Lattice QCD: Summary and Perspective

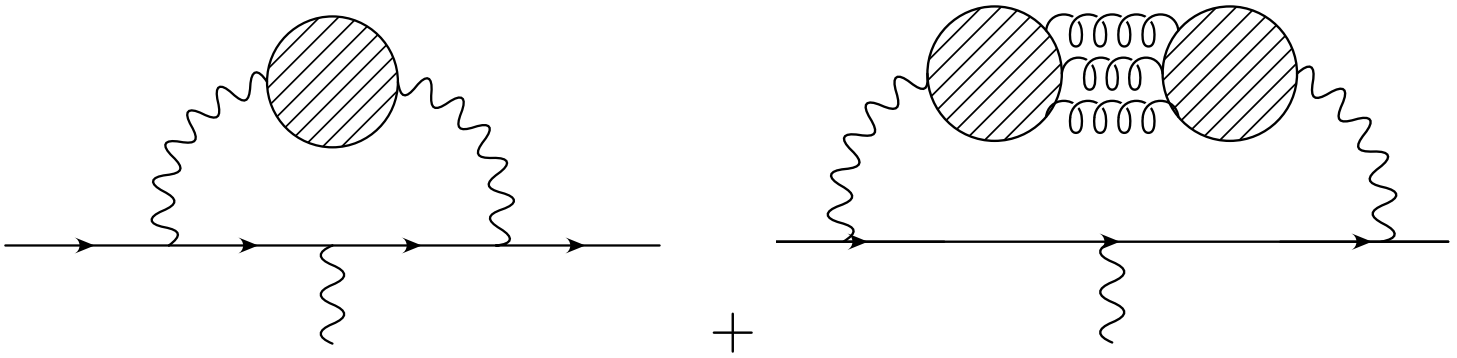
Tom Blum (UConn / RIKEN BNL Research Center)

MITP Workshop on Hadronic contributions to muon g-2
Waldthausen Castle, Mainz

April 1-5, 2014

Outline

Hadronic vacuum polarization (HVP)

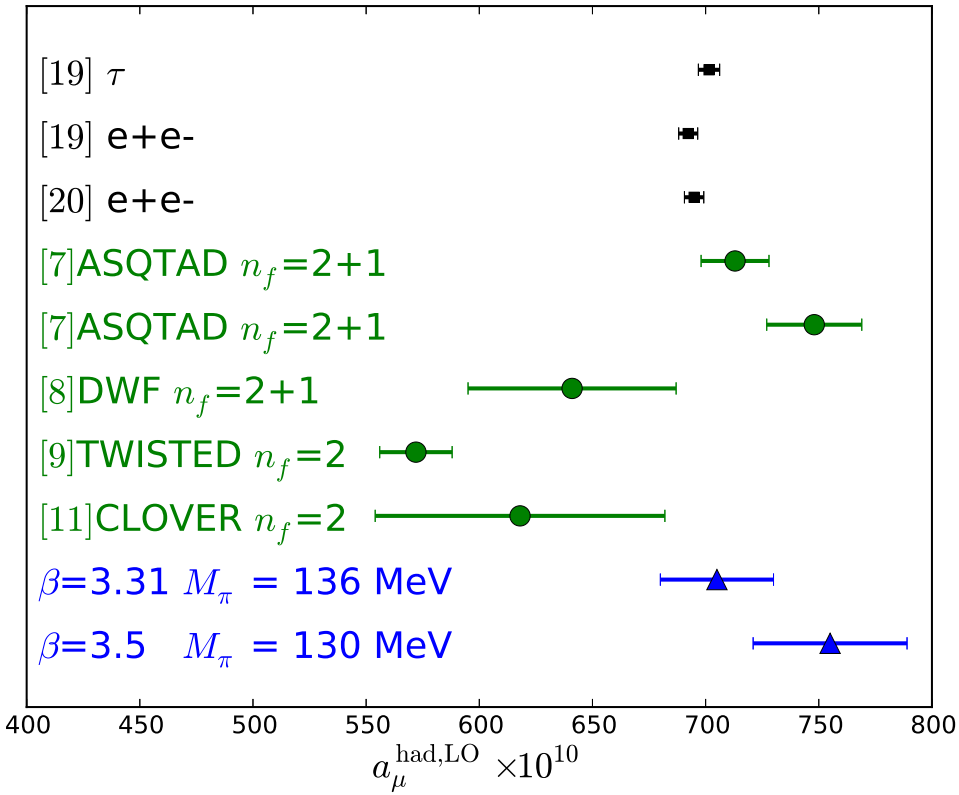


The blobs, which represent all possible intermediate hadronic states, are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for $e^+e^-(\text{and } \tau) \to \text{hadrons}$ $a_\mu^{\text{had}(2)} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\sigma_{\text{total}}(s)$
- first principles using lattice QCD, $a_\mu^{(2)\text{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\,\Pi(Q^2)$ [Lautrup and de Rafael 1969, Blum 2002]

a_μ(HVP) published lattice results

Lattice: $\sim$ 3-10% quoted errors, but incomplete, Experiment: 0.6% errors



Plot from Laurent Lellouch

Talks this week

- Santi Peris: Fits and related systematics for the Hadronic Vacuum Polarization
- Laurent Lellouch: Leading-order Hadronic Contribution to g-2
- Harvey Meyer: The Hadronic Vacuum Polarization from Lattice QCD: the Role of Quark Disconnected Contributions
- Karl Jansen: The Hadronic Vacuum Polarization Contribution Using Analytic Continuation
- Taku Izubuchi: Hadronic Light-by-Light: What can lattice QCD achieve?
- plus afternoon contributions by E. Shintani and G. Herdoiza

