



# The contribution of axial-vector mesons to hyperfine structure of muonic hydrogen

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## ABSTRACT

The contribution from the axial-vector meson exchange to the potential of the muon–proton interaction in muonic hydrogen induced by anomalous axial-vector meson coupling to two photon state is calculated. It is shown that such contribution to the hyperfine splitting in muonic hydrogen is large and important for a comparison with precise experimental data. In the light of our result, the proton radius “puzzle” is discussed.

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## 1. Introduction

Seven years ago, the CREMA (Charge Radius Experiments with Muonic Atoms) Collaboration [1] measured very precisely the Lamb shift of muonic hydrogen. This event opened a new era of precise investigation of the energy spectrum of simple atoms. Furthermore, in the new experiments of this Collaboration with muonic deuterium and ions of muonic helium a charge radii of light nuclei were obtained with very high precision [2–4]. In the case of muonic hydrogen and muonic deuterium it was shown that obtained values of the charged radii are significantly different from those which were extracted from experiments with electronic atoms and in the scattering of the electrons with nuclei and were recommended for using by the CODATA [5]. At present, several experimental groups plan to measure the hyperfine structure (HFS) of various muonic atoms with more high precision [6–8]. This will make it possible to better understand the existing “puzzle” of the proton charge radius, to check the Standard Model with greater accuracy and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state. One way to overcome the crisis situation is a deeper theoretical analysis of the fine and hyperfine structure of muonic atom spectrum, in the verification of previously calculated contributions and the more accurate construction of the particle interaction op-

erator in quantum field theory, the calculation of new corrections whose value for muonic atoms can increase substantially in comparison with electronic atoms. The expected results will allow to get also a new very important information about the forces which are responsible for the structure of atoms. From the theory side it is urgently needed to study the possible effects of exchanges between muon and proton which can contribute to hyperfine structure of muonic hydrogen. One of such effects was considered in recent papers [9–12]. It arises from the effective pion exchange between muon and proton induced by coupling of the pion to two photons (see Fig. 1 (left)). Despite the fact that numerically such contribution was found to be rather small, it can be important for the interpretation of new data.

In this Letter we consider the additional contribution to hyperfine structure of muonic hydrogen which is related to the axial-vector mesons exchanges (see Fig. 1 (right)). We would like to point out, that one can expect the important contribution of this exchange to spin dependent part of muon–proton interaction because the exchange particle has the spin one. Furthermore, it is also well known that in the channel with quantum number  $1^{++}$  axial anomaly effects can play an important role and, particularly, these effects might be considered as a cornerstone to solve so-called “proton spin crisis” [13].

## 2. Axial-vector meson exchange contribution to muon–proton interaction induced by axial anomaly

One-photon exchange interaction in quantum electrodynamics gives the leading order contribution to the interaction operator in

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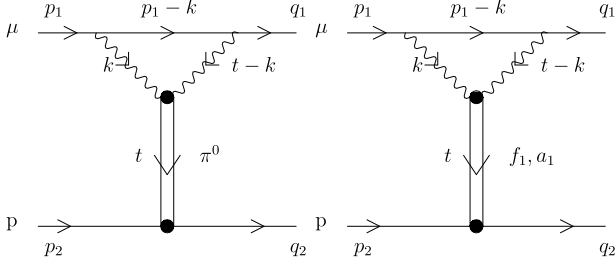


Fig. 1. Muon–proton interaction induced by mesonic exchange.

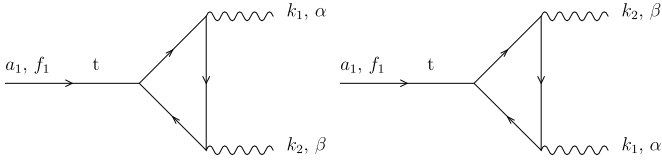


Fig. 2. Coupling of axial-vector mesons to two photons.

