HADRONIC LIGHT-BY-LIGHT CONTRIBUTION: THE (RESONANCE) LAGRANGIAN APPROACH

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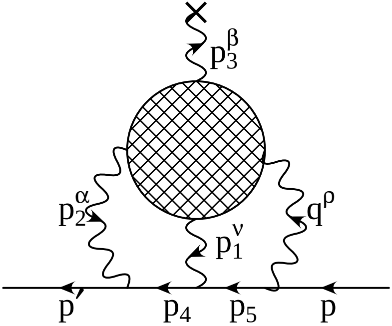
Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction, Mainz 1-5 April 2014

Overview

1. General
2. π^0-exchange
3. π-loop: new stuff is here

HLbL: the main object to calculate



- Muon line and photons: well known
- The blob: fill in with hadrons/QCD
- Trouble: low and high energy very mixed
- Double counting needs to be avoided: hadron exchanges versus quarks

The overall

$$a_\mu^{\rm HLbL} = \frac{-1}{48 m_\mu} {\rm tr}[(\not{p} + m_\mu) M^{\lambda\beta}(0)\,(\not{p} + m_\mu)[\gamma_\lambda, \gamma_\beta]]\,.$$

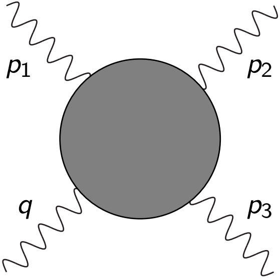
$$M^{\lambda\beta}(p_3) = |e|^6 \int \frac{{\rm d}^4 p_1}{(2\pi)^4} \int \frac{{\rm d}^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m_\mu^2)\,(p_5^2 - m_\mu^2)}$$
$$\times \left[\frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right] \gamma_\alpha (\not{p}_4 + m_\mu) \gamma_\nu (\not{p}_5 + m_\mu) \gamma_\rho\,.$$

- We used: $\Pi^{\rho\nu\alpha\lambda}(p_1, p_2, p_3) = -p_{3\beta} \frac{\delta \Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\,.$
- Can calculate at $p_3 = 0$ but must take derivative
- derivative: improves convergence
- Four point function of $V_i^\mu(x) \equiv \sum_i Q_i\ [\bar{q}_i(x)\gamma^\mu q_i(x)]$

$$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) \equiv i^3 \int {\rm d}^4 x \int {\rm d}^4 y \int {\rm d}^4 z\, e^{i(p_1\cdot x + p_2 \cdot y + p_3 \cdot z)} \times$$
$$\langle 0 | T \left(V_a^\rho(0) V_b^\nu(x) V_c^\alpha(y) V_d^\beta(z) \right) |0\rangle$$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) \quad =$



Actually we really need $\left.\dfrac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$

General properties

$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$:

- In general 138 Lorentz structures (but only 28 contribute to $g-2$)
- Using $q_\rho \Pi^{\rho\nu\alpha\beta} = p_{1\nu}\Pi^{\rho\nu\alpha\beta} = p_{2\alpha}\Pi^{\rho\nu\alpha\beta} = p_{3\beta}\Pi^{\rho\nu\alpha\beta} = 0$ 43 gauge invariant structures
- Bose symmetry relates some of them
- All depend on p_1^2, p_2^2 and q^2, but before derivative and $p_3 \to 0$ also $p_3^2, p_1 \cdot p_2, p_1 \cdot p_3$
- Compare HVP: one function, one variable
- General calculation from experiment difficult; but see other contributions at this workshop
- In four photon measurement: lepton contribution

General properties

$\int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4}$ plus loops inside the hadronic part

- 8 dimensional integral, three trivial,
- 5 remain: $p_1^2, p_2^2, p_1 \cdot p_2, p_1 \cdot p_\mu, p_2 \cdot p_\mu$
- Rotate to Euclidean space:
  - Easier separation of long and short-distance
  - Artefacts (confinement) in models smeared out.
- More recent: can do two more using Gegenbauer techniques Knecht-Nyffeler, Jegerlehner-Nyffeler,JB–Zahiri-Abyaneh–Relefors
- P_1^2, P_2^2 and Q^2 remain
- study $a_\mu^{\mathrm{X}} = \int dl_{P_1} dl_{P_2} a_\mu^{\mathrm{XLL}} = \int dl_{P_1} dl_{P_2} dl_Q a_\mu^{\mathrm{XLLQ}}$ $l_P = \ln\left(P/\mathrm{GeV}\right)$, to see where the contributions are
- Study the dependence on the cut-off for the photons

Gegenbauer

- $P_1,\ P_2,\ Q = P_1 + P_2,\ P_4 = P_\mu - P_1,\ P_5 = P_\mu + P_2$
- Average over muon-direction using:

$$\left\langle \frac{1}{(P_4^2+m^2)(P_5^2+m^2)} \right\rangle_\mu = \delta X\,, \qquad \left\langle \frac{P\cdot P_1}{P_5^2+m^2} \right\rangle_\mu = \frac{1}{8}\,\delta P_1\cdot P_2 r_2^2\,,$$

$$\left\langle \frac{P\cdot P_2}{P_4^2+m^2} \right\rangle_\mu = \frac{1}{8}\,\delta P_1\cdot P_2 r_2^2\,, \qquad \left\langle \frac{1}{P_4^2+m^2} \right\rangle_\mu = \frac{1}{2}\,\delta r_1\,,$$

$$\left\langle \frac{1}{P_5^2+m^2} \right\rangle_\mu = \frac{1}{2}\,\delta r_2\,.$$

$$\delta = \frac{1}{m^2}\,, \qquad r_i = 1 - \sqrt{1+\frac{4m^2}{P_i^2}}\,, \qquad X = \frac{1}{P_1 P_2 \sin\theta}\,\mathrm{atan}\left(\frac{z\sin\theta}{1-z\cos\theta}\right)$$

$$\cos\theta = \frac{P_1\cdot P_2}{P_1 P_2}\,, \qquad z = \frac{P_1 P_2}{4m^2} r_1 r_2\,. \qquad \rho_1 = P_1^2,\quad \rho_2 = P_2^2 \quad \rho_3 = P_1\cdot P_2.$$

