Polarizability effects at leading $(Z\alpha)^5$ and next-to-leading $(Z\alpha)^6 \ln Z\alpha$ order

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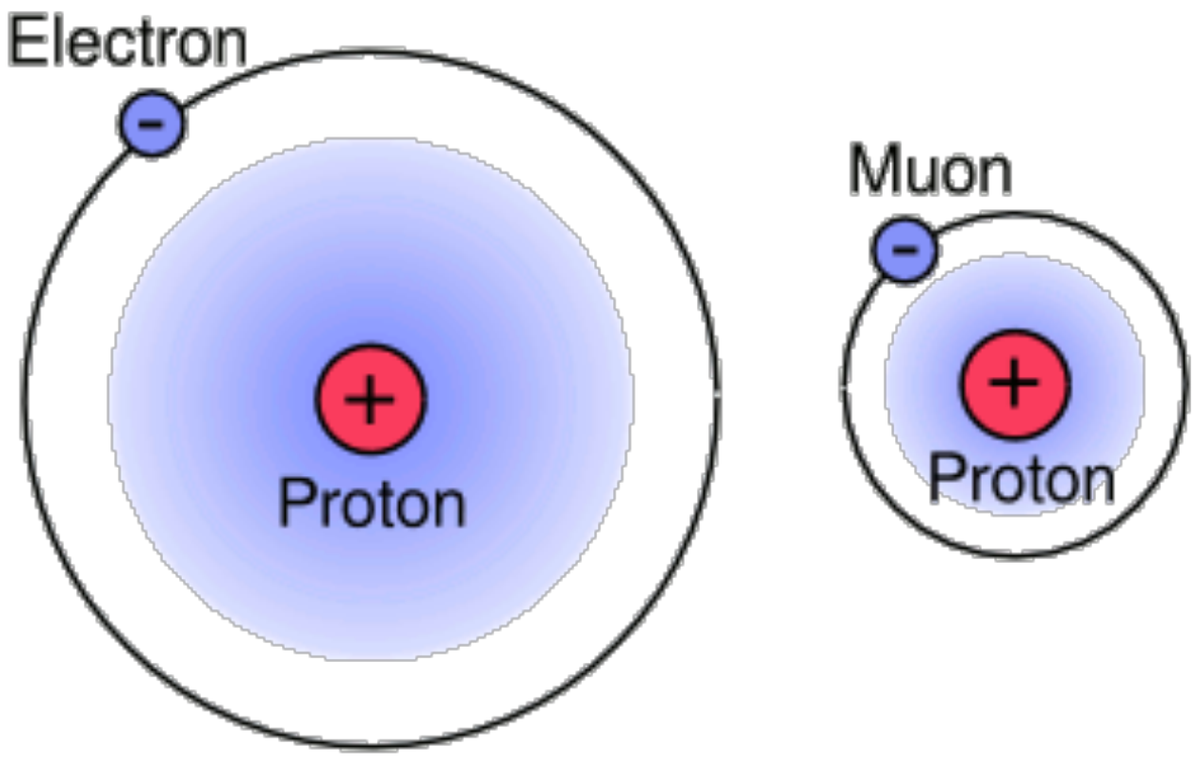
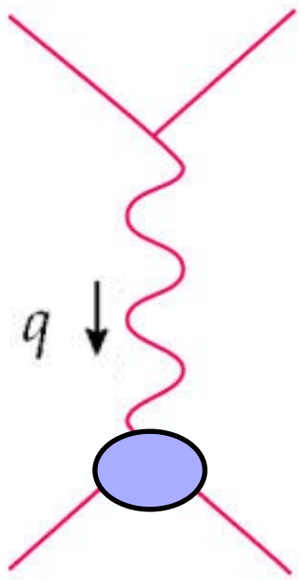
@ Proton Radius Workshop, MITP, Mainz
July 23 — 27, 2018

Muonic hydrogen more sensitive to proton structure





$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}[G_E(-\vec{q}^{\,2}) - 1] = \frac{2}{3}\pi\alpha\, r_E^2 + O(\vec{q}^{\,2})$$

$$\Delta E_{nl}^{(\mathrm{FS})} = \left\langle nlm\right| \delta V^{(1\gamma)} \left|nlm\right\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, r_E^2\, \frac{\alpha^3 m_r^3}{\pi n^3} + O(\alpha^5)$$

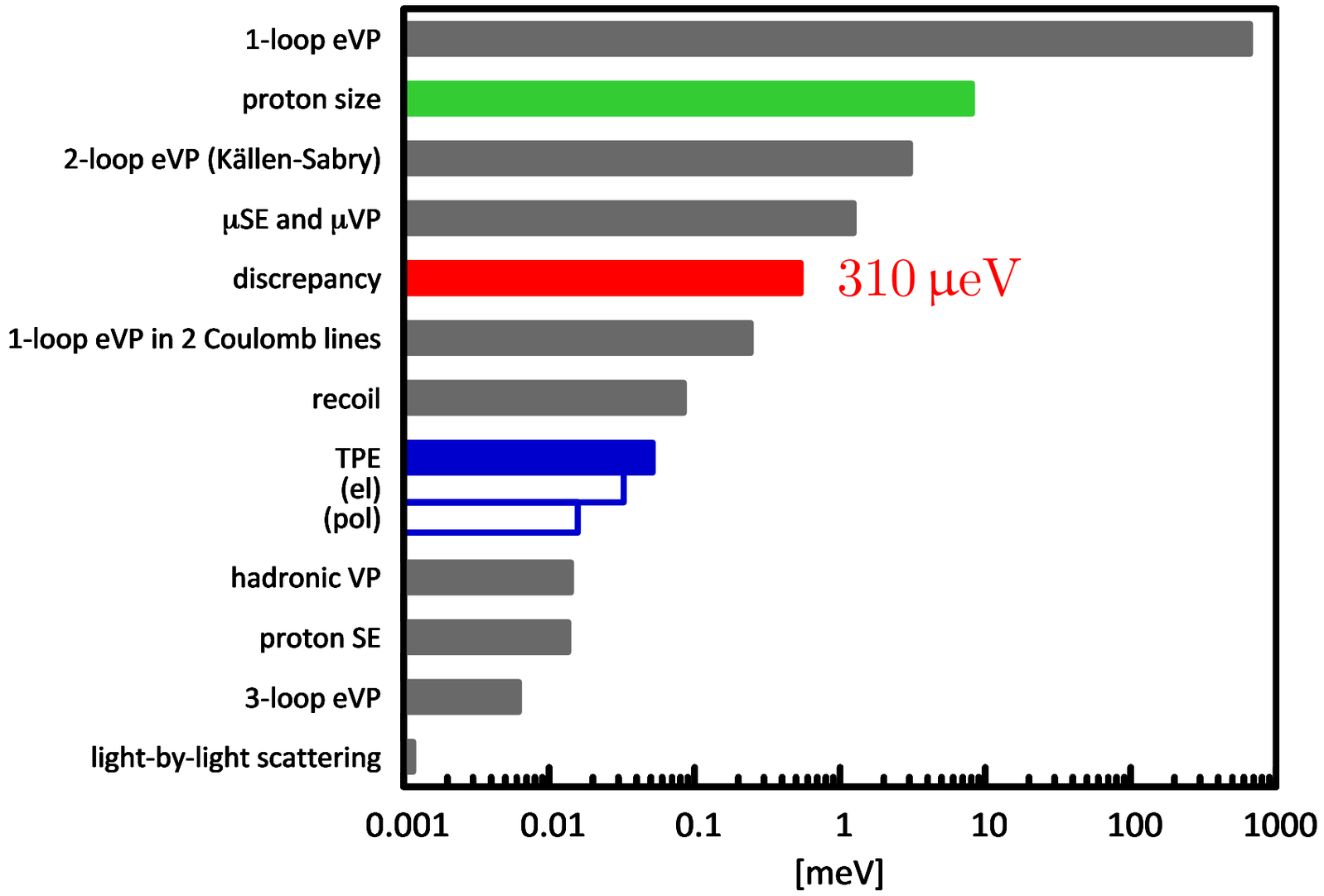
wave function at origin

Muonic Hydrogen Lamb shift

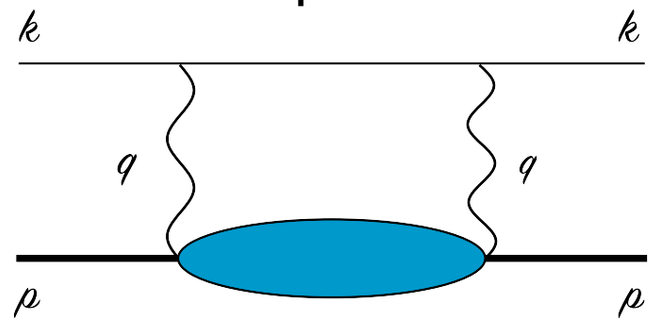
$$\Delta E_{\rm LS}^{\rm th} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

theory uncertainty: $2.5\,\mu eV$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).



subleading effects of proton structure proposed to resolve the puzzle



$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

A. De Rujula, Phys. Lett. B693 (2010)

G. A. Miller, Phys. Lett. B718 (2013)

Large 3rd Zemach moment (Friar radius)?

Correction due to proton's charge distribution:

1st order PT

$$E_{\rm LS}^{\langle eFF\rangle} = -\frac{\pi Z\alpha}{3a^3}\int_0^\infty {\rm dr}\, r^4 e^{-r/a}\rho_E(r),$$
$$= -\frac{Z\alpha}{12a^3}\sum_{k=0}^{\infty}\frac{(-Z\alpha m_r)^k}{k!}\langle r^{k+2}\rangle_E = -\frac{(Z\alpha)^4 m_r^3}{12}\Big(\langle r^2\rangle_E - Z\alpha m_r\langle r^3\rangle_E\Big)+\cdots$$

Moments of charge distribution:

$$\langle r^N\rangle \equiv \int d\vec{r}\, r^N\rho(r) = \frac{(N+1)!}{\pi}\int_{t_0}^{\infty} dt\,\frac{{\rm Im}\, G_E(t)}{t^{1+N/2}}.$$

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2nd order PT

$$\Delta E_{2S}^{{\rm FF}(2)} = -\alpha^5 m_r^4\,\frac{2}{\pi}\int_0^\infty dk\left\{\frac{1}{\pi}\int_{t_0}^\infty \frac{dt}{t}\,\frac{1}{t+k^2}\,{\rm Im}\,G_E(t)\right\}^2 + O(\alpha^6)$$
$$= -\alpha^5 m_r^4\,\frac{2}{\pi}\int_0^\infty \frac{dk}{k^4}\left\{G_E(-k^2)-1\right\}^2 + O(\alpha^6).$$

$$\Delta E_{2P-2S}^{(2)} = -\frac{1}{12}\,\alpha^5 m_r^4\left(\langle r^3\rangle - \frac{1}{2}\langle r^3\rangle_{(2)}\right)$$

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De Rujula's scenario means beakdown of the expansion:

PHYSICAL REVIEW A **91**, 040502(R) (2015)

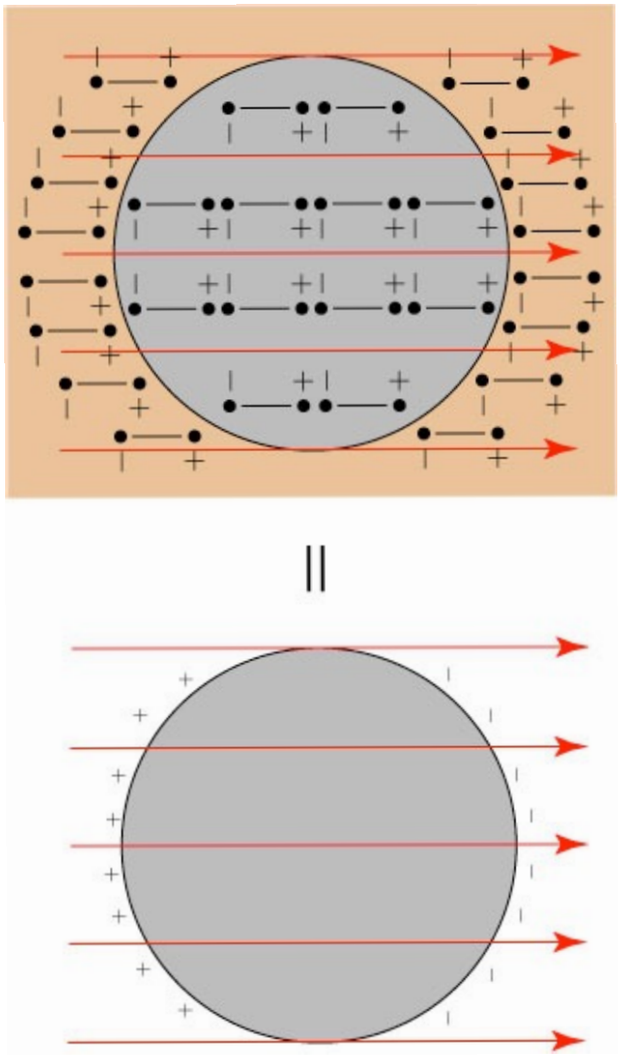
Breakdown of the expansion of finite-size corrections to the hydrogen Lamb shift in moments of charge distribution

Franziska Hagelstein and Vladimir Pascalutsa

(Large) polarizability correction?

