SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

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mitp
Mainz Institute for Theoretical Physics

Summary: Hadronic light-by-light contribution to $(g-2)_\mu$

Marc Vanderhaeghen (JGU Mainz)

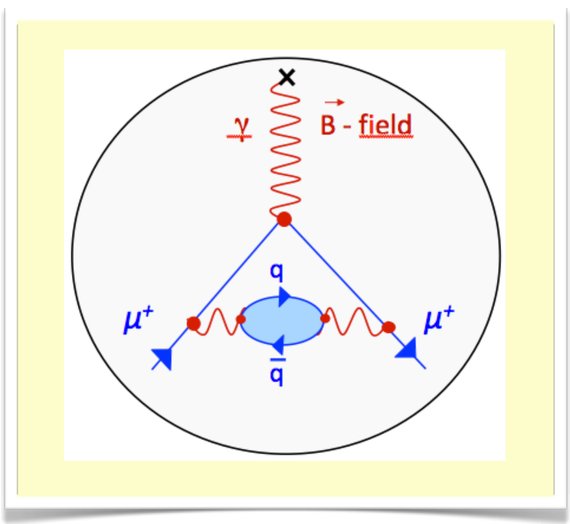
MITP $(g-2)_\mu$ workshop: Schloss Waldthausen, Mainz, April 1-5, 2014

Aim: constraining the hadronic corrections to $(g-2)_\mu$

$a_\mu^{exp} - a_\mu^{SM} = (24.9 \pm 8.7) \cdot 10^{-10}$ (3 σ)

New FNAL $(g-2)_\mu$ expt. (2016):
$\delta a_\mu^{exp} = 1.6 \times 10^{-10}$

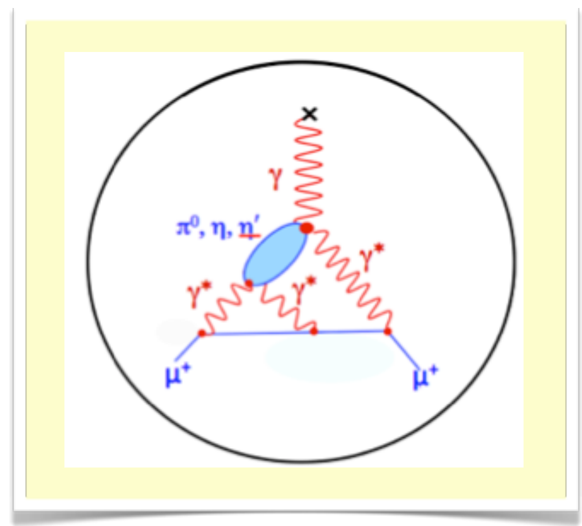
hadronic vacuum polarization



$a_\mu^{had, VP} = (692.3 \pm 4.2) \times 10^{-10}$

cross section measurements of
$e^+ e^-$ -> hadrons

hadronic light-by-light scattering

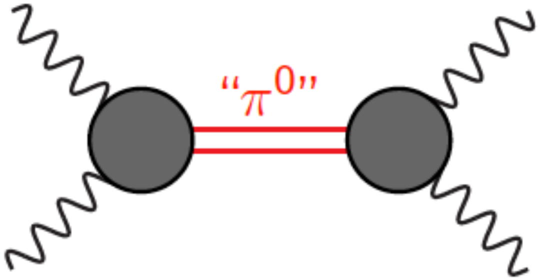


$a_\mu^{had, LbL} = (11.6 \pm 4.0) \times 10^{-10}$

meson transition FF measurements
and theory developments

π^0 exchange

Bijnens, Nyffeler



- BPP: Bijnens et al. $a_\mu^{\pi^0} = 5.9(0.9) \times 10^{-10}$
- Nonlocal quark model: $a_\mu^{\pi^0} = 6.27 \times 10^{-10}$

  A. E. Dorokhov, W. Broniowski, Phys.Rev.**D78** (2008)073011. [0805.0760]
- DSE model: $a_\mu^{\pi^0} = 5.75 \times 10^{-10}$

  Goecke, Fischer and Williams, Phys.Rev.D**83**(2011)094006[1012.3886]
- LMD+V: $a_\mu^{\pi^0} = (5.8 - 6.3) \times 10^{-10}$

  M. Knecht, A. Nyffeler, Phys. Rev. **D65**(2002)073034, [hep-ph/0111058]
- Formfactor inspired by AdS/QCD: $a_\mu^{\pi^0} = 6.54 \times 10^{-10}$

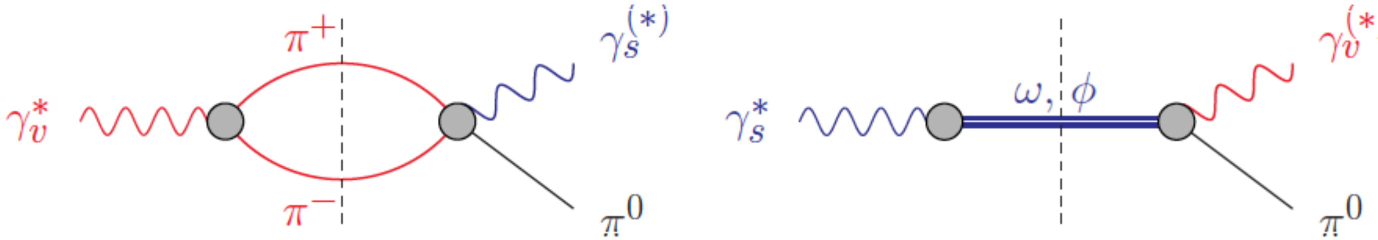
  Cappiello, Cata and D'Ambrosio, Phys.Rev.D**83**(2011)093006 [1009.1161]
- Chiral Quark Model: $a_\mu^{\pi^0} = 6.8 \times 10^{-10}$

  D. Greynat and E. de Rafael, JHEP **1207** (2012) 020 [1204.3029].
- Constraint via magnetic susceptibility: $a_\mu^{\pi^0} = 7.2 \times 10^{-10}$