muonic hydrogen. The potential of hyperfine interaction has the following form [14]:

$$\Delta V_B^{hfs} = \frac{8\pi\alpha\mu_p}{3m_\mu m_p} (\mathbf{S}_p \mathbf{S}_\mu) \delta(\mathbf{r}) - \frac{\alpha\mu_p(1+a_\mu)}{m_\mu m_p r^3} [(\mathbf{S}_p \mathbf{S}_\mu) - 3(\mathbf{S}_p \mathbf{n})(\mathbf{S}_p \mathbf{n})] + \frac{\alpha\mu_p}{m_\mu m_p r^3} \left[ 1 + \frac{m_\mu}{m_p} - \frac{m_\mu}{2m_p \mu_p} \right] (\mathbf{L} \mathbf{S}_p) \quad (1)$$

where  $m_\mu$ ,  $\mathbf{S}_\mu$  and  $m_p$ ,  $\mathbf{S}_p$  are masses and spins of muon and proton, correspondingly,  $\mu_p$  is the proton magnetic moment. The potential (1) gives the main contribution of order  $\alpha^4$  to the hyperfine structure of muonic atom. Precision calculation of the hyperfine structure of the spectrum, which is necessary for a comparison with experimental data, requires the consideration of various corrections to the vacuum polarization, nuclear structure and recoil, and relativistic corrections [14–17]. We calculate further the contribution to HFS which is determined by the axial-vector  $f_1(1285)$ ,  $a_1(1260)$  and  $f_1(1420)$  meson exchanges shown in Fig. 1 (right).

The coupling of the axial-vector meson to two photon state is possible through anomalous triangle diagram, shown in Fig. 2. The general structure of this vertex takes the form [18–20]:

$$T^{\mu\nu\alpha} = 4\pi i\alpha \varepsilon_{\rho\sigma\tau\alpha} \left\{ \left[ v \left( A_3 k_1^\tau - \tilde{A}_3 k_2^\tau \right) + k_2^2 A_4 k_1^\tau - k_1^2 \tilde{A}_4 k_2^\tau \right] g^{\mu\rho} g^{\sigma\nu} + A_3 k_1^\nu k_1^\rho k_2^\sigma g^{\tau\mu} - \tilde{A}_3 k_2^\mu k_1^\rho k_2^\sigma g^{\tau\nu} + A_4 k_2^\nu k_1^\rho k_2^\sigma g^{\tau\mu} - \tilde{A}_4 k_1^\mu k_1^\rho k_2^\sigma g^{\tau\nu} \right\}. \quad (2)$$

where  $A_i \equiv A_i(t^2, k_1^2, k_2^2)$ ,  $\tilde{A}_i \equiv A_i(t^2, k_2^2, k_1^2)$ . Another form of the tensor describing the transition from initial state of two virtual photons with four-momenta  $k_1$ ,  $k_2$  to an axial-vector meson  $A$  ( $J^{PC} = 1^{++}$ ) with the mass  $M_A$  is presented in [21]<sup>1</sup>:

$$T^{\mu\nu\alpha} = 4\pi i\alpha \varepsilon_{\rho\sigma\tau\alpha} \times \left[ R^{\mu\rho}(k_1, k_2) R^{\nu\sigma}(k_1, k_2) (k_1 - k_2)^\tau v F_{A\gamma^*\gamma^*}^{(0)}(k_1^2, k_2^2) + \right.$$

$$\left. + R^{\nu\rho}(k_1, k_2) \left( k_1^\mu - \frac{k_1^2}{v} k_2^\mu \right) k_1^\sigma k_2^\tau F_{A\gamma^*\gamma^*}^{(1)}(k_1^2, k_2^2) + R^{\mu\rho}(k_1, k_2) \left( k_2^\nu - \frac{k_2^2}{v} k_1^\nu \right) k_2^\sigma k_1^\tau F_{A\gamma^*\gamma^*}^{(1)}(k_2^2, k_1^2) \right], \quad (3)$$

$$v = (k_1 k_2) = \frac{1}{2} [(k_1 + k_2)^2 - k_1^2 - k_2^2] = \frac{1}{2} [t^2 - k_1^2 - k_2^2],$$

$$R^{\mu\nu} = R^{\nu\mu}$$

$$= -g^{\mu\nu} + \frac{1}{X} \left[ (k_1 k_2) (k_1^\mu k_2^\nu + k_1^\nu k_2^\mu) - k_1^2 k_2^\mu k_2^\nu - k_2^2 k_1^\mu k_1^\nu \right],$$

$$X = (k_1 k_2)^2 - k_1^2 k_2^2 = \frac{1}{4} \left[ t^4 - 2t^2(k_1^2 + k_2^2) + (k_1^2 - k_2^2)^2 \right].$$

