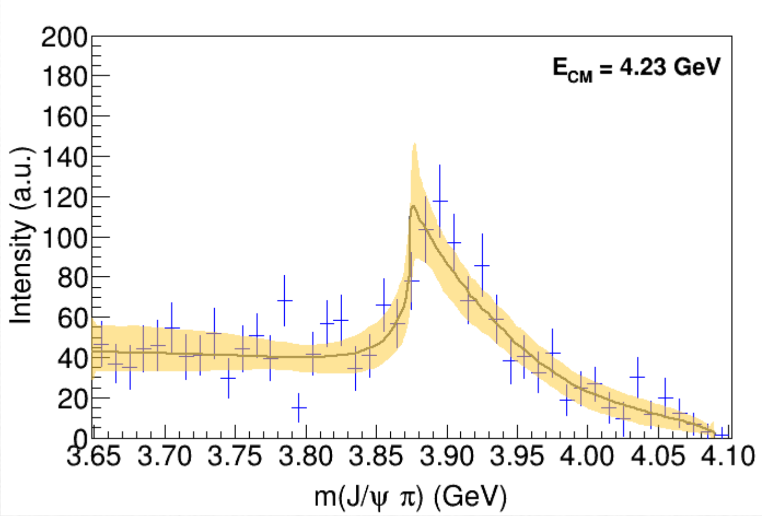

E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
0
10
20
30
40
50
3.90
3.95
4.00
4.05
4.10
m($\bar{D}$D*) (GeV)


E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
0
10
20
30
40
50
60
70
80
90
3.90
3.95
4.00
4.05
4.10
m($\bar{D}$D*) (GeV)


E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
0
20
40
60
80
100
120
0.4
0.6
0.8
1.0
1.2
m(π π) (GeV)

Fit: IV+tr.


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
0
20
40
60
80
100
120
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0
4.1


E_{CM} = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
0
20
40
60
80
100
120
140
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3.70
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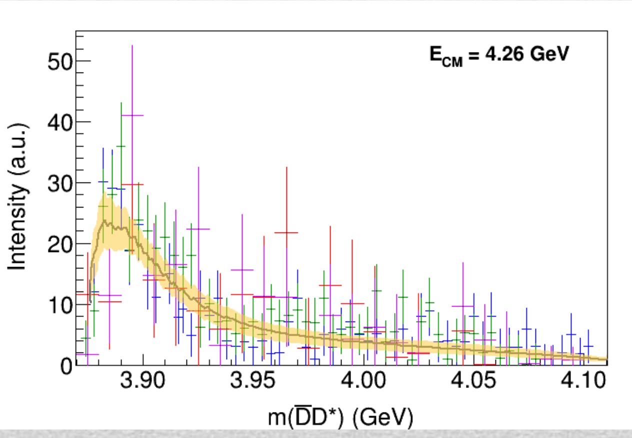
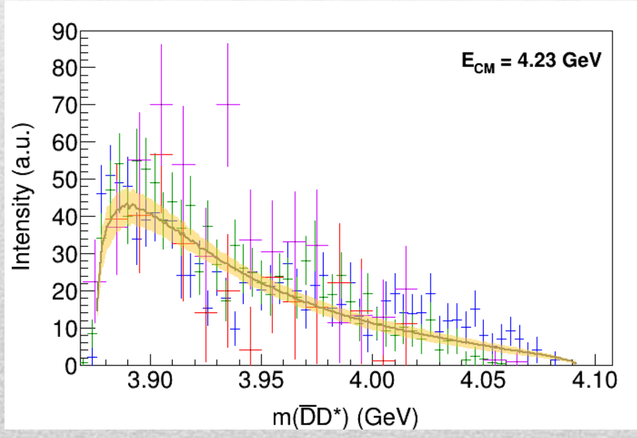
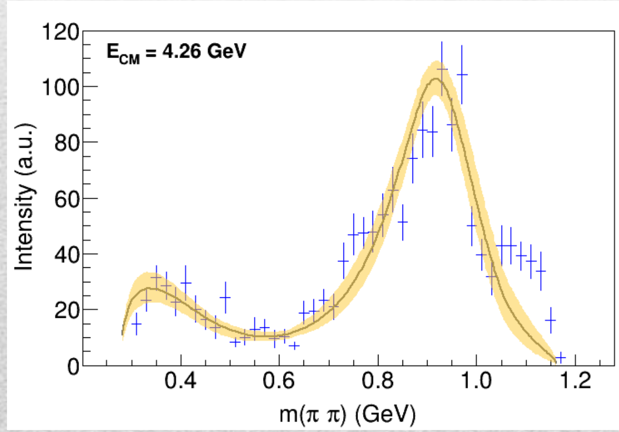
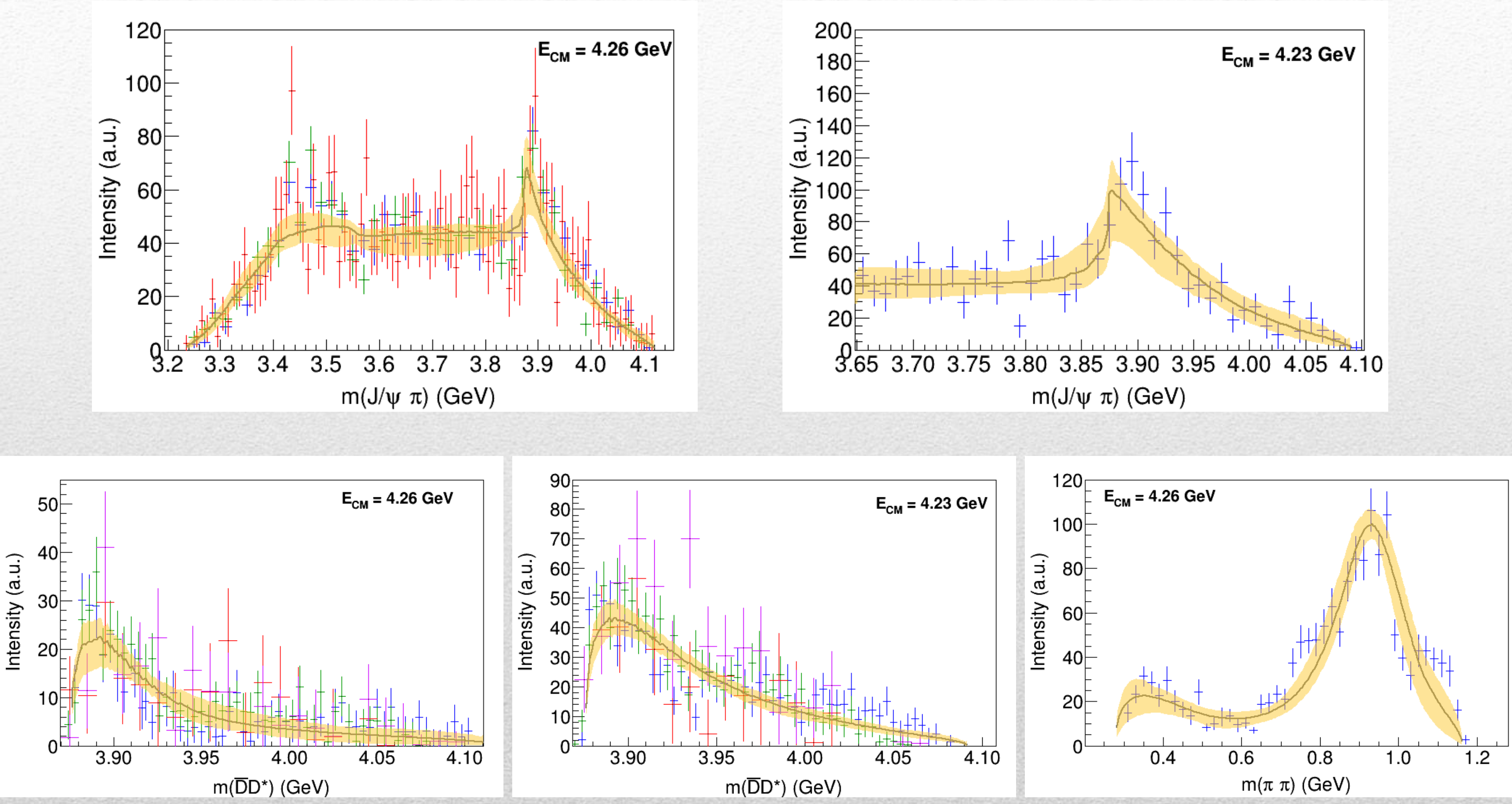
3.90
3.95
4.00
4.05
4.10


E_{CM} = 4.26 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)
0
10
20
30
40
50
3.90
3.95
4.00
4.05
4.10

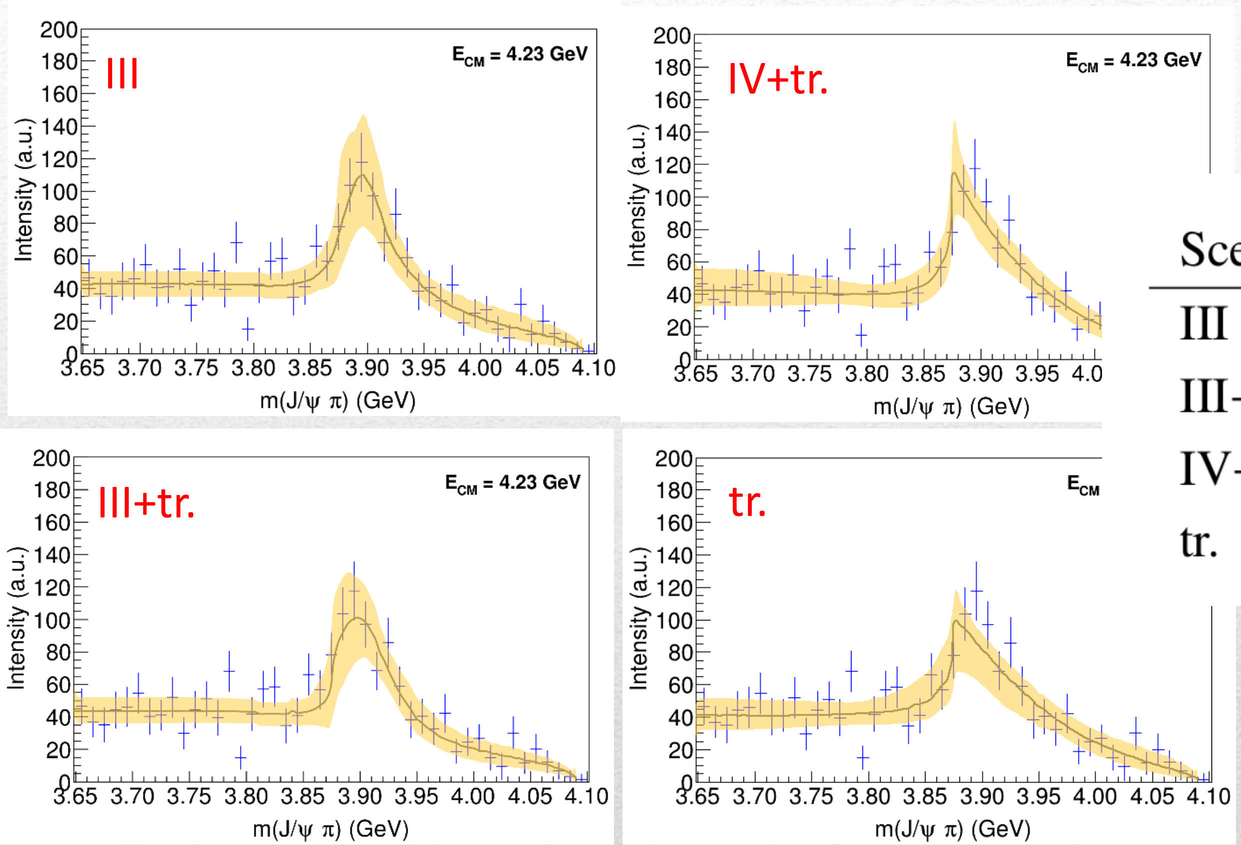

E_{CM} = 4.23 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)
0
10
20
30
40
50
60
70
80
90
3.90
3.95
4.00
4.05
4.10


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)
0
20
40
60
80
100
120
0.4
0.6
0.8
1.0
1.2

Fit: tr.