Gegenbauer

$$\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3) \equiv$$
$$\Pi^1(p_1,p_2,p_3)g^{\rho\nu}g^{\alpha\beta} + \Pi^2(p_1,p_2,p_3)g^{\rho\alpha}g^{\nu\beta} + \Pi^3(p_1,p_2,p_3)g^{\rho\beta}g^{\nu\alpha}$$
$$+ \Pi^{1jk}(p_1,p_2,p_3)g^{\rho\nu}p_j^\alpha p_k^\beta + \Pi^{2jk}(p_1,p_2,p_3)g^{\rho\alpha}p_j^\nu p_k^\beta$$
$$+ \Pi^{3jk}(p_1,p_2,p_3)g^{\rho\beta}p_j^\nu p_k^\alpha + \Pi^{4jk}(p_1,p_2,p_3)g^{\nu\alpha}p_j^\rho p_k^\beta$$
$$+ \Pi^{5jk}(p_1,p_2,p_3)g^{\nu\beta}p_j^\rho p_k^\alpha + \Pi^{6jk}(p_1,p_2,p_3)g^{\alpha\beta}p_j^\rho p_k^\nu$$
$$+ \Pi^{ijkm}(p_1,p_2,p_3)p_i^\rho p_j^\nu p_k^\beta p_m^\alpha$$

- Use Ward identities to rewrite in the $\Pi^{ijkm}(p_1,p_2,p_3)$
- is redundant (81 rather than 43), but easiest to implement and can be done without negative powers of momenta
- $\delta/\delta p_{3\lambda}$, $p_3 \to 0$
- $\Pi^{3jkm}(p_1,p_2,p_3)$, $\Pi^{i3km}(p_1,p_2,p_3)$, $\Pi^{ij3m}(p_1,p_2,p_3)$ $(\delta/\delta p_{3\lambda})(\Pi^{ijk1}(p_1,p_2,p_3) - \Pi^{ijk2}(p_1,p_2,p_3))$: 32 left
- $a_\mu = \dfrac{\alpha^3}{2\pi^2}\int P_1^2 dP_1^2 P_2^2 dP_2^2 \sin\theta\, d\cos\theta\, A_\Pi(P_1,P_2,\cos\theta)$

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$A_\Pi(P_1, P_2, \cos\theta)$