The relation between  $A_i$ ,  $\tilde{A}_i$  and form factors in (3) is the following:

$$F_{A\gamma^*\gamma^*}^{(1)}(t^2, k_1^2, k_2^2) = \frac{v}{X} \left[ k_2^2 (\tilde{A}_3 - A_4) + v (\tilde{A}_4 - A_3) \right], \quad (4)$$

$$F_{A\gamma^*\gamma^*}^{(1)}(t^2, k_2^2, k_1^2) = \frac{v}{X} \left[ k_1^2 (A_3 - \tilde{A}_4) + v (A_4 - \tilde{A}_3) \right]$$

$$F_{A\gamma^*\gamma^*}^{(0)}(t^2, k_1^2, k_2^2) = \frac{1}{v(k_1^2 - k_2^2)} \left[ (v A_3 + k_2^2 A_4) (k_1^2 + v) - (k_1^2 \tilde{A}_4 + v \tilde{A}_3) (k_2^2 + v) \right].$$

The form factors  $F_{A\gamma^*\gamma^*}^{(0)}(k_1^2, k_2^2)$  and  $F_{A\gamma^*\gamma^*}^{(1)}(k_1^2, k_2^2)$  entering in (3) are dependent on the squares of the 4-momenta of virtual photons. With increasing  $k_1^2$ ,  $k_2^2$ , these functions must decrease rapidly to ensure the ultraviolet convergence of the loop integral in the interaction amplitude.

We should mention that in the opposite to the case of pion coupling to two photons, axial-vector meson can not decay into two real photons, according to the Landau–Yang theorem [22,23]. Nevertheless, the coupling of  $1^{++}$  mesons to two photons is still possible in the case when one or both photons are virtual. For small values of relative momenta of particles in the initial and final states and small value of transfer momentum  $t$  between muon and proton, the transition amplitude presented in (2) takes a simple form

$$T^{\mu\nu\alpha} = 8\pi i\alpha \varepsilon_{\mu\nu\alpha\tau} k^\tau k^2 F_{AV\gamma^*\gamma^*}^{(0)}(t^2, k^2, k^2), \quad (5)$$

where  $k = k_1 = -k_2$ . To extract HFS part of the interaction in the case of the S-states the following projection operators are used for states with spin  $S = 0$  and  $S = 1$  [24]:

$$\hat{\Pi}_{S=0}[u(0)\bar{v}(0)]_{S=0} = \frac{1 + \gamma^0}{2\sqrt{2}} \gamma_5,$$

$$\hat{\Pi}_{S=1}[u(0)\bar{v}(0)]_{S=1} = \frac{1 + \gamma^0}{2\sqrt{2}} \hat{\varepsilon}, \quad (6)$$

where  $\varepsilon^\mu$  is the polarization vector of  $^3S_1$  state. The amplitude of the muon–proton interaction presented in Fig. 1 (right) has the following structure:

$$i\mathcal{M} = [\bar{l}(q_1) \Gamma_\alpha^{(\mu)} l(p_1)] \mathcal{D}^{\alpha\beta}(t) [\bar{N}(p_2) \Gamma_\beta^{(p)} N(q_2)], \quad (7)$$

where the vertex operator in the proton line is fixed by the Hamiltonian of nucleon–axial-vector meson interaction

$$H_I(a_1 NN) = g_{a_1 NN} \bar{N} \boldsymbol{\tau} \gamma_\mu \gamma_5 N \mathbf{a}_1^\mu, \quad (8)$$

for  $a_1$  exchange and

$$H_I(f_1 NN) = g_{f_1 NN} \bar{N} \gamma_\mu \gamma_5 N f_1^\mu \quad (9)$$

<sup>1</sup> The only difference between our expression (3) and their work is related to the normalized factor  $1/M_A^2$  used in [21].

for  $f_1$  exchange. The vertex operator in the lepton line  $\Gamma_\alpha^{(\mu)}$  is fixed by the integration over  $k$  in photonic loop, and  $\mathcal{D}^{\alpha\beta}(t)$  is the axial-vector meson propagator. It is easy to show that

$$\Gamma_\alpha^{(\mu)} \sim \gamma_\alpha \gamma_5, \quad (10)$$

and using the relations [13]

$$\begin{aligned} \bar{N}(P, S_p) \gamma_\tau \gamma_5 N(P, S_p) &= 2S_p^\tau, \\ \bar{l}(q, S_\mu) \gamma_\tau \gamma_5 l(q, S_\mu) &= 2S_\mu^\tau, \end{aligned} \quad (11)$$