Concept of polarizabilities

- A dielectric system in external e.m. field is polarized, e.g. in a uniform electric field:



- induced electric dipole polarization:

$$\vec{P} = \alpha_{E1}\,\vec{E}$$

(linear dielectric)

electric polarizability

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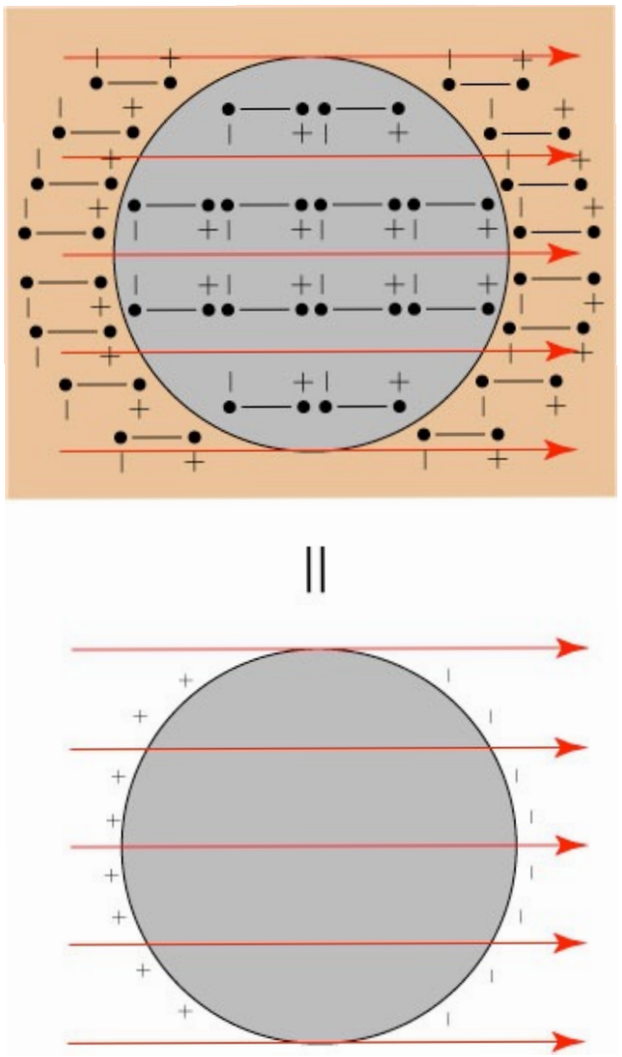

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(linear dielectric)

electric polarizability

- for polarization induced by magnetic field:

$$\vec{P} = \beta_{M1}\,\vec{B}$$

magnetic polarizability

Nucleon is different from e.g. an atom

Proton: $V \sim \langle r_p \rangle^3 \approx 0.6\ \mathrm{fm}^3$, cf. experiment: $\alpha_{E1p}^{(exp.)} = (11 \pm 1) \times 10^{-4}\,\mathrm{fm}^3$

1000 times "stiffer" than hydrogen!

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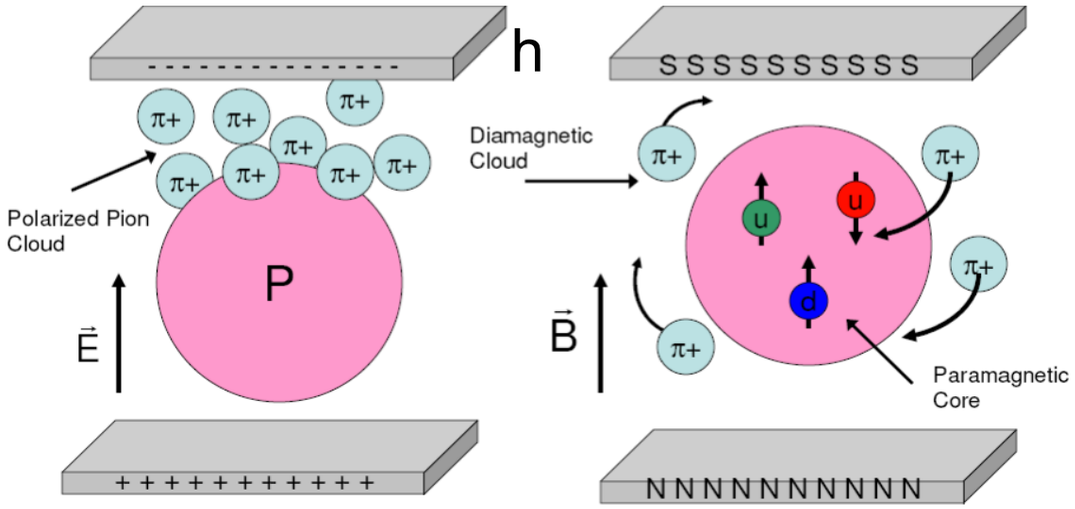

diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$

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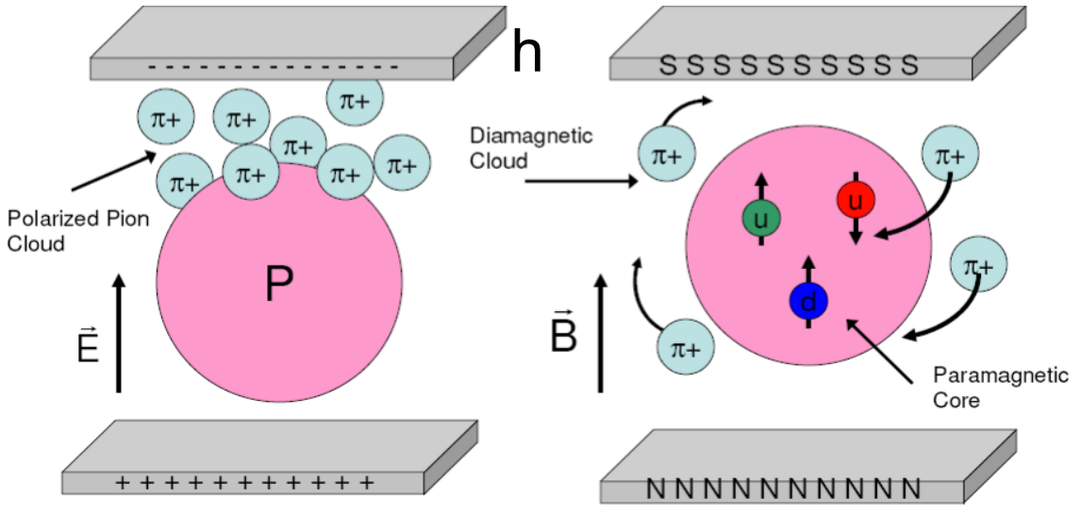

diamagnetic: $\beta_{M1} < 0$
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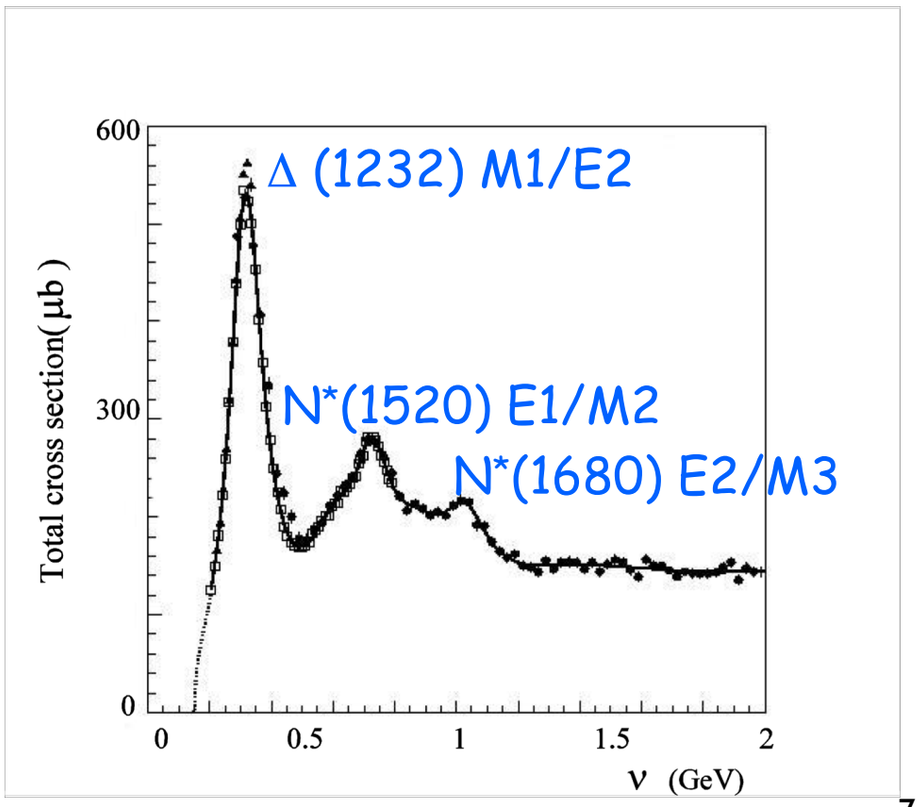
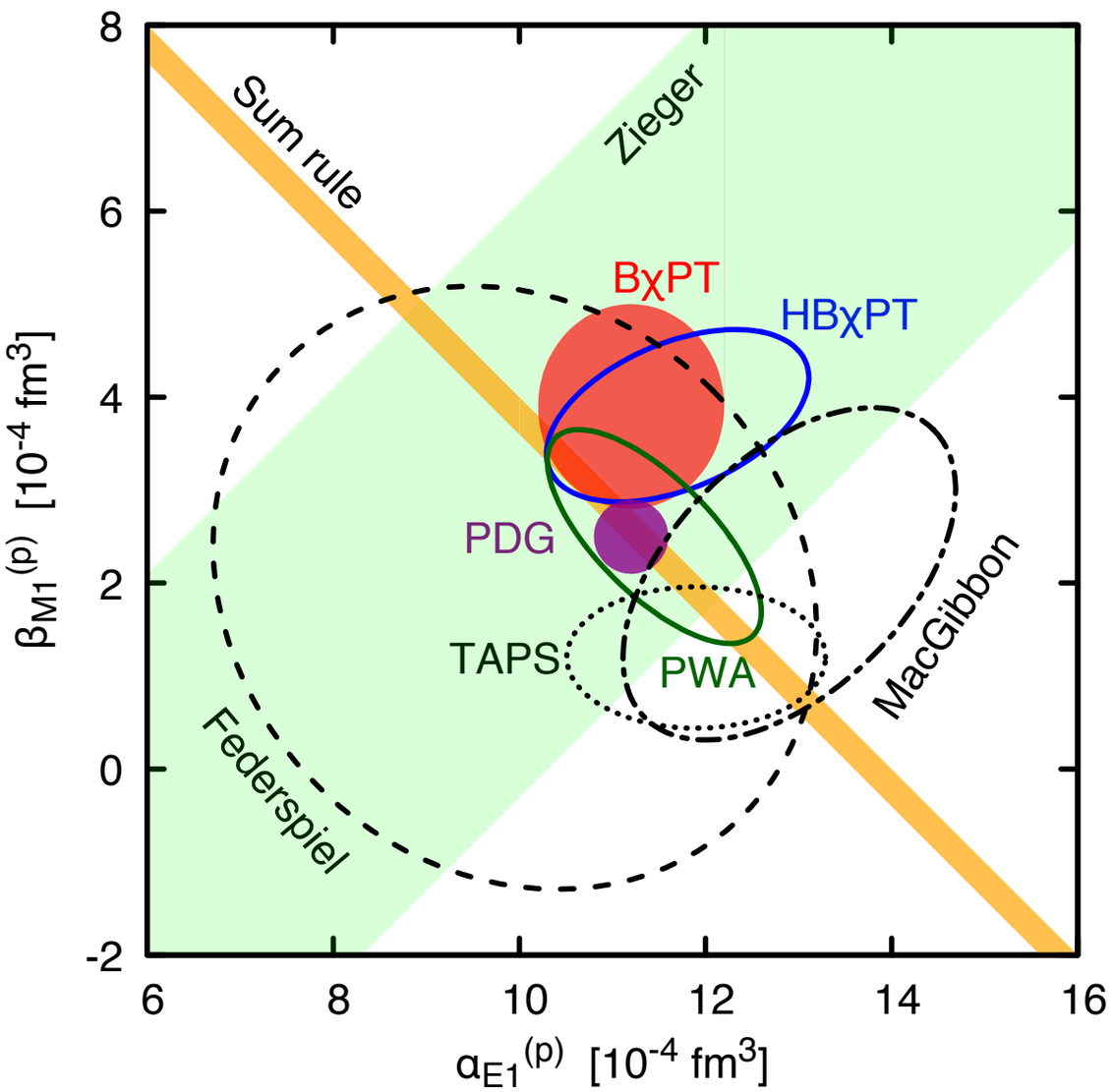
$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \text{fm}^3$$

