The role of experimental data on the HLBL

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THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

Work in collaboration with
<u>Pablo Sanchez-Puertas</u>

Mainz, XXth April 2014

JOHANNES GUTENBERG
UNIVERSITÄT MAINZ

Open questions from the theory side

(in my opinion)

- How to link the general calculation with experiment?
- A particular counting scheme is used (not a systematical expansion) using ChPT-p counting and large-Nc counting:
  - large-Nc enhanced pieces seem dominant: important 1/Nc corrections (just started)
  - ChPT enhanced pieces less dominant but role of pions polarizability may be important (cross-check)
- Double counting needs to be avoided: hadron exchanges versus quark-loops
- New general approaches: Dispersion Relations, CχQM, lattice
- Role of offshellness effects (Ballpark and η-η' suggest ~20% enhancement)
- MV discussion of the "external vertex"
- Role of excited vector states (beyond VMD): syst. improve of models

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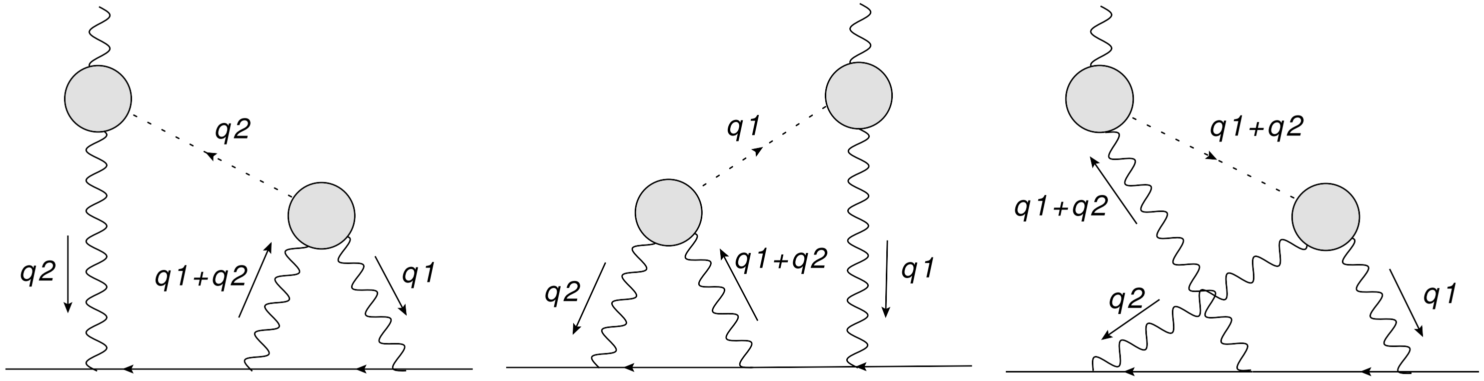
Dissection of the HLBL contribution



$$a_\mu^{LbL;P} = -e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2[(p+q_1)^2-m^2][(p-q_2)^2-m^2]}$$

$$\times \left(\frac{F_{P^*\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2) F_{P^*\gamma^*\gamma^*}(q_2^2, q_2^2, 0)}{q_2^2 - M_P^2} T_1(q_1, q_2; p) \right.$$

Use data from
the Transition Form Factor

$$\left. + \frac{F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_P^2} T_2(q_1, q_2; p) \right)$$