  A. Nyffeler, Phys. Rev. D **79** (2009) 073012 [0901.1172].
- All in reasonable agreement

dispersive analysis of $\pi^0 \to \gamma^* \gamma^*$ transition FF

B. Kubis



- isovector photon: 2 pions

  $\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$

  all determined in terms of pion–pion P-wave phase shift

  + Wess–Zumino–Witten anomaly for normalisation

- isoscalar photon: 3 pions

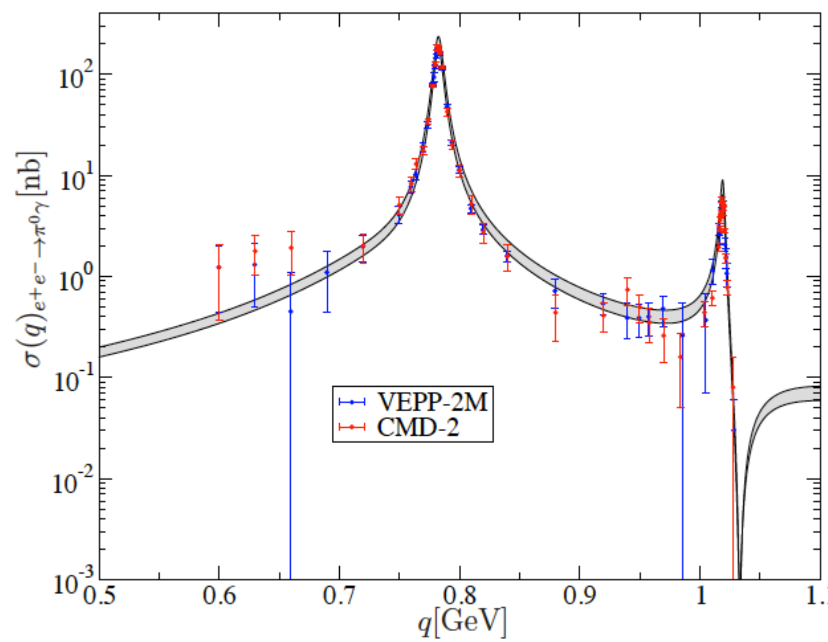
  dominated by narrow resonances ω, ϕ

comparision to $e^+e^- \to \pi^0\gamma$ data

Hoferichter, Kubis, Leupold, Niecknig, Schneider

enters the $\pi^0 \to e^+e^-$ rare decay

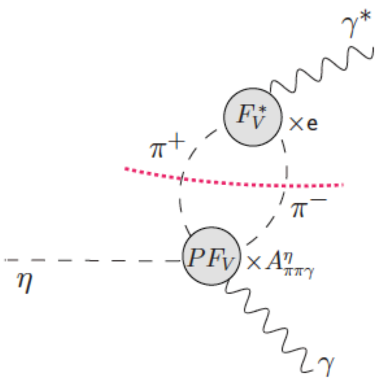
P. Sanchez Puertas



Allows for parameter free prediction for isovector part of slope

$\eta \to \gamma^* \gamma$

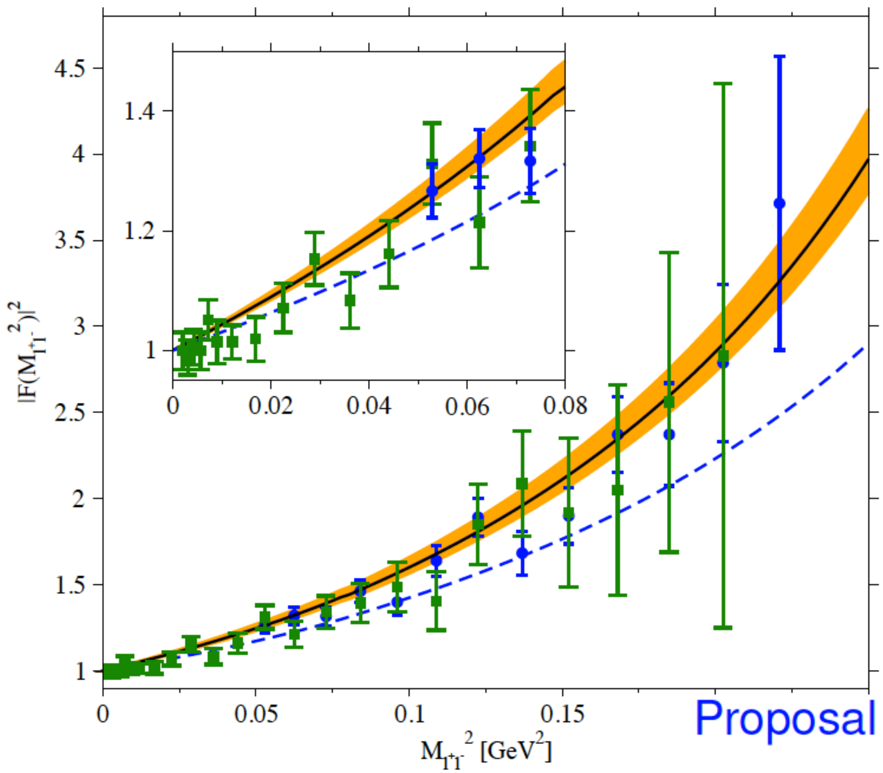
Ch. Hanhart



$$F_{\eta\gamma^\star\gamma}(Q^2,0) \equiv 1 + \Delta F^{(I=1)}_{\eta\gamma^\star\gamma}(Q^2,0) + \Delta F^{(I=0)}_{\eta\gamma^\star\gamma}(Q^2,0)$$

$$\Delta F^{(I=1)}_{\eta\gamma^\star\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^\infty ds'\, \sigma_\pi(s')^3\, P(s')\, \frac{|F_V(s')|^2}{s'-Q^2-i\epsilon}$$

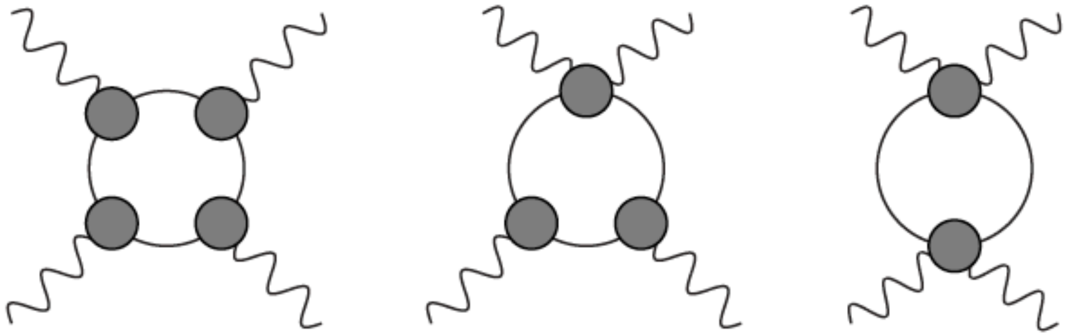
$\Delta F^{(I=0)}_{\eta\gamma^\star\gamma}$ needs to be modeled



