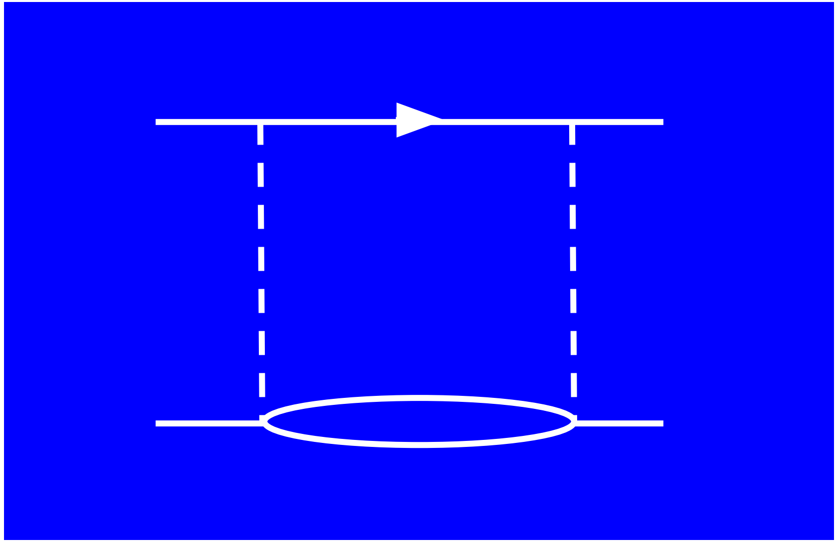
Presented By: Oscar Javier Hernandez

The deuteron-radius puzzle is alive: A new analysis of nuclear structure uncertainties



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Sonia Bacca

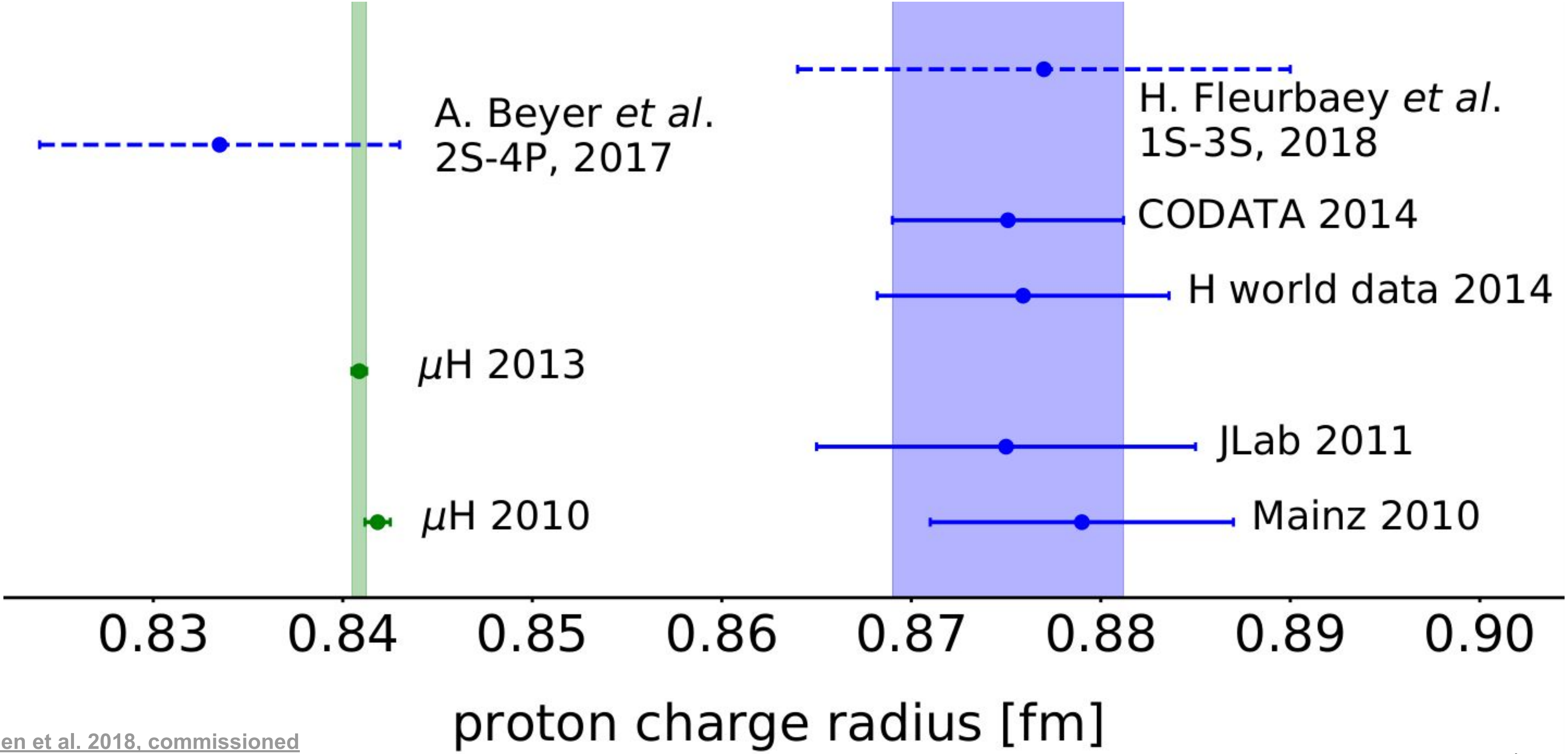
Nir Barnea

THE UNIVERSITY OF BRITISH COLUMBIA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

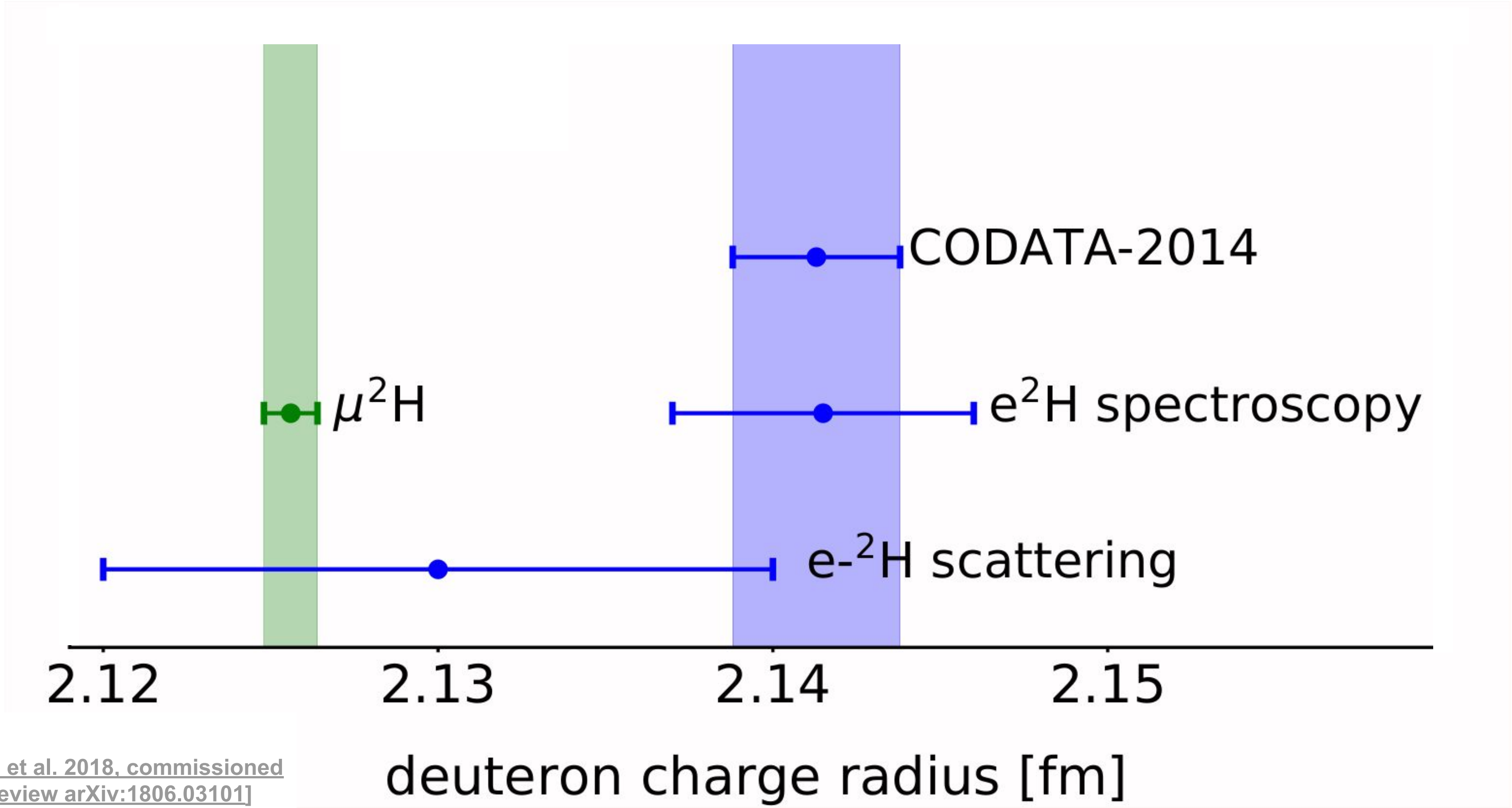
TRIUMF

The growing proton radius puzzle



[Chen et al. 2018, commissioned JPG review arXiv:1806.03101]

There is a discrepancy between eD and μD data



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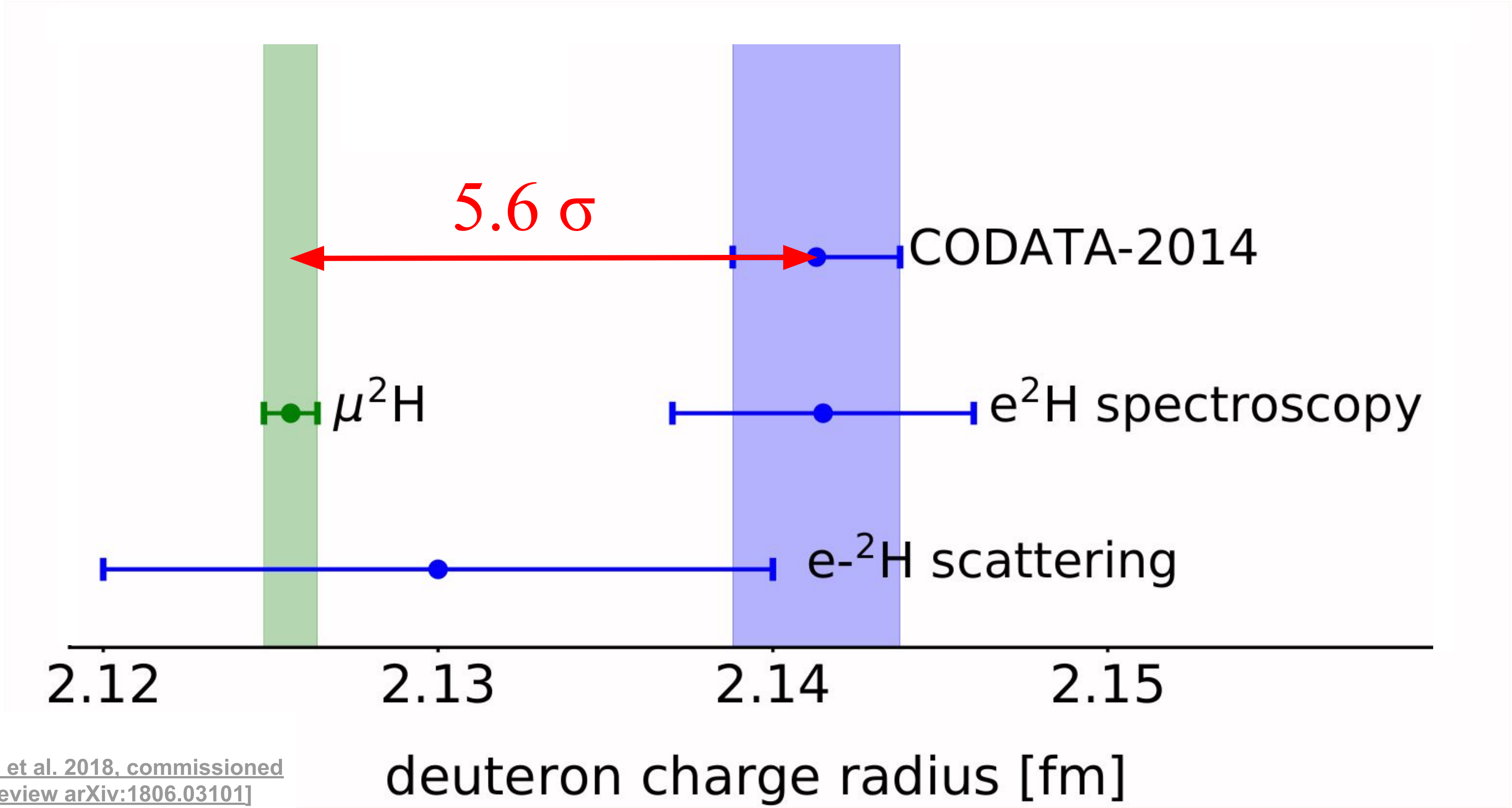

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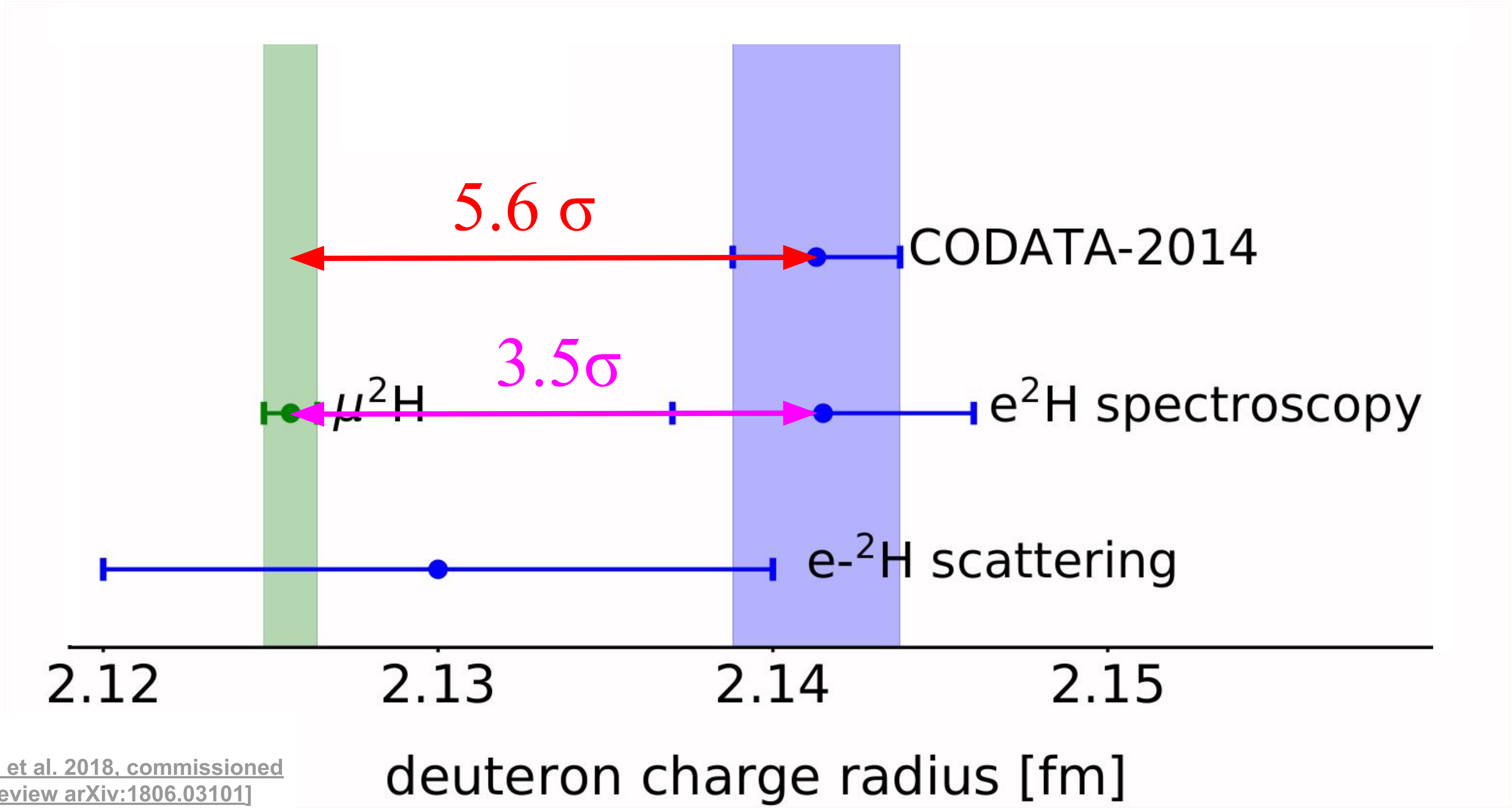

