Single meson contributions to the muon's anomalous magnetic moment

Vladyslav Pauk

Johannes Gutenberg University

Mainz, Germany

(g-2) MITP Workshop

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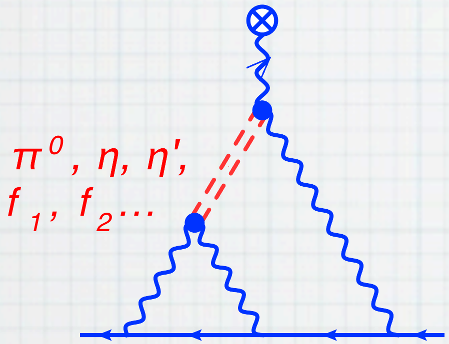
Single meson LbL contribution to $(g-2)_\mu$: state of art



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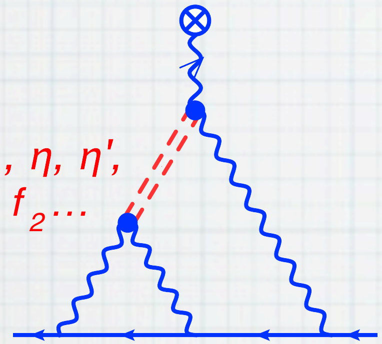

Hadronic LbL contribution to $(g-2)_\mu$ due to single meson exchanges:

	pseudo-scalars	axial-vectors	scalars	tensors
BPP	85 ± 13	2.5 ± 1.0	-7 ± 2	-
HKS	82.7 ± 6.4	1.7 ± 1.7	-	-
MV	114 ± 10	22 ± 5	-	-
KN	83 ± 12	-	-	-
J	93.9 ± 12.4	28.1 ± 5.6	-6.0 ± 1.2	-

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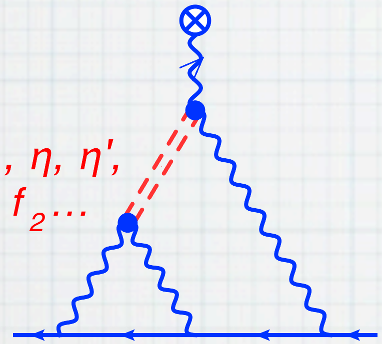

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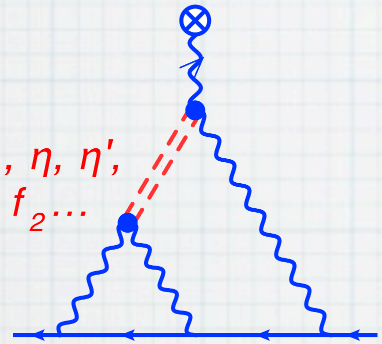

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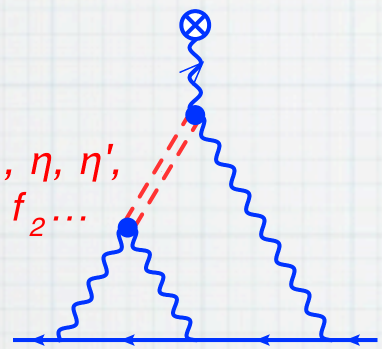

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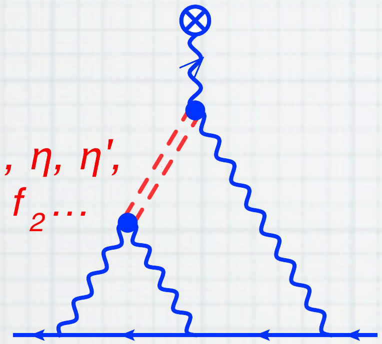

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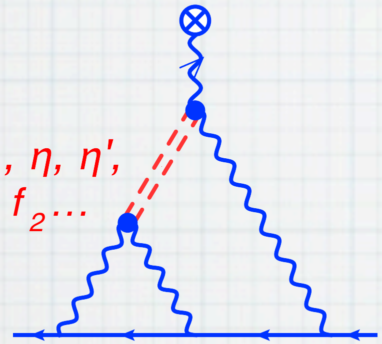

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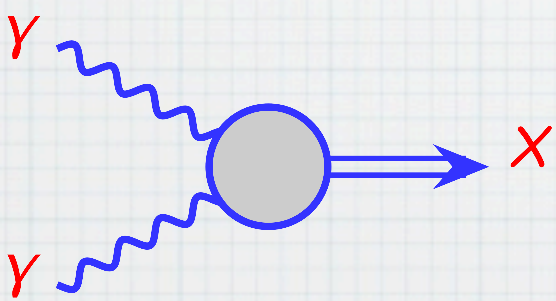
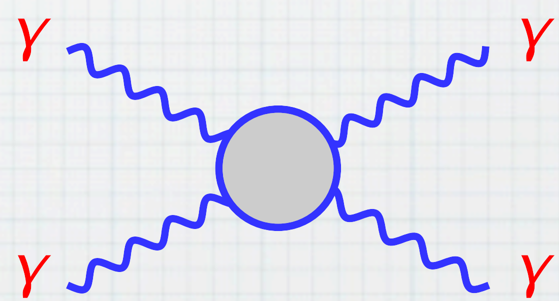
JN (Jegerlehner, Nyffeler) — Large-N_c and data on form factors + off-shell effects
OPE and short-distance constraints

Light-by-light scattering





Elastic light-by-light scattering

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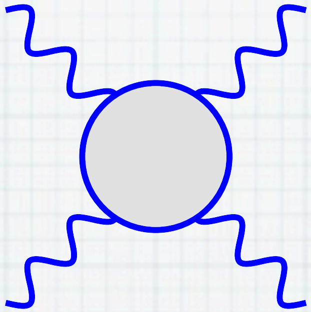
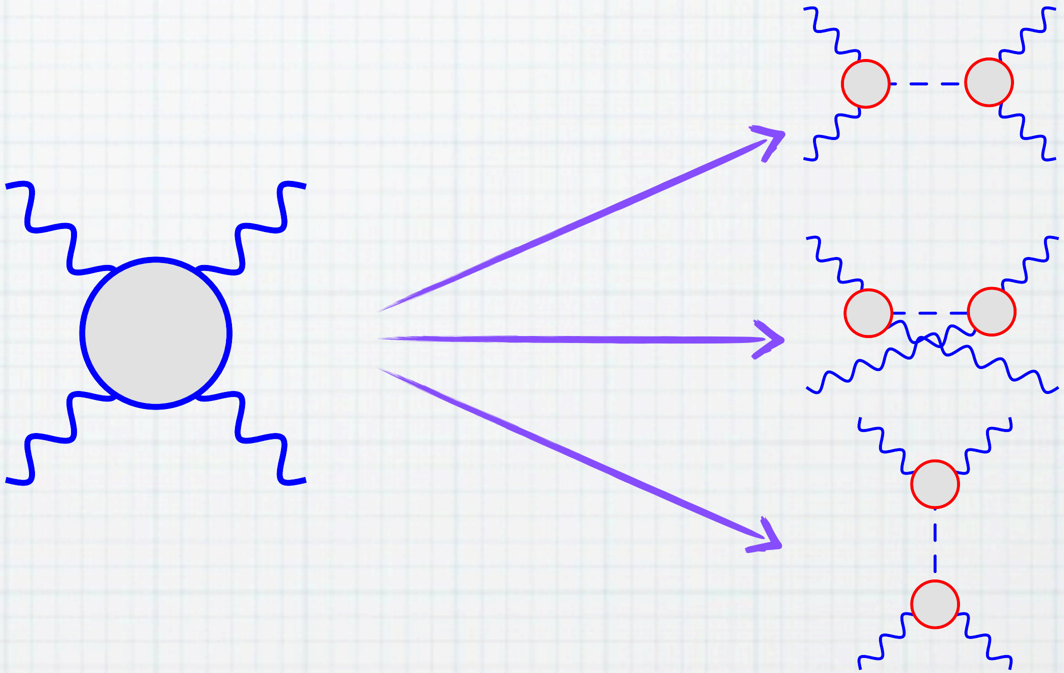


Elastic light-by-light scattering

Single meson exchange

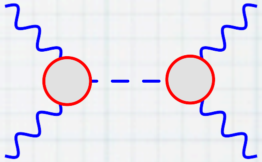


Elastic light-by-light scattering

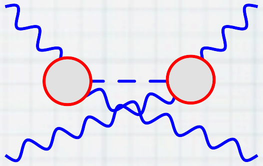
Single meson exchange

$$(ie)^4\Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$

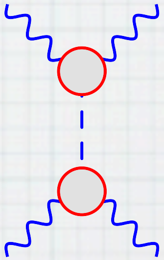
$$= \mathcal{M}_{\mu\nu,\{\alpha\}}(q_1, k-q_1-q_2)\frac{iP^{\{\alpha\},\{\beta\}}(k-q_2)}{(k-q_2)^2-M^2}\mathcal{M}_{\lambda\sigma,\{\beta\}}(q_2,-k)$$



$$+ \mathcal{M}_{\mu\sigma,\{\alpha\}}(q_1,-k)\frac{iP^{\{\alpha\},\{\beta\}}(k-q_1)}{(k-q_1)^2-M^2}\mathcal{M}_{\nu\lambda,\{\beta\}}(k-q_1-q_2,q_2)$$



$$+ \mathcal{M}_{\mu\lambda,\{\alpha\}}(q_1,q_2)\frac{iP^{\{\alpha\},\{\beta\}}(q_1+q_2)}{(q_1+q_2)^2-M^2}\mathcal{M}_{\nu\sigma,\{\beta\}}(k-q_1-q_2,-k)$$