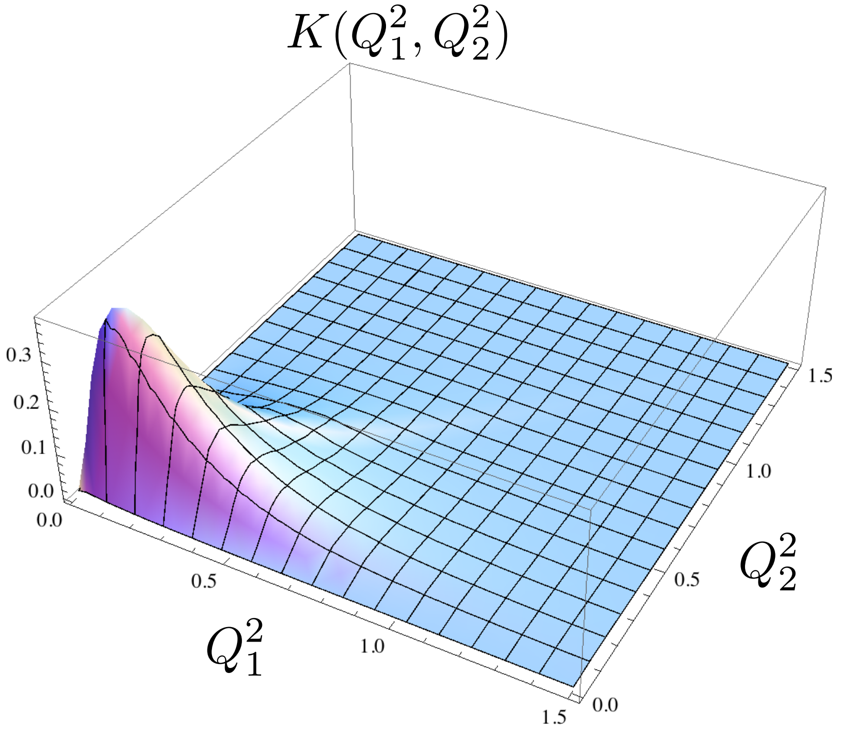
Dissection of the HLBL contribution

- Extraction of meson TFF and HLBL
  - Using CLEO, CELLO, BaBar and Belle to obtain the TFF Low-energy Constants, constrain hadronic model and estimation of π^0-HLBL

$$a_\mu^{LbyL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim P_1^0(Q_1^2, Q_2^2)$

(main energy range from 0 to 1 GeV^2)



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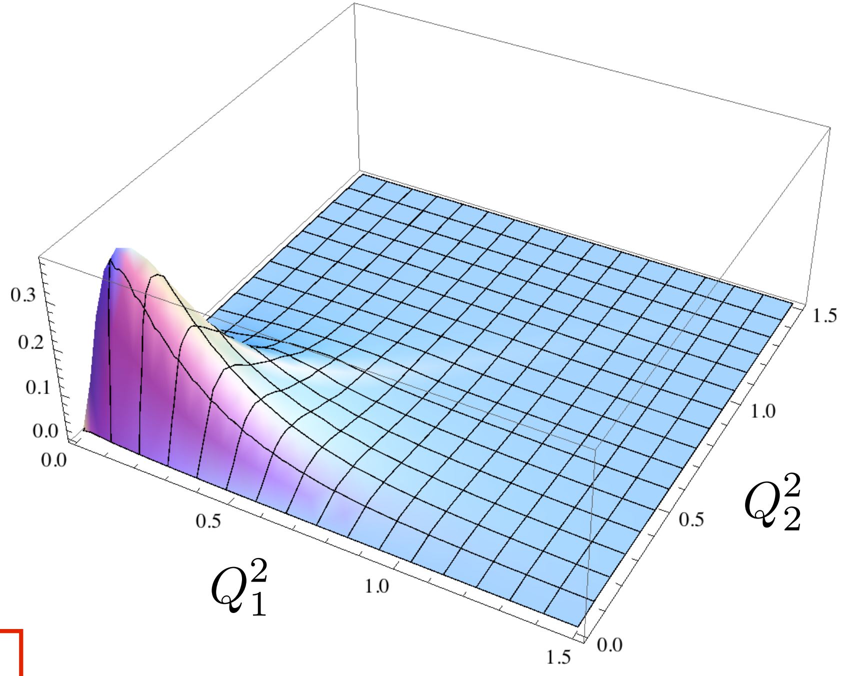
$K(Q_1^2, Q_2^2)$



Systematic error from approach:

$P_1^0(Q_1^2, Q_2^2)$ vs $P_2^1(Q_1^2, Q_2^2) \longrightarrow \boxed{5\%}$

(convergence guaranteed by Pomerenke's theorem) [P.M.,Peris,'07]

Dissection of the HLBL contribution

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Use hadronic models
constrained with
chiral and large-Nc arguments

Use data from
the Transition Form Factor
for numerical integral

Dissection of the HLBL contribution

Use hadronic models
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$$F(0) = \frac{1}{4\pi^2 f_\pi}, \qquad F(Q^2) \to \frac{6 f_\pi}{N_c Q^2} + \cdots$$

