Hadronic Light-by-Light: What can Lattice QCD achieve ?

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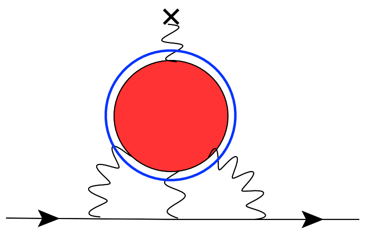
Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction, April 02, 2014, Waldthausen Castle near

HVP like approach on lattice ?



- Calculate 4pt of EM currents

$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\, d^4x_2\, d^4x_3\, \exp[-i(k_1\cdot x_1 + k_2\cdot x_2 + k_3\cdot x_3)] \times \underline{\langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle}$$

- One needs to calc. or fit all (q, k1,k2,k3) combination

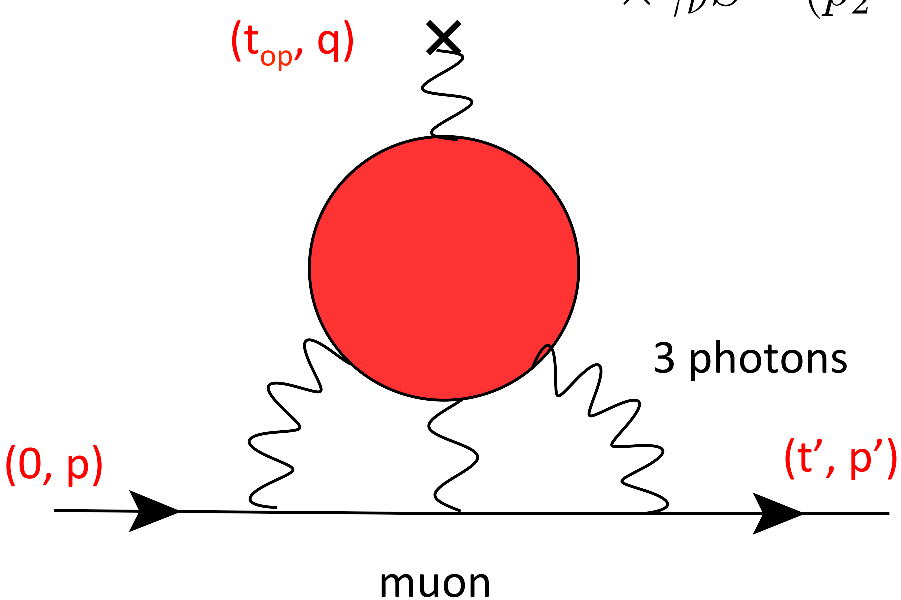
$$\Gamma^{(\mathrm{Hlbl})}_\mu(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\, k_2^2\, k_3^2} \times \gamma_\nu S^{(\mu)}(p_2+k_2)\gamma_\rho S^{(\mu)}(p_1+k_1)\gamma_\sigma$$

- Need to repeat (Volume)^3 times !

Our strategy

- Muon on lattice, photon on lattice, and let lattice calculate the form factor

$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times \gamma_\nu S^{(\mu)}(p_2+k_2)\gamma_\rho S^{(\mu)}(p_1+k_1)\gamma_\sigma$$



- set spacial momentum for
  - external EM vertex q
  - in- and out- muon p, p'

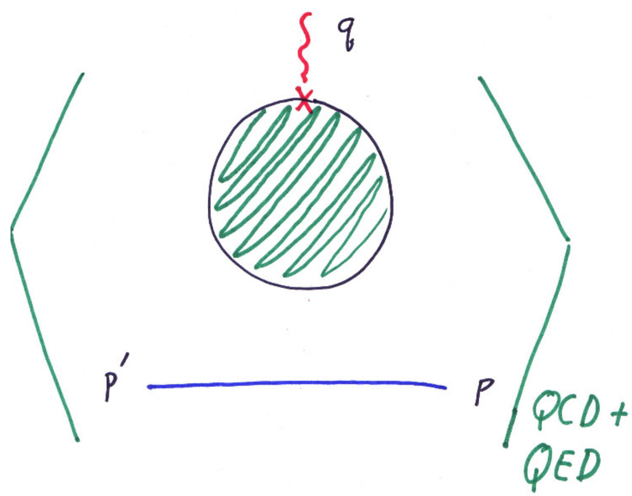
  q = p-p'

- set time slice of muon source(t=0), sink(t') and operator (t_{op})

- take large time separation for ground state matrix element

HLbL from Lattice : QCD+QED on lattice

- A naïve calculation

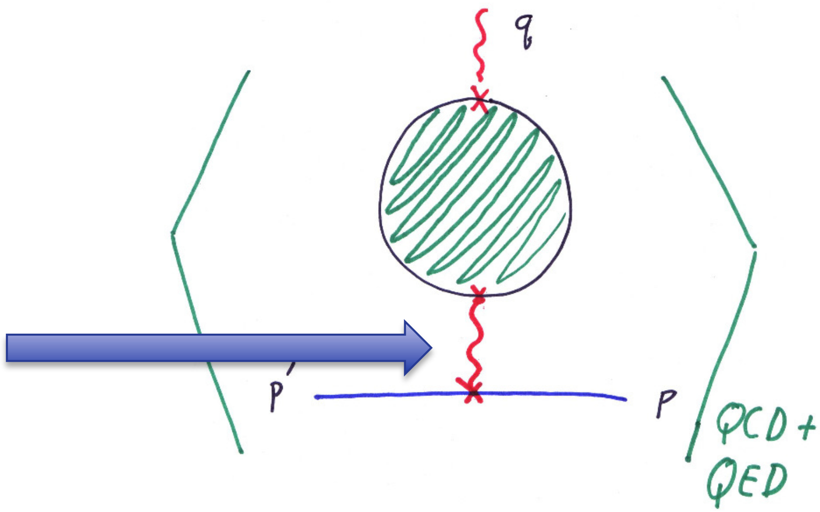


- First problem:

3 photo makes statistical fluctuation larger
→ treat one photon propagator out of three exact "analytic photon"

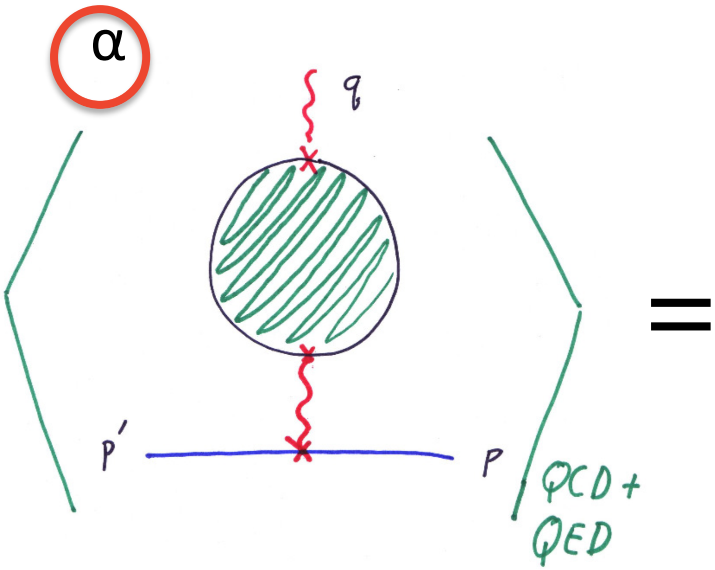
HLbL on Lattice : Analytic photon

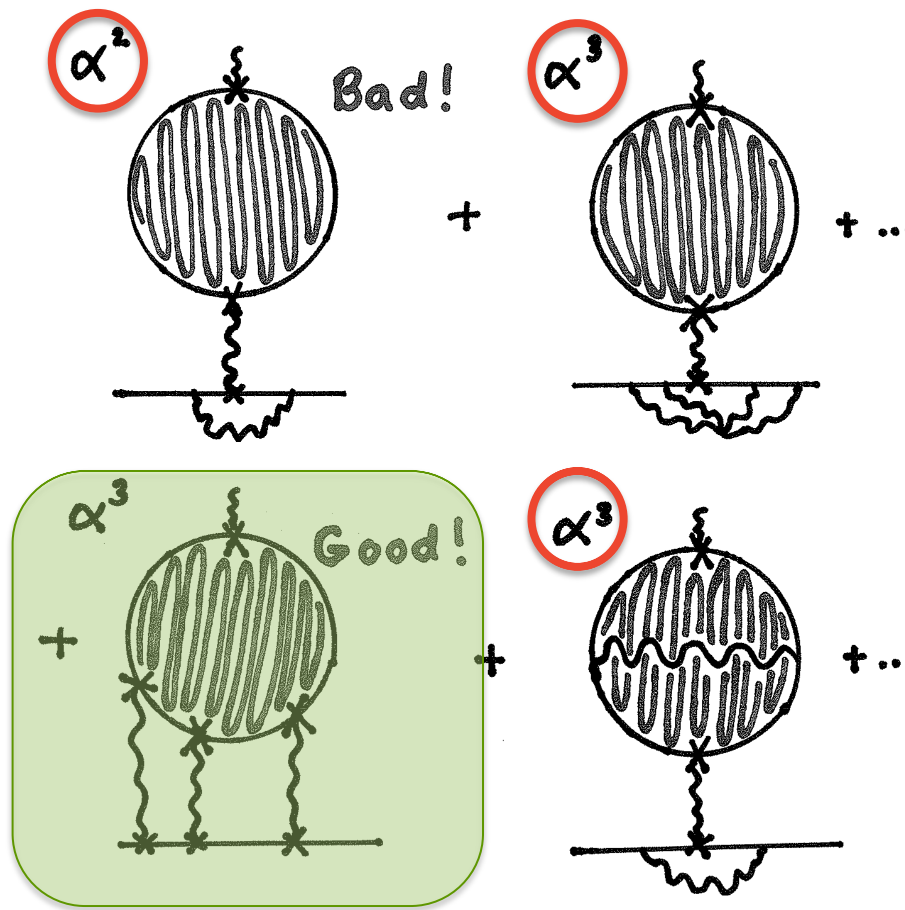
- With the analytic photon propagator, which has zero statistical error



- There is lower or equal orders (α^2, α^3) unwanted diagram, which should be treated as photon wave function renormalization or EM vertex correction

Unwanted diagrams





The subtraction works for diagram by diagram and config-by-config. This is also important for noise reduction.

