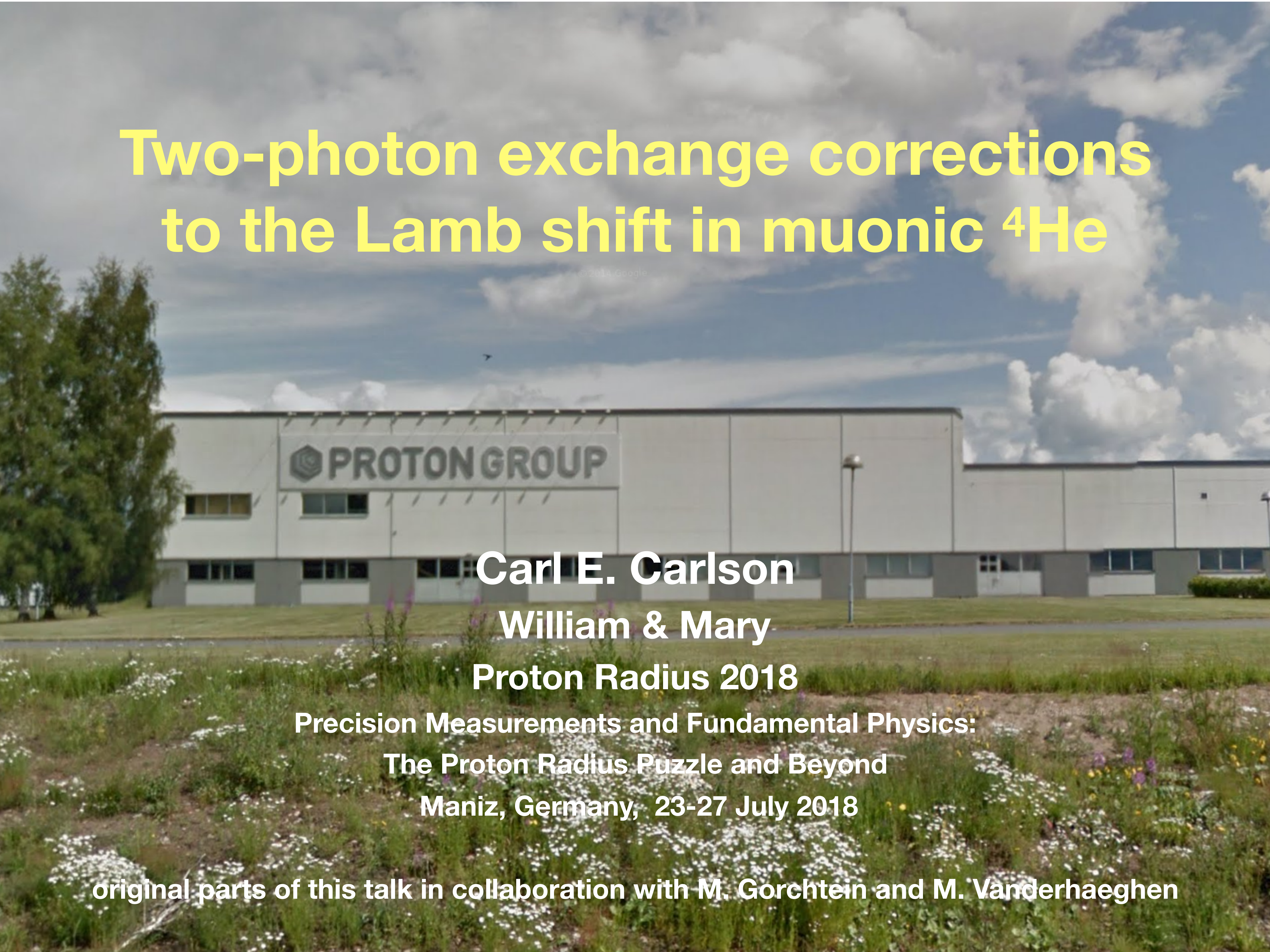


Two-photon exchange corrections to the Lamb shift in muonic ^{4}He

Carl E. Carlson

William & Mary

Proton Radius 2018

Precision Measurements and Fundamental Physics:

The Proton Radius Puzzle and Beyond

Maniz, Germany, 23-27 July 2018

original parts of this talk in collaboration with M. Gorchtein and M. Vanderhaeghen

Intro

- Purpose of this talk: report on a dispersive calculation of the two photon exchange (TPE) contributions to the Lamb shift in muonic ^{4}He.

  - Introduction and motivation

  - Calculational procedure

  - Comparison to other ways of doing the calculation

    - Nuclear theory - Hernandez later

    - Effective field theory - Pineda later

  - Results

- Find result comparable to existing nuclear potential calculation, but somewhat larger in magnitude.

Introduction and motivation

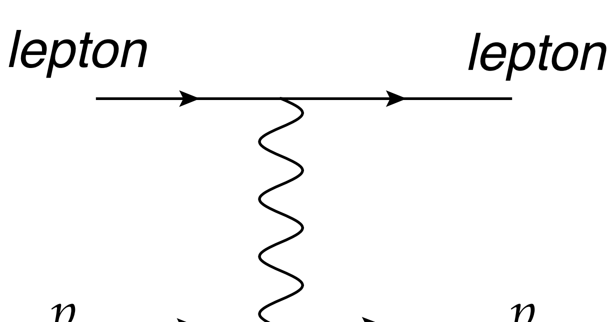
- There is a proton radius problem.
- Will not elaborate.
- The most accurate measurements come from measuring the Lamb shift energy spitting in muonic hydrogen, μH.
- Most of the Lamb shift comes from QED. But the experimental value also gets contributions from the finite size of the proton, so that accurate measurements allow determining the proton charge radius.

Introduction and motivation

- Good results require calculation of corrections to the Lamb shift, beyond pure QED and proton size effects. Chief among them is TPE.

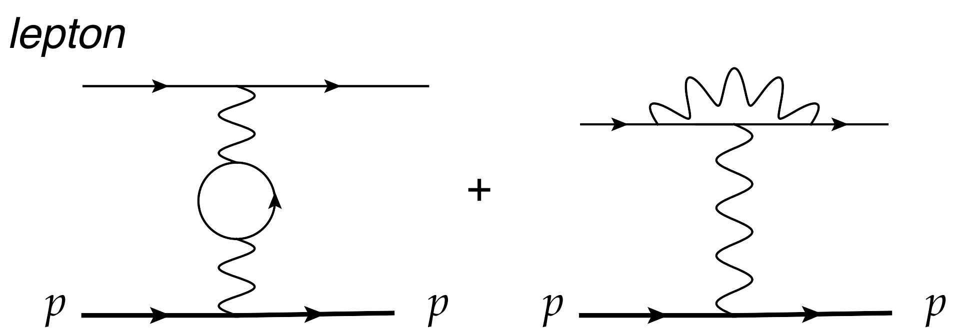
- Also three photon exchange—hear Pachucki soon.

- In pictures,

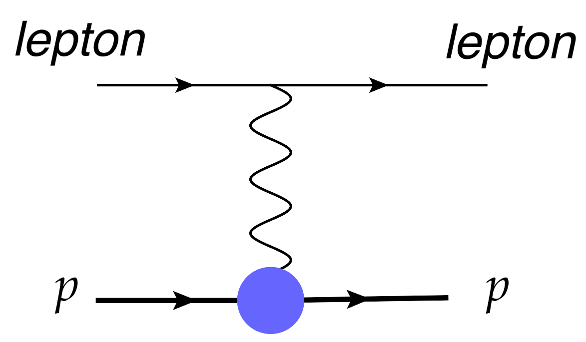


Leading term, with pointlike protons, gives big level splittings for different principle quantum numbers, but no 2S-2P splitting (which is the Lamb shift).

