π^0 to $\gamma\gamma$ transition form factor in lattice QCD

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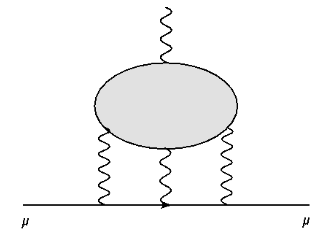
Reference: ES et al. arXiv:1102.5544, Feng et al. PRL109, 182001(2012)

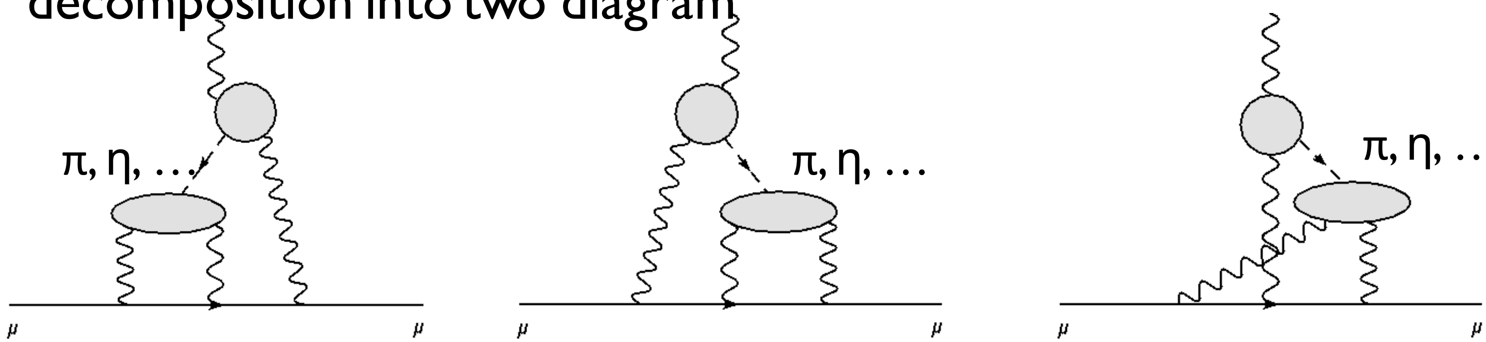
Relation to hadronic L-by-L diagram

Next-to-leading order of hadronic contribution, 3 off-shell vertex and 1 on-shell vertex



- Direct calculation: hard task !
- Indirect calculation: meson pole saturation

  decomposition into two diagram



<u>Pion pole dominance model:</u> M. Knecht, A. Nyffeler (2002)

$$a_\mu^{\rm LbyL} = \int_{p_1}\int_{p_2} \left[p_1^2 p_2^2 (p_1+p_2)^2\right]^{-1} \left[((p+p_1)^2 - m_\mu^2)((p-p_2)^2 - m_\mu^2\right]^{-1} \Pi_{\pi^0\gamma^*\gamma^*}(p_1,p_2,p)$$

$$\Pi_{\pi^0\gamma^*\gamma^*}(p_1,p_2,p) \sim f_{\pi^0\gamma^*\gamma^*} f_{\pi^0\gamma^*\gamma}(p_2^2 - m_\pi^2)^{-1},\ f_{\pi^0\gamma^*\gamma^*} f_{\pi^0\gamma^*\gamma}((p_1+p_2)^2 - m_\pi^2)^{-1}$$

$\leftarrow f_{\pi^0\gamma^*\gamma^*}(p_1^2, p_2^2)$ plays important role

Transition form factor from 3pt function

- Definition

$$\int d^4x_1 d^4x_2 e^{-ip_1x_1-ip_2x_2}\langle\pi^0(q)|V_\nu^{\rm EM}(p_2)V_\mu^{\rm EM}(p_1)|0\rangle = \varepsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta f_{\pi^0\gamma^*\gamma^*}(p_1,p_2)$$

Transition form factor is given from 3pt function as <PVV>.