[Baldin sum rule (1960)]



Static polarizabilities of the proton



- **TAPS:** fit to TAPS/MAMI data based on fixed-*t* DRs of L'vov et al.
  Olmos de Leon et al., *EPJA (2001)*

- **BChPT:** "postdiction"
  Lensky & VP, *EPJC (2010)*
  Lensky, McGovern & VP, *EPJC (2015)*

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  Krupina, Lensky & VP, *PLB (2018)*

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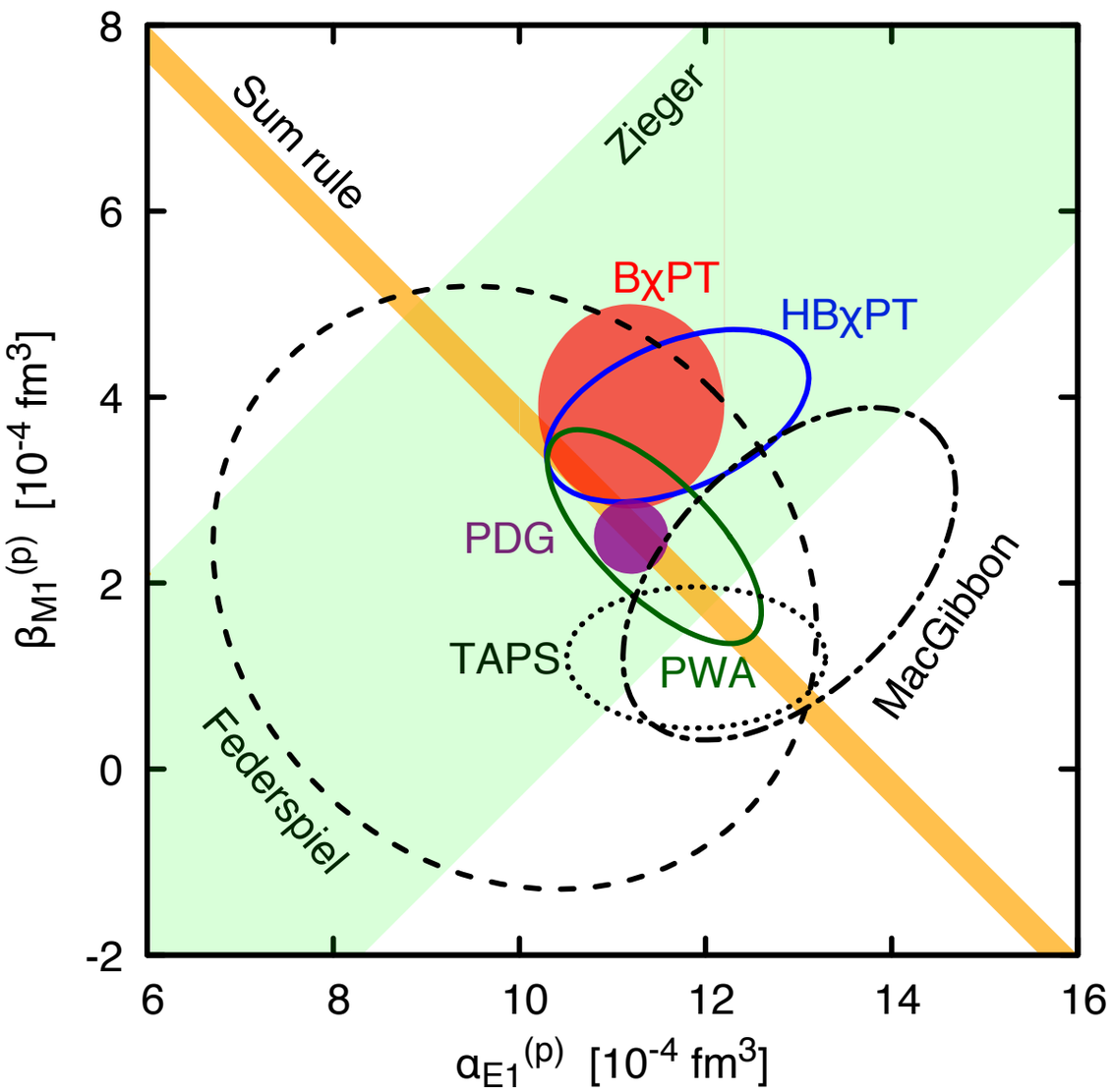

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Partial-Wave Analysis (PWA):
differences between DR and ChPT extractions are due to database inconsistencies, improvements — new experiments — are needed!

Polarizability potential (long-range part)

$$V(r) = -\frac{\alpha\,\alpha_{E1}}{2r^4}$$

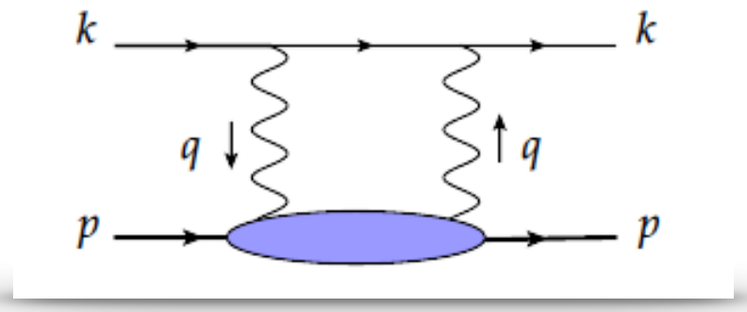
$$\overline{r^{-4}} = \langle nl|r^{-4}|nl\rangle = \frac{{}^1\!/_2\, Z^4\left[3n^2 - l(l+1)\right]}{n^5(l+{}^3\!/_2)(l+1)(l+{}^1\!/_2)l(l-{}^1\!/_2)}$$

divergent for S-waves,
short-range part is needed!

$(Z\alpha)^5$: forward two-photon exchange (TPE)

$(Z\alpha)^6 \ln Z\alpha$: off-forward two-photon exchange

polarizability correction at leading order



empirically known 'inelastic'

unknown 'subtraction'

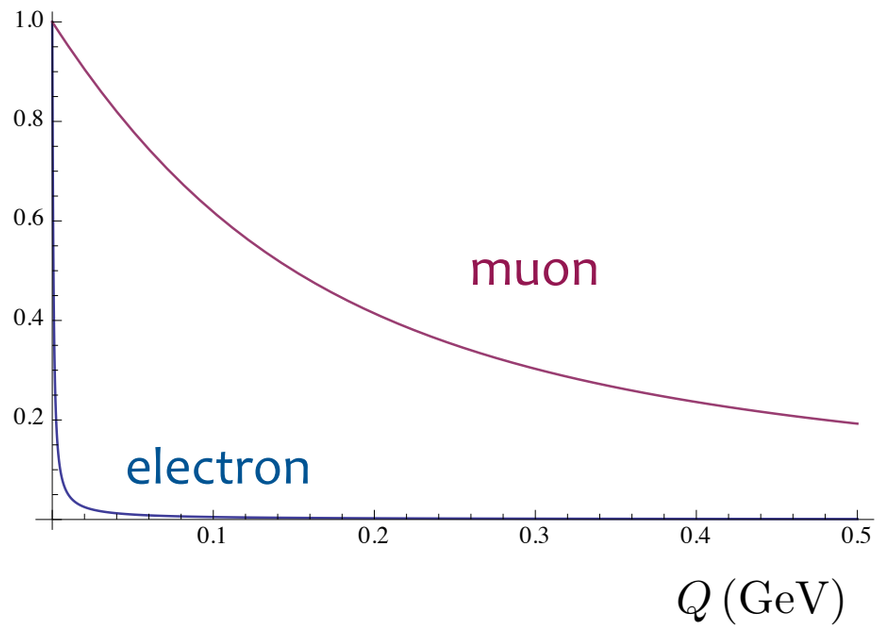
$$\Delta E_{nS}^{(\mathrm{pol})} = -4\alpha_{em}\phi_n^2 \int_0^\infty \frac{dQ}{Q^2}\, w\left(Q^2/4m_\ell^2\right) \left[T_2^{(\mathrm{NB})}(0,Q^2) - T_1^{(\mathrm{NB})}(0,Q^2)\right]$$

where unpolarized, forward Doubly-Virtual Compton scattering (VVCS) amplitude:

$$\phi_n^2(0) = m_r^3\alpha^3/(\pi n^3)$$

$$\begin{aligned} T^{\mu\nu}(p,q) &= \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|Tj^\mu(x)j^\nu(0)|p\rangle \\ &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) \\ &+ \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu,Q^2) \end{aligned}$$

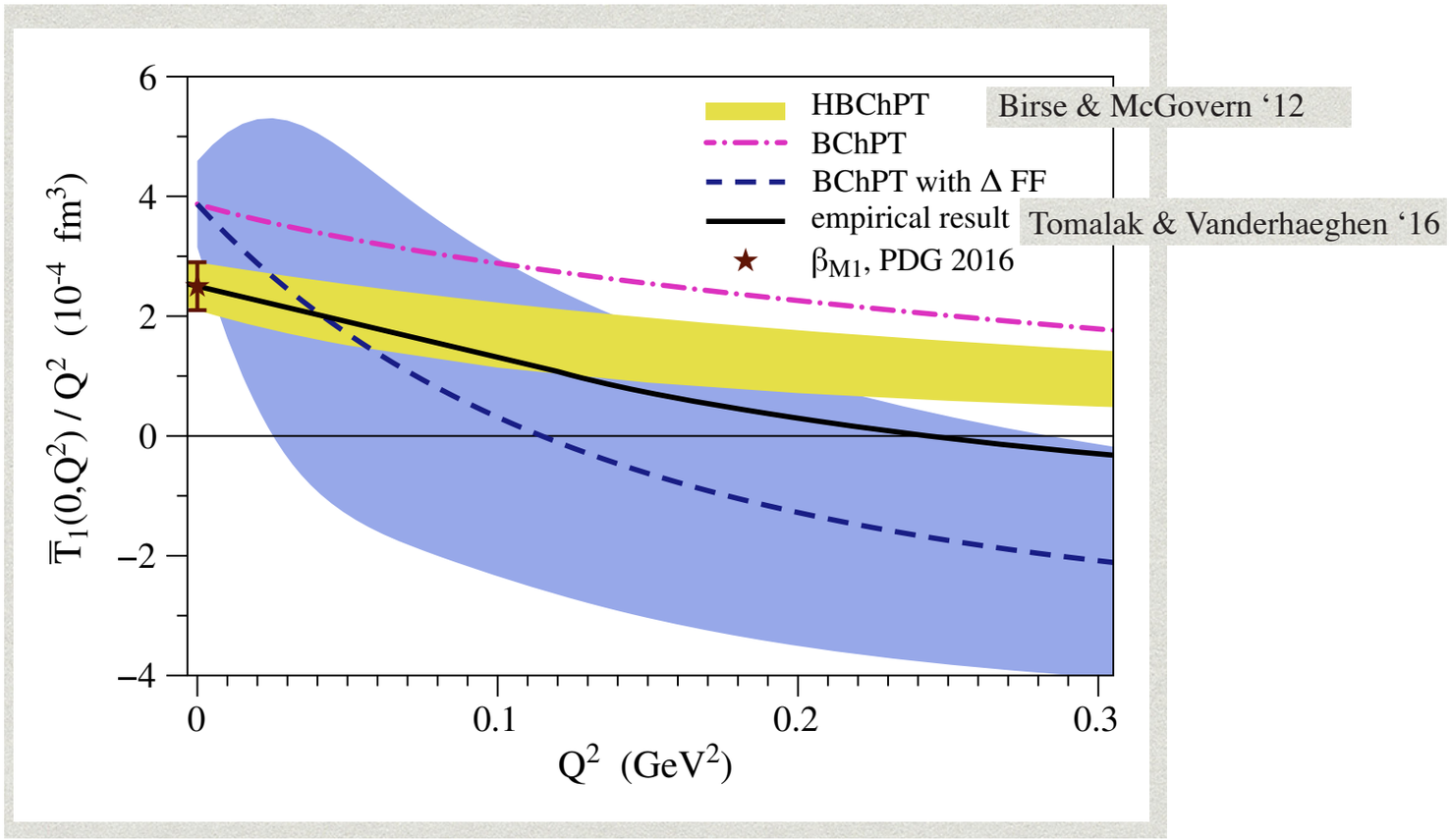
$$w_\ell(Q) = \sqrt{1+\frac{Q^2}{4m_\ell^2}} - \frac{Q}{2m_\ell}$$