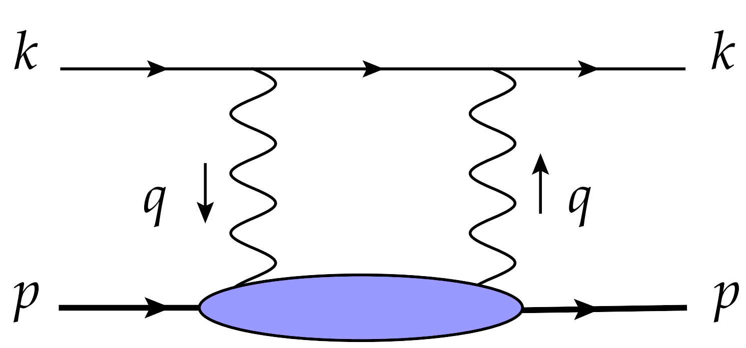
Introduction and motivation



Examples of QED corrections that split the 2S and 2P states



Corrections from proton structure. Can calculate perturbatively. Obtain result proportional to proton charge radius-squared, R_E^2.



Two photon exchange. Involves hadronic physics. Blob on bottom can be just proton ("elastic") or more complicated, such as resonance, nucleon-pion state, or 3q state ("inelastic").

Introduction and motivation

- In equations,

$$\Delta E_L^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r^3 Z^4 \alpha^4}{12} R_E^2 + \Delta E_{\text{TPE}}$$
$$= 206.0336(15) - 5.2275(10) R_E^2 + 0.0332(20)$$

(units are meV and fm)

- with faith,

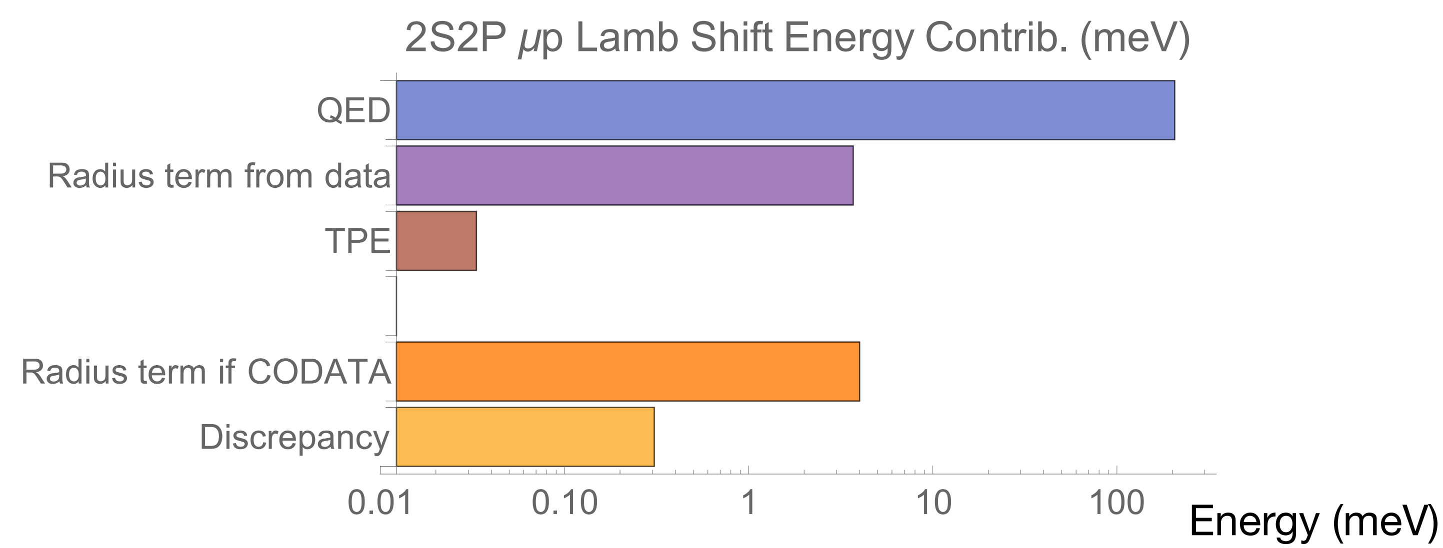
$$\Delta E_L^{\text{theo}} = \Delta E_L^{\text{expt}} = 202.3706\,(23)\ \text{meV}$$

- this leads to

$$R_p = 0.84087\,(32)\ \text{fm}$$

Introduction and motivation

- Thinking about sizes of terms, and uncertainties therein,



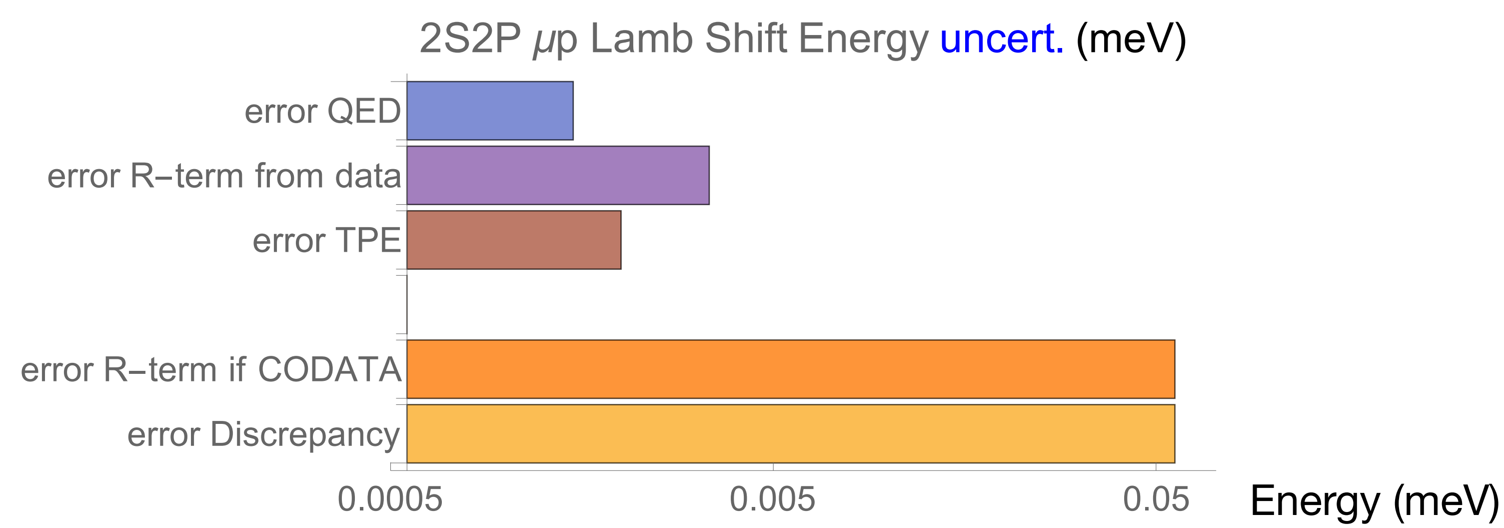
- The first three bars are from QED and TPE calculations, and the energy in the radius term measured using overall Lamb data

