Single particle exchanges in HLbL

Leading LbL contribution from PS mesons:

$$a_\mu[\pi^0,\eta,\eta'] \sim (93.91 = [63.14 + 14.87 + 15.90] \pm 12.40) \times 10^{-11}$$

Expected contribution from axial mesons: Melnikov-Vainshtein form-factors

$$a_\mu[a_1, f_1', f_1] \sim (28.13 = [1.9 + 19.38 + 1.74] \pm 5.63) \times 10^{-11}$$

Expected contribution from $q\bar{q}$ scalars:

- ideal mixing:

$$a_\mu[a_0, f_0', f_0] \sim (-6.01 = [-0.17 - 5.19 - 0.66] \pm 1.20) \times 10^{-11}$$

- nonet symmetry (same masses):

$$a_\mu[a_0, f_0', f_0] \sim (-6.13 = [-0.17 - 4.99 - 0.99] \pm 1.20) \times 10^{-11}$$

depending slightly on assuming nonet symmetry, ideal mixing

$$\mathcal{J}(q_2, q_2) \equiv \int \mathrm{d}^4x\, \mathrm{d}^4y\, \mathrm{e}^{\mathrm{i}\,(xq_1+yq_2)} \langle 0|j_\mu(x)\, j_\nu(y)\, j_\rho(0)|\gamma\rangle$$

- dominated by single particle exchanges of hadrons

$$\boxed{\gamma\gamma H\ ;\quad H = \pi^0, \eta, \eta', a_1, f_1', f_1, a_0, f_0', f_0, \cdots}$$

$\gamma\gamma H$ form factors depend ob 3 invariants q_1^2, q_2^2 and $q_1 q_2$ $\Rightarrow$single hadron exchanges to a_μ 3-dimensional integral over $Q_1 = |Q_1|$, $Q_2 = |Q_2|$ and $t = \cos\theta$:

$$\begin{aligned} a_\mu(\mathrm{LbL}; H) \quad &= \quad -\frac{2\alpha^3}{3\pi^2} \int_0^\infty \mathrm{d}Q_1 \mathrm{d}Q_2 \int_{-1}^{+1} \mathrm{d}t\ \sqrt{1-t^2}\, Q_1^3\, Q_2^3 \\ &\times \left(F_1\, P_6\, I_1(Q_1, Q_2, t) + F_2\, P_7\, I_2(Q_1, Q_2, t)\right) \end{aligned}$$

where $P_6 = 1/(Q_2^2 + m_H^2)$, and $P_7 = 1/(Q_3^2 + m_H^2)$ denote the Euclidean single particle exchange propagators and where the integration kernels I_1 and I_2 are are known analytic functions

$$
\begin{aligned}
F_1 &= \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_2^2, q_1^2, q_3^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_2^2, q_2^2, 0) \,, \\
F_2 &= \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)\, \mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q_3^2, q_3^2, 0) \,.
\end{aligned}
$$

Axial exchanges: a_1, f_1', f_1

Axial exchanges

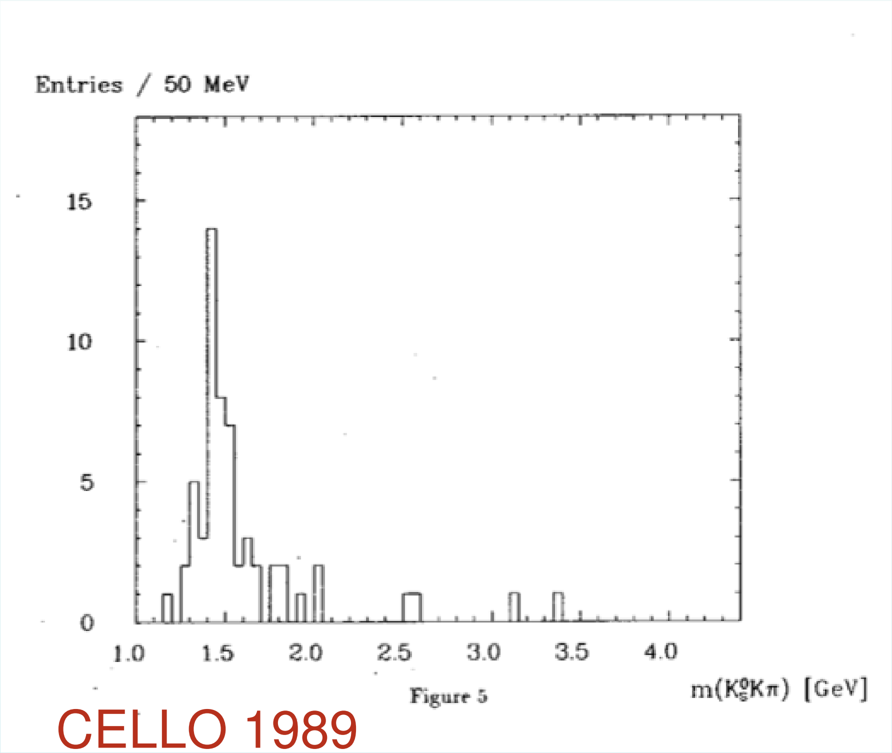
Landau-Yang Theorem: $\mathcal{A}$ (axial meson $\to \gamma\gamma$)=0