$$
\begin{aligned}
&\Pi^{1131} && (-1/6\, \rho_3^2 r_2^2 \delta - 2/3\, \rho_1\rho_3 r_2\delta + 8/3\, \rho_1\rho_3 X - \rho_1^2 r_1\delta - 4/3\, \rho_1^2\rho_3 X\delta - 2\, \rho_1^2\rho_2 X\delta)\\
+&\Pi^{1132} && (+2/3\rho_3 + 1/3\rho_2\rho_3 r_2\delta - 1/6\rho_2\rho_3 r_2^2\delta - 2/3\rho_1\rho_3 r_1\delta - 1/6\rho_1\rho_3 r_1^2\delta - 2/3\rho_1\rho_2 r_2\delta\\
& && +1/3\rho_1\rho_2 r_1\delta + 8/3\rho_1\rho_2 X - 4/3\rho_1\rho_2\rho_3 X\delta + 2/3\rho_1\rho_2^2 X\delta - 4/3\rho_1^2\rho_2 X\delta)\\
+&\Pi^{1231} && (-2/3\rho_3^2 r_2\delta - 1/6\rho_2\rho_3 r_2^2\delta - 2/3\rho_1\rho_3 r_1\delta - 4/3\rho_1\rho_3^2 X\delta + 1/3\rho_1\rho_2 r_2\delta\\
& && +8/3\rho_1\rho_2 X - 4/3\rho_1\rho_2\rho_3 X\delta + 2/3\rho_1^2\rho_2 X\delta)\\
+&\Pi^{1232} && (-2/3\rho_3^2 r_1\delta - 2/3\rho_2 - 2/3\rho_2\rho_3 r_2\delta + 8/3\rho_2\rho_3 X - 4/3\rho_2\rho_3^2 X\delta - 1/3\rho_2^2 r_2\delta\\
& && -1/3\rho_1\rho_2 r_1\delta - 4/3\rho_1\rho_2\rho_3 X\delta - 2/3\rho_1\rho_2^2 X\delta)\\
+&\Pi^{1311} && (+1/3\rho_1\rho_3 r_2\delta + 1/3\rho_1^2 r_1\delta + 2/3\rho_1^2\rho_3 X\delta + 2/3\rho_1^2\rho_2 X\delta)\\
+&\Pi^{1312} && (-2/3\rho_3^2 r_2\delta + 4/3\rho_3^2 X - 1/12\rho_2\rho_3 r_2^2\delta - 4/3\rho_1\rho_3 r_1\delta - 1/12\rho_1\rho_3 r_1^2\delta\\
& && -4/3\rho_1\rho_3^2 X\delta + 1/2\rho_1\rho_2 r_2\delta + 1/6\rho_1\rho_2 r_1\delta + 4/3\rho_1\rho_2 X - 8/3\rho_1\rho_2\rho_3 X\delta + 1/3\rho_1\rho_2^2 X\delta + \rho_1^2\rho_2 X\delta)\\
+&\Pi^{1322} && (-2/3\rho_2 - 2/3\rho_2\rho_3 r_2\delta + 8/3\rho_2\rho_3 X - 1/3\rho_2^2 r_2\delta - 2\rho_1\rho_2 r_1\delta - 4/3\rho_1\rho_2\rho_3 X\delta - 4\rho_1\rho_2^2 X\delta)\\
+&\Pi^{2131} && (-2/3\rho_1 - 2/3\rho_1\rho_3 r_1\delta + 8/3\rho_1\rho_3 X - 2\rho_1\rho_2 r_2\delta - 4/3\rho_1\rho_2\rho_3 X\delta - 1/3\rho_1^2 r_1\delta - 4\rho_1^2\rho_2 X\delta)\\
+&\Pi^{2231} && (-2/3\rho_3^2 r_1\delta + 4/3\rho_3^2 X - 4/3\rho_2\rho_3 r_2\delta - 1/12\rho_2\rho_3 r_2^2\delta - 4/3\rho_2\rho_3^2 X\delta - 1/12\rho_1\rho_3 r_1^2\delta\\
& && +1/6\rho_1\rho_2 r_2\delta + 1/2\rho_1\rho_2 r_1\delta + 4/3\rho_1\rho_2 X - 8/3\rho_1\rho_2\rho_3 X\delta + \rho_1\rho_2^2 X\delta + 1/3\rho_1^2\rho_2 X\delta)\\
+&\Pi^{2232} && (+1/3\rho_2\rho_3 r_1\delta + 1/3\rho_2^2 r_2\delta + 2/3\rho_2^2\rho_3 X\delta + 2/3\rho_1\rho_2^2 X\delta)\\
+&\Pi^{2311} && (-2/3\rho_3^2 r_2\delta - 2/3\rho_1 - 2/3\rho_1\rho_3 r_1\delta + 8/3\rho_1\rho_3 X - 4/3\rho_1\rho_3^2 X\delta - 1/3\rho_1\rho_2 r_2\delta\\
& && -4/3\rho_1\rho_2\rho_3 X\delta - 1/3\rho_1^2 r_1\delta - 2/3\rho_1^2\rho_2 X\delta)\\
+&\Pi^{2312} && (-2/3\rho_3^2 r_1\delta - 2/3\rho_2\rho_3 r_2\delta - 4/3\rho_2\rho_3^2 X\delta - 1/6\rho_1\rho_3 r_1^2\delta + 1/3\rho_1\rho_2 r_1\delta + 8/3\rho_1\rho_2 X\\
& && -4/3\rho_1\rho_2\rho_3 X\delta + 2/3\rho_1\rho_2^2 X\delta)
\end{aligned}
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\begin{aligned}
&+\Pi^{2321}\,(+2/3\rho_3 - 2/3\rho_2\rho_3 r_2\delta - 1/6\rho_2\rho_3 r_2^2\delta + 1/3\rho_1\rho_3 r_1\delta - 1/6\rho_1\rho_3 r_1^2\delta + 1/3\rho_1\rho_2 r_2\delta \\
&\qquad -2/3\rho_1\rho_2 r_1\delta + 8/3\rho_1\rho_2 X - 4/3\rho_1\rho_2\rho_3 X\delta - 4/3\rho_1\rho_2^2 X\delta + 2/3\rho_1^2\rho_2 X\delta) \\
&+\Pi^{2322}\,(-1/6\rho_3^2 r_1^2\delta - 2/3\rho_2\rho_3 r_1\delta + 8/3\rho_2\rho_3 X - \rho_2^2 r_2\delta - 4/3\rho_2^2\rho_3 X\delta - 2\rho_1\rho_2^2 X\delta) \\
&+\Pi^{3111}\,(+1/6\rho_3^2 r_2^2\delta - 2/3\rho_1 - 4/3\rho_1\rho_3 r_2\delta + 1/2\rho_1\rho_3 r_2^2\delta - 1/3\rho_1\rho_2 r_2\delta - \rho_1^2 r_2\delta \\
&\qquad -1/3\rho_1^2 r_1\delta - 8/3\rho_1^2\rho_3 X\delta - 2/3\rho_1^2\rho_2 X\delta - 2\rho_1^3 X\delta) \\
&+\Pi^{3112}\,(+4/3\rho_3 + 2/3\rho_2\rho_3 r_2\delta + 1/6\rho_2\rho_3 r_2^2\delta + 2/3\rho_1 + 2/3\rho_1\rho_3 r_1\delta - 1/3\rho_1\rho_3 r_1^2\delta \\
&\qquad -8/3\rho_1\rho_3 X + 2/3\rho_1\rho_2 r_1\delta - 8/3\rho_1\rho_2 X + 4/3\rho_1\rho_2\rho_3 X\delta + 4/3\rho_1\rho_2^2 X\delta + 1/3\rho_1^2 r_1\delta) \\
&+\Pi^{3121}\,(+2\rho_1 + \rho_1^2 r_1\delta) + \Pi^{3122}(+2\rho_2 + \rho_2^2 r_2\delta) \\
&+\Pi^{3211}\,(+4/3\rho_3 - 8/3\rho_3^2 X + 2/3\rho_2\rho_3 r_2\delta + 2/3\rho_1 + 2/3\rho_1\rho_3 r_1\delta - 1/6\rho_1\rho_3 r_1^2\delta \\
&\qquad -8/3\rho_1\rho_3 X + 1/3\rho_1\rho_2 r_2\delta + 1/3\rho_1\rho_2 r_1\delta + 4/3\rho_1\rho_2\rho_3 X\delta + 2/3\rho_1\rho_2^2 X\delta + 1/3\rho_1^2 r_1\delta + 2/3\rho_1^2\rho_2 X\delta) \\
&+\Pi^{3212}\,(+4/3\rho_3 - 8/3\rho_3^2 X + 2/3\rho_2 + 2/3\rho_2\rho_3 r_2\delta - 1/6\rho_2\rho_3 r_2^2\delta - 8/3\rho_2\rho_3 X \\
&\qquad +1/3\rho_2^2 r_2\delta + 2/3\rho_1\rho_3 r_1\delta + 1/3\rho_1\rho_2 r_2\delta + 1/3\rho_1\rho_2 r_1\delta + 4/3\rho_1\rho_2\rho_3 X\delta + 2/3\rho_1\rho_2^2 X\delta + 2/3\rho_1^2\rho_2 X\delta) \\
&+\Pi^{3221}\,(+4/3\rho_3 + 2/3\rho_2 + 2/3\rho_2\rho_3 r_2\delta - 1/3\rho_2\rho_3 r_2^2\delta - 8/3\rho_2\rho_3 X + 1/3\rho_2^2 r_2\delta + 2/3\rho_1\rho_3 r_1\delta \\
&\qquad +1/6\rho_1\rho_3 r_1^2\delta + 2/3\rho_1\rho_2 r_2\delta - 8/3\rho_1\rho_2 X + 4/3\rho_1\rho_2\rho_3 X\delta + 4/3\rho_1^2\rho_2 X\delta) \\
&+\Pi^{3222}\,(+1/6\rho_3^2 r_1^2\delta - 2/3\rho_2 - 4/3\rho_2\rho_3 r_1\delta + 1/2\rho_2\rho_3 r_1^2\delta - 1/3\rho_2^2 r_2\delta - \rho_2^2 r_1\delta - 8/3\rho_2^2\rho_3 X\delta \\
&\qquad -2\rho_2^3 X\delta - 1/3\rho_1\rho_2 r_1\delta - 2/3\rho_1\rho_2^2 X\delta) \\
&+\Pi^{D111}\,(-1/3\rho_1\rho_3 + 2/3\rho_1\rho_3^2 X - 1/6\rho_1\rho_2\rho_3 r_2\delta + 1/24\rho_1\rho_2\rho_3 r_2^2\delta - 1/6\rho_1^2\rho_3 r_1\delta \\
&\qquad +1/24\rho_1^2\rho_3 r_1^2\delta - 1/12\rho_1^2\rho_2 r_2\delta - 1/12\rho_1^2\rho_2 r_1\delta - 2/3\rho_1^2\rho_2 X - 1/3\rho_1^2\rho_2\rho_3 X\delta \\
&\qquad -1/6\rho_1^2\rho_2^2 X\delta - 1/6\rho_1^3\rho_2 X\delta)
\end{aligned}
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\begin{aligned}
&+\Pi^{D121}\,(+1/3\rho_3^2 - 2/3\rho_3^3X + 1/6\rho_2\rho_3^2r_2\delta - 1/24\rho_2\rho_3^2r_2^2\delta + 1/6\rho_1\rho_3^2r_1\delta - 1/24\rho_1\rho_3^2r_1^2\delta\\
&\qquad +1/12\rho_1\rho_2\rho_3r_2\delta + 1/12\rho_1\rho_2\rho_3r_1\delta + 2/3\rho_1\rho_2\rho_3X + 1/3\rho_1\rho_2\rho_3^2X\delta + 1/6\rho_1\rho_2^2\rho_3X\delta\\
&\qquad +1/6\rho_1^2\rho_2\rho_3X\delta)\\
&+\Pi^{D122}\,(+2/3\rho_2\rho_3 - 4/3\rho_2\rho_3^2X + 1/3\rho_2^2\rho_3r_2\delta - 1/12\rho_2^2\rho_3r_2^2\delta + 1/3\rho_1\rho_2\rho_3r_1\delta\\
&\qquad -1/12\rho_1\rho_2\rho_3r_1^2\delta + 1/6\rho_1\rho_2^2r_2\delta + 1/6\rho_1\rho_2^2r_1\delta + 4/3\rho_1\rho_2^2X + 2/3\rho_1\rho_2^2\rho_3X\delta\\
&\qquad +1/3\rho_1\rho_2^3X\delta + 1/3\rho_1^2\rho_2^2X\delta)\\
&+\Pi^{D211}\,(-2/3\rho_1\rho_3 + 4/3\rho_1\rho_3^2X - 1/3\rho_1\rho_2\rho_3r_2\delta\\
&\qquad +1/12\rho_1\rho_2\rho_3r_2^2\delta - 1/3\rho_1^2\rho_3r_1\delta + 1/12\rho_1^2\rho_3r_1^2\delta - 1/6\rho_1^2\rho_2r_2\delta - 1/6\rho_1^2\rho_2r_1\delta\\
&\qquad -4/3\rho_1^2\rho_2X - 2/3\rho_1^2\rho_2\rho_3X\delta - 1/3\rho_1^2\rho_2^2X\delta - 1/3\rho_1^3\rho_2X\delta)\\
&+\Pi^{D221}\,(-1/3\,\rho_3^2 + 2/3\,\rho_3^3X - 1/6\,\rho_2\rho_3^2r_2\delta + 1/24\,\rho_2\rho_3^2r_2^2\delta - 1/6\,\rho_1\rho_3^2r_1\delta + 1/24\,\rho_1\rho_3^2r_1^2\delta\\
&\qquad -1/12\,\rho_1\rho_2\rho_3r_2\delta - 1/12\,\rho_1\rho_2\rho_3r_1\delta - 2/3\,\rho_1\rho_2\rho_3X - 1/3\,\rho_1\rho_2\rho_3^2X\delta\\
&\qquad -1/6\,\rho_1\rho_2^2\rho_3X\delta - 1/6\,\rho_1^2\rho_2\rho_3X\delta)\\
&+\Pi^{D222}\,(+1/3\,\rho_2\rho_3 - 2/3\,\rho_2\rho_3^2X + 1/6\,\rho_2^2\rho_3r_2\delta - 1/24\,\rho_2^2\rho_3r_2^2\delta + 1/6\,\rho_1\rho_2\rho_3r_1\delta\\
&\qquad -1/24\,\rho_1\rho_2\rho_3r_1^2\delta + 1/12\,\rho_1\rho_2^2r_2\delta + 1/12\,\rho_1\rho_2^2r_1\delta + 2/3\,\rho_1\rho_2^2X + 1/3\,\rho_1\rho_2^2\rho_3X\delta\\
&\qquad +1/6\,\rho_1\rho_2^3X\delta + 1/6\,\rho_1^2\rho_2^2X\delta)\,.
\end{aligned}
$$