- <PVV> in Euclid space (space-like momentum) ES, et al., 0912.0253, 1102.5544

$$\int d^4x\int d^4y e^{-ip_1x-ip_2y}\langle 0|T\{V^{\rm EM}(x)V^{\rm EM}(y)P(0)\}\rangle$$
$$\simeq -\frac{f_\pi m_\pi^2}{Q^2+m_\pi^2}\varepsilon_{\alpha\beta\mu\nu}p_{1\,\alpha}p_{2\,\beta}f_{\pi^0\gamma^*\gamma^*}(p_1,p_2)$$

Assuming the ground state of pion is leading contribution ($f_{\pi'}$ is as O(m)).

- Analytic continuation (time-like momentum) Dudek, Edwards (2006), Feng et al.(2012), Karl's talk

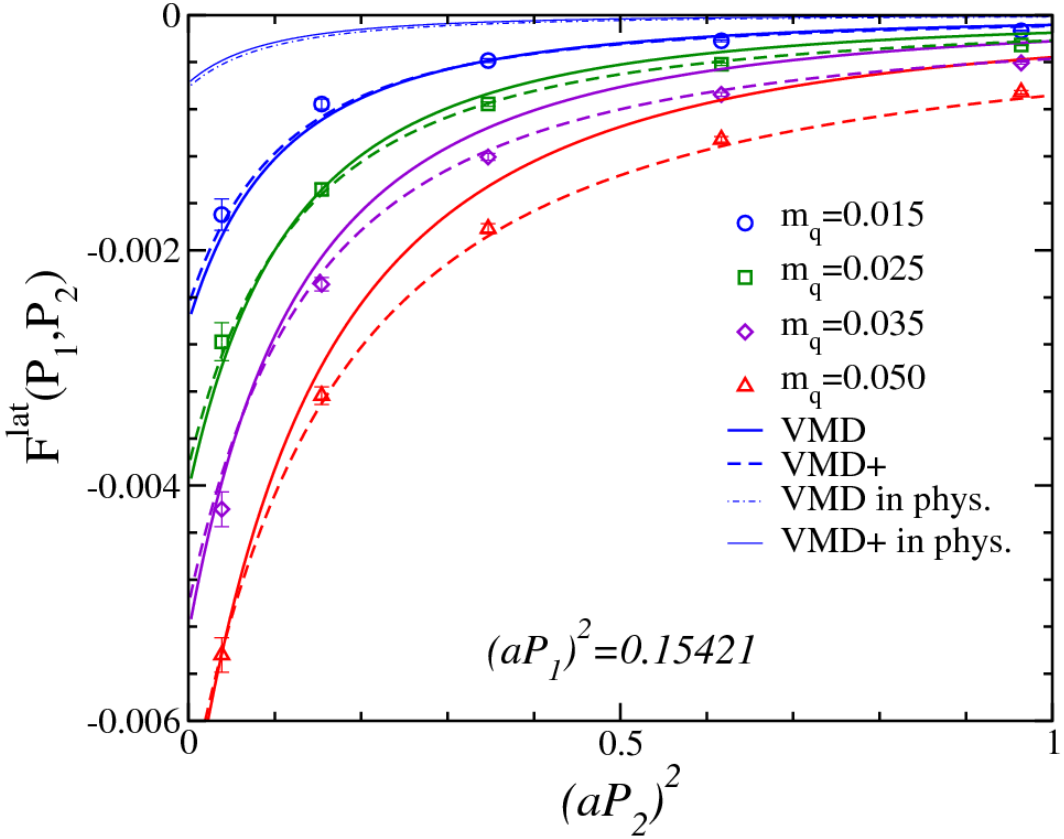
$$M_{\mu\nu}(p_1,p_2)=\lim_{t_{1,2}-t_\pi\to\infty}\frac{1}{\frac{\phi_{\pi,\vec q}}{2E_{\pi,\vec q}}e^{-E_{\pi,\vec q}(t_2-t_\pi)}}\times\int dt_1 e^{\omega(t_1-t_2)}C_{\mu\nu}(t_1,t_2,t_\pi),$$

Ground state of pion by $t_{1,2}-t_\pi\to\infty$
Photon energy ω is arbitral parameter → $p_{1,2}$ is controllable.

