Proton rms-radius from (e,e): a lacking consideration

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Recently published

review "Proton charge radius from electron scattering"
published in special issue of *ATOMS* 6, 2
"High precision measurements of fundamental constants"

Discussed

many aspects of R-determinations
studied/repeatd many analyses of other authors
goal: to understand reasons for differences

Select determinations that respect criteria (discussed below)
take grand average of R's from different models

Find $R = 0.887 \pm 0.012 fm$

Today: discuss only special aspect(s)
related to selection of approaches considered

Observation

scatter of results for rms-radius R

occurs even when using same data

indicates model-dependence of fits

Observation

scatter particularly large for q-space parameterizations

fits done without consideration of $\rho(r)$

actually: most parameterizations do not even correspond to a density!

Before addressing role/difficulties with $\rho(r)$

discuss example

Fit of e-p data (Bernauer) for $q < 2fm^{-1}$

includes all data sensitive to R

Parameterization [m/n]Pade: $G(q) = (1 + \sum^m a_i q^{2i})/(1 + \sum^n b_i q^{2i})$

successful in fits up to largest q_{max}: Kelly, Arrington, IS, ...

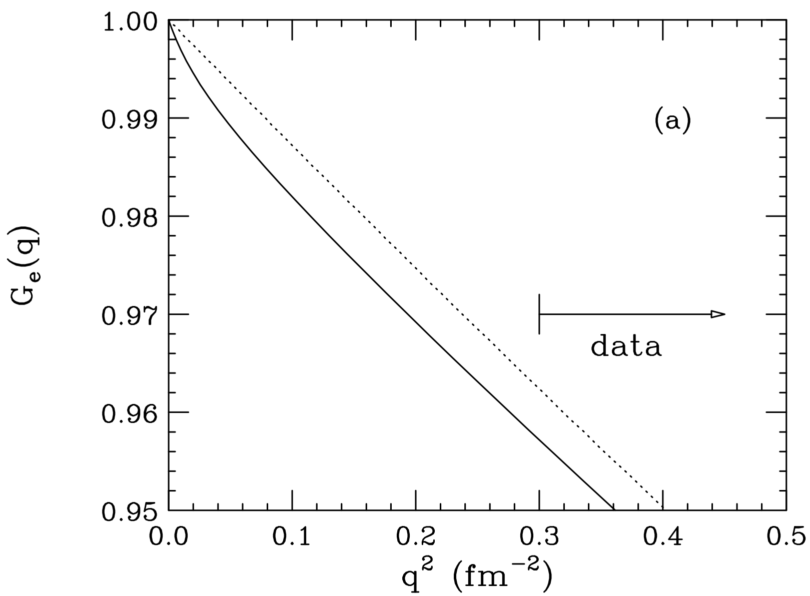
For $q_{max} = 2fm^{-1}$ $m = 1$, $n = 3$ enough for good fit

gives χ^2 as low as bestfit of Bernauer

has none of frequent diseases: poles, incorrect $q = \infty$ limit

Yields $R = 1.48fm$!

Reason: curvature of $G(q)$ at *very* low q, below q_{min}



Note: above $0.2fm^{-2}$ Pade and standard fit differ by a constant 0.5%

(note expanded scale)

Pade and standard fit have same χ^2 as data floating

How does [1/3]Pade generate $R = 1.48 fm$?

- a_1 and b_1 strongly coupled
- both large can produce behavior shown in figure

Is $1.48 fm$ reasonable?

- large coefficients cannot be excluded
  - some fits in literature have huge coefficients
- parameters are not physical
- [1/3]Pade as good a parameterization as any q-space parameterization