ABJ and BL

$$\boxed{F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2}{m_\rho^2 + Q^2},}$$

$$f_\pi =? \quad m_\rho =?$$

$f_\pi = 92.21(14)\ \mathrm{MeV}$ from PDG

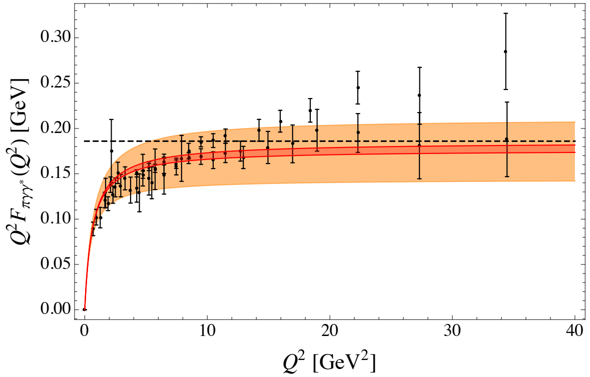
$f_\pi = f_0 = 88.1(4.1)\ \mathrm{MeV}$ from lattice [Ecker et al '14]

$f_\pi = 93(1)\ \mathrm{MeV}$ from $\Gamma_{\pi^0\gamma\gamma}$

$$m_\rho^2 = \frac{24\pi^2 f_\pi^2}{N_c},$$

imposing BL

$$m_\rho = (775 \pm \Delta m_{\rho, N_c \to \infty})\ \mathrm{MeV}$$



$$m_\rho \sim 780 - 820\ \mathrm{MeV}$$

$$m_\rho = (775 \pm \Gamma/2)\ \mathrm{MeV}$$

Dissection of the HLBL contribution

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ABJ and BL

$$F(Q^2) = \frac{1}{4\pi^2 f_\pi} \frac{m_\rho^2 m_{\rho'}^2 + 24 f_\pi^2 \pi^2 Q^2 / N_c}{(m_\rho^2 + Q^2)(m_{\rho'}^2 + Q^2)}$$

$f_\pi = 92.21(14)$ MeV from PDG

$f_\pi = f_0 = 88.1(4.1)$ MeV from lattice [Ecker et al '14]

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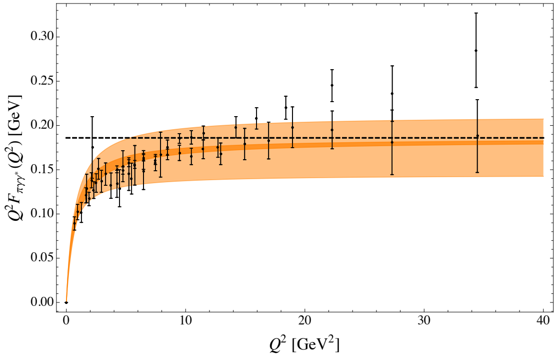

$m_\rho \sim 780 - 820$ MeV

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Use data from
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for numerical integral

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Dissection of the HLBL contribution

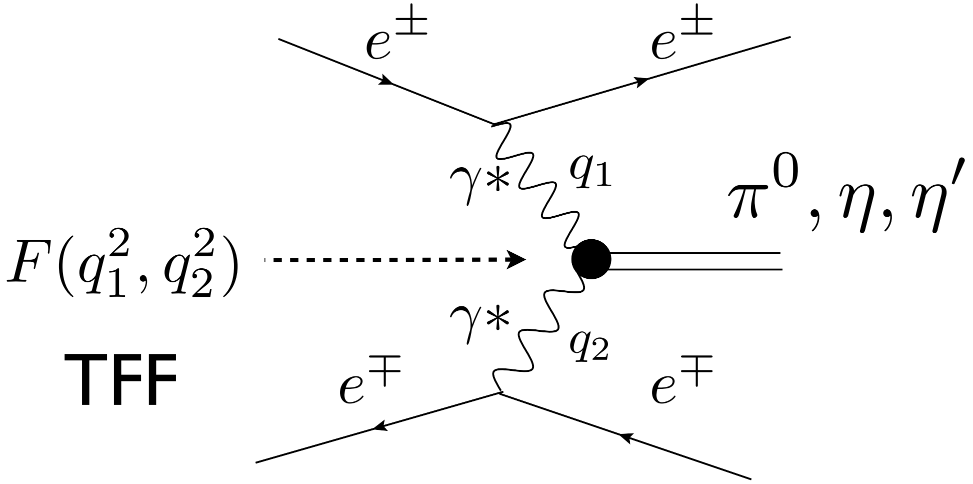
~~Use data from~~
~~the Transition Form Factor~~
~~for numerical integral~~