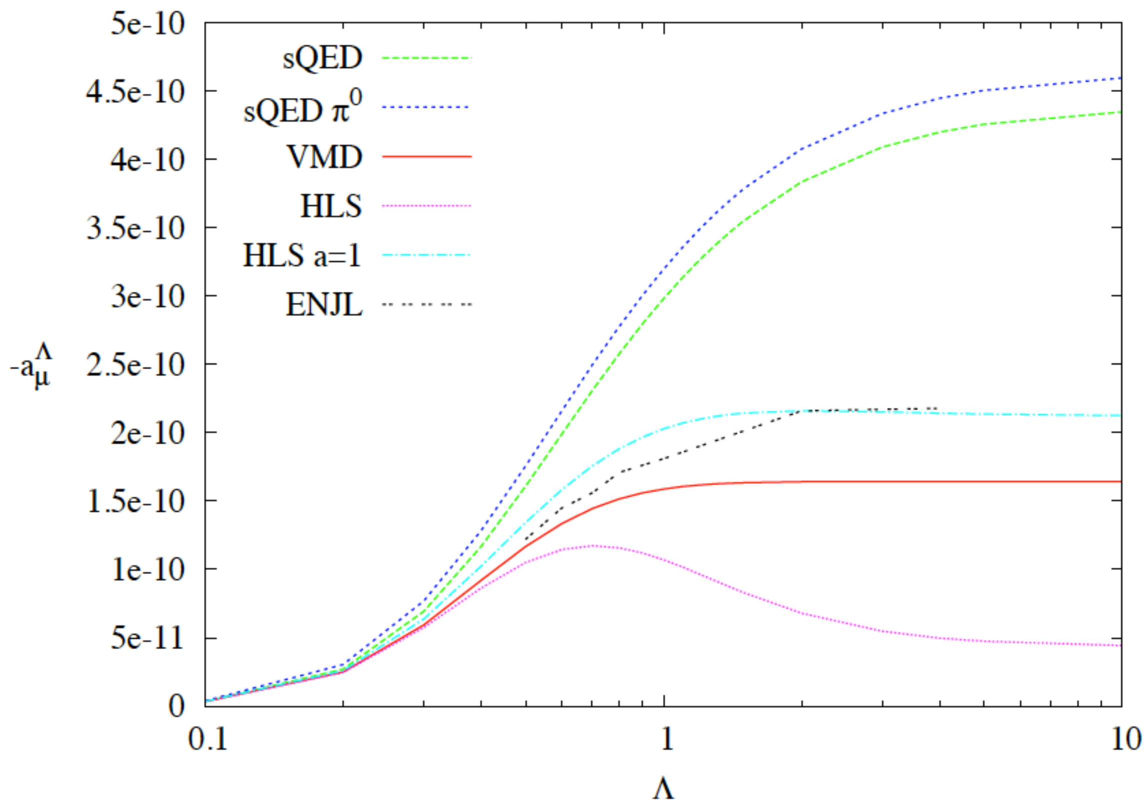
Proposal to experiment: measure $e^+e^- \to \eta\pi^+\pi^-$

The method outlined here allows one to relate this to the isovector piece of $\eta \to \gamma^*\gamma^*$

π loop



- A bare π-loop (sQED) give about $-4 \cdot 10^{-10}$
- The $\pi\pi\gamma^*$ vertex is always done using VMD
- $\pi\pi\gamma^*\gamma^*$ vertex two choices:
  - Hidden local symmetry model: only one γ has VMD
  - Full VMD
  - Both are chirally symmetric
  - $\pi\pi\gamma^*\gamma^*$ for $q_1^2 = q_2^2$ has a short-distance constraint from the OPE as well.
  - HLS does not satisfy it
  - full VMD does: so probably better estimate
- For BPP stopped at 1 GeV but within 10% of higher Λ



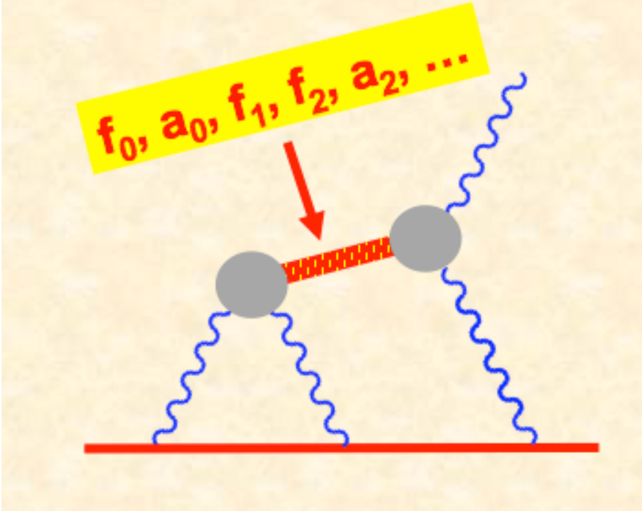
H. Bijnens

$P_1, P_2, Q \leq \Lambda$

- $L_9 + L_{10} \neq 0$ gives an enhancement of 10-15% Bijnens, Zahiri Abyaneh
- To do it fully need to get a model: include a_1
- But all models with reasonable L_9 and L_{10} fall way inside the error quoted earlier $(-1.9 \pm 1.3)\ 10^{-10}$
- Tentative conclusion: Use hadrons only below about 1 GeV: $a_\mu^{\pi-\mathrm{loop}} = (-2.0 \pm 0.5)\ 10^{-10}$