- Fourth bar is what the radius term energy would be if the CODATA, electron based, value of the radius were correct

- Fifth bar is difference (discrepancy) between measured proton size energy and CODATA predicted proton energy

Introduction and motivation

- The uncertainties in each term:



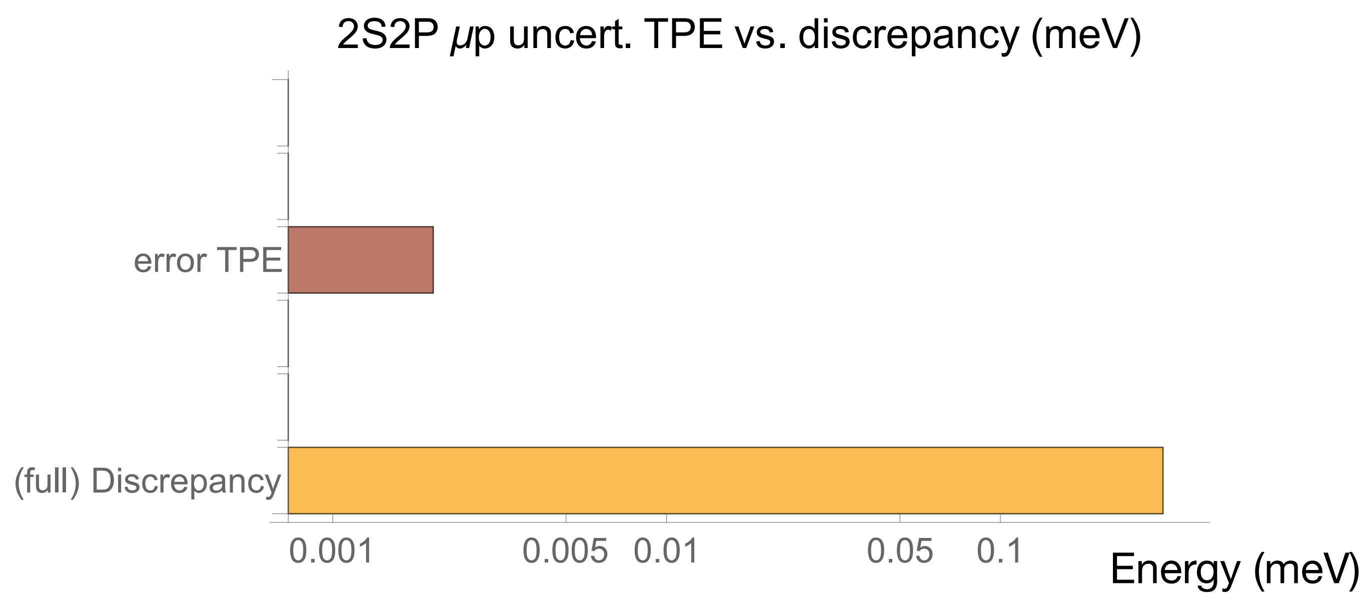
- Big, relatively speaking, uncertainty in energy expected using CODATA radius because CODATA radius has big error, by applicable standards,

$$R_E(\text{CODATA}) = 0.8751(61)\ \text{fm} \qquad [\ 0.70\%\ ]$$

- Uncertainty in discrepancy follows

Introduction and motivation

- Most interesting from viewpoint of this talk is uncertainty in TPE compared for full discrepancy,



- Conclude: all is calculationally well. The uncertainty in the critical TPE correction is much smaller than the size of the discrepancy that has been discovered.
  Smaller by factor ≈150.

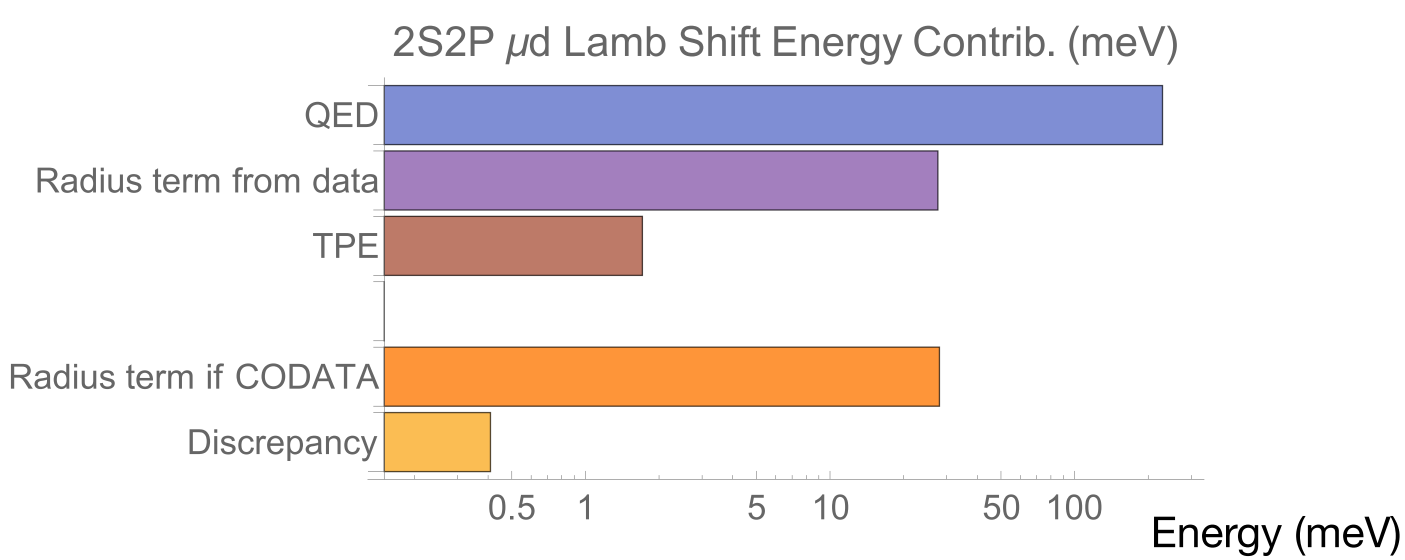
Introduction and motivation

- May get better idea of cause of discrepancy with studies of additional nuclei, as *d*, ^{3}He, ^{4}He.