E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
0
20
40
60
80
100
120
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
4.0
4.1
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
0
20
40
60
80
100
120
140
160
180
200
3.65
3.70
3.75
3.80
3.85
3.90
3.95
4.00
4.05
4.10
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
0
10
20
30
40
50
3.90
3.95
4.00
4.05
4.10
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
0
10
20
30
40
50
60
70
80
90
3.90
3.95
4.00
4.05
4.10
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)
0
20
40
60
80
100
120
0.4
0.6
0.8
1.0
1.2

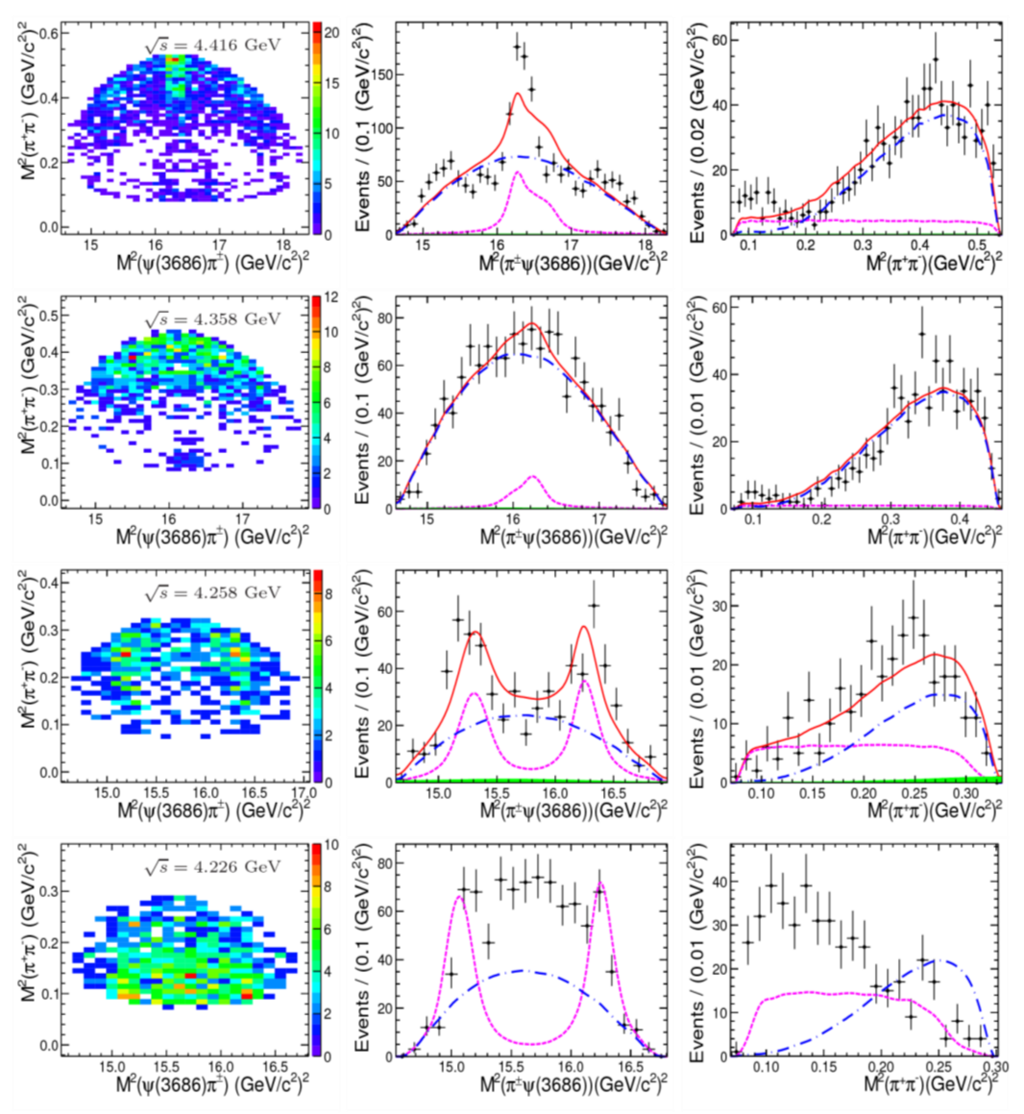
Fit summary



Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

Data can hardly distinguish these scenarios.
Lattice QCD can actually provide the scattering matrix as an input to this analysis

More complicated Dalitz plots

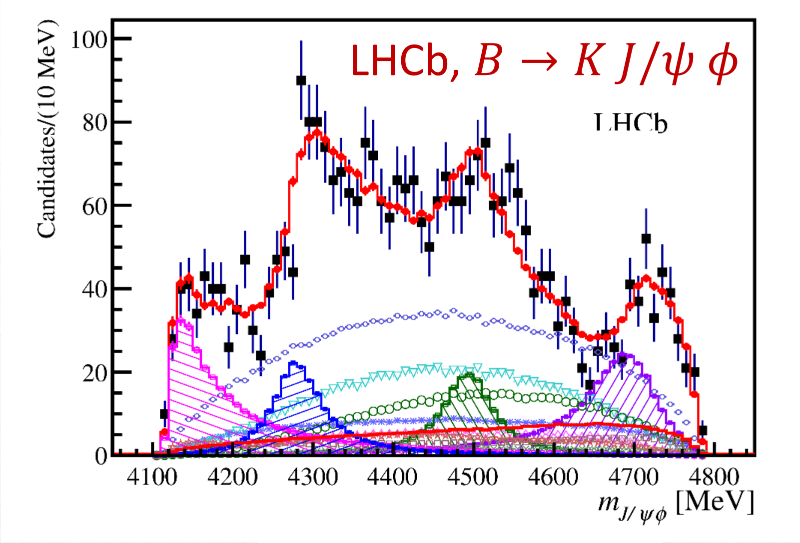


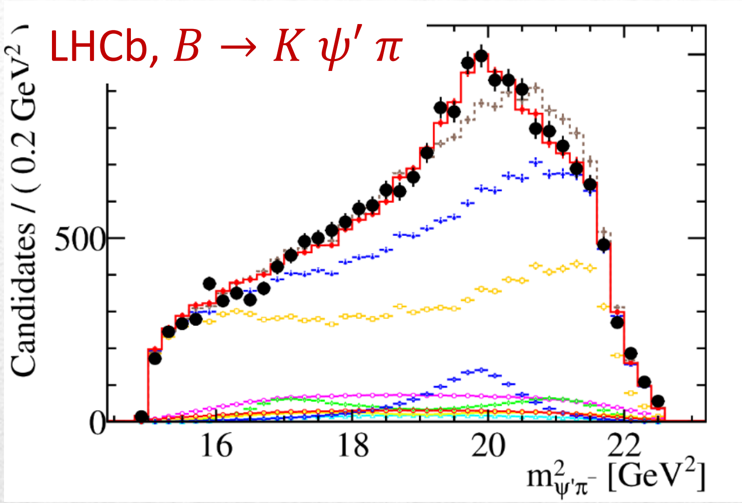
BESIII, PRD96, 032004

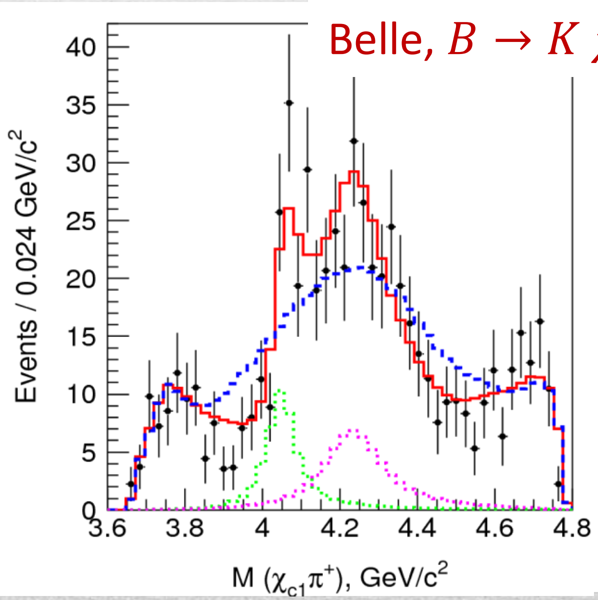
In the reaction $e^+e^- \to \psi'\pi^+\pi^-$, the situation looks even more obscure

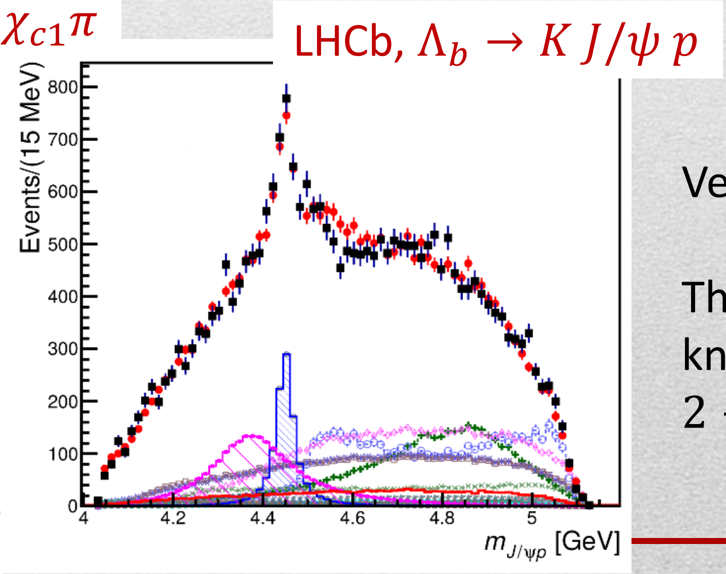
Data refused to be fitted with any simple model

More complicated Dalitz plots


LHCb, $B \to K\, J/\psi\, \phi$


LHCb, $B \to K\, \psi'\, \pi$


Belle, $B \to K\, \chi_{c1}\pi$


LHCb, $\Lambda_b \to K\, J/\psi\, p$

Very complicated Dalitz plots

They can all benefit of the knowledge of the underlying $2 \to 2$ scattering amplitude

Outlook

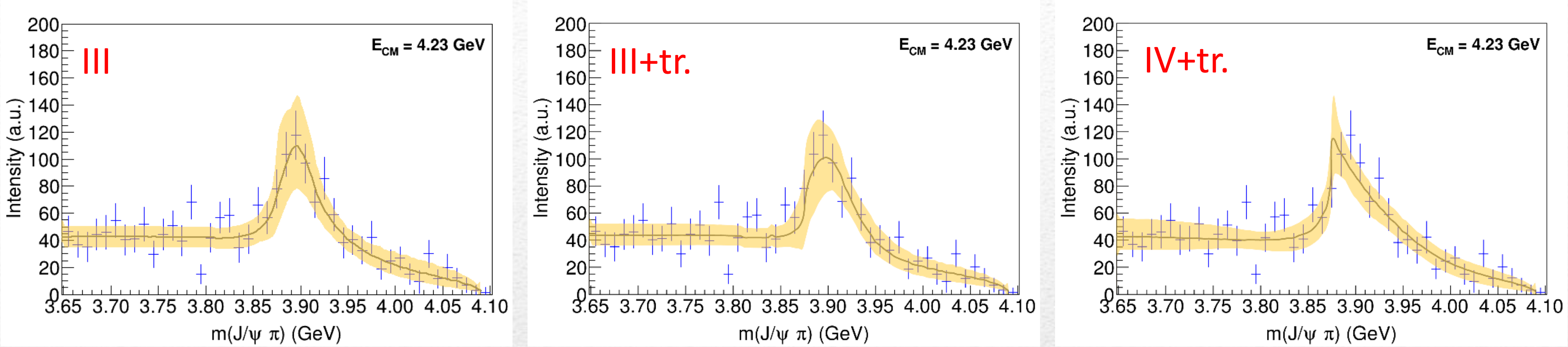
- The light sector: the 3π system
  - The $a_1(1260)$
  - The hybrid π_1
  - The $a_1(1420)$
- The heavy sector: XYZ
  - The $X(3872)$ and the Y states
  - Two-body subchannels: Z_cs and Z_bs
  - Complicated Dalitz plots

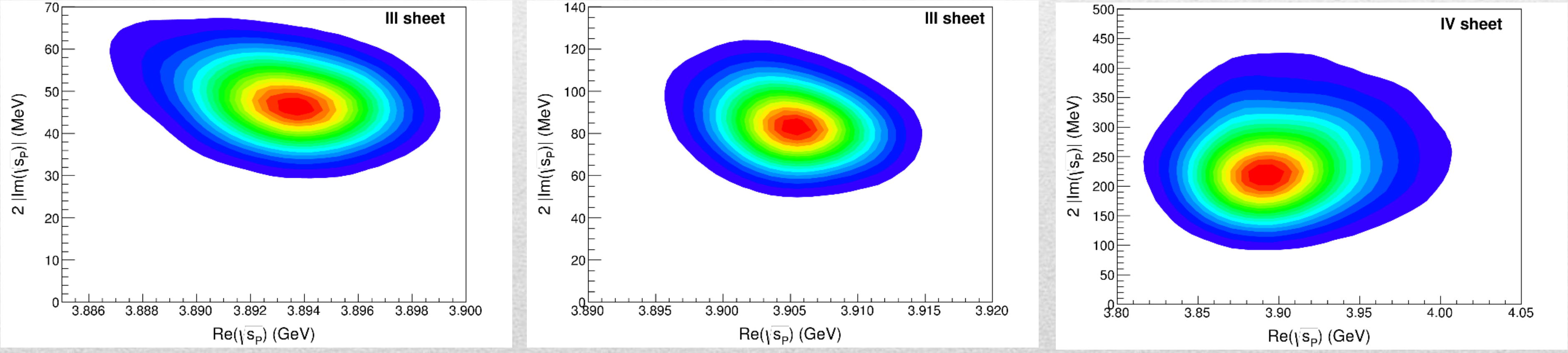
Lattice can disentangle the scattering from the production mechanism
Three body dynamics AND coupled channels