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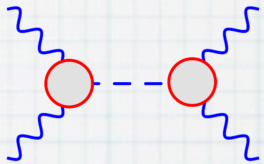

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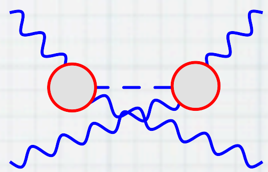

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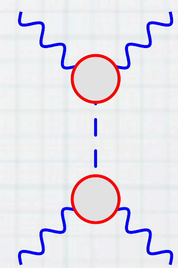
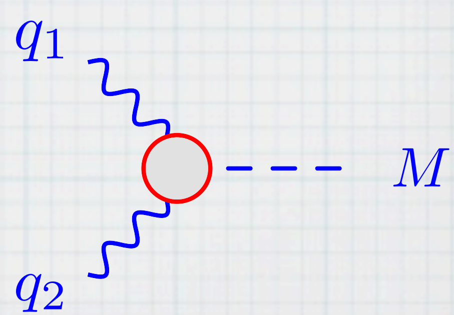


Meson transition amplitudes

$$\mathcal{M}_{\mu\nu,\{\alpha\}}(q_1,q_2) = \sum_h e^2 M^{(h)}_{\mu\nu,\{\alpha\}} F^{(h)}_{\mathcal{M}\gamma^*\gamma^*}(q_1^2, q_2^2, (q_1+q_2)^2)$$

covariant decomposition

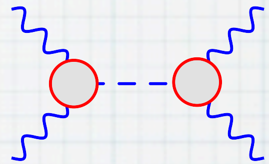
non-perturbative information



Elastic light-by-light scattering

Single meson exchange

$$
\begin{aligned}
&(ie)^4 \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) \\
&= \mathcal{M}_{\mu\nu,\{\alpha\}}(q_1, k-q_1-q_2) \frac{iP^{\{\alpha\},\{\beta\}}(k-q_2)}{(k-q_2)^2 - M^2} \mathcal{M}_{\lambda\sigma,\{\beta\}}(q_2, -k)
\end{aligned}
$$



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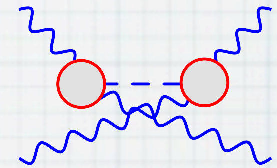
$$



$$
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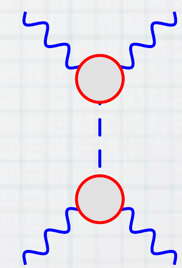

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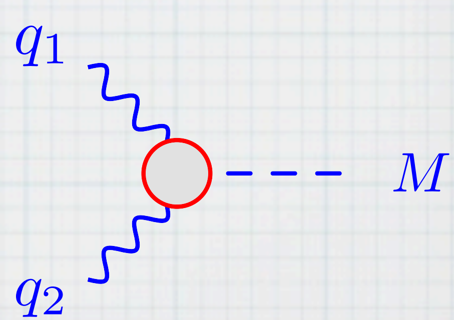

constant form factor
(corresponds to a pole approximation)

$$
\longrightarrow \quad F^{(h)}_{\mathcal{M}\gamma^*\gamma^*}(q_1^2, q_2^2, (q_1+q_2)^2) = F^{(h)}_{\mathcal{M}\gamma^*\gamma^*}(q_1^2, q_2^2, M^2)
$$

Scalar meson transition amplitude

$$\mathcal{M}^{(\mathcal{S})}{}_{\mu\nu}(q_1, q_2) = \mathcal{M}^{(T)}{}_{\mu\nu}(q_1, q_2) + \mathcal{M}^{(L)}{}_{\mu\nu}(q_1, q_2)$$

++ (--) 00

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transverse part

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normalization

$$[F_{\mathcal{S}\gamma^*\gamma^*}(0,0)]^2 = \frac{1}{M}\frac{4}{\pi\alpha^2}\Gamma_{\gamma\gamma}$$

	M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]
$f_0(980)$	980 ± 10	0.29 ± 0.07
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5
$a_0(980)$	980 ± 20	0.3 ± 0.1

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monopole form factor

$$\frac{F_{\mathcal{S}\gamma^*\gamma^*}(q_1^2,q_2^2)}{F_{\mathcal{S}\gamma^*\gamma^*}(0,0)} = \frac{1}{(1-q_1^2/\Lambda_{\rm mon}^2)}\,\frac{1}{(1-q_2^2/\Lambda_{\rm mon}^2)}$$

vector mesons

hadronic scale

$$\Lambda_{\rm mon} = 1 - 2\ GeV$$

Scalar meson transition amplitude

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vector mesons
hadronic scale

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$f_0(500)$ is not included due to large width:
full treatment including $\pi^+\pi^-$ continuum is required

Axial-vector meson transition amplitude

$$\mathcal{M}^{(\mathcal{A})}{}_{\mu\nu\alpha}(q_1, q_2) = i\frac{e^2}{M^2} A(q_1^2, q_2^2) \times \varepsilon_{\rho\nu\tau\alpha} \left[\left(q_1^2 g_\mu^\rho - q_1^\rho q_{1\,\mu} \right) q_2^\tau - \left(q_2^2 g_\mu^\rho - q_2^\rho q_{2\,\mu} \right) q_1^\tau \right]$$

non-relativistic quark model

Cahn (1987)

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non-relativistic quark model

Cahn (1987)

Transition form factor

L3 Collaboration

for 2γ decay widths $\Gamma_{\gamma\gamma}$ and dipole masses Λ_A entering the FF, we use the experimental results from the L3 Collaboration.

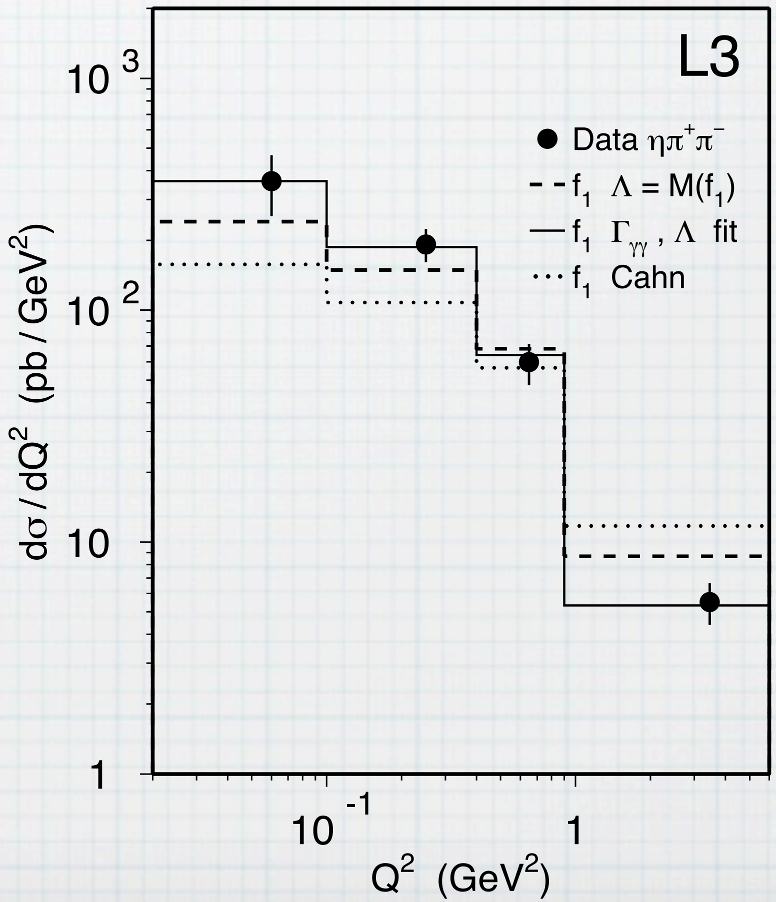
dipole parametrization $A \to \gamma\gamma$ transition FF:

$$\frac{A(q_1^2, 0)}{A(0,0)} = \frac{1}{(1 - q_1^2/\Lambda_A^2)^2}$$

normalization

$$[A(0,0)]^2 = \frac{12}{\pi\alpha^2} \frac{1}{m_A^2} \Gamma_{\gamma\gamma}$$

	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78



Tensor meson transition amplitude and LbL sum rules

$$\mathcal{M}^{(\mathcal{T})}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$
$$= \mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$
$$+ \mathcal{M}^{(1,0)}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$
$$+ \mathcal{M}^{(0,1)}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$
$$+ \mathcal{M}^{(0,T)}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$
$$+ \mathcal{M}^{(0,L)}{}_{\mu\nu\alpha\beta}(q_1, q_2)$$