- "Only" 28 contribute
- Full formula fairly "short"

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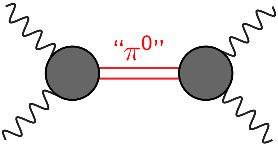
π^0 exchange



- "π^0" $= 1/(p^2 - m_\pi^2)$
- The blobs need to be modelled, and in e.g. ENJL contain corrections also to the $1/(p^2 - m_\pi^2)$
- Pointlike has a logarithmic divergence
- Numbers π^0, but also η, η'

π^0 exchange

- BPP: $a_\mu^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- Nonlocal quark model: $a_\mu^{\pi^0} = 6.27 \times 10^{-10}$

  A. E. Dorokhov, W. Broniowski, Phys.Rev.**D78** (2008)073011. [0805.0760]
- DSE model: $a_\mu^{\pi^0} = 5.75 \times 10^{-10}$

  Goecke, Fischer and Williams, Phys.Rev.D**83**(2011)094006[1012.3886]
- LMD+V: $a_\mu^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$

  M. Knecht, A. Nyffeler, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Formfactor inspired by AdS/QCD: $a_\mu^{\pi^0} = 6.54 \times 10^{-10}$

  Cappiello, Cata and D'Ambrosio, Phys.Rev.D**83**(2011)093006 [1009.1161]
- Chiral Quark Model: $a_\mu^{\pi^0} = 6.8 \times 10^{-10}$

  D. Greynat and E. de Rafael, JHEP **1207** (2012) 020 [1204.3029].
- Constraint via magnetic susceptibility: $a_\mu^{\pi^0} = 7.2 \times 10^{-10}$

  A. Nyffeler, Phys. Rev. D **79** (2009) 073012 [0901.1172].
- All in reasonable agreement

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π^0 exchange: most recent addition

- Kampf Novotny 1104.3137, Roig, Guevara, López Castro, 1401.4099
- RχT: study VVP Green function, $e^+e^- \to \omega\pi^0$ and $\pi\gamma^*\gamma$ tgransition form-factor
- VVP, $V\gamma P$ vertices.
- Lagrangians Kampf Novotny 1104.3137 Roig Sanz-Cillero 1312.6206
- Small violation of Brodsky-Lepage in $\pi\gamma^*\gamma$
- Include vector and pseudo-scalar nonet
- Short distance constraints require $F_V = \sqrt{3}F$ (KSRF $\sqrt{2}$)

$$F_V = \sqrt{3}F\,, \quad c_{125} = 0\,, \quad c_{1256} = -\frac{N_C M_V}{32\sqrt{2}\pi^2 F_V} \sim -3.26\cdot 10^{-2}\,, \quad c_{1235} = 0\,,$$

$$d_{123} = \frac{F^2}{8F_V^2} = \frac{1}{24}\,, \quad d_3 = -\frac{N_C M_V^2}{64\pi^2 F_V^2} \sim -0.112\,, \quad d_s = \frac{\sqrt{2}M_V c_{1256} - 2d_3 F_V}{F_{V_1}} = 0\,,$$