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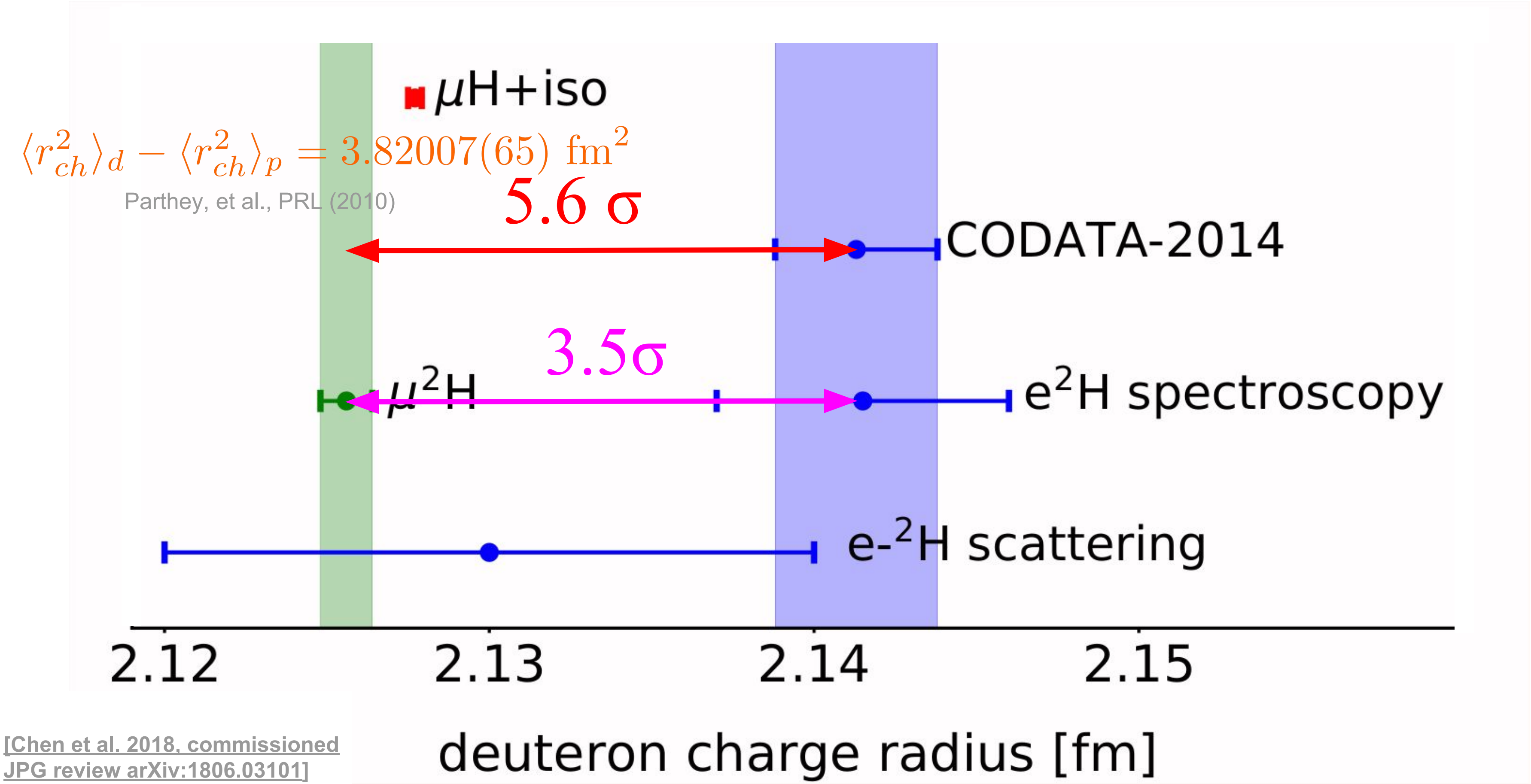

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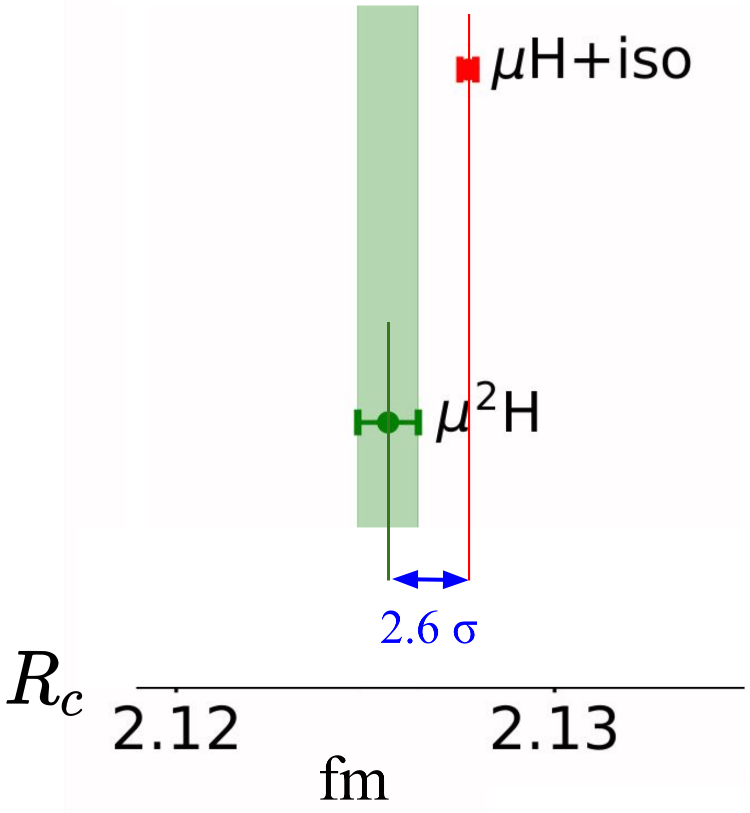
There is a discrepancy between eD and μD data

$$\Delta E_{LS} = \delta_{QED} + \delta_{FS}(R_c) + \delta_{TPE}$$



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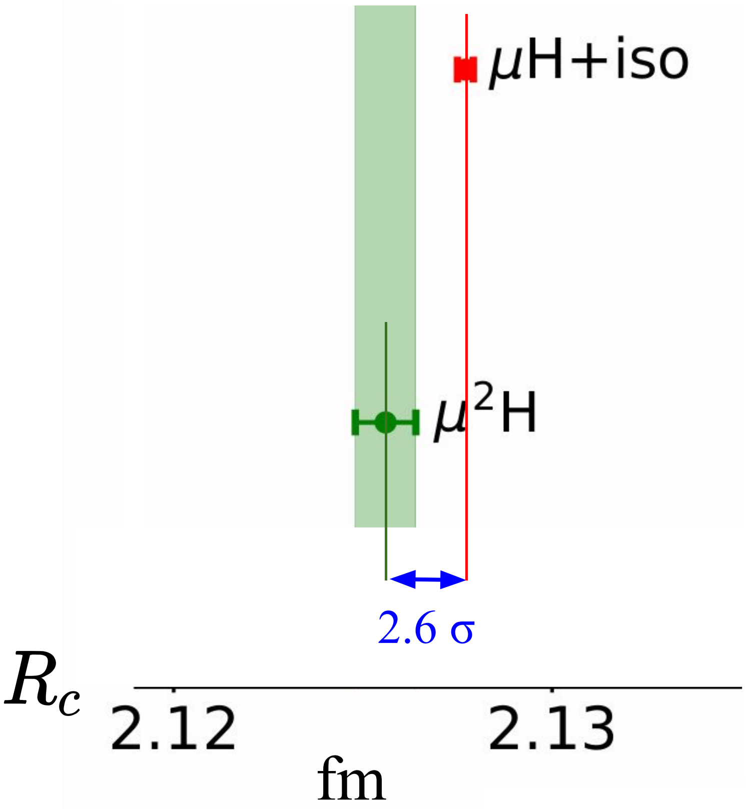

$$\Delta E_{LS} = \delta_{QED} + \delta_{FS}(R_c) + \delta_{TPE}$$

$$\delta_{TPE}(Our\ Work) = -1.718(22)\ meV$$ [2014]

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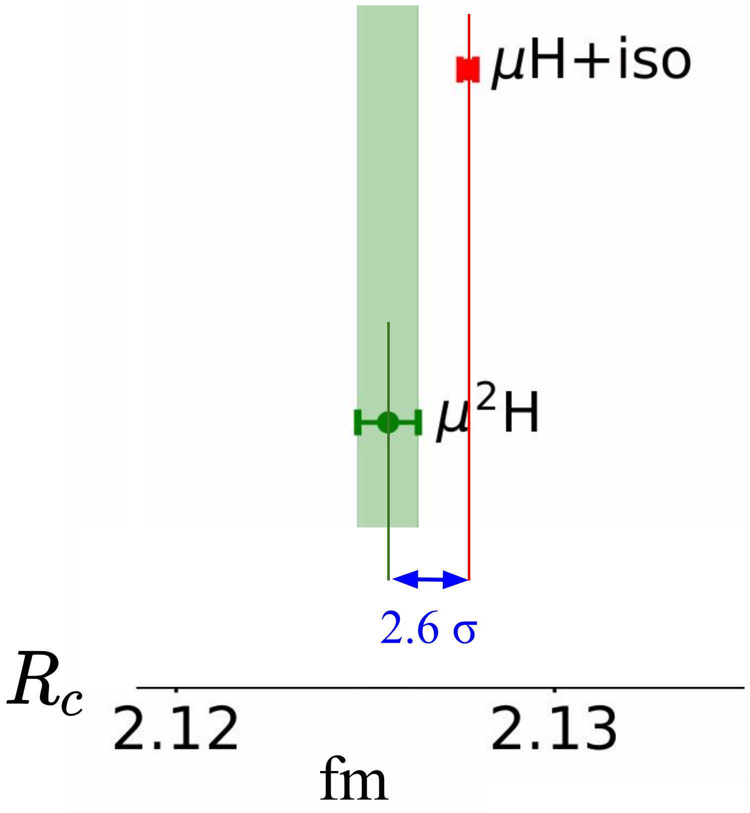

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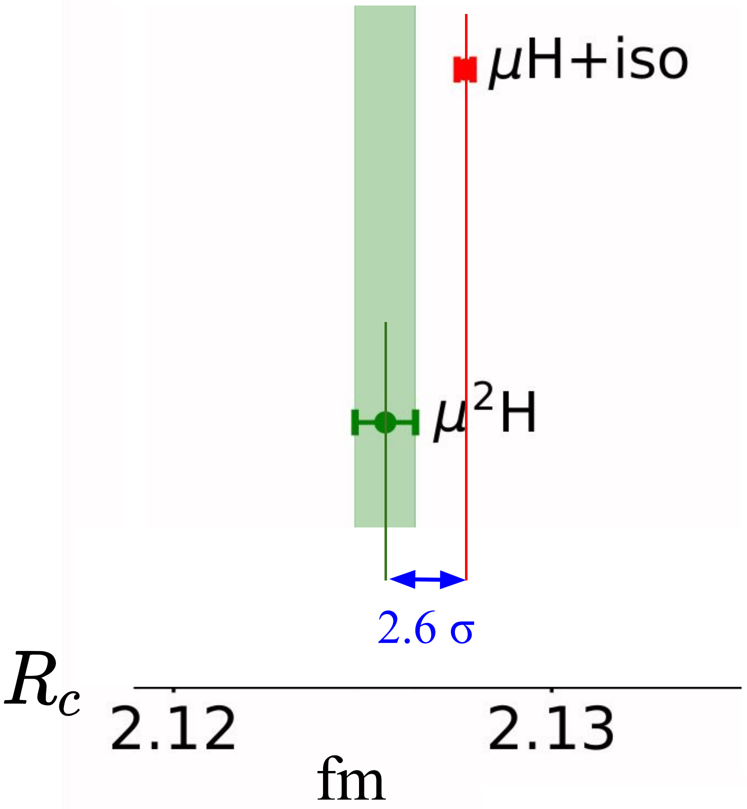

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$$\delta_{TPE} = \delta_{QED} + \delta_{FS}(R_c) - \Delta E_{LS}$$

➡ $$\delta_{TPE}(Exp.) = -1.7638(68)\ meV$$

[Pohl *et. al.* Science, Vol 353, 6300, 2016]

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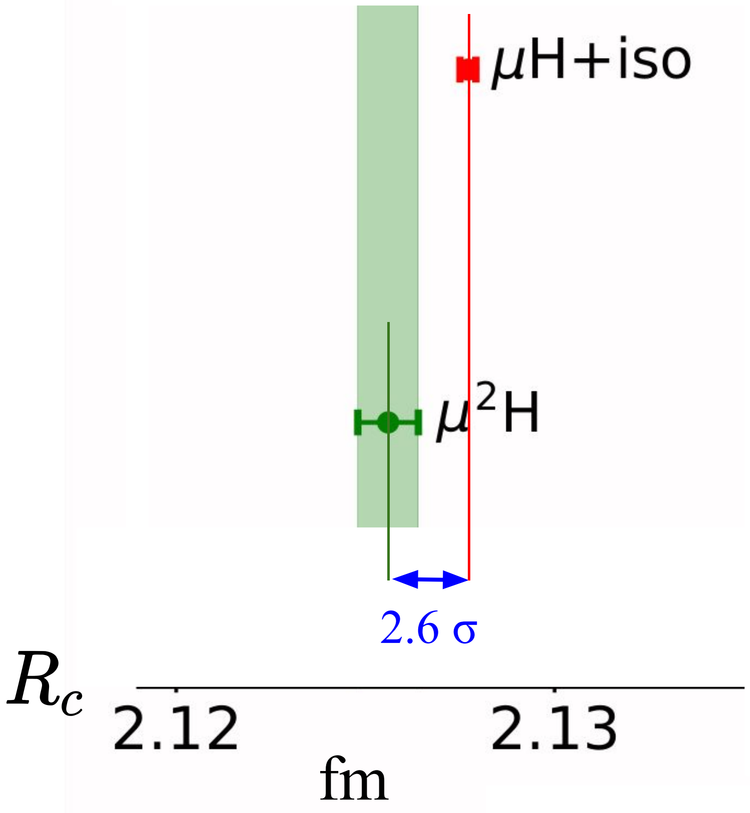

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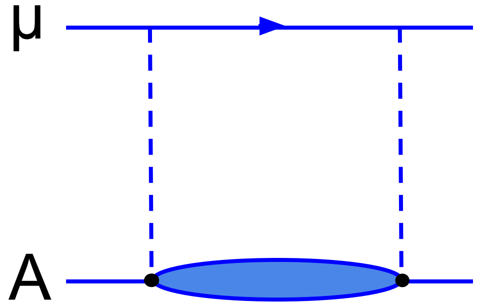
[Pohl *et. al.* Science, Vol 353, 6300, 2016]

- Theoretical TPE is 6 times larger than experimental uncertainty
- A thorough analysis may shed light on difference and the deuteron puzzle

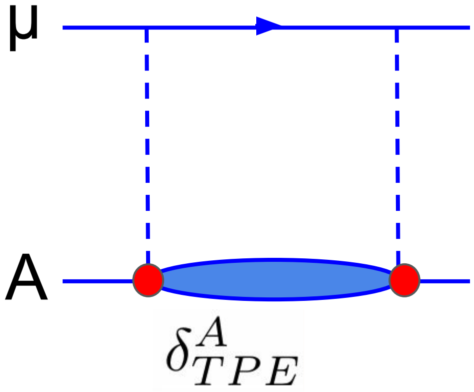
The two-photon exchange



δ^{A}_{TPE}

$$\delta_{TPE} =$$

The two-photon exchange



$$\delta_{TPE} = \underbrace{\delta^A_{TPE}}_{\text{Nuclear}} + \underbrace{\delta^N_{TPE}}_{\text{Nucleonic}}$$