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$$\mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta} = e^2 \frac{\nu}{m_T} T^{(2)}(Q_1^2, Q_2^2) \times \left[R^{\mu\alpha}(q_1,q_2) R^{\nu\beta}(q_1,q_2) + \frac{s}{8X} R^{\mu\nu}(q_1,q_2)(q_1-q_2)^\alpha (q_1-q_2)^\beta\right]$$

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Sum rules

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2 = 0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s + Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s + Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2 = 0}$$

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$$+ \mathcal{M}^{(0,1)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(0,T)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(0,L)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$

$$\mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta} = e^2 \frac{\nu}{m_T} T^{(2)}(Q_1^2, Q_2^2) \times \left[R^{\mu\alpha}(q_1,q_2) R^{\nu\beta}(q_1,q_2) + \frac{s}{8X} R^{\mu\nu}(q_1,q_2)(q_1-q_2)^\alpha (q_1-q_2)^\beta \right]$$

Sum rules

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

at finite Q_1^2 the SRs imply information on meson transition form-factors:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

Tensor meson transition amplitude and LbL sum rules

$$\mathcal{M}^{(\mathcal{T})}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$= \mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(1,0)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(0,1)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(0,T)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$
$$+ \mathcal{M}^{(0,L)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$

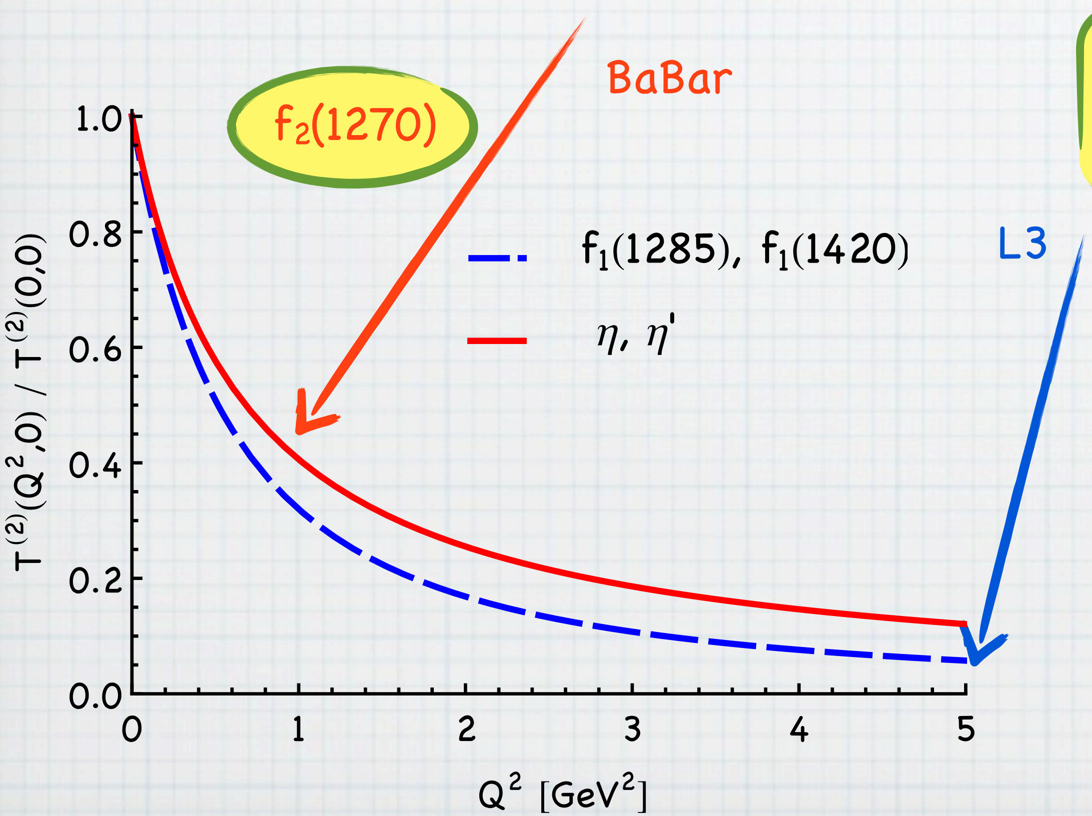
$$\mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta} = e^2 \frac{\nu}{m_T} T^{(2)}(Q_1^2, Q_2^2) \times \left[R^{\mu\alpha}(q_1,q_2) R^{\nu\beta}(q_1,q_2) + \frac{s}{8X} R^{\mu\nu}(q_1,q_2)(q_1-q_2)^\alpha (q_1-q_2)^\beta \right]$$

Sum rules

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

at finite Q_1^2 the SRs imply information on meson transition form-factors:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$



Tensor meson transition amplitude and LbL sum rules

$$\mathcal{M}^{(\mathcal{T})}{}_{\mu\nu\alpha\beta}(q_1,q_2) = \mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta}(q_1,q_2) + \mathcal{M}^{(1,0)}{}_{\mu\nu\alpha\beta}(q_1,q_2) + \mathcal{M}^{(0,1)}{}_{\mu\nu\alpha\beta}(q_1,q_2) + \mathcal{M}^{(0,T)}{}_{\mu\nu\alpha\beta}(q_1,q_2) + \mathcal{M}^{(0,L)}{}_{\mu\nu\alpha\beta}(q_1,q_2)$$

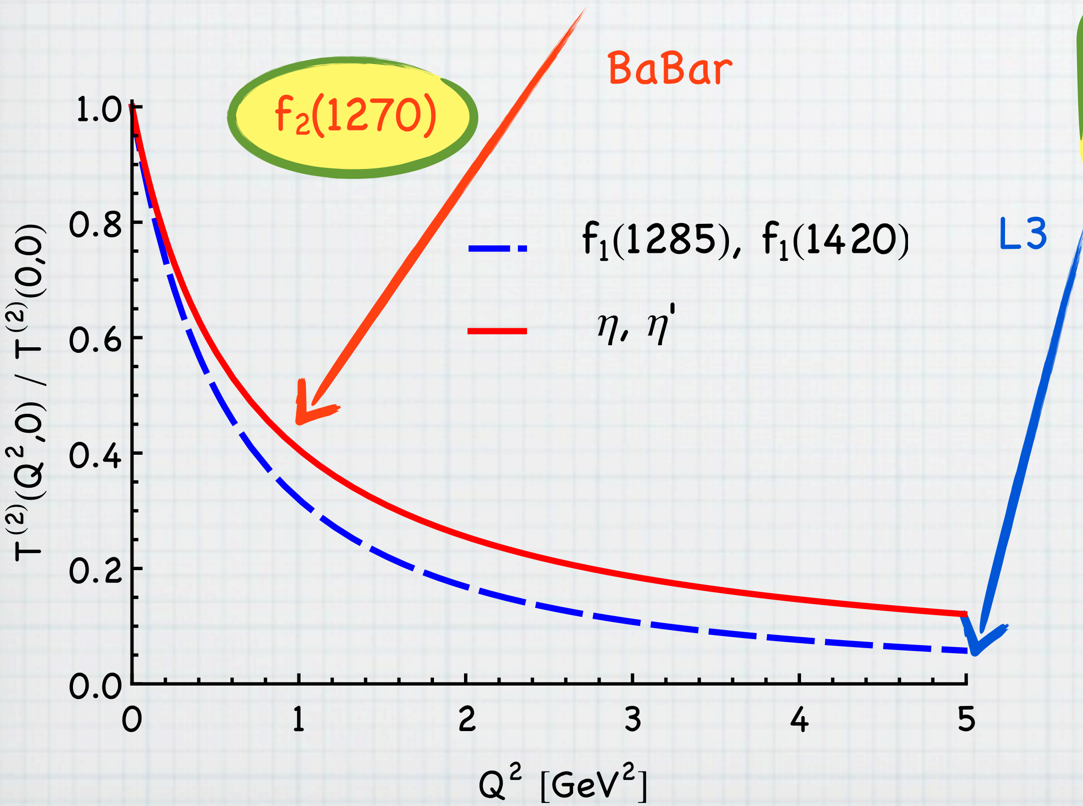
$$\mathcal{M}^{(2)}{}_{\mu\nu\alpha\beta} = e^2 \frac{\nu}{m_T} T^{(2)}(Q_1^2, Q_2^2) \times \left[R^{\mu\alpha}(q_1,q_2) R^{\nu\beta}(q_1,q_2) + \frac{s}{8X} R^{\mu\nu}(q_1,q_2)(q_1-q_2)^\alpha (q_1-q_2)^\beta \right]$$