$$C_{\mu\nu}(t_1,t_2,t_\pi)\equiv\int d^3\vec x e^{-i\vec p_1\cdot\vec x}\int d^3\vec z e^{i\vec q\cdot\vec z}\times\langle\Omega|T\{j_\mu(\vec x,t_1)j_\nu(\vec 0,t_2)\pi^0(\vec z,t_\pi)\}|\Omega\rangle,$$

Fourier transformation for <PVV> in spatial momentum

Space-like momentum



- Overlap fermion, exact chiral
- $16^3 \times 32$, a^{-1} = 1.67 GeV
- m_π = 400 -- 800 MeV
- Fitting with VMD(+)

$$F^{\rm VMD} = \langle PVV\rangle/\varepsilon p_1 p_2 = -\frac{m_\pi^2}{Q^2+m_\pi^2} X_a G_V(p_1) G_V(p_2)$$

1-param X_a

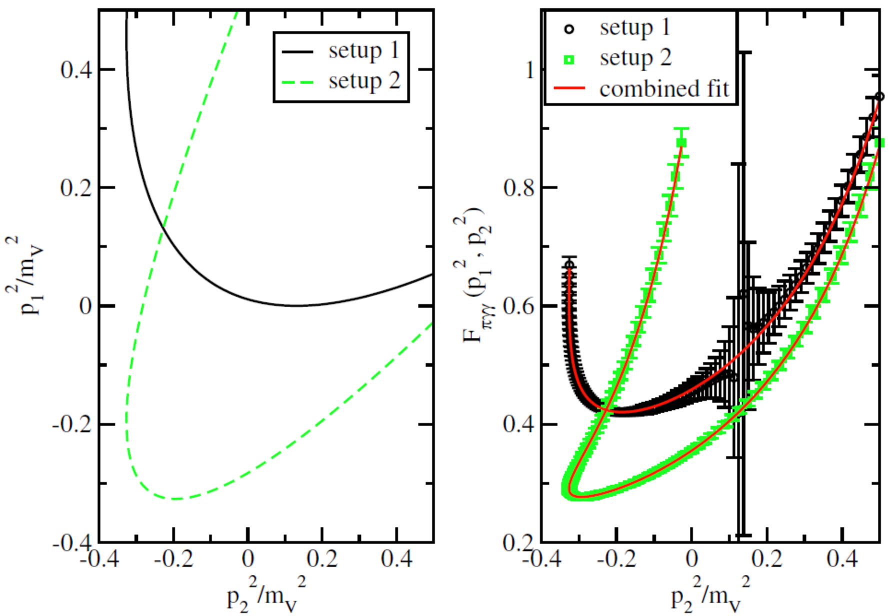
$$F^{\rm VMD+} = -\frac{m_\pi^2}{Q^2+m_\pi^2} X_a \Big[c_3 G_V(p_1) G_V(p_2) + \frac{c_4-c_3}{2}(G_V(p_1)+G_V(p_2)) + 1-c_4\Big]$$

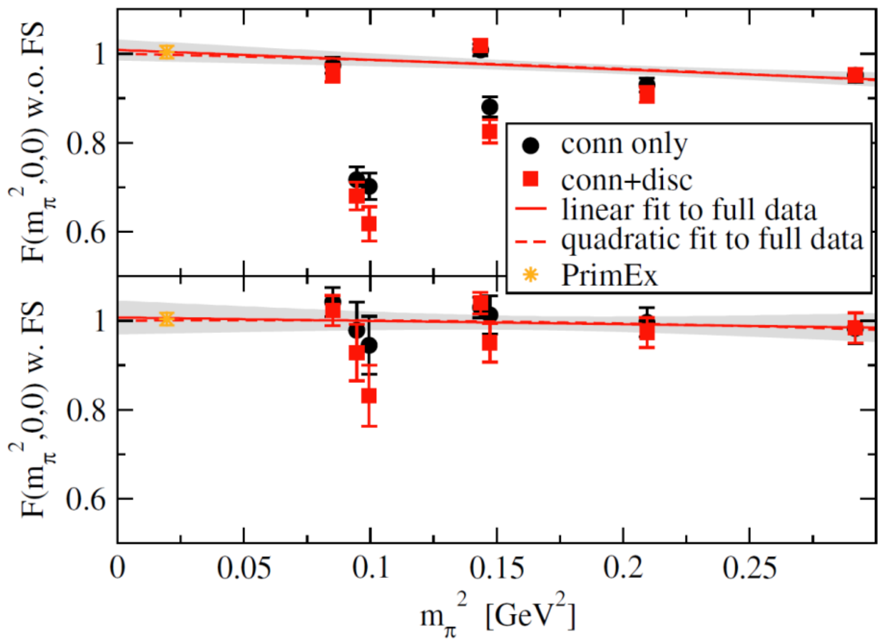
4-params: $\{c_4^0, 2\text{-}c_4^0\}$ + $mc_{3,4}^m$ and X_a