Lattice can provide the $2 \to 2$ scattering amplitude that can be used as input in the phenomenological models

BACKUP

Pole extraction



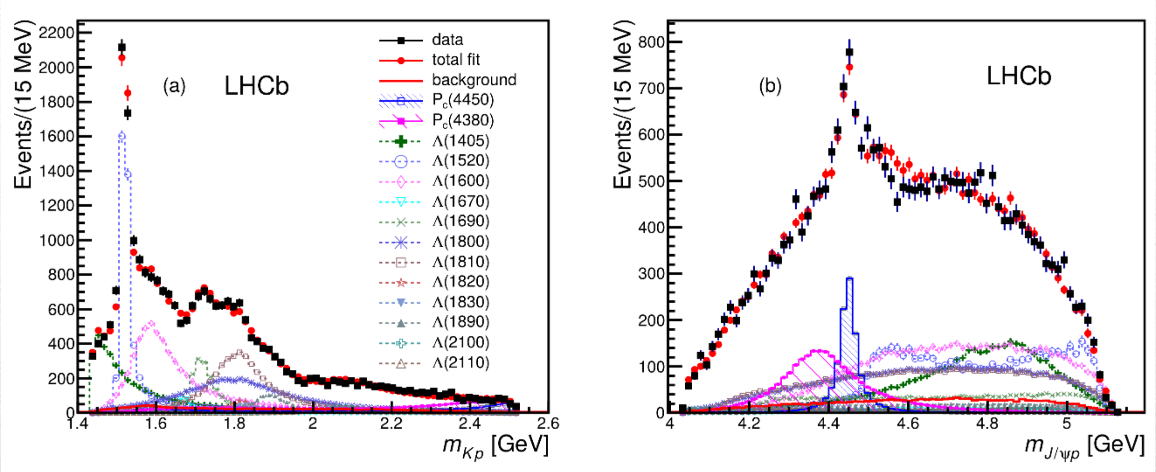


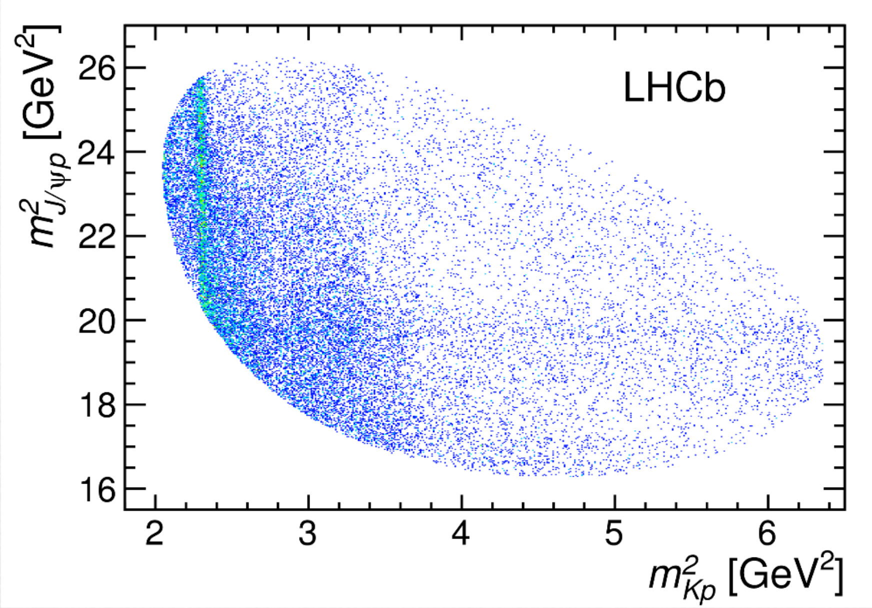
Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

Pentaquarks!





LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

Two states seen in $\Lambda_b \to (J/\psi\, p)\, K^-$, evidence in $\Lambda_b \to (J/\psi\, p)\, \pi^-$

$$M_1 = 4380 \pm 8 \pm 29 \text{ MeV}$$
$$\Gamma_1 = 205 \pm 18 \pm 86 \text{ MeV}$$
$$M_2 = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$$
$$\Gamma_2 = 39 \pm 5 \pm 19 \text{ MeV}$$

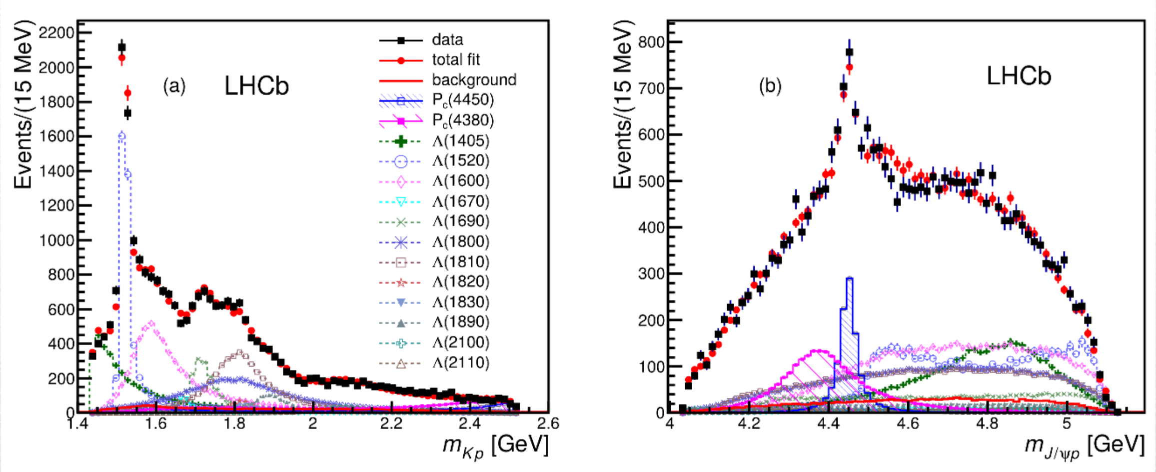
Quantum numbers

$$J^P = \left(\frac{3}{2}^-, \frac{5}{2}^+\right) \text{ or } \left(\frac{3}{2}^+, \frac{5}{2}^-\right) \text{ or } \left(\frac{5}{2}^+, \frac{3}{2}^-\right)$$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by Λ^* (model dependence?)

No obvious threshold nearby

Pentaquarks!



LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

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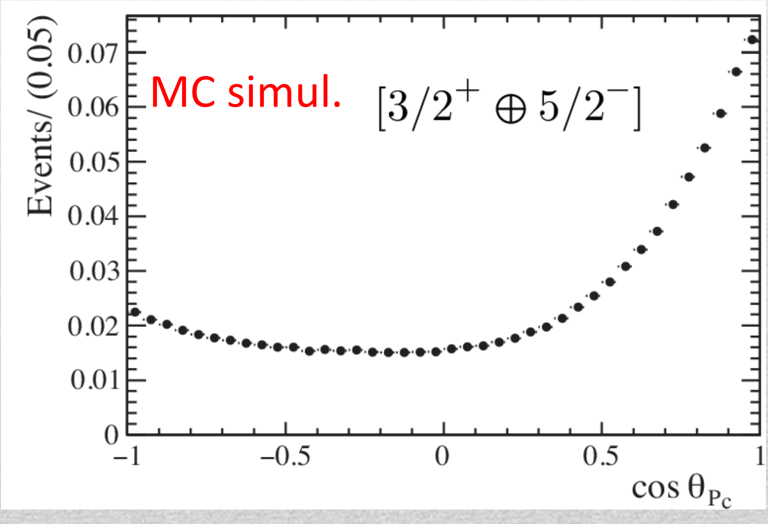

MC simul.

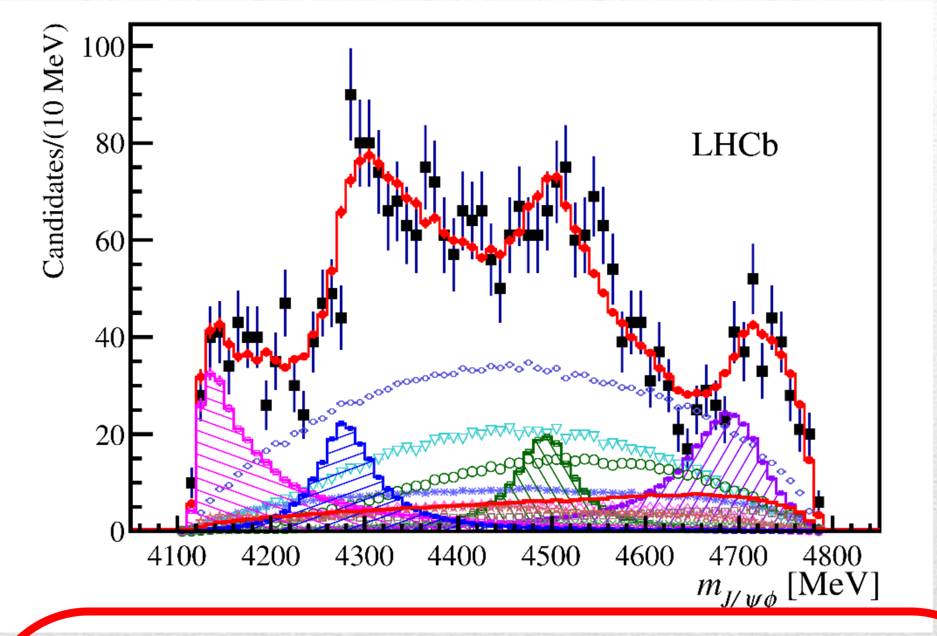
Quantum numbers

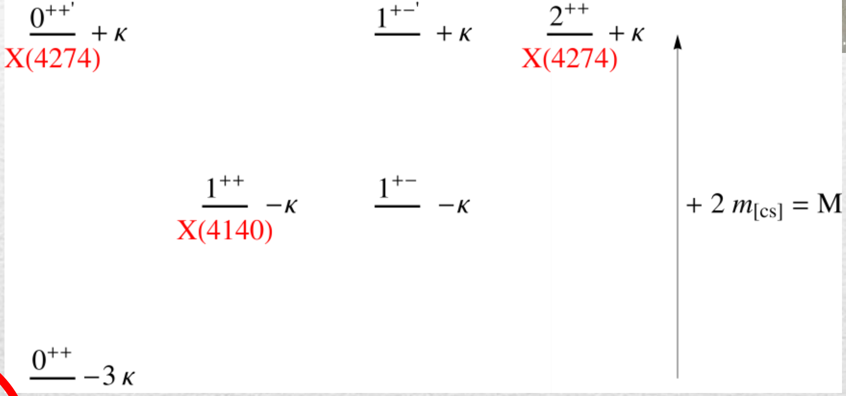
$$J^P = \left(\frac{3}{2}^-, \frac{5}{2}^+\right) \text{ or } \left(\frac{3}{2}^+, \frac{5}{2}^-\right) \text{ or } \left(\frac{5}{2}^+, \frac{3}{2}^-\right)$$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by Λ^* (model dependence?)

No obvious threshold nearby

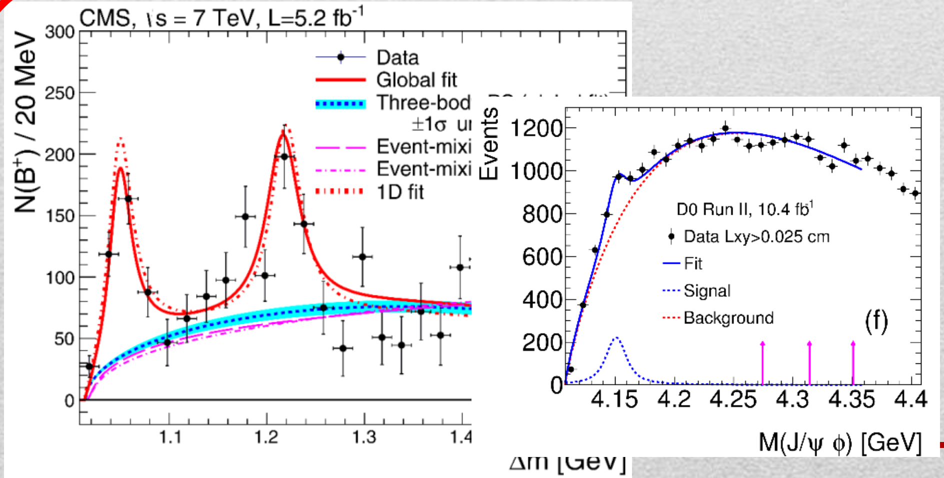
Tetraquark: the $c\bar{c}s\bar{s}$ states