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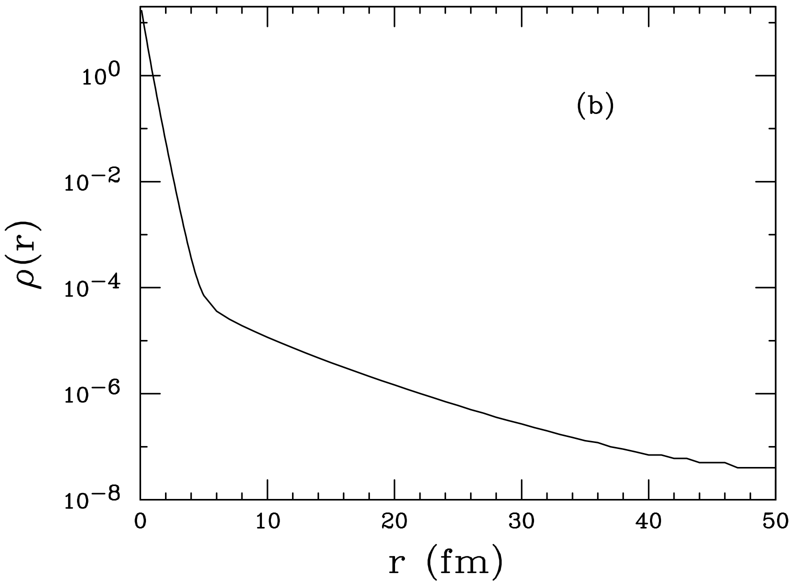
[1/3]Pade as good a parameterization as any q-space parameterization

The big difference

[1/3]Pade does have Fourier transform

can calculate $\rho(r)$

simple look at $\rho(r)$ excludes fit as unphysical



Elimination only possible because [1/3]Pade *has* FT

without considering $\rho(r)$ would not know about disease

Could published $G(q)$'s *without* look at $\rho(r)$ have similar problems?

or similar, 10 times smaller, problems (not obvious in $G(q)$)?

$\rho(r)$ ignored in almost all analyses of e-p data

R not obtained from $\int \rho(r)\, r^4\, dr$

R obtained from slope of $G(q=0)$ despite obvious problems:

– not measurable

– must extrapolate (always dangerous)

– doubly dangerous as need *slope* of extrapolated $G(q)$

Reason: $G(q)$= FT($\rho(r)$) only valid for non-relativistic recoil

modified if recoil relativistic

can ignore 'complication' if restrict attention to $q = 0$ slope

(illusion as extrapolate from finite q)

Relativistic corrections: 2 types (Licht70,Mitra77,Ji91,Holzwarth96)

1. Electron sees moving proton

must describe scattering in Breit frame (Lorentz contraction)

can be taken into account by using $\tilde{q}$ instead of q

$$\tilde{q} = q/\sqrt{1 + q^2/4M^2}$$

2. For composite systems additional correction

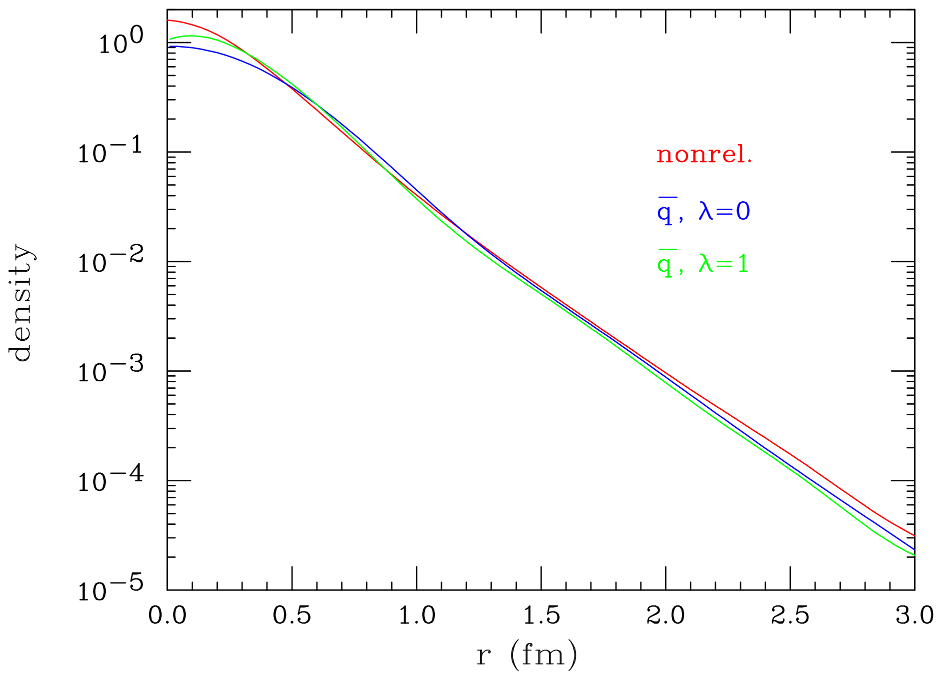
$$G \rightarrow \tilde{G} = G_e(1 + q^2/4M^2)^\lambda$$