NB stands for non-Born, i.e. w/o elastic FFs

$$\begin{aligned} T_1^{(\mathrm{NB})}(0,Q^2) &\simeq Q^2\beta_{M1} \\ T_2^{(\mathrm{NB})}(0,Q^2) &\simeq Q^2(\alpha_{E1}+\beta_{M1}), \qquad \text{for low } Q \end{aligned}$$

Subtraction function



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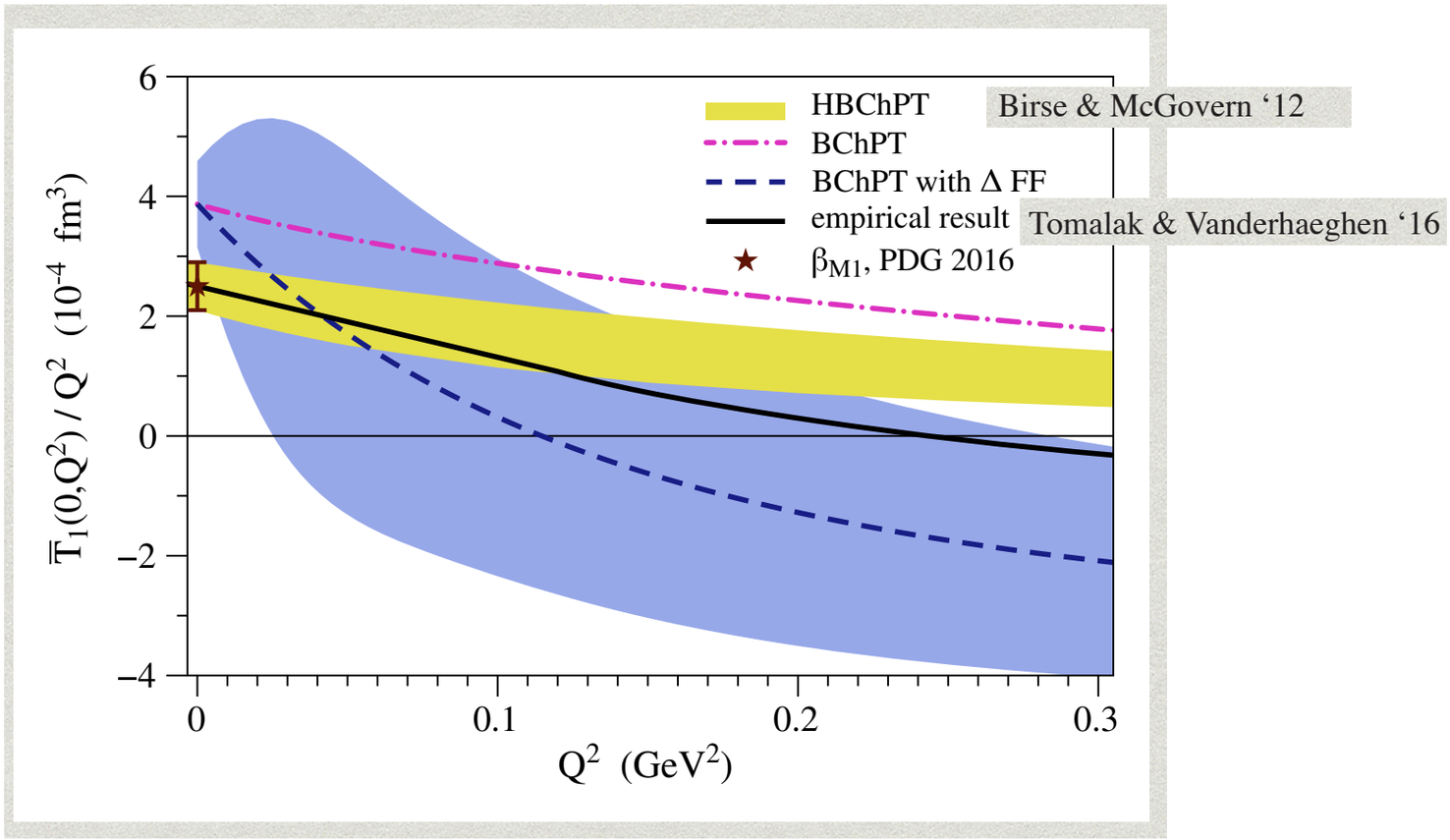
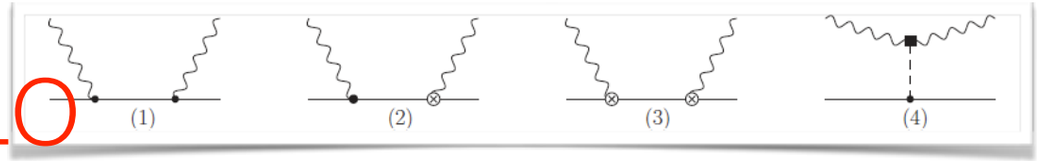


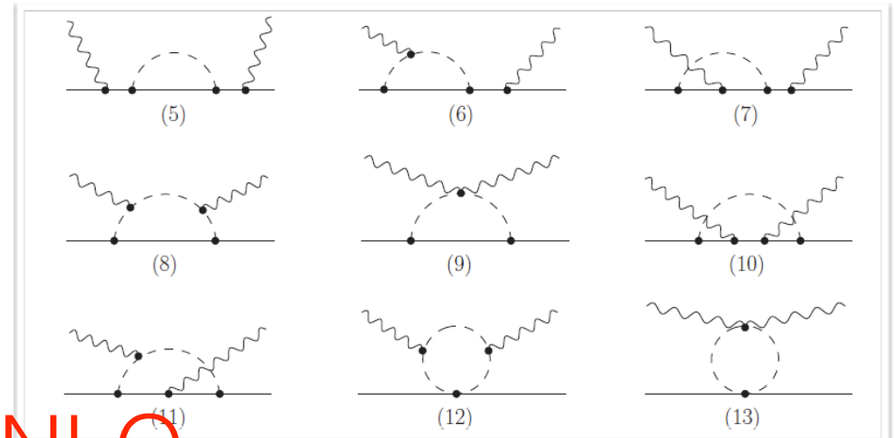
How to estimate uncertainty?

Compton scattering

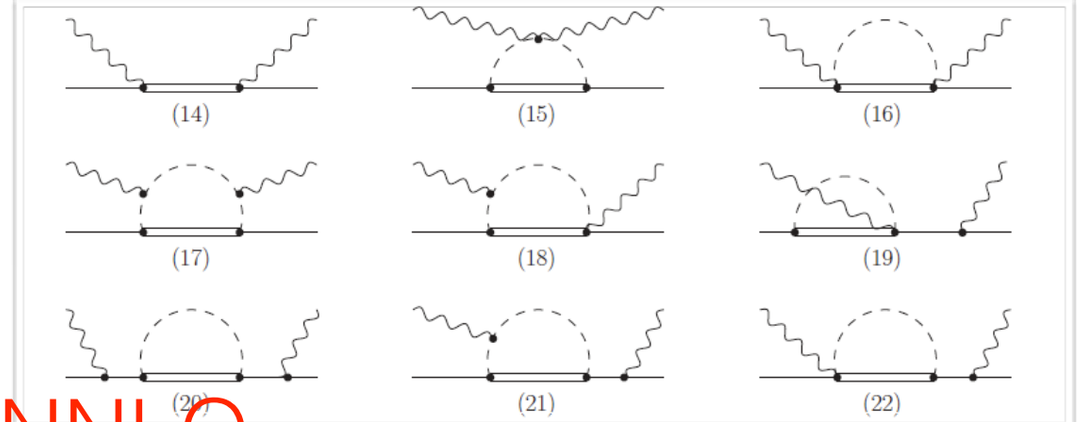
LO



Lensky & V.P., EPJC (2010);
Lensky, McGovern & V.P., EPJC (2015)