- Note short-distance matching must be done in many channels, JB,Gamiz,Lipartia,Prades, hep-ph/0304222: with finite number of resonances this requires compromises

π^0 exchange: most recent addition

$$\mathcal{F}_{\pi^0\gamma\gamma}(p^2,q^2,0) = \frac{2}{3F}\left[-\frac{N_C}{8\pi^2}+\frac{4F_V^2 d_3(p^2+q^2)}{(M_V^2-p^2)(M_V^2-q^2)}+2\sqrt{2}\frac{F_V}{M_V}\frac{p^2c_{1256}-q^2c_{125}}{M_V^2-p^2}\right.$$
$$\left.+2\sqrt{2}\frac{F_V}{M_V}\frac{q^2c_{1256}-p^2c_{125}}{M_V^2-q^2}\right].$$

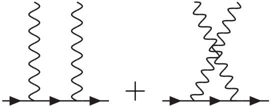
$$\mathcal{F}_{\pi^0\gamma\gamma}(p^2,q^2,r^2) = \frac{2r^2}{3F}\left[-\frac{N_C}{8\pi^2 r^2}+4F_V^2\frac{d_3(p^2+q^2)}{(M_V^2-p^2)(M_V^2-q^2)r^2}+\frac{4F_V^2 d_{123}}{(M_V^2-p^2)(M_V^2-q^2)}\right.$$
$$-2\sqrt{2}\frac{F_V}{M_V}\frac{r^2c_{1235}-p^2c_{1256}+q^2c_{125}}{(M_V^2-p^2)r^2}-2\sqrt{2}\frac{F_V}{M_V}\frac{r^2c_{1235}-q^2c_{1256}+p^2c_{125}}{(M_V^2-q^2)r^2}+\frac{64P_1}{M_P^2-r^2}$$
$$\left.-\frac{16\sqrt{2}P_2F_V}{(M_V^2-p^2)(M_P^2-r^2)}-\frac{16\sqrt{2}P_2F_V}{(M_V^2-q^2)(M_P^2-r^2)}+\frac{16F_V^2P_3}{(M_V^2-p^2)(M_V^2-q^2)(M_P^2-r^2)}\right],$$

- Kampf Novotny 1104.3137 $\quad a_\mu^{\pi^0} = (6.58 \pm 0.12)\times 10^{-10}$
- Roig, Guevara, López Castro, 1401.4099 $\quad a_\mu^{\pi^0} = (6.65 \pm 0.19)\times 10^{-10}$

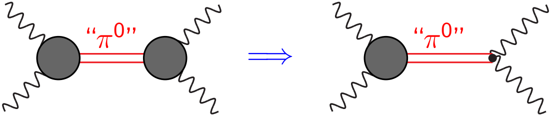
  No r^2 (i.e. pole) $\quad a_\mu^{\pi^0} = (5.75 \pm 0.05)\times 10^{-10}$

MV short-distance: π^0 exchange

- K. Melnikov, A. Vainshtein, Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment revisited, Phys. Rev. **D70** (2004) 113006. [hep-ph/0312226]
- take $P_1^2 \approx P_2^2 \gg Q^2$: Leading term in OPE of two vector currents is proportional to axial current
- $\Pi^{\rho\nu\alpha\beta} \propto \frac{P_\rho}{P_1^2} \langle 0 | T\left(J_{A\nu} J_{V\alpha} J_{V\beta}\right) | 0 \rangle$
- J_A comes from



- AVV triangle anomaly: extra info
- Implemented via setting one blob = 1



- $a_\mu^{\pi^0} = 7.7 \times 10^{-10}$

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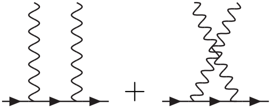
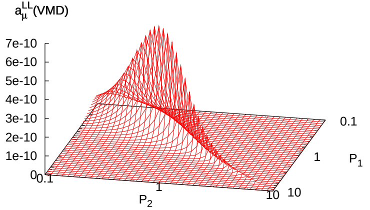
π^0 exchange

- The pointlike vertex implements shortdistance part, not only π^0-exchange
- 

  Are these part of the quark-loop? See also in
  Dorokhov,Broniowski, Phys.Rev. D78(2008)07301
- BPP quarkloop $+\ \pi^0$-exchange $\approx$ MV π^0-exchange

π^0 exchange

- Which momentum regimes important studied: JB and J. Prades, Mod. Phys. Lett. A **22** (2007) 767 [hep-ph/0702170]
- $a_\mu = \int dl_1 dl_2 a_\mu^{LL}$ with $l_i = \log(P_i/GeV)$



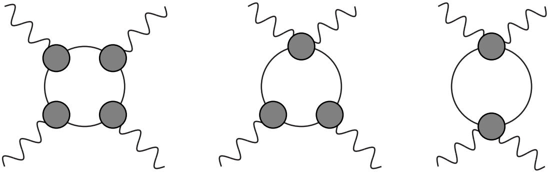
Which momentum regions do what:
volume under the plot $\propto a_\mu$

Pseudoscalar exchange

- Point-like VMD: π^0 η and η' give 5.58, 1.38, 1.04.
- Roig et al. 6.65, 2.03, 1.75
- Models that include $U(1)_A$ breaking give similar ratios
- Pure large N_c models use this ratio
- The MV argument should give some enhancement over the full VMD like models
- Total pseudo-scalar exchange is about $a_\mu^{PS} = 8 - 10 \times 10^{-10}$
- AdS/QCD estimate (includes excited pseudo-scalars) $a_\mu^{PS} = 10.7 \times 10^{-10}$

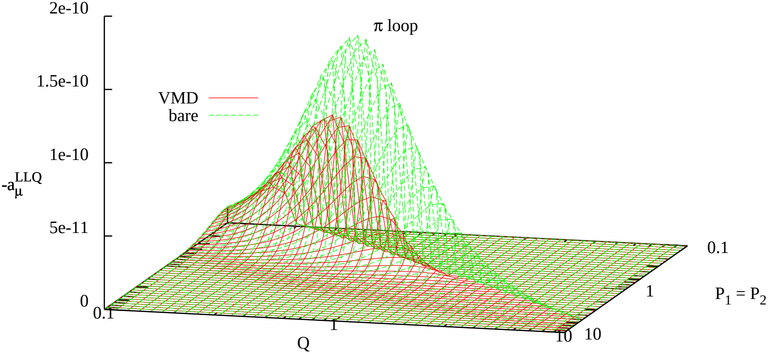
  D. K. Hong and D. Kim, Phys. Lett. B **680** (2009) 480 [0904.4042]

