On the disconnected diagram contribution to a_μ^{HLO}

Harvey Meyer

SRFN

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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Outline

- disconnected diagrams: definition
- theoretical expectations
- comparison of the vector correlator with a phenomenological study
- finite-volume effects on the correlator

Basic relations

For a quark flavor f, set $j_\mu^f = \bar{f}\gamma_\mu f$.

E.m. current in the 2+1 flavor theory with masses $m_l = m_u = m_d$ and m_s:

$$j_\mu^\gamma = \underbrace{\tfrac{1}{2}(j_\mu^u - j_\mu^d)}_{\equiv j^\rho,\ (I=1)} + \underbrace{\tfrac{1}{6}(j_\mu^u + j_\mu^d - 2j_\mu^s)}_{I=0}.$$

$$G^{\gamma\gamma}(x_0) = -\frac{1}{3}\int d^3\boldsymbol{x}\,\left\langle j_k^\gamma(x) j_k^\gamma(0)\right\rangle,$$

$$G^l_{\rm conn}(x_0) = -\frac{1}{3}\int d^3\boldsymbol{x}\,\left\langle j_k^l(x) j_k^l(0)\right\rangle_{\rm conn}, \quad G^{\rho\rho}(x_0) = \frac{1}{2}G^l_{\rm conn}(x_0),$$

$$G^s_{\rm conn}(x_0) = -\frac{1}{3}\int d^3\boldsymbol{x}\,\left\langle j_k^s(x) j_k^s(0)\right\rangle_{\rm conn},$$

$$G^l_{\rm disc}(x_0) = -\frac{1}{3}\int d^3\boldsymbol{x}\,\left\langle j_k^l(x)\, j_k^l(0)\right\rangle_{\rm disc}.$$

$$G^{ls}_{\rm disc}(x_0) = -\frac{1}{3}\int d^3\boldsymbol{x}\,\left\langle (j_k^l(x) - j_k^s(x))(j_k^l(0) - j_k^s(0))\right\rangle_{\rm disc}.$$

$$\Rightarrow \quad G^{\gamma\gamma}(x_0) = \frac{10}{9}G^{\rho\rho}(x_0) + \frac{1}{9}G^s_{\rm conn}(x_0) + \frac{1}{9}G^{ls}_{\rm disc}(x_0).$$

The disconnected diagram contribution, more explicitly

Calculate the following Wick-disconnected correlators,

$$\delta_{ki}\ f^{ls}(x_0-y_0) = a^6\Big\langle\Big(\sum_{\boldsymbol{x}}(\hat{j}^l_k(x)-\hat{j}^s_k(x))\Big)\ \Big(\sum_{\boldsymbol{y}}(\hat{j}^l_i(y)-\hat{j}^s_i(y))\Big)\Big\rangle_{\rm disc}$$

$$\delta_{ki}\ f^{l}(x_0-y_0) = a^6\Big\langle\Big(\sum_{\boldsymbol{x}}\hat{j}^l_k(x)\Big)\ \Big(\sum_{\boldsymbol{y}}\hat{j}^l_i(y)\Big)\Big\rangle_{\rm disc}$$

Here $\hat{j}_\mu$ refers to the local vector current on the lattice, which receives a renormalization factor $Z_V(g_0)$. In terms of quark contractions,

$$\delta_{ki}\ f^l(x_0-y_0) = a^6\Big\langle\Big(\sum_{\boldsymbol{x}}{\rm Tr}\{\gamma_k D^{-1}(x,x)\}\Big)\ \Big(\sum_{\boldsymbol{y}}{\rm Tr}\{\gamma_i D^{-1}(y,y)\}\Big)\Big\rangle_{\rm gauge\ fields}.$$

Then

$$G^{ls}_{\rm disc}(x_0) = -\frac{Z_V(g_0)^2}{L^3}f^{ls}(x_0),$$

$$G^{l}_{\rm disc}(x_0) = -\frac{Z_V(g_0)^2}{L^3}f^{l}(x_0).$$

Extracting $a_\mu^{\rm HLO}$

$$a_\mu^{\rm HLO} = 4\alpha^2 m_\mu \int_0^\infty \mathrm{d}t\, t^3\, G^{\gamma\gamma}(t)\, \tilde K(t), \tag{1}$$

$$\tilde K(t) \equiv \frac{2}{m_\mu t^3} \int_0^\infty \frac{\mathrm{d}\omega}{\omega}\, K_E(\omega^2) \left[\omega^2 t^2 - 4\sin^2\left(\tfrac{\omega t}{2}\right)\right]. \tag{2}$$