	X_a	c_3^m	c_4^0	c_4^m
VMD	0.0260(6)	-	-	-
VMD+	0.0243(29)	-8.4(8.6)	1.20(21)	-1.3(5.8)

- X_a is consistent with anomaly factor, $1/4\pi$=0.02533.
- VMD+ is in good agreement with data below 1 GeV^2

Analytic continuation





- $p_1 = (\omega, \mathbf{p_1})$, $p_2 = (E_\pi - \omega, \mathbf{p_2})$
- Continuous behavior as $p_{1,2}$ controlled by ω
- Fitting with VMD function including higher order

$$\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2) + \sum_m c_m[(p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)] + \sum_{m,n} c_{m,n}(p_1^2)^m (p_2^2)^n,$$

- Suffering large finite size correction to form factor and pion decay constant.
- Consistent value of with PrimEx 7.82(22) GeV
  $\Gamma_{\pi\gamma\gamma} = 7.83(31)(49)$ GeV
- Easily extend to off-shell photon

Perspective

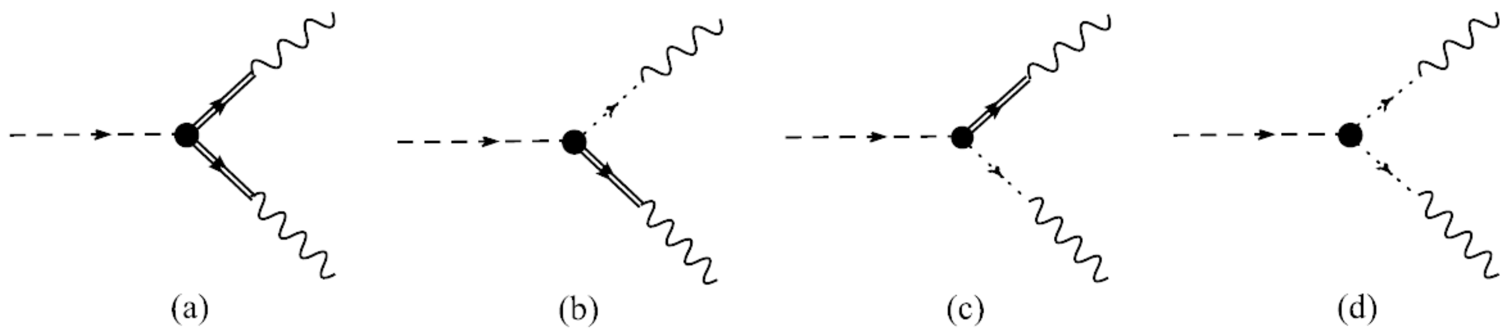
- Large volume is important for investigation of $p_{1,2}$ dependence, and correction to finite size effect.
- Pion mass should be close to physical point

  Check of chiral behavior and remove chiral extrapolation
- Disconnected contribution

  Isospin breaking will be significant.