one can see that the interaction (7) contains the spin–spin interaction,  $\mathcal{M} \sim \mathbf{S}_p \mathbf{S}_\mu$  which contributes to hyperfine splitting. Performing the projection of the amplitude (7) to the two particle states with the help of (6), we obtain that the numerator of the one-meson exchange amplitude (see Fig. 1 (right)) contains a trace of the product of the Dirac gamma-matrices and numerous convolutions by the Lorentz indices:

$$\begin{aligned} \mathcal{N}_{\mathcal{A}} &= 8\pi i \alpha \varepsilon_{\rho\sigma\tau\alpha} g^{\mu\rho} g^{\nu\sigma} k^\tau k^2 g^{\alpha\beta} F_{AV\gamma^*\gamma^*}^{(0)}(t^2, k^2, k^2) g_{AVNN} \times \\ & \quad \text{Tr} \left[ (\hat{q}_1 + m_\mu) \gamma^\nu (\hat{p}_1 - \hat{k} + m_\mu) \gamma^\mu (\hat{p}_1 + m_\mu) \times \right. \\ & \quad \left. \frac{1 + \gamma_0}{2\sqrt{2}} \hat{\varepsilon}(\hat{p}_2 - m_p) \gamma_\beta \gamma_5 (\hat{q}_2 - m_p) \hat{\varepsilon}^* \frac{1 + \gamma_0}{2\sqrt{2}} \right], \end{aligned} \quad (12)$$

where  $p_{1,2}$  ( $q_{1,2}$ ) are initial (final) momenta of the muon and proton. For the case of spin zero state the substitution  $\hat{\varepsilon} \rightarrow \gamma_5$  should be done in (12). Introducing the total and relative momenta of particles in the initial and final states  $p = (0, \mathbf{p})$  and  $q = (0, \mathbf{q})$ , and taking into account their smallness in the bound state ( $|\mathbf{p}| \sim \mu\alpha$ ,  $|\mathbf{q}| \sim \mu\alpha$ ) ( $\mu$  is the reduced mass), we can obtain the leading order contribution to  $\mathcal{N}_{\mathcal{A}}$  which does not have terms proportional to the powers of transfer momentum  $t = p - q$ . Our result for hyperfine part of the potential is the following:

$$\begin{aligned} \Delta V_{AV}^{hfs}(\mathbf{p} - \mathbf{q}) & \quad (13) \\ &= -\frac{32\alpha^2 g_{AVpp}}{3\pi^2(\mathbf{t}^2 + M_A^2)} \int id^4k \frac{(2k^2 + k_0^2)}{k^2(k^2 - 2m_\mu k_0)} F_{AV\gamma^*\gamma^*}^{(0)}(0, k^2, k^2). \end{aligned}$$

After an analytical integration in (13) over angular variables more simple formula for the potential can be obtained:

$$\begin{aligned} \Delta V_{AV}^{hfs}(\mathbf{p} - \mathbf{q}) & \quad (14) \\ &= -\frac{32\alpha^2 g_{AVpp}}{3\pi^2(\mathbf{t}^2 + M_A^2)} \int_0^\infty dk^2 L_\mu(k^2) F_{AV\gamma^*\gamma^*}^{(0)}(0, k^2, k^2), \end{aligned}$$

$$L_\mu(k^2) = \frac{\pi^2}{8m_\mu^4} \left[ k^2(k^2 - 6m_\mu^2) - (k^2 - 8m_\mu^2) \sqrt{k^2(k^2 + 4m_\mu^2)} \right],$$

where the kernel  $L_\mu(k^2)$  behaves as  $\sim 2\pi^2 \sqrt{k^2}/m_\mu$  for small  $k^2$  while the asymptotic value for large  $k^2$  is  $9\pi^2/4$ . Therefore  $L_\mu(k^2)$  effectively suppresses the region of small  $k^2$ .

### 3. Model estimations

One of the main ingredients in (13) is the form factor of transition of  $1^{++}$  meson to two photons  $F_{AV\gamma^*\gamma^*}(t^2, k^2, k^2)$ . Unfortunately, at present we have only few experimental data on it [25–27]. In the paper [25] of the L3 Collaboration the reaction  $e^+e^- \rightarrow e^+e^-\gamma^*\gamma^* \rightarrow e^+e^-f_1(1285) \rightarrow e^+e^-\eta\pi^+\pi^-$  was studied and  $f_1(1285)$  transition form factor was measured for the case when one of the photons is real and another one is virtual. In [26]