Contribution	HKS	BPP	KN	MV	PdRV	N/JN
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	99±16
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13
axial vectors	1.7±1.7	2.5±1.0	–	22± 5	15±10	22± 5
scalars	–	−6.8±2.0	–	–	−7± 7	−7± 2
quark loops	9.7±11.1	21± 3	–	–	2.3	21± 3
total	89.6±15.4	83±32	80±40	136±25	105±26	116±39

axial-vector, scalar, tensor meson contributions



➡ **Axial vector** meson contribution re-evaluated V. Pauk, F. Jegerlehner

Landau-Yang theorem constraint built in correctly

Use available data on $f_1(1285)$, $f_1(1420)$

$$a_\mu[f_1', f_1] \sim (6.4 = [5.0 + 1.4] \pm 2.0) \times 10^{-11}$$

Pauk, Vdh (2013)

$$a_\mu[a_1, f_1', f_1] \sim (7.55 = [1.89 + 5.19 + 0.47] \pm 2.71) \times 10^{-11}$$

Jegerlehner

➡ **Tensor** meson contributions evaluated

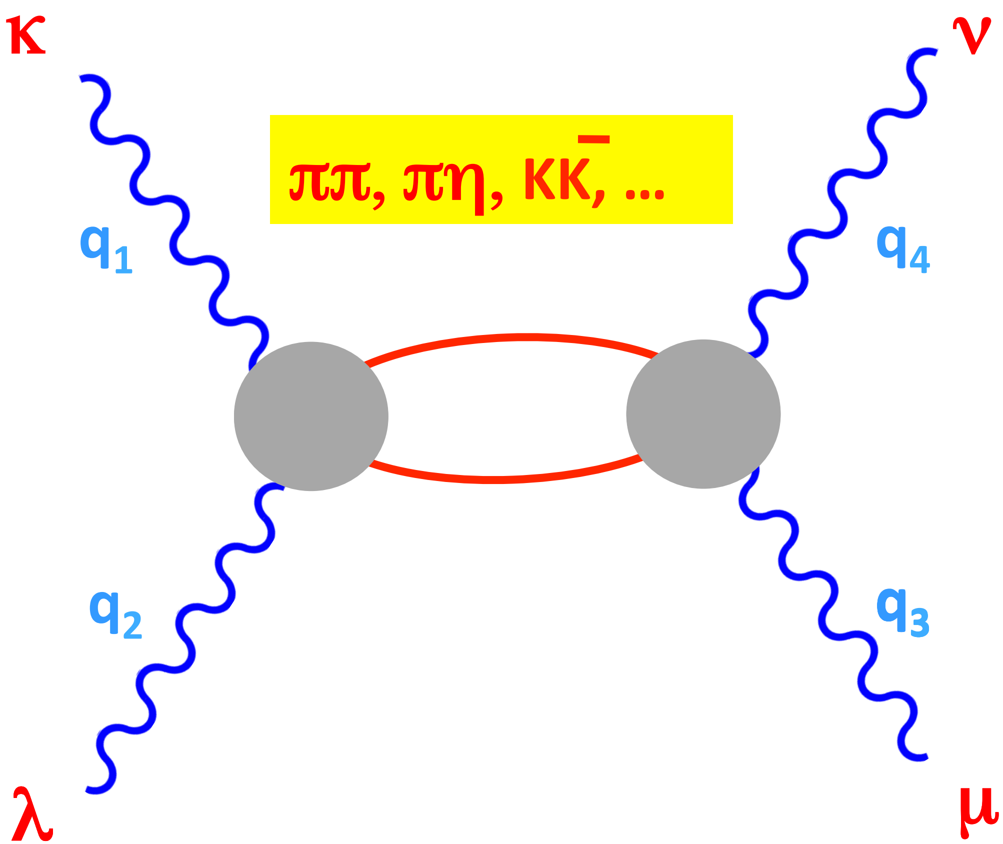
$$a_\mu[f_2', f_2, a_2', a_2] \sim (1.1 = [0.79 + 0.07 + 0.22 + 0.02] \pm 0.1) \times 10^{-11}$$

$$a_\mu[a_0, f_0', f_0] \sim (-3.1 = [-0.63 - 1.84 - 0.61] \pm 0.8) \times 10^{-11}$$

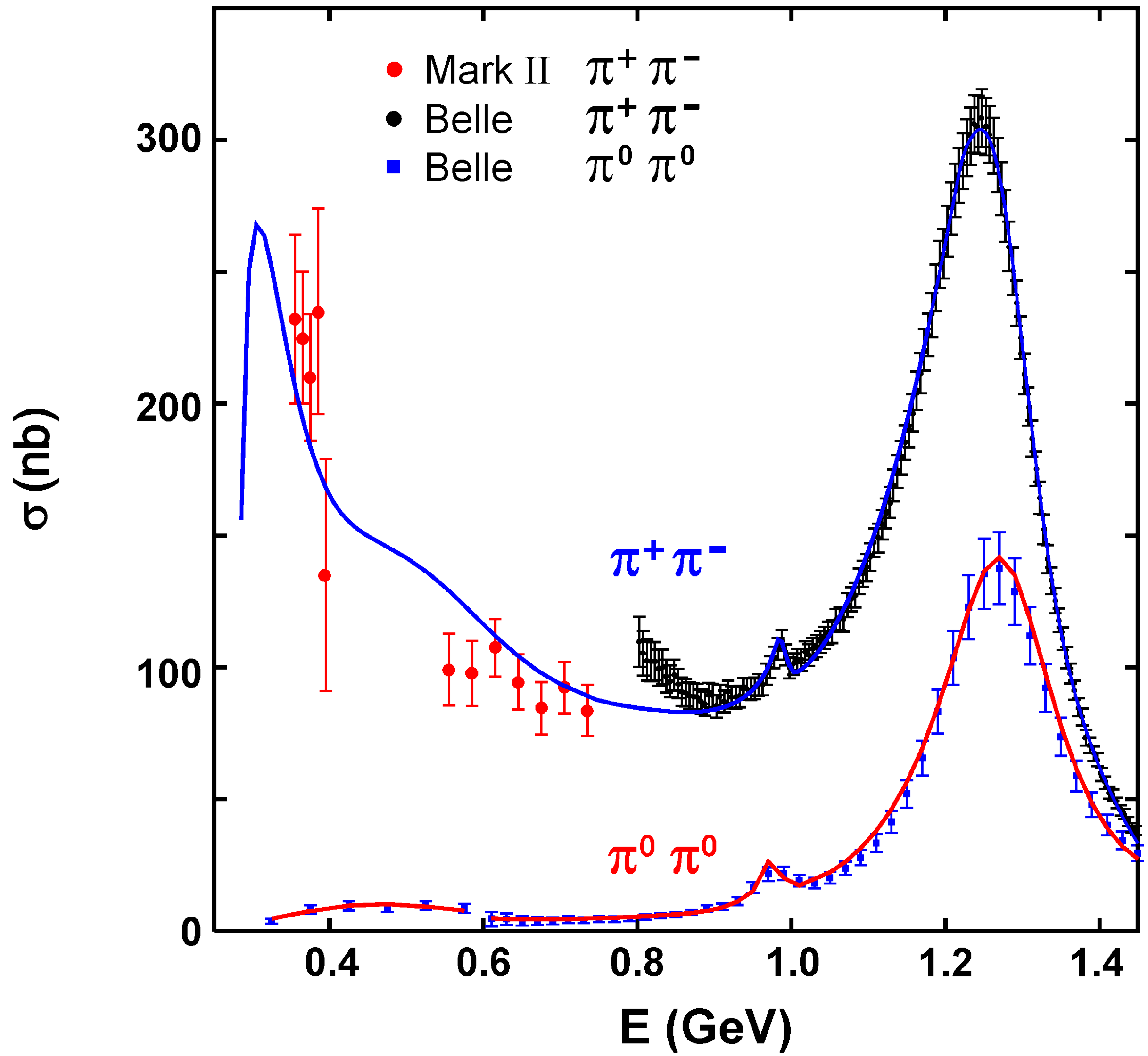
2-meson channels:

Input discontinuity into dispersion relations: γγ -> ππ, ...

M. Pennington



Integrated cross-section

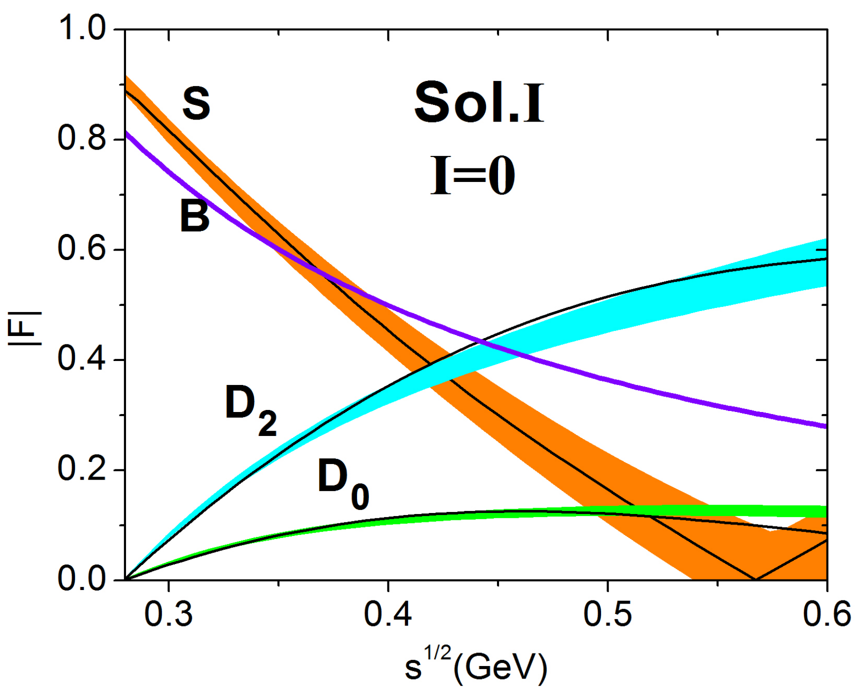


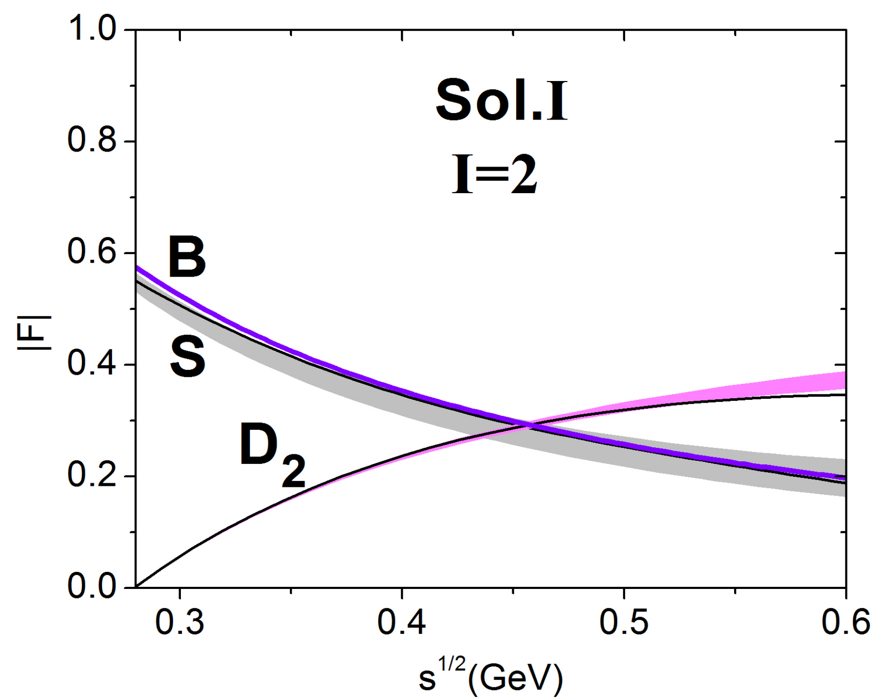
dispersion analysis: γγ -> ππ

new calculation
reduces bands
of uncertainty

Dai & Pennington

using Pelaez et al
inputs





Unusual feature: large D-waves near threshold, I=2 as large as I=0

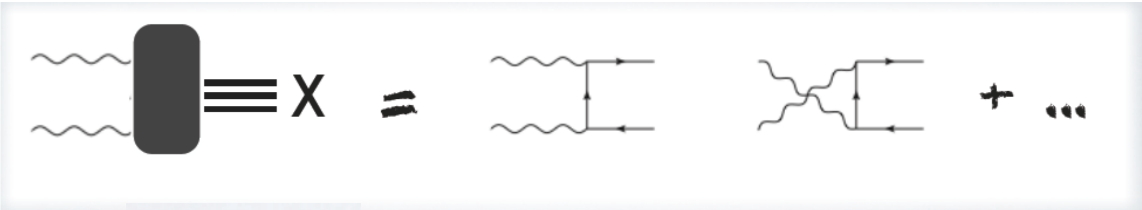
constraints from forward LbL sum rules

Pascalutsa, Pauk, Vdh (2011,2012)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_0 - \sigma_2\right]_{Q_2^2=0},$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_\parallel + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0},$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2}\right]_{Q_2^2=0}.$$



non-PT

cancellation of (pseudo)scalar and tensor meson contributions

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds\, \Delta\sigma/s$ [nb]
π^0	134.98	$(7.8 \pm 0.6) \times 10^{-3}$	-195.0 ± 15.0
η	547.85	0.51 ± 0.03	-190.7 ± 11.2
η'	957.66	4.30 ± 0.15	-301.0 ± 10.5
Sum η, η'			-492 ± 22