The kernel $\tilde K(t)$ is dimensionless, proportional to t at small t and to $1/t$ at large t.

$$K_E(s) = \frac{1}{m_\mu^2} \cdot \hat s \cdot Z(\hat s)^3 \cdot \frac{1 - \hat s Z(\hat s)}{1 + \hat s Z(\hat s)^2}, \tag{3}$$

$$Z(\hat s) = -\frac{\hat s - \sqrt{\hat s^2 + 4\hat s}}{2\hat s}, \quad \hat s = \frac{s}{m_\mu^2}. \tag{4}$$

Spectral representation

$$G^{\gamma\gamma}(t) = \int_0^\infty \mathrm{d}\omega\, \omega^2 \rho(\omega^2) e^{-\omega|t|},$$

$$\rho(s) = \frac{R(s)}{12\pi^2}, \qquad R(s) \equiv \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{4\pi\alpha(s)^2/(3s)}.$$

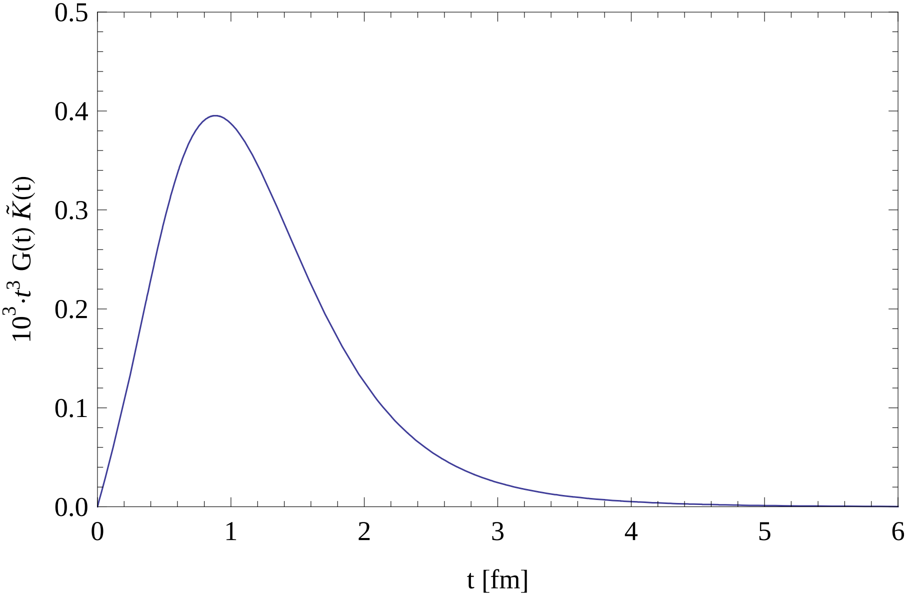
The Euclidean correlator is related to the R-ratio measured in e^+e^- collisions via (1107.4388)

$$G^{\gamma\gamma}(x_0) = \frac{1}{12\pi^2} \int_0^\infty d\omega\, \omega^2 e^{-\omega|x_0|} R(\omega^2).$$

- if exclusive channels are measured experimentally, can do flavor separation in a model independent way
- introduce $R_1(s)$ defined as $R(s)$, but final state required to be **isovector**,

$$G^{\rho\rho}(x_0) = \frac{1}{12\pi^2} \int_0^\infty d\omega\, \omega^2 e^{-\omega|x_0|} R_1(\omega^2).$$

Integrand for $a_\mu^{\rm HLO}$ (using pheno. R, 1107.4388)