S. Peris (fitting systematics)

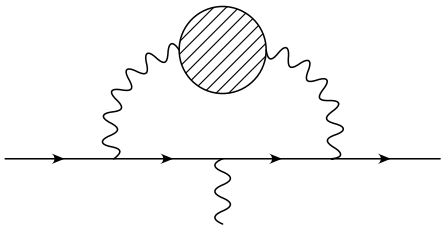
Fits and Related Systematics for the Hadronic Vacuum Polarization on the Lattice

Santi Peris (U.A. Barcelona)

Based on coll. with Christopher Aubin (Fordham U.), Tom Blum (Connecticut U.), Maarten Golterman (SFSU) & Kim Maltman (York U. & U. Adelaide)

S. Peris (fitting systematics)

Introduction



Lautrup-de Rafael '69
Blum '02

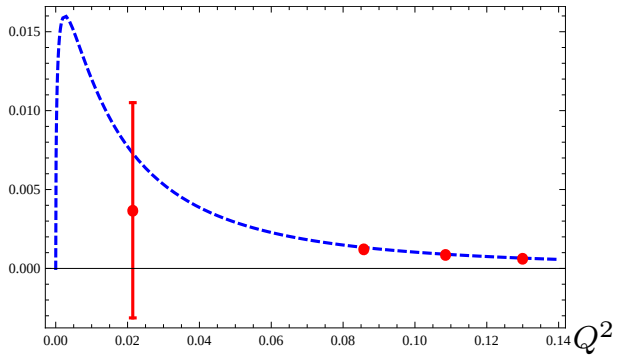
$$(g-2)_\mu^{HVP} \sim \int_0^\infty dQ^2 \underbrace{f(Q^2)}_{\text{known}} \Big[\Pi(Q^2) - \Pi(0)\Big]$$

$$\Pi(Q^2) - \Pi(0) \Longrightarrow (g-2)_\mu^{HVP}$$

★ integrand strongly peaked at $Q^2 \sim m_\mu^2/4 \sim 0.003\ \text{GeV}^2$

if no good data in region of curvature $\Longrightarrow$ possibly wrong results !

(g-2) integrand



(even with good χ^2)

Need reliable fitting function !

how to test this theoretical error?

S. Peris (fitting systematics)

Fitting functions

Aubin, Blum, Golterman, SP '12

★ Padés, model independent, they enjoy a convergence theorem for $N \to \infty$:

$$\Pi(Q^2) = \Pi(0) + Q^2 \underbrace{\left(a_0 + \sum_{r=1}^{N} \frac{a_r}{Q^2 + b_r} \right)}_{\text{Pade}}$$

$\Pi(0), a's$ and $b's$ are fitting parameters.

★ VMD is not a Pade, since you fix $b_1 = M_\rho^2$. (true $\Pi(Q^2)$ has cut starting at $4m_\pi^2$...)

We have: $a_0 \neq 0 \Longrightarrow [N, N]$ Pade; $a_0 = 0 \Longrightarrow [N-1, N]$ Pade.

For instance:

- $\frac{a_1}{Q^2+b_1}$ is a [0,1] Pade $\Longrightarrow \Pi(Q^2) = \Pi(0) + Q^2 \left(\frac{a_1}{Q^2+b_1} \right)$
- $a_0 + \frac{a_1}{Q^2+b_1}$ is a [1,1] Pade $\Longrightarrow \Pi(Q^2) = \Pi(0) + Q^2 \left(a_0 + \frac{a_1}{Q^2+b_1} \right)$

etc...

S. Peris (fitting systematics)

Model Fits

Golterman, Maltman, SP '13

"Exact result": $(g-2)_\mu^{HVP}|_{Q^2 \leq 1\ \mathrm{GeV}^2} = 1.2059 \times 10^{-7}$.

Fit interval $0 < Q^2 \leq 1\ \mathrm{GeV}^2$, (49 points).

Pull = (exact - fit) / error

	$(g-2)_\mu \times 10^7$	Error$\times 10^7$	χ^2/dof	Pull
VMD	1.3201	0.0052	2189/47	-
VMD+	1.0658	0.0076	67.4/46	18
$[0,1]$	0.8703	0.0095	285/46	-
$[1,1]$	1.116	0.022	61.4/45	4
$[1,2]$	1.182	0.043	55.0/44	0.5
$[2,2]$	1.177	0.058	54.6/43	0.5

- VMD has a bad χ^2 and $(g-2)$.
- VMD+ also gets it wrong although the χ^2 is good $\Longrightarrow$ DANGER !
- Pades [1,2] and [2,2] get it right, but the error is $\sim 4\%$.

L. Lellouch

Leading hadronic contribution to $g_\mu - 2$ and lattice QCD

Laurent Lellouch

CPT Marseille
CNRS/INP & Aix-Marseille U.