- Deuteron case here

- TPE effects are non-relativistically polarizability effects, and are much bigger for deuteron than for proton

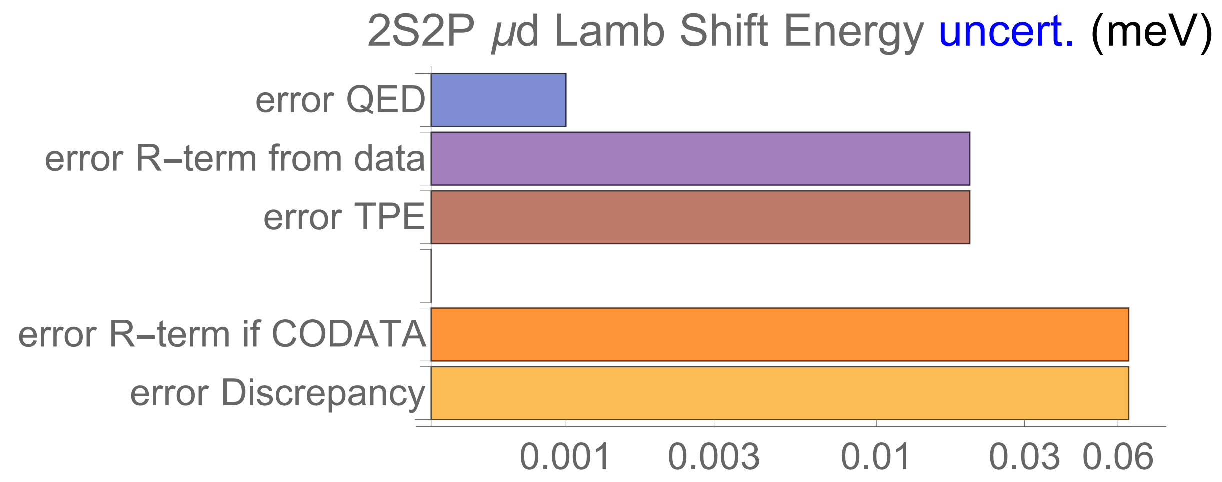
- Analogous plots are



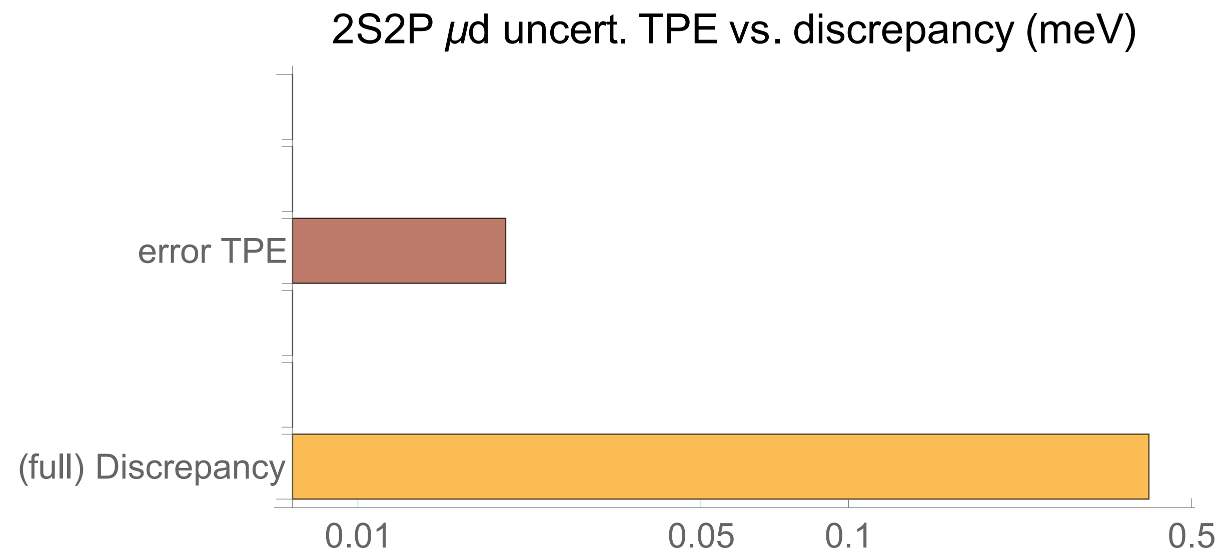
Credits for TPE: Hernandez, Ji, Bacca, Nevo Dinur, Barnea, Pachucki, Weinczek. (CC & co. examined different method.)

Introduction and motivation

- μd uncertainties



- “Interesting plot,”



- True still that TPE uncertainty is much smaller than size of discrepancy.
  But now only by factor ≈20.

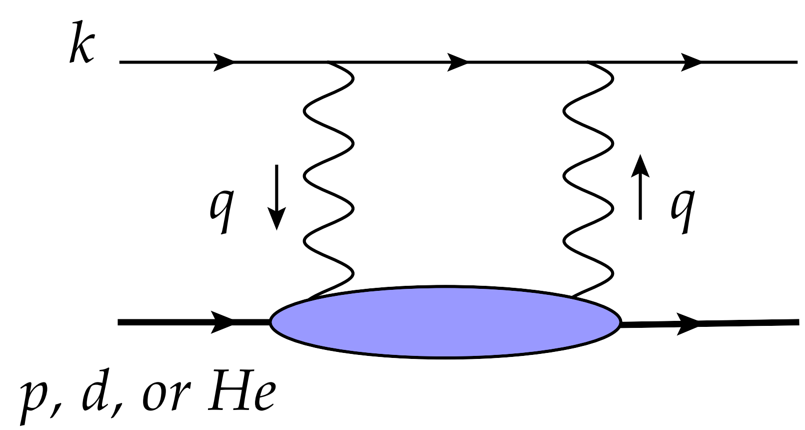
Introduction and motivation

- For ^{3}He and ^{4}He plots, official data not published.
- Work on ^{4}He TPE in anticipation.
- May make some comments later about possible beyond the standard model predictions.

The calculation

- We (CC, MG, MV) calculate using dispersion relations.

- Need

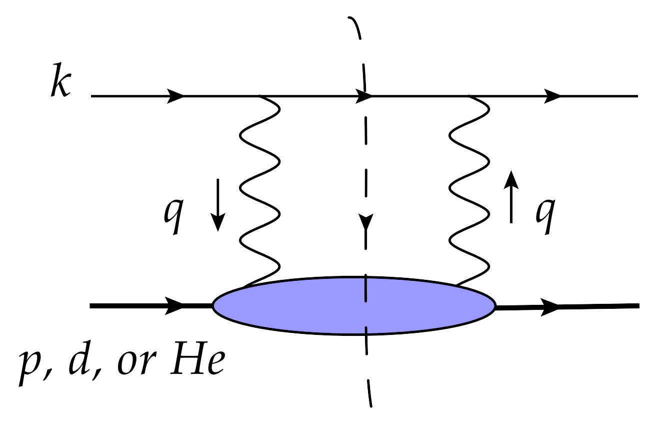


- Can set external 3-momenta to zero for this calculation.

- Lower part of diagram is forward, off-shell Compton scattering. Given (spin-independent part) in terms of two Compton amplitudes, $T_1(\nu,Q^2)$ and $T_2(\nu,Q^2)$. But these amplitudes are not directly known.

The calculation

- But imaginary part comes from situation where intermediate lepton & hadronic states are on-shell.



- Intermediate matter on-shell: LHS of diagram is scattering amplitude, RHS is its conjugate. *I.e.*, is cross section. Obtain from lepton-^{4}He scattering data.

- Blob can be elastic, quasi-elastic (QE for ^{4}He breakup into nucleons only), or deep inelastic (pion production region).

The calculation

- Obtain all of $T_{1,2}$ from imaginary part and Cauchy formula

- Useful,

$$\text{Im } T_1(\nu, Q^2) = \frac{1}{4M_\alpha} F_1(\nu, Q^2)$$
$$\text{Im } T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2)$$

- $F_{1,2}$ are structure functions gotten from e-^{4}He scatt. data,

The calculation

- For DIS region, Bosted & Christy have fitted higher energy data and give analytic forms for $F_{1,2}$.