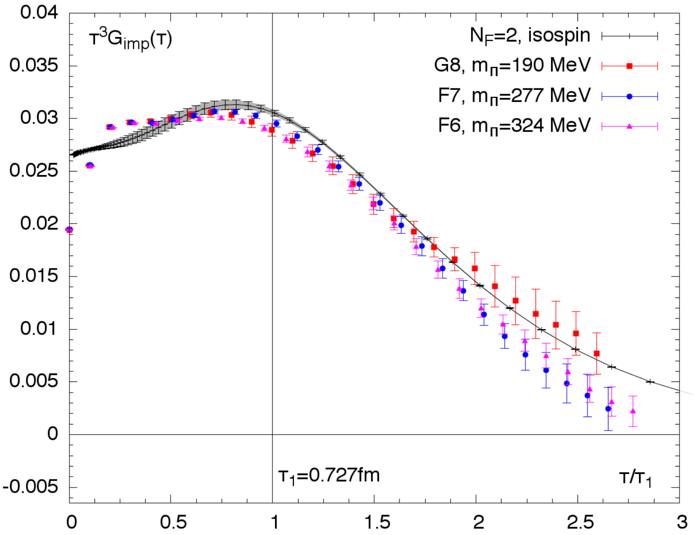


CLS Ensembles

β	label	lat. dimensions	$m_\pi[\text{MeV}]$
5.5	N_5	$48^3 \times 96$	440
	N_6	$48^3 \times 96$	340
	O_7	$64^3 \times 128$	270
5.3	F_6	$48^3 \times 96$	310
	F_7	$48^3 \times 96$	270
	G_8	$64^3 \times 128$	190
5.2	A_4	$32^3 \times 64$	380
	A_5	$32^3 \times 64$	330

$m_\pi L > 4$ in all ensembles.

Comparison of $\tau^3 G_1(\tau)$ with phenomenology 1312.0035



Pheno. flavor separation by F. Jegerlehner

Disconnected diagrams: theory expectations

The contribution of massless Wick-disconnected diagrams is known to the first order where they contribute, namely α_s^3. In the R-ratio, the contribution is

$$R(s)|_{\rm disc} = -1.240\Big(\sum_f Q_f\Big)^2 \,\left(\frac{\alpha_s}{\pi}\right)^3 .$$

As a rough estimate, we can treat α_s as a constant and start the integration from some minimal frequency ω_0,

$$G^l_{\rm disc}(x_0) \quad \simeq \quad \frac{-1.240}{12\pi^2}\left(\frac{\alpha_s}{\pi}\right)^3 \int_{\omega_0}^\infty d\omega\,\omega^2\, e^{-\omega|x_0|}$$

If we set $\omega_0 = 6{\rm fm}^{-1} = 1.18{\rm GeV}$, $\alpha_s/\pi = 0.09$ we get

$$|x_0|^3\cdot G^l_{\rm disc}(x_0=0.25{\rm fm}) \simeq -1.2\cdot 10^{-5} \simeq -2.4\cdot 10^{-4}\,|x_0|^3\cdot G^l_{\rm conn}(x_0).$$

NB. The absolute statistical error on this quantity is currently about 10^{-3}
$\Rightarrow$ no signal.

Disc. contribution to $G^{\gamma\gamma}$: both OZI rule and $SU(3)_{\rm f}$ breaking

Short distances: The correlator is expected to be order

$$G^{ls}_{\rm disc}(x_0) \sim \alpha_s^3(1/x_0) \cdot \frac{m_s^2 - m_l^2}{x_0}$$

As far as I know, the coefficient is at present unknown; mass effects in the vector correlator have been computed up to order α_s^2 (Chetyrkin et al. arXiv:hep-ph/9704222).

The fact that the disc. contribution is $SU(3)_{\rm f}$ breaking means that they are even more suppressed at short distances.