C. E. Carlson et al. Phys. Rev. A 89, 022504 (2014).
J. J. Krauth, et al. Ann. of Phy. 366, 168 (2016).
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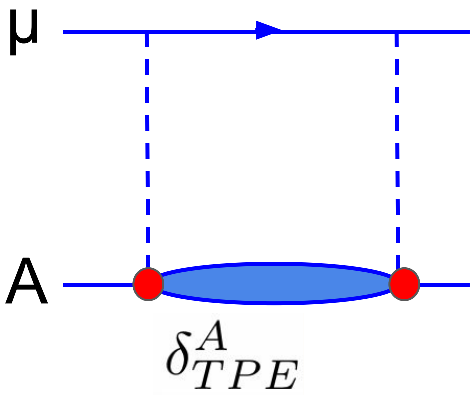

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$$\delta^{A}_{TPE} = \delta^{A}_{\text{pol}} + \delta^{A}_{Zem}$$

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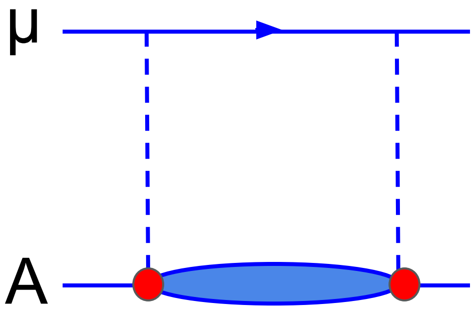

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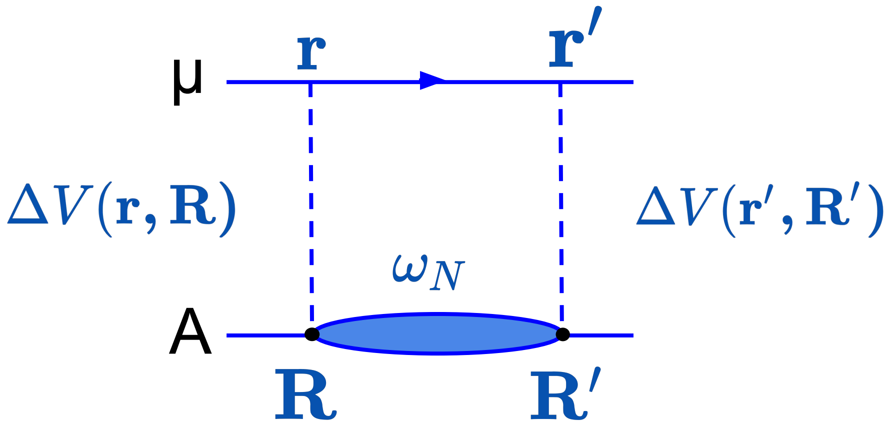
$$\delta^A_{TPE} = \delta^A_{\text{pol}} + \delta^A_{Zem}$$

$$\delta^A_{Zem} = \frac{\pi}{3} m_r (Z\alpha)^2 \phi^2(0) \iint d^3R d^3R' |\mathbf{R} - \mathbf{R}'|^3 \rho^{\mathbf{p}}_{\mathbf{0}}(\mathbf{R}) \rho^{\mathbf{p}}_{\mathbf{0}}(\mathbf{R}')$$

The evaluation of the nuclear polarizabilty



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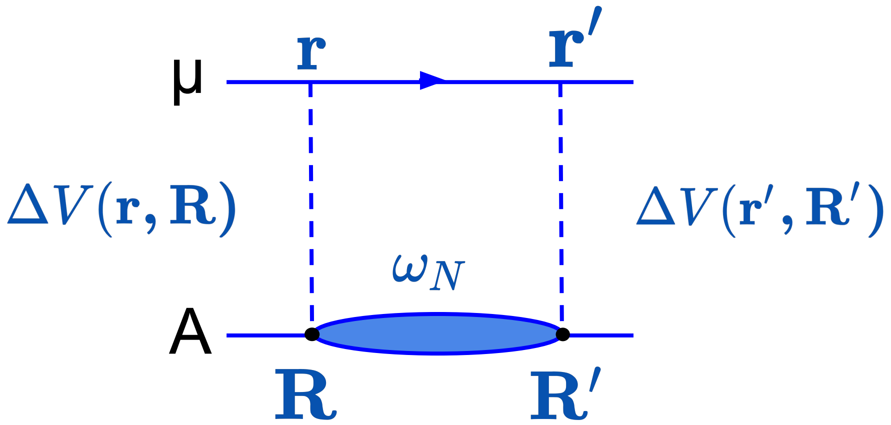

$$\delta^A_{\text{pol}} = \sum_{N \neq N_0} \int \int d^3 R d^3 R' \rho^{p*}_N (\mathbf{R}) \mathbf{W}(\mathbf{R}, \mathbf{R}', \omega_{\mathrm{N}}) \rho^{\mathrm{p}}_{\mathrm{N}} (\mathbf{R}'),$$

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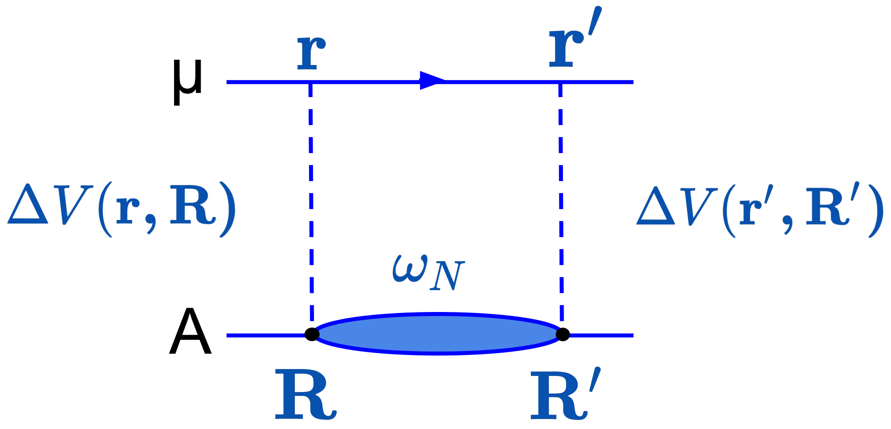

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$$W(\mathbf{R},\mathbf{R}',\omega_\mathrm{N}) = -\mathbf{Z}^2 \int \mathbf{d^3r\,d^3r'}\,\boldsymbol{\Delta}\mathbf{V}(\mathbf{r},\mathbf{R})\langle\mu|\mathbf{r}\rangle\langle\mathbf{r}|\frac{1}{\mathbf{H}_\mu+\omega_\mathrm{N}-\epsilon_\mu}|\mathbf{r}'\rangle\langle\mathbf{r}'|\mu\rangle\boldsymbol{\Delta}\mathbf{V}(\mathbf{r}',\mathbf{R}')$$

The expansion of the matrix element

- Expand the matrix element in terms of scale parameter

$$W(\mathbf{R},\mathbf{R}',\omega_\mathrm{N}) = \frac{2\pi}{3}(\mathrm{Z}\alpha)^2\phi^2(0)\sqrt{\frac{2\mathrm{m_r}}{\omega_\mathrm{N}}}\left[|\mathbf{R}-\mathbf{R}'|^2 - \frac{1}{4}\sqrt{2\mathrm{m_r}\omega_\mathrm{N}}|\mathbf{R}-\mathbf{R}'|^3 + \frac{1}{10}\mathrm{m_r}\omega_\mathrm{N}|\mathbf{R}-\mathbf{R}'|^4 + \cdots\right],$$

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- The scale parameter of the expansion is

$$\eta \equiv \sqrt{2m_r\omega_N}|\mathbf{R}-\mathbf{R}'| \approx \sqrt{m_r/m_p} \approx \sqrt{m_\mu/m_p} \ll 1$$

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- Perform a multipole expansion of η , leading term

$$|\mathbf{R}-\mathbf{R}'|^2 \to -\frac{8\pi}{3}\mathbf{R}\mathbf{R}'\,\mathbf{Y}_1(\hat{\mathbf{R}})\cdot\mathbf{Y}_1(\hat{\mathbf{R}}'),$$

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$\Rightarrow$

$$\delta_{D1}^{(0)} = -\frac{16\pi^2}{9}(Z\alpha)^2\phi^2(0)\int_0^\infty d\omega\sqrt{\frac{2m_r}{\omega}}S_{D_1}(\omega),$$

The expansion of the matrix element

- Sub-sub leading terms are given by

$$|\mathbf{R}-\mathbf{R}'|^4 \to \frac{10}{3}\mathbf{R}^2\mathbf{R}'^2 + \frac{32\pi}{15}\mathbf{R}^2\mathbf{R}'^2\mathbf{Y}_2(\hat{\mathbf{R}})\cdot\mathbf{Y}_2(\hat{\mathbf{R}}') - \frac{16\pi}{3}(\mathbf{R}^2+\mathbf{R}'^2)\mathbf{R}\mathbf{R}'\mathbf{Y}_1(\hat{\mathbf{R}})\cdot\mathbf{Y}_1(\hat{\mathbf{R}}')$$

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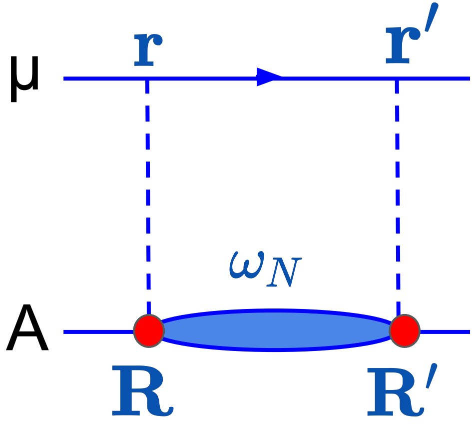
$$\Rightarrow$$

$$\delta^{(2)}_{R^2} = \frac{4\pi}{9} m_r^2 (Z\alpha)^2 \phi^2(0) \int_0^\infty d\omega \sqrt{\frac{\omega}{2m_r}} S_{R^2}(\omega)$$

$$\delta^{(2)}_{Q} = \frac{64}{225}\pi^2 m_r^2 (Z\alpha)^2 \phi^2(0) \int_0^\infty d\omega \sqrt{\frac{\omega}{2m_r}} S_{Q_2}(\omega)$$

$$\delta^{(2)}_{D1D3} = -\frac{64}{45}\pi^2 m_r^2 (Z\alpha)^2 \phi^2(0) \int_0^\infty d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1 D_3}(\omega)$$

Finite size corrections



- Insertion of form factors into the nuclear vertices

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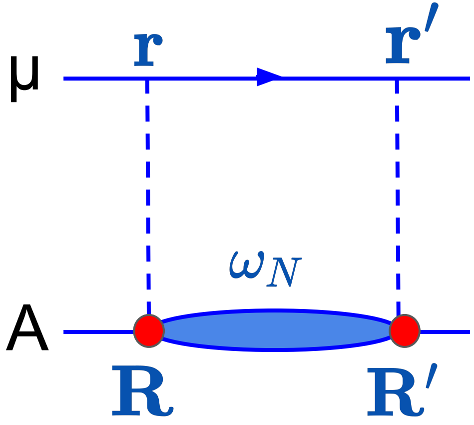

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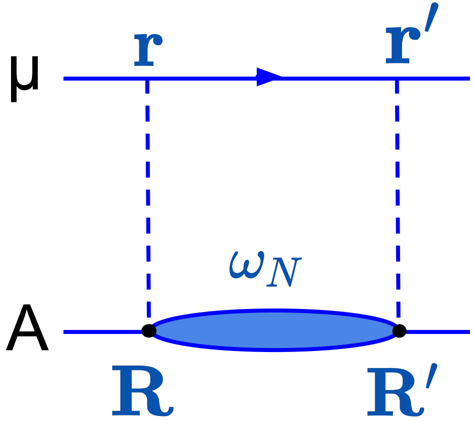
$$\delta_{Z1}^{(1)} = 8\pi m_r (Z\alpha)^2 \phi^2(0) \iint d^3R\, d^3R' \, |\mathbf{R} - \mathbf{R}'| \rho_0^{\mathrm{p}}(\mathbf{R}) \left[\frac{2}{\beta^2} \rho_0^{\mathrm{p}}(\mathbf{R}') - \lambda \rho_0^{\mathrm{n}}(\mathbf{R}') \right]$$

$$\delta_{NS}^{(2)} = -\frac{128}{9} \pi^2 m_r^2 (Z\alpha)^2 \phi^2(0) \left[\frac{2}{\beta^2} + \lambda \right] \int_0^\infty d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1}(\omega).$$

Additional corrections



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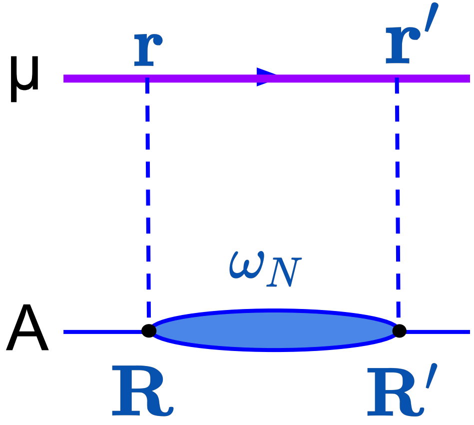

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$$\delta_C^{(0)} = -\frac{16\pi^2}{9}(Z\alpha)^3\phi^2(0)\int_0^\infty d\omega \frac{m_r}{\omega}\ln\frac{2(Z\alpha)^2 m_r}{\omega} S_{D_1}(\omega)$$

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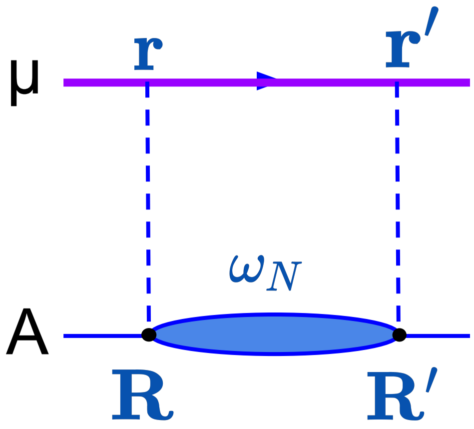

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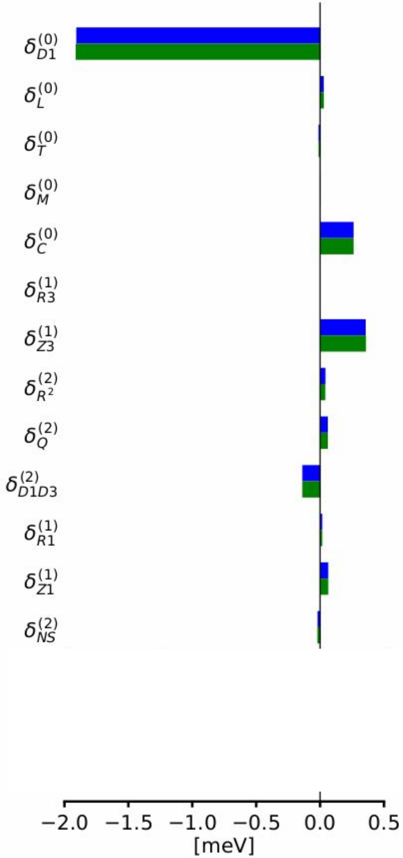
- Relativistic corrections

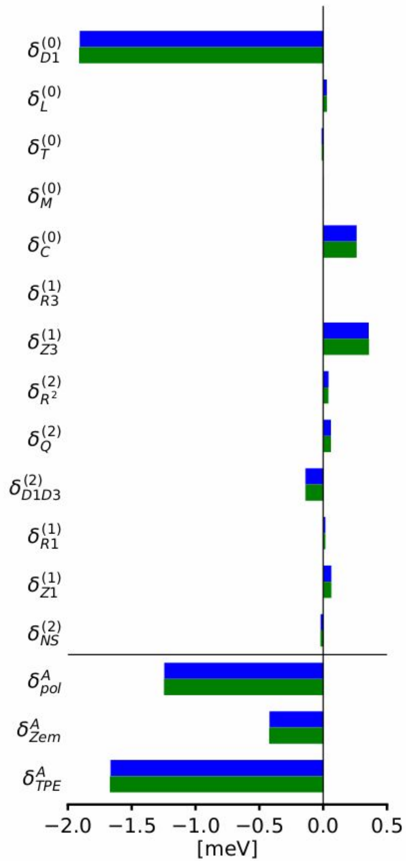
$$\delta_L^{(0)}\ , \quad \delta_T^{(0)}\ , \quad \delta_M^{(0)}$$