$$\cancel{F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



Dissection of the HLBL contribution

~~Use data from the Transition Form Factor for numerical integral~~

$$\cancel{F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

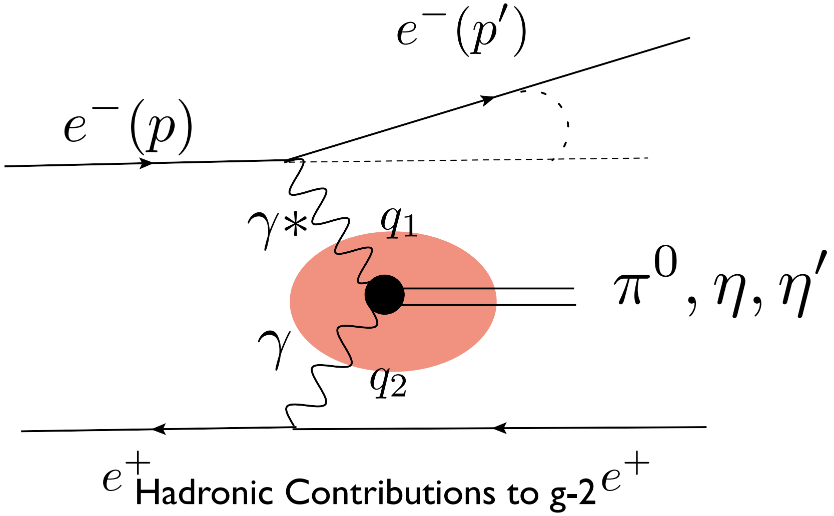
$$\cancel{F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)}$$

$$\Downarrow$$

Use data from the Transition Form Factor to constrain your hadronic model

$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

single-tag method



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$$\cancel{F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)}$$

⇓

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⇓

Use data from
the Transition Form Factor
to constrain your
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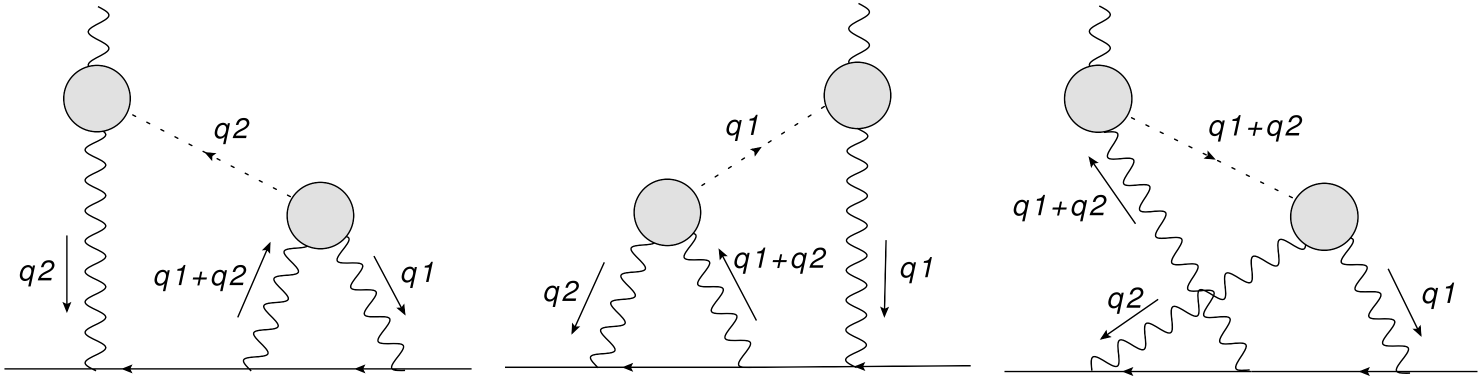
How??

Nice synergy between experiment and theory

Dissection of the HLBL contribution



its calculation requires info. on the pseudoscalar form factors

$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P\left(1 + b_P\frac{Q^2}{m_P^2} + c_P\frac{Q^4}{m_P^4} + \dots\right)$$

$$|F_{P\gamma\gamma}(0)|^2 = \frac{64\pi}{(4\pi\alpha)^2}\frac{\Gamma(P\to\gamma\gamma)}{m_P^3}$$

$\nearrow$ $\Gamma_{P\to\gamma\gamma}$

$\uparrow$ slope

$\uparrow$ curvature

Constrain Hadronic Models

P.M. '12

P.M., Vanderhaeghen'12

Escribano, P.M., P. Sanchez-Puertas,

Our proposal
use Padé Approximants

[P.M.'12; R. Escribano, P.M., P. Sanchez-Puertas, '13]

$$F_{P\gamma*\gamma}(Q^2,0) = \underset{\nearrow \atop \Gamma_{P\to\gamma\gamma}}{a_0^P}\left(1 + \underset{\uparrow \atop \text{slope}}{b_P}\frac{Q^2}{m_P^2} + \underset{\uparrow \atop \text{curvature}}{c_P}\frac{Q^4}{m_P^4} + \ldots\right)$$