different theories give, for charge-form factor, $\lambda = 0$ or 1

Numerical effect of relativistic corrections

start from [3/5]Pade fit of *world* data up to $10 fm^{-1}$, calculate

- $\rho(r)$ non-relativistically
- $\rho(r)$ using $\tilde{q}$
- $\rho(r)$ using $\tilde{q}$ and $\lambda = 1$



Result

important change at $r \sim 0$
hardly affects shape of $\rho(r > 1 fm)$

Despite relativistic corrections shape at large r remains well-defined, unambiguous
this $\rho(r)$ strongly affects R! (see below)

What do we know about $\rho(r > 1fm)$?

1. **Cloudy bag-type models**

    $r < 1fm$ complicated quark/gluon structure

    $r > 1fm$ dominated by Fock component with lowest separation energy: π+n

    asymptotic w.f. of pion $W_{-\eta,3/2}(2\kappa r)/r$

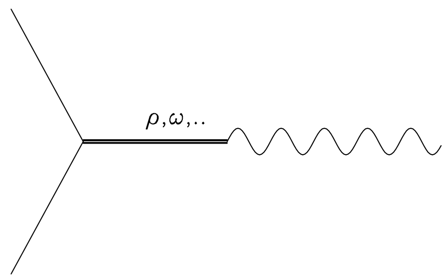
    can be used to calculate *shape* of $\rho(r)$ (norm from fit to (e,e))

    complications ($\pi + \Delta$) studied, small effect

    used extensively for $A \geq 2$

2. **Vector Dominance Model**

    basic assumption of VDM

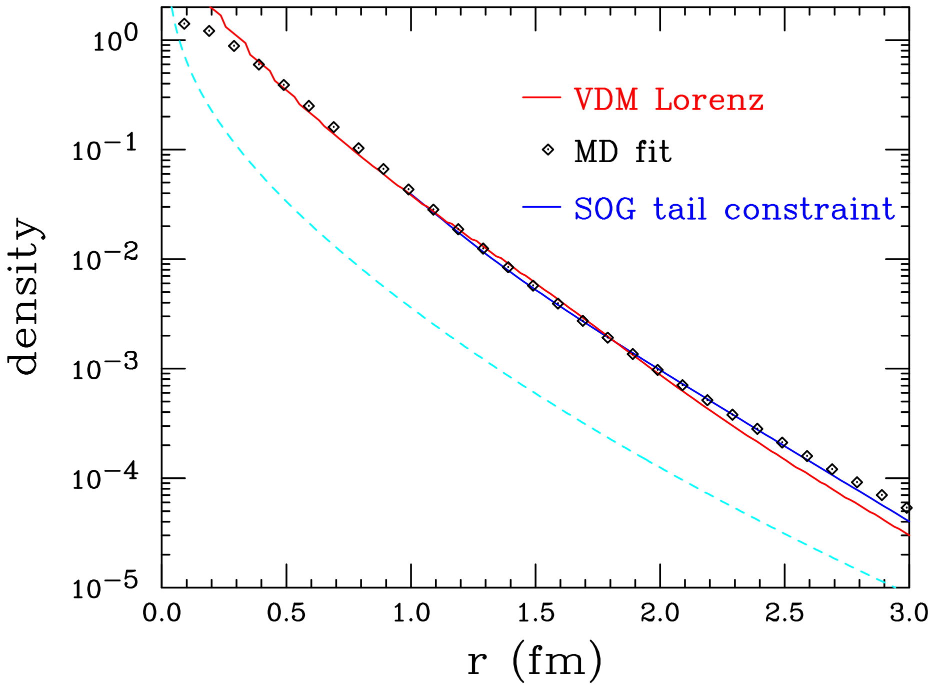


using known vector mesons and coupling constants

using dispersion relations to calculate 2π *ect* contributions (longest range)