Results for μD



	AV18	χ^{EFT}
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.02	-0.021

Results for μD



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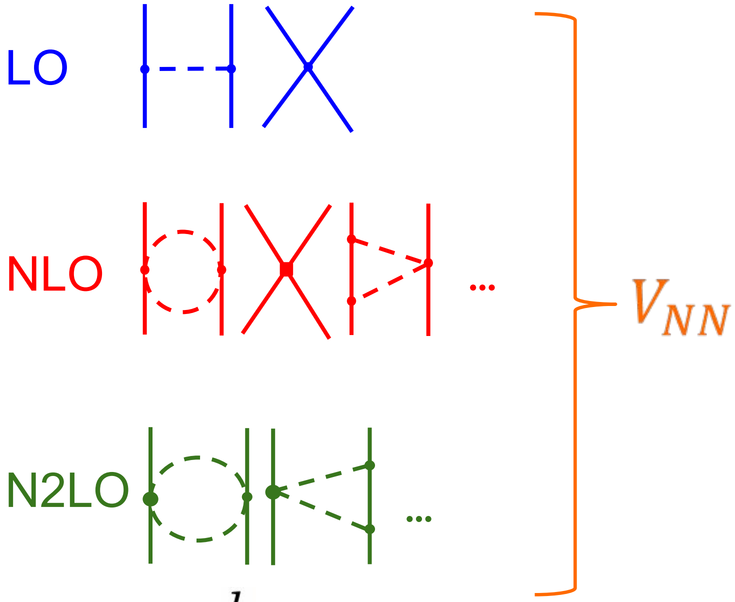
$\delta^{(1)}_{R1}$	0.017	0.017
$\delta^{(1)}_{Z1}$	0.064	0.064
$\delta^{(2)}_{NS}$	-0.02	-0.021
δ^{A}_{pol}	-1.245	-1.247
δ^{A}_{Zem}	-0.421	-0.423
δ^{A}_{TPE}	-1.666	-1.671

$$\delta^A_{pol} = \sum \delta^k$$

$$\delta^A_{TPE} = \delta^A_{\text{pol}} + \delta^A_{Zem}$$

Improving the uncertainty estimates

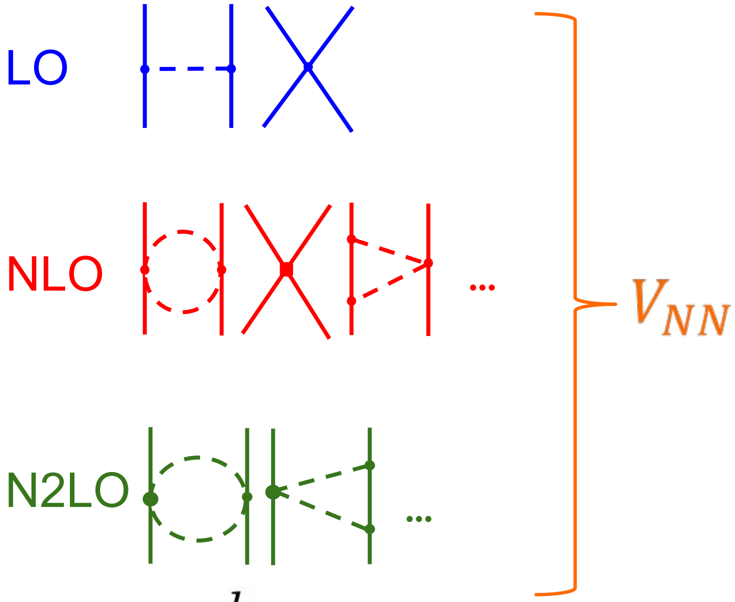
$$\delta^{A}_{TPE}\left(c_\mu, \Lambda, T^{Max}_{Lab}, k\right)$$



$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_\nu \left(\frac{Q}{\Lambda_\chi}\right)^{\nu}$$

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Improving the uncertainty estimates



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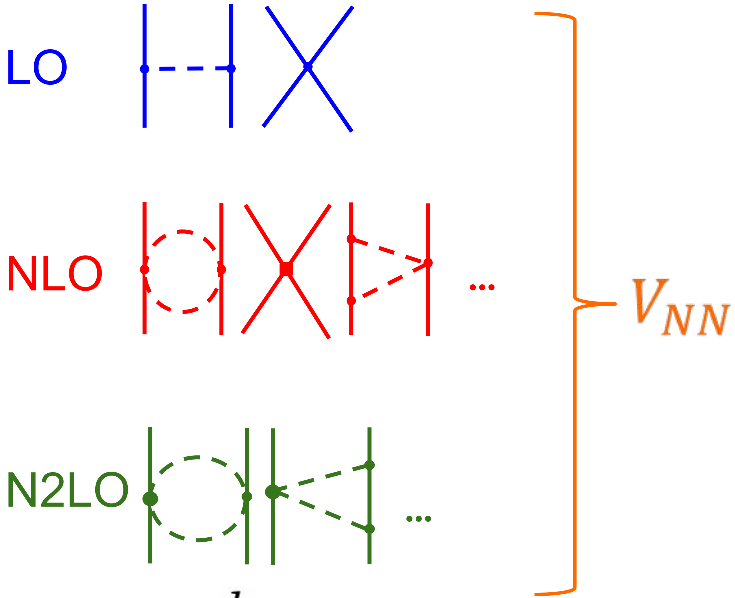
- Use N2LO potentials fit simulatenously to NN and πN data

Statistical uncertainties: c_μ

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Improving the uncertainty estimates



$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_\nu \left(\frac{Q}{\Lambda_\chi}\right)^\nu$$

$$\delta_{TPE}^{A}\left(c_\mu, \Lambda, T_{Lab}^{Max}, k\right)$$

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Statistical uncertainties: c_μ

Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

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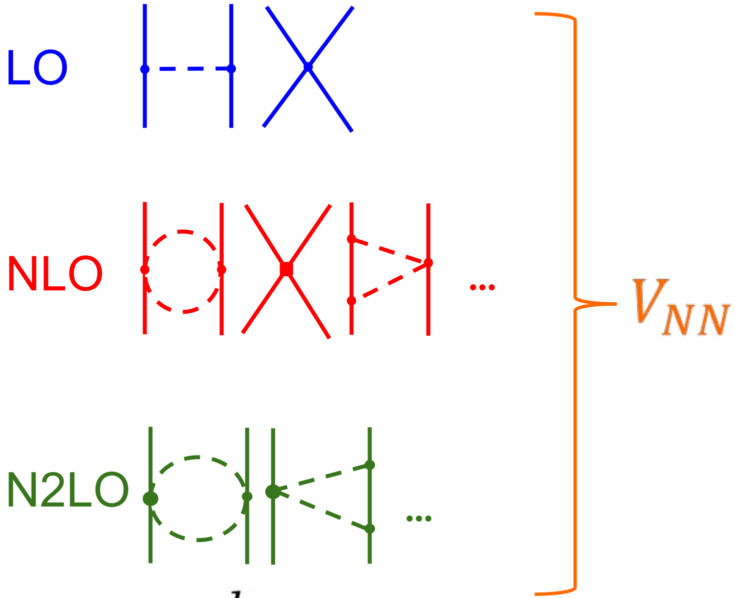

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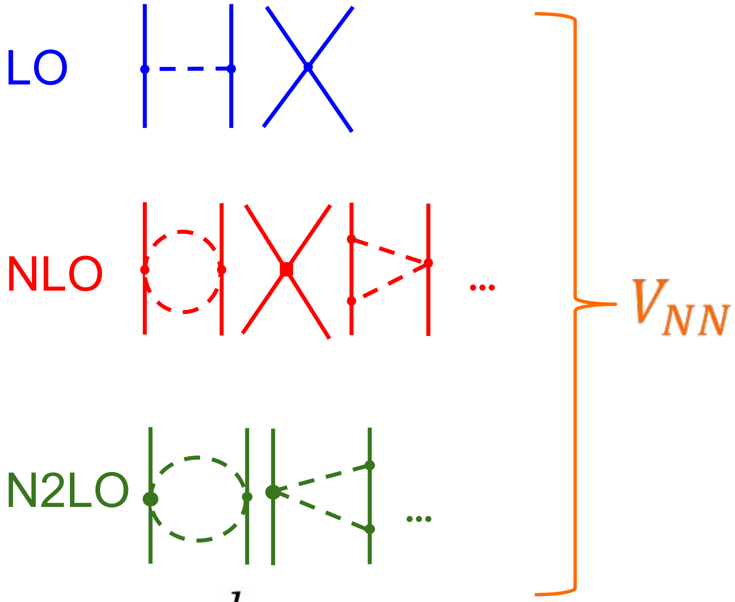
Systematic uncertainties: $\Lambda, T^{Max}_{Lab}, k$

$\eta, \rho, \vec{j}$

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Improving the uncertainty estimates



$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_{\nu} \left(\frac{Q}{\Lambda_{\chi}}\right)^{\nu}$$

$$\delta_{TPE}^{A}\,(c_{\mu}, \Lambda, T_{Lab}^{Max}, k)$$

- Use N2LO potentials fit simulatenously to NN and πN data

Statistical uncertainties: c_{μ}

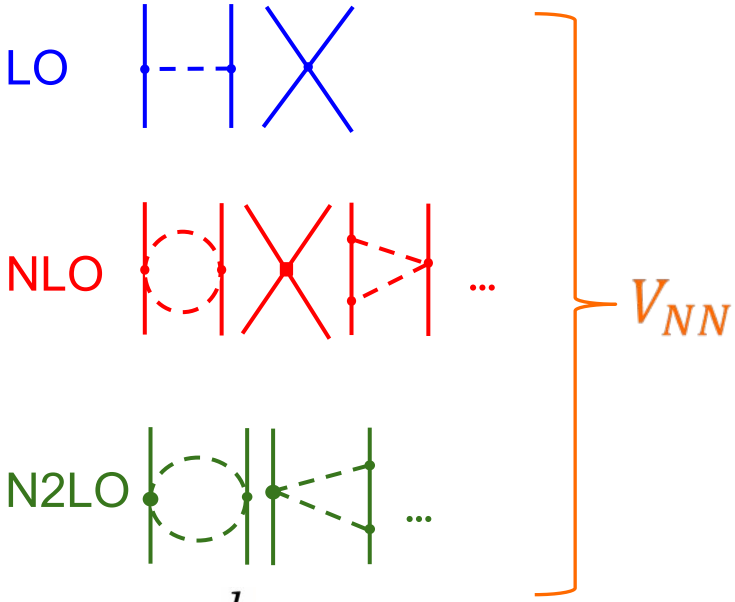
Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

$\eta, \rho, \vec{j}$

Single Nucleon: δ_{TPE}^{N}

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Improving the uncertainty estimates



$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu}$$

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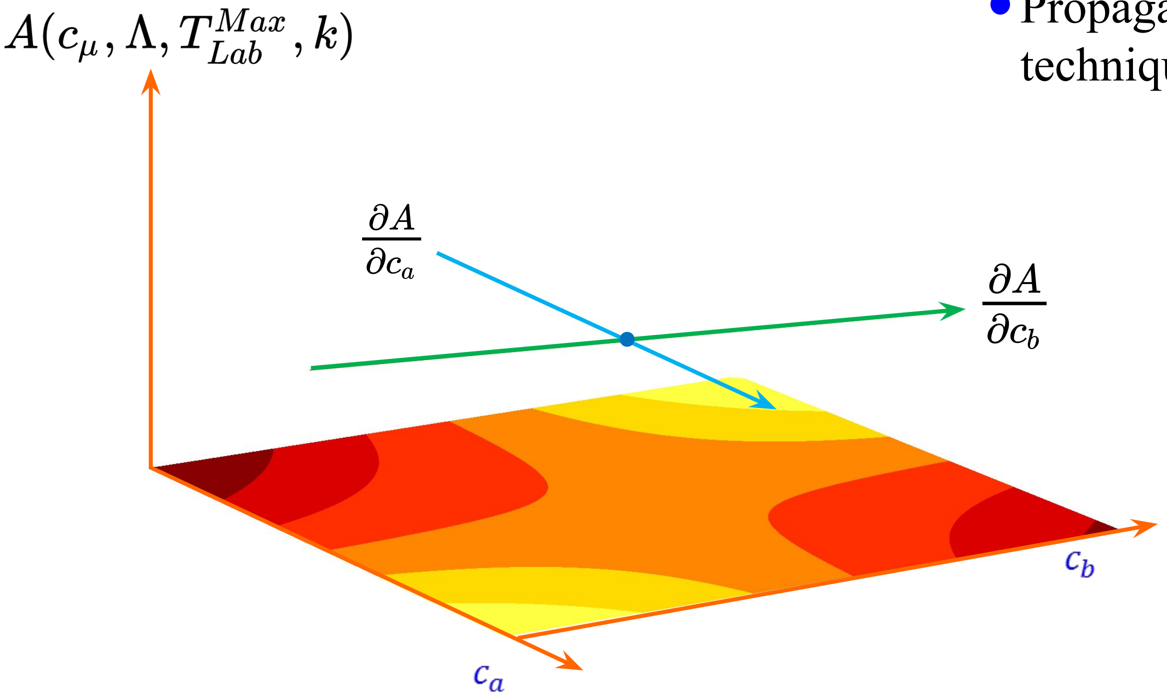
Single Nucleon: δ^{N}_{TPE}

Higher Order Corrections: $O(\alpha^{6})$

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

Statistical uncertainties



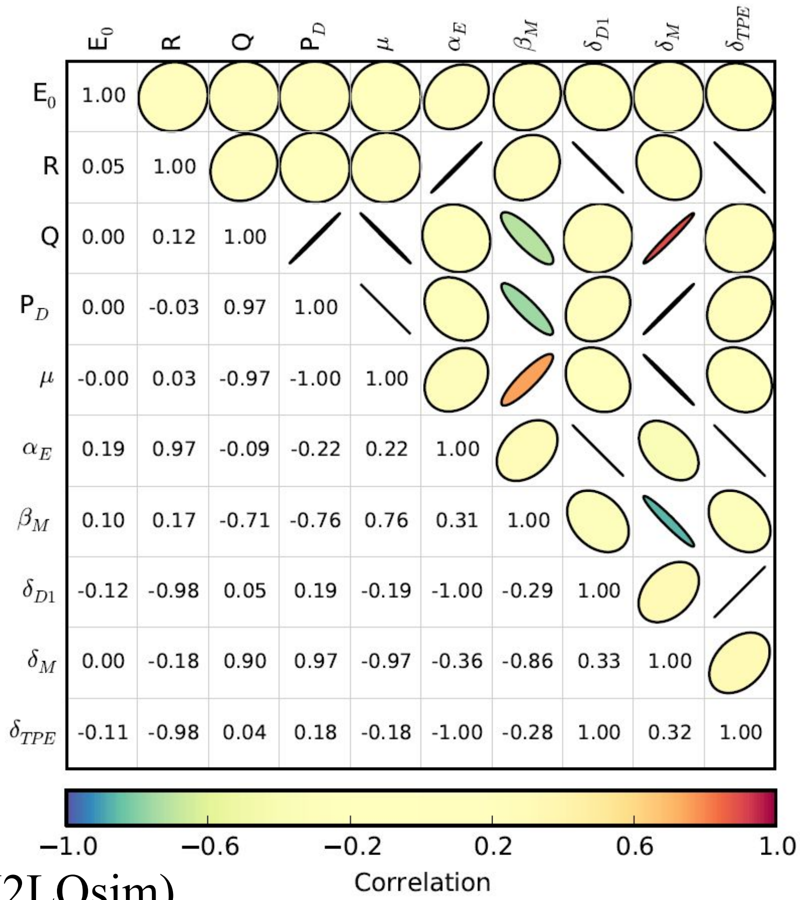
- Propagate uncertainty using standard techniques