Space-like momentum (cont)

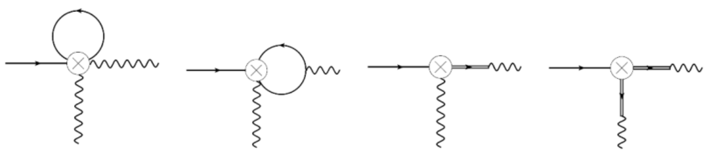
- VMD based fitting function



$$
\begin{aligned}
F^{\mathrm{VMD}+} &= -\frac{m_\pi^2}{Q^2+m_\pi^2} X_a \Big[c_3 G_V(p_1) G_V(p_2) && \longleftarrow \text{(a)} \\
&+ \frac{c_4 - c_3}{2} \left(G_V(p_1) + G_V(p_2) \right) && \longleftarrow \text{(b),(c)} \\
&+ 1 - c_4 \Big] && \longleftarrow \text{(d)}
\end{aligned}
$$

Space-like momentum (cont)

- One-loop correction

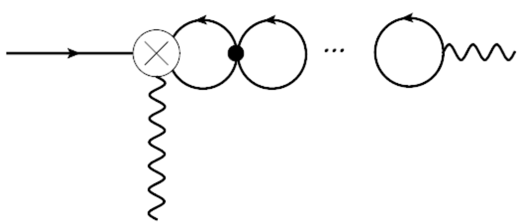


$$\Gamma^{\text{1loop+VMD}}(m_\pi^2,P_1^2.P_2^2) = 1+\frac{1}{f_\pi^2}\Big(-\frac{4}{3}\Delta_\pi(m_\pi^2)+J(m_\pi^2,P_1^2)+J(m_\pi^2,P_2^2)\Big) -\frac{256\pi^2}{3}m_\pi^2 y_1+\frac{64\pi^2}{3}\left(\frac{P_1^2}{P_1^2+m_V^2}+\frac{P_2^2}{P_2^2+m_V^2}\right)y_2 +\frac{P_1^2P_2^2}{(P_1^2+m_V^2)(P_2^2+m_V^2)}y_3$$

$$\Delta_\pi(m_\pi^2) = \frac{m_\pi^2}{16\pi^2}\ln\frac{m_\pi^2}{\mu^2},$$

$$J(m_\pi^2,P^2) = \frac{2}{3}\left[\frac{P^2}{64\pi^2}\sigma^2\left\{-1+\ln\frac{m_\pi^2}{\mu^2}+2\sigma\tanh^{-1}\frac{1}{\sigma}\right\}-\frac{P^2+6m_\pi^2}{96\pi^2}\right]$$