Subtraction

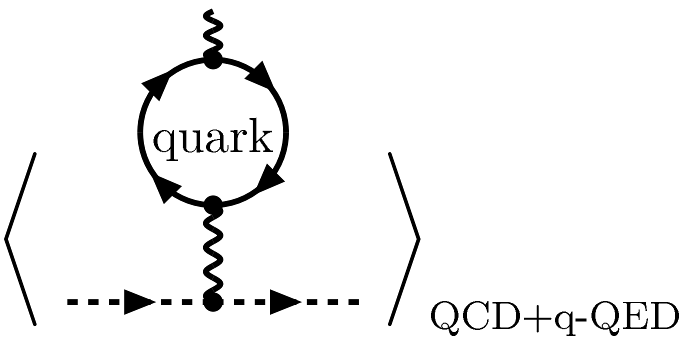
First multiply the analytic photo between
- quark loop part
- lepton line part

then take QCD+QED ensemble average

First take QCD+QED average for
- quark loop part
- lepton line part

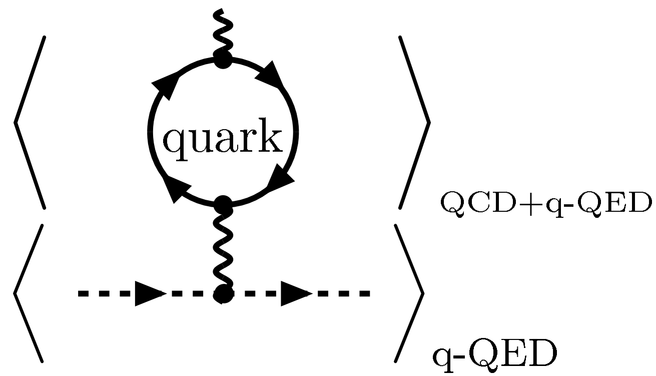
then multiply the analytic photon

unsubtracted term

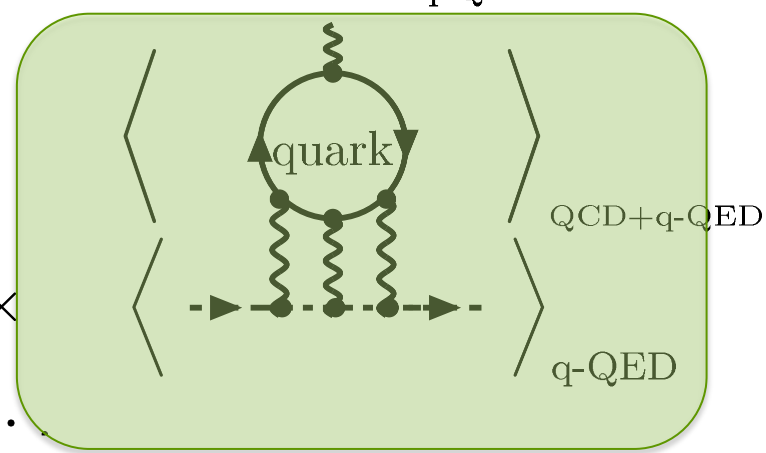


$=$

Subtraction term



$+\,3\times$



$+\cdots$

$$
\begin{aligned}
&\langle \psi(t',\mathbf{p}')\, j_\mu(t_{op},\mathbf{q})\, \overline{\psi}(0,\mathbf{p})\rangle_{\mathrm{HLbL}} \\
&= -\sum_{q=u,d,s}(Q_q e)^2 \sum_k \Big\{\langle \gamma_\mu S_q(t_{op},-\mathbf{q};k)\gamma_\nu S_q(k;t_{op},-\mathbf{q}) \\
&\quad \frac{\delta_{\nu\rho}}{\hat{k}^2} G(t',\mathbf{p}';-k)\gamma_\rho G(-k;0,-\mathbf{p})\rangle_{\mathrm{QCD+QED}} \\
&\quad - \langle \gamma_\mu S_q(t_{op},-\mathbf{q};k)\gamma_\nu S_q(k;t_{op},-\mathbf{q})\rangle_{\mathrm{QCD+QED}} \\
&\quad \frac{\delta_{\nu\rho}}{\hat{k}^2}\langle G(t',\mathbf{p}';-k)\gamma_\rho G(-k;0,-\mathbf{p})\rangle_{\mathrm{QED}}\Big\}\,, \qquad (1)
\end{aligned}
$$

[Blum, TI, Shintani PRD 88 (2013) 094503]

Covariant Approximation Averaging (CAA)
a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} + \mathcal{O}^{(\mathrm{rest})}$$

Lattice Symmetry

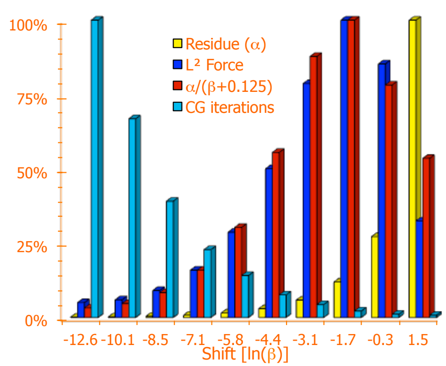
unbiased imporved

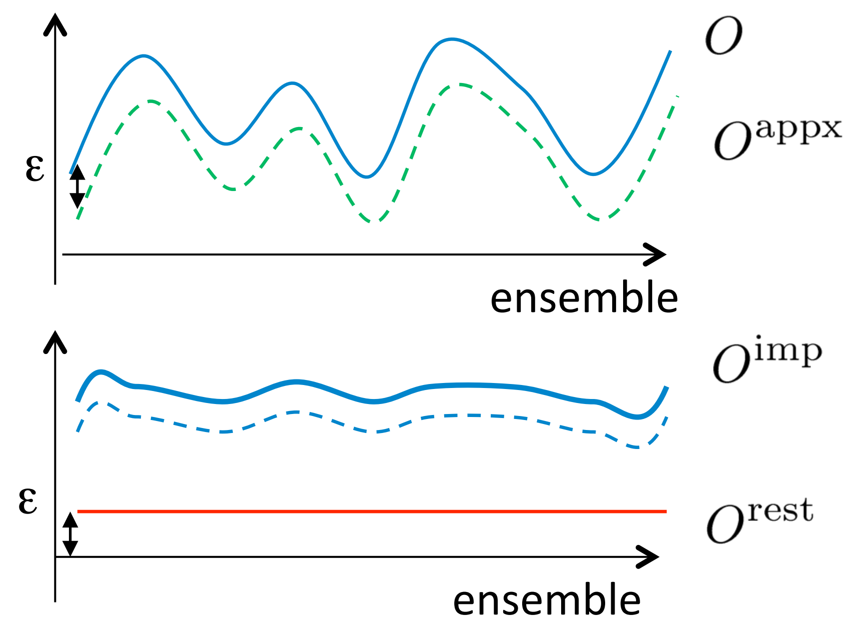
$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}$$

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(\mathrm{imp})}$ has smaller error
  $O^{(\mathrm{appx})}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply



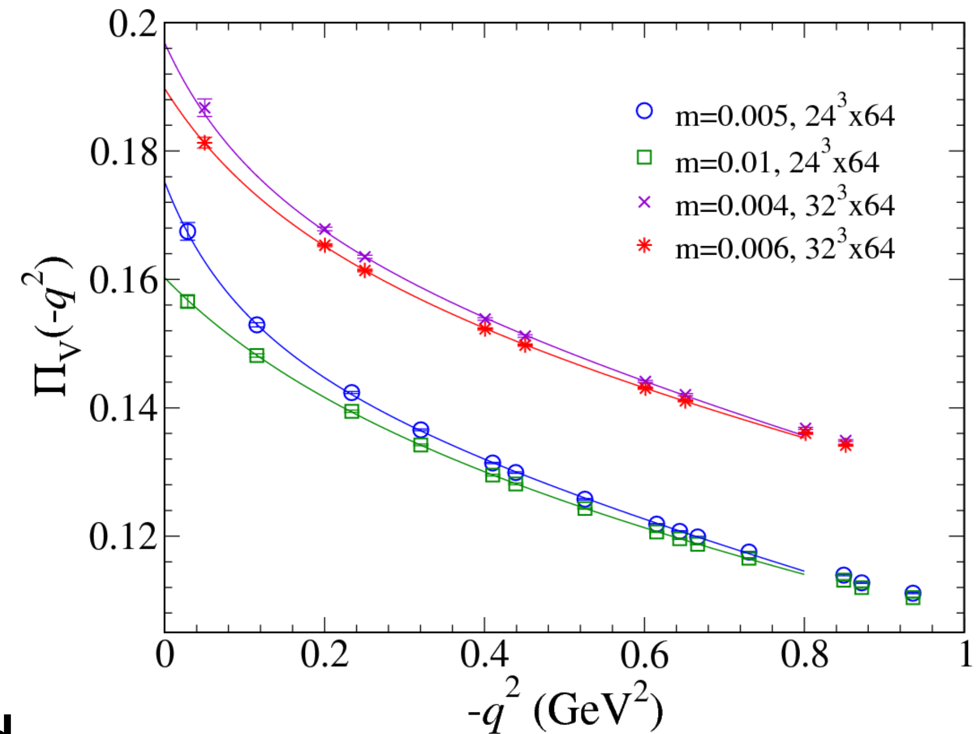


RBC/UKQCD DWF AMA Results

- Two lattice spacings a = 0.11, 0.088fm, Mpi=0.28-0.33 GeV
  All stat err < 0.7%
  q_min = 2 m(mu)
- Applied [2,1] Pade,