π-loop



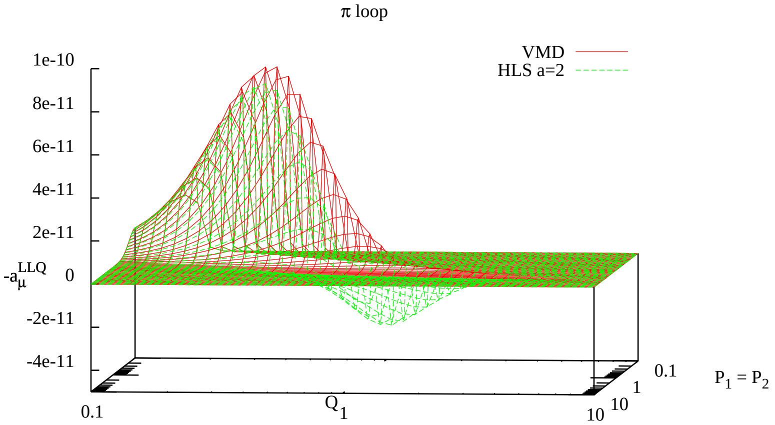
- A bare π-loop (sQED) give about $-4 \cdot 10^{-10}$
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model: only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - The HLS model used has problems with π^+-π^0 mass difference (due to not having an a_1)
- Final numbers quite different: -0.45 and -1.9 $(\times 10^{-10})$
- For BPP stopped at 1 GeV but within 10% of higher Λ

π loop: Bare vs VMD



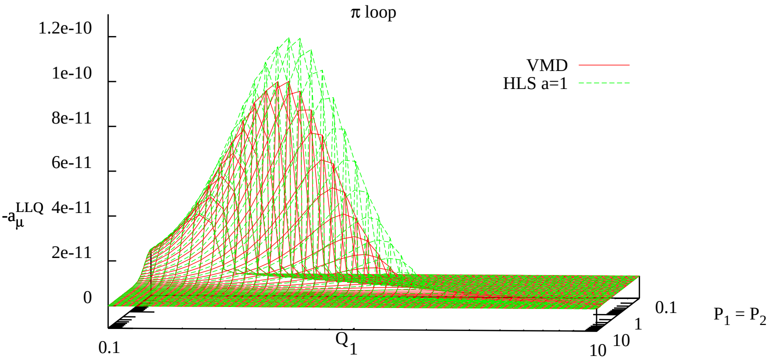
- plotted a_μ^{LLQ} for $P_1 = P_2$
- $a_\mu = \int dl_{P_1} dl_{P_2} dl_Q \, a_\mu^{LLQ}$
- $l_Q = \log(Q/1 \text{ GeV})$

π loop: VMD vs HLS



Usual HLS, $a = 2$

π loop: VMD vs HLS



HLS with $a = 1$, satisfies more short-distance constraints

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π loop

- $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
- HLS does not satisfy it
- full VMD does: so probably better estimate
- Ramsey-Musolf suggested to do pure ChPT for the π loop K. T. Engel, H. H. Patel and M. J. Ramsey-Musolf, "Hadronic light-by-light scattering and the pion polarizability," Phys. Rev. D **86** (2012) 037502 [arXiv:1201.0809 [hep-ph]].
- Later added a_1 Engel and Ramsey-Musolf, arXiv:1309.2225

- Polarizability ($L_9 + L_{10}$) up to 10%, charge radius 30% at low energies
- Both HLS and VMD have charge radius effect but not polarizability

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π loop: L_9, L_{10}

- ChPT for muon $g-2$ at order p^6 is not powercounting finite so no prediction for a_μ exists.
- But can be used to study the low momentum end of the integral over P_1, P_2, Q
- The four-photon amplitude is finite still at two-loop order (counterterms start at order p^8)
- Add L_9 and L_{10} vertices to the bare pion loop JB-Zahiri-Abyaneh
- Program the Euler-Heisenberg plus NLO result of Ramsey-Musolf et al. into our programs for a_μ
- Bare pion-loop and L_9, L_{10} part in limit $p_1, p_2, q \ll m_\pi$ agree with Euler-Heisenberg plus next order analytically

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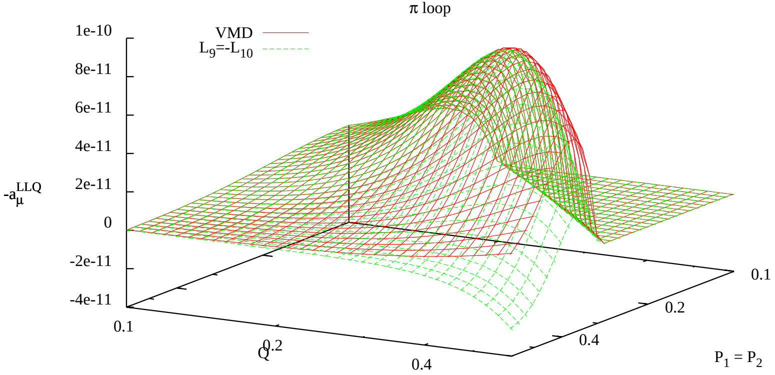
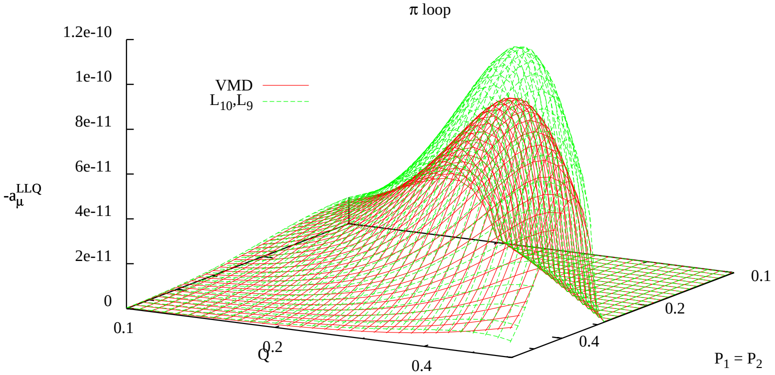
π loop: VMD vs charge radius



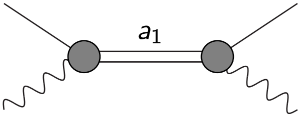
low scale, charge radius effect well reproduced