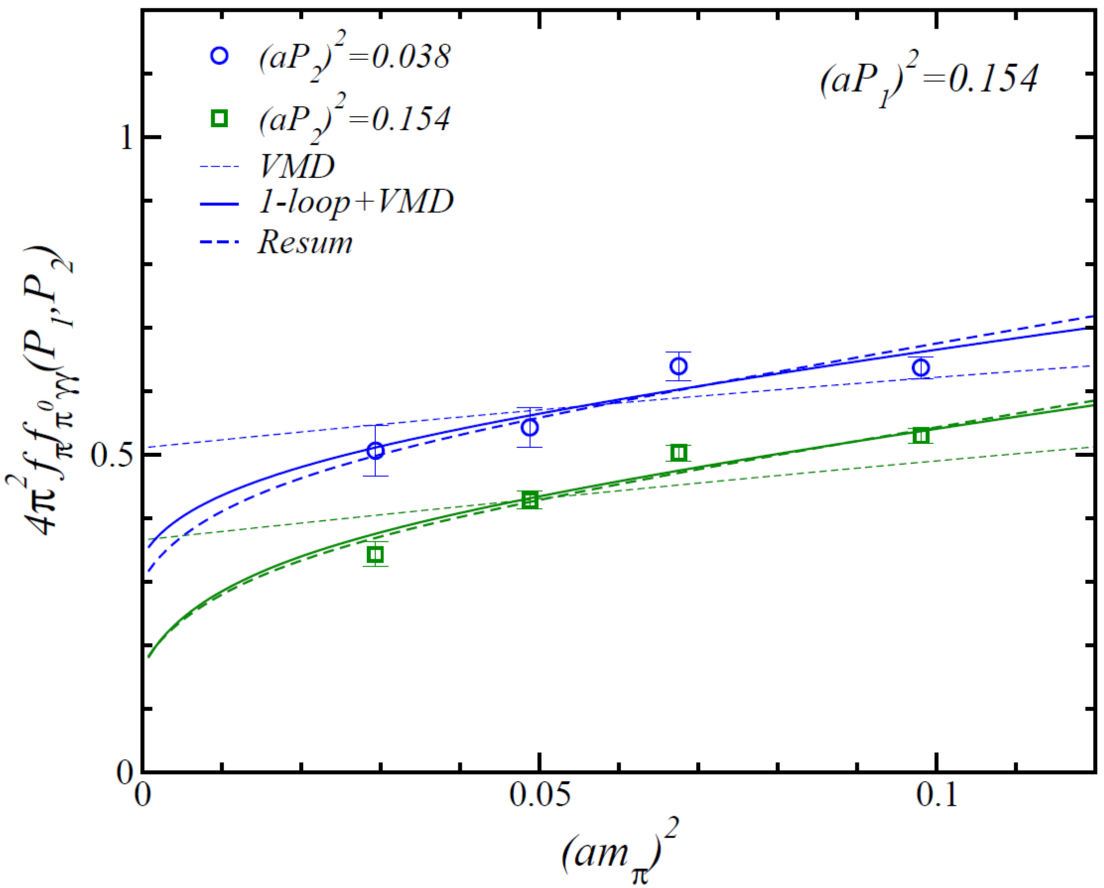
- Resummation



$$\Gamma^{\text{resum}}(m_\pi^2,P_1^2.P_2^2) = 1-\frac{2}{f_\pi^2}\Delta_\pi(m_\pi^2)-\frac{256\pi^2}{3}m_\pi^2 z_1 +\frac{L(m_\pi^2,P_1^2,z_2)}{1-2L(m_\pi^2,P_1^2,z_2)}+\frac{L(m_\pi^2,P_2^2,z_2)}{1-2L(m_\pi^2,P_2^2,z_2)},$$

$$L(m_\pi^2,P^2,z_2) = \frac{1}{f_\pi^2}\left(\frac{1}{3}\Delta_\pi(m_\pi^2)+J(m_\pi^2,P^2)\right)+\frac{64\pi^2}{3}P^2 z_2,$$

Space-like momentum (cont)