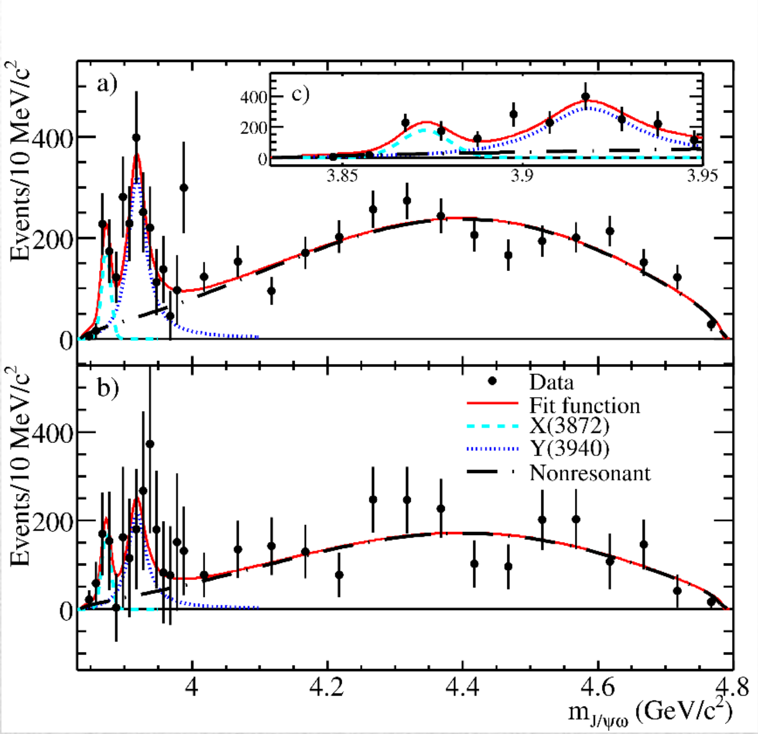
Good description of the spectrum **but** one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with 0^{++} and 2^{++})

Maiani, Polosa and Riquer, PRD 94, 054026



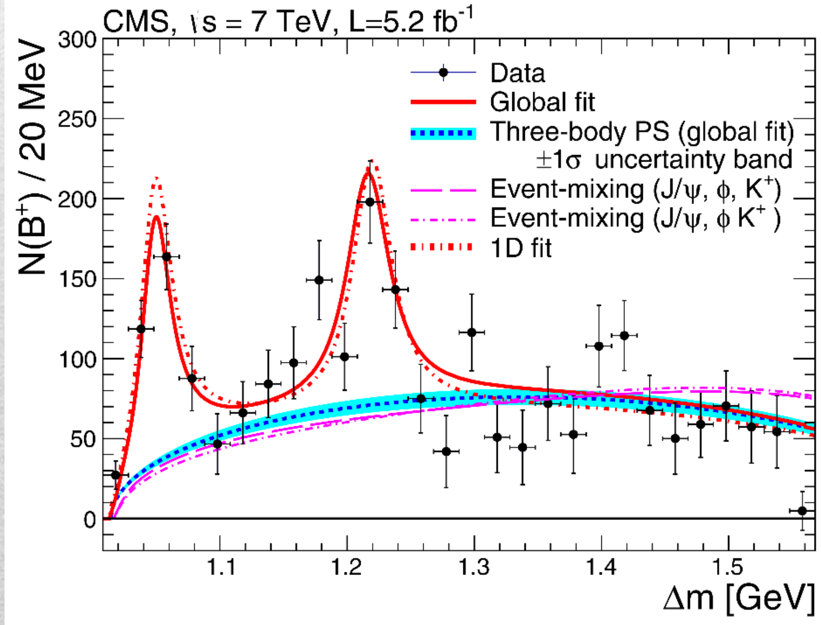
Much narrower than LHCb! Look for prompt!

Other beasts



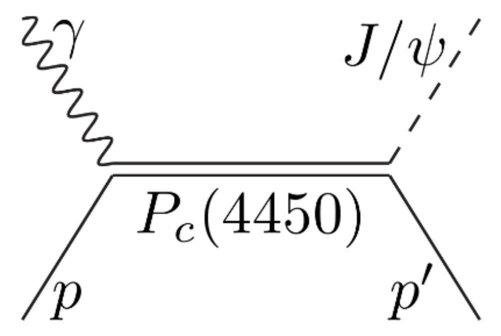
$X(3915)$, seen in $B \to X\,K \to J/\psi\,\omega$
and $\gamma\gamma \to X \to J/\psi\,\omega$
$J^{PC} = 0^{++}$, candidate for $\chi_{c0}(2P)$
But $X(3915) \nrightarrow D\bar{D}$ as expected,
and the hyperfine splitting
$\mathrm{M}(2^{++}) - \mathrm{M}(0^{++})$ too small

One/two peaks seen in $B \to XK \to J/\psi\,\phi\,K$, close to threshold



P_c photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.



Hadronic vertex EM vertex

$$\langle \lambda_\psi \lambda_{p'} | T_r | \lambda_\gamma \lambda_p \rangle = \frac{\boxed{\langle \lambda_\psi \lambda_{p'} | T_\text{dec} | \lambda_R \rangle} \boxed{\langle \lambda_R | T^\dagger_\text{em} | \lambda_\gamma \lambda_p \rangle}}{M_r^2 - W^2 - i\Gamma_r M_r}$$

Hadronic part

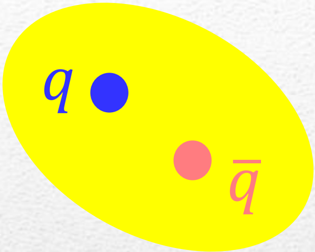
- 3 independent helicity couplings,
  → approx. equal, $g_{\lambda_\psi, \lambda_{p'}} \sim g$
- g extracted from total width and (unknown) branching ratio

Vector meson dominance relates the radiative width to the hadronic width

$$\Gamma_\gamma = 4\pi\alpha\, \Gamma_{\psi p} \left(\frac{f_\psi}{M_\psi} \right)^2 \left(\frac{\bar{p}_i}{\bar{p}_f} \right)^{2\ell+1} \times \frac{4}{6}$$

Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Dictionary – Quark model

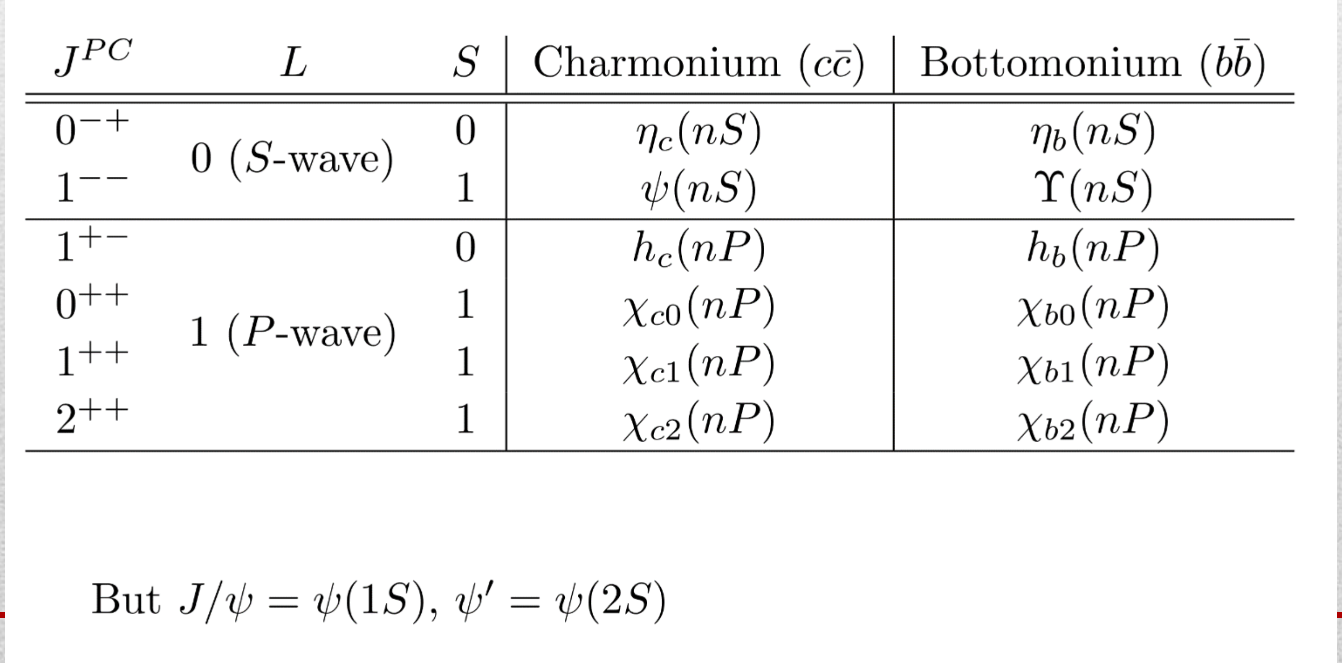


L = orbital angular momentum
S = spin $q + \bar{q}$

J = total angular momentum
= exp. measured spin

I = isospin = 0 for quarkonia

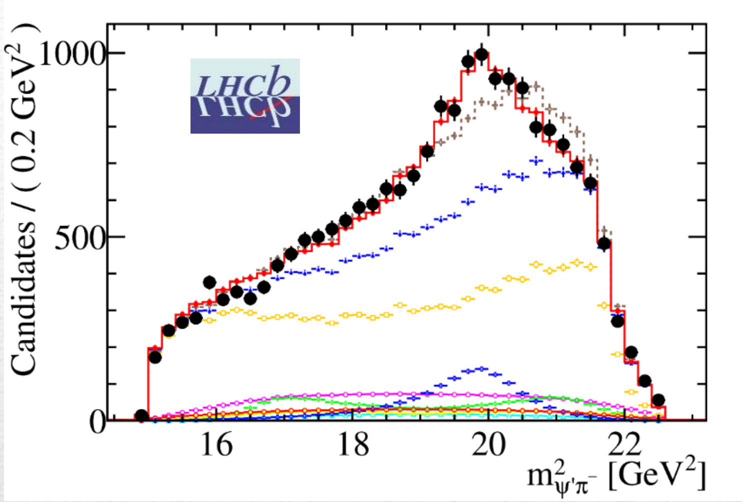
$$L - S \leq J \leq L + S$$
$$P = (-1)^{L+1}, C = (-1)^{L+S}$$
$$G = (-1)^{L+S+I}$$

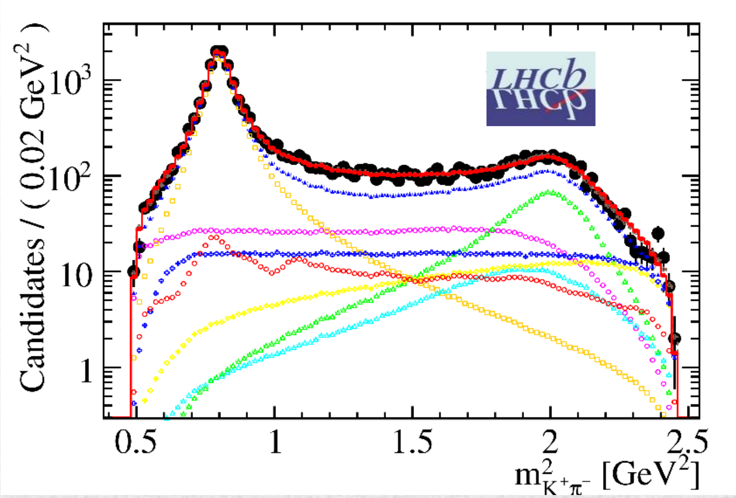


J^{PC}	L	S	Charmonium ($c\bar{c}$)	Bottomonium ($b\bar{b}$)
0^{-+}	0 (S-wave)	0	$\eta_c(nS)$	$\eta_b(nS)$
1^{--}		1	$\psi(nS)$	$\Upsilon(nS)$
1^{+-}	1 (P-wave)	0	$h_c(nP)$	$h_b(nP)$
0^{++}		1	$\chi_{c0}(nP)$	$\chi_{b0}(nP)$
1^{++}		1	$\chi_{c1}(nP)$	$\chi_{b1}(nP)$
2^{++}		1	$\chi_{c2}(nP)$	$\chi_{b2}(nP)$

But $J/\psi = \psi(1S)$, $\psi' = \psi(2S)$

Charged Z states: $Z(4430)$

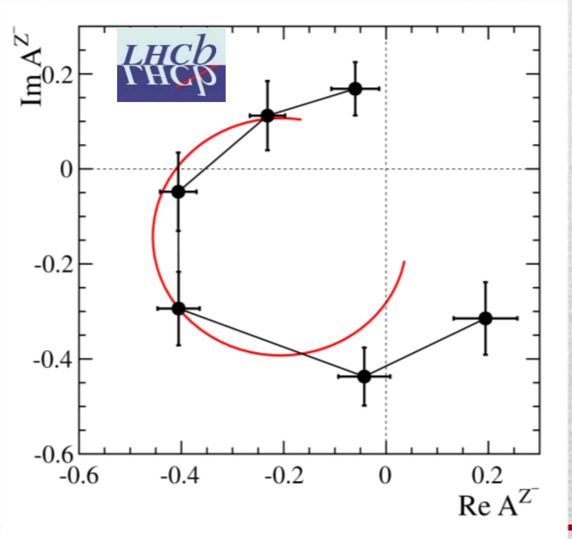




$$Z(4430)^+ \to \psi(2S)\,\pi^+$$
$$I^G J^{PC} = 1^+ 1^{+-}$$

$$M = 4475 \pm 7^{+15}_{-25}\text{ MeV}$$
$$\Gamma = 172 \pm 13^{+37}_{-34}\text{MeV}$$

Far from open charm thresholds

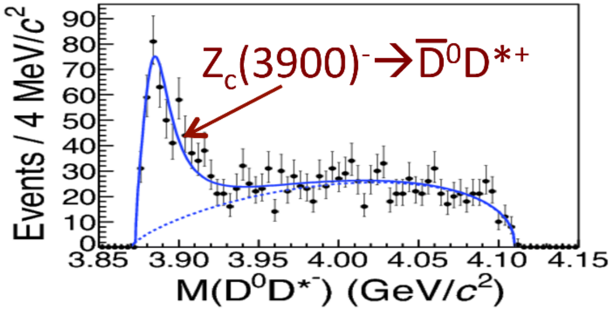


If the amplitude is a free complex number, in each bin of $m^2_{\psi'\pi^-}$, the resonant behaviour appears as well

$Y(4260) \to \bar{D}D_1$?