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We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P^N_M(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

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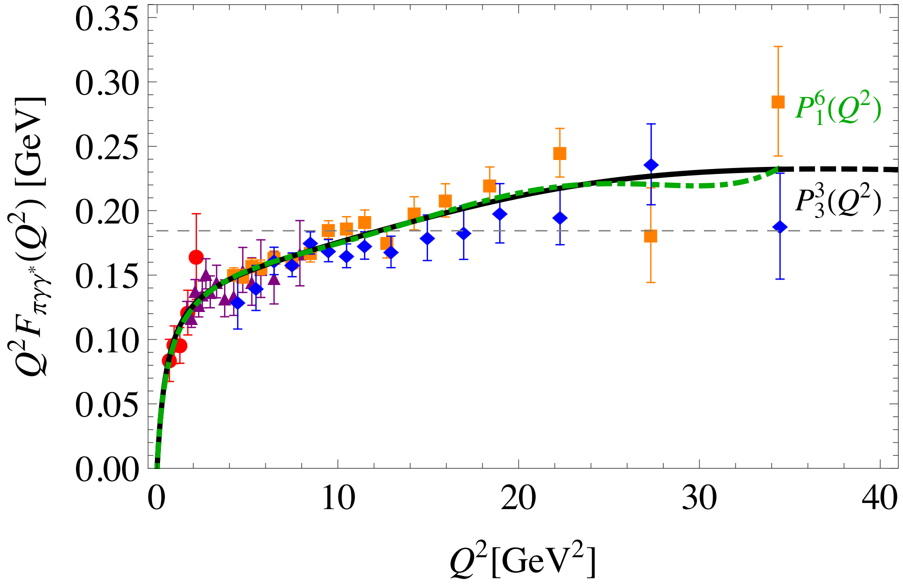
$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \ldots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \ldots \end{array}$$

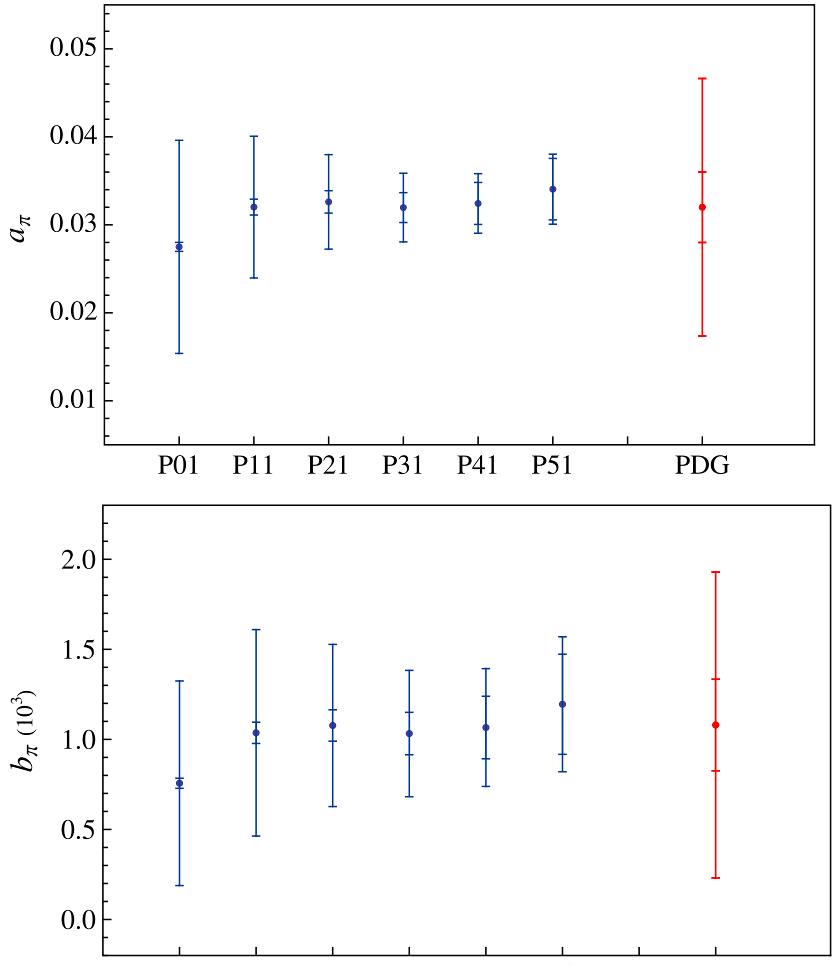
π^0-TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12