$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

$$\mathrm{Cov}(A, B) = \boldsymbol{J}_A \mathrm{Cov}(c_\mu) \boldsymbol{J}_B^T$$

$$\sigma_{A,stat} = \sqrt{\mathrm{Cov}(A, A)}$$

Correlation analysis

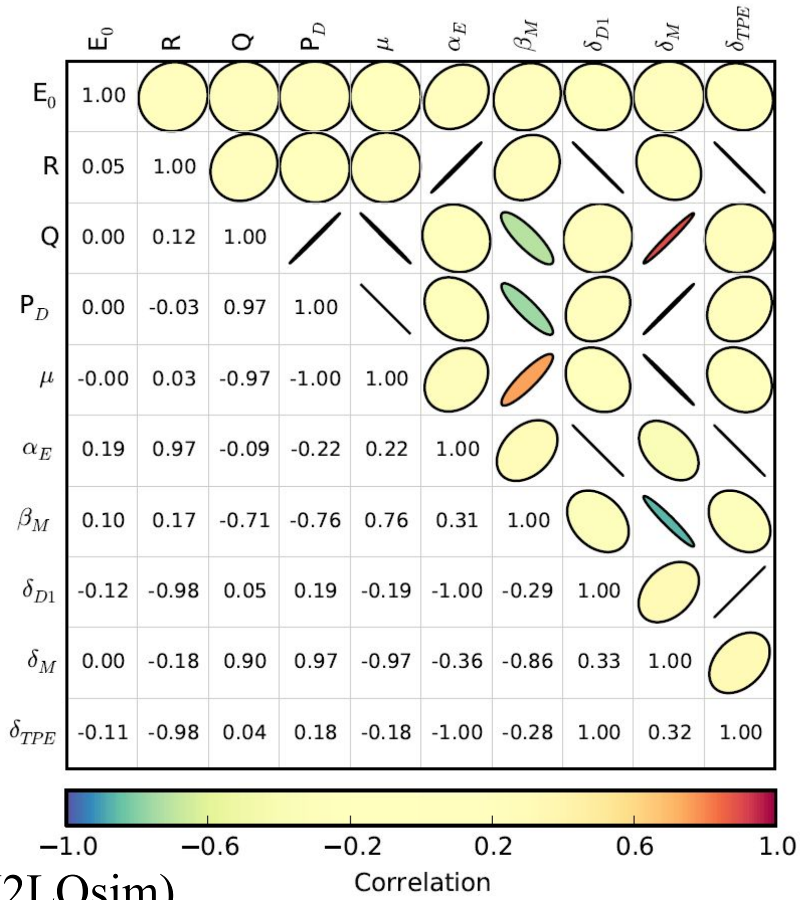


(N2LOsim)

- Serves as a check of the error propagation formalism

$$\rho(A,B) = \frac{Cov(A,B)}{\sigma_A \sigma_B}$$

Correlation analysis



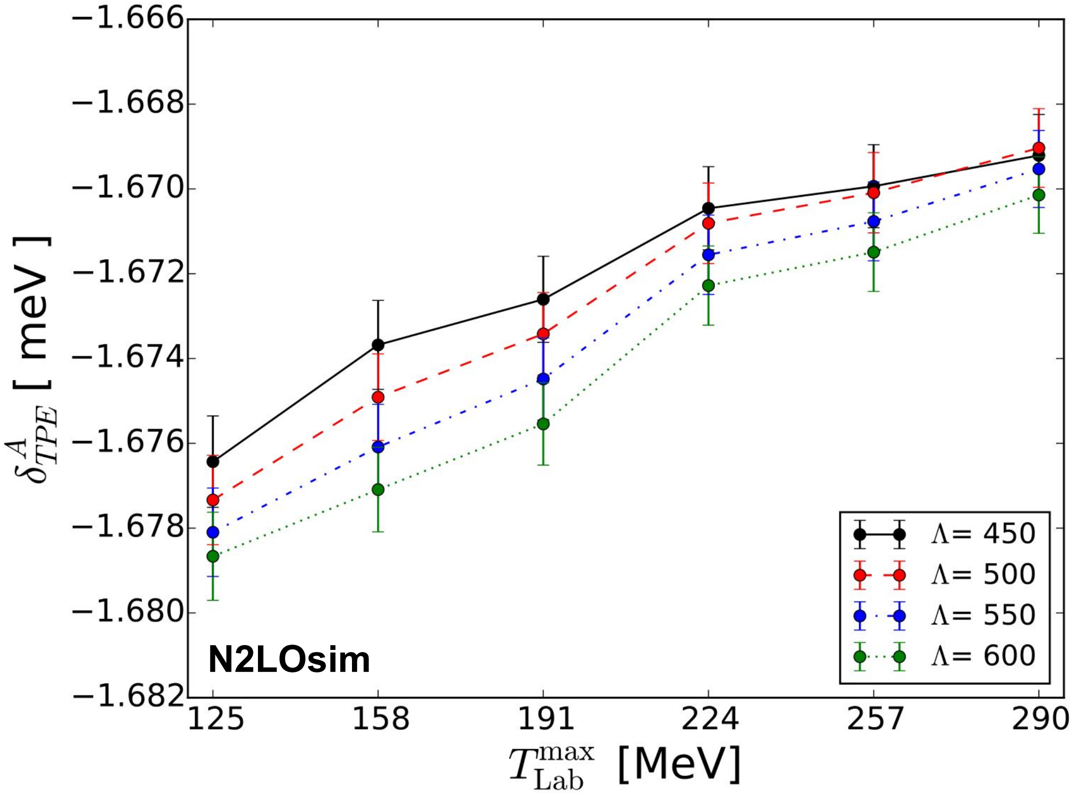
(N2LOsim)

- Serves as a check of the error propagation formalism

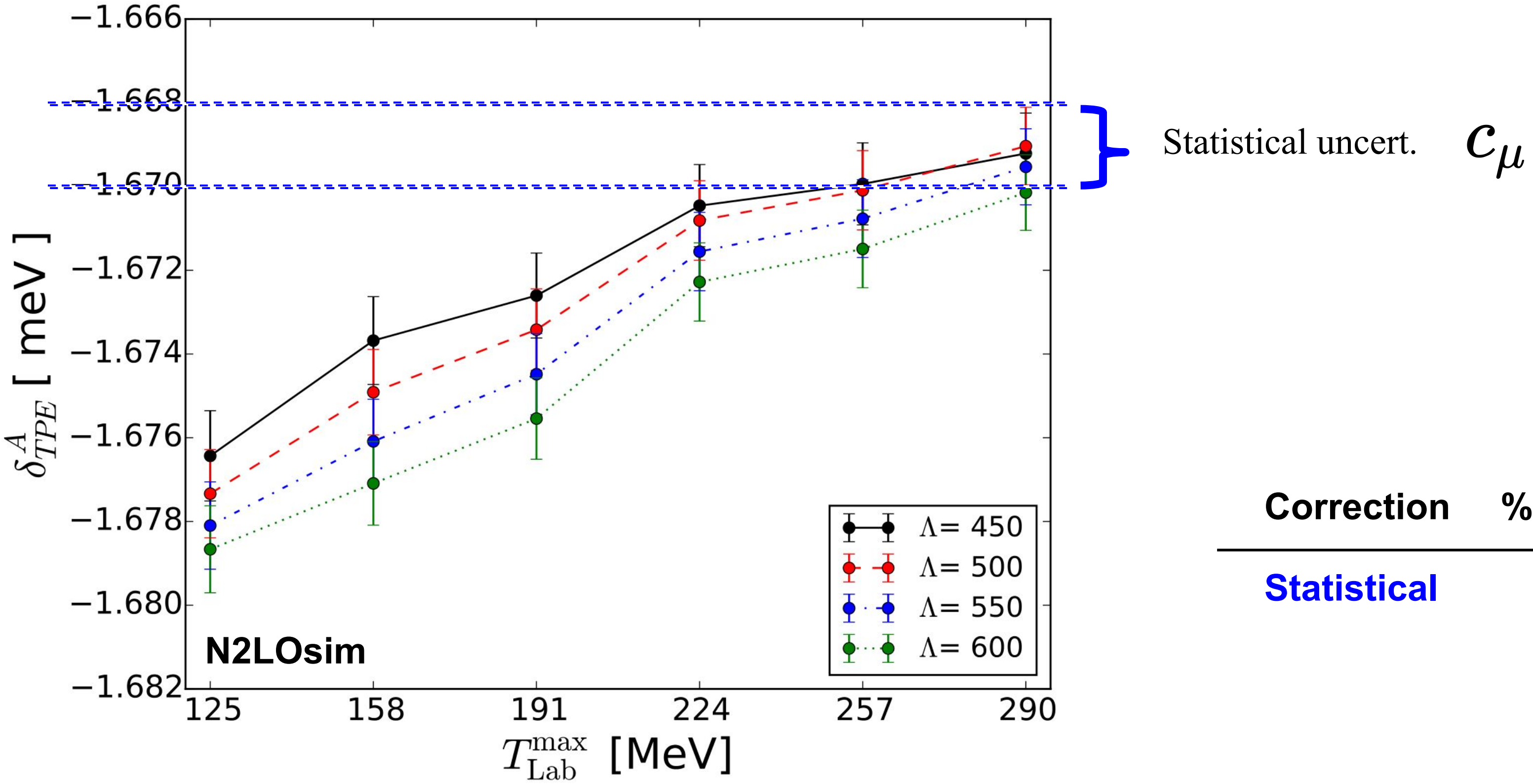
$$\rho(A, B) = \frac{Cov(A, B)}{\sigma_A \sigma_B}$$

- We observe expected correlations between
  - $\{P_d, \mu_d\}$
  - $\{R(^2\mathrm{H}), \alpha_E\}$
  - $\{R(^2\mathrm{H}), \delta_{TPE}\}$

Statistical uncertainties

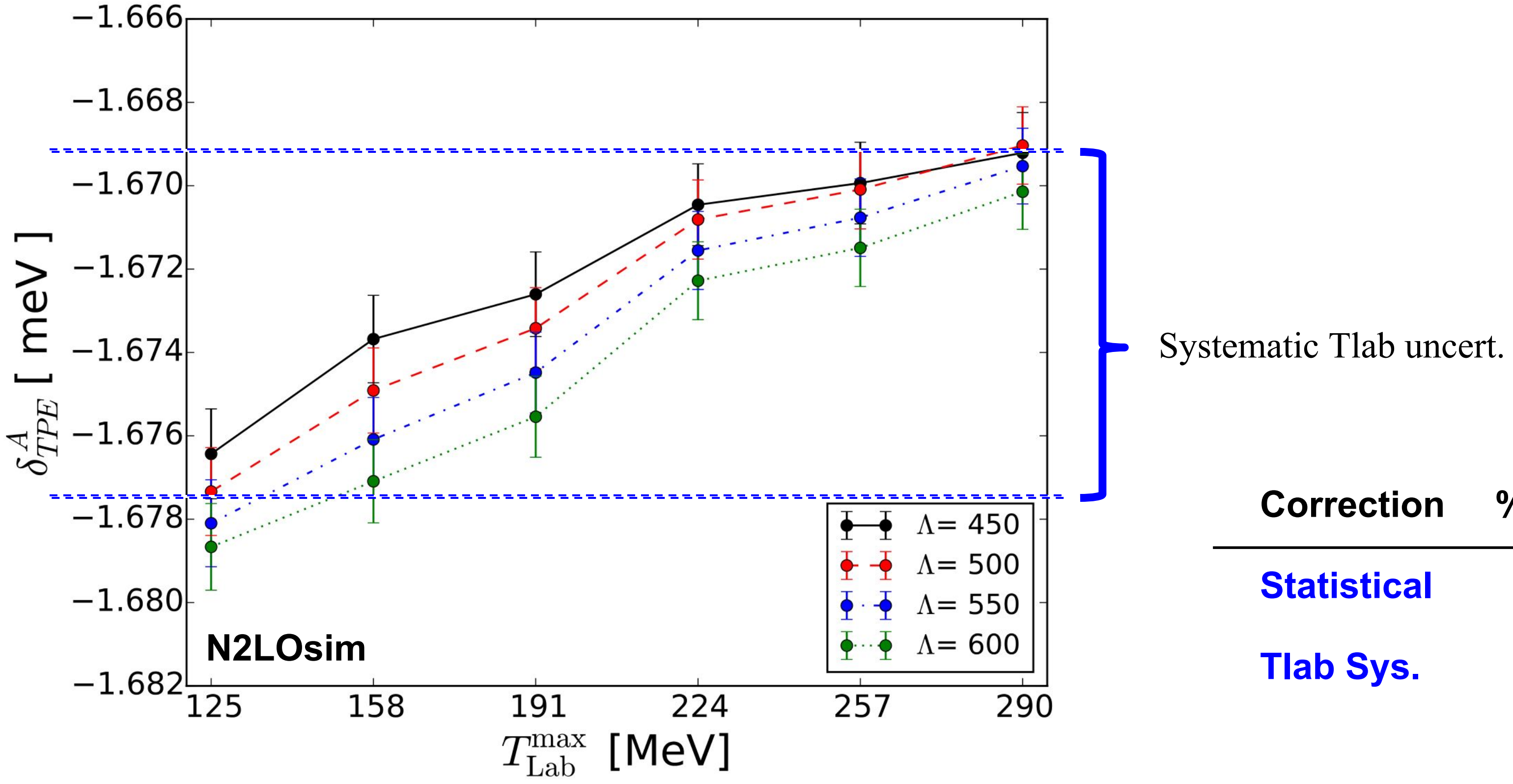


Statistical uncertainties



Correction	% Uncert.
Statistical	0.06

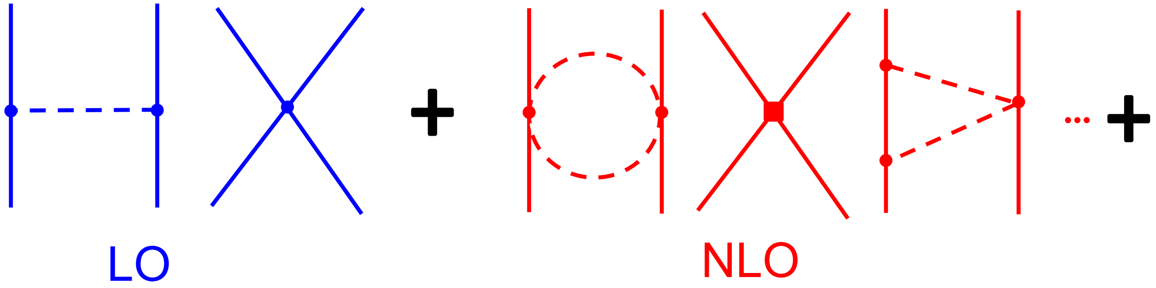
Sytematic Tlab uncertainties



Correction	% Uncert.
Statistical	0.06
Tlab Sys.	0.2

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} =$$



- Expand observable in the same Chiral EFT pattern,

$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_{\nu}(p) Q^{\nu} \qquad Q = max\left\{ \frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b} \right\}$$

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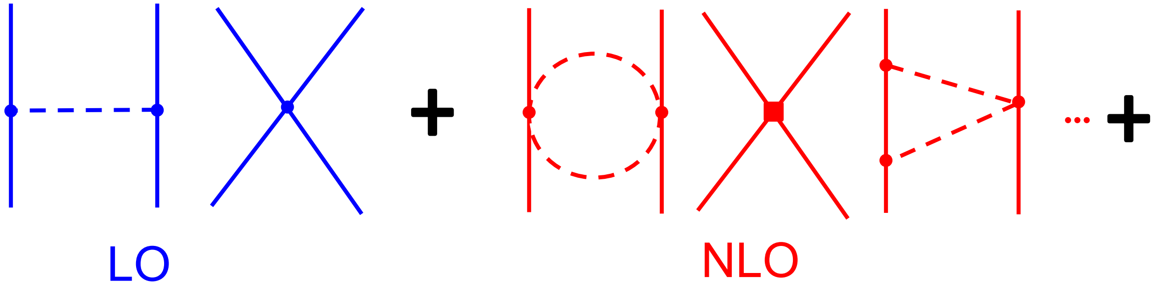