Sum rules

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_2 - \sigma_0]_{Q_2^2=0}$$

at finite Q_1^2 the SRs imply information on meson transition form-factors:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

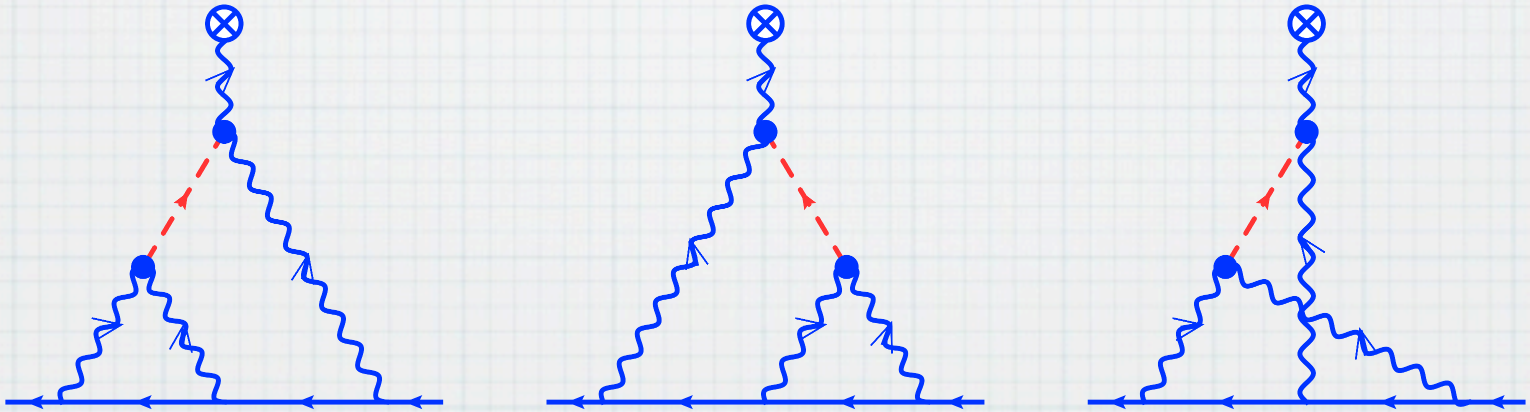


$$\frac{T(q_1^2,0)}{T(0,0)} = \frac{1}{(1 - q_1^2/\Lambda_T^2)^2}$$

$$\Lambda_T = 1.5\ GeV$$

	M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35
$f_2(1565)$	1562 ± 13	0.70 ± 0.14
$a_2(1320)$	1318.3 ± 0.6	1.00 ± 0.06
$a_2(1700)$	1732 ± 16	0.30 ± 0.05

Single-meson contributions to the $(g-2)_\mu$



Projection technique

$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{1}{(p+q_1)^2 - m^2} \frac{1}{(p'-q_2)^2 - m^2} \times T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$

leptonic tensor:

$$T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) = \mathrm{Tr}\left[(\not p + m)\Lambda^\sigma(p',p)(\not p' + m) \times \gamma^\lambda(\not p' - \not q_2 + m)\gamma^\nu(\not p + \not q_1 + m)\gamma^\mu\right]$$

projector:

$$\Lambda_\mu(p',p) = \frac{m^2}{k^2(4m^2-k^2)} \times \left[\gamma_\mu + \frac{k^2+2m^2}{m(k^2-4m^2)}(p'+p)_\mu\right]$$

Projection technique

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$$T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) = \mathrm{Tr}\left[(\not p + m)\Lambda^\sigma(p',p)(\not p' + m) \times \gamma^\lambda(\not p' - \not q_2 + m)\gamma^\nu(\not p + \not q_1 + m)\gamma^\mu\right]$$

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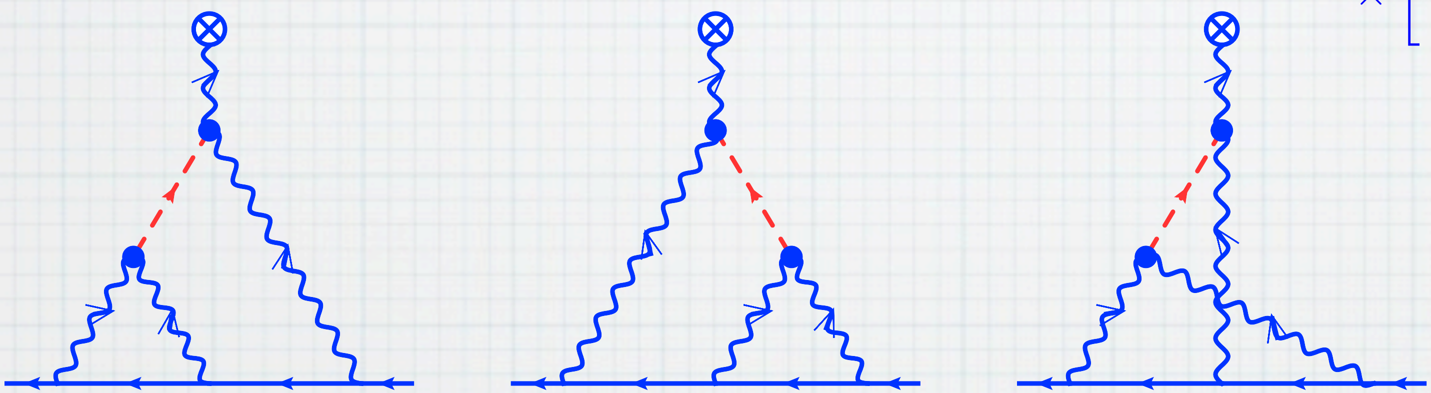


Projection technique

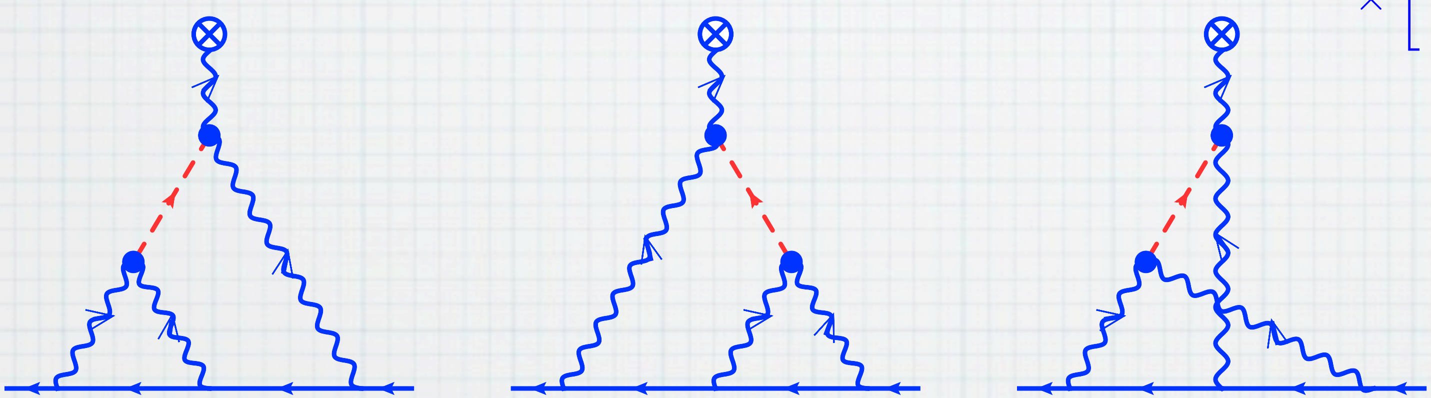
$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{1}{(p+q_1)^2 - m^2} \frac{1}{(p'-q_2)^2 - m^2}$$
$$\times T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$

leptonic tensor:

$$T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) = \mathrm{Tr}\left[(\not p + m)\Lambda^\sigma(p', p)(\not p' + m) \times \gamma^\lambda(\not p' - \not q_2 + m)\gamma^\nu(\not p + \not q_1 + m)\gamma^\mu\right]$$

projector:

$$\Lambda_\mu(p', p) = \frac{m^2}{k^2(4m^2 - k^2)} \times \left[\gamma_\mu + \frac{k^2 + 2m^2}{m(k^2 - 4m^2)}(p' + p)_\mu\right]$$



$$a_\mu^{LbL} = \lim_{k\to 0} -e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (k-q_1-q_2)^2 \left[(p+q_1)^2 - m^2\right]\left[(p+k-q_2)^2 - m^2\right]}$$
$$\times \left[\frac{F_{\mathcal{M}\gamma^*\gamma^*}(q_1^2, (k-q_1-q_2)^2) F_{\mathcal{M}\gamma^*\gamma^*}(k^2, q_2^2)}{(k-q_2)^2 - M^2} T_1(q_1, k-q_1-q_2, q_2)\right.$$
$$\left. + \frac{F_{\mathcal{M}\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\mathcal{M}\gamma^*\gamma^*}((k-q_1-q_2)^2, k^2)}{(q_1+q_2)^2 - M^2} T_2(q_1, k-q_1-q_2, q_2)\right]$$