Ina Lorenz, Bonn group

Comparison



large-r densities very similar

with or without rel. corrections

given by *understood physics*

shape should be $\pm$ respected in fits of data

shape-constraint most helpful in R-determination, see review in *Atoms* 6 (2018) 2

as $r > 1fm$ contributes ∼50% to R

Today: consider *much more elementary constraint*

$\rho(r > 3.5fm) \sim 0$ for practical purposes

this minimal constraint *is* important when aiming at R!

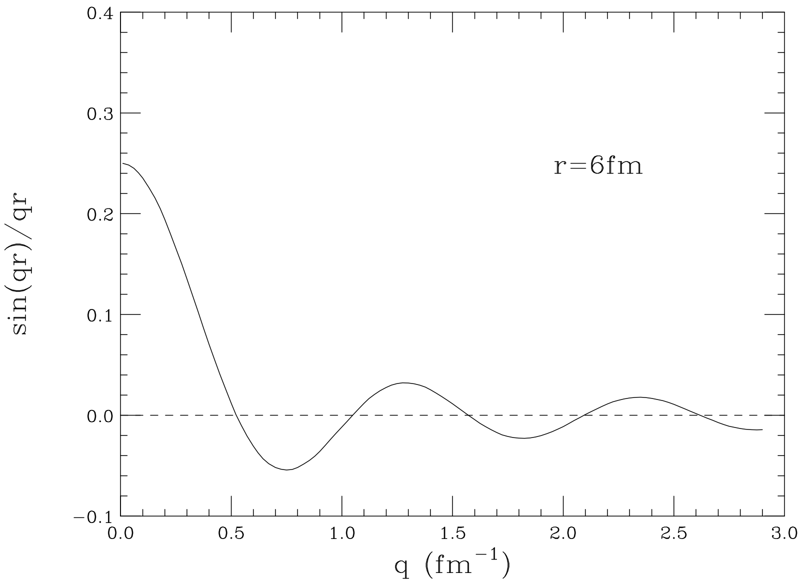
Take seriously as rel.corr. do not generate/remove contribution at $r > 3.5fm$

Non-zero ρ at large r problematic for R

large r have large weight in calculation of R

give largest contribution at small q: $G(q) = \int sin(qr)/qr \; \rho(r) r^2 dr$

Example: $\Delta G(q)$ for charge at $6fm$



1% contribution to R

$\Delta G(q > 0.5 fm^{-1}) < 0.0001$

biggest contribution at $q < 0.5 fm^{-1}$

this region is not covered by data

amplitude of $sin(qr)/qr$ term poorly determined

contributions from $r > 3.5fm$ add noise (model dependence) to R-determination

Important question: do published fits respect $\rho(r > 3.5fm) = 0$?

$\rho(r > 3.5fm) = 0$ easy to enforce/verify with

- parameterizations in r-space
- parameterizations in q-space falling faster than q^{-4}

  can take FT to check, eliminate if $\rho(r > 3.5) \neq 0$

Problem: what about all these $G(q)$'s that do *not* have a FT?

i.e. the vast majority of published $G(q)$'s

could contain $sin(qr)/qr$ components corresponding to $r > 3.5fm$

would give unphysical contributions to R

which would be poorly constrained by data

How can be verified?

how can make sure that *most elementary property* $\rho(r > 3.5fm) = 0$ respected?