- For elastic contribution, can calculate with known (measured) form factors (“Born calculation”). BTW, get

$$T_1^{\text{Born}}(\nu, Q^2) = -\frac{Z^2 G^2(Q^2)}{4\pi M_\alpha},$$

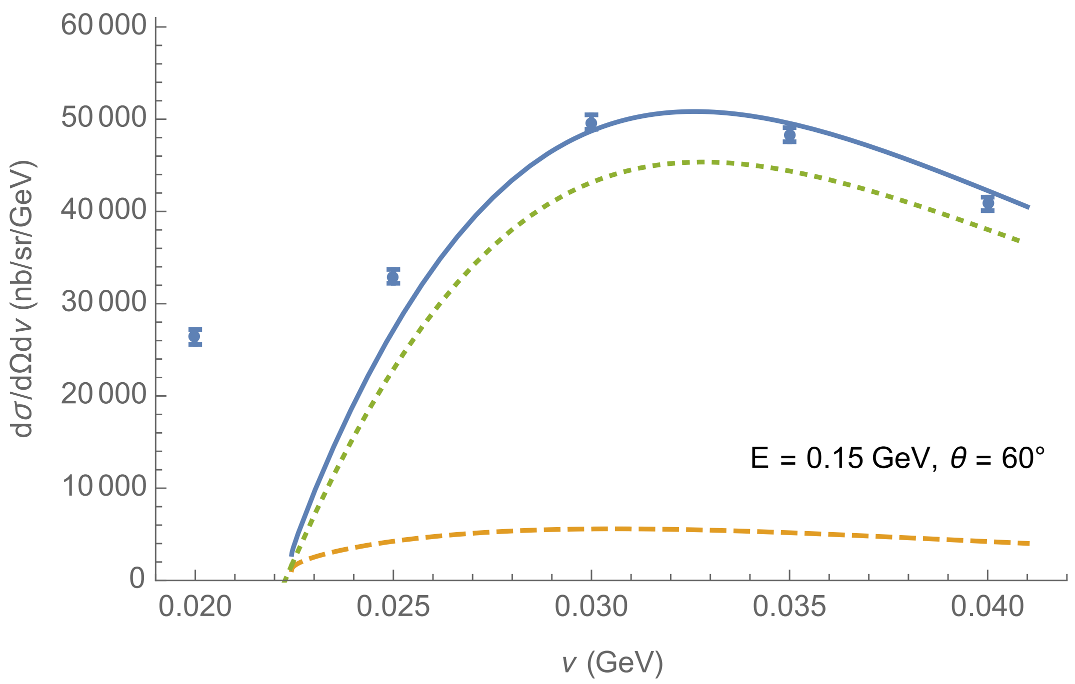
  $G(Q^2)$ being ^{4}He elastic form factor (used fit from Ingo Sick)

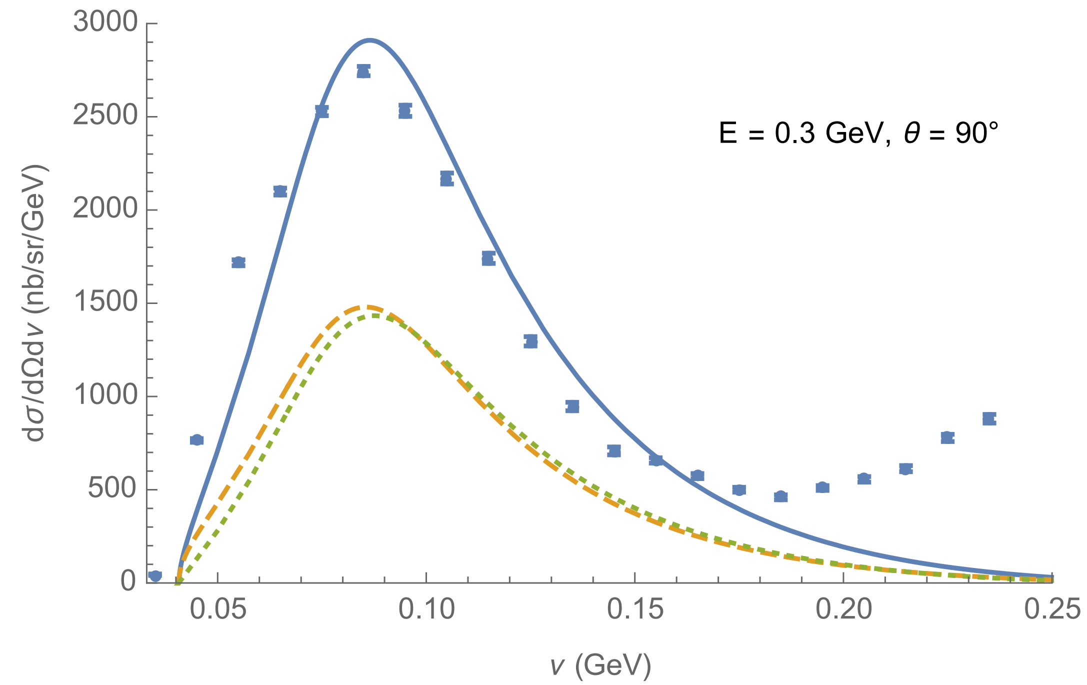
  Shows no pole term, and no falloff with ν.

  Dispersion relation for T_1 needs subtraction.

- For QE region, no known fits, so we make our own. Data cataloged by Donal Day and catalog available on web.

Some fit results, QE part only

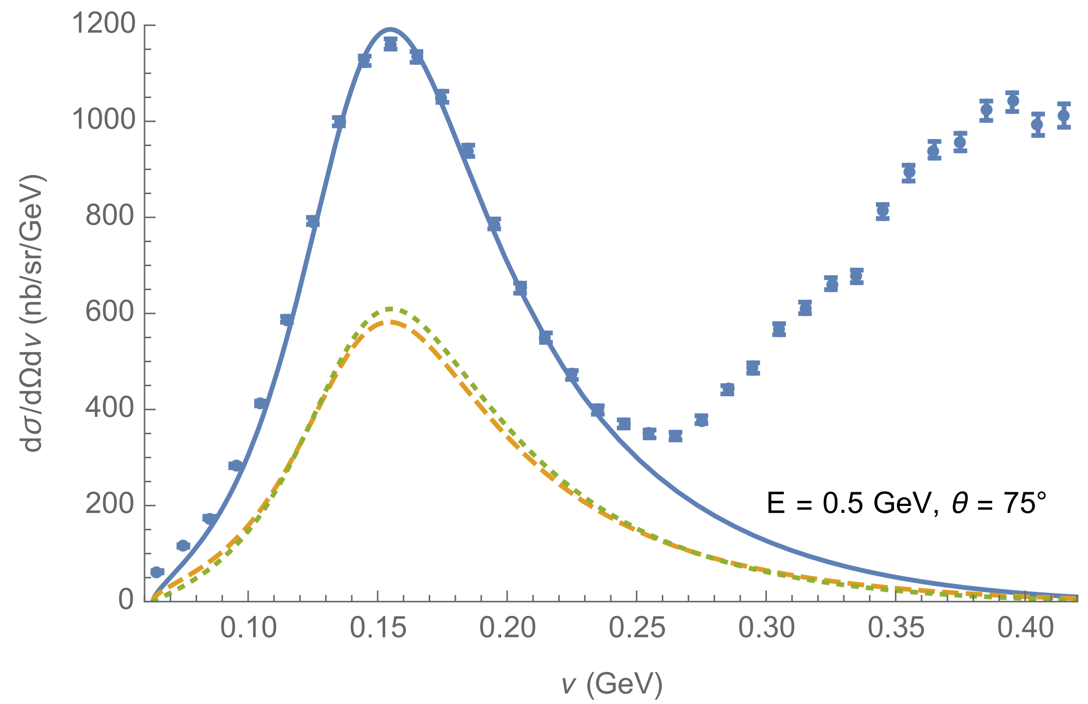




$\nu_{\text{pion prod thr.}} = 0.14$ GeV,

$Q^2|_{\text{right edge}} = 0.017$ GeV^2

$\nu_{\text{pion prod thr.}} = 0.16$ GeV



$\nu_{\text{pion prod thr.}} = 0.18$ GeV

The calculation

- Dispersion relations for T_1 and T_2,

$$\text{Re } T_1(\nu, Q^2) = T_1(0,Q^2) + \frac{\nu^2}{2\pi M_\alpha}\mathscr{P}\int_{\nu_{th}}^{\infty} \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)} F_{1\alpha}(\nu', Q^2)\,.$$