Budapest-Marseille-Wuppertal collaboration (BMWc)
Special thanks to E. Gregory, R. Malak & C. McNeile

(work in progress)

L. Lellouch

Zero-momentum moments and Padé's of $\hat{\Pi}(Q^2)$

HPQCD propose to compute on lattice (arXiv:1403.1778):

$$G_{2n} = a^4 \sum_{x_\mu} x_0^{2n} \langle J_i^{\rm EM}(x) J_i^{\rm EM}(0) \rangle$$

which give coefficients of Taylor expansion

$$\hat{\Pi}(Q^2) = \sum_{n=1}^{\infty} \Pi_n Q^{2n}, \qquad \Pi_n \equiv (-1)^{n+1} \frac{G_{2n+2}}{(2n+2)!}$$

Use to obtain Padé

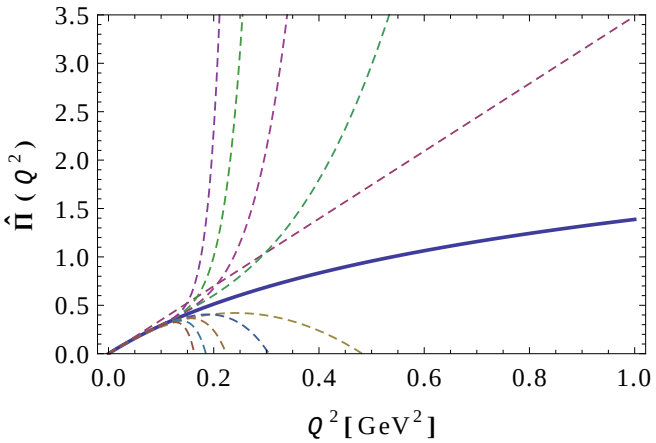
$$\hat{\Pi}_{[N/D]}(Q^2) \equiv \frac{\sum_{n=0}^{N} a_n Q^{2n}}{1 + \sum_{n=1}^{D} b_n Q^{2n}}$$

by matching term-by-term to Taylor expansion up to order $N + D$

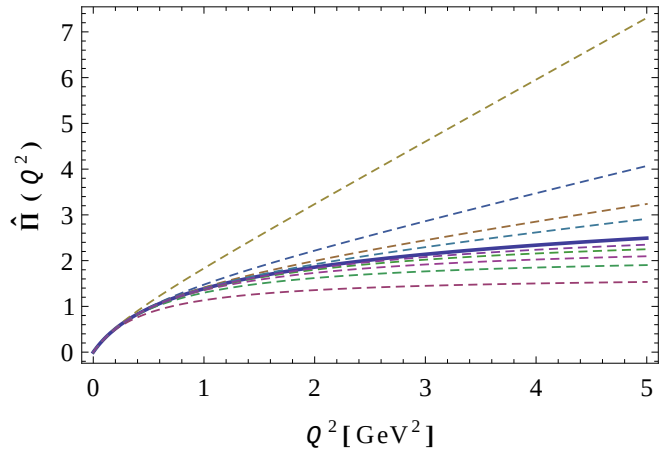
Perform calculation for connected s and c quark contributions

L. Lellouch

Convergence and residual errors of Padé's (cont'd)

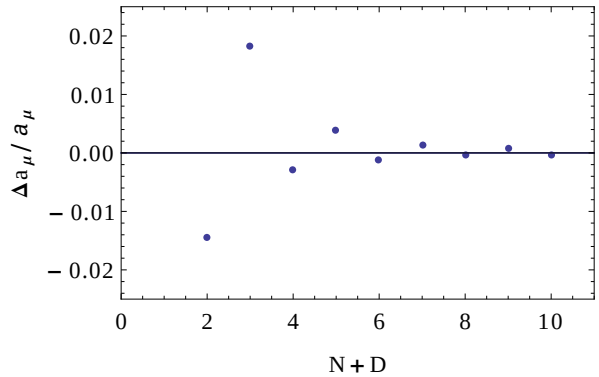


$\longrightarrow$



Taylor expansions for $N = 1, \cdots, 10$

$[N, N-1]$ & $[N, N]$ Padé's: $[1,1] \to [5,5]$



- $\Delta a_\mu / a_\mu \lesssim 0.003$ from $\hat{\Pi}_{[2,2]}(Q^2)$
- Requires G_{10}, i.e. 10th moment of $\Pi_{\mu\nu}$!

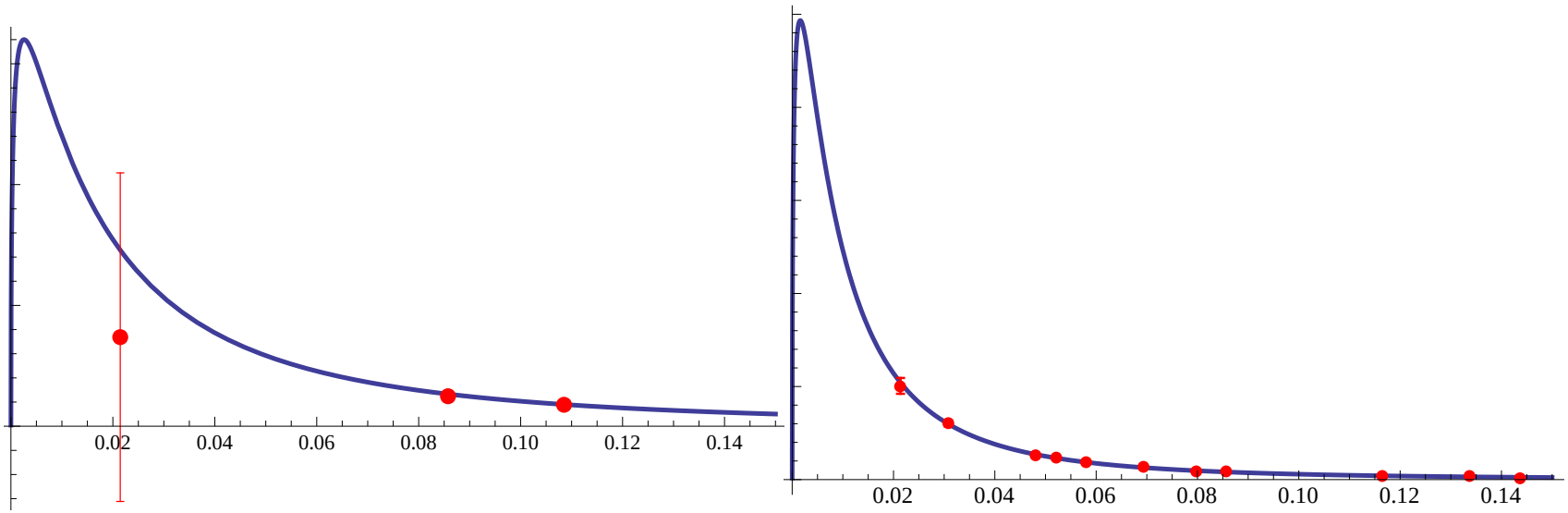
$\Rightarrow$ statistics and systematics?