$P_1^N(Q^2)$ up to N=6



$P_N^N(Q^2)$ up to N=3



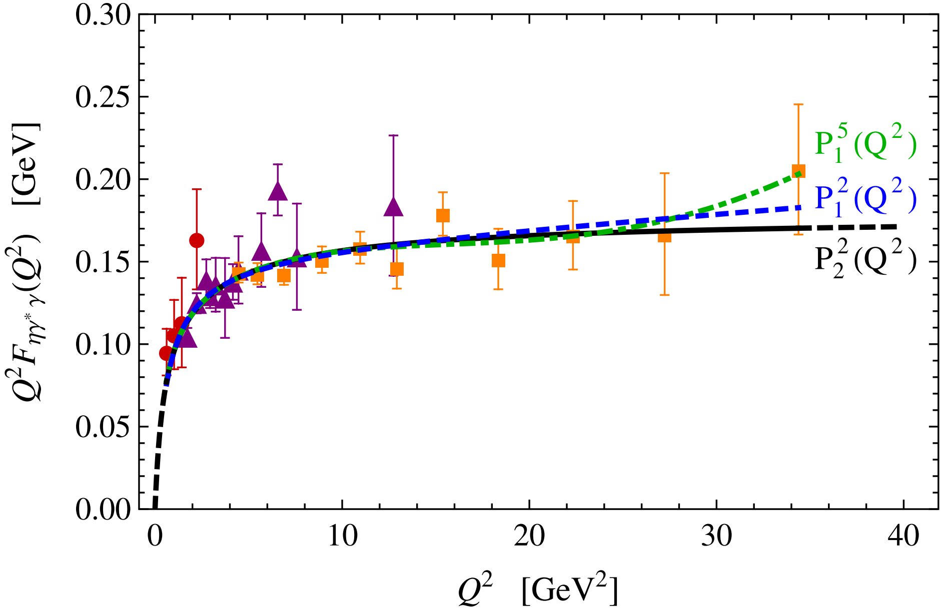
[P.M, '12]

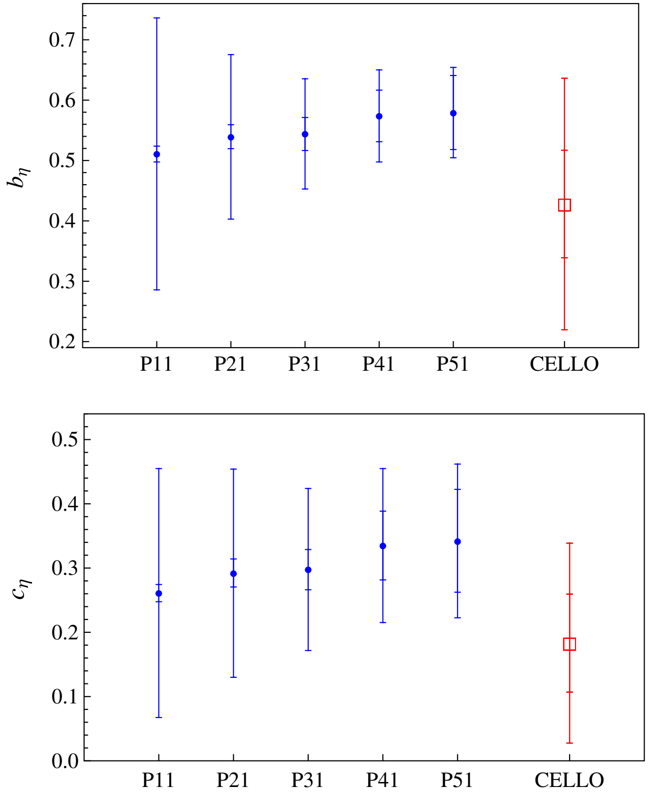
η-TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11+ $\Gamma_{\eta\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