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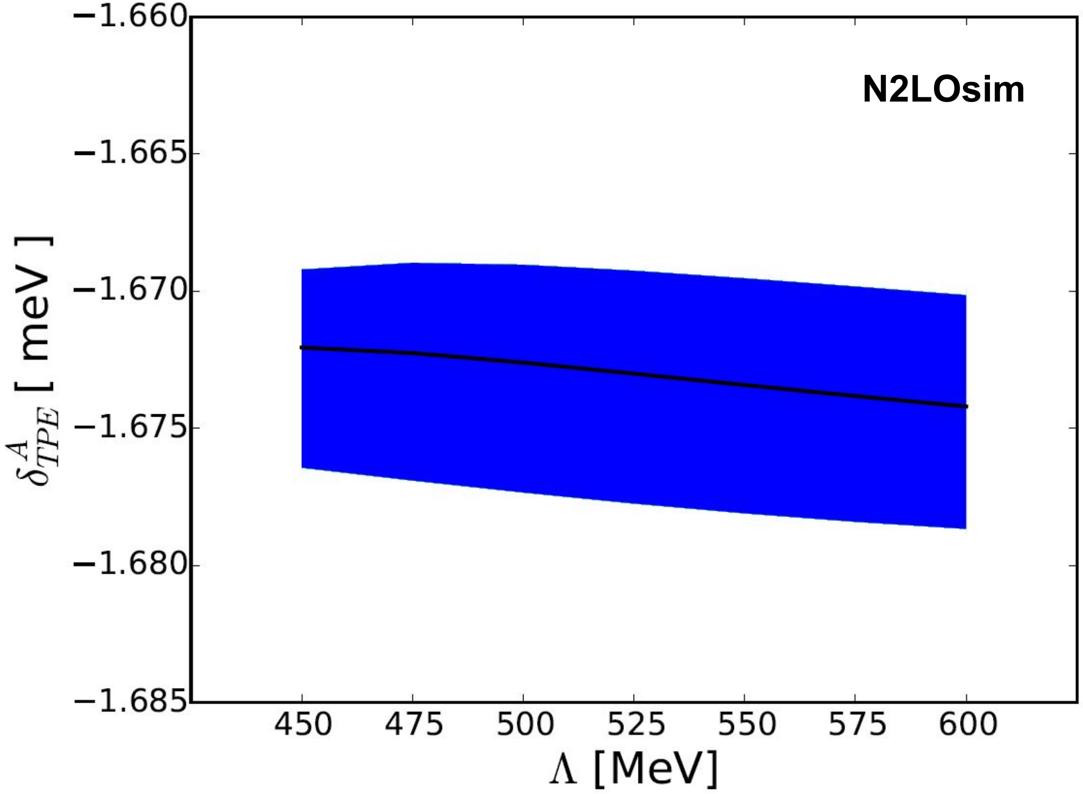
- Truncation uncertainty can then be calculated according to

$$\sigma_{A,sys}^{N^kLO}(p) \approx Q \cdot |A_0 Q^{k+1} \beta_{k+1}|$$

$$\sigma_{A,sys}^{N^kLO}(p) = A_0 Q^{k+2}\ max\{|\beta_0|, ..., |\beta_{k+1}|\}$$

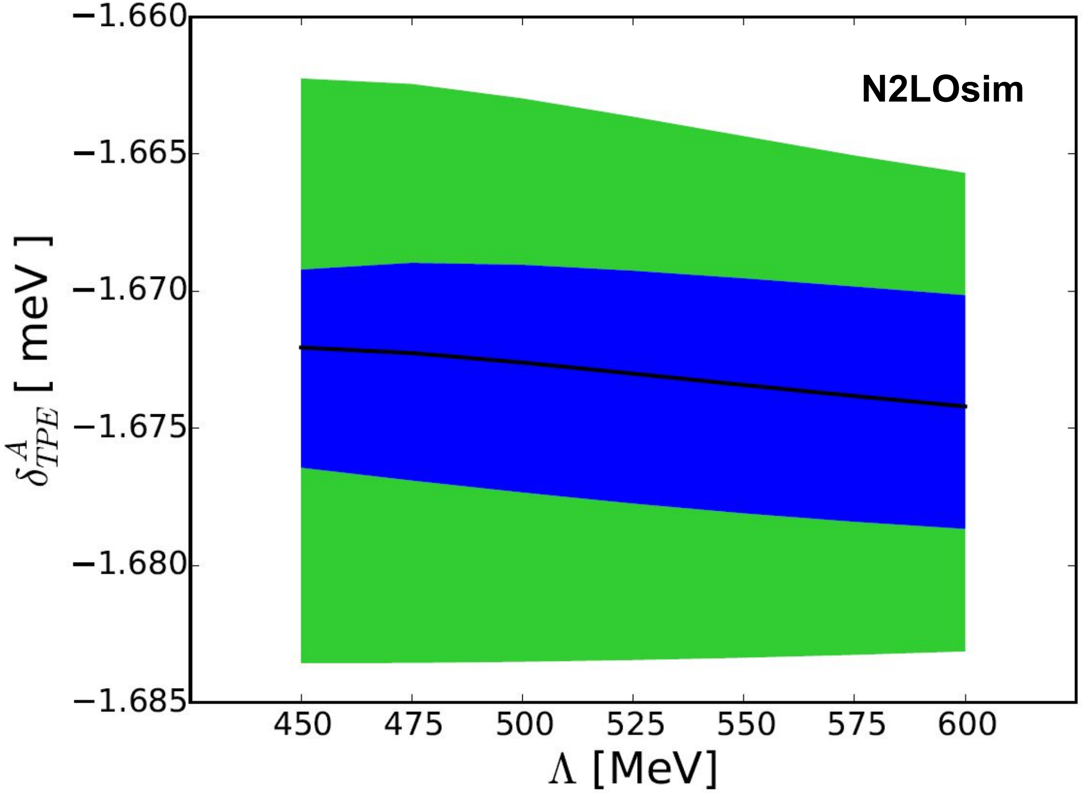
Chiral truncation uncertainties



- Tlab variation
- Chiral truncation estimate

$$\sigma_{A,sys}^{N^kLO}(p) = A_0 Q^{k+2}\ max\{|\beta_0|, ..., |\beta_{k+1}|\}$$

Chiral truncation uncertainties



- Tlab variation
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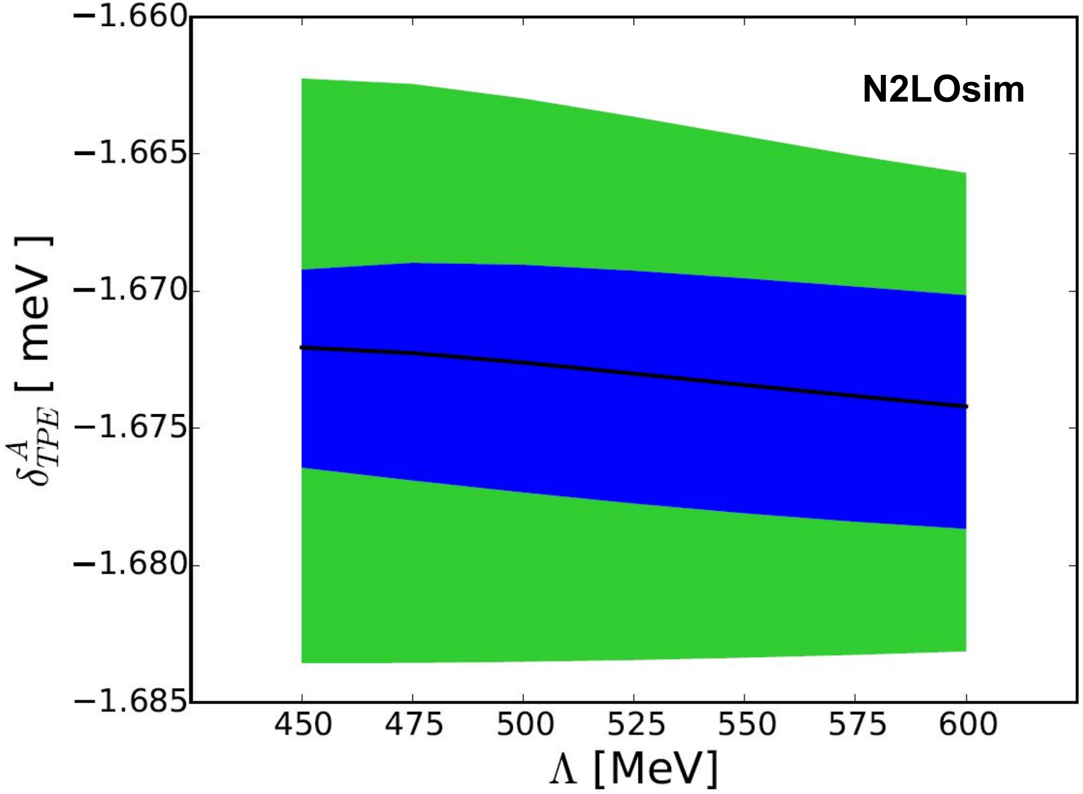
$$\sigma^{N^k LO}_{A,sys}(p) = A_0 Q^{k+2}\ max\{|\beta_0|, ..., |\beta_{k+1}|\}$$

- Estimate momentum scale of TPE

$$\langle\omega\rangle_{D1} = \frac{\int d\omega\,\omega\sqrt{\frac{2m_r}{\omega_N}}\,S_{D1}(\omega)}{\int d\omega\sqrt{\frac{2m_r}{\omega_N}}\,S_{D1}(\omega)}.$$

$$\langle\omega\rangle_{D1} \approx 7\ \mathrm{MeV}$$

Chiral truncation uncertainties



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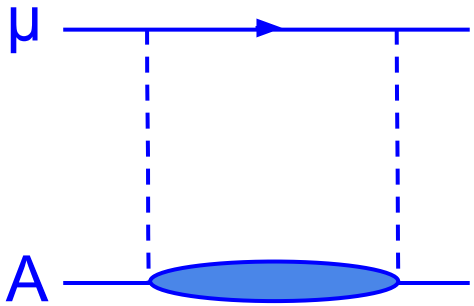
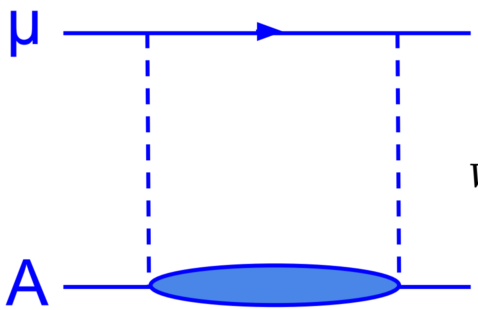
Correction	% Uncert.
Chiral Trunc.	0.4

η-expansion uncertainty



$$\delta^A_{\rm pol} = \sum_{N\neq N_0} \int\int d^3R d^3R' \rho^{p*}_N(\mathbf{R}) \mathbf{W}(\mathbf{R},\mathbf{R}',\omega_\mathbf{N}) \rho^{\rm p}_\mathbf{N}(\mathbf{R}'),$$

η-expansion uncertainty



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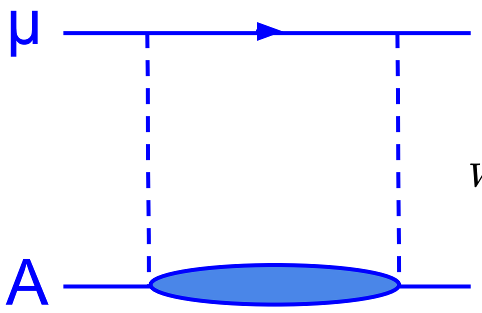
$$W(\mathbf{R},\mathbf{R}',\omega_N) = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3}\left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2+2m_r\omega_N}(1-e^{i\boldsymbol{q}\cdot\boldsymbol{R}})(1-e^{-i\boldsymbol{q}\cdot\boldsymbol{R}'})$$

η-expansion uncertainty



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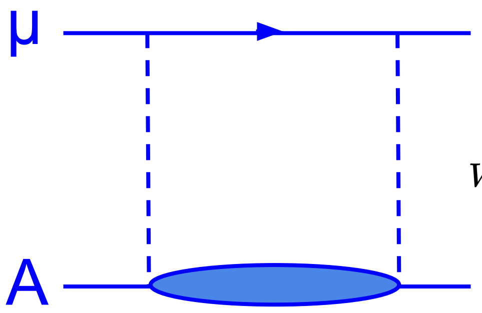
[insert intermediate nuclear states]

$$\sum_{N\neq N_0}|N\rangle\langle N|$$

η-expansion uncertainty



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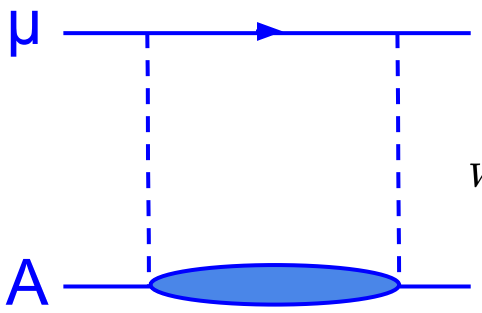
[insert intermediate nuclear states] $\sum_{N\neq N_0} |N\rangle\langle N|$

[leading to the non-relativistic result] $\delta^A_{NR} = -8(Z\alpha)^2|\phi(0)|^2 \int dq \int d\omega\; K_{NR}(q,\omega) S_L(q,\omega)$

η-expansion uncertainty



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[leading to the non-relativistic result] $\delta^A_{NR} = -8(Z\alpha)^2|\phi(0)|^2 \int dq \int d\omega\, K_{NR}(q,\omega)S_L(q,\omega)$

Full treatment

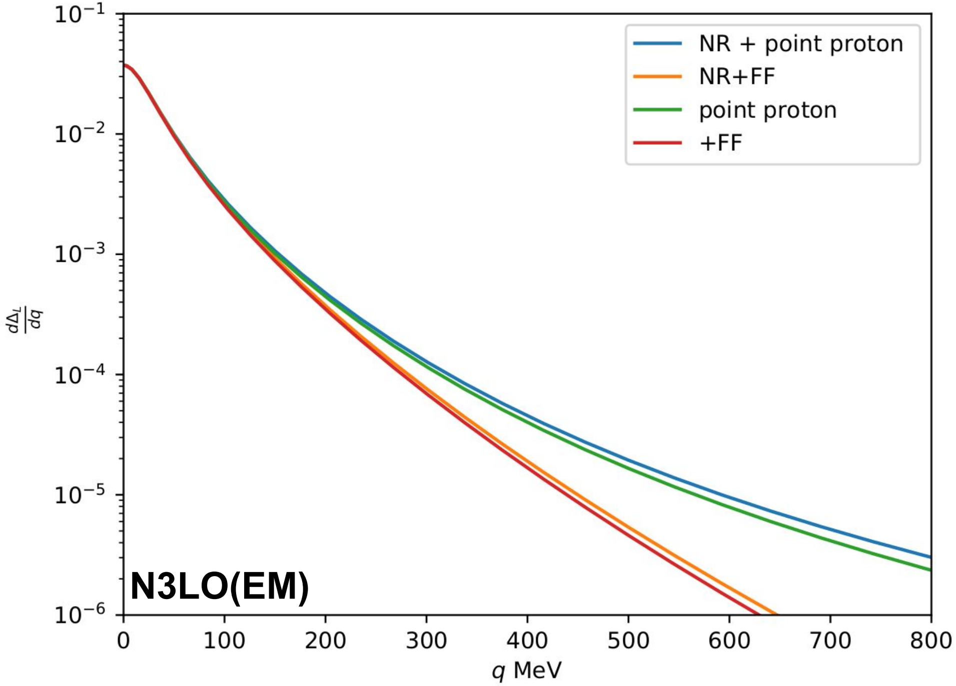
$$\delta^A_{TPE} = -8(Z\alpha)^2|\phi(0)|^2 \int dq \int d\omega \left[K_L(q,\omega)S_L(q,\omega) + K_T(q,\omega)S_T(q,\omega) + K_S(q,\omega)S_T(0,\omega)\right]$$

$$\delta^A_{TPE} = \Delta_L + \Delta_T$$

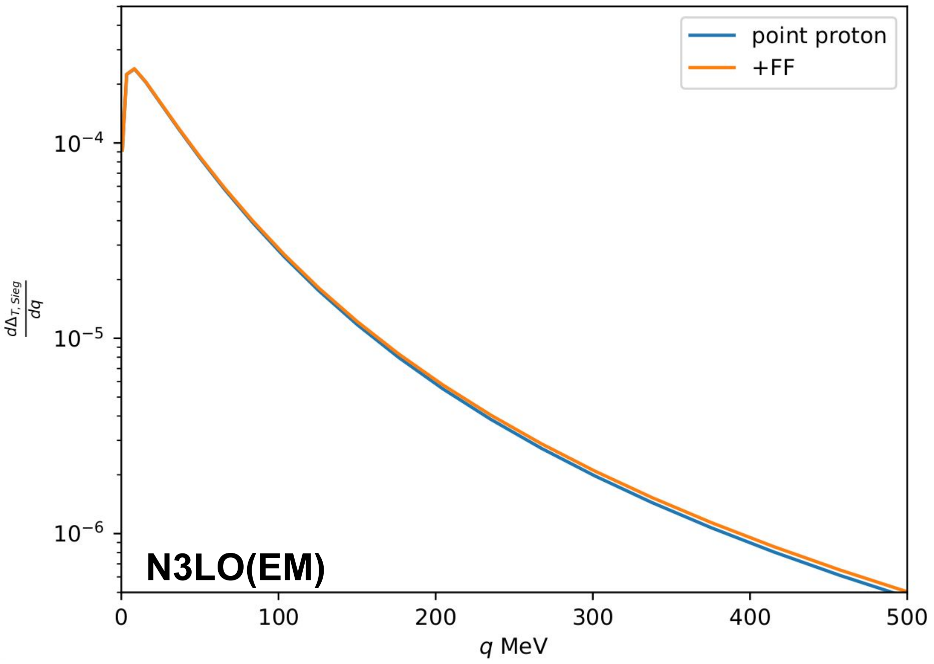
η-less expansion: Integrated response functions

$$\frac{d\Delta}{dq} = -8(Z\alpha)^2 |\phi(0)|^2 \int_{\omega_0}^{\infty} d\omega \, K(q,\omega) S(q,\omega),$$