$\Rightarrow$ unphysical poles, . . . ?

Working Group Discussion (fitting systematics)

- Many methods to get to small q^2/large Euclidean time. No clear best approach, should pursue all for now
  - Large volume, twisted b.c.'s, analytic continuation, mixed (time-mom) representation
- Model independent (*e.g.*, Padé) fits desirable
- Good chi-sq is not enough if extrapolating to $q = 0$
- Covariance matrix can be ill-conditioned

Working Group Discussion (fitting systematics)



Plots from Maarten Golterman. Huge improvement from AMA and twisting, but still an extrapolation!

Working Group Discussion

- Lowest moments useful for $\Pi(0)$
- How does it compare to Rome method for $\Pi(0)$ (formally based on twisting)? In practice need 4 point function
- HPQCD used successfully for strange, charm, but higher moments very noisy for light quarks
- Error reduction: subtract N_t^n contribution?

H. Meyer (disconnected quark diagrams)

On the disconnected diagram contribution to a_μ^{HLO}

Harvey Meyer

SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

HIM

SRFN

MITP "g-2" workshop, Mainz April 4, 2014

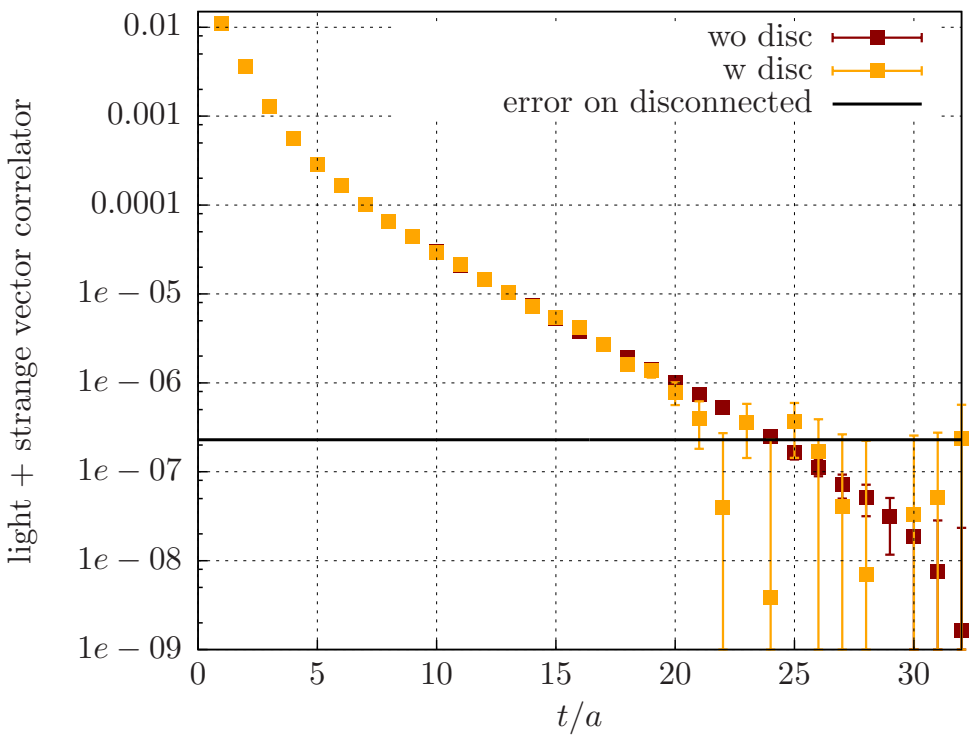
JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Thanks to members of Mainz lattice group, Anthony Francis, Vera Gülpers, Gregorio Herdoiza, Georg von Hippel, Hanno Horch, Benjamin Jäger, Hartmut Wittig

H. Meyer (disconnected quark diagrams)

Lattice data (Vera Gülpers et al.)