$e^+e^- \to Y(4260) \to \pi^-\bar{D}^0D^{*+}$

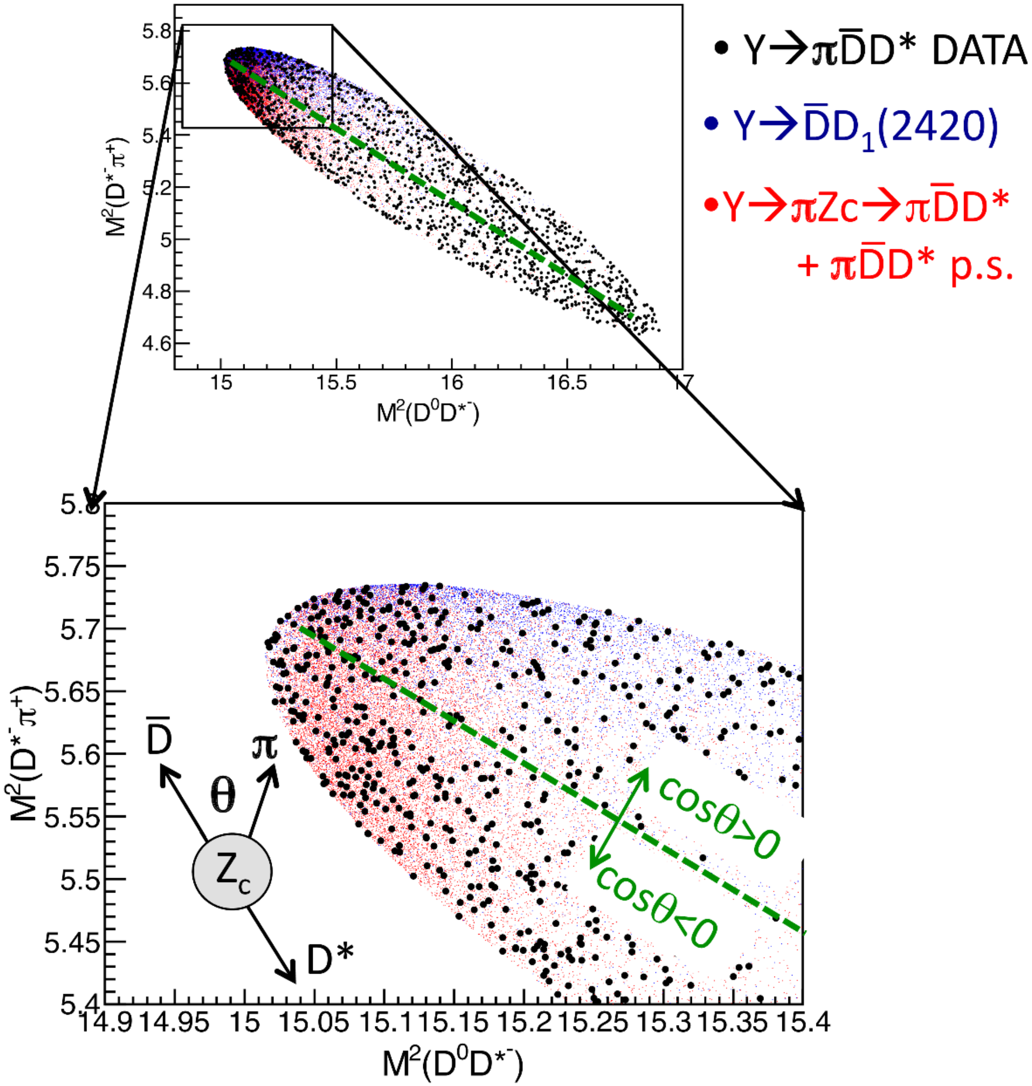
BESIII PRL 112, 022001



$$\mathcal{A} = \frac{N_{|\cos\theta|>0.5} - N_{|\cos\theta|<0.5}}{N_{|\cos\theta|>0.5} + N_{|\cos\theta|<0.5}}$$

	DD_1 MC	Z_c+ps MC	data
$\mathcal{A}$	0.43±0.04	0.02±0.02	0.12+0.06

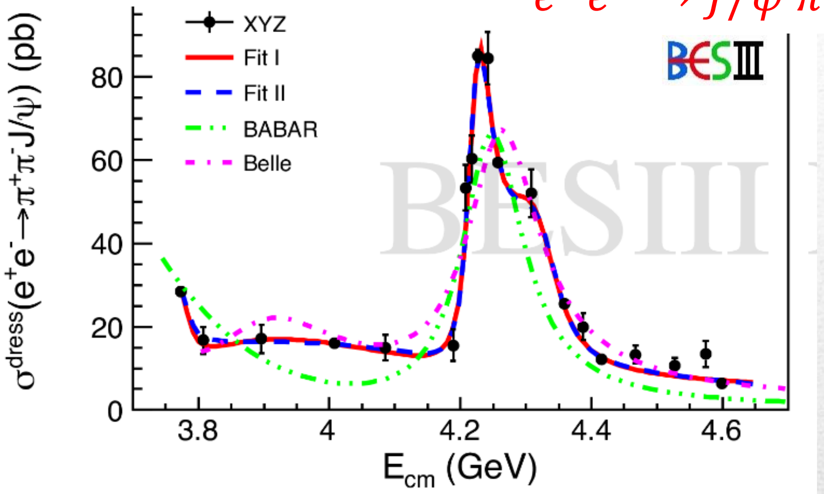
Not a lot of room for $\bar{D}D_1(2410)$



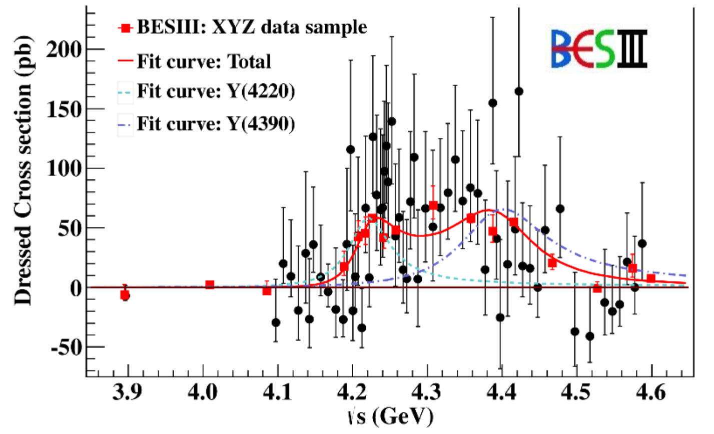
Vector Y states in BESIII

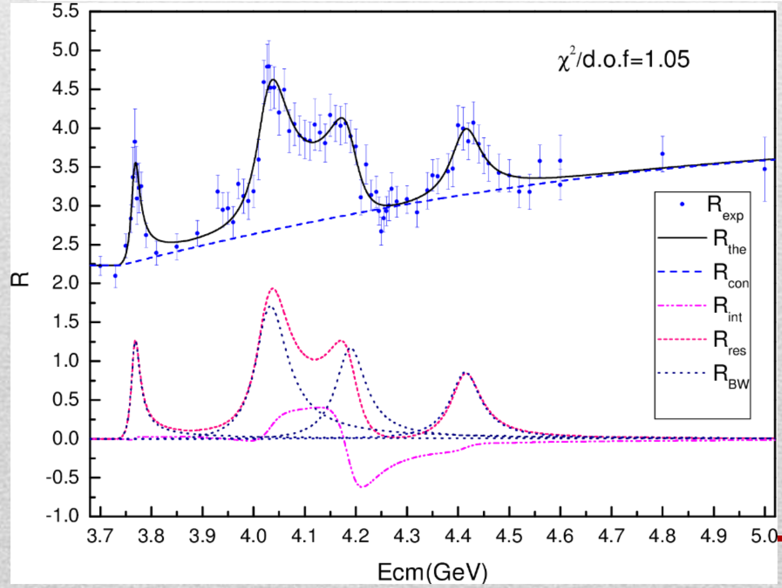
BESIII, 1611.01317

$e^+e^- \to J/\psi\,\pi\pi$

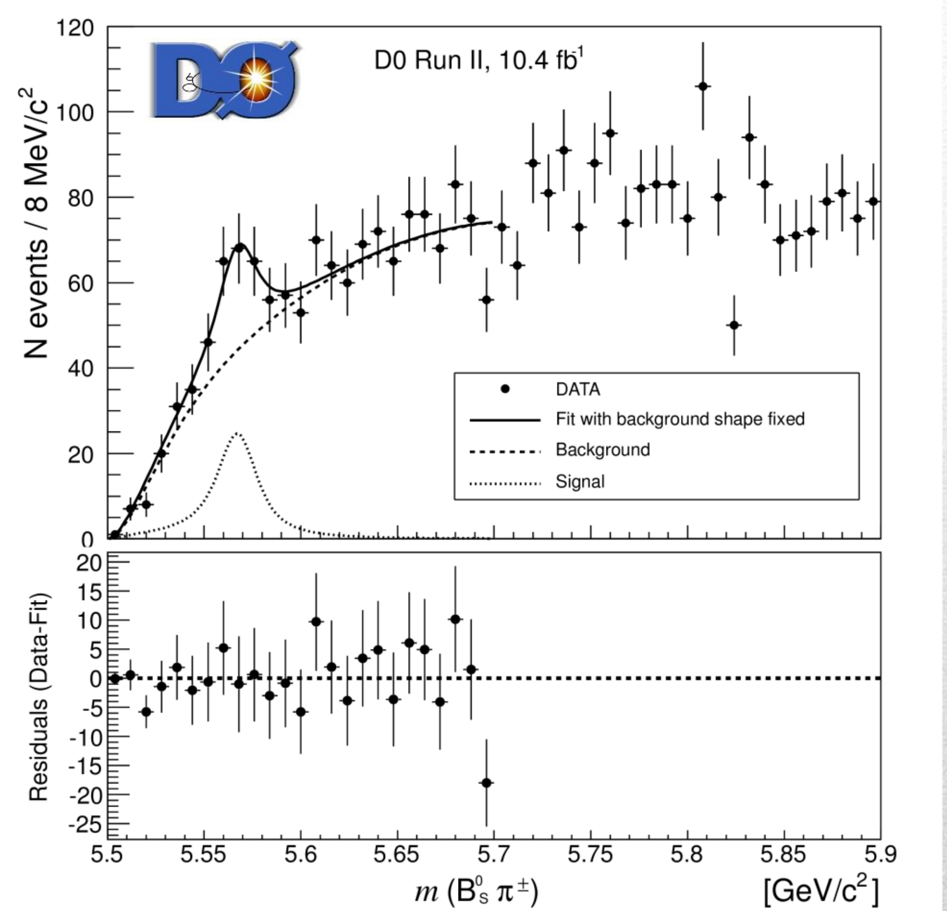


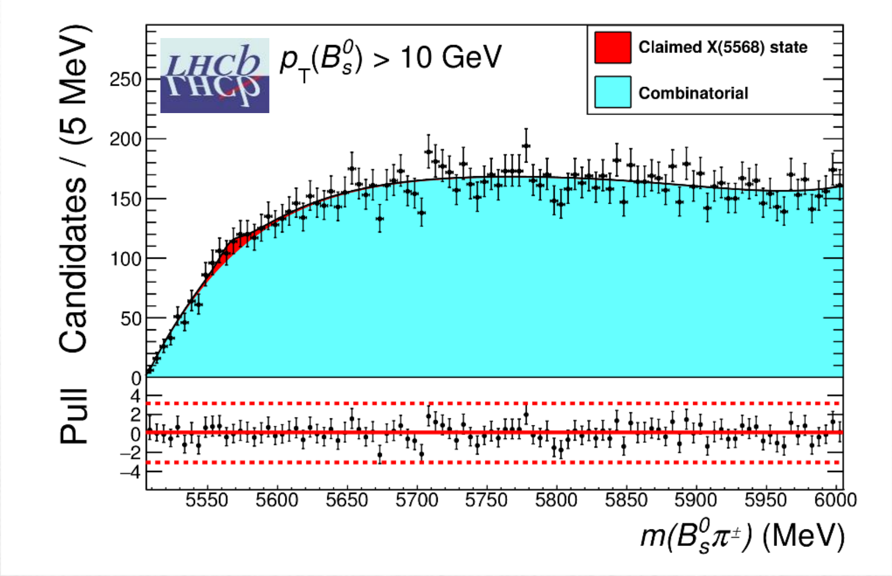
$e^+e^- \to h_c\,\pi\pi$ BESIII, 1611.07044