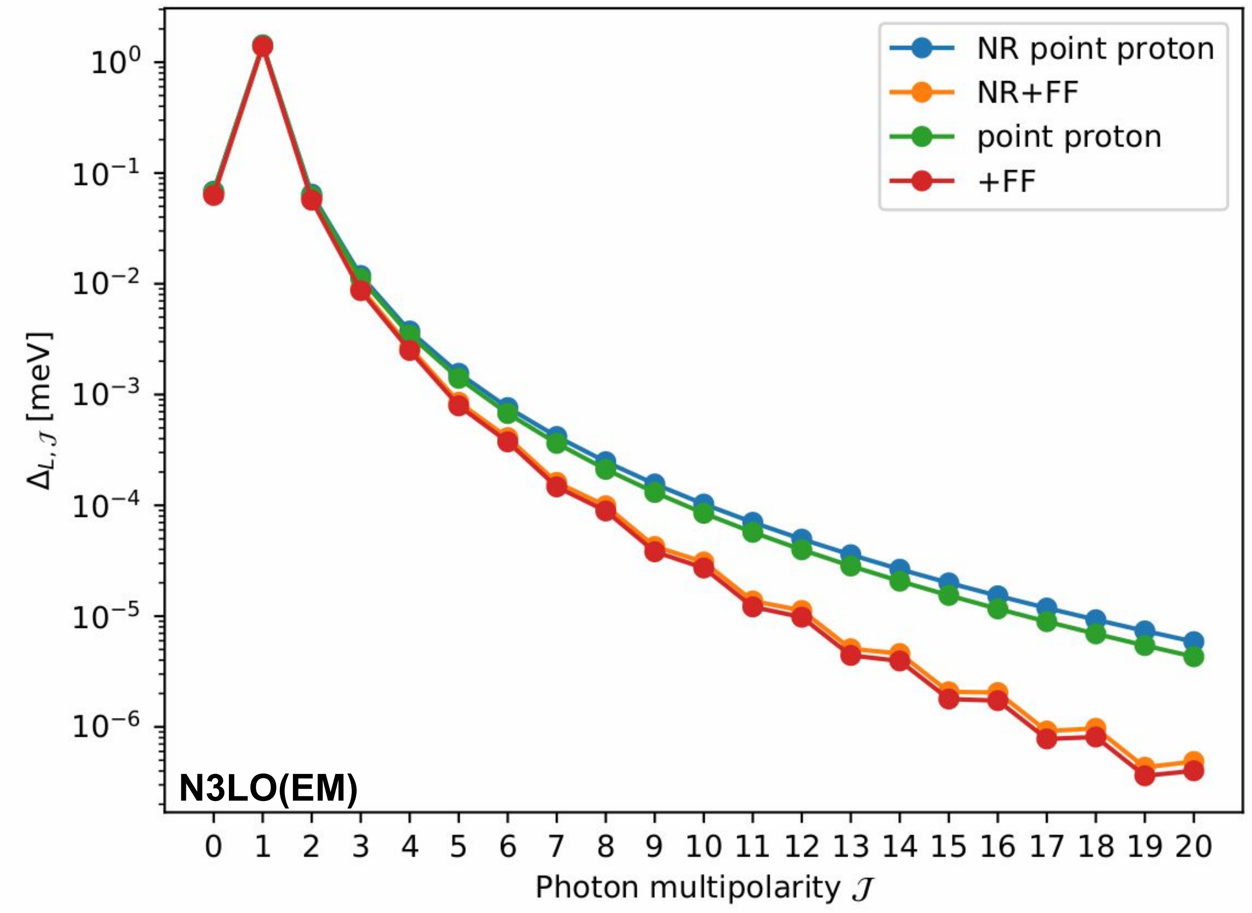
Longitudinal



Transverse

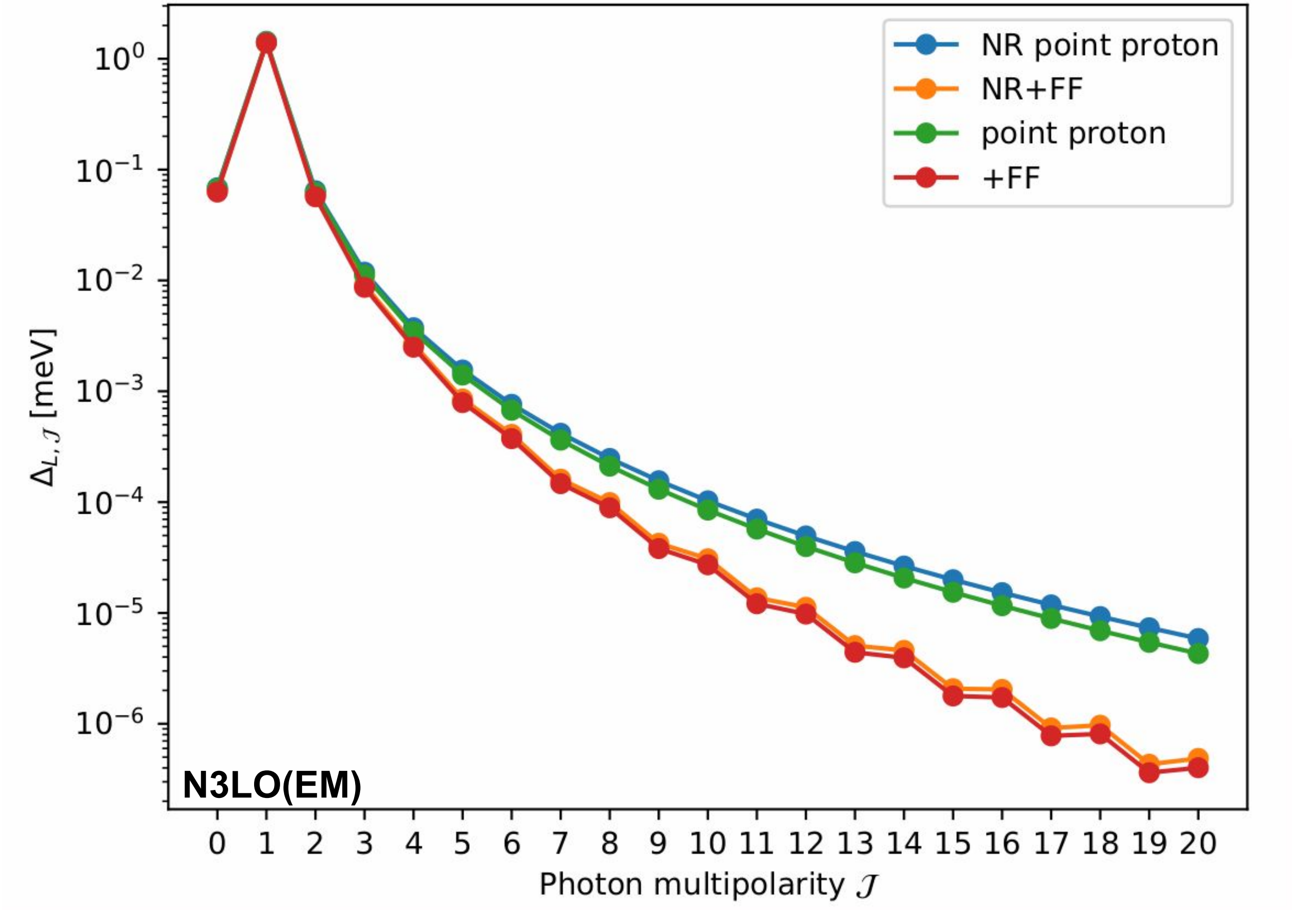


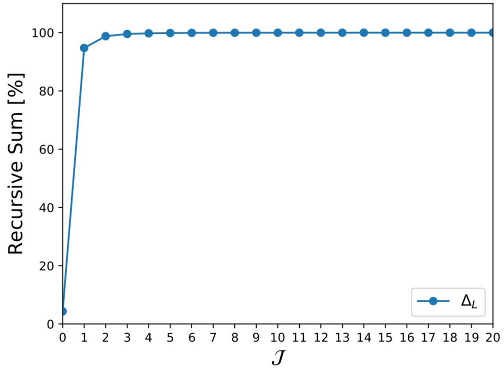
η-less expansion: Results



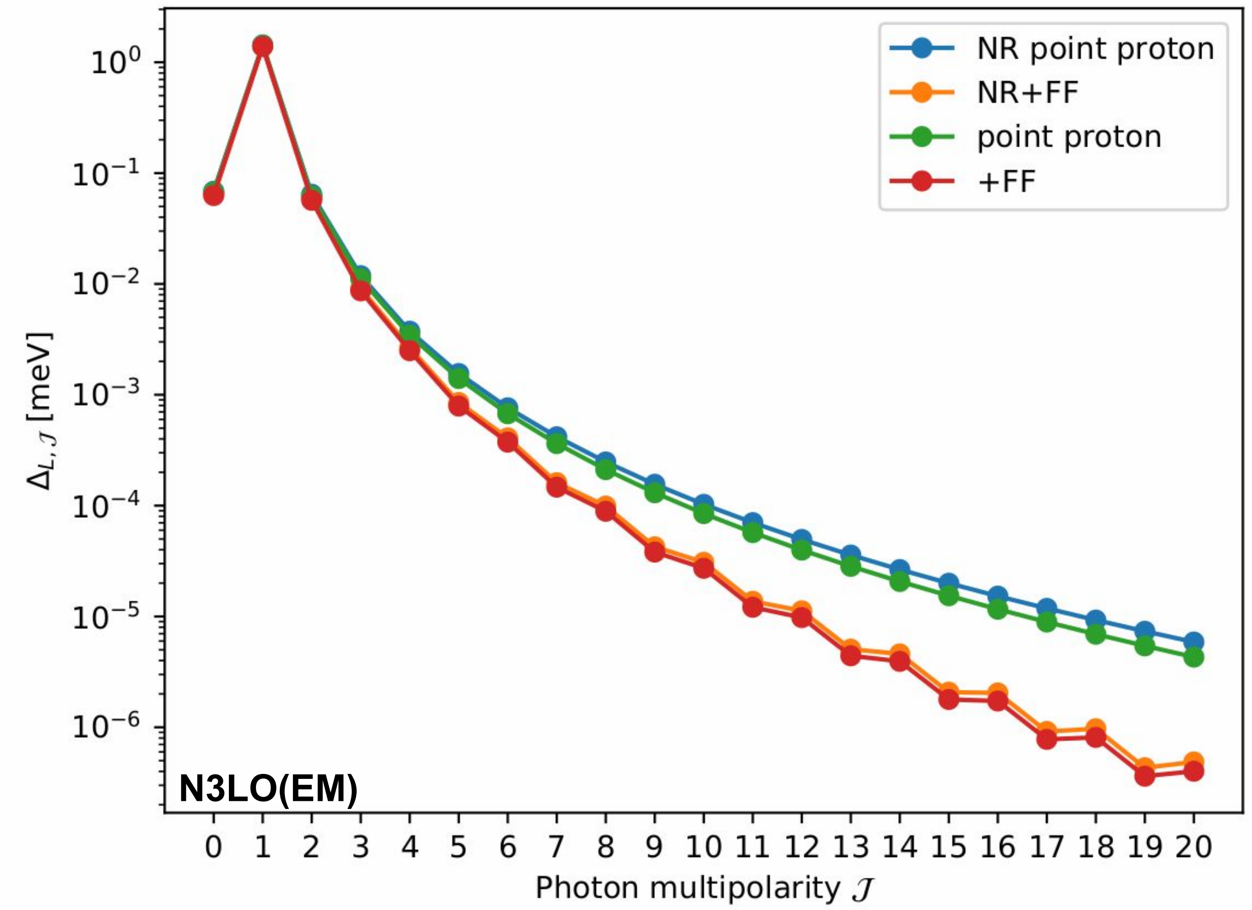
η-less expansion: Results

$$\Delta_L = \sum_{\mathcal{J}} \Delta_{L,\mathcal{J}}$$





η-less expansion: Results



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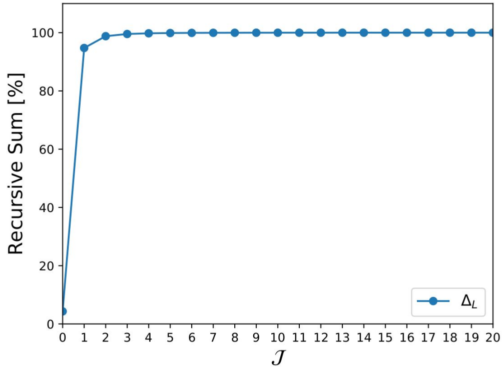
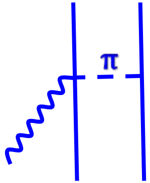


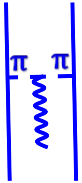
$$\delta^A_{pol} = -1.590\, meV$$
$$\Delta_L = -1.588\, meV$$

Correction	% Uncert.
η Exp.	0.3

Additional uncertainties

Two body currents + relativistic corr.





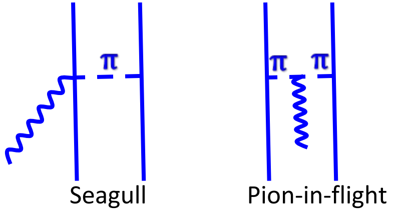
$$\rho, \vec{j}$$

Correction	% Uncert.
NLO MEC	0.05
Rel. Corr.	0.05

Additional uncertainties

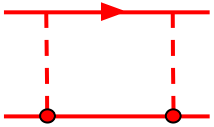
* C. E. Carlson et al. Phys. Rev. A 89, 022504 (2014).
J. J. Krauth, et al. Ann. of Phy. 366, 168 (2016).
R.J. Hill, G. Paz Phys. Rev. D, 95 (2017)

Two body currents + relativistic corr.