e.g. $Z^0 \not\to \gamma\gamma$, while $Z^0 \to \gamma e^+ e^-$ ✓

Why $a_\mu[a_1, f_1', f_1] \sim 25 \times 10^{-11}$ so large?

- untagged $\gamma\gamma \to f()$ no signal!
- single-tag $\gamma^*\gamma \to f()$ strong peak is $Q^2 \gg m_f^2$

$\sigma(\gamma^*\gamma \to f_1 \to K_s^0 K\pi)$



Sparse data so far, new measurements important; in particular momentum dependent $\Gamma(a_1 \to \gamma\gamma^*)$ etc.
Expected contribution from axial mesons:

$$a_\mu[a_1, f_1', f_1] \sim (28.13 \pm 5.63) \times 10^{-11}$$

Axial vertex function:

$$
\begin{aligned}
T_A^{\mu\nu\tau}(q_1,q_2) &= \mathrm{i}\, P_\alpha Q_\beta \Big\{ Q^\tau\, \varepsilon^{\mu\nu\alpha\beta}\, F_A(q_1^2,q_2^2) \\
&+ \{(Pq_1)\, Q^\mu - (Qq_1)\, P^\mu\}\; \varepsilon^{\nu\alpha\beta\tau}\, F'_A(q_1^2,q_2^2) \\
&+ \{(Pq_2)\, Q^\nu - (Qq_2)\, P^\nu\}\; \varepsilon^{\mu\alpha\beta\tau}\, F''_A(q_1^2,q_2^2) \Big\}
\end{aligned}
$$

with $P = (q_1 + q_2)\,, Q = (q_1 - q_2)/2$.

Axial exchange:

$$
T_A^{\mu\nu\tau}(q_1,q_2)\, T_A^{*\rho\lambda\tau'}(k,k+q_3)\,(-g_{\tau\tau'} + P_\tau P_{\tau'}/[q_3^2])\,\frac{1}{q_3^2 - M_A^2}
$$

$$
F_A(q_1^2,q_2^2)\, F_A(0,q_3^2)
$$

OPE inspired Melnikov-Vainshtein approach: identify

$$F_A(q_1^2,q_2^2) \to \Phi_i(Q_1^2,Q_2^2)\ ;\quad F_A(0,q_3^2) \to w_T^{(i)}(Q_3^2)$$

$$
\begin{aligned}
w_T^{(3)} &= \frac{1}{M_{a_1}^2 - M_\rho^2}\Big[\frac{M_{a_1}^2 - m_\pi^2}{Q_3^2 + M_\rho^2} - \frac{M_\rho^2 - m_\pi^2}{Q_3^2 + M_{a_1}^2})\Big]\frac{Q_3^2}{Q_3^2 + M_\rho^2} \\
&\simeq \left(\frac{1}{Q_3^2} - \frac{m_\pi^2}{Q_3^4} + \cdots\right) \\
\Phi_3 &= \frac{\mathrm{asym}(Q_2^2, Q_1^2)\, M_\rho^4}{(Q_1^2 + M_\rho^2)\,(Q_2^2 + M_\rho^2)}\ , \quad W_3 = \frac{1}{4} \\
w_T^{(ud)} &= \frac{1}{M_{f_1}^2 - M_\omega^2}\Big[\frac{M_{f_1}^2 - m_\eta^2/5}{Q_3^2 + M_\omega^2} - \frac{M_\omega^2 - m_\eta^2/5}{Q_3^2 + M_{f_1}^2})\Big]\frac{Q_3^2}{Q_3^2 + M_\omega^2} \\
&\simeq \left(\frac{1}{Q_3^2} - \frac{m_\eta^2/5}{Q_3^4} + \cdots\right) \\
\Phi_{ud} &= \frac{\mathrm{asym}(Q_2^2, Q_1^2)\, M_\omega^4}{(Q_1^2 + M_\omega^2)\,(Q_2^2 + M_\omega^2)}\ , \\
& \quad W_8 = \frac{2}{3}\ , \quad W_{ud}^{\mathrm{ideal}} = \frac{25}{36}
\end{aligned}
$$

