Lattice QCD studies of the hadronic vacuum polarisation function

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PRISMA JG|U SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

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$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- leading order (LO) contribution

$$\int \mathrm{d}^4x \, e^{iQx} \, \langle J_\mu(x) \, J_\nu(0) \rangle = (Q_\mu Q_\nu - Q^2 \, \delta_{\mu\nu}) \, \Pi(Q^2)$$

$$J_\mu(x) = \sum_{f=1}^{N_\mathrm{f}} Q_f \, \overline{\psi}_f(x) \gamma_\mu \psi_f(x)$$

$Q_f \in \{-1/3, 2/3\}$

- $\Pi(Q^2)$: photon vacuum polarization function (VPF)

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- $\Pi(Q^2)$: photon vacuum polarization function (VPF)

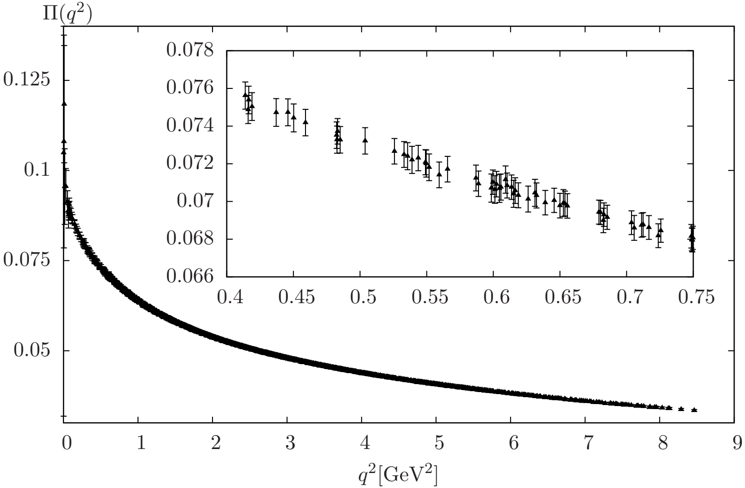
$$\Delta\alpha_{\mathrm{QED}}(Q^2) = 4\pi\alpha\,\big(\Pi(Q^2) - \Pi(0)\big)$$

- Adler function $D(Q^2)$:

$$\frac{D(Q^2)}{Q^2} = 12\pi^2\,\frac{d\,\Pi\,(q^2)}{dq^2} = -\frac{3\pi}{\alpha}\,\frac{d}{dq^2}\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(q^2)$$

$$Q^2 = -q^2$$

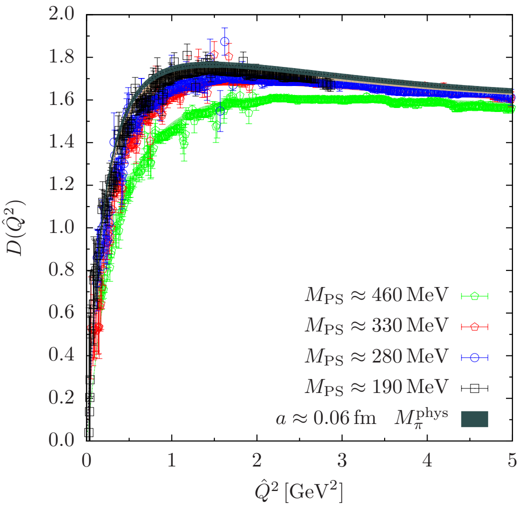
Vacuum polarization function



twisted boundary conditions

Adler function : light-quark mass dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$



u, d

Padé approximants

Adler function : light-quark mass dependence

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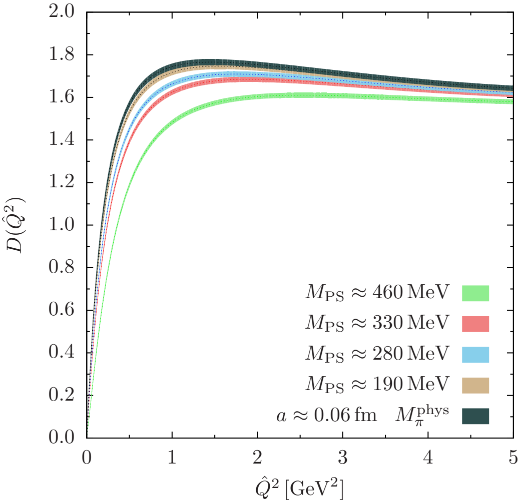