Projection technique

$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{1}{(p+q_1)^2 - m^2} \frac{1}{(p'-q_2)^2 - m^2} \times T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$

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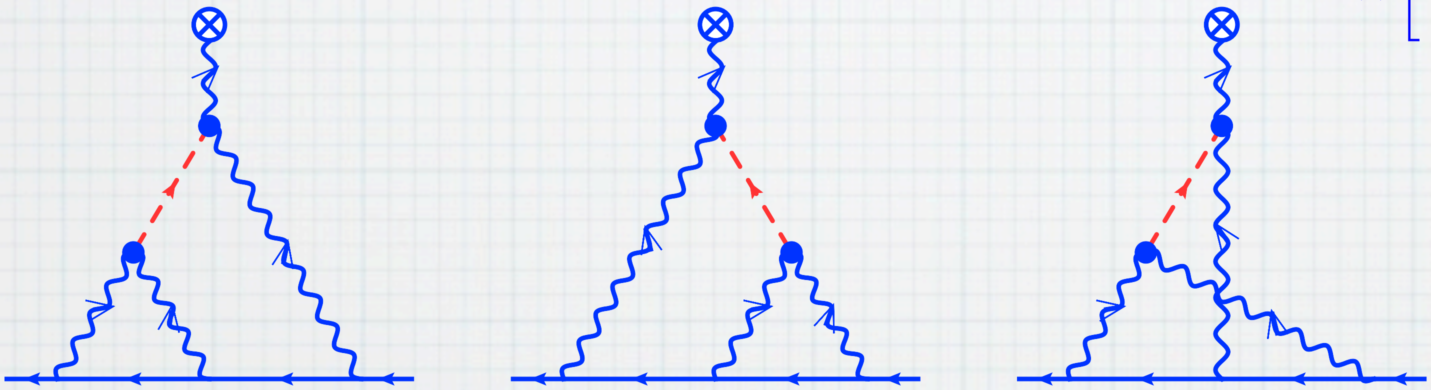
$$T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2) = \mathrm{Tr}\left[(\not p + m)\Lambda^\sigma(p', p)(\not p' + m) \times \gamma^\lambda(\not p' - \not q_2 + m)\gamma^\nu(\not p + \not q_1 + m)\gamma^\mu\right]$$

projector:

$$\Lambda_\mu(p', p) = \frac{m^2}{k^2(4m^2 - k^2)} \times \left[\gamma_\mu + \frac{k^2 + 2m^2}{m(k^2 - 4m^2)}(p' + p)_\mu\right]$$



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Angular integration

$$R^{\mu\nu}(q_1,q_2) \equiv -g^{\mu\nu} + \frac{1}{X}\left\{(q_1\cdot q_2)\,(q_1^\mu\, q_2^\nu + q_2^\mu\, q_1^\nu) - q_1^2\, q_2^\mu\, q_2^\nu - q_2^2\, q_1^\mu\, q_1^\nu\right\}$$

Angular integration

$$R^{\mu\nu}(q_1,q_2) \equiv -g^{\mu\nu} + \frac{1}{X}\left\{(q_1\cdot q_2)\,(q_1^\mu\, q_2^\nu + q_2^\mu\, q_1^\nu) - q_1^2\, q_2^\mu\, q_2^\nu - q_2^2\, q_1^\mu\, q_1^\nu\right\}$$

$$X \equiv (q_1\cdot q_2)^2 - q_1^2 q_2^2 \quad \longrightarrow \quad (q_1\cdot k)^i (q_2\cdot k)^j, \quad \frac{(q_1\cdot k)^i (q_2\cdot k)^j}{(q_2\cdot k)^2 - q_2^2 k^2}, \quad \frac{(q_1\cdot k)^i (q_2\cdot k)^j}{[(q_2\cdot k)^2 - q_2^2 k^2]^2}$$

for scalars and tensors

the k→0 limit not defined!

Angular integration

$$R^{\mu\nu}(q_1,q_2) \equiv -g^{\mu\nu} + \frac{1}{X}\{(q_1\cdot q_2)\,(q_1^\mu\,q_2^\nu + q_2^\mu\,q_1^\nu) - q_1^2\,q_2^\mu\,q_2^\nu - q_2^2\,q_1^\mu\,q_1^\nu\}$$

$$X \equiv (q_1\cdot q_2)^2 - q_1^2 q_2^2 \quad \longrightarrow \quad (q_1\cdot k)^i(q_2\cdot k)^j, \quad \frac{(q_1\cdot k)^i(q_2\cdot k)^j}{(q_2\cdot k)^2 - q_2^2k^2}, \quad \frac{(q_1\cdot k)^i(q_2\cdot k)^j}{[(q_2\cdot k)^2 - q_2^2k^2]^2}$$

for scalars and tensors

the k→0 limit not defined!

3dim angular averaging

(muon's rest frame)

$$\int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi}\, a_\mu(\mathbf{k}) = a_\mu$$

Angular integration

$$R^{\mu\nu}(q_1,q_2) \equiv -g^{\mu\nu} + \frac{1}{X}\left\{(q_1\cdot q_2)\,(q_1^\mu\, q_2^\nu + q_2^\mu\, q_1^\nu) - q_1^2\, q_2^\mu\, q_2^\nu - q_2^2\, q_1^\mu\, q_1^\nu\right\}$$

$$X \equiv (q_1\cdot q_2)^2 - q_1^2 q_2^2 \quad \longrightarrow \quad (q_1\cdot k)^i (q_2\cdot k)^j, \quad \frac{(q_1\cdot k)^i (q_2\cdot k)^j}{(q_2\cdot k)^2 - q_2^2 k^2}, \quad \frac{(q_1\cdot k)^i (q_2\cdot k)^j}{[(q_2\cdot k)^2 - q_2^2 k^2]^2}$$

for scalars and tensors

the k→0 limit not defined!

3dim angular averaging (muon's rest frame)

$$\int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi}\, a_\mu(\mathbf{k}) = a_\mu$$

angular integration can be performed analytically using Legendre polynomials

$$Q_i^0 = Q_i \cos\psi_i, \qquad |\mathbf{Q}_i| = Q_i \sin\psi_i$$

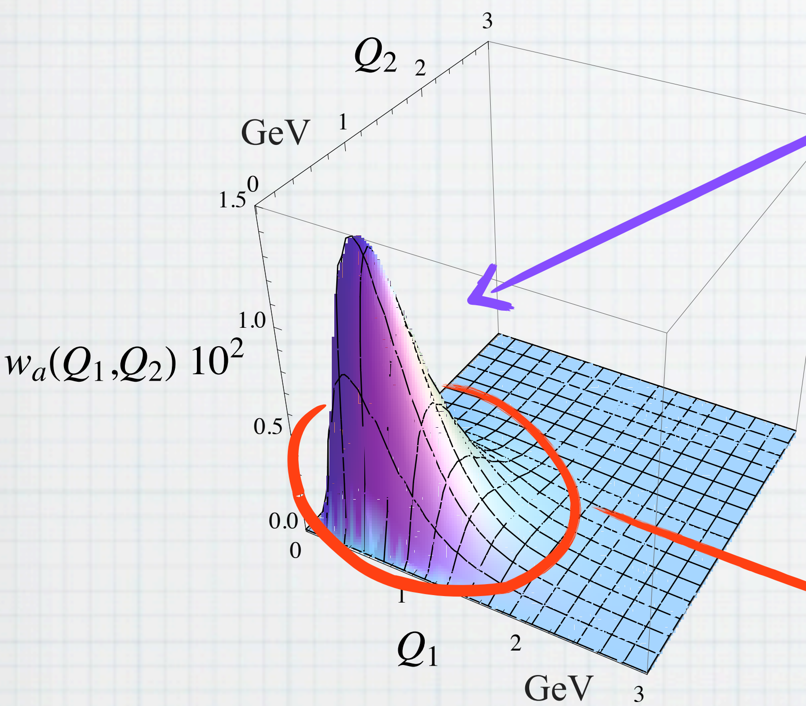
$$a_\mu^{LbL} = -\frac{4\alpha^3}{\pi^3}(2J+1)|F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2 \int_0^\infty dQ_1 \int_0^\pi d\psi_1 \int_0^\infty dQ_2 \int_0^\pi d\psi_2 \frac{1}{Q_1^2/\Lambda^2+1}\frac{1}{Q_2^2/\Lambda^2+1}$$
$$\times \frac{1}{Q_2^2+M^2}\frac{\sin^2\psi_1 \sin^2\psi_2}{Q_1+2im\cos\psi_1}\left[2\,\frac{\tilde{T}_1(Q_1,Q_2,\psi_1,\psi_2)}{Q_2-2im\cos\psi_2} + Q_2\tilde{T}_2(Q_1,Q_2,\psi_1,\psi_2)\right]$$