can't fit with

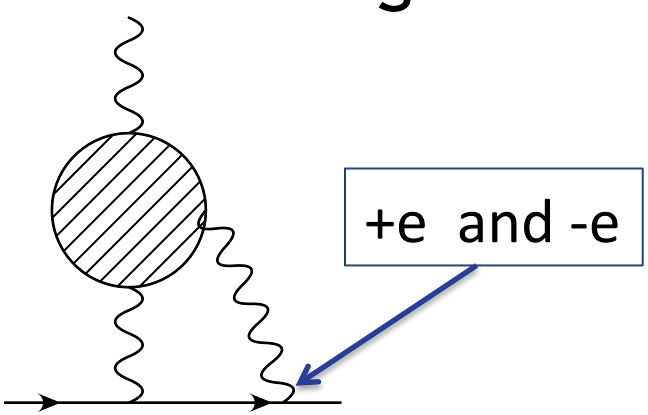
b1 >= 4 Mpi^2 bound



Lattice	m_u	$\Pi_V(0)$	a_0 (GeV^{-2})	a_1	b_1 (GeV^2)	χ^2/dof	$4m_\pi^2$ GeV^2
$24^3\times64$	0.005	0.1752(2)	0.0325(2)	0.0407(1)	0.139(1)	2.7(4)	0.44
	0.01	0.1603(2)	0.0219(3)	0.0434(4)	0.408(7)	0.4(1)	0.71
$32^3\times64$	0.004	0.197(2)	0.026(3)	0.052(3)	0.227(37)	0.08(7)	0.31
	0.006	0.190(3)	0.027(7)	0.043(11)	0.253(25)	0.4(5)	0.44

α^2 noise & Furry's theorem

- target : α^3
- Analytic photon and the subtraction removes the unwanted diagrams, α, α^2, α^3, not in the statistical way, < M_1 > - <M_2> =0, but in an exact way : M_1 – M_2 = 0 for each QED+QCD configuration
- There is also remaining unwanted diagram in α^2 , which is zero by Furry's theorem. This cancellation is stochastic, α^2 M, <M> → 0. By averaging "+e" and "-e" of stochastic photon coupling only to lepton, the unwanted diagram is exactly removed
- Also done by averaging over "+p" and "-p" of lepton.



Form factor extraction

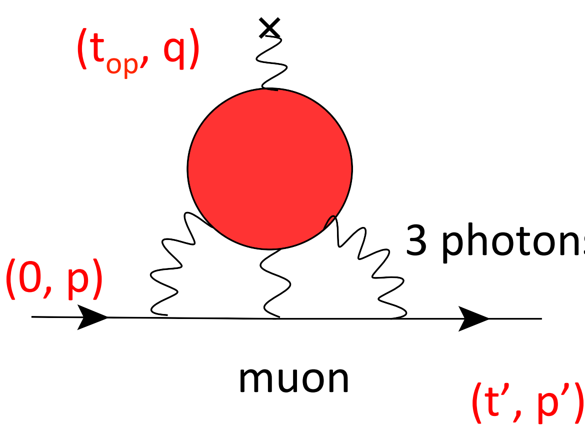
- Rest is the standard calculation to extract matrix elements/form factors from 3pt

$$\lim_{t' \gg t_{op} \gg 0} \langle \psi(t', \mathbf{p}')\, j_\mu(t_{op}, \mathbf{q})\, \overline{\psi}(0, \mathbf{p}) \rangle_{\mathrm{HLbL}} = \frac{\langle 0|\psi(0,\mathbf{p}')|\mathbf{p}', s'\rangle}{2E'V} \boxed{\langle p', s'|\Gamma_\mu|p, s\rangle} \frac{\langle \mathbf{p}, s|\overline{\psi}(0,\mathbf{p})|0\rangle}{2EV} \times e^{-E'(t'-t_{\mathrm{op}})} e^{-Et_{\mathrm{op}}},$$

$$\langle p', s'|\Gamma_\mu|p, s\rangle \equiv \bar{u}(p', s')\left(F_1(q^2)\gamma_\mu + \boxed{i\frac{F_2(q^2)}{2m_\mu}[\gamma_\mu, \gamma_\nu]q_\nu}\right) u(p, s)$$

- Except it's noisy (disconnected quark loop)
  → AMA.

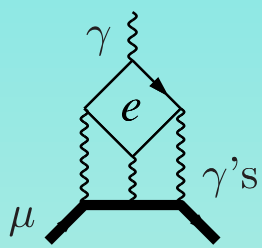
$$F_2(0) = a_\mu$$



QED only study

[Saumitra Chowdhury Ph.D. Uconn 2009]

- DWF, e=1, m_μ / m_e = 40
- Continuum QED perturbation = 1.63x10^{-4}



$$a_\mu^{(6)}(\mathrm{lbl}, e) = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \frac{59}{270}\pi^4 - 3\,\zeta(3) - \frac{10}{3}\pi^2 + \frac{2}{3} + O\left(\frac{m_e}{m_\mu}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^3$$

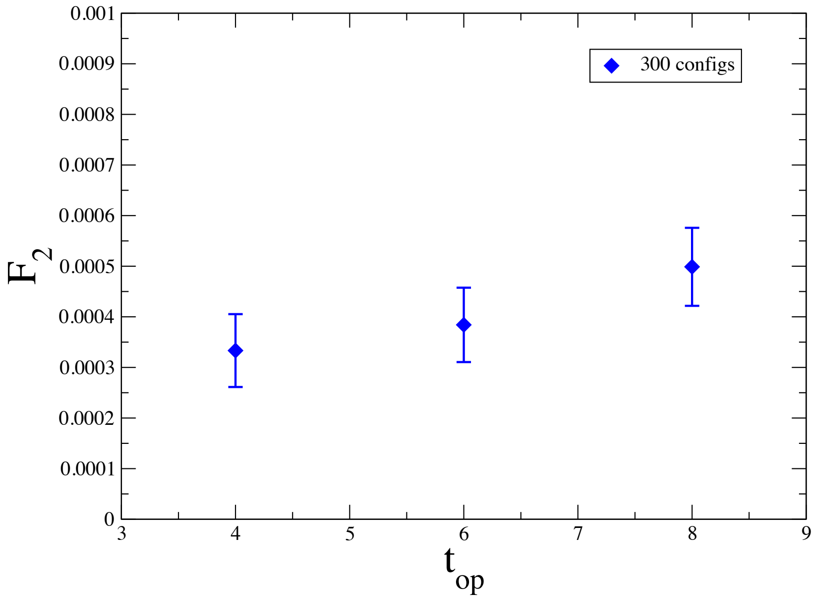
- a m_μ = 0.4 , q = 2 Π / L

$F_2(q_{min}^2)$ = [3.96 (70)] x 10^{-4} on $(16)^3$

= [1.19 (32)] x 10^{-4} on $(24)^3$

QED test [Chowdhury Ph. D. thesis, UConn, 2009]

$F_2 = (3.96 \pm 0.70) \times 10^{-4}$



- $m_\mu / m_e = 40$
- $e = 1$
- $16^3 \times 32$ lattice size
- lowest non-zero momentum only ($|\mathbf{p}|/m_\mu \approx 1$)
- **stat error only**

- Expected size of enhancement (compared to $m_\mu / m_e = 1$)
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ ($e = 1$)
- roughly consistent with PT result, large finite volume effect

tsep=0,12 , Feynman & Columb gauge

QCD+QED studies

- Nf=2+1 (u,d,s) chiral quark,
  DWF QCD [RBC/UKQCD]
- a=0.114 fm, 24^3x64 $(2.7\ \mathrm{fm})^3$, m_π = 329 MeV,
  ~300 QCD configuration (1 QED per 1 QCD)
- m_μ =~ 190 MeV, e=1
- 0.11 GeV^2 <= Q^2 <= 0.31 GeV^2
- Use All Mode Averaging (AMA)
  - 6^3 = 216 source locations per one configration
  - AMA approximation : "sloppy CG" with r(stop) = 10^{-4}
  - ~ 700 configuration, ~150 K measurements

- $F_2(q_{min}^2)$ = [-2.2(8)] x 10^{-5} on $(1.8\ \mathrm{fm})^3$, m_π = 420 MeV

also checked α^3 scaling by changing e=0.84, 1.19
$\rightarrow$ subtraction works

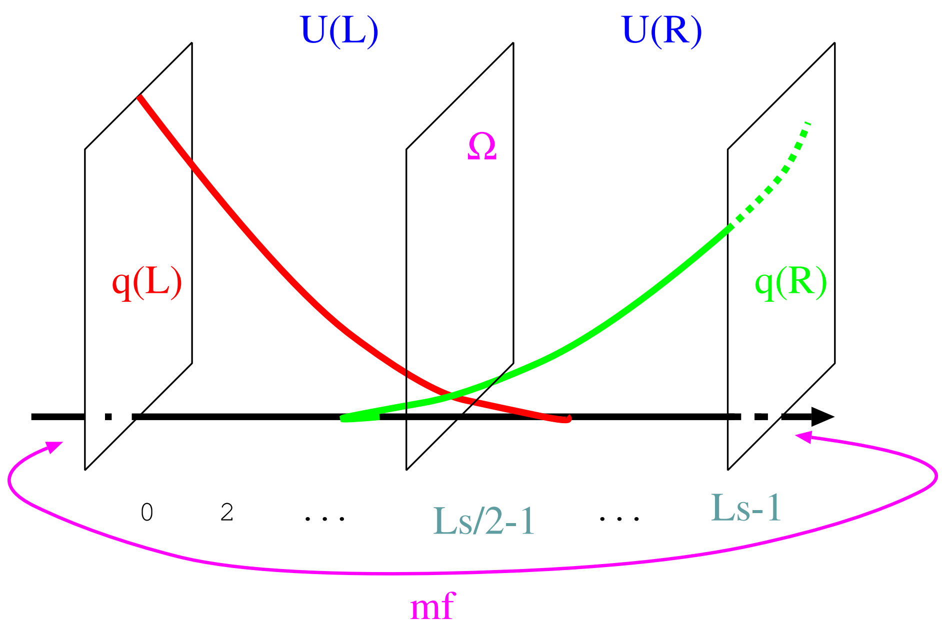
Domain Wall Quarks (for up, down, and strange)