π loop: VMD vs L_9 and L_{10}

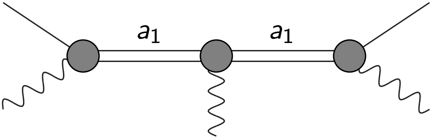


- $L_9 + L_{10} \neq 0$ gives an enhancement of 10-15%
- To do it fully need to get a model: include a_1

Include a_1

- $L_9 + L_{10}$ effect is from 

- But to get gauge invariance correctly need



Include a_1

- Consistency problem: full a_1-loop?
- Treat a_1 and ρ classical and π quantum: there must be a π that closes the loop
  Argument: integrate out ρ and a_1 classically, then do pion loops with the resulting Lagrangian
- To avoid problems: representation without a_1-π mixing
- Check for curiosity what happens if we add a_1-loop

Include a_1

- Use antisymmetric vector representation for a_1 and ρ
- Fields $A_{\mu\nu}$, $V_{\mu\nu}$ (nonets)
- Kinetic terms: $-\frac{1}{2}\left\langle\nabla^\lambda V_{\lambda\mu}\nabla_\nu V^{\nu\mu}-\frac{1}{2}V_{\mu\nu}V^{\mu\nu}\right\rangle$
$$-\frac{1}{2}\left\langle\nabla^\lambda A_{\lambda\mu}\nabla_\nu A^{\nu\mu}-\frac{1}{2}A_{\mu\nu}A^{\mu\nu}\right\rangle$$
- Terms that give contributions to the L_i^r:
$$\frac{F_V}{2\sqrt{2}}\langle f_{+\mu\nu}V^{\mu\nu}\rangle+\frac{iG_V}{\sqrt{2}}\langle V^{\mu\nu}u_\mu u_\nu\rangle+\frac{F_A}{2\sqrt{2}}\langle f_{-\mu\nu}A^{\mu\nu}\rangle$$
- $L_9=\frac{F_V G_V}{2M_V^2}$, $L_{10}=-\frac{F_V^2}{4M_V^2}+\frac{F_A^2}{4M_A^2}$
- Weinberg sum rules: (Chiral limit)
$$F_V^2=F_A^2+F_\pi^2 \qquad F_V^2M_V^2=F_A^2M_A^2$$
- VMD for $\pi\pi\gamma$: $F_VG_V=F_\pi^2$

$V_{\mu\nu}$ only

- $\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)$ is not finite
  (but was also not finite for HLS)
- But $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ also not finite
  (but was finite for HLS)
- Derivative one finite for $G_V = F_V/2$
- Surprise: $g-2$ identical to HLS with $a = \frac{F_V^2}{F_\pi^2}$
- Yes I know, different representations are identical BUT they do differ in higher order terms and even in what is higher order
- Same comments as for HLS numerics

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$V_{\mu\nu}$ and $A_{\mu\nu}$

- Add a_1
- Calculate a lot
- $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite for:
  - $G_V = F_V = 0$ and $F_A^2 = -2F_\pi^2$
  - If adding full a_1-loop $G_V = F_V = 0$ and $F_A^2 = -F_\pi^2$
- Clearly unphysical (but will show some numerics anyway)

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$V_{\mu\nu}$ and $A_{\mu\nu}$

- Start by adding $\rho a_1 \pi$ vertices
- $\lambda_1 \langle [V^{\mu\nu}, A_{\mu\nu}]\,\chi_- \rangle + \lambda_2 \langle [V^{\mu\nu}, A_{\nu\alpha}]\, h_\mu{}^\nu \rangle$
$+\lambda_3 \langle i\,[\nabla^\mu V_{\mu\nu}, A_{\nu\alpha}]\, u_\alpha \rangle + \lambda_4 \langle i\,[\nabla_\alpha V_{\mu\nu}, A_{\alpha\nu}]\, u^\mu \rangle$
$+\lambda_5 \langle i\,[\nabla^\alpha V_{\mu\nu}, A_{\mu\nu}]\, u_\alpha \rangle + \lambda_6 \langle i\,[V^{\mu\nu}, A_{\mu\nu}]\, f_-{}^\alpha{}_\nu \rangle$
$+\lambda_7 \langle i V_{\mu\nu} A^{\mu\rho} A^\nu{}_\rho \rangle$
- All lowest dimensional vertices of their respective type
- Not all independent, there are three relations
- Follow from the constraints on $V_{\mu\nu}$ and $A_{\mu\nu}$ (thanks to Stefan Leupold)

$V_{\mu\nu}$ and $A_{\mu\nu}$: big disappointment

- Work a whole lot
- $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ not obviously finite
- Work a lot more
- Prove that $\left.\frac{\delta\Pi^{\rho\nu\alpha\beta}(p_1,p_2,p_3)}{\delta p_{3\lambda}}\right|_{p_3=0}$ finite, only same solutions as before
- Try the combination that show up in $g-2$ only
- Work a lot
- Again, only same solutions as before
- Small loophole left: after the integration for $g-2$ could be finite but many funny functions of m_π, m_μ, M_V and M_A show up.

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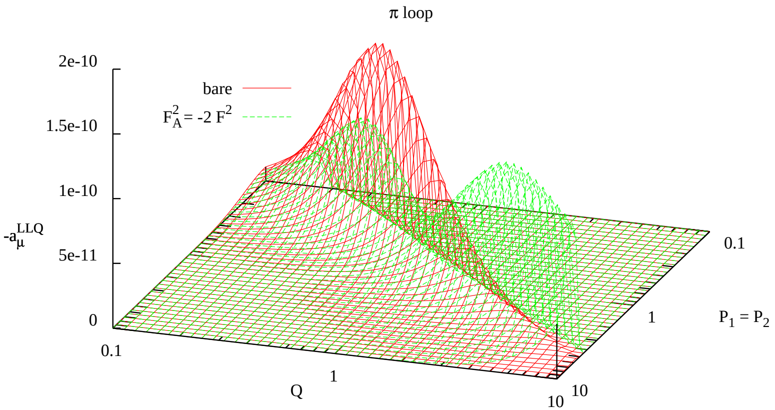
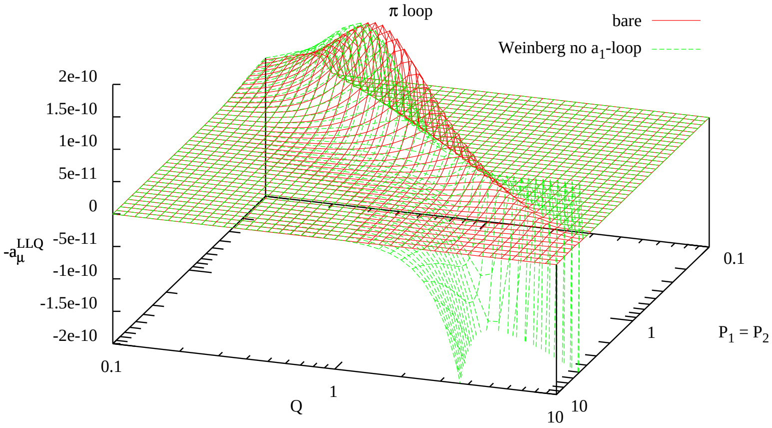
π loop: add a_1 and $F_A^2 = -2F_\pi^2$