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds\, \Delta\sigma/s$ narrow res. [nb]	$\int ds\, \Delta\sigma/s$ Breit-Wigner [nb]
$a_2(1320)$	1318.3	1.00 ± 0.06	134 ± 8	137 ± 8
$f_2(1270)$	1275.1	3.03 ± 0.35	448 ± 52	479 ± 56
$f_2'(1525)$	1525	0.081 ± 0.009	7 ± 1	7 ± 1
Sum f_2, f_2'			455 ± 53	486 ± 57

$$\int_{4m_\pi^2}^{\infty} \frac{ds}{s} \left[\sigma_2 - \sigma_0 \right] = 0$$

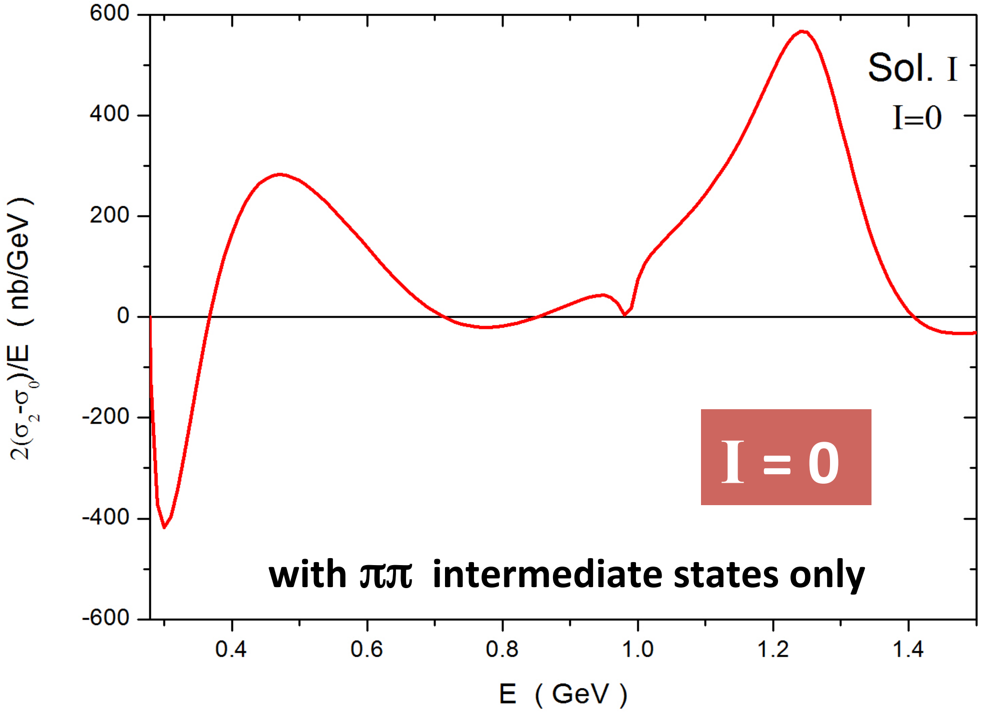
define
$\bar{\sigma}_\lambda = \sigma_\lambda - B_\lambda$

$$\int_{2m_\pi}^{\infty} \frac{2dE}{E} \left[\bar{\sigma}_2 - \bar{\sigma}_0 \right] = 0$$

for each hadronic isospin, I

Dai, Pennington

$\Delta\bar{\sigma}_\lambda$

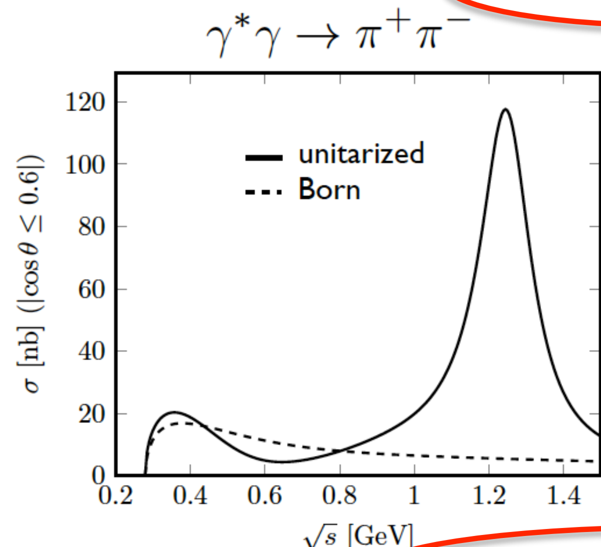


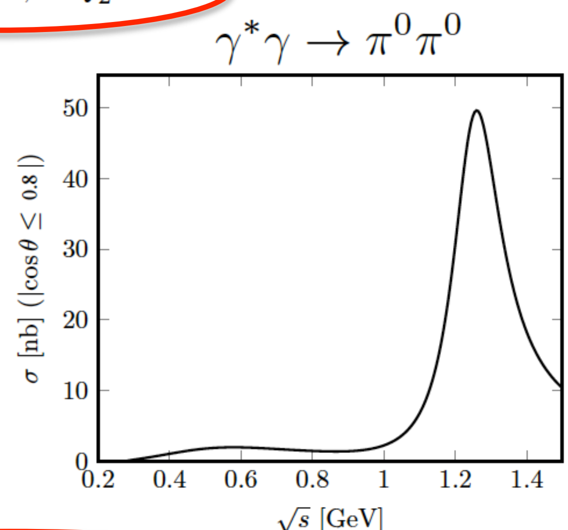
- Sum rules: may be used as a consistency test on models !
- Allow to constrain so-far unmeasured contributions, e.g. tensor mesons for virtual photons

