Positronium resonance contribution to the electron $g-2$

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Background

Recently, Mishima (MG) (G. Mishima, arXiv:1311.7109 [hep-ph]) pointed out

- Positronium $\sim e^- e^+$ in the vector channel gives the contribution to the electron $g-2$ (a_e)
    - that is essentially nonperturbative and cannot be captured by the perturbation theory,
    - that is comparable to $O(\alpha^5)$ perturbative contribution.
    - the new additional correction is relevant even to the present precision of the electron $g-2$
- Fael and Passera (FP) (M. Fael and M. Passera, arXiv:1402.1575 [hep-ph]) indicated the mistakes in Mishima's paper, and draw the qualitatively same conclusion.
- Afterwards, two papers (K. Melnikov, A. Vainshtein and M. Voloshin, arXiv:1402.5690 [hep-ph]; M. I. Eides, arXiv:1402.5860 [hep-ph]) insist that there are no sizable contribution.

Purpose

- All of the four papers analyze the issue from the same perspective;
  - Since the positronium in the vector channel is long-lived $\Gamma_p = O(\alpha^6)$ compared to the bounding energy $\Delta E_p = O(\alpha^2)$.
  - *the decay effect can be neglected and the positronium can be dealt with as absolutely stable bound states*.
- I analyze the issue *from a quite different perspective*;
  - We analyze the positronium *resonance* contribution based on full order of QED.
  - I demonstrate that the positronium resonance contribution to a_e is negligbly small ($\sim O(\alpha^7)$).
  - I discuss that there is no quantum field theory that realizes abosolutely stable positroniums.

What is the basis of the state space of QED?

Here, QED means the quantum electrodynamics *with electron only*.

- One of the most important points of my discussion according to full QED is the correct understanding on the state space of QED: It consists of the following basis vectors
  - $N = 0$ sector; vacuum $|0\rangle$.
  - $N = 1$ sector ($\Leftrightarrow$ poles in the spectral functions);
    - $Q = 0 : \gamma$. This is irrelevant to $\Pi(q^2)$, which enters in the *1PI contribution* to the two point function of the electromagnetic currents.
    - $Q = -1$: e^-.
    - $Q = +1$: e^+.
    - *no stable bound states*.
  - $N \geq 2$ sector (multi-particle states, which are composed of one particle states). In particular, $N \geq 2$ in the channel of $Q = 0$ and $J = 1 \Leftrightarrow$ branch cuts of $\Pi(s)$, and *there is no poles* !
    - 3γ, 5γ $n\gamma$ ($\Leftrightarrow$ cuts starting from $0+$)
    - e^+e^-, $e^+e^- + \gamma$, $e^+e^- + n\gamma$ ($\Leftrightarrow$ cuts starting from $s = (2m_e)^2$)
    - cuts starting from $(4m_e)^2$ and so on.

What is the relavant states for leptonic $\Pi(q^2)$?

- We insert the complete set of states

$$
\begin{aligned}
&\mathrm{i}\left(\delta_\mu{}^\nu q^2 - q_\mu q^\nu\right)\Pi(q^2) \\
&\quad = \int d^4x\, e^{iq\cdot x}\,\langle 0|\,T j_\mu(x) j^\nu(0)\,|0\rangle_{1\mathrm{PI}} \\
&\quad = \int d^4x\, e^{iq\cdot x}\,\Big\{\theta(x^0)\,\langle 0|\,j_\mu(x)\,|n\rangle\,\widetilde{\eta}_{nn'}\,\langle n'|\,j^\nu(0)\,|0\rangle \\
&\qquad\qquad\qquad + \theta(-x^0)\,(\cdots)\Big\}_{1\mathrm{PI}}\,,
\end{aligned}
$$

where the nontrivial contribution comes from n such that $\langle 0|\,j_\mu(0)\,|n\rangle \neq 0$, i.e. e^-e^+, $e^-e^+\gamma$, $\cdots$, $\gamma\gamma\gamma$, $\gamma\gamma\gamma\gamma\gamma$, $\cdots$.

What is the relavant states for leptonic $\Pi(q^2)$?

- Using the dispersive expression

$$\frac{\Pi_{\mathrm{R}}(q^2+\mathrm{i}\epsilon)}{q^2+\mathrm{i}\,\epsilon} = -\frac{1}{\pi}\int_{0+}^{\infty}\frac{ds}{s}\,\frac{\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)}{q^2-s+\mathrm{i}\epsilon}\,,$$

  which is ensured by the analyticity of $\Pi_{\mathrm{R}}(q^2)$ on the Riemann surface, the vacuum polarization contribution $a_e[\mathrm{vp}]$ can be expressed as the superposition of the contribution of $a_e(s)$ from the massive vector boson with mass squared s weighted by $\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)$;

$$a_e[\mathrm{vp}] = \int_{0+}^{\infty}\frac{ds}{s}\,\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)\,a_e(s)\,.$$

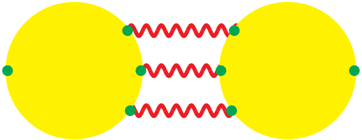
- Recall that $\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)$ is associated with branch cuts of $\Pi_{\mathrm{R}}(s)$.
- *Which branch cuts are associated with the vector-like positronium resonance contribution* to a_e ?

What is the cuts associated with the positronium contribution to $\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)$?

- *The branch cut associated with* e^-e^+ which starts from $\sqrt{s}=2m_e$ *is not* associated with the positronium resonance contribution, because
  - The support of $\mathrm{Im}\,\Pi_{\mathrm{R}}(s+\mathrm{i}\,0)$ corresponding to the positronium resonace is centered at $\sqrt{s_p}=2m_e-O(\alpha)$ with the very narrow width $\Delta s_p\propto\Gamma_p\sim O(\alpha^6)$.
- Indeed, the cuts associated with $\gamma\gamma\gamma$, $\gamma\gamma\gamma\gamma\gamma$, $\cdots$, which starts from $\sqrt{s}=0$ *should be responsible to* the positronium *resonance* contribution to a_e. These cuts are overlooked in the paper by Mishima.

What type of Feynman diagrams is responsible to positronium contribution to a_e ?

- Since QED does not contain the instantons and so on, we can idenfity a set of Feynman diagrams which cause the dynamics responsible to the phenomenon of our interest.
- Needless to say, it is impossible to single out the positronium resonace contribution only. What we can say is which Feynman diagrams are connected with the positronium resonance contribution.
- The positroniums may contribute to a_e through the following particular type of the vacuum polarization, where 3γ state appears as an intermediate state;



What type of Feynman diagrams is responsible to positronium contribution to a_e ?

- Each of the blob parts contain at least one photon exchange. Recall that the bounding energy of the positroniums at the leading order is calculated by the Schrödinger equation with the potential term, which is given by the non-relativistic limit of the amplitude given by the one-photon exchange between e^- and e^+.
- Therefore, the positronium resonance contribution to a_e starts from $O(\alpha^7)$.
- As a reference, I quote the value of $O(\alpha^5)$ with the same topology of diagrams

$$a_e[\mathrm{I(j)}, e \text{ only}] = 0.000\ 3950\ (87) \left(\frac{\alpha}{\pi}\right)^5 ,$$

which is much smaller than the full $O(\alpha^5)$ result of a_e.