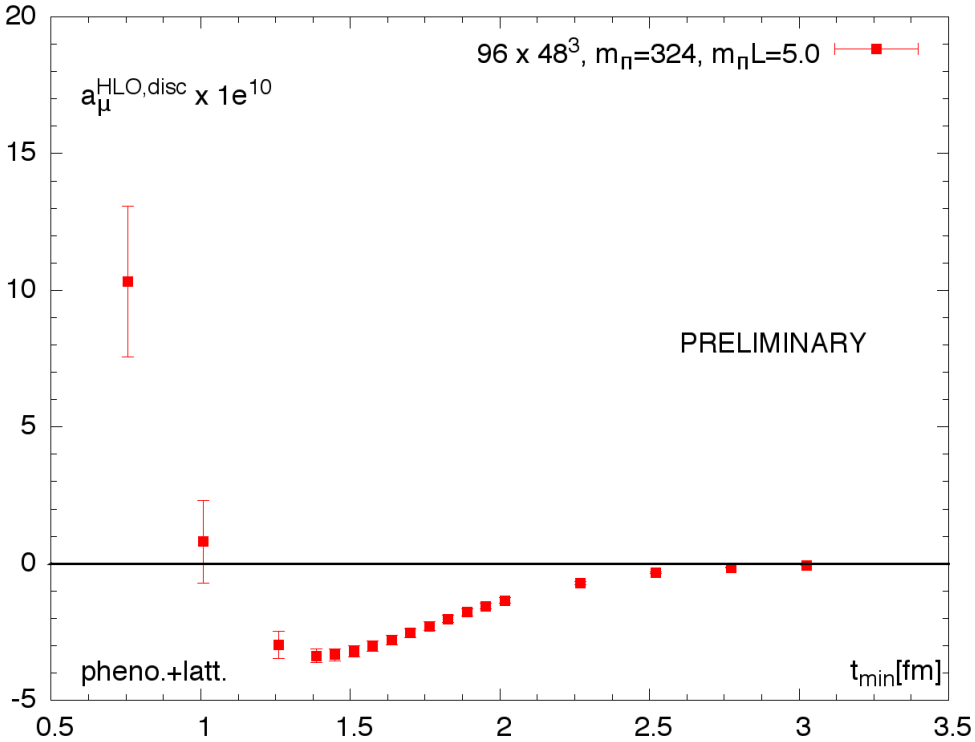
$m_\pi = 450\,\mathrm{MeV}$ $\qquad$ $15a \simeq 1\,\mathrm{fm}$



NB. ∼1000 configurations, time-diluted stochastic sources, 3 per time-slice

H. Meyer (disconnected quark diagrams)

Estimate of the disconnected diagram contribution to $a_\mu^{\rm HLO}$



$$a_{\mu,\rm disc}^{\rm HLO} \simeq 4\alpha^2 m_\mu \int_{t_{\rm min}}^{\infty} dt\, \tilde{K}(t)\, t^3 \Big[G^{\rho\rho}(t) \Big(\frac{G^{\gamma\gamma}(t) - G^{\rho\rho}(t)}{G^{\rho\rho}(t)} - \frac{1}{9} \Big)_{\rm pheno} - \frac{1}{9} G^{s}_{\rm conn}(t) \Big]$$

H. Meyer (disconnected quark diagrams)

Outlook

- disconnected diagram contributions are very small compared to the connected diagrams up to fairly long distances
- however, the ratio becomes $-1/9$ of the isovector part at long distances.
- for the time being, the transition between the two regimes can be studied by combining lattice and experimental data
- it appears very unlikely that the disconnected diagram contribution to a_μ^{HLO} is more than 3%.

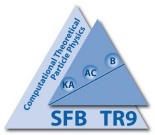
Working Group Discussion (disconnected diagrams)

- Difficult! Mainz group: 1000 configs * 64 time slices * 3 hits $\approx$ 200,000 measurements, no signal, but (absolute) error is also very small.
- Contribution is small. HM: conservatively less than 3%. We are comfortable with this estimate
- Discussion of noise reduction ideas: link smearing?

K. Jansen (analytic continuation)

Using analytical continuation for $a_\mu^{\rm hvp}$

Karl Jansen



in collaboration with

Xu Feng, Shoji Hashimoto, Grit Hotzel, Marcus Petschlies, Dru Renner

- **Status of standard $a_\mu^{\rm hvp}$ calculation**
- **Analytical continuation**
- **Example of $a_\mu^{\rm hvp}$**
- **Conclusion**

K. Jansen (analytic continuation)

Alternative method: analytic continuation

Compute HVP function via analytic continuation

$$\bar{\Pi}(K^2)(K_\mu K_\nu - \delta_{\mu\nu} K^2) = \int dt\, e^{\omega t} \int d^3\vec{x}\, e^{i\vec{k}\vec{x}}\, \langle \Omega | T\{ J^E_\mu(\vec{x},t) J^E_\nu(\vec{0},0)\} | \Omega \rangle$$

- $J^E_\mu(X)$ electromagentic current
- $K = (\vec{k}, -i\omega)$, $\vec{k}$ spatial momentum, ω the photon energy (input)

Advantage

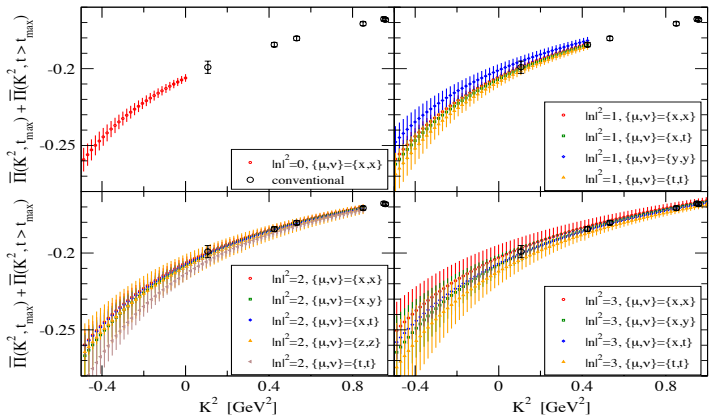
- vary $\omega \rightarrow$ smooth values for $K^2 = -\omega^2 + \vec{k}^2$
- can cover space-like and time-like momentum regions
- can reach small momenta and even zero momentum
- important condition:

$$-K^2 = \omega^2 - \vec{k}^2 < M_V^2\ , \quad \text{or} \quad \omega < E_{\text{vector}}$$