dispersion analysis: $\gamma^*\gamma^*$ -> $\pi\pi$

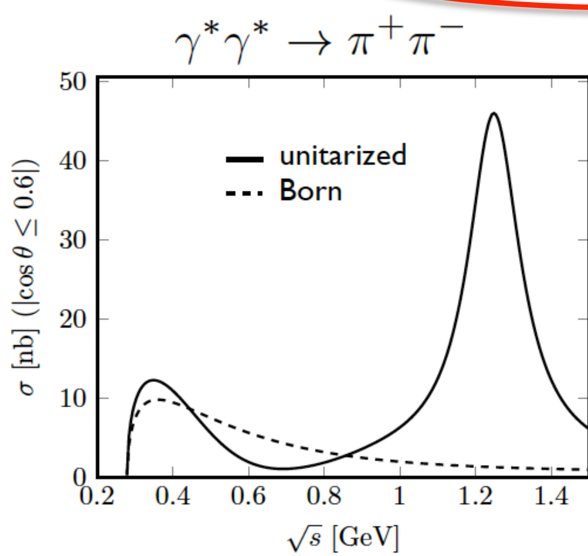
Moussallam (2013)
Hoferichter, Colangelo, Procura, Stoffer (2013)
Asmussen, Masjuan, Vdh (2014)

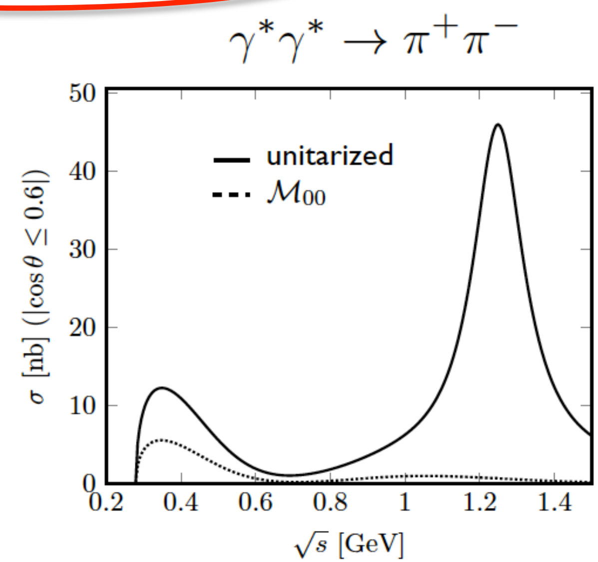
$Q_1^2 = 0.5\ \text{GeV}^2\,, \quad Q_2^2 = 0$





$Q_1^2 = 0.5\ \text{GeV}^2\,, \quad Q_2^2 = 0.5\ \text{GeV}^2$



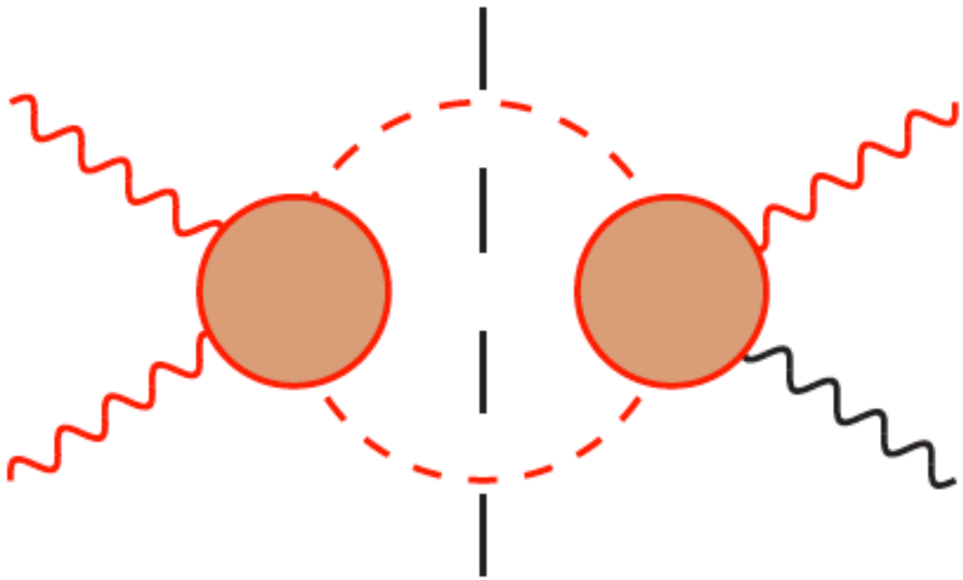


data needed !

Dispersive approach to HLbL

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$



Master formula for a_μ

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s((p+q_1)^2 - m^2)((p-q_2)^2 - m^2)},$$

$$I^{\pi\pi} = \sum_{i\in\{1,2,3,6,14\}} \left(T_{i,s} I_{i,s} + 2T_{i,u} I_{i,u} \right) + 2T_{9,s} I_{9,s} + 2T_{9,u} I_{9,u} + 2T_{12,u} I_{12,u}$$

with $I_{i,(s,u)}$ dispersive integrals and $T_{i,(s,u)}$ integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \left(\frac{1}{s'-s} - \frac{s' - q_1^2 - q_2^2}{\lambda(s', q_1^2, q_2^2)} \right) \mathrm{Im}\bar{h}^0_{++,++}(s'; q_1^2, q_2^2; s, 0),$$

$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s' - q_1^2 - q_2^2)(s'-s)^2} \mathrm{Im}\bar{h}^2_{+-,+-}(s'; q_1^2, q_2^2; s, 0) \left(\frac{75}{8} \right)$$