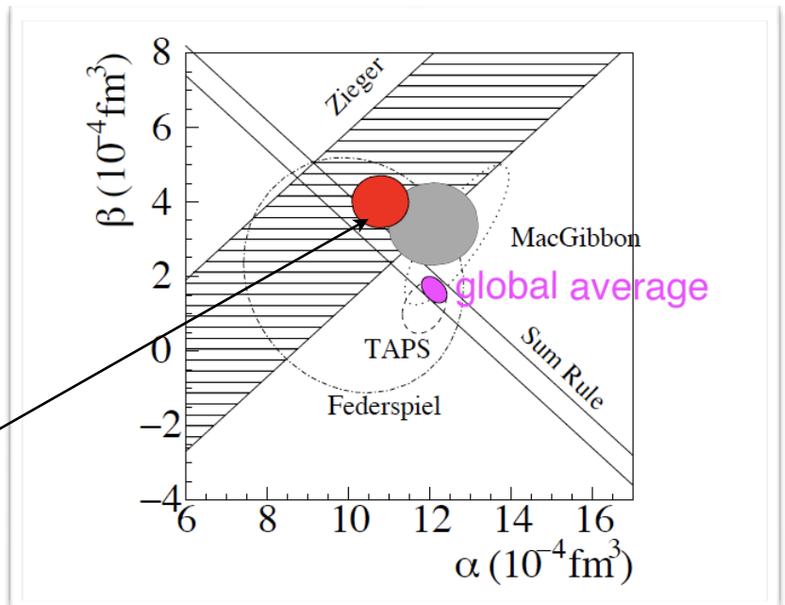


NLO

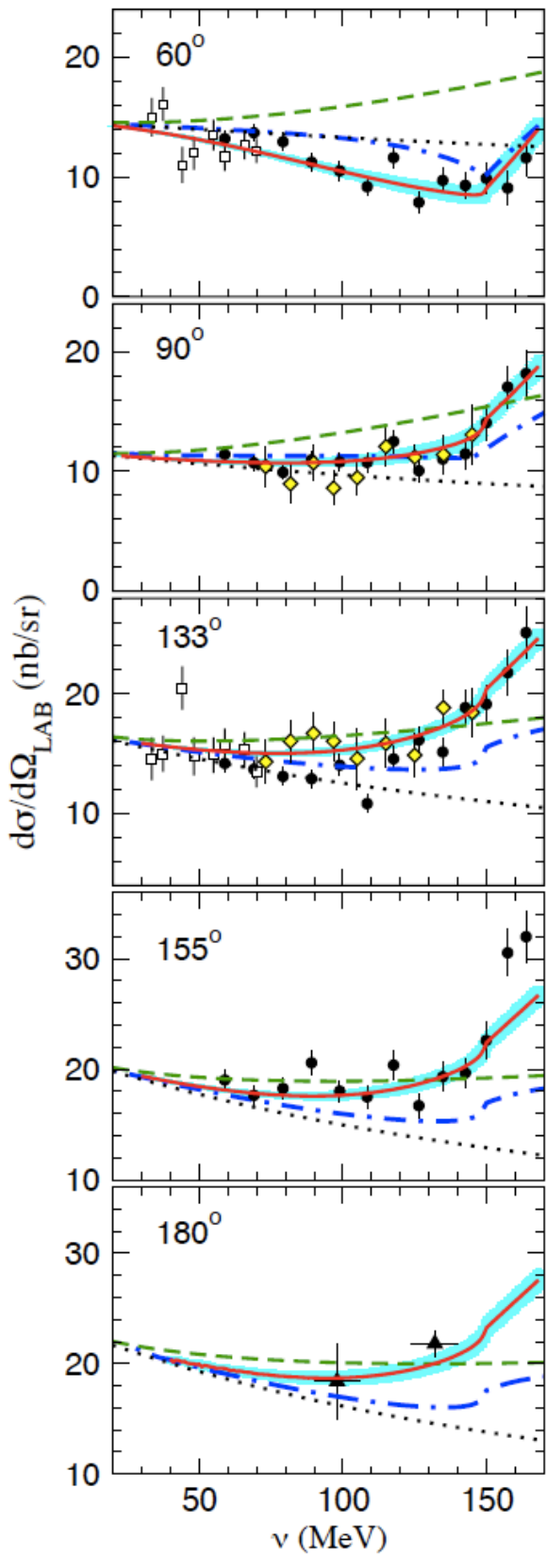


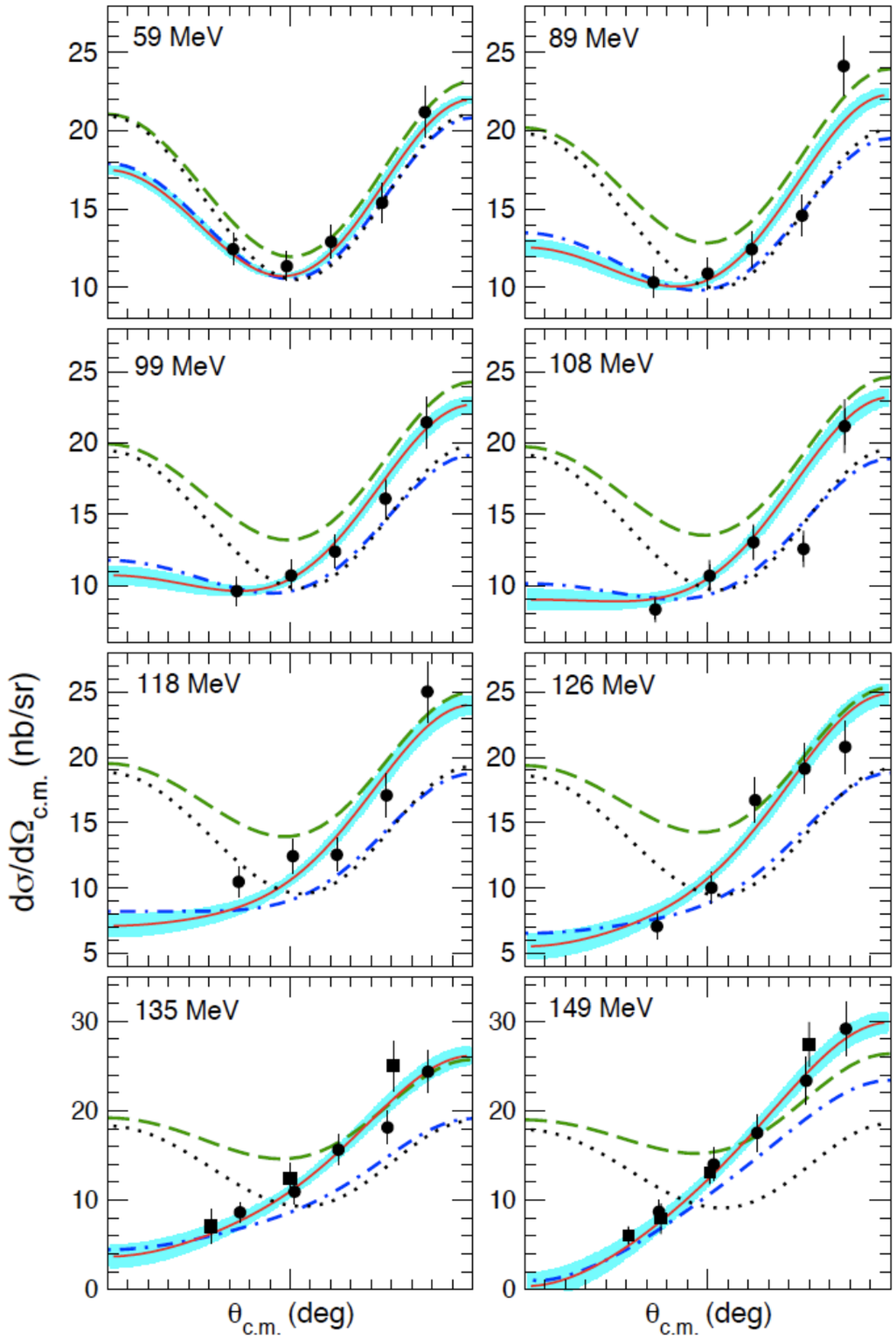
NNLO

$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$ size of the red blob



Unpolarized cross sections





Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

Curves:

..... Klein-Nishina

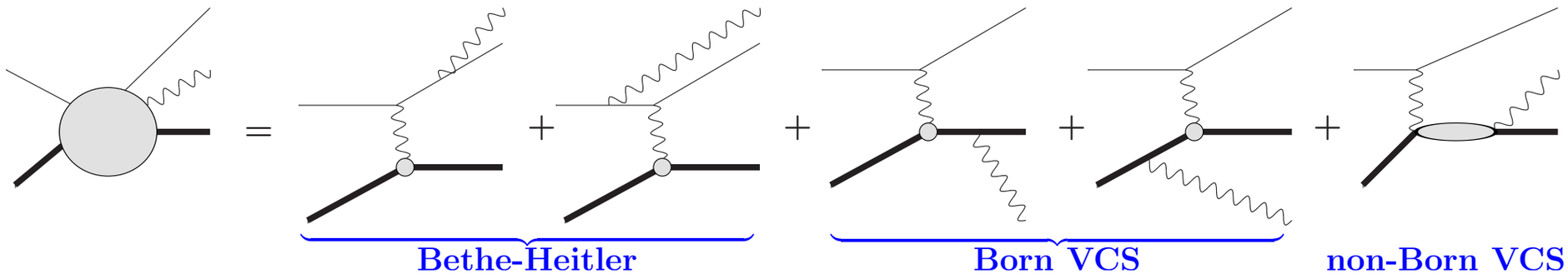
– – – – Born + WZW

– · – · – + p-qube

——— Total NNLO

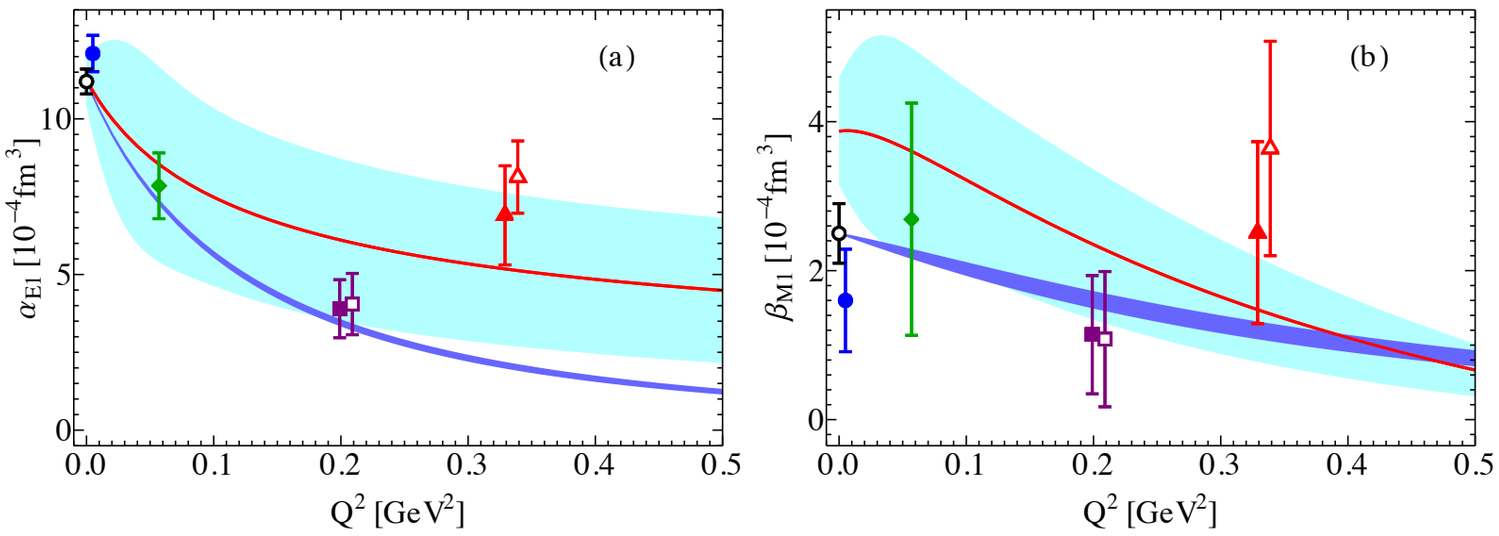
Lensky & V.P.,EPJC (2010)

Virtual Compton scattering (VCS) and generalized polarizabilities (GPs)

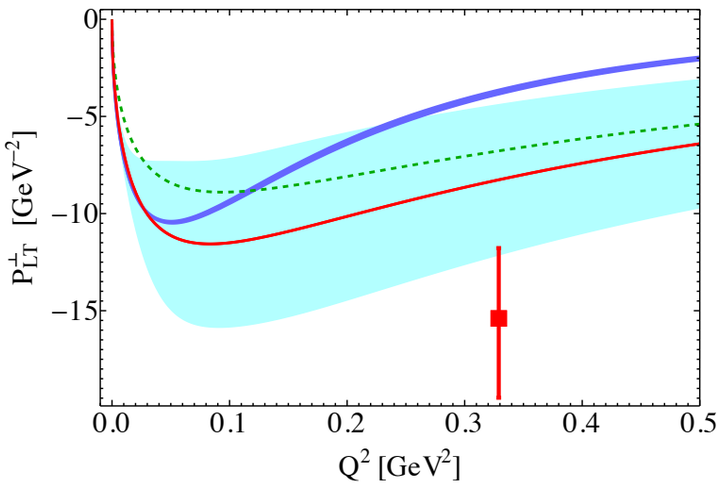


NLO-BChPT: Lensky, VP & Vanderhaeghen, *EPJC (2017) [1612.08626]*

Fixed-*t* DR: Pasquini et al., *PRC (2000); EPJA (2001)*



open circle, PDG 2014 [61]; blue circle, Olmos de León et al [62]; green diamond, MIT-Bates (DR) [7, 8]; green open diamond, MIT-Bates (LEX) [7, 8]; purple solid square, MAMI (DR) [13]; purple open square, MAMI (LEX) [13]; red solid triangle, MAMI1 (LEX) [9]; red solid inverted triangle, MAMI1 (DR) [11]; red open triangle, MAMI2 (LEX) [10]. Some of the data points are shifted to the right in order to enhance their visibility; namely, Olmos de León, MIT-Bates (LEX), MAMI LEX, MAMI1 DR and MAMI2 LEX sets have the same values of Q^2 as PDG, MIT-Bates (DR), MAMI DR, and MAMI1 LEX, respectively.

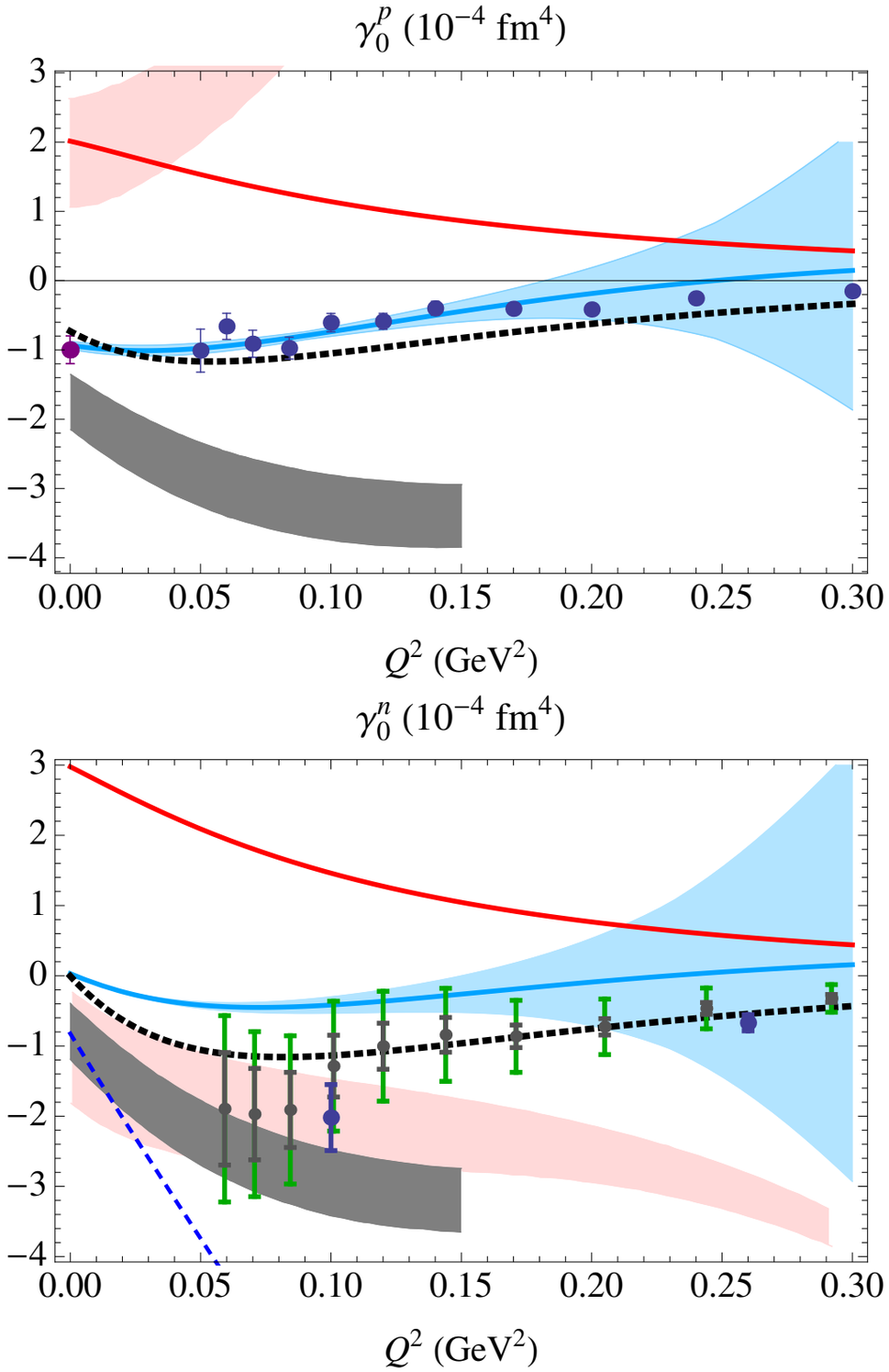


preliminary MAMI data:

L. Corea, H. Fonvieille,
H. Merkel et al. [A1 Coll.]

Forward spin polarizability at finite Q

figures from: Alarcon, Hagelstein, Lensky & V.P., in prep.



