HADRONIC LIGHT-BY-LIGHT CONTRIBUTION: QUARK LOOP

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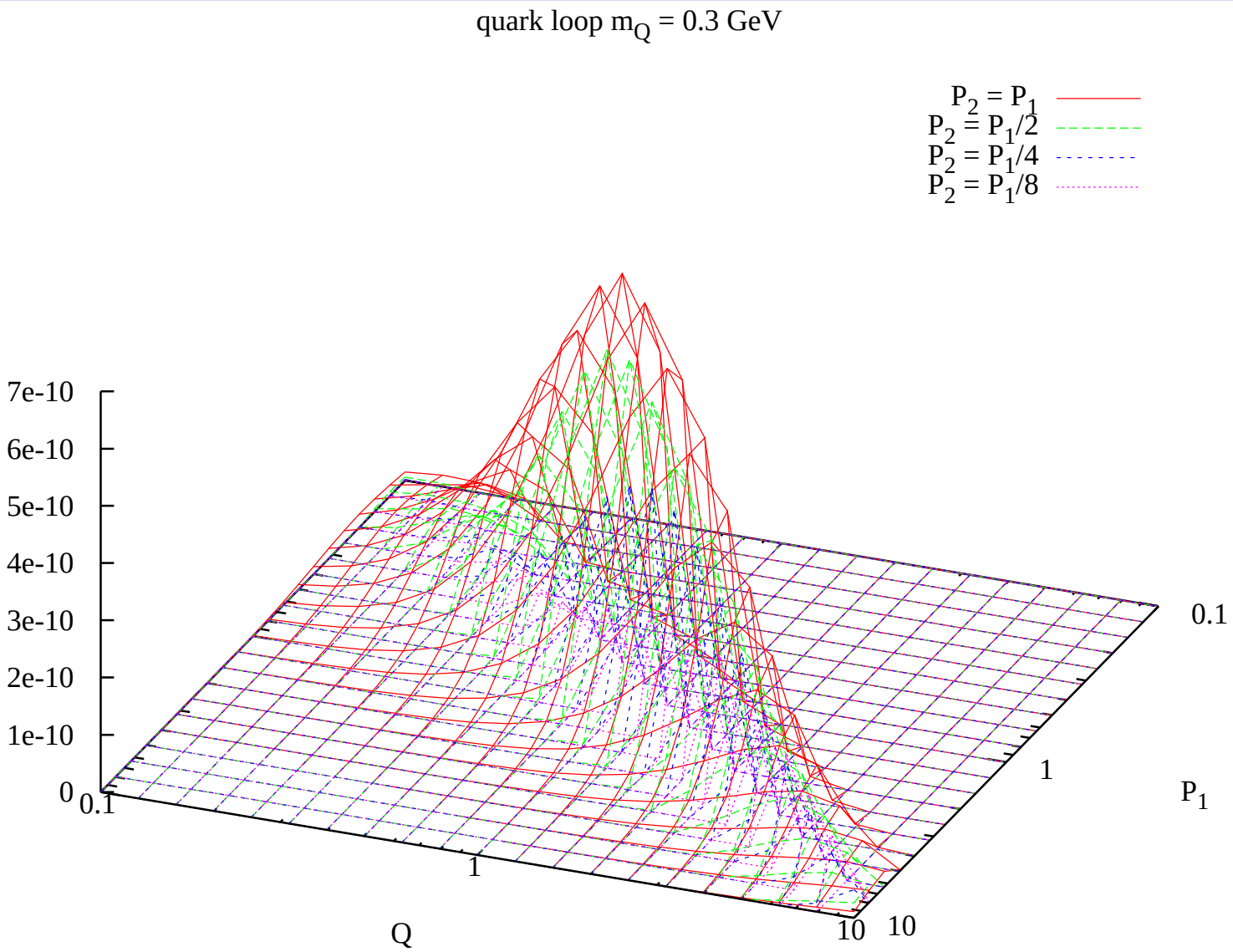
Pure quark loop

Cut-off Λ (GeV)	$a_\mu \times 10^7$ Electron Loop	$a_\mu \times 10^9$ Muon Loop	$a_\mu \times 10^9$ Constituent Quark Loop
0.5	2.41(8)	2.41(3)	0.395(4)
0.7	2.60(10)	3.09(7)	0.705(9)
1.0	2.59(7)	3.76(9)	1.10(2)
2.0	2.60(6)	4.54(9)	1.81(5)
4.0	2.75(9)	4.60(11)	2.27(7)
8.0	2.57(6)	4.84(13)	2.58(7)
Known Results	2.6252(4)	4.65	2.37(16)