Flavored $X(5568)$

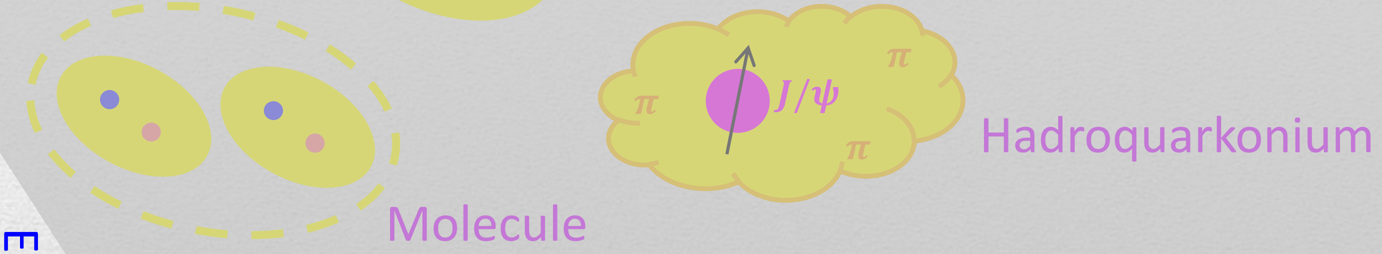




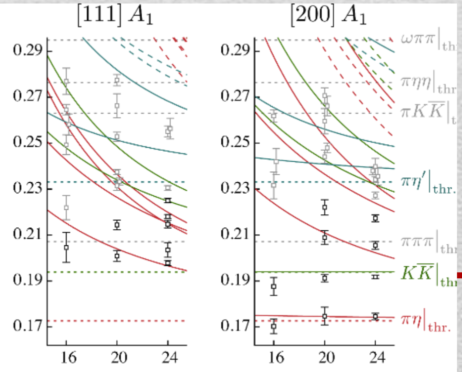
- A flavored state seen in $B_s^0\,\pi$ invariant mass by D0 (both $B_s^0 \to J/\psi\,\phi$ and $\to D_s\mu\nu$),
- not confermed by LHCb or CMS
- (different kinematics? Compare differential distributions)

Controversy to be solved

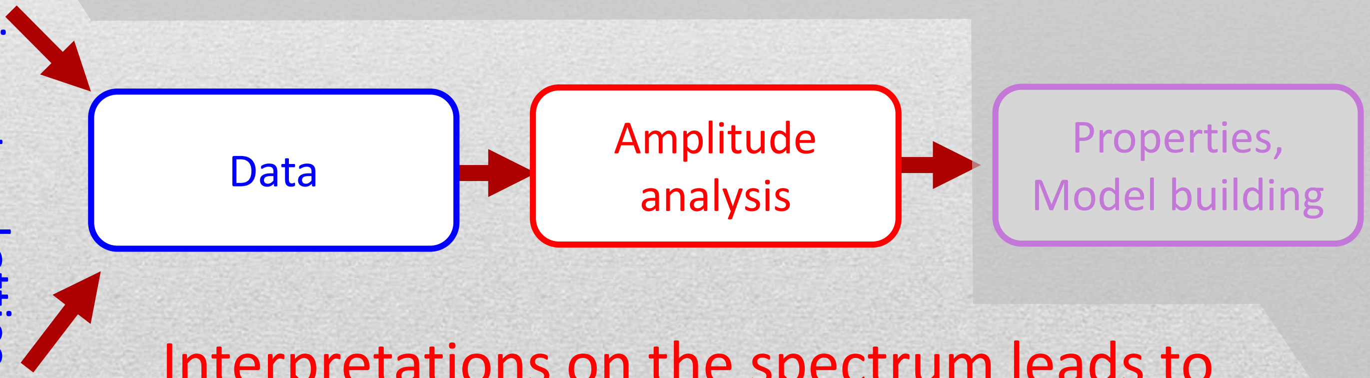
Hadron Spectroscopy



Experiment



Lattice QCD



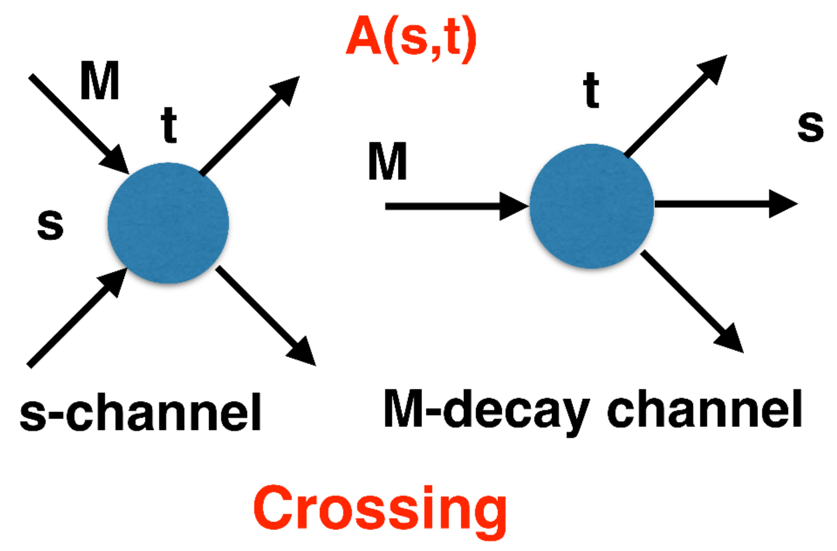
Data

Amplitude analysis

Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

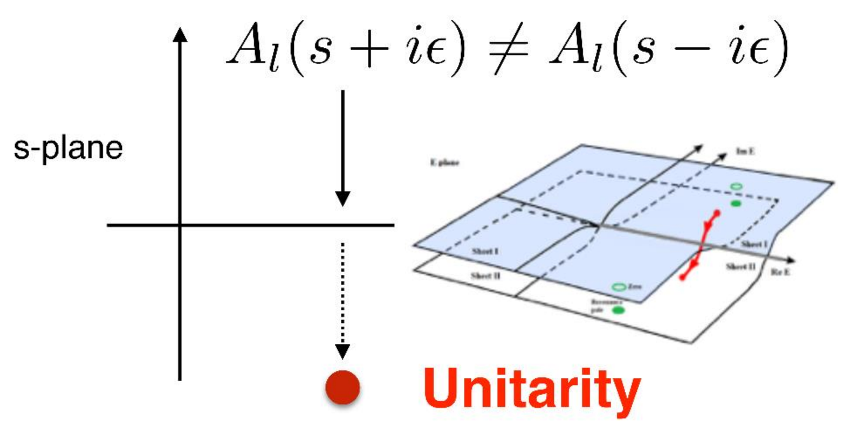
S-Matrix principles



$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon)$$

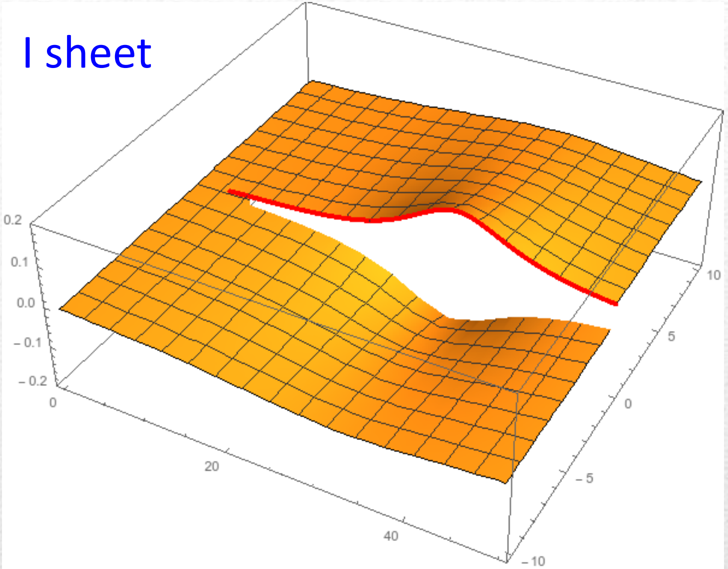


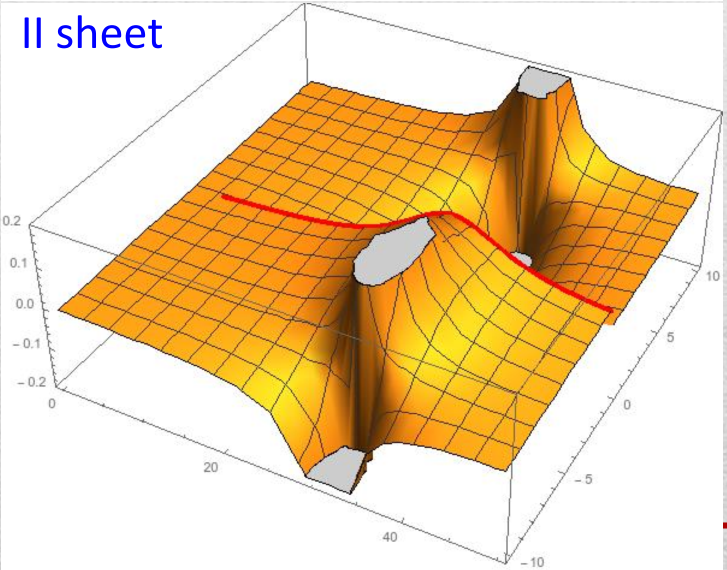
$$A_l(s + i\epsilon) \neq A_l(s - i\epsilon)$$

These are constraints the amplitudes have to satisfy, but do not fix the dynamics

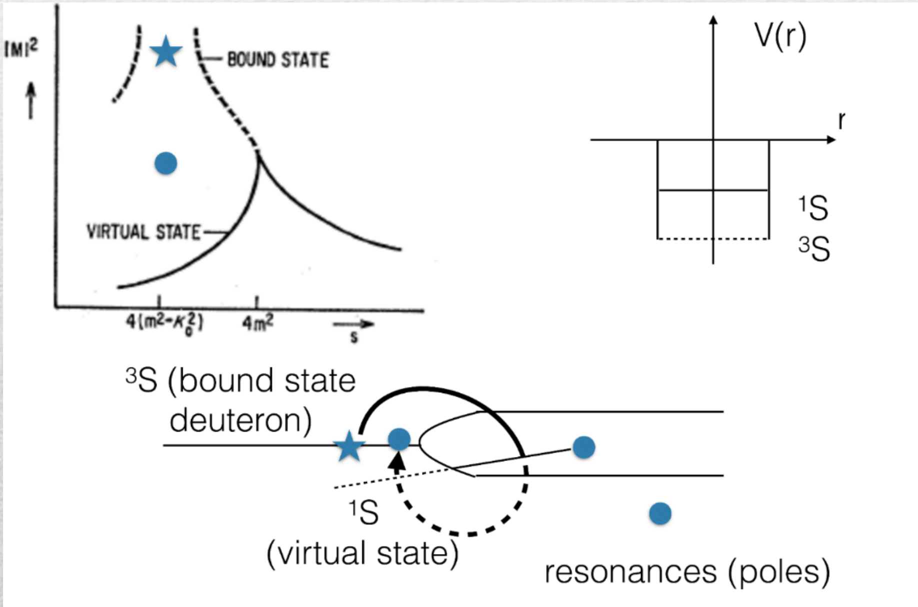
Resonances (QCD states) are poles in the unphysical Riemann sheets