$$\text{Re } T_2(\nu, Q^2) = \frac{1}{2\pi}\mathscr{P}\int d\nu' \frac{F_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

- How can we get the subtraction function $T_1(0,Q^2)$? Not from data: $\nu = 0$ and spacelike Q^2 is unphysical.

The Gorchtein sum rule

- Believe:
  - For $F_{1\text{nucleus}}(\nu,Q^2)$ there is a gap region between the QE region and the DIS region where the structure function is zero (or nearly so).
  - For high energies, including the gap region, the nucleus can be treated as a sum of protons and neutrons

The Gorchtein sum rule

- Enough to work the existing dispersion relation into

$$T_{1\alpha}(0,Q^2) = \text{Born part} + \frac{1}{2\pi M_\alpha}\int_{\nu_{th}}^{\nu_1}\frac{d\nu}{\nu}F_{1\alpha}(\nu,Q^2)$$
$$+\frac{Q^2}{e^2}\left(Z\beta_{Mp}(Q^2) + N\beta_{Mn}(Q^2)\right) - \frac{1}{4\pi M_N}\left(ZF_{1p}^2(Q^2) + NF_{1n}^2(Q^2)\right)$$

- ν_1 is effective upper limit of QE region
- $\beta_{\mathrm{Mp}}(0) = \beta_{\mathrm{Mp}}$ = magnetic polarizability of proton;
use same Q^2 dependence as in proton case.
- BTW, for $Q^2 = 0$, GSR works into Bethe-Levinger sum rule,

$$NZ = 2\int_{\nu_{th}}^{\nu_1}\frac{d\nu}{\nu}F_{1\alpha}(\nu,0)$$

constrains/tests our fits

Comparison

- TPE for nuclear cases also possible using nuclear theory for the elastic and QE parts. See Hernandez, Ji, Bacca, Nevo Dinur, and Barnea, and Pachucki and Weinczek. One uses nuclear potentials to find the wave functions, and then proceed.

- Done for *d*, ^{3}He, and ^{4}He.

- Dispersive calculation can also work for *p*, and has been done for *d* (with some uncertainty) and ^{3}He (with good results), and now for ^{4}He.

- Both methods can thought of as starting from data, but are very different.

Comparison

Dispersive

- Data from *e*-hadron scattering.
- Directly gives imaginary part of necessary amplitudes
- Use Cauchy integral formula (dispersion relations) to find real part and energy.

Potential models

- Data is from *pp* and *np* scattering.
- Convert to NN potentials. Also some NNN used.
- Use potential in wave equation to find wave functions, etc.

Results

- Energies in meV

	Totals	Subtotals
E^{elastic}	-6.85986	
$E^{\text{elas, nonsubt.}}$		-6.89774
$E^{\text{elas, subt}}$		0.0378778
E^{QE}	-5.09527	
$E^{\text{QE, nonsubt}}$		-6.33547
$E^{\text{QE, subt}}$		1.2402
E^{DIS}	-0.28263	
$E^{\text{DIS, nonsubt}}$		-0.421752
$E^{\text{DIS, subt}}$		0.139122
— — — — — —	— — — — —	
E (total TPE)	-12.2378	

PRELIMINARY

Results

- Energies in meV

	Totals	Subtotals	Hernandez et al. (2016)
E^{elastic}	-6.85986		–6.83(28)
$E^{\text{elas, nonsubt.}}$		-6.89774	
$E^{\text{elas, subt}}$		0.0378778	
E^{QE}	-5.09527		–2.36(14)
$E^{\text{QE, nonsubt}}$		-6.33547	
$E^{\text{QE, subt}}$		1.2402	
E^{DIS}	-0.28263		–0.38(22)
$E^{\text{DIS, nonsubt}}$		-0.421752	
$E^{\text{DIS, subt}}$		0.139122	
— — — — — —	— — — — —		— — — — —
E (total TPE)	-12.2378		–9.58(38)

PRELIMINARY

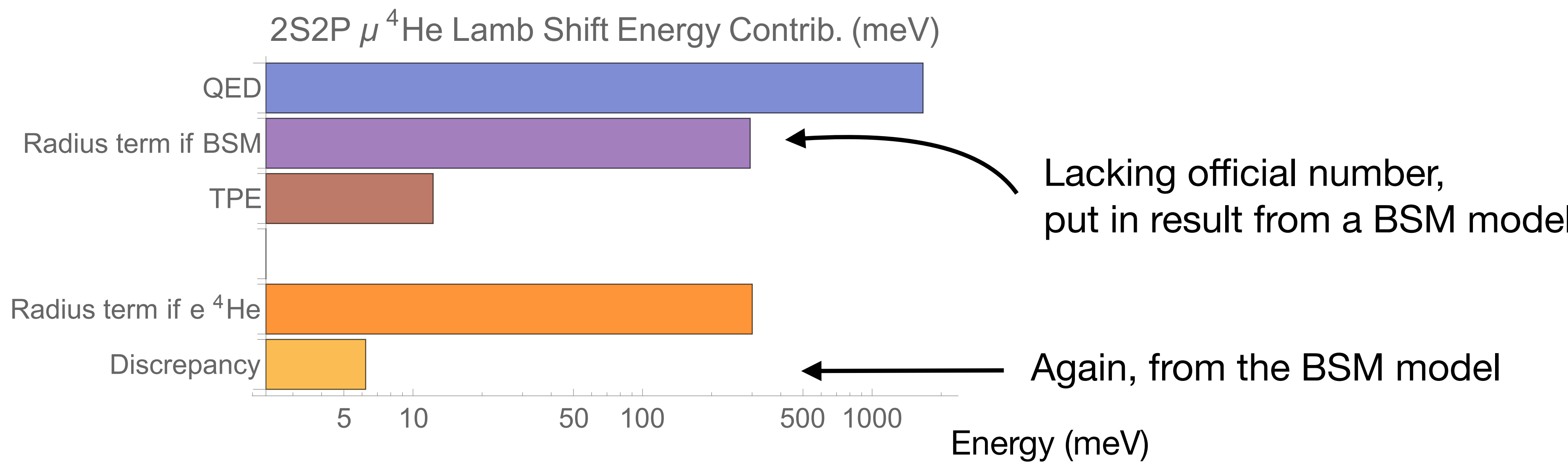
Reminder results for ^{3}He

- Energies in meV

	Totals (^{3}He)	Subtotals	Hernandez et al. (2016)
E^{elastic}	−10.93(27)		−11.01(24)
$E^{\text{elas, nonsubt.}}$			
$E^{\text{elas, subt}}$			
E^{QE}	−4.11(42)		−4.17(17)
$E^{\text{QE, nonsubt}}$		−5.50(40)	
$E^{\text{QE, subt}}$		1.39(12)	
E^{DIS}	−0.10(4)		−0.28(12)
$E^{\text{DIS, nonsubt}}$		−0.31(2)	
$E^{\text{DIS, subt}}$		0.21(3)	
— — — — — —	— — — — —		— — — — —
E (total TPE)	−15.14 (49)		−15.46 (39)