Disconnected diagrams at long distances

Recall

$$G^{\gamma\gamma}(x_0) \quad = \quad \frac{10}{9}G^{\rho\rho}(x_0) + \frac{1}{9}G^{s}_{\rm conn}(x_0) + \frac{1}{9}G^{ls}_{\rm disc}(x_0).$$

Simple observation (1306.2532): at late Eucl. time, 2π states dominate $\Rightarrow I = 1$ piece dominates,

$$G^{\gamma\gamma}(x_0) = G^{\rho\rho}(x_0)\Big(1 + \mathrm{O}(e^{-m_\pi x_0})\Big).$$

In that regime,

$$G^{ls}_{\rm disc}(x_0) = -\Big(G^{s}_{\rm conn}(x_0) + G^{\rho\rho}(x_0)\Big)\Big(1 + \mathrm{O}(e^{-m_\pi x_0})\Big).$$

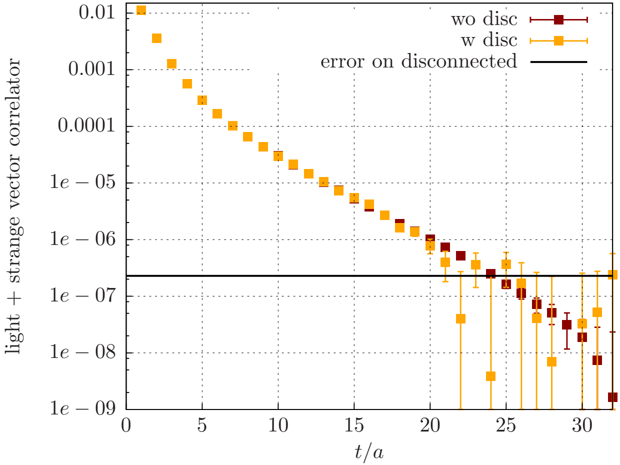
For $x_0 \to \infty$, the disconnected diagrams are of the same order as the connected.

- at what x_0 is this regime reached?
- what is the contribution of this regime to $a_\mu^{\rm HLO}$?

See A. Jüttner and M. Della Morte JHEP 11 (2010) 154 for an estimate, using ChPT.

Lattice data (Vera Gülpers et al.)

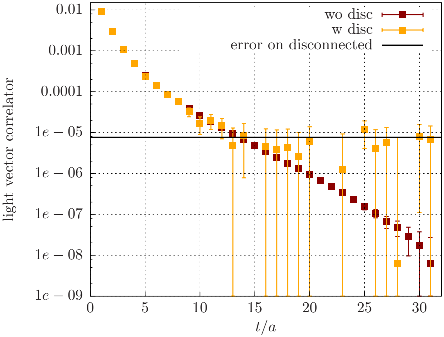
$m_\pi = 450\,\text{MeV} \qquad 15a \simeq 1\,\text{fm}$



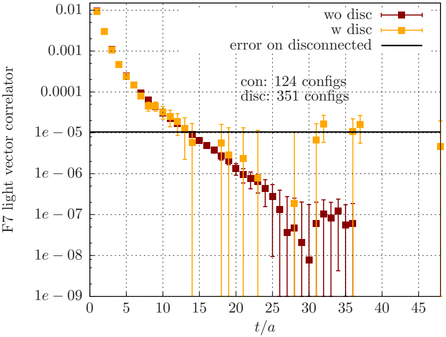
NB. ∼1000 configurations, time-diluted stochastic sources, 3 per time-slice

Quark mass dependence G^l_{conn} and G^l_{disc} (light-quark sector)

$m_\pi = 450\,\mathrm{MeV}$



$m_\pi = 275\,\mathrm{MeV}$



$15a \simeq 1\,\mathrm{fm}$

Can we estimate the disconnected contribution?

Recall

$$G^{\gamma\gamma}(x_0) \quad = \quad \frac{10}{9} G^{\rho\rho}(x_0) + \frac{1}{9} G^{s}_{\rm conn}(x_0) + \frac{1}{9} G^{ls}_{\rm disc}(x_0).$$

Contribution of disc. diagrams relative to $G^{\rho\rho}(x_0)$:

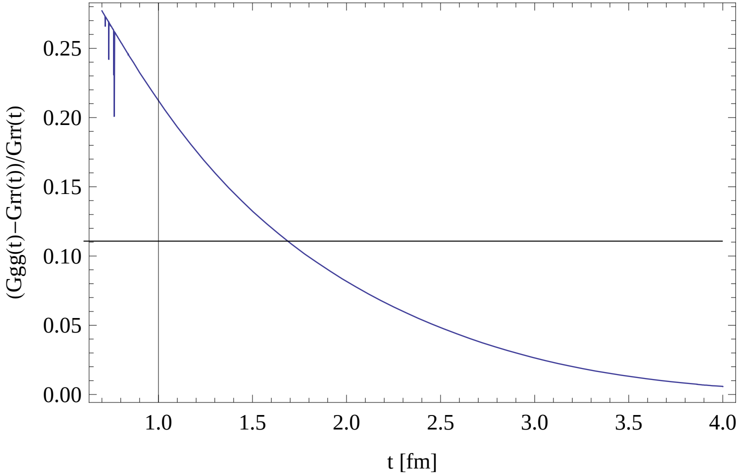
$$\frac{1}{9}\frac{G^{ls}_{\rm disc}(x_0)}{G^{\rho\rho}(x_0)} = \frac{G^{\gamma\gamma}(x_0) - G^{\rho\rho}(x_0)}{G^{\rho\rho}(x_0)} - \frac{1}{9}\left(1 + \frac{G^{s}_{\rm conn}(x_0)}{G^{\rho\rho}(x_0)}\right) \stackrel{x_0\to\infty}{\longrightarrow} -\frac{1}{9}.$$

- up to 1fm or so, direct lattice calculation shows that the disc. diagrams are negligible
- at long distances, take the first term on the RHS from phenomenology,

$$\frac{G^{\gamma\gamma}(x_0) - G^{\rho\rho}(x_0)}{G^{\rho\rho}(x_0)} = \frac{\int_0^\infty d\omega\,\omega^2\,(R(\omega^2) - R_1(\omega^2))\,e^{-\omega x_0}}{\int_0^\infty d\omega\,\omega^2\,R_1(\omega^2)\,e^{-\omega x_0}}.$$

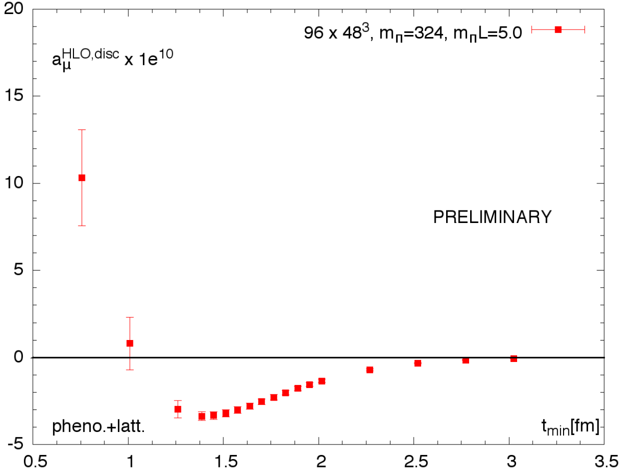
- second term from the lattice

Pheno. estimate of the first term $\frac{G^{\gamma\gamma}(x_0)-G^{\rho\rho}(x_0)}{G^{\rho\rho}(x_0)}$



Lesson: the asymptotic regime where $\frac{G^{ls}_{\rm disc}(x_0)}{G^{\rho\rho}(x_0)} = -1$ is gradually reached between $x_0 = 2\,$fm and 4fm.

Estimate of the disconnected diagram contribution to $a_\mu^{\rm HLO}$



$$a_{\mu,\rm disc}^{\rm HLO} \simeq 4\alpha^2 m_\mu \int_{t_{\rm min}}^{\infty} dt\, \tilde{K}(t)\, t^3 \left[G^{\rho\rho}(t) \left(\frac{G^{\gamma\gamma}(t) - G^{\rho\rho}(t)}{G^{\rho\rho}(t)} - \frac{1}{9} \right)_{\rm pheno} - \frac{1}{9} G^s_{\rm conn}(t) \right]$$

Outlook

- disconnected diagram contributions are very small compared to the connected diagrams up to fairly long distances
- however, the ratio becomes $-1/9$ of the isovector part at long distances.
- for the time being, the transition between the two regimes can be studied by combining lattice and experimental data
- it appears very unlikely that the disconnected diagram contribution to $a_\mu^{\rm HLO}$ is more than 3%.