$P_1^N(Q^2)$ up to N=4





$P_N^N(Q^2)$ up to N=2

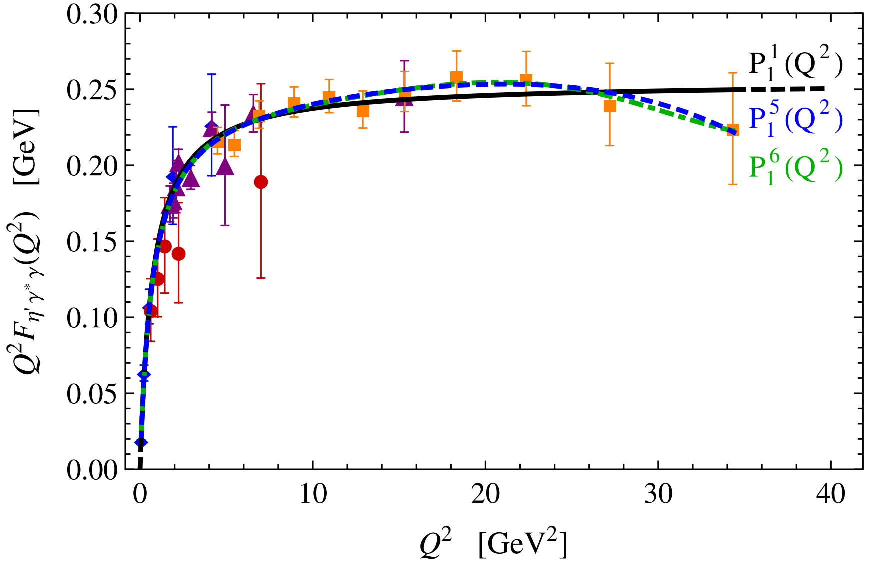
$$\lim_{Q^2\to\infty} Q^2F_{\eta\gamma*\gamma}(Q^2,0) = 0.164(2)GeV$$

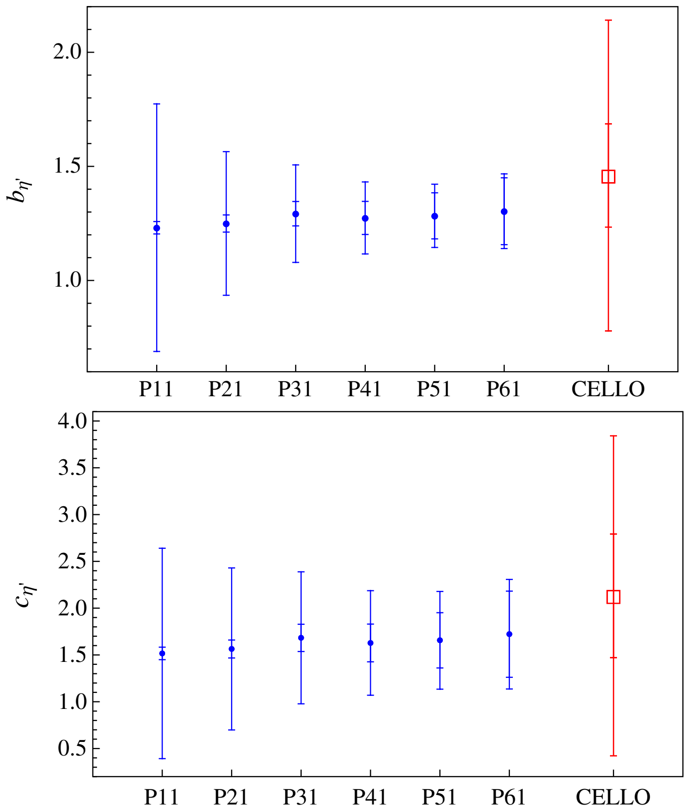
η'-TFF

Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11+ $\Gamma_{\eta'\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

$P_1^N(Q^2)$ up to N=5





$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4)GeV$$

Dissection of the HLBL contribution

a la Knecht-Nyffeler

Central value:

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:

$$F_\pi = 92.4 \text{ MeV}$$

$$m_\rho = 769 \text{ MeV}$$

$$m_{\rho'} = 1465 \text{ MeV}$$

$$h_1 = 0\,(\text{ BL limit})$$

$$h_5 = 6.93 \text{ GeV}^4$$

$$h_2 = -10 \text{ GeV}^2$$

$$a_\mu^{\text{HLBL},\pi} = 6.3\times 10^{-10}$$

Preliminary, using exp data

$$\Gamma_{\pi^0\to\gamma\gamma}$$

$$m_\rho = 775 \text{ MeV}$$

$$h_1 = 0\,(\text{ BL limit})$$

$$h_2 = -10 \text{ GeV}^2$$

slope

curvature

$$a_\mu^{\text{HLBL},\pi} = 7.5\times 10^{-10}$$

Dissection of the HLBL contribution

a la Knecht-Nyffeler

Error budget:

$$F^{VMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\frac{M_V^2}{(q_1^2-M_V^2)}\frac{M_V^2}{(q_2^2-M_V^2)}$$

$$F^{LMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{(q_1^2+q_2^2)-c_V}{(q_1^2-M_V^2)(q_2^2-M_V^2)}$$