Can be done by borrowing old idea from F. Lenz

Model-independent information from (e,e)

F. Lenz, Z. Physik 222 (1969) 491

Densities with same first moment function

$$T(Q) = \int_0^Q r(Q')dQ'$$

with Q = integrated charge between radii 0 and r

give same σ

Convenient representation

$$\rho(r) = \sum \frac{p_i}{r_i^2}\,\delta(r - r_i) \qquad \longrightarrow \qquad T(Q_i) = \sum_{j=1}^{i} p_j\, r_j$$

With enough δ-functions at $0 < r < r_{max}$ can represent $T(Q)$ to any accuracy desired

Consequence: can represent $G(q)$ with $\sum p_i\; sin(qr_i)/(qr_i)$

Decomposition of $G(q)$ into $sin(qr)/qr$-components
allows localization of charge in r *without* need for FT

Components $sin(qr_j)/qr_j$ with $r_j > 3.5 fm$ would imply unphysical contributions

Test of published $G(q)$'s

use $q_{max} = 1.5 fm^{-1}$ (covers range sensitive to R)

generate pseudo-data from $G(q)$

select r_i's uniformly distributed over range 0 ... $7 fm$
have tried several r-ranges for $r > 3.5$ contributions
range 3.5...$7 fm$ covers most relevant region

fit with $\sum p_i \, sin(qr_i)/qr_i$

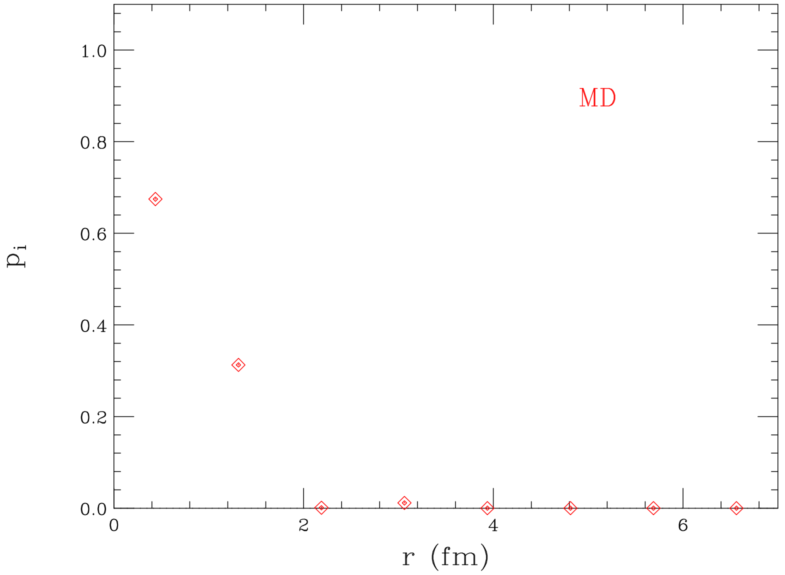
to avoid overfitting with correlated p_i:
constrain $p(r_i > 3.5 fm)$ to either > 0 or < 0

check $\sum p_i$ for $r_i > 3.5 fm$

Results

1. For G's corresponding to ρ's with $\rho(r > 3.5 fm) \sim 0$

contribution to R of $\sum p_i$'s for $r > 3.5 fm$ typically 0.8% (discretization noise)



fits used: MD, Pade, Laguerre, Borisyuk, VDM Mergel

2. Fits $G(q)$ *not* corresponding to $\rho(r)$

contribution to R of $\sum p_i$ for $r > 3.5 fm$ between 0 and ±20%

typically contribution $\sim \pm 10\%$!