[Kaplan, Shamir, Blum & Soni]

- 4D lattice quark utilizing an ''extra dimension'', L_s. (expensive)
- Almost perfect chiral symmetry

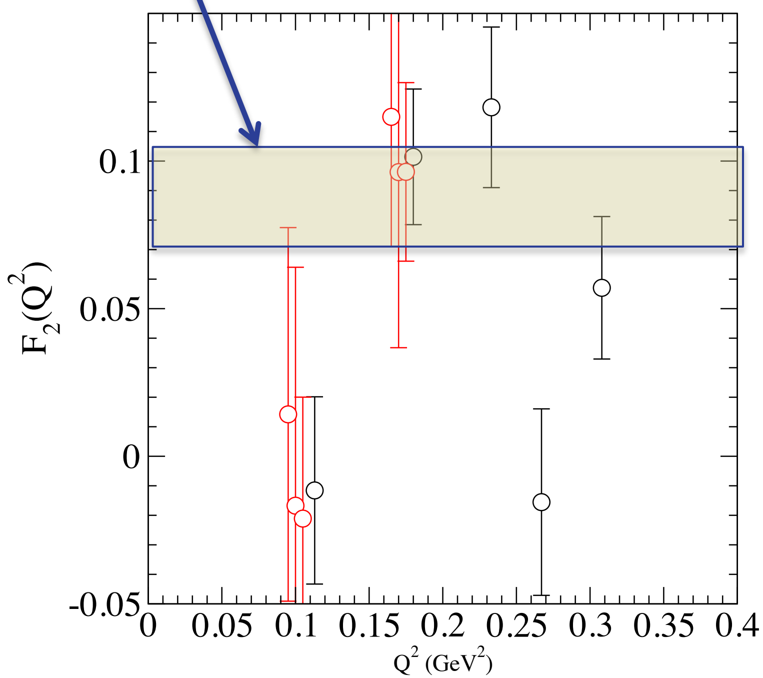
  Small unphysical mixing for the Weak Matrix Elements
  Error from discretization is small $\mathcal{O}(a^2\Lambda^2_{QCD}) \sim$ a few %.
  Chiral extrapolation is simpler, continuum like.

- Unitary theory (at long distance).



HLBL from Nf=2+1 DWF

a_μ (Model) = 0.084 (20) x $(\alpha/\pi)^3$

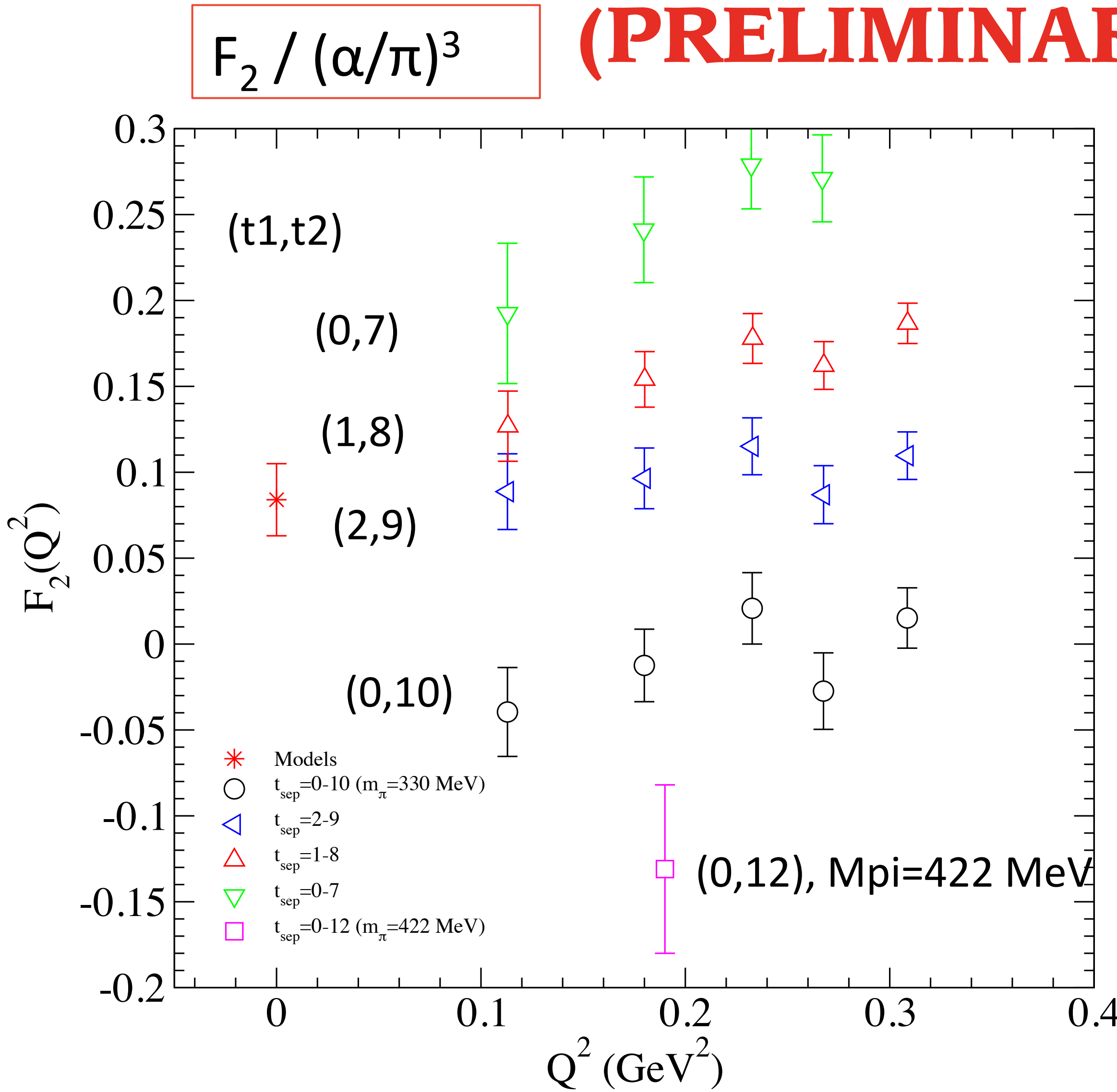


tsep=0-10

$m_\pi \approx 329$ MeV
$m_\mu \approx 190$ MeV

- 157+190+375=722 configs, lowest 2 momenta
- 375 configs, highest 3 momenta
- Two points zero within large errors for momenta in 1 direction
- Statistics? 6, 24, 24, 6, 48 mom. combinations for $Q^2 = 1, 2, 3, 4, 5$
- Different directions correlated, cancelations

HLBL from Nf=2+1 DWF (PRELIMINARY)



24^3 lattice size

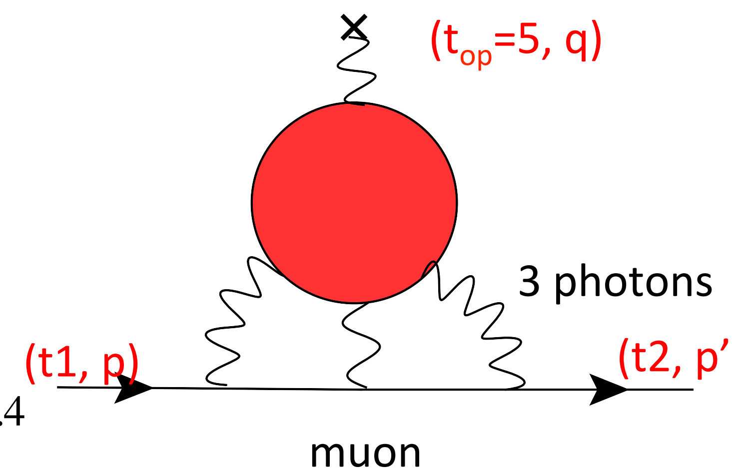
$Q^2 = 0.11$ and 0.18 GeV^2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

multiply

$$(2/3)^4 + (-1/3)^4 + (-1/3)^4$$



HLBL from Nf=2+1 DWF
(PRELIMINARY)