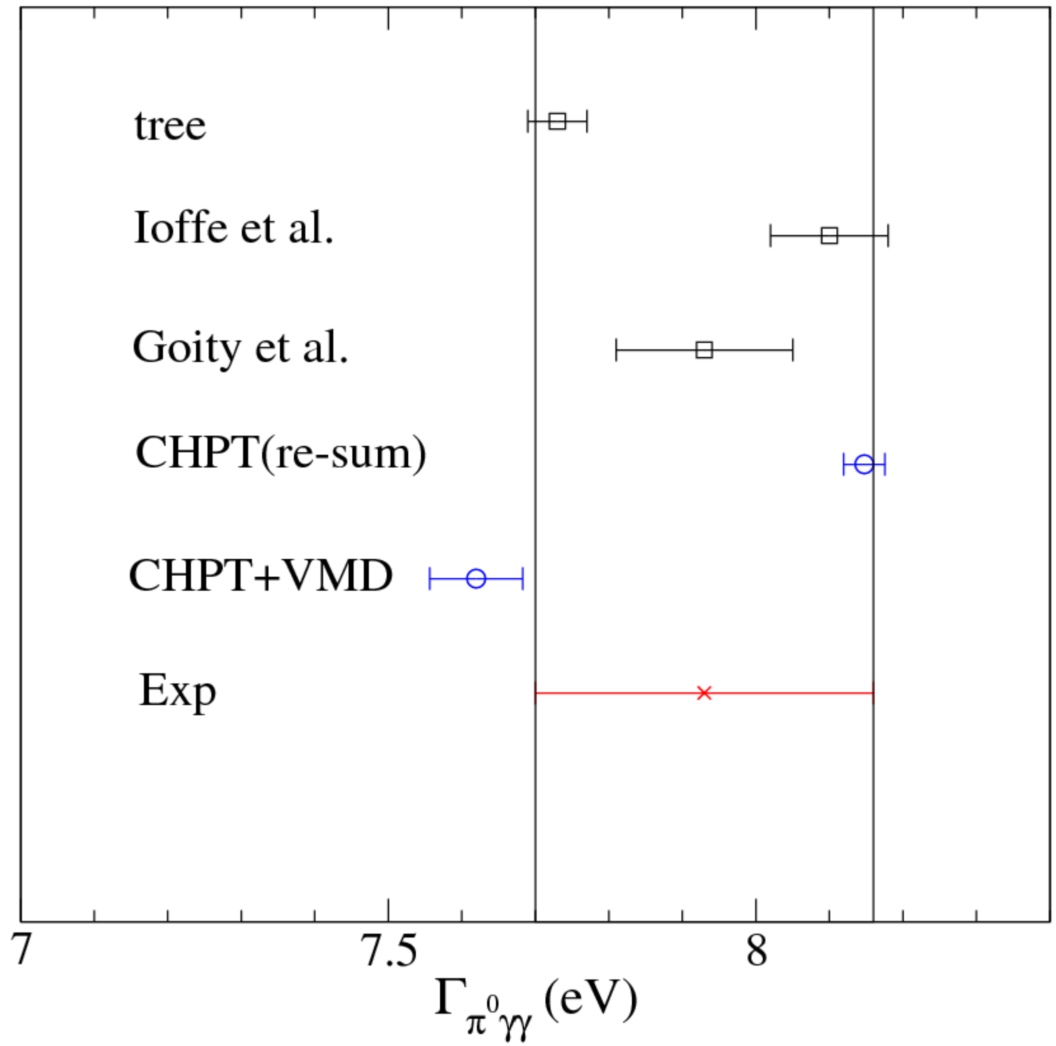
- VMD is only applicable to momentum below p_2 ~ 0.4 GeV^2
- Correction with pion loop is working for improvement of chi^2 fitting in 400 -- 800 MeV pion.
- More small momentum is needed for detailed investigation.

Decay width

$$\Gamma_{\pi^0\gamma\gamma} = \frac{m_\pi^3 \alpha_e^2}{64\pi^3 f_\pi^2} \Lambda^2_{\pi^0\gamma\gamma}$$

$$= 8.15(3)\,\mathrm{eV} \;:\; \mathrm{CHPT}$$

$$= 7.62(6)\,\mathrm{eV} \;:\; \mathrm{CHPT+VMD}$$



- observable: $M_{\mu\nu}(p_1,p_2) = \int d^4x \, e^{ip_1x} \langle 0|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$
- analytic continuation

$$M_{\mu\nu}(p_1,p_2) = \lim_{t_{1,2}-t_\pi\to\infty} \frac{1}{\frac{\phi_{\pi,\vec{q}}}{2E_{\pi,\vec{q}}} e^{-E_{\pi,\vec{q}}(t_2-t_\pi)}} \int dt_1 \, e^{\omega(t_1-t_2)} C_{\mu\nu}(t_1,t_2,t_\pi)$$