u, d

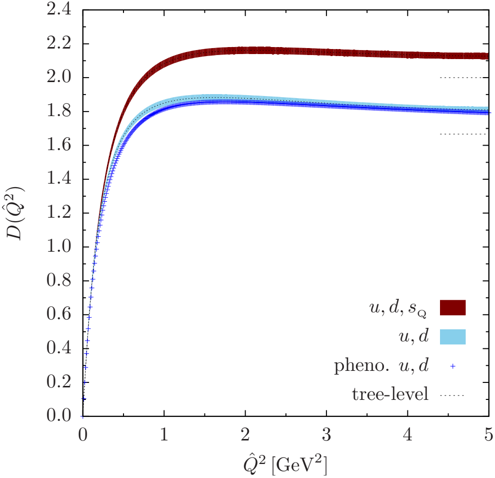
Padé approximants

Adler function : light-quark flavour contributions

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha^{\rm had}_{\rm QED}(q^2)$$

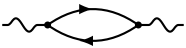
$a \to 0$

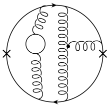
$M_\pi^{\rm phys}$



u, d and $u, d, s_{\rm Q}$

$u, d, s, \ldots$





pheno. model

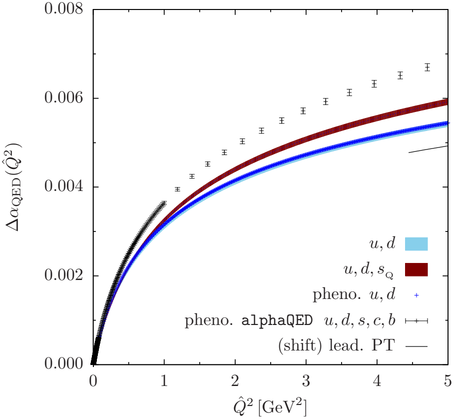
[D. Bernecker & H. Meyer, 1107.4388]

Adler function provides an alternative way to determine $a_\mu^{\rm had}$

running QED coupling: $\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

$$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2) = 4\pi\alpha\,\left(\Pi(Q^2) - \Pi(0)\right)$$



u, d

u, d, s_Q

running QED coupling: $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

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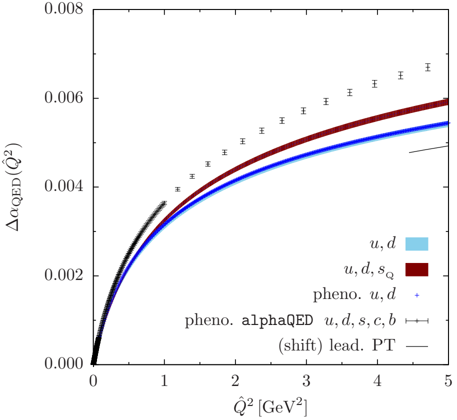


u, d

u, d, s_Q

[PRELIMINARY]

- $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(M_0 = 2.5\,\mathrm{GeV})$ [10^{-3} units]

  u, d: 5.62(06)(17)(02)(12) [19] [4%]

  5.60(06) [1%]

  Pheno. [D. Renner et al., 1206.3113]

- $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(1.0\,\mathrm{GeV})$

  u, d: 3.07(05)(07)(02)(05) [09] [3%]

  u, d, s_Q: 3.29(05)(03)(11)(03) [12] [4%]

  u, d, s, c, b: 3.64(04) [1%]

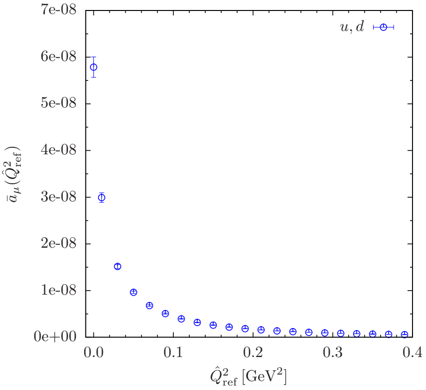
  Pheno. [alphaQED package, F. Jegerlehner]

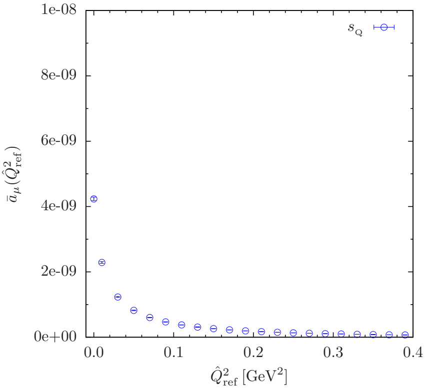
- difference $\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(2\mathrm{GeV}) - \Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(1.0\,\mathrm{GeV})$

  u, d: 1.99(02)(08)(01)(00) [08] [4%]

  u, d, s_Q: 2.33(02)(08)(00)(02) [08] [4%]