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

$$\Delta F_\pi \Rightarrow 2\Delta a_\mu^{\text{HLBL, P}}$$

$$\Delta\ \text{slope} \Rightarrow 0.75\Delta a_\mu^{\text{HLBL, P}}$$

$$\Delta\ \text{curv.} \Rightarrow 0.5\Delta a_\mu^{\text{HLBL, P}}$$

$$\Delta m_\rho = \Gamma/2 \Rightarrow 1.3\Delta a_\mu^{\text{HLBL,}}$$

Current PDG: $\Delta F_\pi \sim 1.1\%$

$\Delta\ \text{slope} \sim 13\%$

$\Delta\ \text{curvature} \sim 25\%$

Chiral limit $F_0 \to F_\pi \sim 5\%$

1/Nc $\Delta m_\rho \sim 10\%$

$$\boxed{\Delta a_\mu^{\text{HLBL},\pi} \sim 15\%}$$

Dissection of the HLBL contribution

a la Melnikov-Vainshtein

Central value:

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Publication:	Preliminary, using exp data
$F_\pi = 92.4$ MeV	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV	$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	$h_1 = 0\,(\text{ BL limit})$
$h_1 = 0\,(\text{ BL limit})$	$h_2 = -10\ \text{GeV}^2$
$h_5 = 6.93\ \text{GeV}^4$	slope
$h_2 = -10\ \text{GeV}^2$	curvature
$a_\mu^{\text{HLBL},\pi} = 7.7 \times 10^{-10}$	$a_\mu^{\text{HLBL},\pi} = 9.8 \times 10^{-10}$

Dissection of the HLBL contribution

a la Melnikov-Vainshtein

$$F_{\pi^0\gamma^*\gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

$$\Delta F_\pi \Rightarrow 2\Delta a_\mu^{\mathrm{HLBL,\ P}}$$

$$\Delta\ \mathrm{slope} \Rightarrow 1\Delta a_\mu^{\mathrm{HLBL,\ P}}$$

$$\Delta\ \mathrm{curv.} \Rightarrow 2\Delta a_\mu^{\mathrm{HLBL,\ P}}$$

$$\Delta m_\rho = \Gamma/2 \Rightarrow 1.3\Delta a_\mu^{\mathrm{HLBL,\ P}}$$

$$\boxed{\Delta a_\mu^{\mathrm{HLBL},\pi} \sim 30\%}$$

Dissection of the HLBL contribution

a la Padé

R. Escribano, P.M., P. Sanchez-Puertas, 1307.2061

$$F^{P01}_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = P^0_1(Q_1^2,Q_2^2) = a\frac{b}{Q_1^2+b}\frac{b}{Q_2^2+b}$$

$$F^{P12}_{P\gamma^*\gamma^*}(Q_1^2,Q_2^2) = P^1_2(Q_1^2,Q_2^2) = \frac{a+bQ_1^2}{(Q_1^2+c)(Q_1^2+d)}\frac{a+bQ_2^2}{(Q_2^2+c)(Q_2^2+d)}$$

$$F^{P^0_1}_{P\gamma^*\gamma^*}(p_P^2,q_1^2,q_2^2) = a\frac{b}{q_1^2-b}\frac{b}{q_2^2-b}(1+c\,p_P^2)$$

	b_P	c_P	$\lim_{Q^2\to\infty} Q^2 F_{P\gamma^*\gamma}(Q^2)$	$a_\mu^{\mathrm{HLBL;P}}$
π^0	0.0324(22)	$1.06(27)\cdot 10^{-3}$	$2f_\pi$	$6.49(56)\cdot 10^{-10}$
η	0.60(7)	0.37(12)	0.160(24)GeV	$1.25(15)\cdot 10^{-10}$
η'	1.30(17)	1.72(58)	0.255(4)GeV	$1.27(19)\cdot 10^{-10}$

$F_{P\gamma\gamma^*}(Q_1^2,Q_2^2)$	η	η'	Total
$P^0_1(Q_1^2,Q_2^2)$	1.25(15)	1.21(12)	8.96(59)
$P^1_2(Q_1^2,Q_2^2)$	1.27(19)	1.22(12)	9.00(74)
Eq. (13)	1.44(19)	1.27(29)	8.84(35)
Eq. (14)	1.38(16)	1.22(9)	8.48(45)

$$\boxed{a_\mu^{HLBL;PS} = 8.9(6)(4)\times 10^{-10}}$$

Thank you!