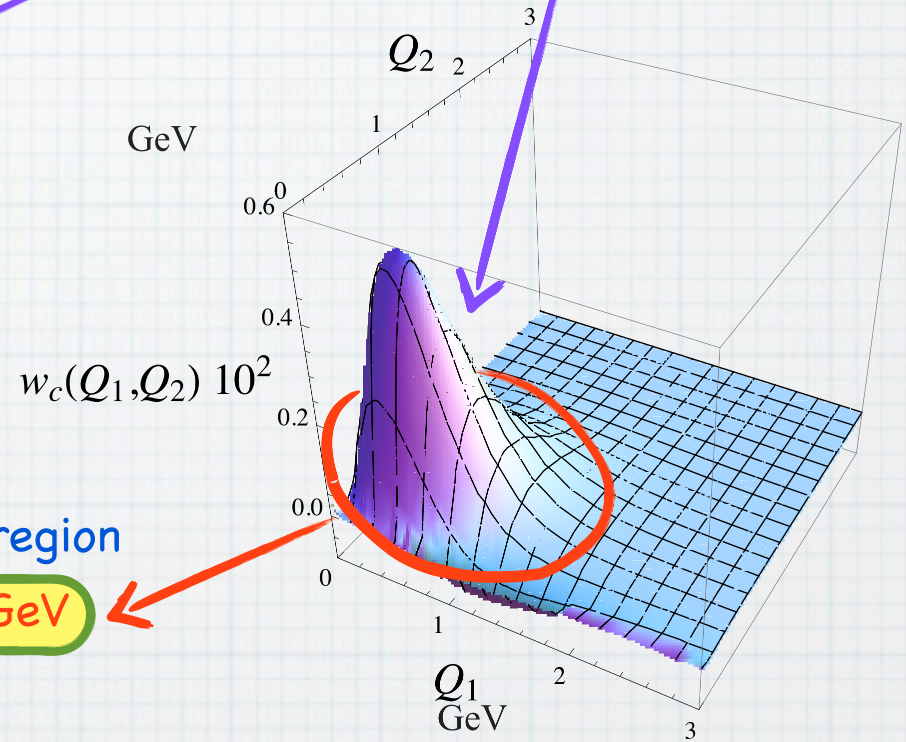
Results: axial-vector mesons

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int \mathrm{d}Q_1 \int \mathrm{d}Q_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$

Results: axial-vector mesons

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int dQ_1 \int dQ_2 \left[2w_a(Q_1,Q_2) + w_c(Q_1,Q_2)\right]$$



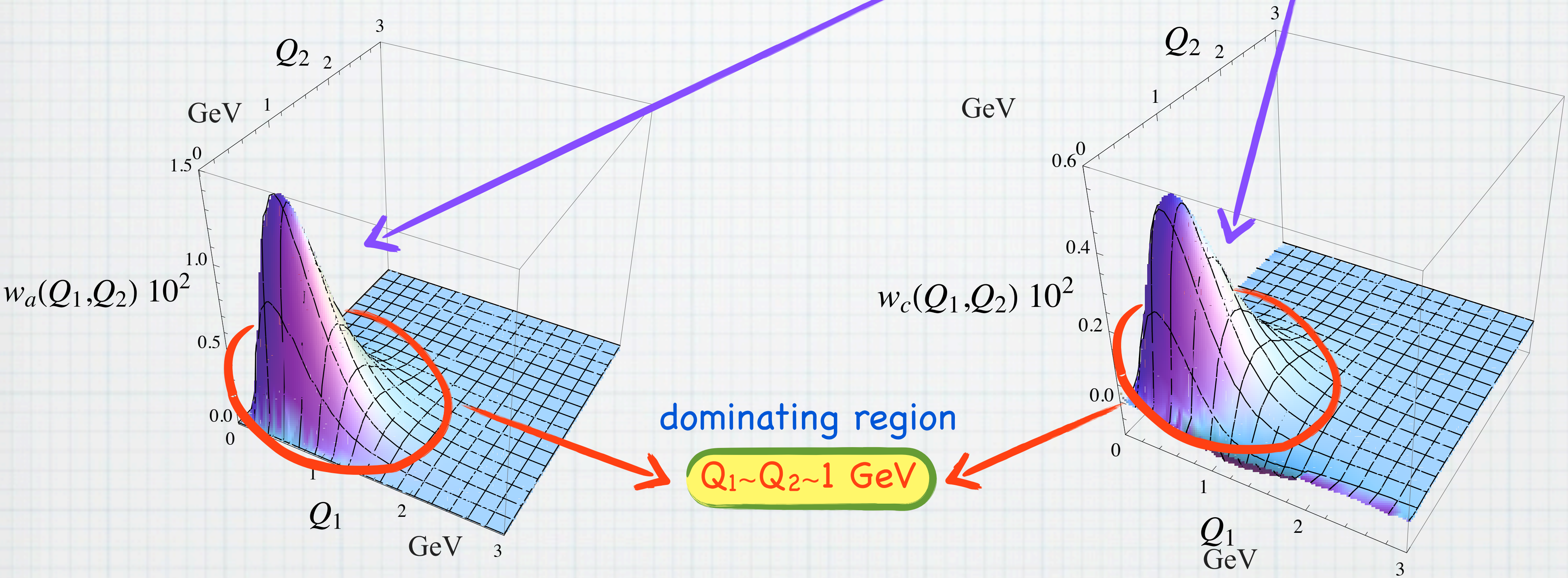


dominating region

$Q_1 \sim Q_2 \sim 1$ GeV

Results: axial-vector mesons

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int dQ_1 \int dQ_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$



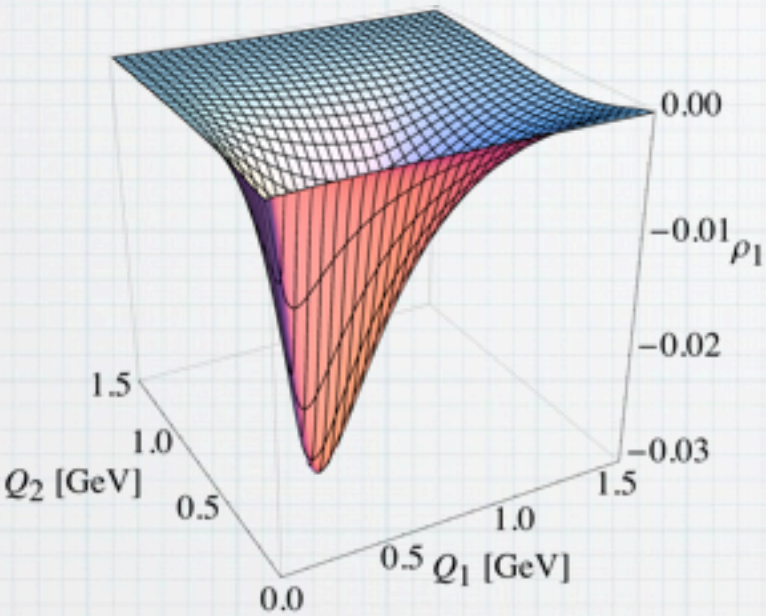
	m_A [MeV]	$\tilde{\Gamma}_{\gamma\gamma}$ [keV]	Λ_A [MeV]	$a_\mu^{LbL;A} \times 10^{10}$
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	1040 ± 78	$0.50^{+0.20}_{-0.17}$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	926 ± 78	$0.14^{+0.07}_{-0.06}$

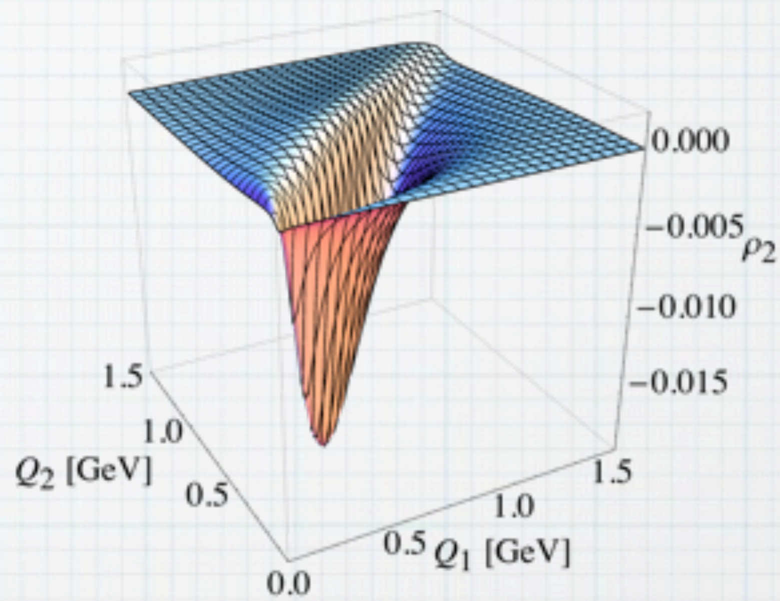
the contribution of the axial-vector pole to the $(g-2)_\mu$