24^3 lattice size
$Q^2 = 0.11$ and $0.18\ \mathrm{GeV}^2$
$m_\pi \approx 329$ MeV
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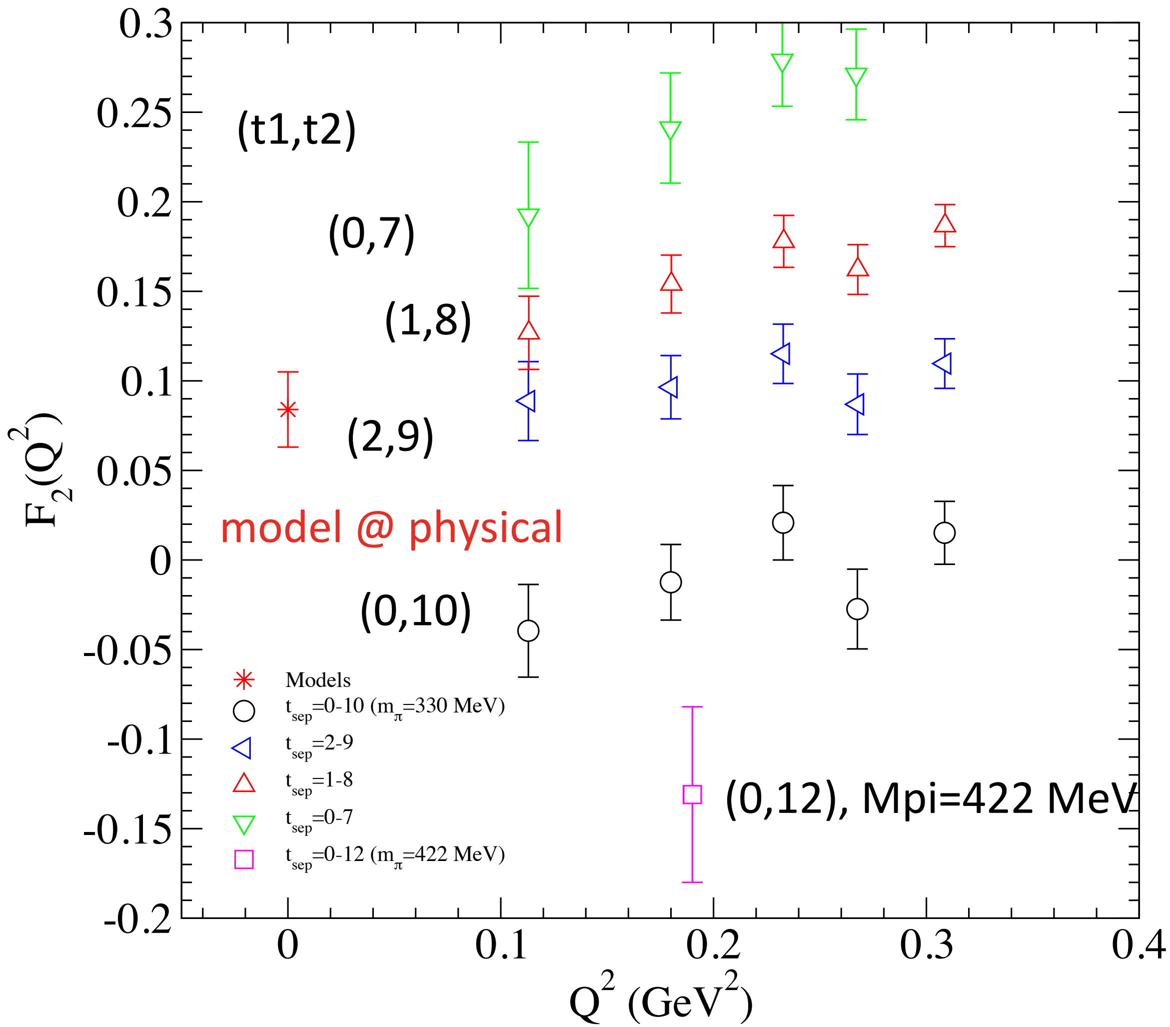
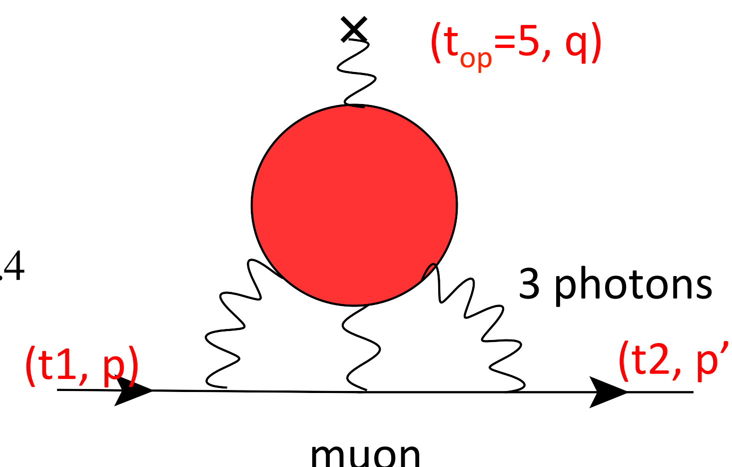


- Statistically resolved
- Roughly same order to the model calculation
- heavier pion leads negative F2

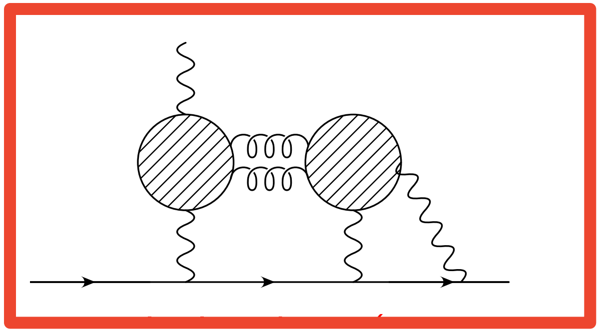
- Significant excited state contamination
  μ, μ+γ, μ+μ~μ, …. ?



Summary of HLBL

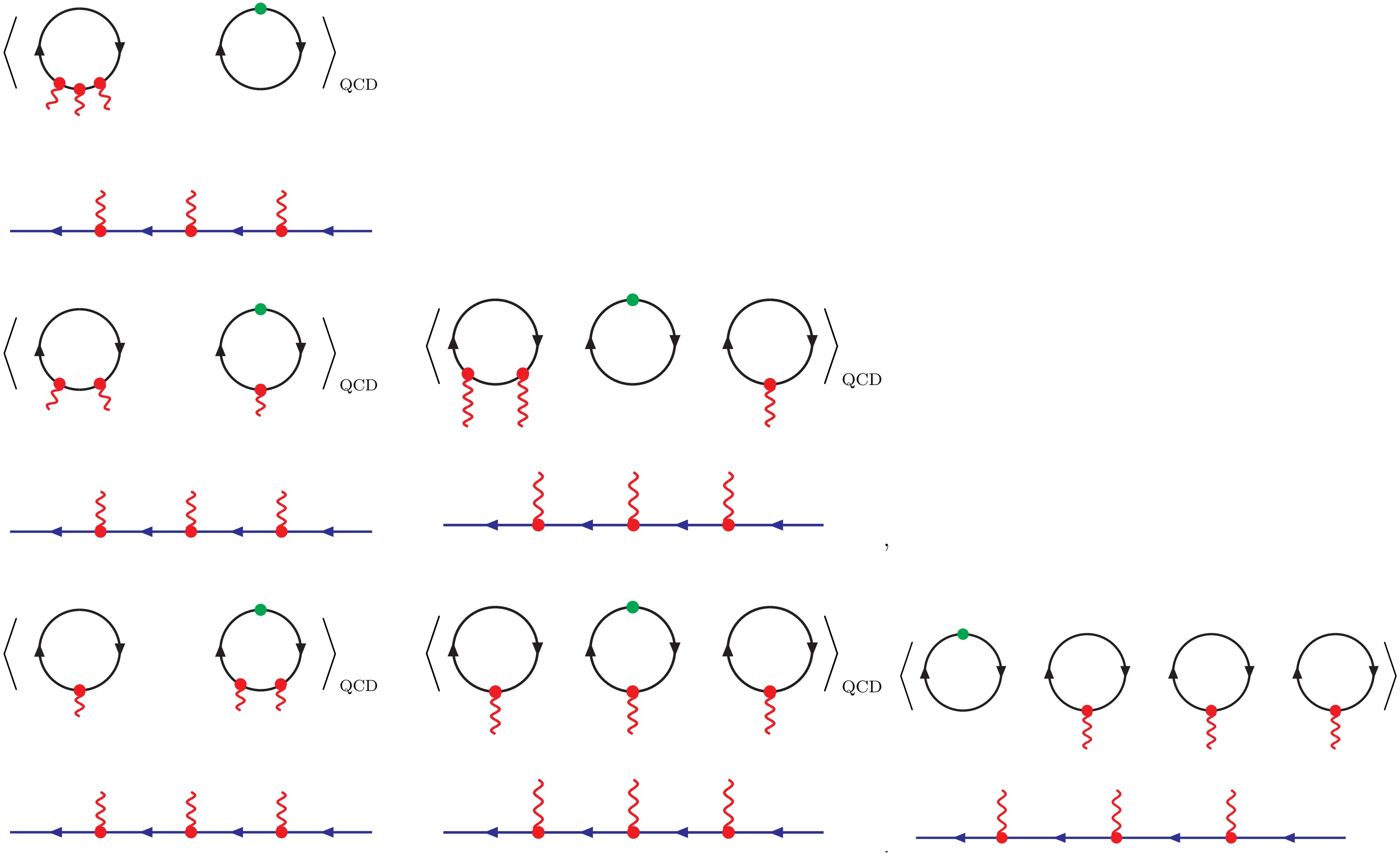
- Statistically significant signal may be starting to show up. Very encouraging !
- Still various source of systematic error
  - Excited state contamination (ROM123 method)
  - Finite volume effect (mass less photon)
  - heavier mass : m_π=330 MeV, m_μ=190 MeV
  - (2.7 fm)^3, 1/a= 1.7 GeV
  - $Q^2 \rightarrow 0$ extrapolation
  - Lack of disconnected diagrams

  → three ways
    - *Add another valence loops*
    - *Re-weight in α(sea) [T. Ishikawa]*
    - *Dynamical QCD+QED*

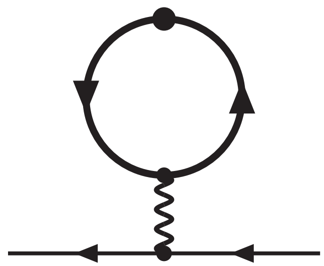
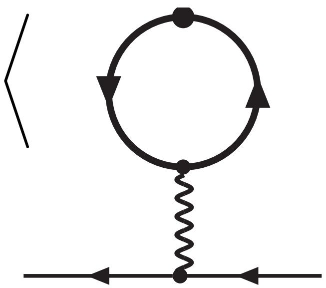