fits used: Lee, HH, polynomial Bernauer, pol. Griffioen, inv. polynomial

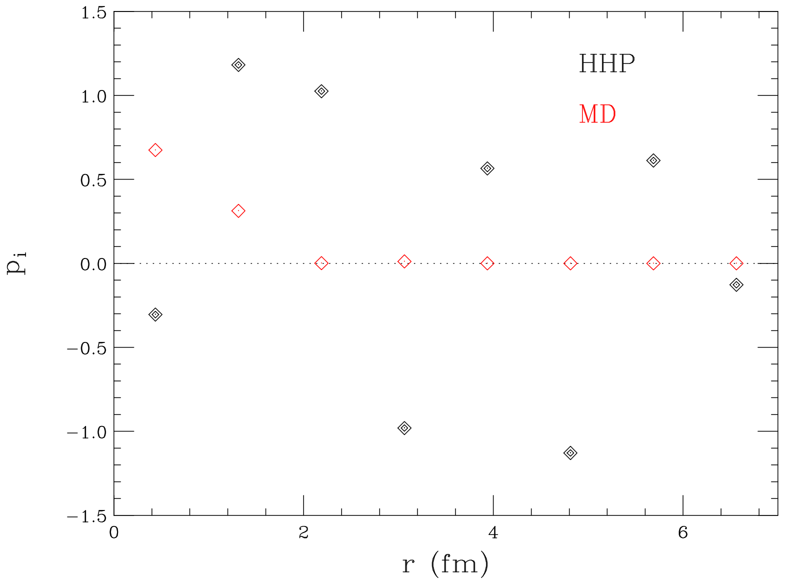
3. Special case: polynomials in q^2

get good fit only when at least one $p_i < 0$
confirms old insight of disease of low-order $\sum p_i q^{2i}$

4. Very special case: Horbatsch, Hessels, Pineda

extremely small r^4, disagrees with data (and common sense)
can only be fit with strange combination of p_i's

	$r^4/(r^2)^2$	$r^6/(r^2)^3$
Dipole	2.50	11.6
Bernauer fit	4.32	64.2
HHP	1.25	14.5



Conclusion

Parameterizations *without* $\rho(r)$ often contain $sin(qr)/qr$ contributions for $r > 3.5 fm$

they give significant contributions to R which

– depend on model used to parameterize $G(q)$

– are poorly constrained by data as main effect occurs at $q < q_{min}$

These unphysical contributions add 'noise' to R-determinations

can be avoided by parameterizing $\rho(r)$ instead of $G(q)$

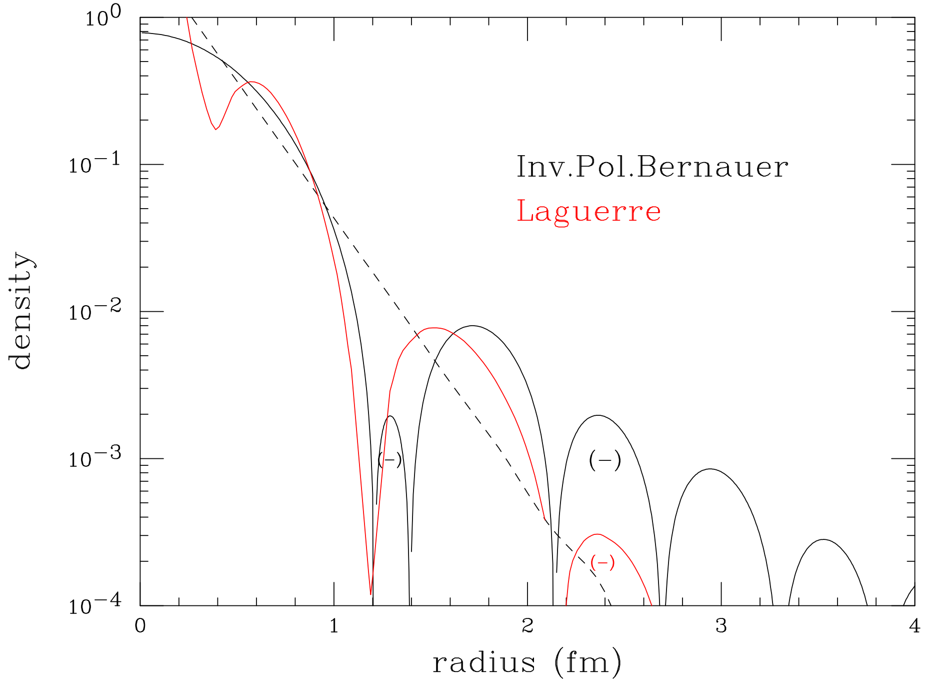
and imposing $\rho(r > 3.5 fm) = 0$

...... fixes problem of unphysical contributions to R even if $\rho(r)$ ignored

...... fixes the major disease of R-determinations from $G(q)$-parameterizations

Look at $\rho(r)$ also useful for

- **plausibility of fit**
- **potential problems with data**



Difference data world ... Bernauer

Ratio of cross section Bernauer/(Fit world) using Laguerre fit