for ^{4}He

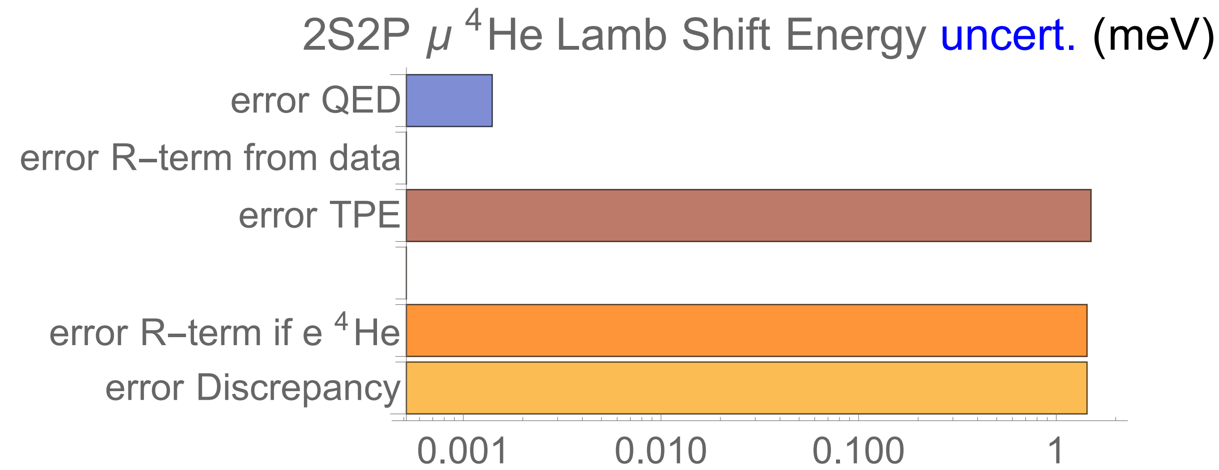
- Thinking about sizes of terms, and uncertainties therein, plot corresponding to previous ones,



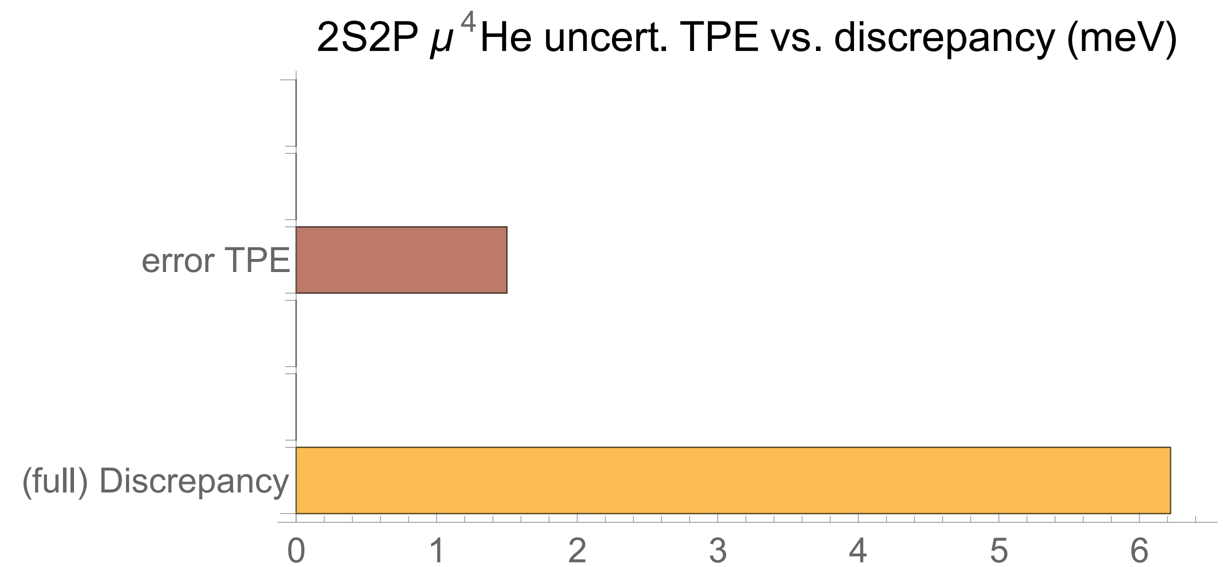
- Electron scattering ^{4}He radius is 1.681(4) fm [0.25%]

for ^{4}He

- μ 4He uncertainties (for $\delta E_{TPE} \approx 1.5$ meV)



- “Interesting plot,”



- TPE uncertainty no longer much smaller than size of discrepancy.
- But still smaller than energy deficit expected if puzzle has BSM explanation (and new force particle is somewhat heavy).
- But about same size as uncertainty in prediction from ^{4}He radius measured in electron scattering.

short BSM comment

- If the energy deficit seen in μH (0.307 meV) is due to the exchange of a new BSM boson, then by some expectation it scales to new nucleus like Z^4 and (reduced mass)^3. For ^{4}He,

$$\Delta E^{\alpha}_{\text{L, BSM}} = Z^4 \left(\frac{m_r^{\alpha}}{m_r^{p}}\right)^3 \Delta E^{p}_{\text{L, BSM}} \approx 6.22\,\text{meV}$$

Final comments

- Calculated two photon exchange energy contributions using dispersion theory and *e* Helium-4 scattering data. Comparable result to existing nuclear potential calculation, but somewhat larger in magnitude.

- "Your results are as good as your theory."

- How good is the theory? (Re: TPE corrections in ^{4}He)

  - One view: crummy. The size of the uncertainty is as large as the uncertainty coming from electronic radius measurements. For *d* or *p* were smaller by factor 20 or 150.