Curves:

MAID (empir.)

LO-HBChPT

NLO-HBChPT

NLO- IRBChPT
[Bernard et al (2006)]

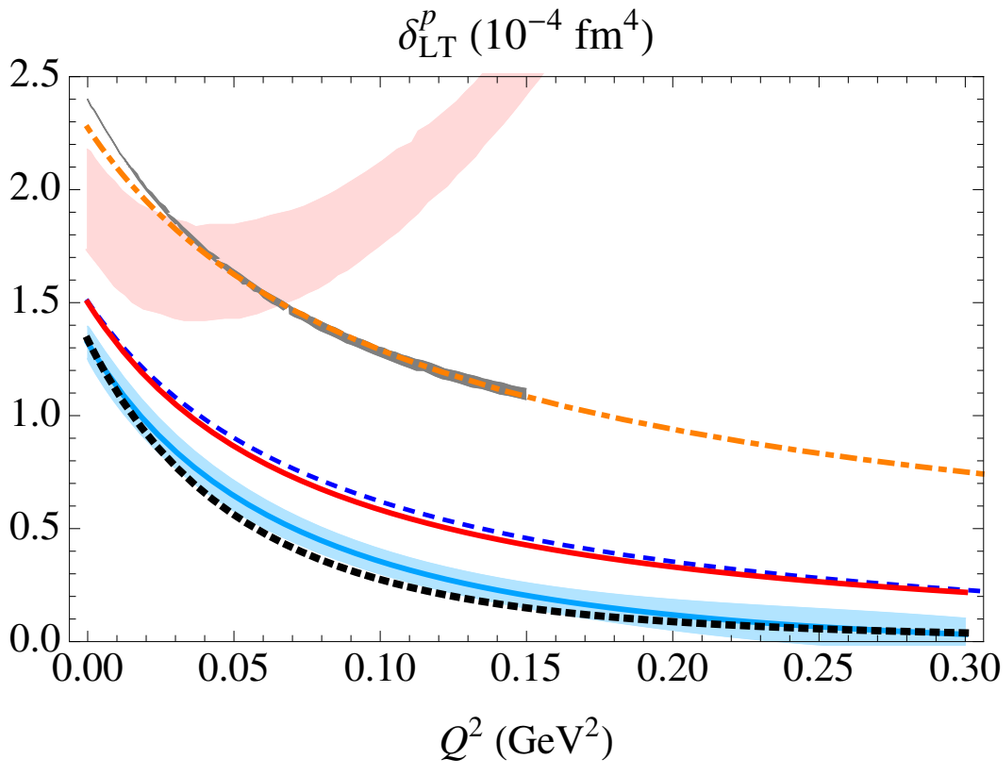
LO-BChPT

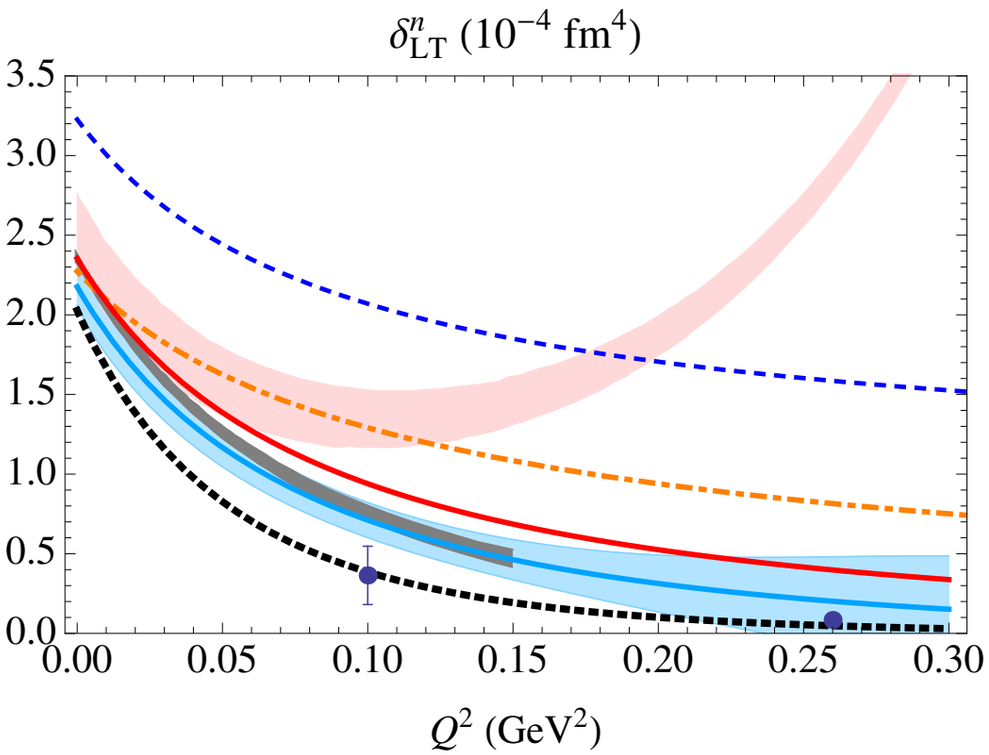
NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

NLO-BChPT
[Bernard et al (2013)]

Data points:
K. Slifer, J.-P. Chen, S. Kuhn, et al
[Jefferson Lab spin program]

LT spin polarizability





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[Bernard et al (2006)]

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NLO-BChPT
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$(Z\alpha)^5$ Polarizability contribution in ChPT

Eur. Phys. J. C (2014) 74:2852
DOI 10.1140/epjc/s10052-014-2852-0

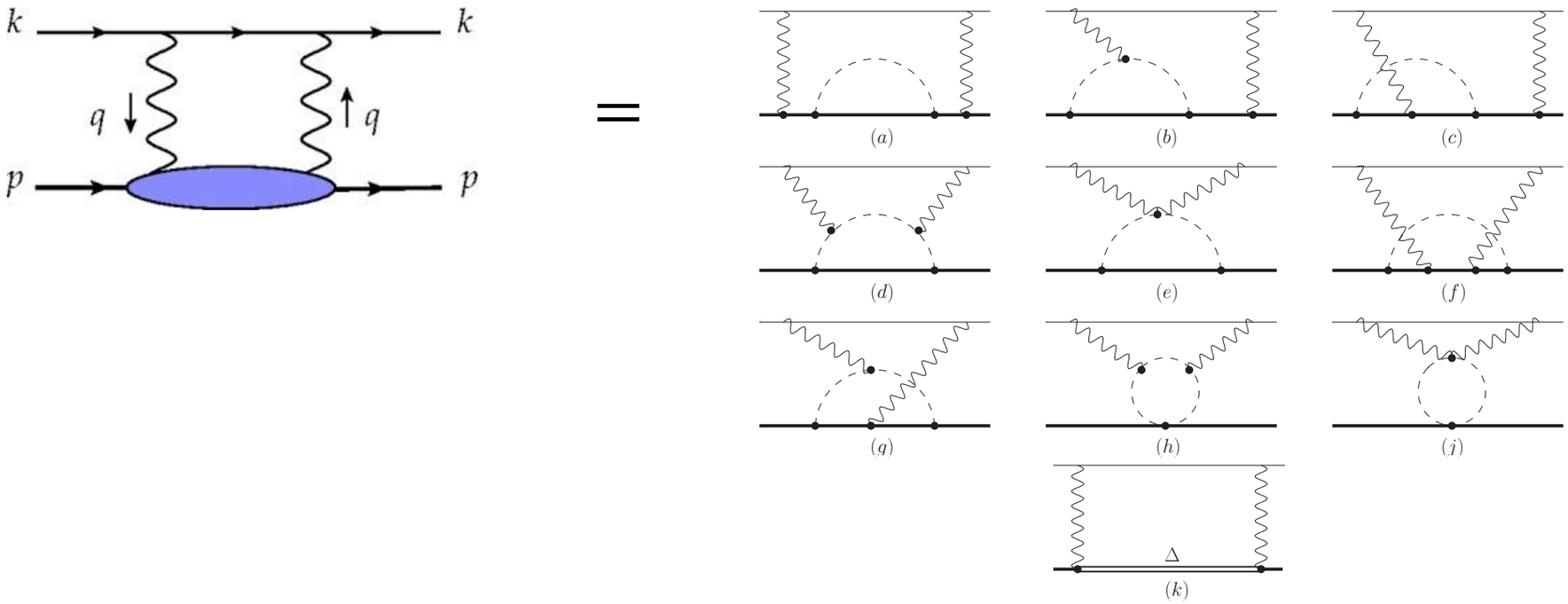
THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

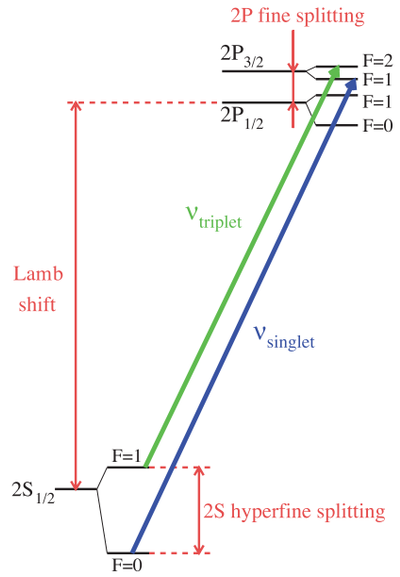
Jose Manuel Alarcón[1,a], **Vadim Lensky**[2,3], **Vladimir Pascalutsa**[1]

[1] Cluster of Excellence PRISMA Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz 55099, Germany
[2] Theoretical Physics Group, School of Physics and Astronomy, University of Manchester, Manchester M13 9PL, UK
[3] Institute for Theoretical and Experimental Physics, Bol'shaya Cheremushkinskaya 25, 117218 Moscow, Russia



with corrections to elastic proton FFs subtracted, i.e. "polarizability" alone

Effect on muonic-hydrogen Lamb shift

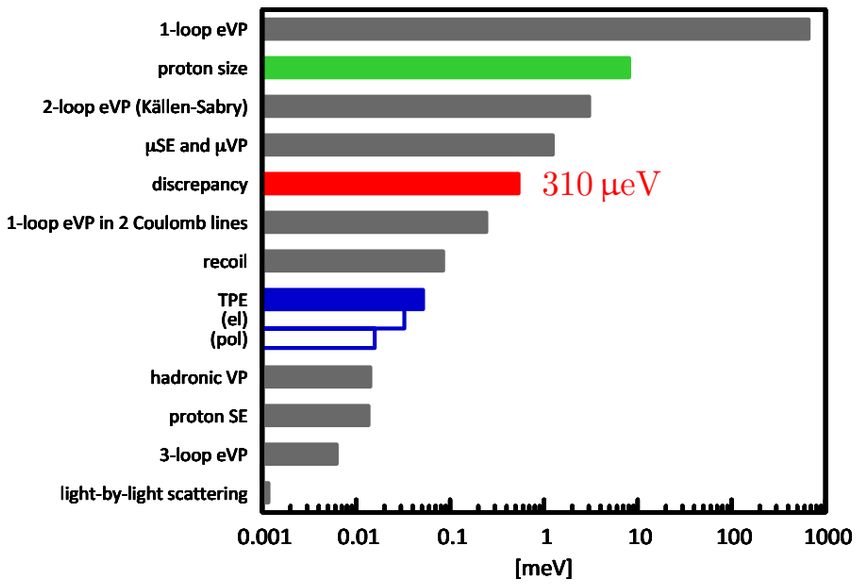


Muonic Hydrogen Lamb shift

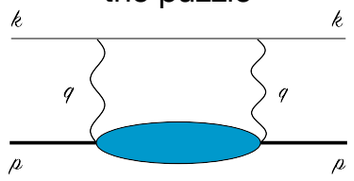
$$\Delta E_{\rm LS}^{\rm th} = 206.0668(25) - 5.2275(10)\,(R_E/{\rm fm})^2$$

theory uncertainty: $2.5\ \mu eV$

numerical values reviewed in: A. Antognini *et al.*, Annals Phys. **331**, 127-145 (2013).