- make use of ideas: (Ji; Meyer; X. Feng, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, J. Noaki, E. Shintani; G. de Divitiis, R. Petronzio, N. Tantalo)

K. Jansen (analytic continuation)

HVP from analytical continuation



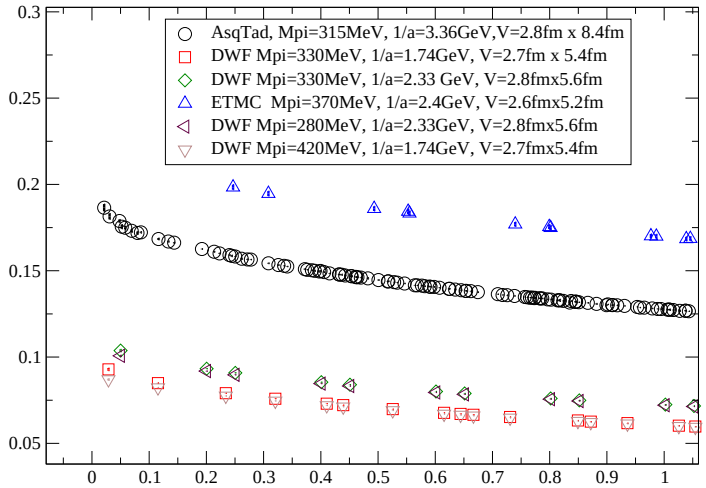
- different $\vec{n}$ lead to consistent results
- agreement with standard calculation
- however, larger errors for $|\vec{n}| > 0$

Working Group Discussion (analytic continuation)

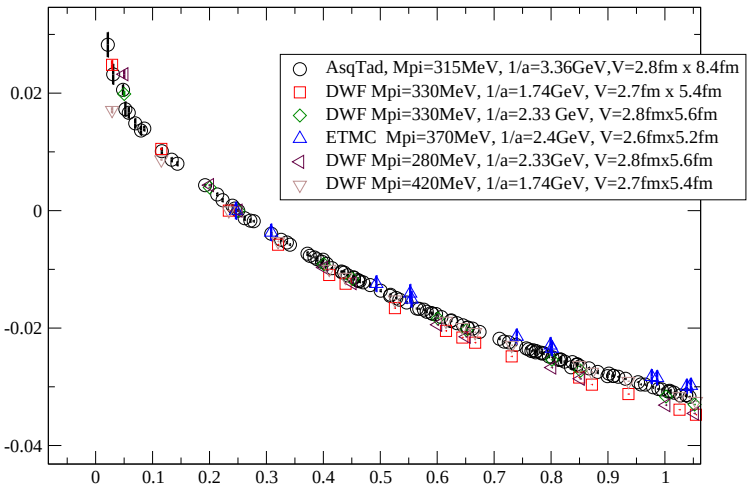
- Difficulty: must integrate time to ∞ but correlator at large time is noisy
- impose cut-off, leads to model dependence (small q^2 problem again)
- Analytic continuation problematic for $a \neq 0$?

Working Group Discussion (compare intermediate results)

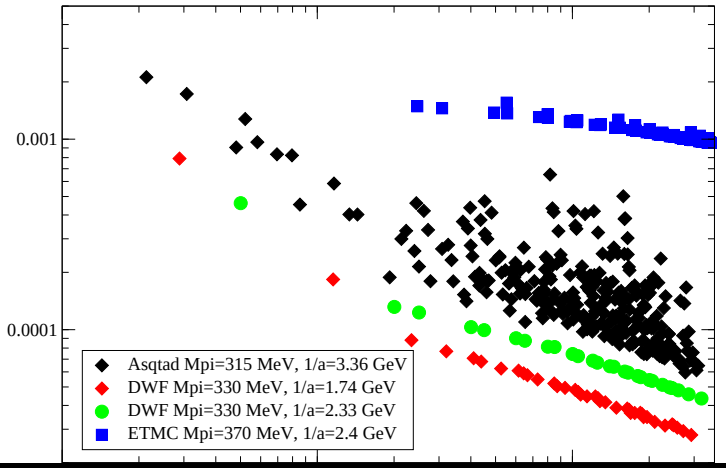
Pi(Q2) - Pi(Q2~0.25 GeV^2)



Pi(Q2) - Pi(Q2~0.25 GeV^2)



Absolute error Pi(Q2)