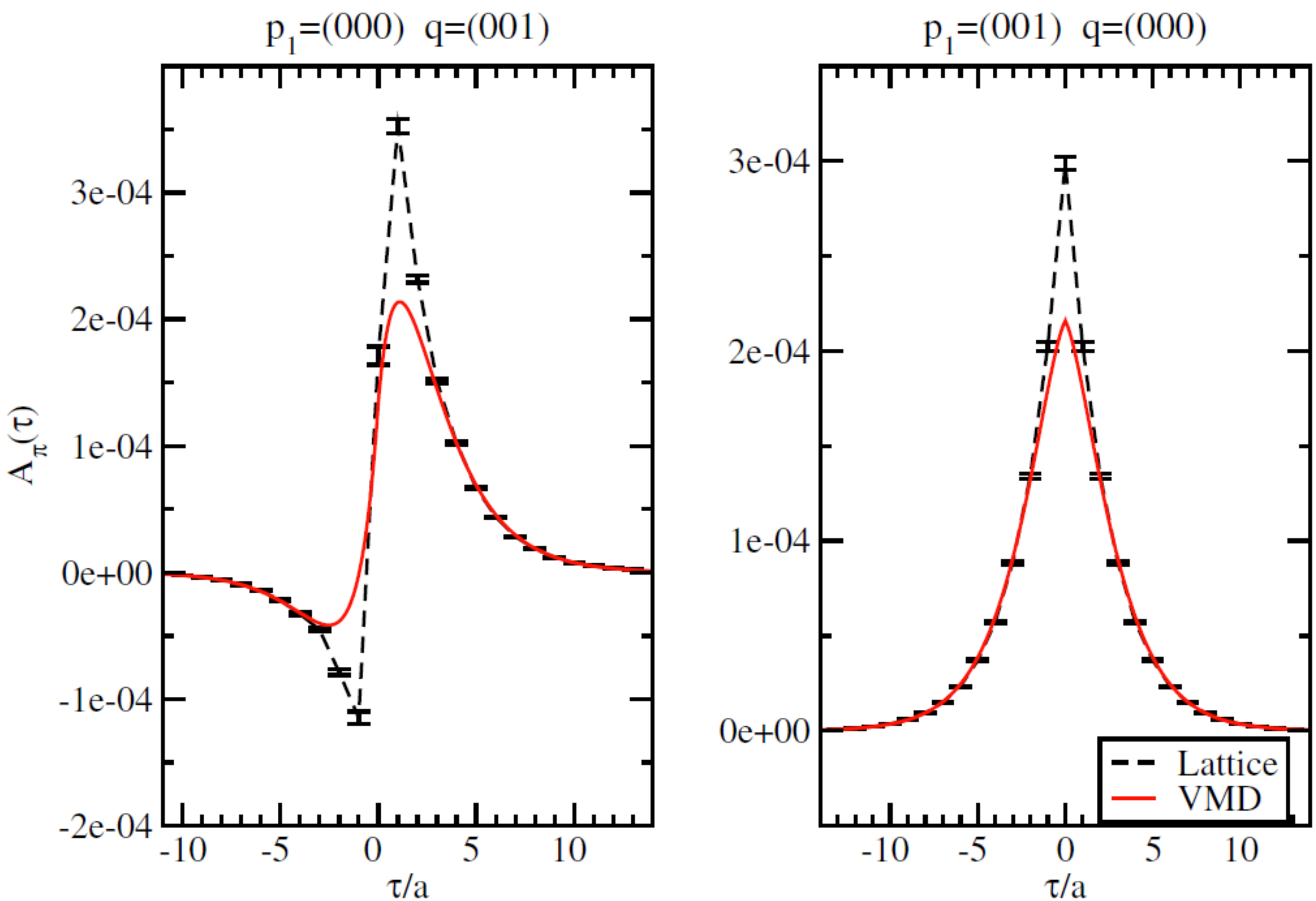
$$C_{\mu\nu}(t_1,t_2,t_\pi) \equiv \int d^3\vec{x}\, e^{-i\vec{p}_1\cdot\vec{x}} \int d^3\vec{z}\, e^{i\vec{q}\cdot\vec{z}} \langle 0|T\{J_\mu(\vec{x},t_1)J_\nu(\vec{y},t_2)\pi^0(\vec{z},t_\pi)\}|0$$

  - large $t_{1,2}-t_\pi$ limit to pick up pion
  - $e^{\omega(t_1-t_2)}$ divergent for $t_1 > t_2$? No, suppression by $C_{\mu\nu}(t_1,t_2,t_\pi)$

- we want to study the t_1-t_2 dependence of $C_{\mu\nu}(t_1,t_2,t_\pi)$
- define amplitude $A_\pi(\tau)$

$$A_\pi(\tau) \equiv \lim_{t-t_\pi\to\infty} \frac{C_{\mu\nu}(t_1,t_2,t_\pi)}{e^{-E_\pi(t-t_\pi)}}, \quad \tau = t_1-t_2, \quad t = \min\{t_1,t_2\}$$

Analytic continuation (cont)



Analytic continuation (cont)