the production of  $f_1(1420)$  was investigated by the same Collaboration in the reaction  $\gamma^*\gamma^* \rightarrow K_S^0 K^\pm \pi^\mp$ . Using these data, we can parameterize the transition form factor for the case of two photons with equal virtualities as

$$F_{AV\gamma^*\gamma^*}^{(0)}(M_A^2, k^2, k^2) = F_{AV\gamma^*\gamma^*}^{(0)}(M_A^2, 0, 0) F_{AV}^2(k^2), \quad (15)$$

where

$$F_{AV}(k^2) = \frac{\Lambda_A^4}{(\Lambda_A^2 - k^2)^2}. \quad (16)$$

It should be mentioned that in comparison with the case of light  $\pi^0$  exchange, the effects of off-shellness for the exchange by massive  $f_1$  mesons might be important. The effect of off-shellness was investigated in [28,29], and in [30] a simple parametrization was proposed. The simplest way to take it into account is to introduce an exponential suppressive factor [30]:

$$\frac{F_{AV\gamma^*\gamma^*}^{(0)}(t^2, 0, 0)}{F_{AV\gamma^*\gamma^*}^{(0)}(M_A^2, 0, 0)} \approx e^{(t^2 - M_A^2)/M_A^2}, \quad (17)$$

which gives the factor  $\sim e^{-1}$  for  $t^2 \approx 0$ . The values of the form factors in (15) for the case of  $f_1(1285)$  and  $f_1(1420)$  can be fixed from the L3 data using the relations given by the nonrelativistic quark model [21]:

$$F_{AV\gamma^*\gamma^*}^{(0)}(M_A^2, 0, 0) = -F_{AV\gamma^*\gamma^*}^{(1)}(M_A^2, 0, 0) \quad (18)$$

and

$$\tilde{\Gamma}_{\gamma^*\gamma^*}(AV) = \frac{\pi\alpha^2 M_A^5}{12} [F_{AV\gamma^*\gamma^*}^{(1)}(M_A^2, 0, 0)]^2, \quad (19)$$

where  $\tilde{\Gamma}_{\gamma^*\gamma^*}(AV)$  is the decay width of axial-vector meson. We would like to mention that according to the nonrelativistic quark model the sign of  $F_{AV\gamma^*\gamma^*}^{(0)}(M_A^2, 0, 0)$  should be positive [31]. Finally, we obtain from the L3 data:

$$\begin{aligned} F_{f_1(1285)\gamma^*\gamma^*}^{(0)}(M_{f_1(1285)}^2, 0, 0) &= (0.266 \pm 0.043) \text{ GeV}^{-2}, \\ F_{f_1(1420)\gamma^*\gamma^*}^{(0)}(M_{f_1(1420)}^2, 0, 0) &= (0.193 \pm 0.041) \text{ GeV}^{-2}. \end{aligned} \quad (20)$$

The value of form factor  $F_{f_1(1285)\gamma^*\gamma^*}^{(0)}(M_{f_1(1285)}^2, 0, 0)$  can be estimated also within nonrelativistic quark model [31] using the relation

$$F_{f_1\gamma^*\gamma^*}^{(0)}(M_{f_1}^2, 0, 0) = 24 \langle e_q^2 \rangle R'(0) \frac{\sqrt{2}}{\sqrt{\pi} M_A^{9/2}}, \quad (21)$$

where  $R'(0)$  is the derivative of the radial wave function at the origin,  $\langle e_q^2 \rangle$  is effective quark charge squared in the bound state.

For the isospin  $I = 1$  state  $(u\bar{u} - d\bar{d})/\sqrt{2}$  ( $a_1$  meson) we have  $\langle e_q^2 \rangle = \sqrt{2}/6$ , and for the isosinglet state  $(u\bar{u} + d\bar{d})/\sqrt{2}$  ( $f_1$  meson)  $\langle e_q^2 \rangle = 5\sqrt{2}/18$ . The value of  $R'(0)$  can be estimated from the decay width  $f_2(1270) \rightarrow \gamma + \gamma$  [32] 3.034 keV by means of the expression

$$\Gamma(f_2(1270) \rightarrow \gamma^*\gamma^*) = \frac{576}{5} \alpha^2 \langle e_q^2 \rangle \frac{|R'(0)|^2}{M_A^4}, \quad (22)$$

assuming that the radial wave functions for  $f_1(1285)$  and  $f_2(1270)$  at the origin are the same. The equation (22) leads to  $R'(0) \approx 0.099 \text{ GeV}^{5/2}$  and

$$F_{f_1(1285)\gamma^*\gamma^*}^{(0)}(M_{f_1(1285)}^2, 0, 0) \approx 0.240 \text{ GeV}^{-2}, \quad (23)$$