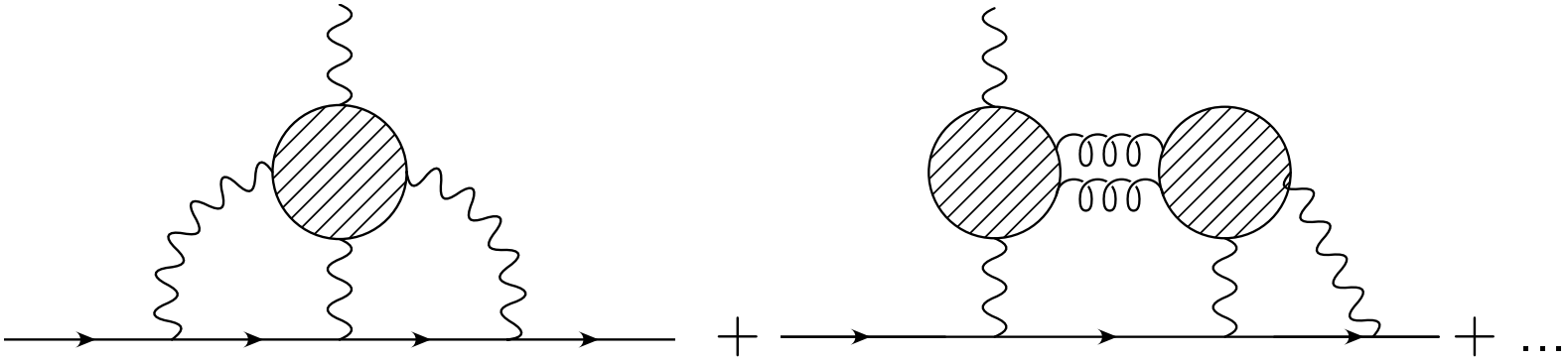


Plots from T. Izubuchi

Talk by Gregorio Herdoiza: Adler function, $\Pi(Q^2) - \Pi(Q^2_{\text{ref}})$

Outline

HLbL



Blobs: all possible hadronic states

Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty

"No dispersion relation" *a'la* vacuum polarization (but see talk by G. Colangelo!)

Dominated by pion pole

Lattice regulator: model independent, approximations systematically improvable

T. Izubuchi (hadronic light-by-light)

Hadronic Light-by-Light: What can Lattice QCD achieve ?

Taku Izubuchi

Tom Blum, Masashi Hayakawa

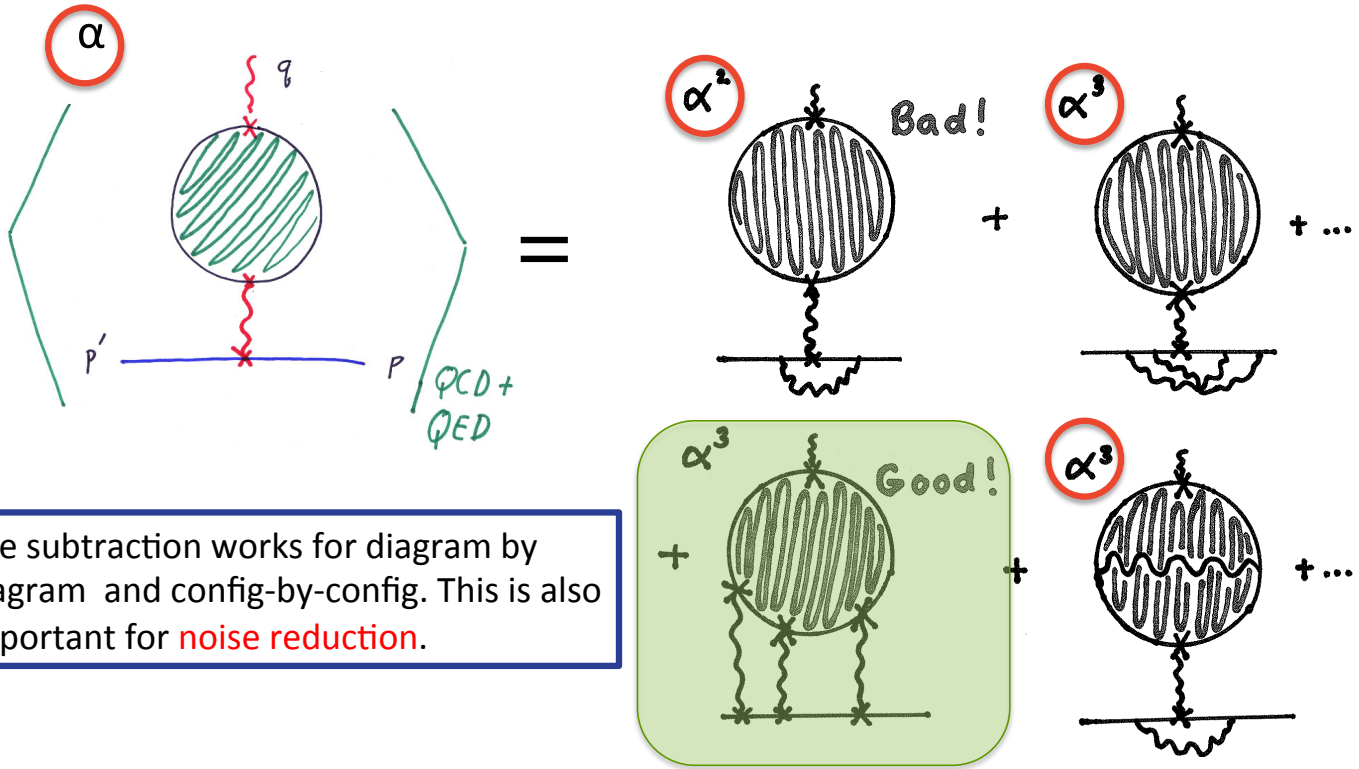
BROOKHAVEN NATIONAL LABORATORY

RIKEN BNL Research Center

Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction, April 02, 2014, Waldthausen Castle near

T. Izubuchi (hadronic light-by-light)

Unwanted diagrams



The subtraction works for diagram by diagram and config-by-config. This is also important for noise reduction.

