Elastic three photon exchange contribution to hyperfine structure

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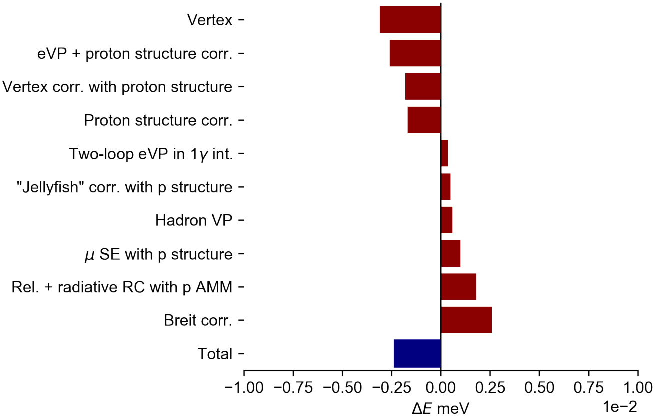
α^2 corrections to 2s μH HFS



Data from Antognini et al. [2013]

Finite size elastic contribution

Total α^6 elastic contribution

$$E_{\text{fns}}^{(6)} = E_L^{(6)} + E_H^{(6)}$$

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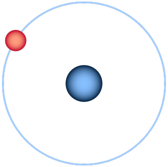
Total α^6 elastic contribution

$$E^{(6)}_{\text{fns}} = E^{(6)}_L + E^{(6)}_H$$

Low energy part $E^{(6)}_L$ potential

$$V(q) = \underbrace{\frac{4}{6}\pi Z\alpha r_c^2}_{V_{E_1}} + \underbrace{\frac{g_p \pi Z\alpha}{2dM}\sigma_p^{ij}\sigma_e^{ij}\left(1 - \frac{r_m^2}{6}q^2\right)}_{V_{M_1}+V_{M_2}}$$



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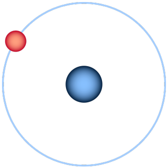


Perturbation theory

$$E^{(6)}_L = 2\langle\phi|\, V_{E_1} \frac{1}{(E_0 - H_0)'} V_{M_1} \,|\phi\rangle + \langle\phi|\, V_{M_2} \,|\phi\rangle$$

Dimensional regularization (as in Pachucki et al. [2018])

$$d = 3 - 2\epsilon$$

$$V(q) = -\frac{4\pi}{q^2} \to V(r) = -\frac{C_\epsilon}{r^{1-2\epsilon}}$$

$$\left\langle \left[\frac{1}{r^4}\right]_\epsilon \right\rangle_{nS} = \frac{8}{n^3}\left(-\frac{5}{3} + \frac{1}{2n} + \frac{1}{6n^2} + \gamma + \Psi(n) - \ln\frac{n}{2}\right) + \phi^2(0)8\pi\left(-\frac{1}{4\epsilon} + \ln Z\alpha + 1\right)$$

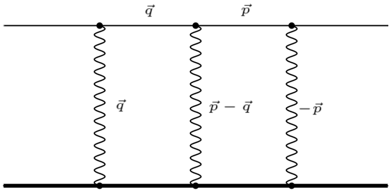
Low energy part $E_L^{(6)}$ result

$$\frac{1}{3}(Z\alpha)^2(mr_c)^2\left(4\left[-\frac{1}{n} - \frac{1}{2} + \gamma - \ln\frac{n}{2} + \Psi(n) - \frac{1}{4\epsilon} + \ln Z\alpha\right] + \frac{r_m^2}{r_c^2 n^2}\right)E_F$$

Finite size elastic contribution

$E_H^{(6)}$

High energy part can be calculated from forward scattering amplitude



High energy part $E_H^{(6)}$ result

$$\frac{4}{3}(Z\alpha)^2(mr_c)^2\left(\frac{1}{4\epsilon}+\gamma+\frac{1}{2}+\ln mr_N\right)E_F$$

Finite size elastic contribution - final result

$E_H^{(6)} + E_L^{(6)}$

$$\Delta E = \frac{1}{3}(Z\alpha)^2 \, (m r_c)^2 \left(4 \left[-\frac{1}{n} + 2\gamma - \ln\frac{n}{2} + \Psi(n) + \ln(m r_N Z\alpha) \right] + \frac{r_m^2}{r_c^2 n^2} \right) E_F$$

Fermi energy

$$E_F = \frac{4}{3n^3} \frac{\mu_r^3 g_p}{mM} \alpha^4$$

Digamma function

$$\Psi(n) = \frac{\Gamma'(n)}{\Gamma(n)}$$

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Example values for some nS states in hydrogen $(r_m \approx r_c)$

1s

- Muonic: $-9.78\,\mu\text{eV}$
- Electronic: $-4.331\,\text{Hz}$

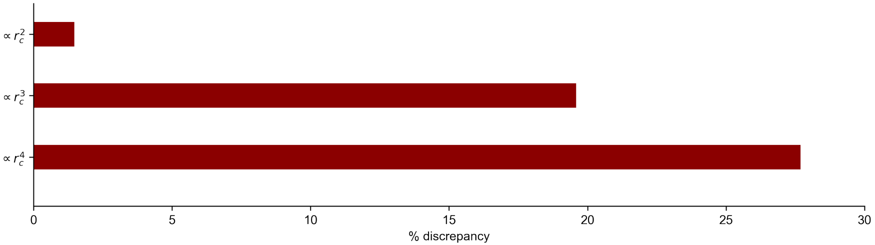
2s

- Muonic: $-1.02\,\mu\text{eV}$
- Electronic: $-0.185\,\text{Hz}$

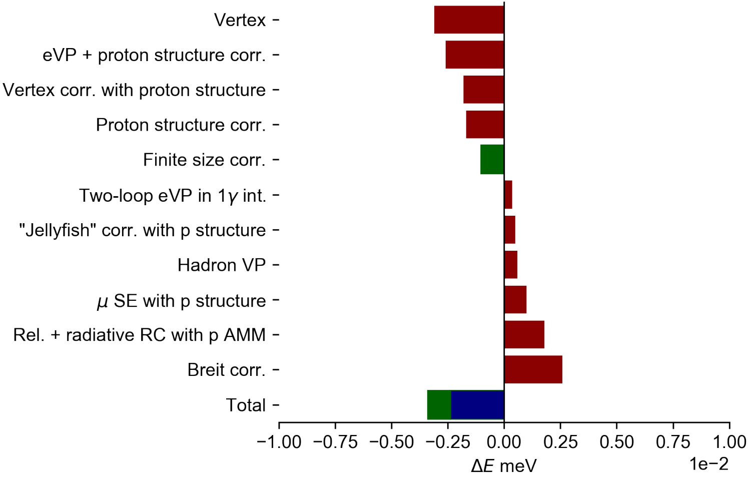
Comparison with numerical results

Indelicato [2013]

	This work	Indelicato	Units
$\propto r_c^2$	-0.0012874	-0.0012687	meV fm^{-2}
$\propto r_c^3$	0.00034126	0.00042442	meV fm^{-3}
$\propto r_c^4$	-0.049389	-0.068306	1×10^{-3} meV fm^{-4}



Comparison with other contributions



Conclusions

Analytical result

Compact expression for elastic fns contribution of order α^6 is given.

Different value

Higher order finite-size contribution for 2s in μH ($-1.018\,\mu\text{eV}$) is $\approx 54\%$ larger than assumed in Antognini et al. [2013] ($-0.659\,\mu\text{eV}$).

Other hydrogenic systems

Presented formula is valid, but inelastic contributions will probably dominate.

References

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