**Table 1**  
Axial-vector meson contributions to hyperfine structure of muonic hydrogen.

AV meson	$I^G(J^{PC})$	$\Lambda_A$ in GeV	$g_{AVpp}$	$F_{AV\gamma^*\gamma^*}^{(0)}$ in $\text{GeV}^{-2}$	$\Delta E^{hfs}(1S)$ in meV	$\Delta E^{hfs}(2S)$ in meV
$f_1(1285)$	$0^+(1^{++})$	1.040	3.40	0.266	$-0.0090 \pm 0.0033$	$-0.0011 \pm 0.0004$
$a_1(1260)$	$1^-(1^{++})$	1.040	5.67	0.160	$-0.0094 \pm 0.0038$	$-0.0012 \pm 0.0005$
$f_1(1420)$	$0^+(1^{++})$	0.926	1.51	0.193	$-0.0019 \pm 0.0011$	$-0.0002 \pm 0.0001$

which is very close to the L3 value (20). Therefore, one can believe that nonrelativistic quark model describes the dynamics of axial-vector mesons rather well. However, below we will use the L3 value for  $F_{f_1(1285)\gamma^*\gamma^*}^{(0)}$  form factor to decrease the dependence of our predictions from the model. Unfortunately, there is no data for  $a_1(1260)$  meson production in  $\gamma^*\gamma^*$  collisions. We estimate  $F_{a_1\gamma^*\gamma^*}^{(0)}(M_{a_1}^2, 0, 0)$  using the diagram presented in Fig. 2 and introducing the value of quark-meson couplings  $g_{a_1qq}$  and  $g_{f_1qq}$ . The chiral symmetry gives the relation  $g_{a_1qq} = g_{f_1qq}$  (see, for example [33]). Finally, the ratio of  $a_1(1260)$  and  $f_1(1285)$  form factors in this case should be equal to the ratio of the effective quark charges squared for  $f_1(1285)$  and  $a_1(1260)$ :

$$\frac{F_{a_1(1260)\gamma^*\gamma^*}^{(0)}(M_{a_1(1260)}^2, 0, 0)}{F_{f_1(1285)\gamma^*\gamma^*}^{(0)}(M_{f_1(1285)}^2, 0, 0)} \approx \frac{3}{5}. \quad (24)$$

Then we obtain from (20) and (24):

$$F_{a_1(1260)\gamma^*\gamma^*}^{(0)}(M_{a_1(1260)}^2, 0, 0) \approx 0.160 \text{ GeV}^{-2}. \quad (25)$$

Our potential (14) of hyperfine interaction can be rewritten in the form:

$$\Delta V_{AV}^{hfs}(\mathbf{p} - \mathbf{q}) = -\frac{32\alpha^2 g_{AVpp} F_{AV\gamma^*\gamma^*}^{(0)}(0, 0, 0)}{3\pi^2(\mathbf{t}^2 + M_A^2)} I\left(\frac{m_\mu}{\Lambda_A}\right), \quad (26)$$

where  $I(m_\mu/\Lambda_A)$  is a convolution of the kernel  $L_\mu(k^2)$  and form-factor  $F_A^2(k^2)$  which are dependent on the muon mass  $m_\mu$  and hadron scale  $\Lambda_A$  correspondingly ( $a_\mu = 2m_\mu/\Lambda_A$ ):

$$I\left(\frac{m_\mu}{\Lambda_A}\right) = -\int_0^\infty dk^2 L_\mu(k^2) F_A^2(k^2) \\ = -\frac{\pi^2 \Lambda_A^2}{4(1 - a_\mu^2)^{5/2}} \left[ 3\sqrt{1 - a_\mu^2} - a_\mu^2(5 - 2a_\mu^2) \ln \frac{1 + \sqrt{1 - a_\mu^2}}{a_\mu} \right]. \quad (27)$$