V.P., M. Vanderhaeghen (2014)

Results: scalar mesons

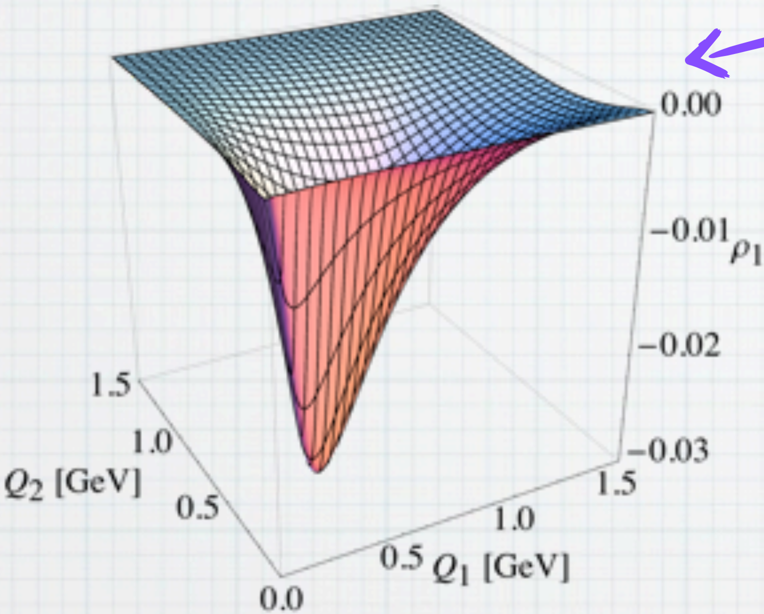
$$a_\mu^{LbL} = -\frac{4\alpha^3}{\pi^3}|F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2 \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \,[2\,\rho_1(Q_1,Q_2) + \rho_2(Q_1,Q_2)]$$





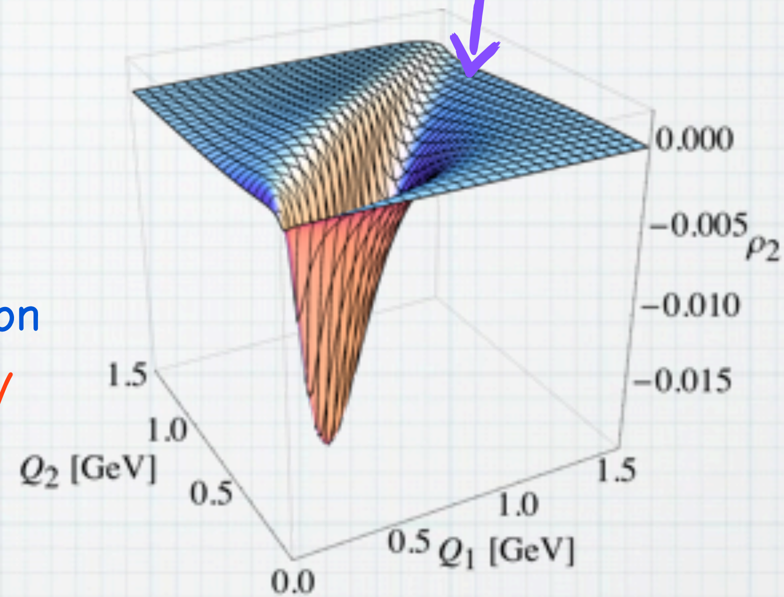
Results: scalar mesons

$$a_\mu^{LbL} = -\frac{4\alpha^3}{\pi^3} |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \, [2\,\rho_1(Q_1,Q_2) + \rho_2(Q_1,Q_2)]$$



dominating region

$Q_1 \sim Q_2 \sim 0.5$ GeV



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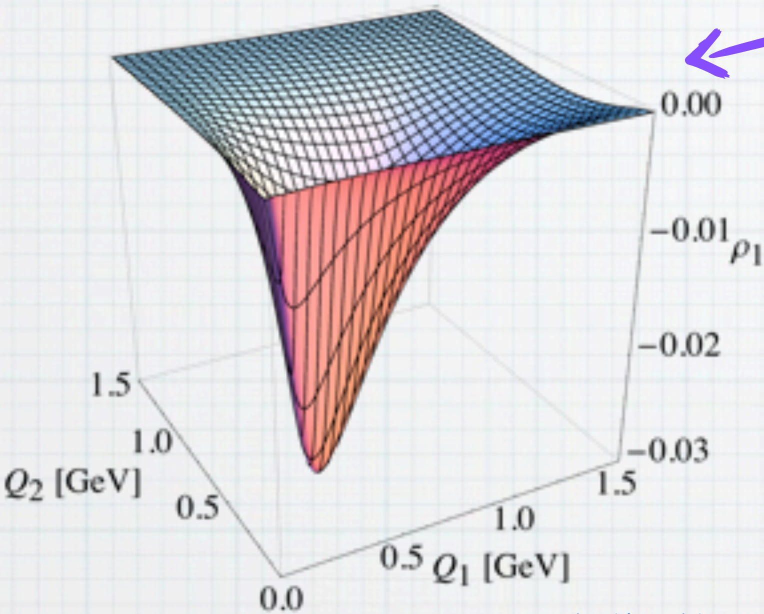

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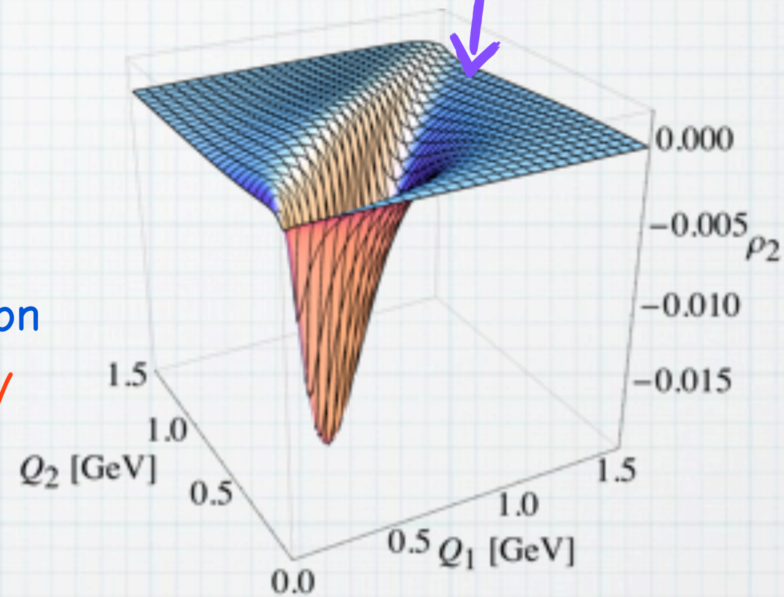
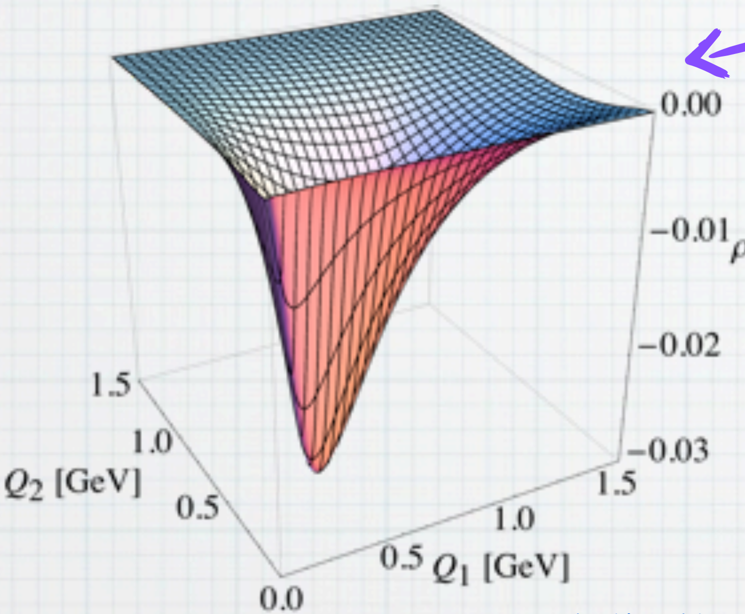


contribution of the narrow scalar resonances

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	a_μ (Λ_{mono} = 1 GeV) [10^{-11}]	a_μ (Λ_{mono} = 2 GeV) [10^{-11}]
$f_0(980)$	980 ± 10	0.29 ± 0.07	-0.19 ± 0.05	-0.61 ± 0.15
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-0.54 ± 0.21	-1.84 ± 0.73
$a_0(980)$	980 ± 20	0.3 ± 0.1	-0.20 ± 0.07	-0.63 ± 0.21
Sum			-0.9 ± 0.2	-3.1 ± 0.8