$\Pi(Q^2)$ & $\bar{a}_\mu^{\mathrm{HLO}}(Q^2_{\mathrm{ref}})$

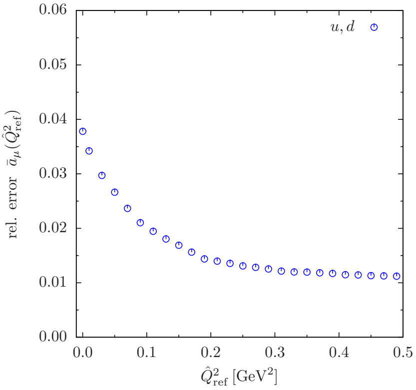


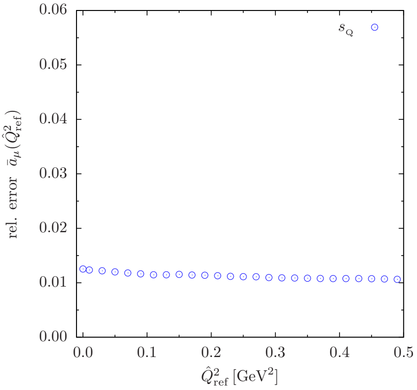


$$\bar{a}_\mu^{\mathrm{HLO}}(Q^2_{\mathrm{ref}}) \equiv 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{\mathrm{ref}}}^{\infty} dQ^2\, f\left(Q^2, m_\mu^2\right) \left[\Pi(Q^2) - \Pi(Q^2_{\mathrm{ref}})\right] \xrightarrow{Q^2_{\mathrm{ref}} \to 0} a_\mu^{\mathrm{HLO}}$$

integrand is peaked at $Q^2 \sim m_\mu^2$ $m_\mu^2 \sim 0.01\,\mathrm{GeV}^2$

$\Pi(Q^2)$ & $\bar{a}_\mu^{\mathrm{HLO}}(Q^2_{\mathrm{ref}})$





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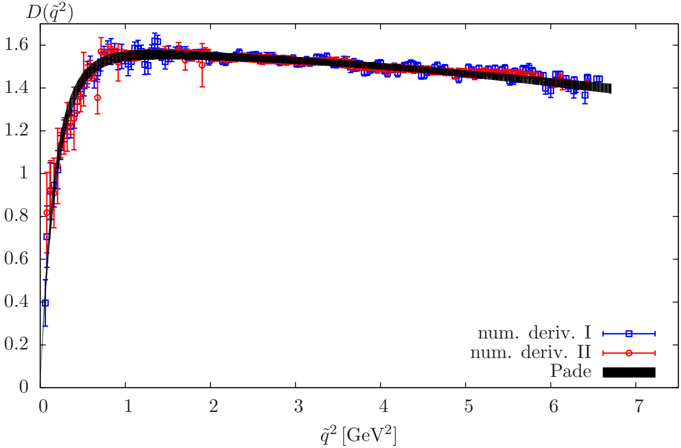
integrand is peaked at $Q^2 \sim m_\mu^2$

$m_\mu^2 \sim 0.01\,\mathrm{GeV}^2$

Comparison of the different methods

$m_\pi = 330\,\text{MeV}, a = 0.050\text{fm}$



Adler function : combined fit

Adler function:

$$D(Q^2) = -12\,\pi^2\,Q^2\,\frac{d\Pi(Q^2)}{dQ^2}$$

- fit form :

$$D(Q^2) = \mathrm{Padé}(Q^2)\ [\,1 \ + \ \mathrm{discr.} \ + \ \mathrm{mass}\,]\ ,$$

$$D(Q^2) = Q^2\left(p_0 + \frac{p_1}{\left(p_2 + Q^2\right)^2} + \frac{p_3}{\left(p_4 + Q^2\right)^2}\right) \times \left[1 + \left(d_1\,a^n + d_2\,(aQ)^n\right) + \left(\frac{c_1}{c_2 + Q^2}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right)\right]\ .$$

$$n = \{1, 2\}$$

- consider 10 ensembles with different a, M_{PS}
- consider VPF with u, d and s_Q [i.e. partial quenching of s]

Light-quark mass dependence

Adler function:

$$D(Q^2) = -12\,\pi^2\,Q^2\,\frac{d\Pi(Q^2)}{dQ^2}$$
$$= c_1 + s(Q^2)\,M_{\mathrm{PS}}^2$$