Making the Fourier transform of (26) and averaging the obtained expression with the wave functions of the 1S and 2S states, we obtain the following contribution to hyperfine splitting coming from the axial-vector exchange:

$$\Delta E_{AV}^{hfs}(1S) = \frac{32\alpha^5 \mu^3 g_{AVpp} F_{AV\gamma^*\gamma^*}^{(0)}(0, 0, 0)}{3M_A^2 \pi^3 \left(1 + \frac{2W}{M_A}\right)^2} I\left(\frac{m_\mu}{\Lambda_A}\right), \quad (28)$$

$$\Delta E_{AV}^{hfs}(2S) = \frac{2\alpha^5 \mu^3 g_{AVpp} F_{AV\gamma^*\gamma^*}^{(0)}(0, 0, 0) \left(2 + \frac{W^2}{M_A^2}\right)}{3M_A^2 \pi^3 \left(1 + \frac{W}{M_A}\right)^4} I\left(\frac{m_\mu}{\Lambda_A}\right), \quad (29)$$

where  $W = \mu\alpha$  and  $\mu$  is the reduced mass.

For numerical estimate we fix the slope of form factors according to the L3 data to  $\Lambda_{f_1(1285)} = 1.040 \pm 0.078$  GeV [25] and  $\Lambda_{f_1(1420)} = 0.926 \pm 0.078$  GeV [26], and assume that  $\Lambda_{a_1(1260)} \approx \Lambda_{f_1(1285)}$ . Unfortunately, there is no direct experimental data on the value of axial-vector meson couplings to the quarks and proton. Therefore, we estimate them using nonrelativistic quark model with chiral symmetry. One of the examples of such model is the NJL model [33]. Within this model the form factor of  $f_1(1285)$  meson can be obtained by the calculation of triangle diagram presented in Fig. 2:

$$F_{f_1(1285)\gamma^*\gamma^*}^{(0)}(M_{f_1(1285)}^2, 0, 0) = \frac{5g_{f_1(1285)qq}}{72\pi^2 m^2}, \quad (30)$$

where  $m$  is the dynamical quark mass related to the spontaneous chiral symmetry breaking.

Another couplings are related to each others by using chiral symmetry and  $SU(6)$ -model for wave function of the proton as follows:

$$g_{a_1(1260)qq} = g_{f_1(1285)qq}, \quad g_{f_1(1285)pp} = g_{f_1(1285)qq}, \\ g_{a_1(1260)pp} = \frac{5}{3} g_{f_1(1285)qq}. \quad (31)$$

In the most versions of quark models which are used in hadron spectroscopy, the value of quark mass is in the interval  $m \sim 0.25 \div 0.35$  GeV. At the central value  $m = 0.300$  GeV we get the following couplings:

$$g_{a_1(1260)qq} = g_{f_1(1285)qq} = g_{f_1(1285)pp} = 3.40 \pm 1.19, \\ g_{a_1(1260)pp} = 5.67 \pm 1.98. \quad (32)$$

The error in determining the interaction constants, which is at least 35 percent, is written out directly in (32). In the case of  $f_1(1420)$  meson one should take into account the singlet-octet mixing effects [26], [34]. The estimation given in [34] shows that the wave function of this meson in flavor space is equal

$$f_1(1420) \approx |\bar{s}s\rangle + \delta|n\bar{n}\rangle, \quad (33)$$

where  $n\bar{n} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$  and  $\delta \approx 0.4 \div 0.5$ . Therefore, for the proton wave function in the OZI limit one can neglect the interaction of strange component of  $f_1(1420)$  with proton and obtain the following estimation:

$$g_{f_1(1420)pp} \approx 1.36 \div 1.70. \quad (34)$$

The central value  $g_{f_1(1420)pp} = 1.51$  is taken for numerical estimate.

Our results for the contribution of the axial-vector mesons to HFS are presented in Table 1. For the case of both 1S and 2S states, the summary contribution of axial-vector meson exchanges is more than an order of magnitude greater than the contribution of pseudoscalar mesons and very important to obtain the total value of the HFS with high precision. We can use the obtained expressions (28)–(29) to estimate the similar contribution to the hyperfine structure of electronic hydrogen. In the case of the 1S



state, the total contribution of the axial vector mesons  $f_1$ ,  $a_1$  is about 0.8 kHz, which is comparable with the error in calculating the contribution to the proton polarizability [35].