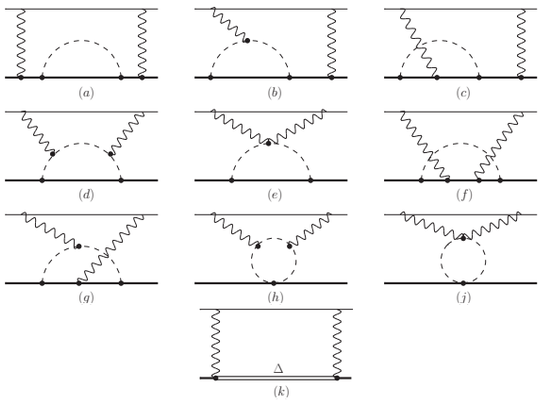


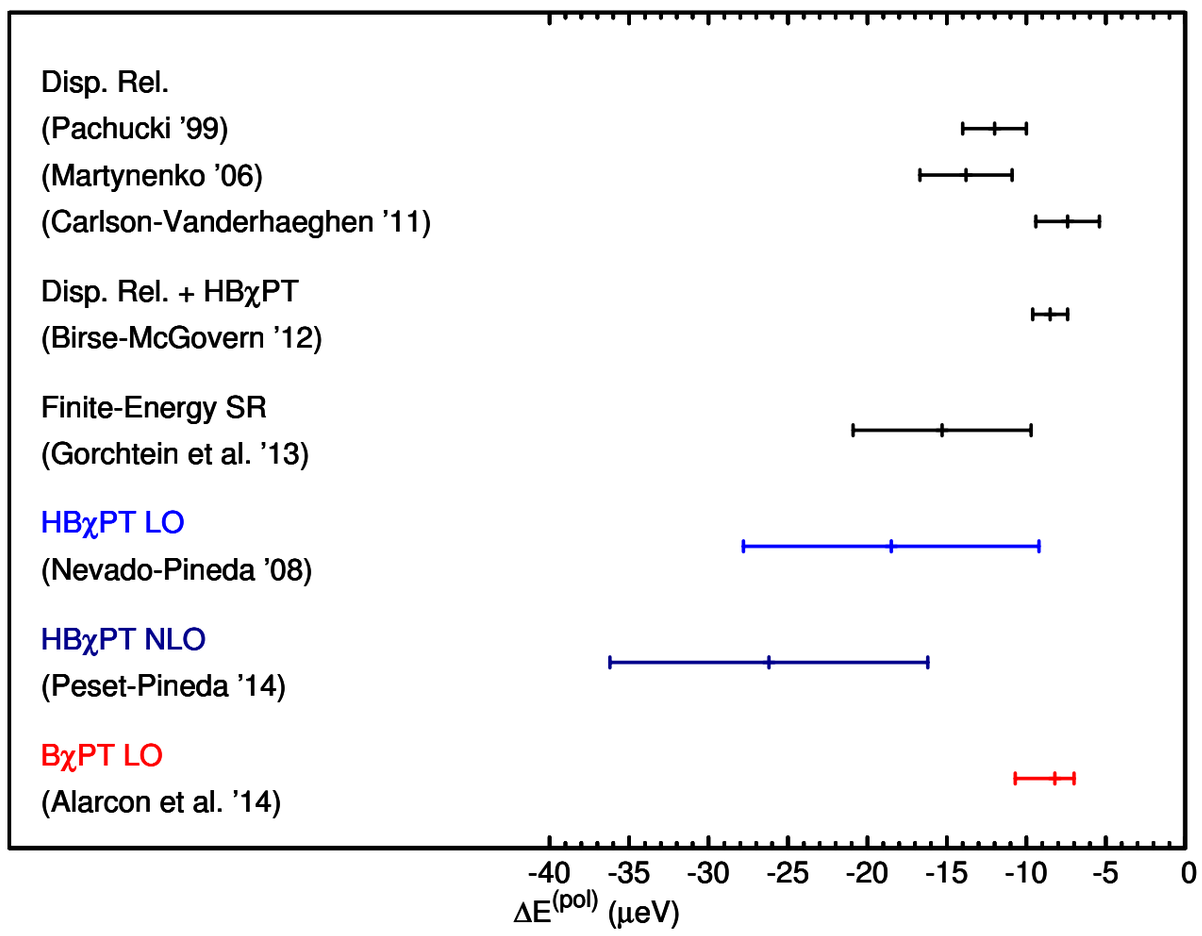
subleading effects of proton structure proposed to resolve the puzzle



$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\rm elastic} + \delta V^{(2\gamma)}_{\rm polariz.}$$

Vladimir Pascalutsa — Nucleon at Very Low Q — NStar 2015 — Osaka, May 25-2, 2015 13





Compiled by: Hagelstein, Miskimen & VP, *Prog. Part. Nucl. Phys. (2016)*

"Effectiveness" of ChPT calculations

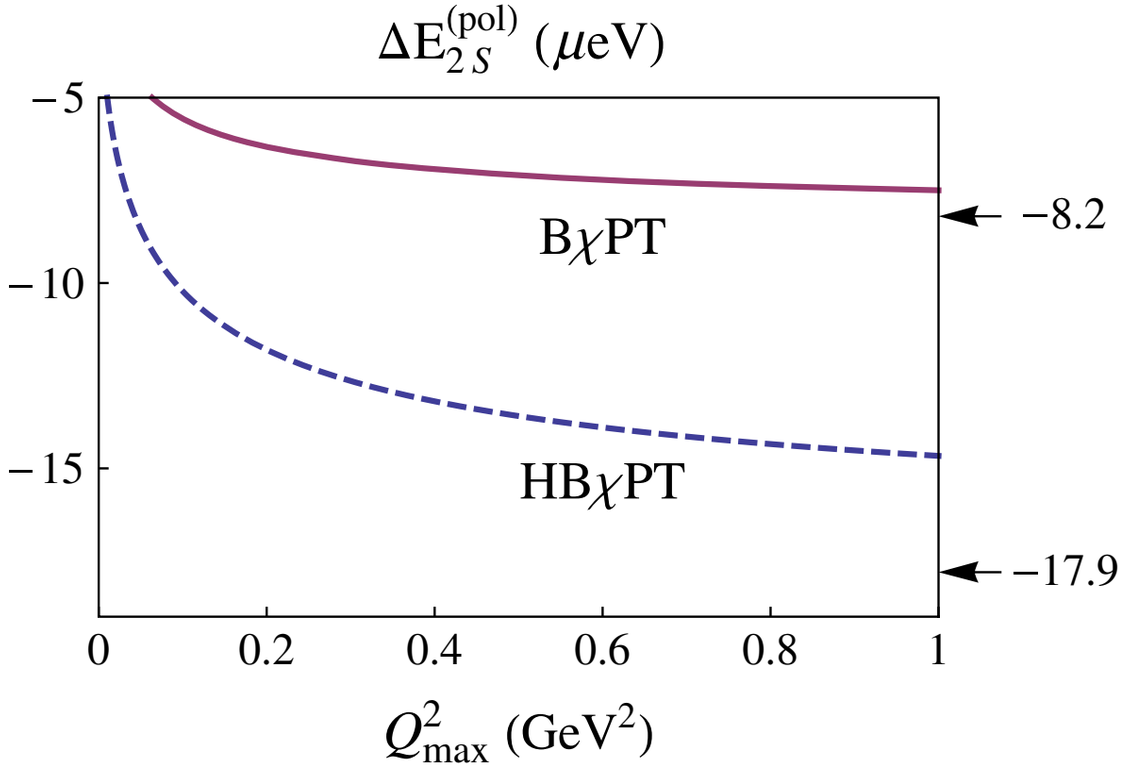


Fig. 4 The polarizability effect on the $2S$-level shift in μH computed in HBχPT and BχPT as a function of the ultraviolet cutoff Q_{max}. The *arrows* on the *right* indicate the asymptotic ($Q_{max} \to \infty$) values

HFS is less (and more) of a problem

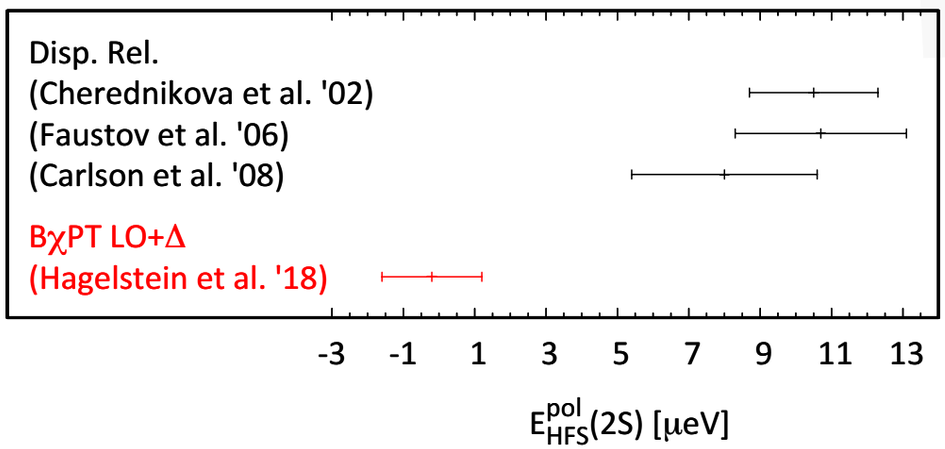
HFS theory status

$$\Delta E_{\rm HFS}(1S) = [1 + \Delta_{\rm QED} + \Delta_{\rm weak+hVP} + \underbrace{\Delta_{\rm Zemach} + \Delta_{\rm recoil} + \Delta_{\rm pol}}_{\Delta_{\rm TPE}}]\Delta E_0^{\rm HFS}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

	μp		$\mu\,^3$He$^+$		
	Magnitude	Uncertainty	Magnitude	Uncertainty	
$\Delta E_0^{\rm HFS}$	182.443 meV	0.1×10^{-6}	1370.725 meV	0.1×10^{-6}	
$\Delta_{\rm QED}$	1.1×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}	
$\Delta_{\rm weak+hVP}$	2×10^{-5}	2×10^{-6}			
$\Delta_{\rm Zemach}$	7.5×10^{-3}	7.5×10^{-5}	3.5×10^{-2}	2.2×10^{-4}	$\leftarrow G_E(Q^2), G_M(Q^2)$
$\Delta_{\rm recoil}$	1.7×10^{-3}	10^{-6}	2×10^{-4}		$\leftarrow G_E, G_M, F_1, F_2$
$\Delta_{\rm pol}$	4.6×10^{-4}	8×10^{-5}	$(3.5 \times 10^{-3})^*$	$(2.5 \times 10^{-4})^*$	$\leftarrow g_1(x,Q^2), g_2(x,Q^2)$

Polarizability effect on HFS in ChPT versus DR



Polarizability contribution at order $(Z\alpha)^6 \ln Z\alpha$

Hagelstein & V.P., in prep.

$$E_{\rm LS} = \frac{4\,\alpha\,(Z\alpha m_r)^4\,\alpha_{E1}}{n^3}\,\ln\frac{Z\alpha m_r}{2nm}$$

$$E_{\rm LS}(\mu{\rm H}) = -0.79 \pm 0.03\,\mu{\rm eV}$$

For deuterium, helium, etc.,
this correction is analogous to "Coulomb distortion"

How to match TPE (Carlson,Gorchtein, Vanderhaeghen)
and nuclear structure (Pachucki, Bacca, ...) calculations?