Pole hunting

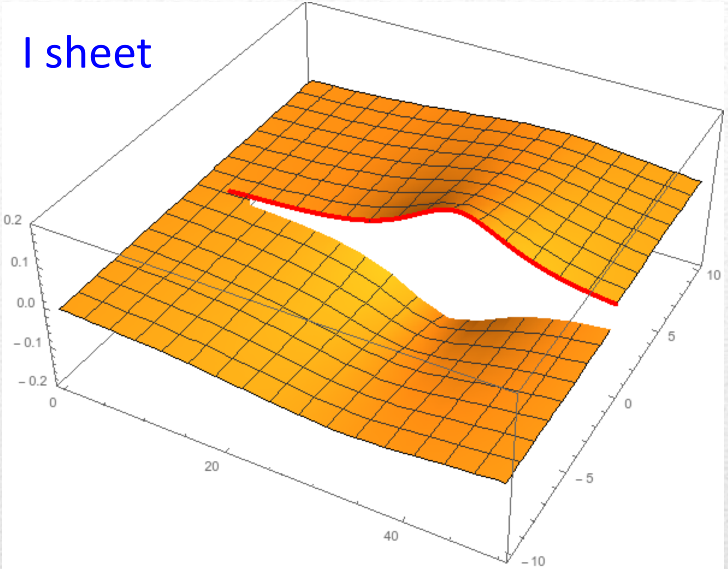


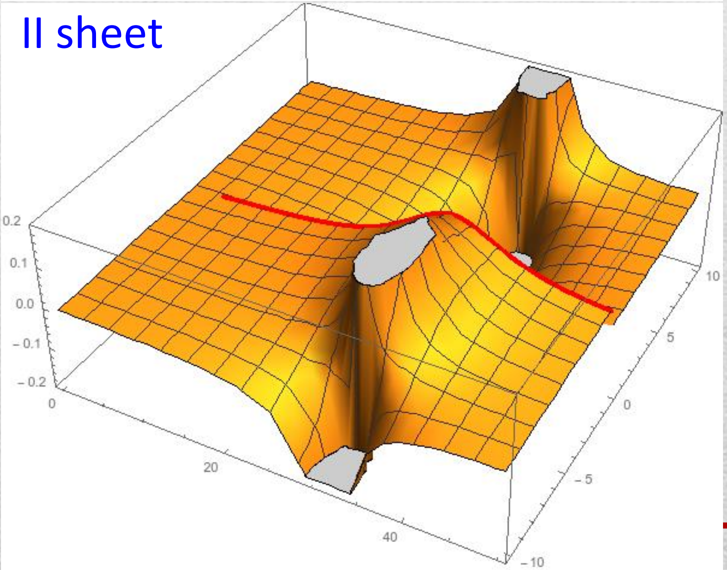


Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet



Pole hunting

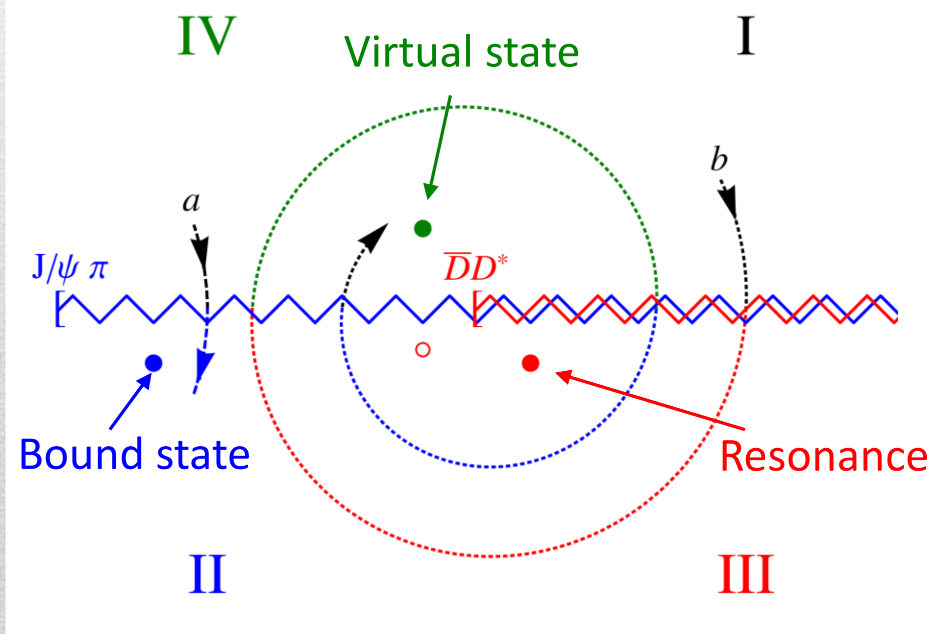




More complicated structure when more thresholds arise: two sheets for each new threshold

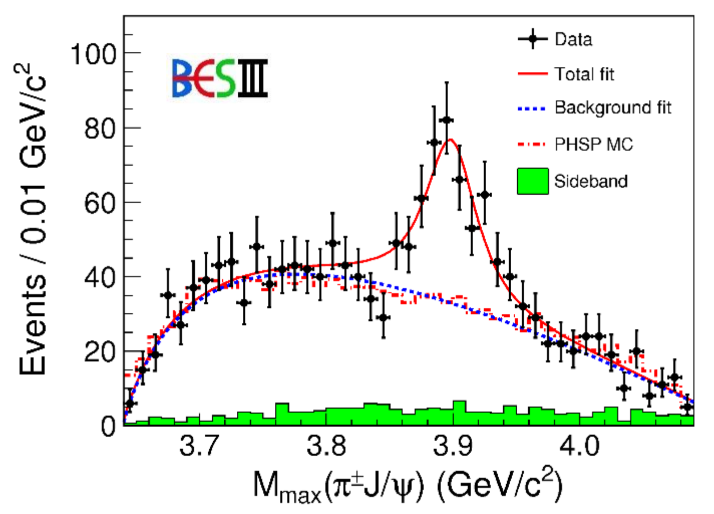
III sheet: usual resonances

IV sheet: cusps (virtual states)

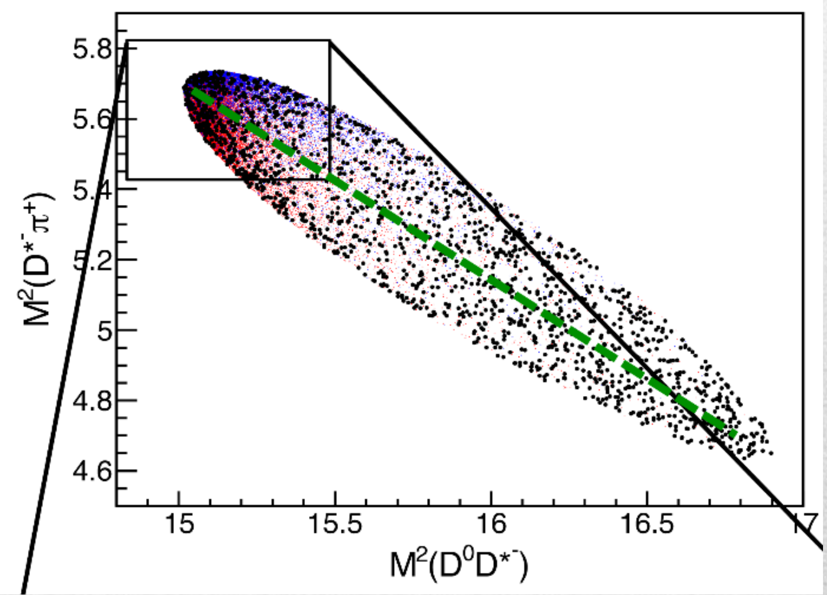


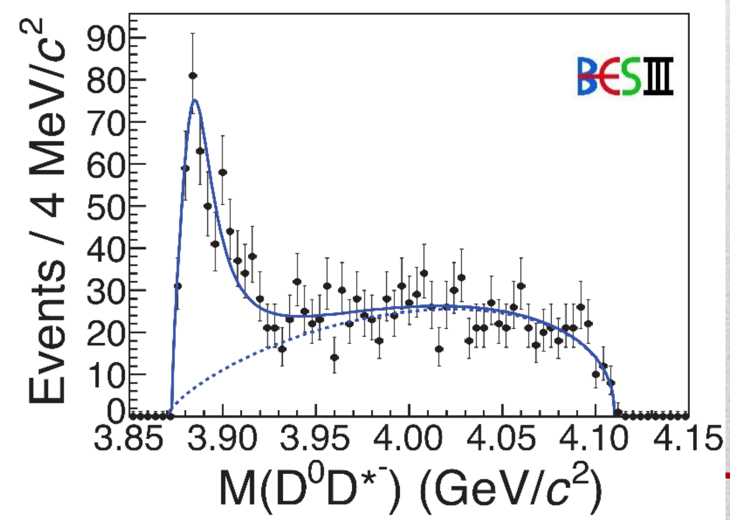
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.



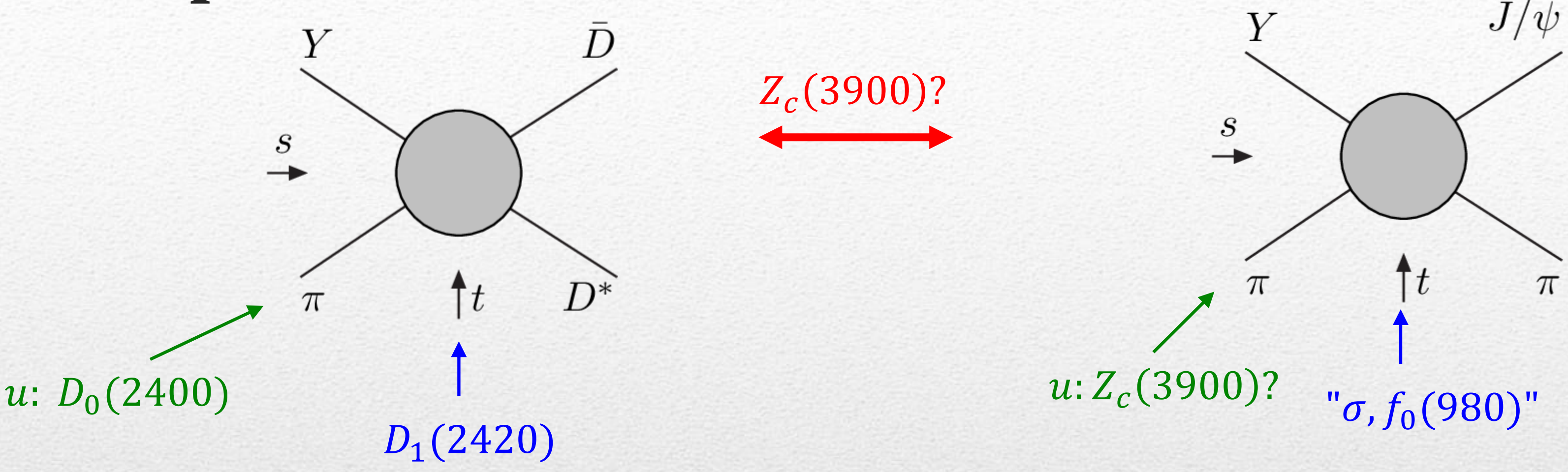
$$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi\,\pi^+\pi^- \text{ and} \to (DD^*)^+\pi^-$$
$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$





Such a state would require a minimal 4q content and would be manifestly exotic

Amplitude model



$Z_c(3900)?$

u: $D_0(2400)$

$D_1(2420)$

u: $Z_c(3900)?$

"$\sigma, f_0(980)$"

$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1)\left(a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u)\right)$$

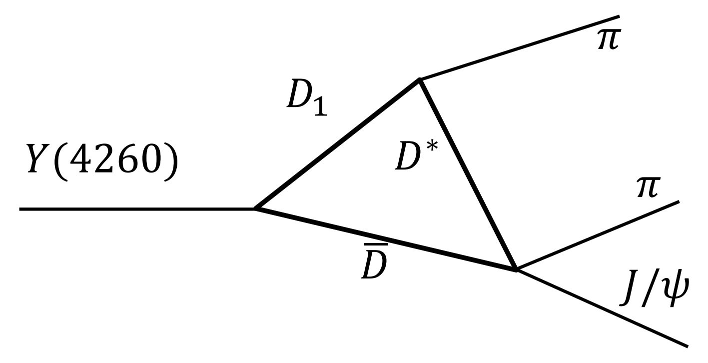
Khuri-Treiman

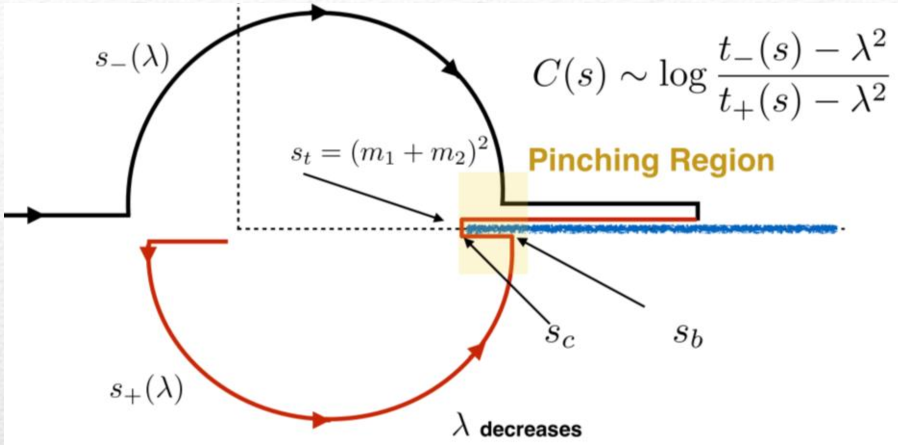
$$f_{0,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi}\int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, P_l(z_s)\left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s)\frac{1}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'-s},$$

$$\boxed{f_i(s,t,u) = 16\pi\left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s)\left(c_j + \frac{s}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s'-s)}\right)\right],}$$

Triangle singularity





Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438), However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)

$$f_{0,i}(s) = b_{0,i}(s) + \frac{t_{ij}}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'-s}$$