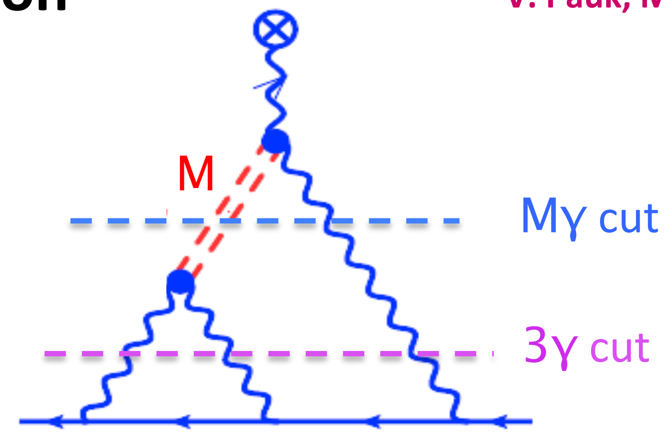
Helicity amplitudes contribute up to $J = 2$ (S and D waves)

a_μ : dispersion relation

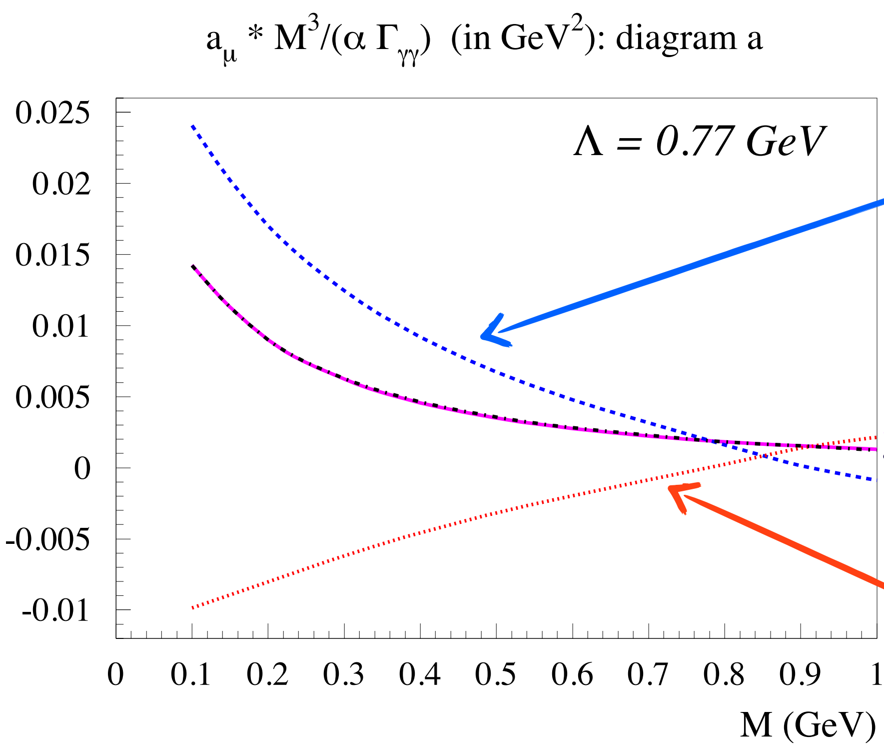
V. Pauk, M.Vdh

$$a_\mu = F_2(0)$$

$$F_2(0) = \frac{1}{2\pi i}\int \frac{dk^2}{k^2}\,\text{Abs}\,F_2(k^2)$$



Proof of principle: meson pole contribution in dispersive approach



2-particle discontinuities

dispersive evaluation

Feynman integral evaluation

3-particle discontinuities

BOTH timelike (e^+e^- -> M γ)

AND spacelike (γ* γ*-> M)

data needed as input

on-shell FF Knecht, Nyffeler (2001)

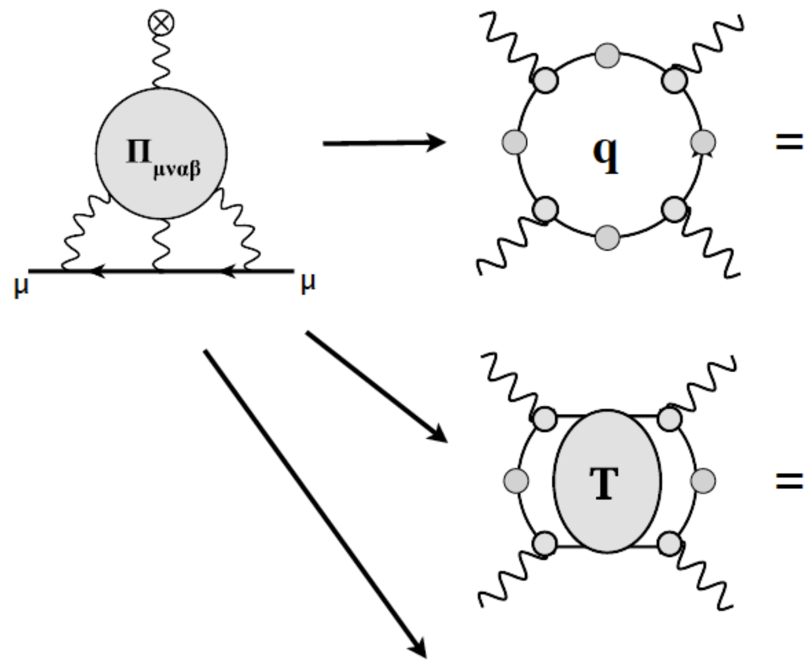
Dyson-Schwinger approach

Ch. Fischer

- calculations at present with effective model for qqg vertex
- comparison for HVP: at 5 - 10%
- for HLbL: potentially large quark loop contribution (10×10^{-10})

  vs 2×10^{-10} in ENJL

  Comparable ?

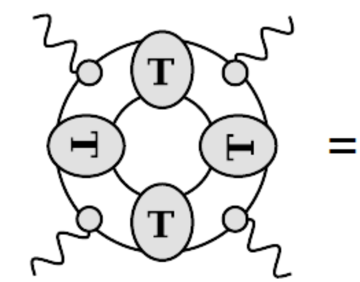


- Roadmap: cross-check the four-point correlator for specific values of virtualities with lattice results
- Check consistency constraints: e.g. sum rules for LbL

  Phenomenology: transition FFs

Derived from gauge invariance!

Goecke, CF, Williams, PRD 83 (2011) 094006
Eichmann and CF, PRD 87 (2013) 3, 036006



HLbL: Outlook

- **Goal: realistic error estimate / reduce to 2×10^{-10} (20% of HLbL)**
- **Important issue: study radiative corrections in γγ physics ,**
  make MC tools available for experiment **H. Czyz**
- **DR tools become available for PS TFF, $\gamma^*\gamma^*$ -> ππ,..., a_μ**

  require close collaboration with experiment
- **Outcome of workshop would be to draft a roadmap for HLbL**
  (document which can be referred to)

  Status

  Strategy of analysis (describe tools available)

  What is needed most (data input, **quantify** required precision)