Conclusions

1. **Neither Friar radius and nor Polarizability corrections are large,** the latter is from chiral PT — low-energy EFT of QCD — which provides a systematic description of Compton processes (RCS, VCS, VVCS)

2. **Polarizability contribution at order** $(Z\alpha)^6 \ln Z\alpha$ **is determined very accurately:**

$$E_{\rm LS}\,(\mu{\rm H}) = -0.79 \pm 0.03\,\mu{\rm eV}$$

3. **Dispersive and BChPT calculations disagree on the size of polarizability contribution to HFS, meaning — a slight shift in Zemach radius**

Compton scattering specifics

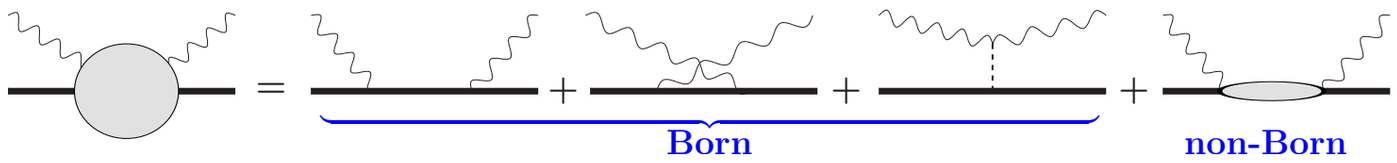


FIG. 1: Mechanisms contributing to real CS: Born and non-Born terms.

Compton scattering specifics

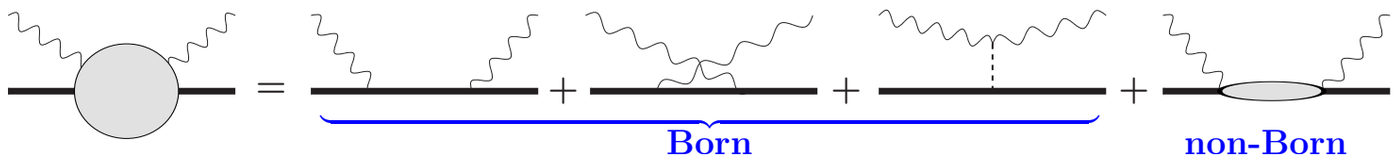


FIG. 1: Mechanisms contributing to real CS: Born and non-Born terms.

- No resonances (below pion production threshold)

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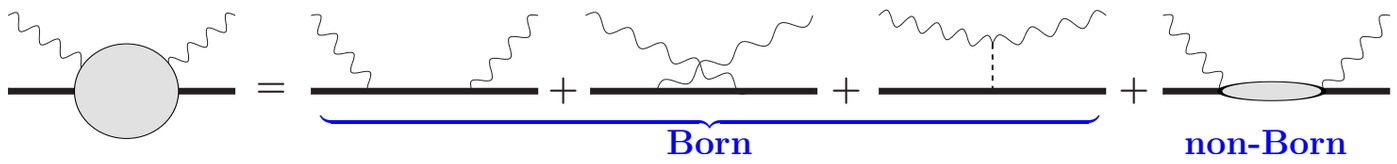

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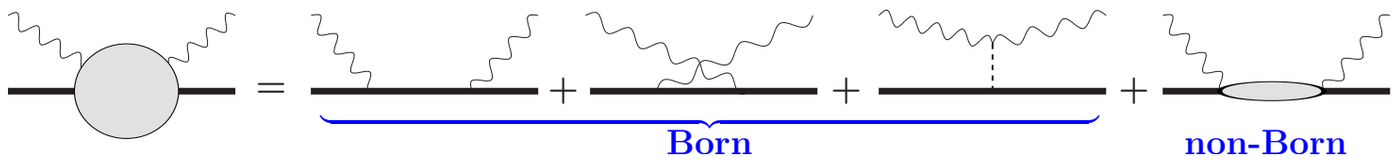

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- Forward-scattering is determined, via the sum rules (photoabsorption cross sections):

  yields linear relations on the multipoles, rather than bilinear

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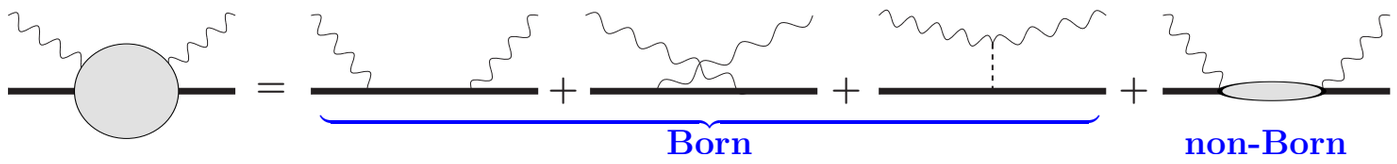

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- Forward-scattering is determined, via the sum rules (photoabsorption cross sections):

  yields linear relations on the multipoles, rather than bilinear

- Not much data (about 100 data points, many from old experiments)

Multipole Expansion

W. Pfeil, H. Rollnik and S. Stankowski, "A partial-wave analysis for proton Compton scattering in the delta(1232) energy region," Nucl. Phys. B **73**, 166 (1974).

and references therein

$$T_{\sigma'\lambda',\sigma\lambda} = \sum_{J=1/2}^{\infty} (2J+1)\, T^{J}_{\sigma'\lambda',\sigma\lambda}(\omega)\, d^{J}_{\sigma'-\lambda',\sigma-\lambda}(\theta)$$

Helicity amplitudes

PW amplitudes

Wigner d-functions

Multipole amplitudes

$$f^{l\pm}_{\rho\rho'}(\omega), \qquad \text{with } \rho,\rho' = E(\text{lectric}), \text{ or } M(\text{agnetic})$$

angular momentum l

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Unitary relation to pi-photoproduction multipoles (between 1 and 2 pion thresholds):

$$\mathrm{Im} f^{\ell\pm}_{EE} = k \sum_c \left| E^{(c)}_{(\ell\pm1)\mp} \right|^2, \qquad \mathrm{Im} f^{\ell\pm}_{MM} = k \sum_c \left| M^{(c)}_{\ell\pm} \right|^2,$$

$$\mathrm{Im} f^{(\ell\pm1)\mp}_{EM} = \mathrm{Im} f^{\ell\pm}_{ME} = \mp k \sum_c \mathrm{Re}\left(E^{(c)}_{\ell\pm} M^{(c)*}_{\ell\pm} \right),$$

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We expand the non-Born piece only, truncated at J=3/2 (only J=1/2, 3/2 are taken into account):

$$f = f^{\mathrm{Born}} + \bar{f} \qquad \bar{f} = (\bar{f}^{1+}_{EE}, \bar{f}^{1-}_{EE}, \bar{f}^{1+}_{MM}, \bar{f}^{1-}_{MM}, \bar{f}^{1+}_{EM}, \bar{f}^{1+}_{ME}, \bar{f}^{2+}_{EE}, \bar{f}^{2-}_{EE}, \bar{f}^{2+}_{MM}, \bar{f}^{2-}_{MM})$$

Dynamic to Static Polarizabilities

$$\begin{pmatrix} \alpha_{E\ell}(\omega) \\ \beta_{M\ell}(\omega) \end{pmatrix} = \frac{[\ell(2\ell-1)!!]^2}{\omega^{2\ell}} \left[(\ell+1)\bar{f}^{\ell+}_{\substack{EE \\ MM}}(\omega) + \ell \bar{f}^{\ell-}_{\substack{EE \\ MM}}(\omega) \right]$$

$$\gamma_{\substack{E\ell E\ell \\ M\ell M\ell}}(\omega) = \frac{2\ell-1}{\omega^{2\ell+1}} \left[\bar{f}^{\ell+}_{\substack{EE \\ MM}}(\omega) - \bar{f}^{\ell-}_{\substack{EE \\ MM}}(\omega) \right],$$

$$\bar{f}^{\ell\pm}_{\substack{EE \\ MM}} \sim \omega^{2\ell}, \qquad \bar{f}^{\ell+}_{\substack{EM \\ ME}} \sim \omega^{2\ell+1}$$

$$\gamma_{\substack{E\ell M(\ell+1) \\ M\ell E(\ell+1)}}(\omega) = 2^{2-\ell} \frac{(2\ell+1)!!}{\omega^{2\ell+1}} \bar{f}^{\ell+}_{\substack{EM \\ ME}}(\omega).$$

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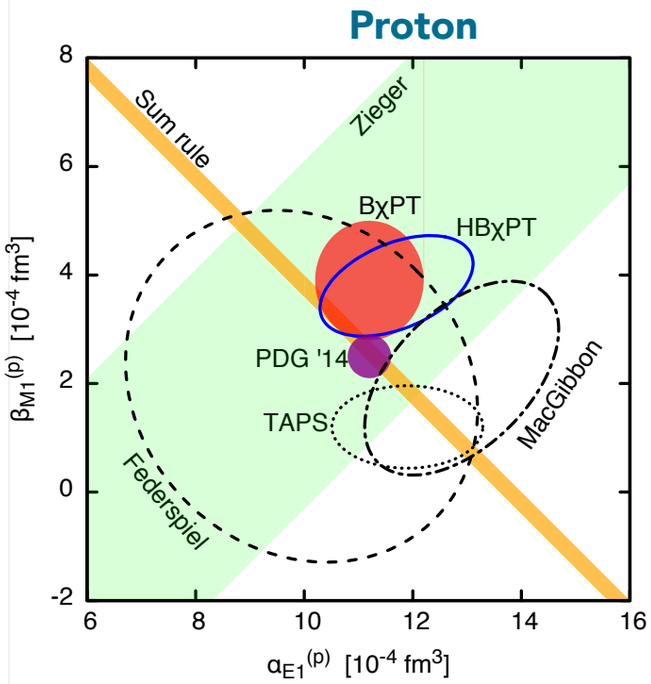
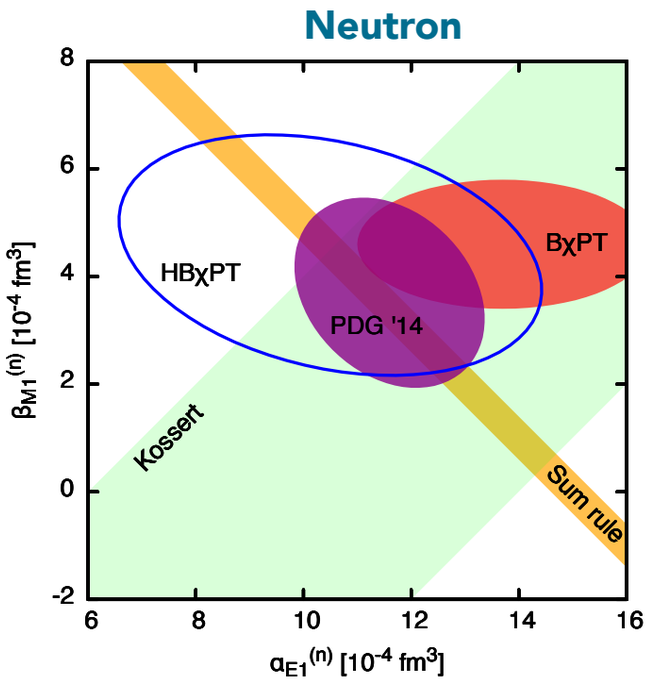
Static Limit, energy=0:





Forward spin polarizability

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

Backward spin polarizability

$$\gamma_\pi = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

Partial-wave analysis (PWA) of Compton scattering data below pion production threshold

Krupina, Lensky & VP, *Phys. Lett. B782 (2018) 34.*

Sum rule determination of forward Compton scattering

PHYSICAL REVIEW D **92,** 074031 (2015)

Evaluation of the forward Compton scattering off protons: Spin-independent amplitude

Oleksii Gryniuk,[1,2] Franziska Hagelstein,[1] and Vladimir Pascalutsa[1]

[1]*Institut für Kernphysik and PRISMA Cluster of Excellence, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany*

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(Received 2 September 2015; published 21 October 2015)

PHYSICAL REVIEW D **94,** 034043 (2016)

Evaluation of the forward Compton scattering off protons. II. Spin-dependent amplitude and observables

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Review

Progress in Particle and Nuclear Physics 88 (2016) 29–97

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Nucleon polarizabilities: From Compton scattering to hydrogen atom

Franziska Hagelstein[a], Rory Miskimen[b], Vladimir Pascalutsa[a,*]

[a] *Institut für Kernphysik and PRISMA Excellence Cluster, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany*

[b] *Department of Physics, University of Massachusetts, Amherst, 01003 MA, USA*

Basic Introduction

IOP Concise Physics

Causality Rules

A light treatise on dispersion relations and sum rules

Vladimir Pascalutsa