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo, Meissner, Wang, Yang PRD92, 071502

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

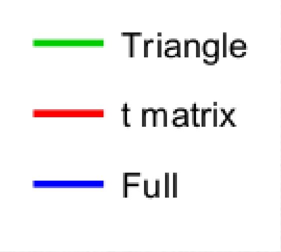
The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

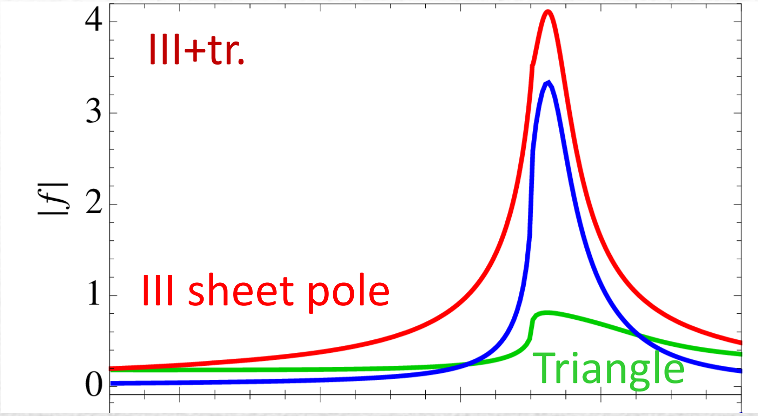
Four different scenarios considered:

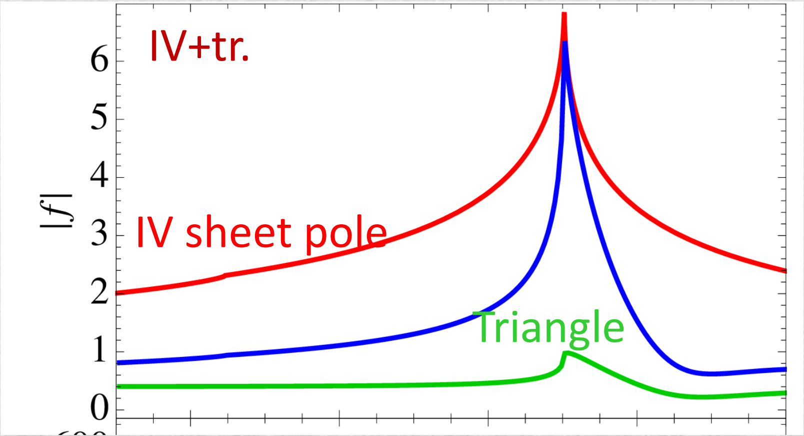
- «III»: the K matrix is $\frac{g_i\,g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

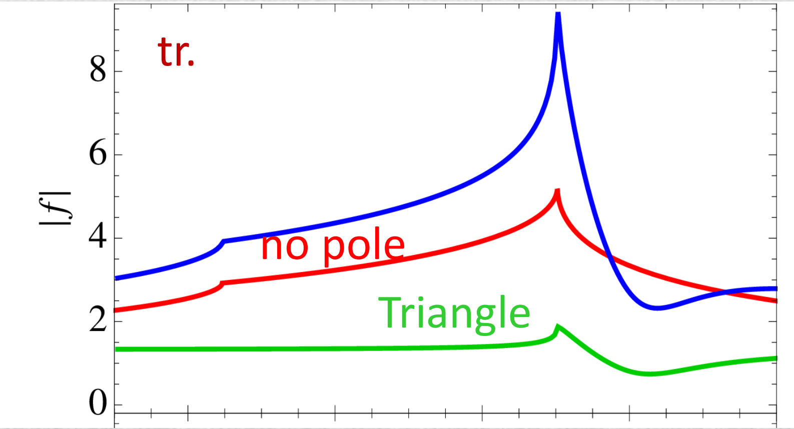
Singularities and lineshapes

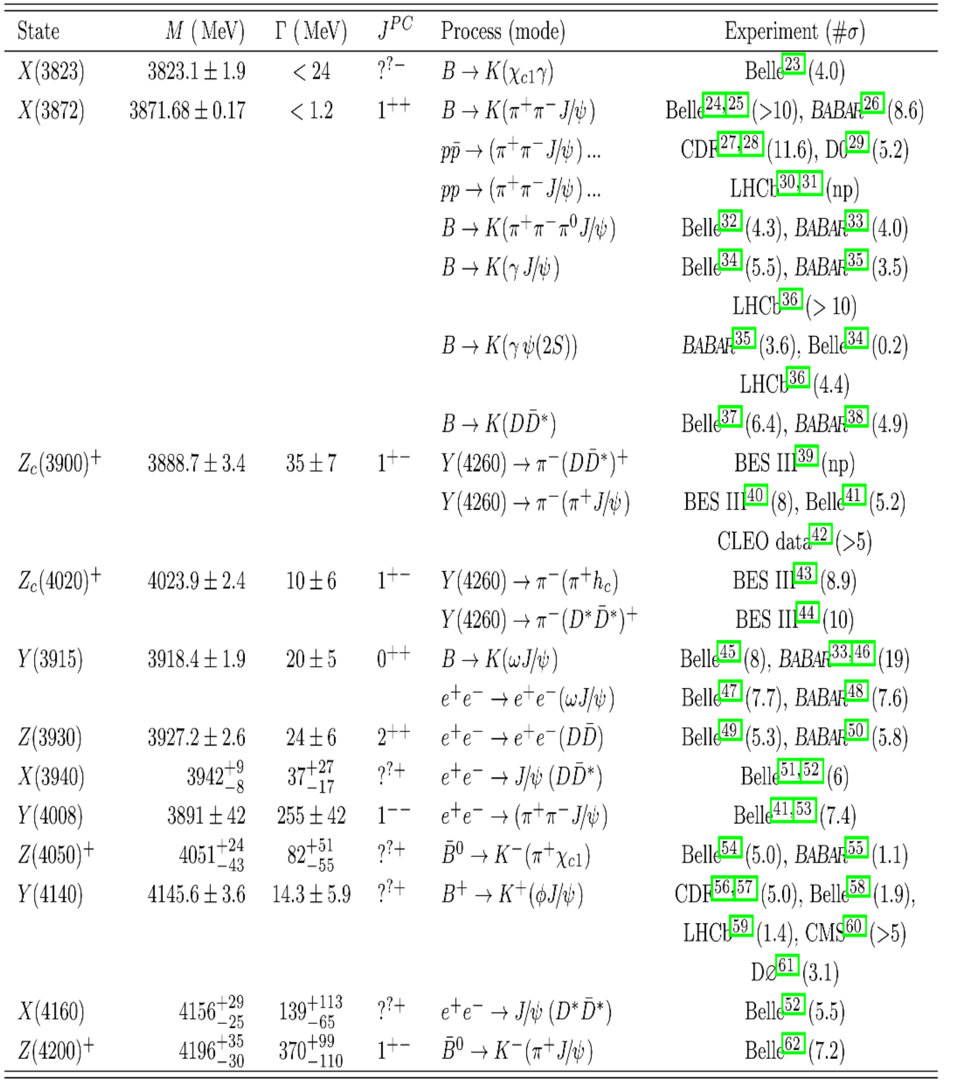
Different lineshapes according to different singularities



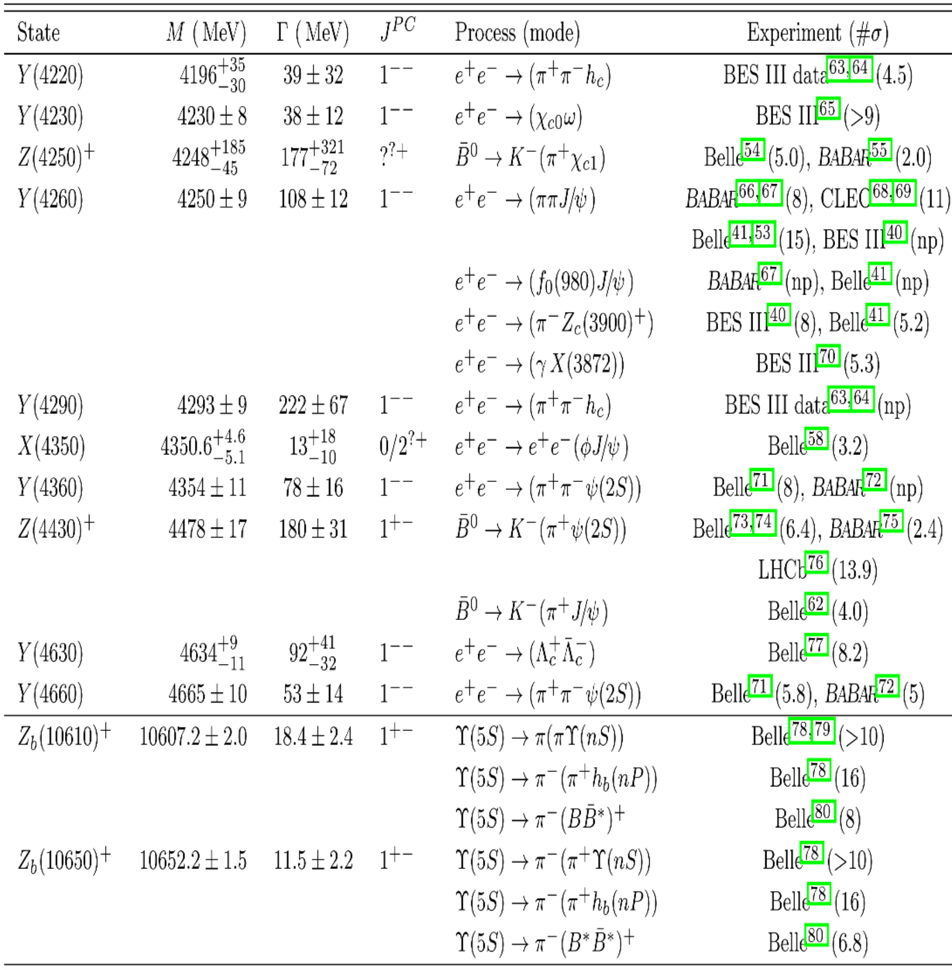








State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$B \to K(\chi_{c1}\gamma)$	Belle[23] (4.0)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle[24,25] (>10), *BABAR*[26] (8.6)
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF[27,28] (11.6), D0[29] (5.2)
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb[30,31] (np)
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle[32] (4.3), *BABAR*[33] (4.0)
				$B \to K(\gamma\, J/\psi)$	Belle[34] (5.5), *BABAR*[35] (3.5) LHCb[36] (> 10)
				$B \to K(\gamma\, \psi(2S))$	*BABAR*[35] (3.6), Belle[34] (0.2) LHCb[36] (4.4)
				$B \to K(D\bar{D}^*)$	Belle[37] (6.4), *BABAR*[38] (4.9)
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III[39] (np)
				$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III[40] (8), Belle[41] (5.2) CLEO data[42] (>5)
$Z_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$Y(4260) \to \pi^-(\pi^+ h_c)$	BES III[43] (8.9)
				$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III[44] (10)
$Y(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \to K(\omega J/\psi)$	Belle[45] (8), *BABAR*[33,46] (19)
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle[47] (7.7), *BABAR*[48] (7.6)
$Z(3930)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle[49] (5.3), *BABAR*[50] (5.8)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\, (D\bar{D}^*)$	Belle[51,52] (6)
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle[41,53] (7.4)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (1.1)
$Y(4140)$	4145.6 ± 3.6	14.3 ± 5.9	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF[56,57] (5.0), Belle[58] (1.9), LHCb[59] (1.4), CMS[60] (>5) DØ[61] (3.1)
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\, (D^*\bar{D}^*)$	Belle[52] (5.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (7.2)



State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$Y(4220)$	4196^{+35}_{-30}	39 ± 32	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63,64] (4.5)
$Y(4230)$	4230 ± 8	38 ± 12	1^{--}	$e^+e^- \to (\chi_{c0}\omega)$	BES III[65] (>9)
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (2.0)
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	*BABAR*[66,67] (8), CLEO[68,69] (11) Belle[41,53] (15), BES III[40] (np)
				$e^+e^- \to (f_0(980) J/\psi)$	*BABAR*[67] (np), Belle[41] (np)
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III[40] (8), Belle[41] (5.2)
				$e^+e^- \to (\gamma\, X(3872))$	BES III[70] (5.3)
$Y(4290)$	4293 ± 9	222 ± 67	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63,64] (np)
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle[58] (3.2)
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle[71] (8), *BABAR*[72] (np)
$Z(4430)^+$	4478 ± 17	180 ± 31	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle[73,74] (6.4), *BABAR*[75] (2.4) LHCb[76] (13.9)
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (4.0)
$Y(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle[77] (8.2)
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle[71] (5.8), *BABAR*[72] (5)
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$	Belle[78,79] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B\bar{B}^*)^+$	Belle[80] (8)
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(5S) \to \pi^-(\pi^+\Upsilon(nS))$	Belle[78] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B^*\bar{B}^*)^+$	Belle[80] (6.8)

Guerrieri, AP, Piccinini, Polosa,
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