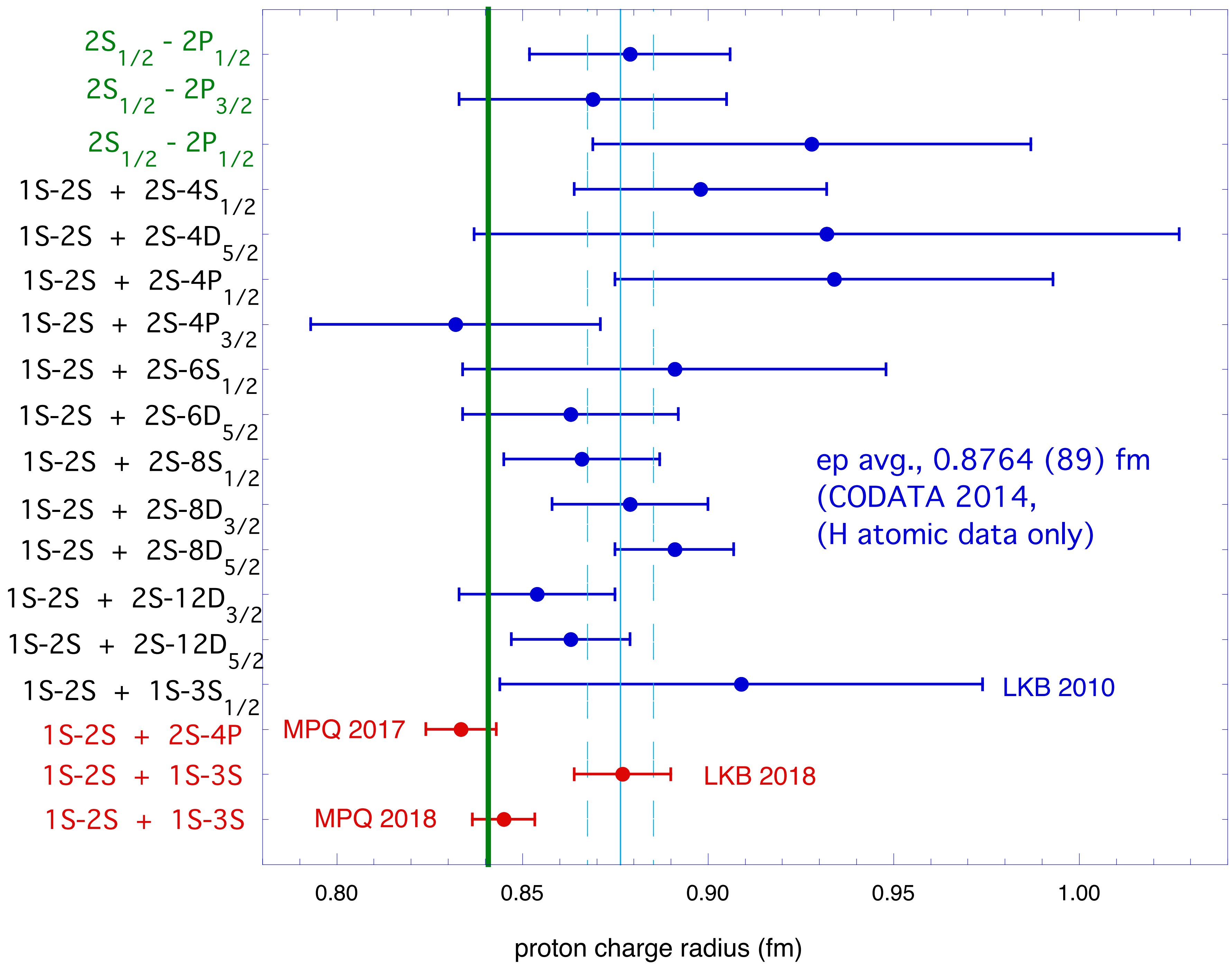
  - Another view: no so bad. Still can make clear distinction between no radius discrepancy and appearance of discrepancy whose size, relatively speaking, is comparable to the proton discrepancy.

- Results are improvable.

The end for today

after the end

Atomic measurements of R_E



Energy equations

$$\Delta E_{nL}^{\text{elas}} = 4Z^2\alpha^2 m_r \phi_{nL}^2(0)\left\{ 4\int_0^\infty \frac{dQ^2}{Q^3}G'(0) - \int_0^\infty \frac{dQ^2}{Q^4}\left[\frac{\gamma_2(\tau_\alpha)}{\sqrt{\tau_\alpha}} - \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}\right]\frac{G^2(Q^2)-1}{M_\alpha - m}\right\}$$

$$\tau_\alpha = \frac{Q^2}{4M_\alpha^2},\ \tau_l = \frac{Q^2}{4m^2},\ \text{and } G'(0) = \left.\frac{dG(Q^2)}{dQ^2}\right|_{Q^2=0}$$

$$\gamma_1(\tau) = (1+\tau)^{1/2}(1-2\tau) + 2\tau^{3/2}$$

$$\gamma_2(\tau) = (1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\tau^{1/2}$$

Energy equations

$$\Delta E_{nL}^{\text{inel}} = -\frac{2\alpha^2}{mM_\alpha}\phi_{nL}^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_0}^\infty \frac{d\nu}{\nu}\left[\tilde{\gamma}_1(\tau,\tau_l)F_1(\nu,Q^2) + \frac{M_\alpha \nu}{Q^2}\tilde{\gamma}_2(\tau,\tau_l)F_2(\nu,Q^2)\right]$$

$$\tilde{\gamma}_1(\tau,\tau_l) = \frac{1}{\tau_l - \tau}\left(\sqrt{\tau_l}\,\gamma_1(\tau_l) - \sqrt{\tau}\,\gamma_1(\tau)\right)$$

$$\tilde{\gamma}_2(\tau,\tau_l) = \frac{1}{\tau_l - \tau}\left(\frac{\gamma_2(\tau)}{\sqrt{\tau}} - \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}\right)$$

$$\tau = \frac{\nu^2}{Q^2}$$

$$\Delta E_{nL}^{\text{subt}} = \frac{4\pi\alpha^2}{m}\phi_{nL}^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\frac{\gamma_1(\tau_l)}{\sqrt{\tau_l}}\left(T_1(0,Q^2) - T_1(0,0)\right)$$

Proton radius from low Q^2 fits

- Proton radius accuracy from few parameter fits to *ep*→*ep*
- Following Jan Bernauer,

$$G_E \sim \sqrt{\frac{d\sigma}{d\Omega}} \sim 1 + \overset{\approx 6\ \mathrm{GeV}^{-2}}{A}Q^2 + \overset{\approx 30\ \mathrm{GeV}^{-4}}{B}Q^4 + \ldots$$

- Linear fit: Say want unknown Q^4 term under 1% of AQ^2. Using A, B estimates,

$$BQ^4 < 1\,\%\,AQ^2 \quad \text{or} \quad Q^2 < 0.01\frac{A}{B} = 0.002\ \mathrm{GeV}^2$$

- Then for proton radius or A to 1%,

$$A \approx \frac{1 - G_E}{Q^2} \quad \text{or} \quad \delta G_E = \delta A\, Q^2 \approx 0.01 A Q^2 \approx 0.01 \times 6 \times 0.002 = 0.012\,\%$$

- Best current data ca. 0.15%. Bernauer: “Good luck”

Proton radius from low Q^2 fits

- The real case: do linear plus quadratic (in Q^2) fits,

≈ 120 GeV^{-6}

$$G_E \sim \sqrt{\frac{d\sigma}{d\Omega}} \sim 1 + AQ^2 + BQ^4 + CQ^6 + \ldots$$

- Say want unknown CQ^6 under 1% of AQ^2,

$$CQ^6 < 1\,\%\,AQ^2 \quad \text{or} \quad Q^2 < \sqrt{0.01\frac{A}{C}} \approx \sqrt{0.0005\ \ldots} \approx 0.02\ \text{GeV}^2$$

- Then for proton radius or A to 1%,

$$\delta A \approx \frac{\delta G_E}{Q^2} \quad \text{or} \quad \delta G_E \approx 0.01 A Q^2 \approx 0.01 \times 6 \times 0.02 = 0.12\,\%$$

- Feasible