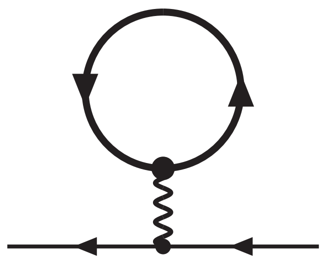
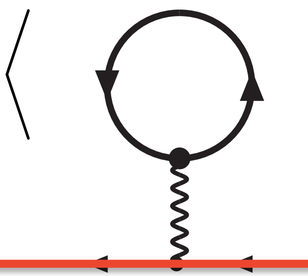
- Current plans (2014-15) :
  - More scrutinizing current data, especially the excited state
  - Mpi=170 MeV, L=5fm connected LbL
  - and dynamical QED or reweighting

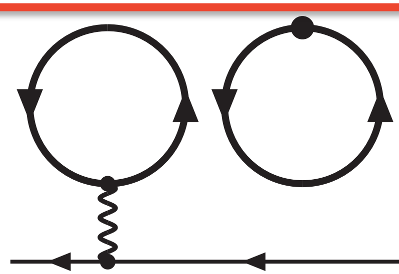
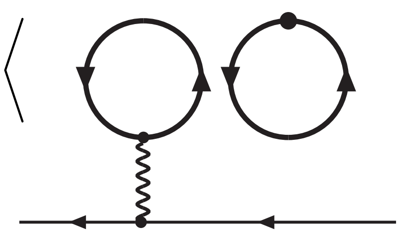
Disconnected diagrams



Similar strategy

$\mathcal{M}_C = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}}$, $\mathcal{S}_C = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}} \Big\langle \quad \Big\rangle_{\text{f-QED}}$.

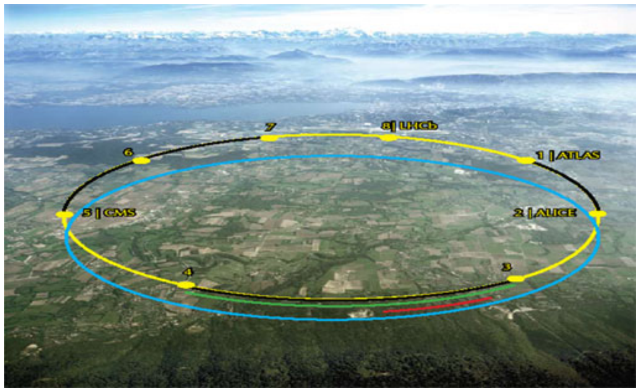
$\mathcal{M}_{C'} = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}}$, $\mathcal{S}_{C'} = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}} \Big\langle \quad \Big\rangle_{\text{f-QED}}$.

$\mathcal{M}_D = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}}$ $\mathcal{S}_D = \Big\langle$  $\Big\rangle_{\text{QCD+f-QED}} \Big\langle \quad \Big\rangle_{\text{f-QED}}$.

Backup slides

Complementarity is crucial

- I experiment <-> [QCD corrections] ⊗ [Standard Model & Beyond]



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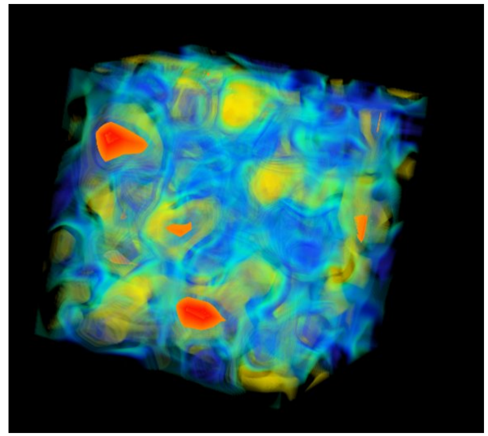

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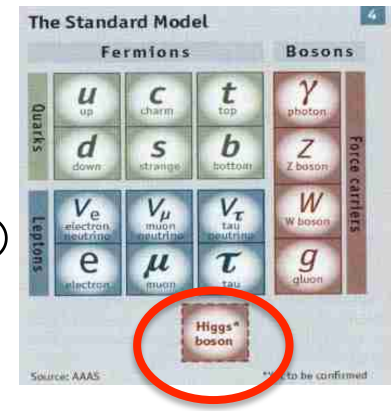
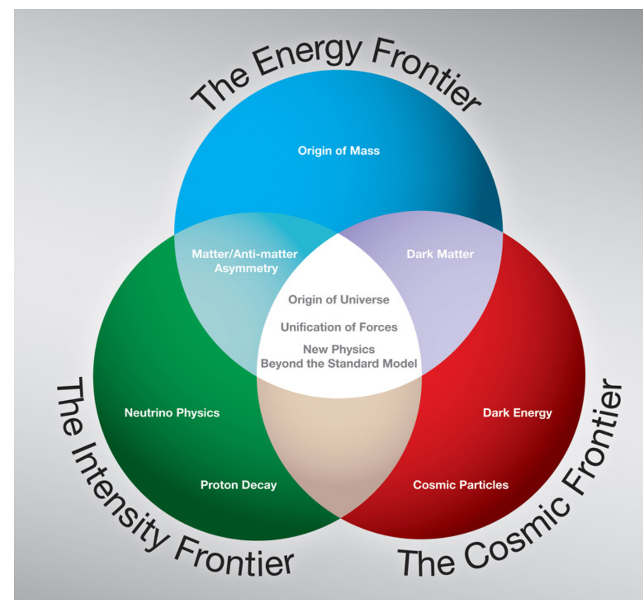


- LHC / ILC, composite Higgs
- Flavor physics K→ππ I=2 & I=0, ε '/ε , $\Delta M(K_L\text{-}K_S)$
  B physics (Super-B)
- Nucleon Electric Dipole Moments
- Dark Matter Search , strangeness of Nucleons

……



Examples of Covariant Approximations (contd.)

- **All Mode Averaging AMA**

  Sloppy CG or Polynomial approximations

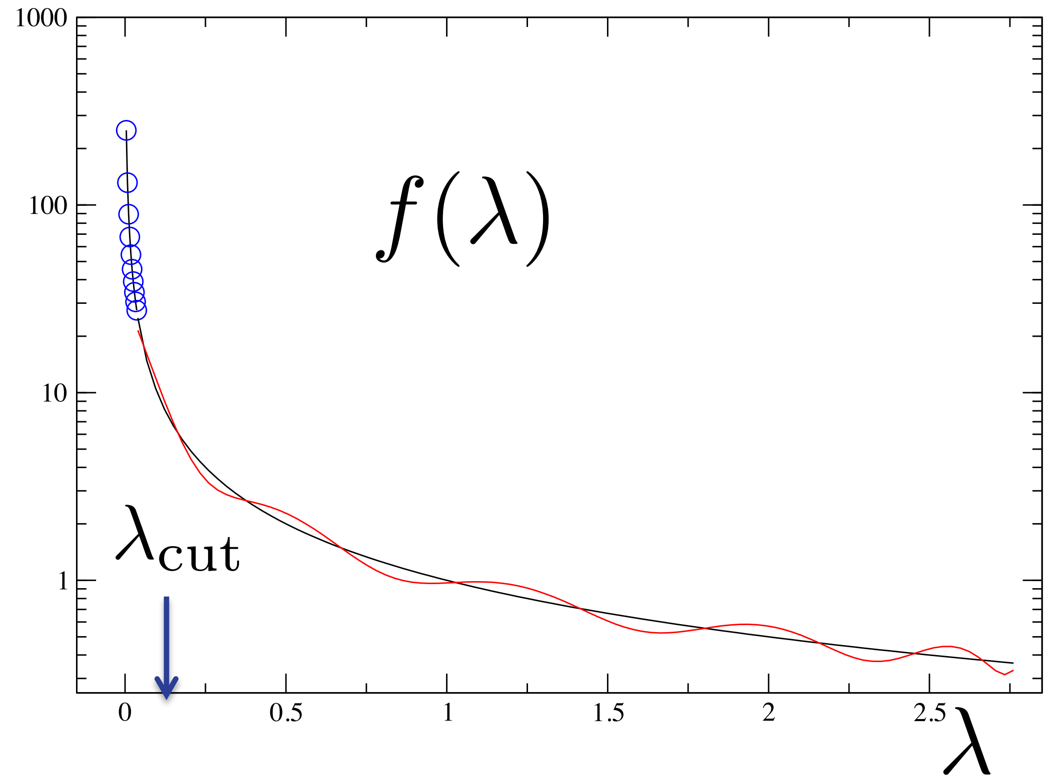
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_\lambda v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unneccesary



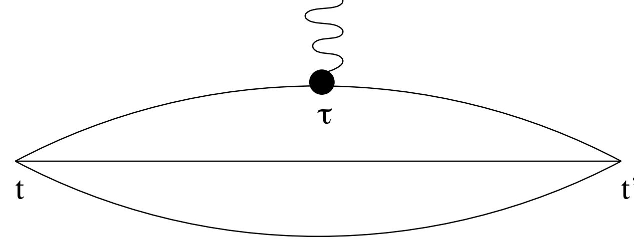
accuracy control :
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