$$
\begin{aligned}
I_1(Q_1,Q_2,t) &= X(Q_1,Q_2)\Big(-8\,P_1\,P_2\,(Q_1\cdot Q_2) \\
+ &\quad 4\,P_1\,P_3\,Q_2^2 - 2\,P_1\left(6 - \frac{2\,(Q_1\cdot Q_2)-Q_2^2}{m_\mu^2}\right) \\
- &\quad 4\,P_2\,P_3\,Q_1^2 + 4\,P_2 - 2\,P_3\left(4+\frac{Q_1^2}{m_\mu^2}\right) + \frac{6}{m_\mu^2}\Big) \\
+ &\quad P_1\,P_2\left(4 + 2\,(1-R_{m1})\,(1+\frac{1}{4}\,(1-R_{m1}))\frac{(Q_1\cdot Q_2)}{m_\mu^2}\right) \\
- &\quad P_1\,P_3\left((1-R_{m2})^2\,\frac{(Q_1\cdot Q_2)}{m_\mu^2}\right) - P_1\,\frac{((1-R_{m1})-4\,(1-R_{m2}))}{m_\mu^2} \\
- &\quad P_2\,P_3\left(4+(1-R_{m1})\frac{Q_1^2}{m_\mu^2} - \frac{1}{2}\,(1-R_{m1})^2\frac{(Q_1\cdot Q_2)}{m_\mu^2}\right) \\
+ &\quad P_2\,\frac{(1-R_{m1})}{m_\mu^2} - 2\,P_3\,\frac{(1-R_{m2})}{m_\mu^2}
\end{aligned}
$$

$$
\begin{aligned}
I_2(Q_1,Q_2,t) &= X(Q_1,Q_2)\Big(+4\,P_1\,P_2\,(Q_1\cdot Q_2) - 2\,P_1\,P_3\,Q_2^2 \\
- \quad & P_1\left(2+\frac{Q_2^2}{m_\mu^2}\right) - 2\,P_2\,P_3\,Q_1^2 - P_2\left(2+\frac{Q_1^2}{m_\mu^2}\right) + 4\,P_3 - \frac{4}{m_\mu^2}\Big) \\
+ \quad & P_1\,P_3\,\Big(2+\frac{1}{4}\,((1-R_{m1})^2+(1-R_{m2})^2)\,\frac{(Q_1\cdot Q_2)}{m_\mu^2} \\
& +\frac{1}{2}\,(1-R_{m2})\frac{Q_2^2}{m_\mu^2}\Big) - P_1\,\frac{((1-R_{m1})+3\,(1-R_{m2}))}{2m_\mu^2} \\
+ \quad & P_2\,P_3\,\Big(2+\frac{1}{4}\,((1-R_{m1})^2+(1-R_{m2})^2)\frac{(Q_1\cdot Q_2)}{m_\mu^2} \\
& +\frac{1}{2}\,(1-R_{m1})\frac{Q_1^2}{m_\mu^2}\Big) - P_2\,\frac{(3\,(1-R_{m1})+(1-R_{m2}))}{2m_\mu^2} \\
+ \quad & P_3\,\frac{(2-R_{m1}-R_{m2})}{2m_\mu^2} - 2\,P_1\,P_2\;.
\end{aligned}
$$

LbL: Present

Expected contribution from axial mesons: Melnikov-Vainshtein form-factors Landau-Yang modified

- ideal mixing:

$$a_\mu[a_1, f_1', f_1] \sim (7.55 = [1.89 + 5.19 + 0.47] \pm 2.71) \times 10^{-11}$$

- nonet symmetry (same masses):

$$a_\mu[a_1, f_1', f_1] \sim (7.58 = [1.89 + 4.89 + 0.70] \pm 2.71) \times 10^{-11}$$

JN09 based on Nyffeler 09: the only result relaxing from pole approximation

$$a_\mu^{\mathrm{LbL;had}} = (102 \pm 38) \times 10^{-11}$$

Summary of results

Contribution	HKS	BPP	KN	MV	PdRV	N/JN
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	99±16
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13
axial vectors	1.7±1.7	2.5±1.0	–	22± 5	15±10	22± 5
scalars	–	−6.8±2.0	–	–	−7± 7	−7± 2
quark loops	9.7±11.1	21± 3	–	–	2.3	21± 3
total	89.6±15.4	83±32	80±40	136±25	105±26	116±39

V. Pauk, M. Vanderhaeghen: meson pole conributions

$a_\mu[a_0, f_0', f_0] \sim (-3.1 = [-0.63 - 1.84 - 0.61] \pm 0.8) \times 10^{-11}$

$a_\mu[f_1', f_1] \sim (6.4 = [5.0 + 1.4] \pm 2.0) \times 10^{-11}$

$a_\mu[f_2', f_2, a_2', a_2] \sim (1.1 = [0.79 + 0.07 + 0.22 + 0.02] \pm 0.1) \times 10^{-11}$