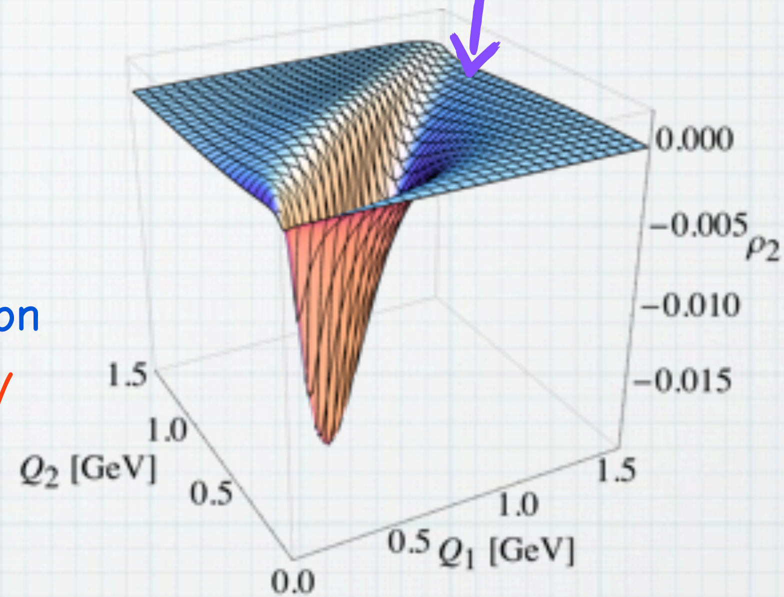
Results: scalar mesons

$$a_\mu^{LbL} = -\frac{4\alpha^3}{\pi^3}|F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2 \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \left[2\,\rho_1(Q_1,Q_2) + \rho_2(Q_1,Q_2)\right]$$



dominating region

$Q_1 \sim Q_2 \sim 0.5$ GeV



contribution of the narrow scalar resonances

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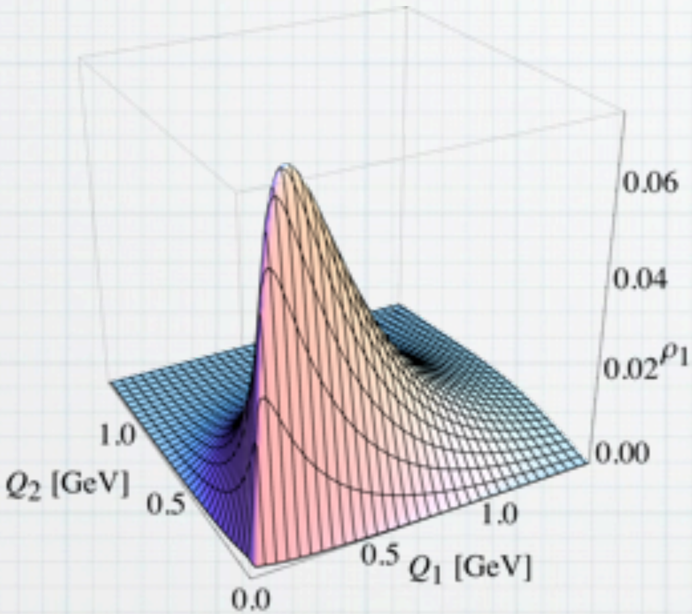
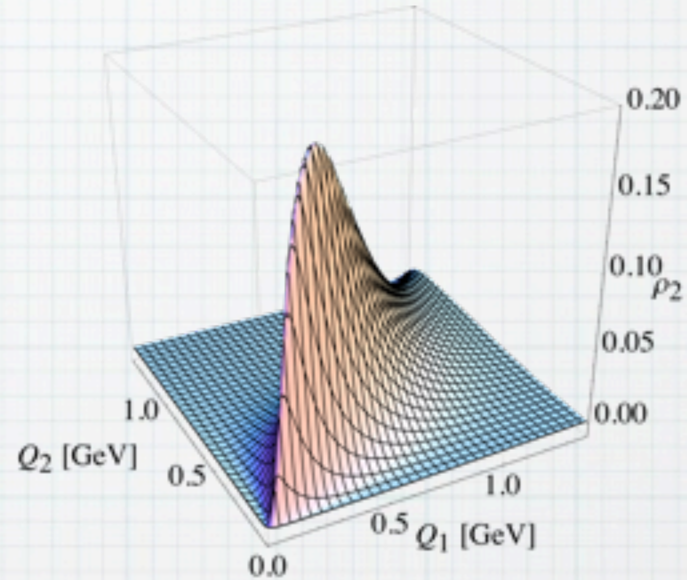
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	-0.54 ± 0.21	-1.84 ± 0.73
$a_0(980)$	980 ± 20	0.3 ± 0.1	-0.20 ± 0.07	-0.63 ± 0.21
Sum			-0.9 ± 0.2	-3.1 ± 0.8

Results: tensor mesons

$$a_\mu^{LbL} = -\frac{20\alpha^3}{\pi^3} |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2 \int_0^\infty \mathrm{d}Q_1 \int_0^\infty \mathrm{d}Q_2 \, [2\,\rho_1(Q_1,Q_2) + \rho_2(Q_1,Q_2)]$$





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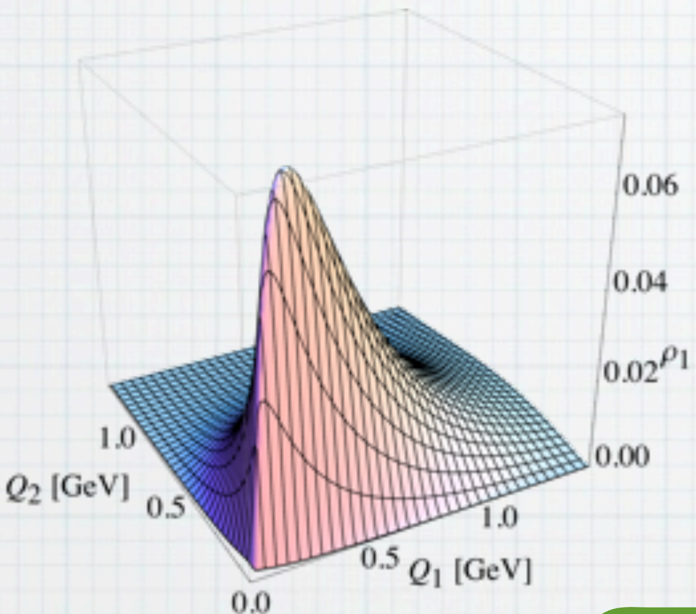
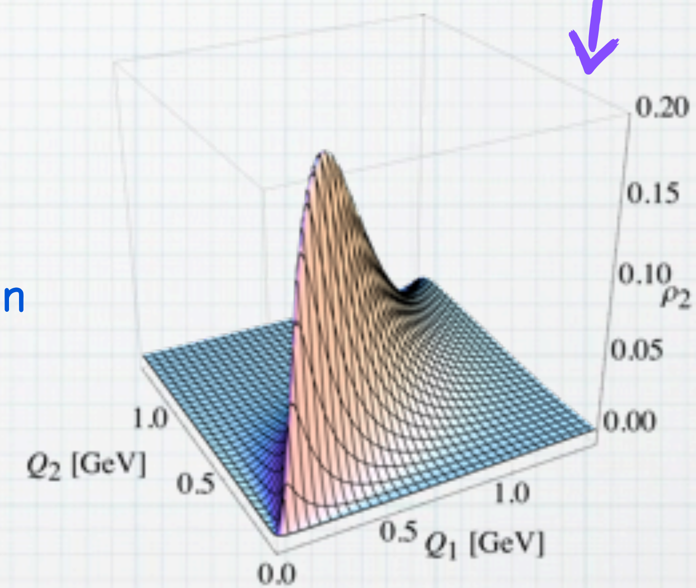


dominating region

$Q_1 \sim Q_2 \sim 0.5$ GeV



contribution of the narrow tensor resonances

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	a_μ ($\Lambda_{dip} = 1.5$ GeV) [10^{-11}]
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	0.79 ± 0.09
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	0.07 ± 0.01
$a_2(1320)$	1318.3 ± 0.6	1.00 ± 0.06	0.22 ± 0.01
$a_2(1700)$	1732 ± 16	0.30 ± 0.05	0.02 ± 0.003
Sum			1.1 ± 0.1

Conclusions & outlook

E821 measurement of $(g-2)_\mu$ (2009)

$a_\mu^{exp} = (11\ 659\ 2089 \pm 63) \times 10^{-11}$

	pseudo-scalars	axial-vectors	scalars	tensors
BPP	85 ± 13	2.5 ± 1.0	-7 ± 2	-
HKS	82.7 ± 6.4	1.7 ± 1.7	-	-
MV	114 ± 10	22 ± 5	-	-
KN	83 ± 12	-	-	-
J	93.9 ± 12.4	28.1 ± 5.6	-6.0 ± 1.2	-
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data on meson form factors in a space-like region

separation of the resonant part from the continuum (Pennington & Colangelo)