- M_Q : 300 MeV
- now known fully analytically
- Us: 5+(3-1) integrals extra are Feynman parameters
- Slow convergence:
  - electron: all at 500 MeV
  - Muon: only half at 500 MeV, at 1 GeV still 20% missing
  - 300 MeV quark: at 2 GeV still 25% missing

Pure quark loop: momentum area



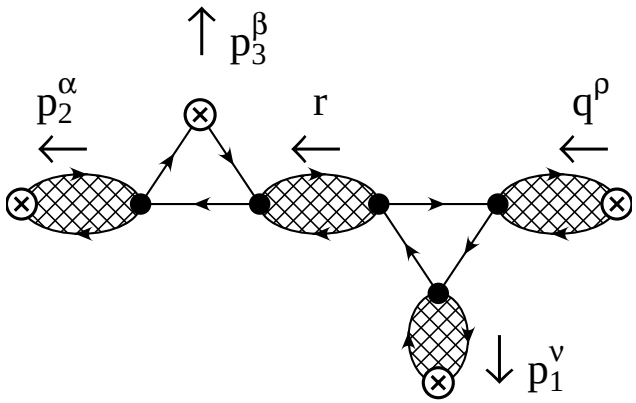
Most from $P_1 \approx P_2 \approx Q$, sizable large momentum part

ENJL quark-loop

Cut-off Λ GeV	$a_\mu \times 10^{10}$ VMD	$a_\mu \times 10^{10}$ ENJL	$a_\mu \times 10^{10}$ masscut	$a_\mu \times 10^{10}$ sum ENJL+masscut
0.5	0.48	0.78	2.46	3.2
0.7	0.72	1.14	1.13	2.3
1.0	0.87	1.44	0.59	2.0
2.0	0.98	1.78	0.13	1.9
4.0	0.98	1.98	0.03	2.0
8.0	0.98	2.00	.005	2.0

- Very stable
- ENJL cuts off slower than pure VMD
- masscut: $M_Q = \Lambda$ to have short-distance and no problem with momentum regions
- Quite stable in region 1-4 GeV

ENJL: scalar



- $$\Pi^{\rho\nu\alpha\beta} = \overline{\Pi}^{VVS}_{ab}(p_1, r) g_S \left(1 + g_S \Pi^S(r)\right) \overline{\Pi}^{SVV}_{cd}(p_2, p_3) \mathcal{V}^{abcd\rho\nu\alpha\beta} + \text{permutations}$$
- $g_S \left(1 + g_S \Pi_S\right) = \frac{g_A(r^2)(2M_Q)^2}{2f^2(r^2)} \frac{1}{M_S^2(r^2) - r^2}$
- $\mathcal{V}^{abcd\rho\nu\alpha\beta}$: ENJL VMD legs
- In ENJL only scalar+quark-loop properly chiral invariant

ENJL: scalar/QL

Cut-off Λ GeV	$a_\mu \times 10^{10}$ Quark-loop VMD	$a_\mu \times 10^{10}$ Quark-loop ENJL	$a_\mu \times 10^{10}$ Scalar Exchange
0.5	0.48	0.78	−0.22
0.7	0.72	1.14	−0.46
1.0	0.87	1.44	−0.60
2.0	0.98	1.78	−0.68
4.0	0.98	1.98	−0.68
8.0	0.98	2.00	−0.68

- ENJL only scalar+quark-loop properly chiral invariant
- Note: ENJL+scalar (BPP) $\approx$ Quark-loop VMD (HKS)
- $M_S \approx 620$ MeV certainly an overestimate for real scalars
- If scalar is σ: related to pion loop part?
- quark-loop: $a_\mu^{ql} \approx 1 \times 10^{-10}$ bare $a_\mu^{ql} = 2.37 \times 10^{-9}$

Other quark loop

- de Rafael-Greynat 1210.3029 $(7.6 - 8.9)\ 10^{-10}$
- Boughezal-Melnikov 1104.4510 $(11.8 - 14.8)\ 10^{-10}$
- Masjuan-Vanderhaeghen 1212.0357 $(7.6 - 12.5)\ 10^{-10}$
- Various interpretations: the full calculation or not
- All (even DSE) have in common that a low quark mass is used for a large part of the integration range