$\rho, \vec{j}$

Single Nucleon Physics



δ^{N}_{TPE}

Correction	% Uncert.	
NLO MEC	0.05	
Rel. Corr.	0.05	
Nucleon*	0.6	1.2

Additional uncertainties

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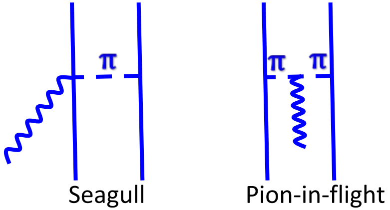
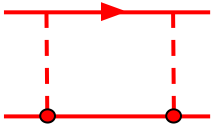


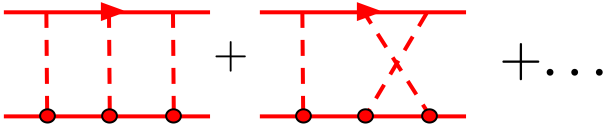
$\rho, \vec{j}$

Single Nucleon Physics



δ^{N}_{TPE}

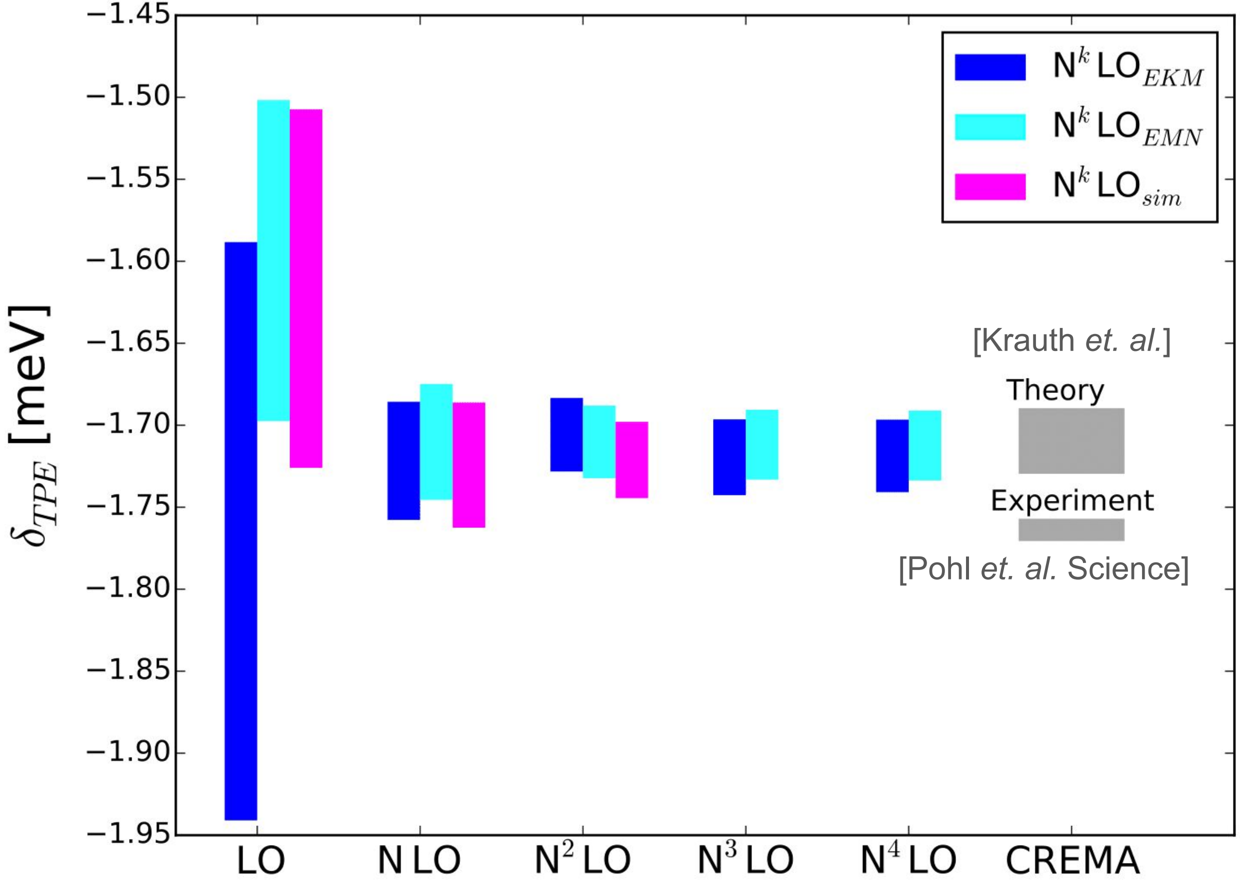
Atomic Physics uncert.



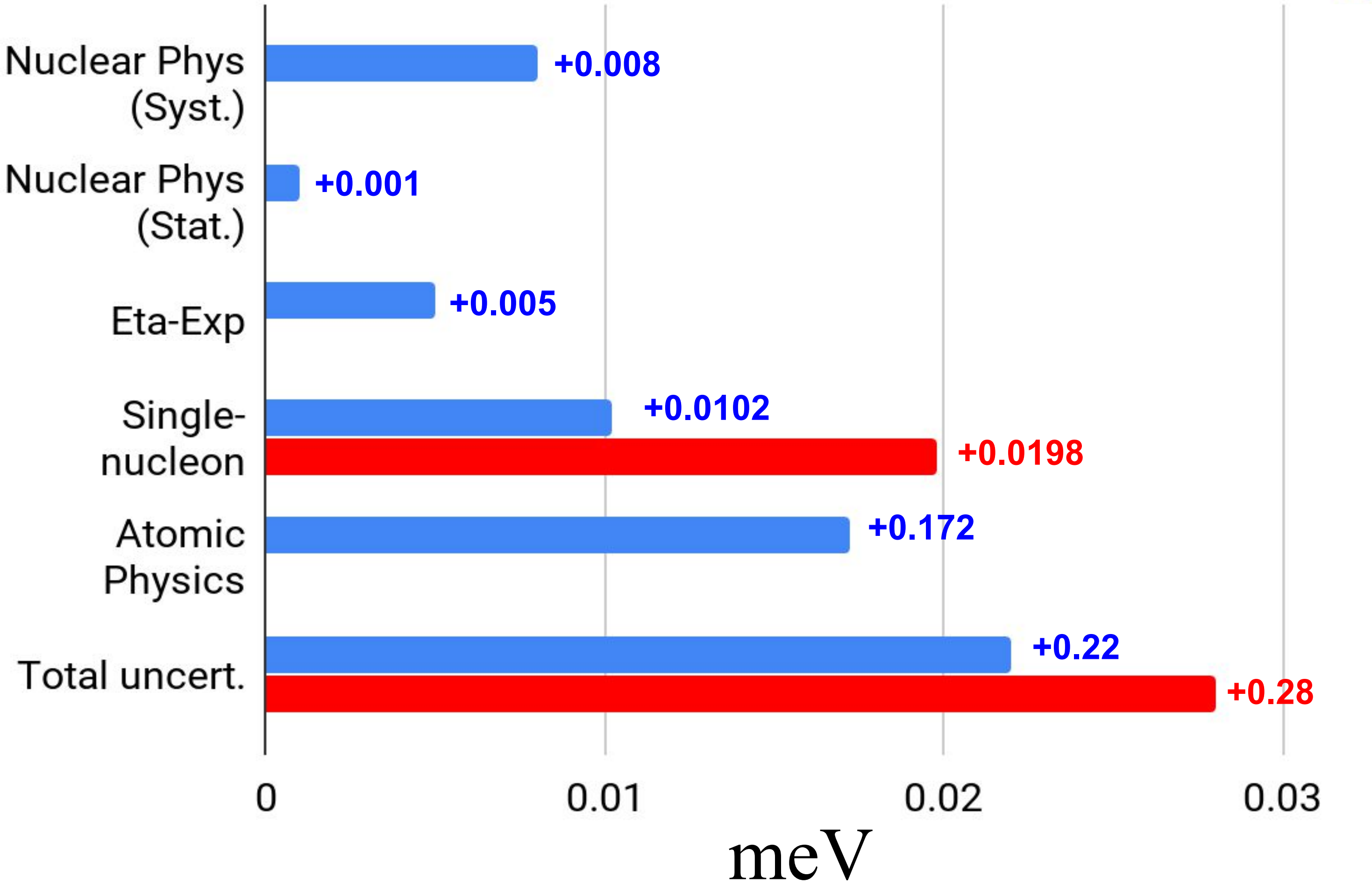
$$\Big\} \, O(\alpha^6)$$

Correction	% Uncert.	
NLO MEC	0.05	
Rel. Corr.	0.05	
Nucleon*	0.6	1.2
Atomic Phys.	1.0	

Uncertainty comparisons



Final uncertainty budget



Summary

Krauth et. al. [2016]

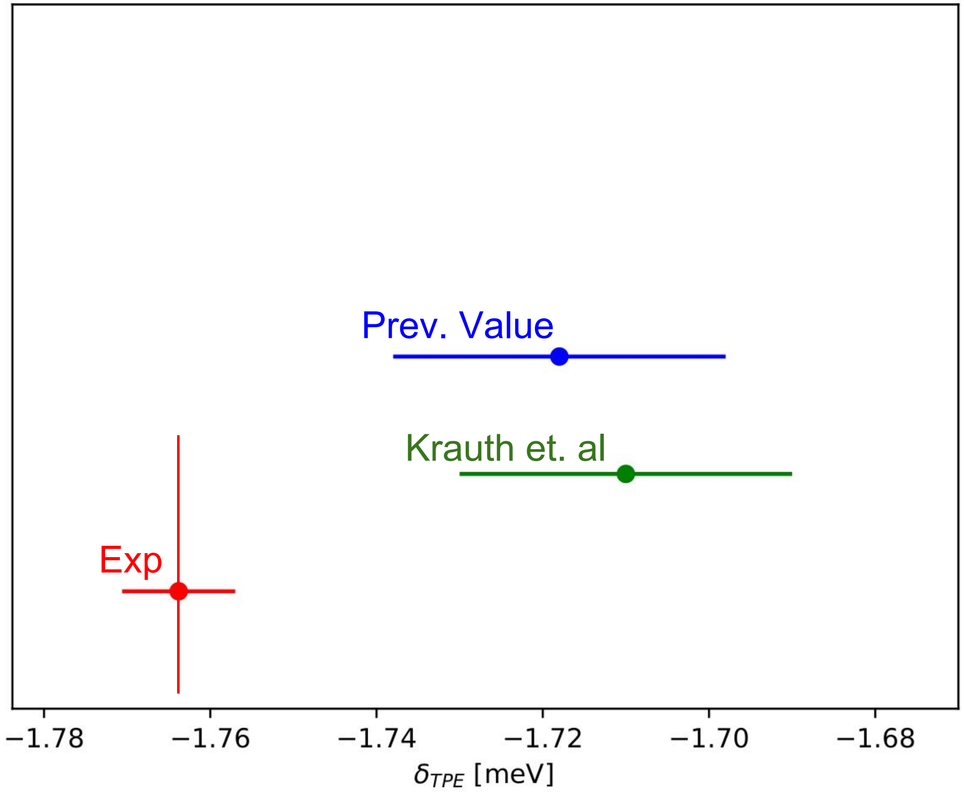
$$\delta_{TPE}(Krauth\ et\ al.) = -1.710(20)\ meV$$

Carlson et. al. [2016]

$$\delta_{TPE}(Carlson\ et\ al.) = -2.01(74)\ meV$$

Previous Value [2014,2016]

$$\delta_{TPE}(Prev\ Work) = -1.718(22)\ meV$$



Experimental

$$\delta_{TPE}(Exp) = -1.7638(68)\ meV$$

Summary

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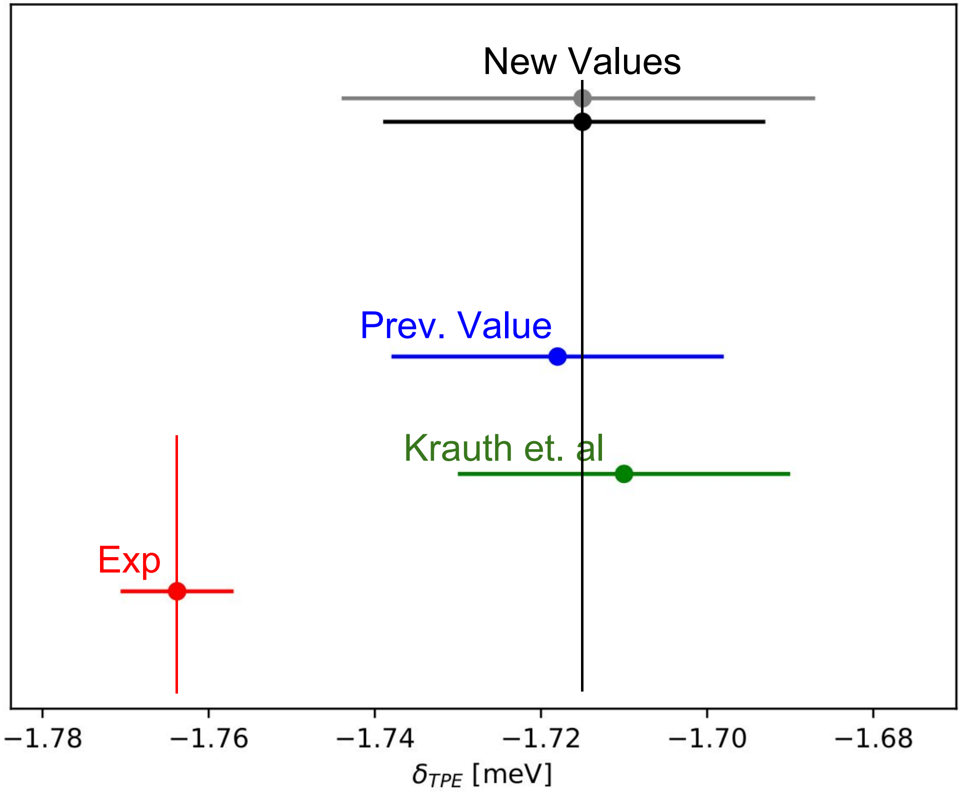
New values [2018]

$$\delta_{TPE} = -1.715^{+22}_{-24}\ meV$$

$$\delta_{TPE} = \qquad {}^{+28}_{-29}\ meV$$

Experimental

$$\delta_{TPE}(Exp) = -1.7638(68)\ meV$$



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Previous Value [2014,2016]

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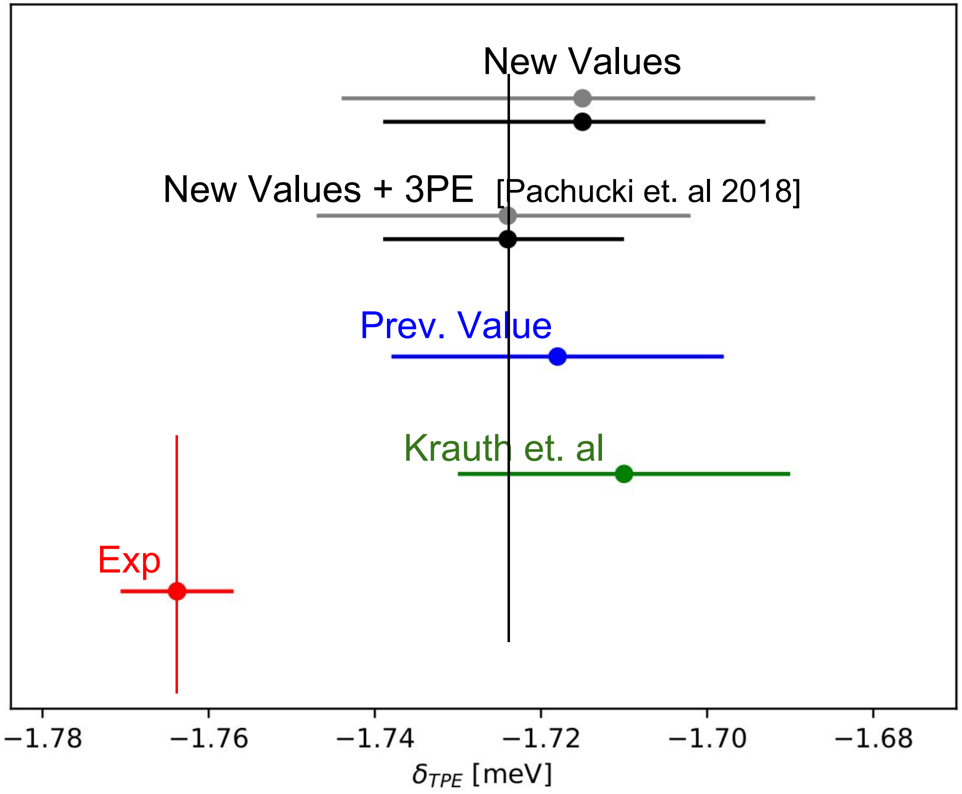
New values+3PE (Pachucki) [2018]

$$\delta_{TPE} = -1.724^{+14}_{-15}\ meV$$

$$\delta_{TPE} = ^{+22}_{-23}\ meV$$

Experimental

$$\delta_{TPE}(Exp) = -1.7638(68)\ meV$$



Outlook

Results:

- Experimental vs theory difference improved by thorough analysis of nuclear TPE uncertainty.
- Uncertainty in TPE cannot solve the 5.6 σ discrepancy.

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- Uncertainty in TPE cannot solve the 5.6 σ discrepancy.

Uncertainty Analysis:

- Reduce atomic physics uncert. $O(\alpha^6)$

	Correction	μH	μ3H	μ3He	μ4He
δ^A_{pol}	**Higher (Zα)**	0.7	0.7	1.5	1.5
	η Exp.	0.3	0.9	0.3	0.2

Etaless Expansion:

- Apply formalism to A=3 systems
- Extend formalism for HFS

Thank you!

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

THE UNIVERSITY OF BRITISH COLUMBIA

TRIUMF

Presented By: Oscar Javier Hernandez