#### 4. Conclusion

A new important contribution to the muon–nucleon interaction is discovered. It is determined by the effective axial-vector meson exchange induced by anomalous axial-vector meson vertex with two photon state. The contribution of this exchange to hyperfine structure of muonic hydrogen is calculated in the framework of quasipotential method in quantum electrodynamics and with the use of the technique of projection operators on states of two particles with a definite spin. It is shown that this contribution is large and should be taking into account for the interpretation of new data on HFS in muonic hydrogen. As has been mentioned above the CREMA Collaboration measured two transition frequencies in muonic hydrogen for the 2S triplet state ( $2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}$ ) and for the 2S singlet state ( $2P_{3/2}^{F=1} - 2S_{1/2}^{F=0}$ ) [2]. From these measurements it is possible to extract the value of hyperfine splitting of the 2S level. The obtained value  $\Delta E_{exp}^{hfs}(2S) = 22.8089(51)$  meV allows to get the value of the Zemach radius with accuracy 3.4%  $r_Z = 1.082(31)^{exp}(20)^{th}$  with the help of following relation:  $\Delta E_{th}^{hfs} = 22.9843(15) - 0.1621(10)R_Z$ . This is in the agreement with another numerical values  $r_Z = 1.086(12)$  fm [36],  $r_Z = 1.045(4)$  fm [37],  $r_Z = 1.047(16)$  fm [38],  $r_Z = 1.037(16)$  fm [39] obtained from electron–proton scattering and from hydrogen and muonium spectroscopy. At present the theory estimates of hadronic corrections to the 1S hyperfine splitting in muonic hydrogen are known with a precision near 400 ppm [3] (see more detailed analysis in a recent paper [40]). We should emphasize that the changing the theoretical value of the HFS on 0.001 meV leads to the changing of the Zemach radius on 0.006 fm. Therefore, our contribution coming from axial-vector meson exchange leads to new value of the radius  $R_Z = 1.067(37)$  fm, which is greater in the comparison with most listed results but still agree with them within errorbars.

The CREMA Collaboration have performed successively several experiments with muonic hydrogen. In the first experiment of 2010 [1], the frequency of a single  $2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}$  transition was measured. To extract a new value of the proton charge radius in this case, there was used the theoretical expression for the hyperfine splitting of the 2S-level in the form:

$$\Delta E_{th}(2P_{3/2}^{F=2} \div 2S_{1/2}^{F=1}) = 209.9779(49) - 5.2262r_p^2 + 0.0347r_p^3. \quad (35)$$

Since in this paper we are just calculating the hyperfine structure of the spectrum, our result could be related to the correction of the proton charge radius. But already in the experiment of 2013 [2] two transition frequencies were measured, which made it possible to find the experimental value of the hyperfine splitting of the 2S-level. The theoretical result was used for the hyperfine structure of the P-levels. Since the hyperfine splitting of the 2S-level can now be considered fixed from the experiment and used further, the theoretical contribution to the HFS obtained in this paper does not lead to a change in the proton charge radius, which remains equal to  $r_p = 0.84087(39)$  fm [2] and differs from the value recommended by CODATA-2014,  $r_p = 0.8751(61)$  fm [5], based on H spectroscopy and electron–proton scattering.

There are a number of uncertainties related to the main used parameters, among which the key role are played by the value of form factor  $F_{AV\gamma^*\gamma^*}^{(0)}(0, 0)$  (or  $R'(0)$ ) and coupling constants  $g_{AVpp}$ . The errors in determining the parameters  $\tilde{\Gamma}_{\gamma\gamma}$  and  $\Lambda_A$ , through

which  $R'(0)$  is expressed, are 25% and 8%, respectively. Another error of about 35% for the mesons  $f_1(1285)$ ,  $a_1(1260)$  and  $f_1(1420)$  is related to the magnitude of the interaction constants of the axial vector mesons with the nucleon. Therefore, from the experimental data and model approximations for constructing the transition form factor and the interaction potential of particles, we estimate the error in calculating the contribution of the axial vector mesons  $f_1(1285)$  to 35%,  $a_1(1260)$  to 40%, and the meson  $f_1(1420)$  in 60%. All theoretical errors are directly indicated in Table 1.

It is necessary to mention that our estimations are mainly based on the data of the L3 Collaboration on the transition form factors of the axial vector mesons in photon–photon interaction. These data are restricted by rather small kinematical region. A new, more detailed measurements of these form factors are urgently needed. Such type experiment is possible, for example by the BE-SIII and BELLEII Collaborations.

We also believe that it is important to investigate this new contribution to the hyperfine structure of the muonic deuterium and muonic helium. In this case it might be even possible to separate contributions coming from  $a_1$  and  $f_1$  mesons due to different isospin structure of these nuclei. The research in this direction is in progress.

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