- Lowers at low energies, $L_9 + L_{10} < 0$ here
- funny peak at a_1 mass

a_1-loop: cases with good L_9 and L_{10}



- Add F_V, G_V and F_A
- Fix values by Weinberg sum rules and VMD in $\gamma^*\pi\pi$
- no a_1-loop

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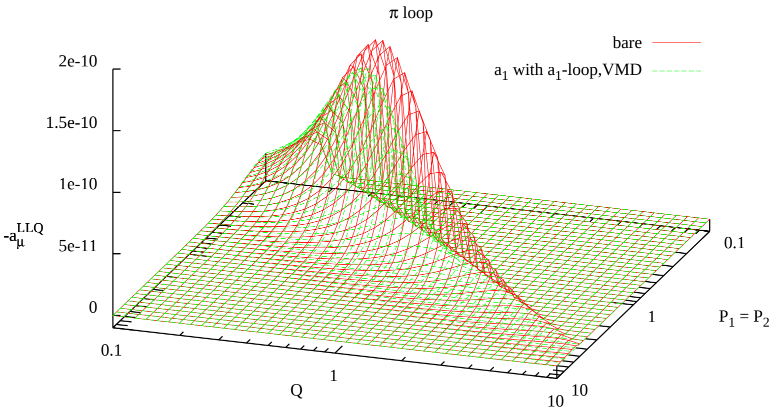
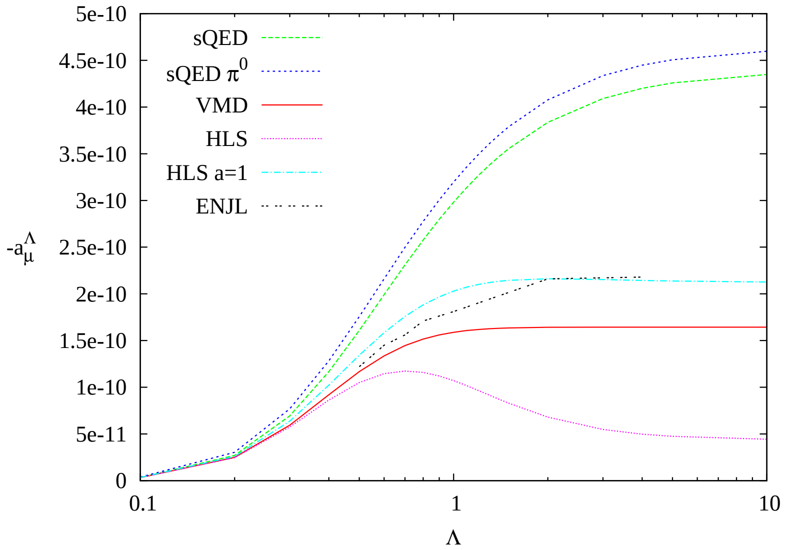
a_1-loop: cases with good L_9 and L_{10}



- Add a_1 with $F_A^2 = +F_\pi^2$ and a_1-loop
- Add the full VMD as done earlier for the bare pion loop

Integration results



$P_1, P_2, Q \leq \Lambda$

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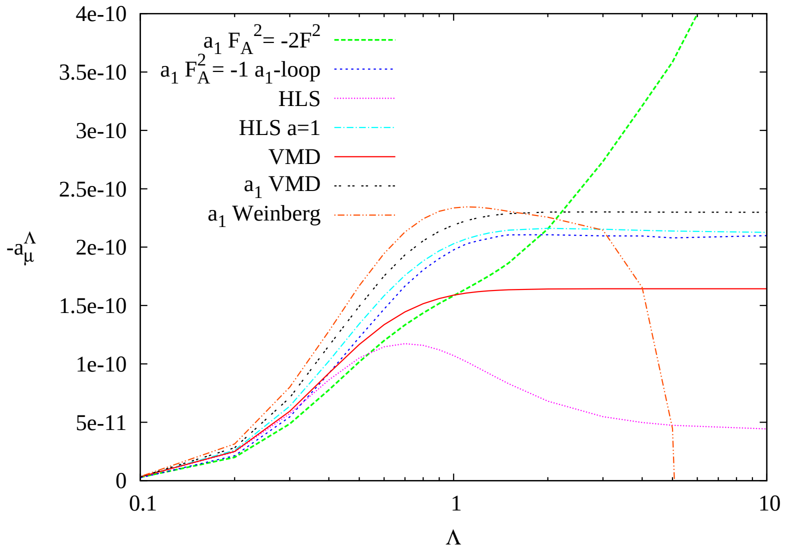

$P_1, P_2, Q \leq \Lambda$

Integration results with a_1

- Problem: get high energy behaviour good enough
- But all models with reasonable L_9 and L_{10} fall way inside the error quoted earlier $(-1.9 \pm 1.3)\ 10^{-10}$
- Tentative conclusion: Use hadrons only below about 1 GeV: $a_\mu^{\pi-\mathrm{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$
- Note that Engel and Ramsey-Musolf, arXiv:1309.2225 is a bit more pessimistic quoting numbers from $(-1.1 \text{ to } -7.1)\ 10^{-10}$

Summary: ENJL vc PdRV

	BPP	PdRV arXiv:0901.0306
quark-loop	$(2.1 \pm 0.3) \cdot 10^{-10}$	—
pseudo-scalar	$(8.5 \pm 1.3) \cdot 10^{-10}$	$(11.4 \pm 1.3) \cdot 10^{-10}$
axial-vector	$(0.25 \pm 0.1) \cdot 10^{-10}$	$(1.5 \pm 1.0) \cdot 10^{-10}$
scalar	$(-0.68 \pm 0.2) \cdot 10^{-10}$	$(-0.7 \pm 0.7) \cdot 10^{-10}$
πK-loop	$(-1.9 \pm 1.3) \cdot 10^{-10}$	$(-1.9 \pm 1.9) \cdot 10^{-10}$
errors	linearly	quadratically
sum	$(8.3 \pm 3.2) \cdot 10^{-10}$	$(10.5 \pm 2.6) \cdot 10^{-10}$

But now with a smaller error on the π-loop