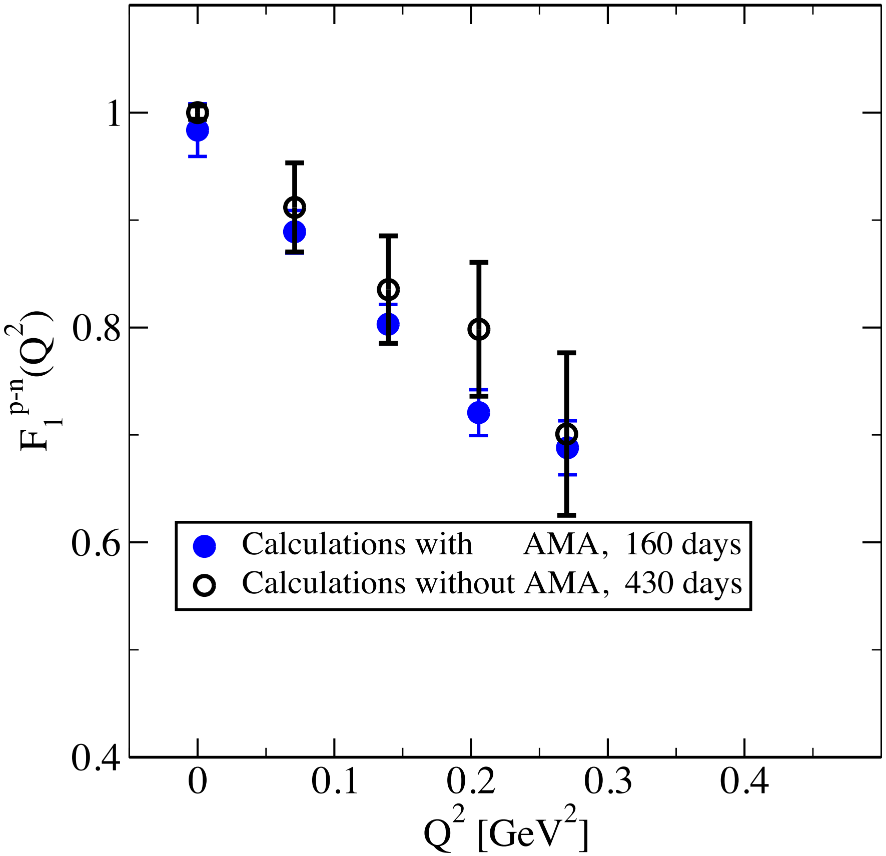
AMA at work

- Target : $V=32^3 \times 64 = (4.6\text{fm})^3 \times 9.6\text{fm}$, Ls=32 Shamir-DWF, a^{-1}=1.37 GeV, Mpi = 170 MeV
- Use Ls=16 Mobius as the approximation
  [Brower, Neff, Orginos, arXiv:1206.5214]

- quark propagator cost on SandyBridge 1024 cores (XSEDE gordon@SDSC)
  - non-deflated CG, r(stop)=1e-8 : ~9,800 iteration, 5.7 hours / prop
  - Implicitly restarting Lanczos of Chebyshev polynomials of even-odd prec operator for 1000 eigenvectors
    [Neff et al. PRD64, 114509 (2001)] : 12 hours
  - deflated CG with 1000 eigenvectors : ~700 iteration, 20 min /prop
  - deflation+sloppy CG, r(stop)=5e-3 : ~125 iteration, 3.2 min /prop
- Multiplicative Cost reduction for General hadrons could combine with {EigCG | AMG} and Distillation:
  x1.2 (Mobius) x 14 (deflation) x 7 (sloppy CG) ~ x 110

AMA at work
[M. Lin]





- $F_1(Q^2)$: tsep = 9 a ~ 1.3 fm

  1 forward + 2 (up and down) seq-props, contraction cost is ~15% of sloppy propagator

- Error bar
  x 2 - 2.7 ~ sqrt(4400/600)
- Total cost reduction upto
  (430 / 160) * (4400/600)
  ~ <u>x19.7</u>

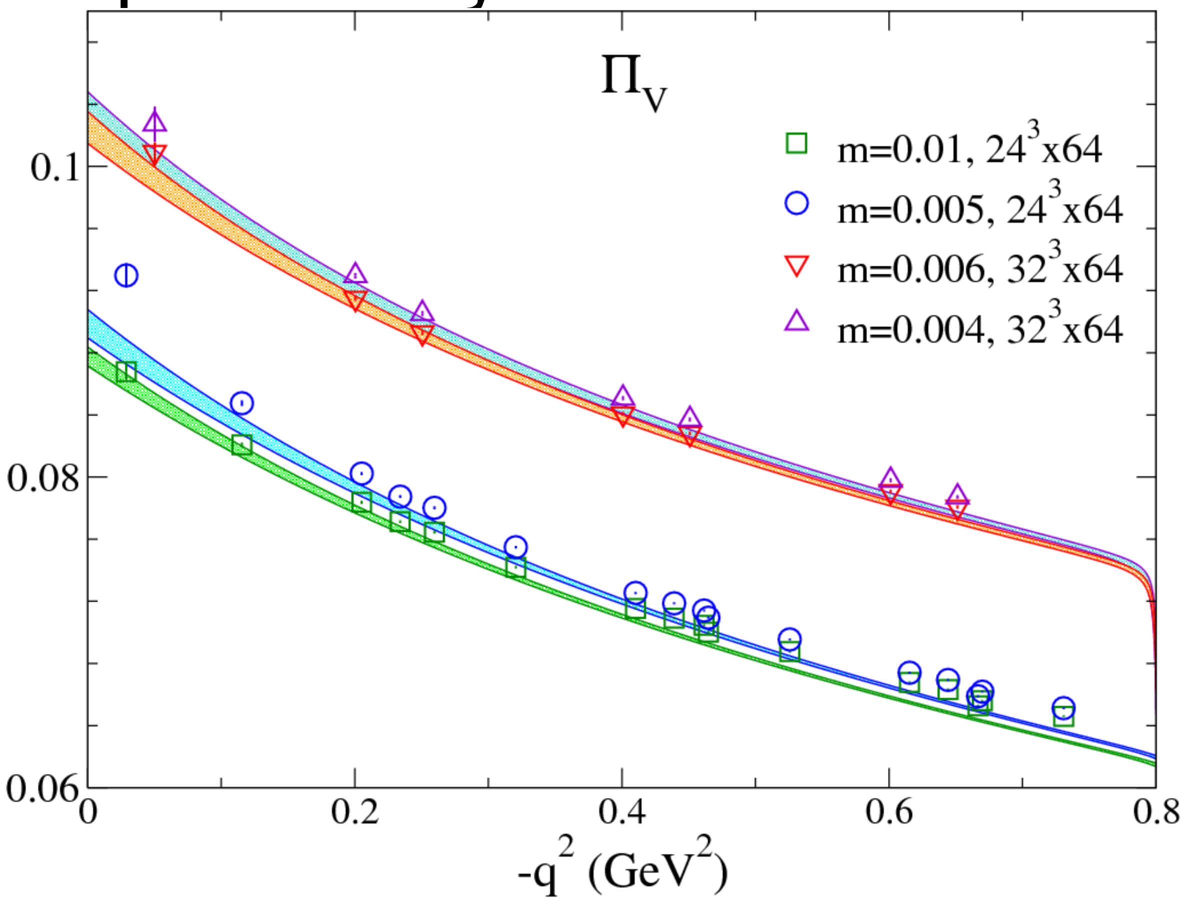
- Note this is still sub-optimal, 4 exact source and without deflation. (would be x30 for 2 exact sources)

- <u>non-deflated CG</u>, 150 config x 4 sources = 600 measurements :
  5.7 * 3 * 4 * 150 config = 10K hours, 430 days

- <u>AMA</u> : 39 config, 4 exact solves / config (perhaps overkill) , N_G=112 sloppy solves
  => 39 x 112 = 4400 AMA measurements :
  (5.7 * 3 * 4 + 12 + 0.06 * 3 * 112) * 39 config = 3.9 K hours, 160 days
  4-exact (68%) + Lanczos (12%) + sloppy CG (20%)

HVP with time-like momentum

- preliminary



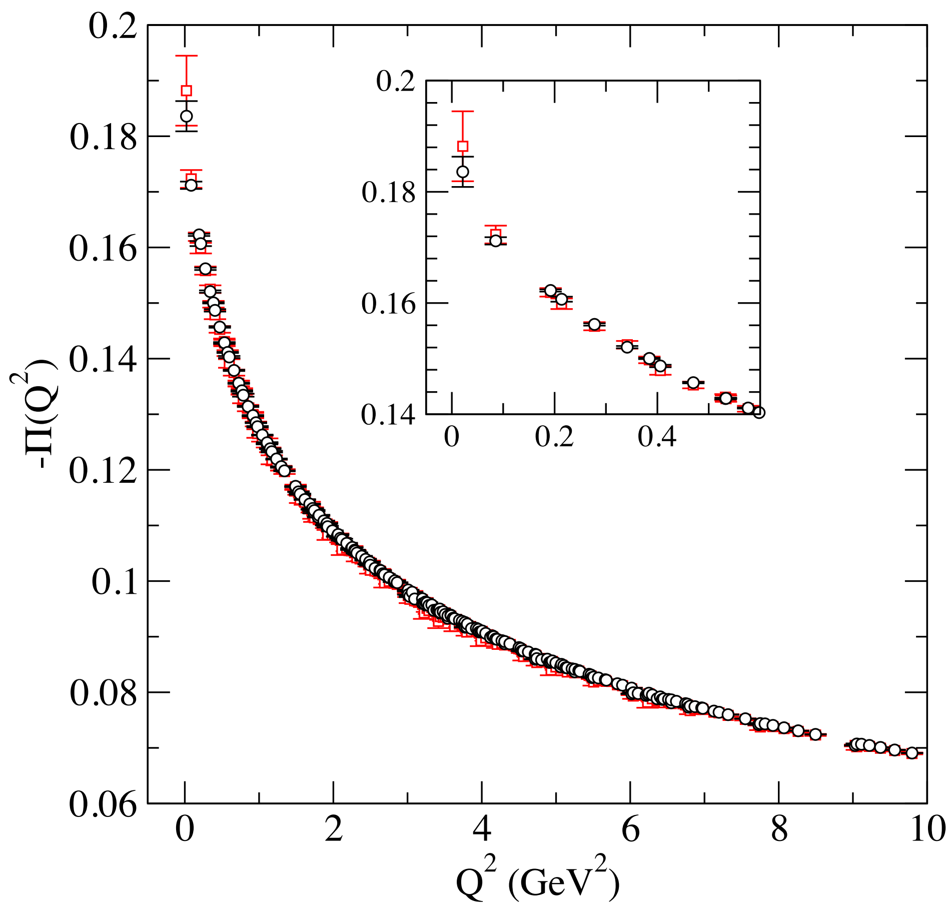
t_{cut} = 9 (24^3), 10 (32^3)
Fitting range at large t
[8,13] (24^3), [10,15] (32^3)

- Similar behavior with results obtained in Euclid momentum
- Slight discrepancy from HVP in space-like momentum, especially for light mass.

More carefully systematic study is necessary !

Improving HVP statistics using AMA

- Staggered Fermion (MILC Asqtad, Mpi=300 MeV) 2.6 -- 20 times smaller error with same cost



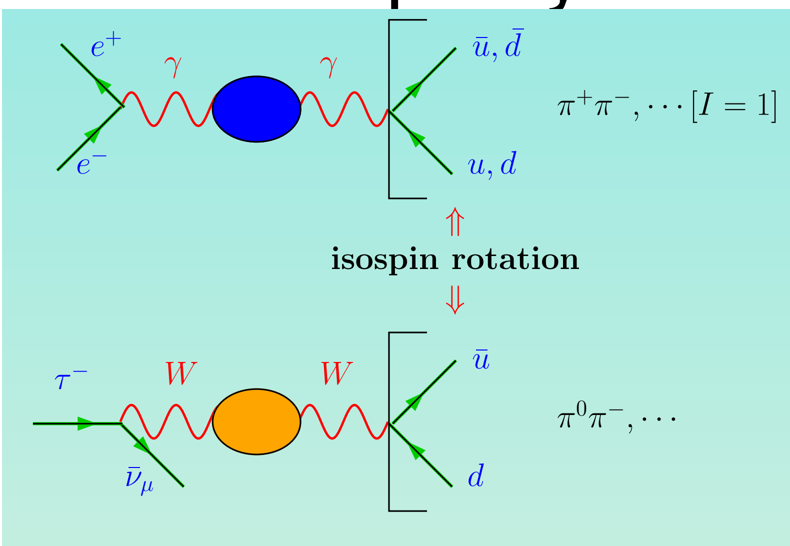
Now getting to
all stat error < 2%

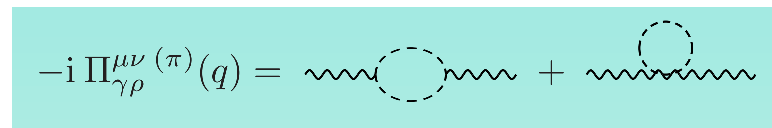
qmin^2 = 1.5 m(mu)

Tau decay puzzle is resolved ?

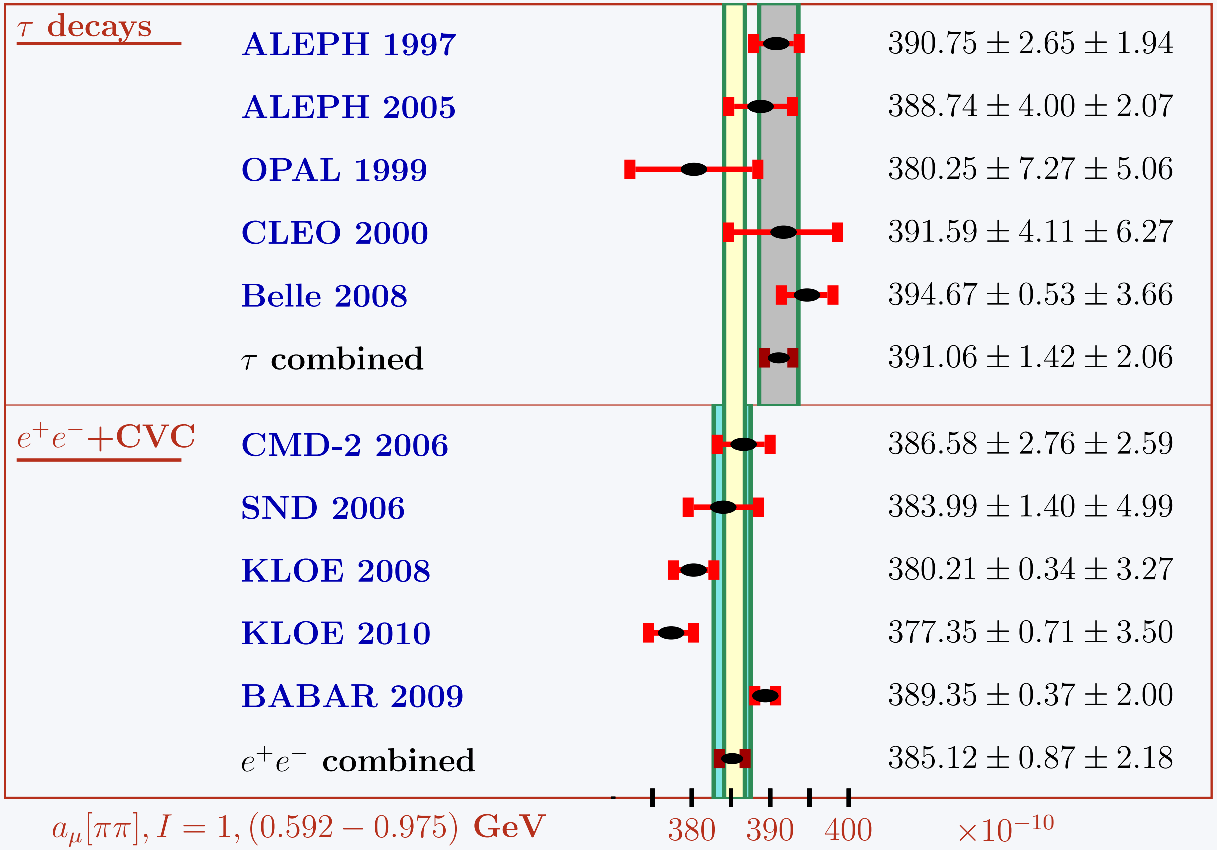
- Use isospin to relate tau decay data

~ 10% discrepancy between tau vs e+e-.

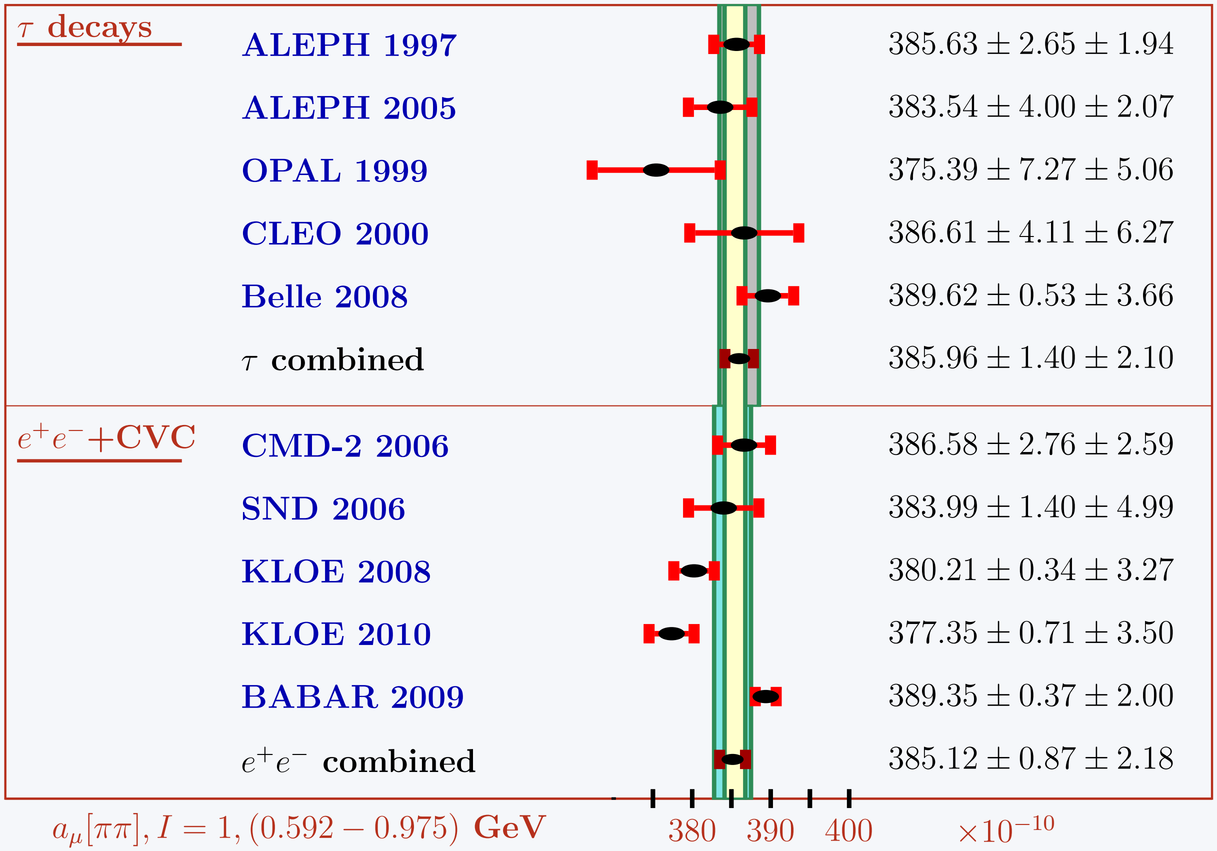




- tau decay needs to fix ρ - γ mixing
  Jegerlehner Szafron



I=1 part of $a_\mu^{\rm had}[\pi\pi]$



I=1 part of $a_\mu^{\rm had}[\pi\pi]$