T. Izubuchi (hadronic light-by-light)

Subtraction

First multiply the analytic photo between
- quark loop part
- lepton line part

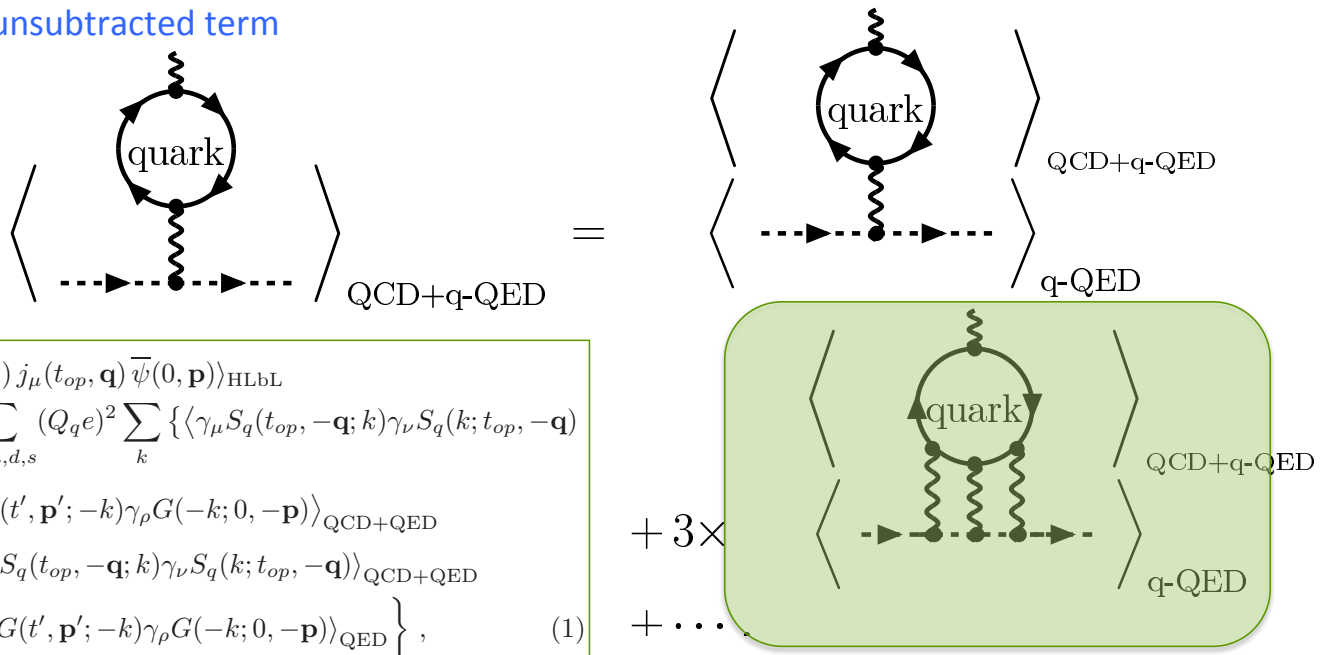
then take QCD+QED ensemble average

First take QCD+QED average for
- quark loop part
- lepton line part

then multiply the analytic photon

unsubtracted term

Subtraction term



$$
\begin{aligned}
&\langle \psi(t',\mathbf{p}')\, j_\mu(t_{op},\mathbf{q})\, \overline{\psi}(0,\mathbf{p})\rangle_{\mathrm{HLbL}} \\
&= -\sum_{q=u,d,s}(Q_q e)^2 \sum_k \Big\{ \big\langle \gamma_\mu S_q(t_{op},-\mathbf{q};k)\gamma_\nu S_q(k;t_{op},-\mathbf{q}) \\
&\quad \frac{\delta_{\nu\rho}}{\hat{k}^2} G(t',\mathbf{p}';-k)\gamma_\rho G(-k;0,-\mathbf{p})\big\rangle_{\mathrm{QCD+QED}} \\
&\quad - \langle \gamma_\mu S_q(t_{op},-\mathbf{q};k)\gamma_\nu S_q(k;t_{op},-\mathbf{q})\rangle_{\mathrm{QCD+QED}} \\
&\quad \frac{\delta_{\nu\rho}}{\hat{k}^2} \langle G(t',\mathbf{p}';-k)\gamma_\rho G(-k;0,-\mathbf{p})\rangle_{\mathrm{QED}} \Big\}\,, \qquad (1)
\end{aligned}
$$

T. Izubuchi (hadronic light-by-light)

[Blum, TI, Shintani PRD 88 (2013) 094503]

Covariant Approximation Averaging (CAA) a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} + \mathcal{O}^{(\mathrm{rest})}$$

Lattice Symmetry

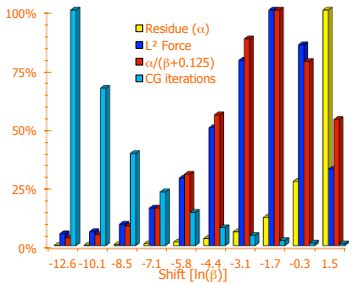
unbiased imporved

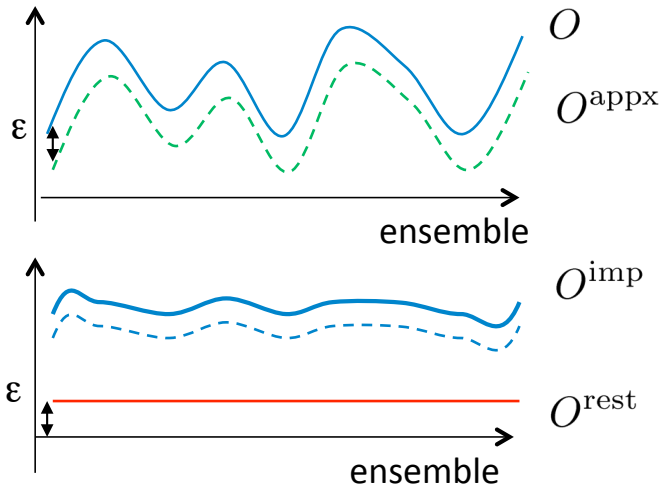
$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}$$

Expensive : infrequently measured

Cheap : frequently measured

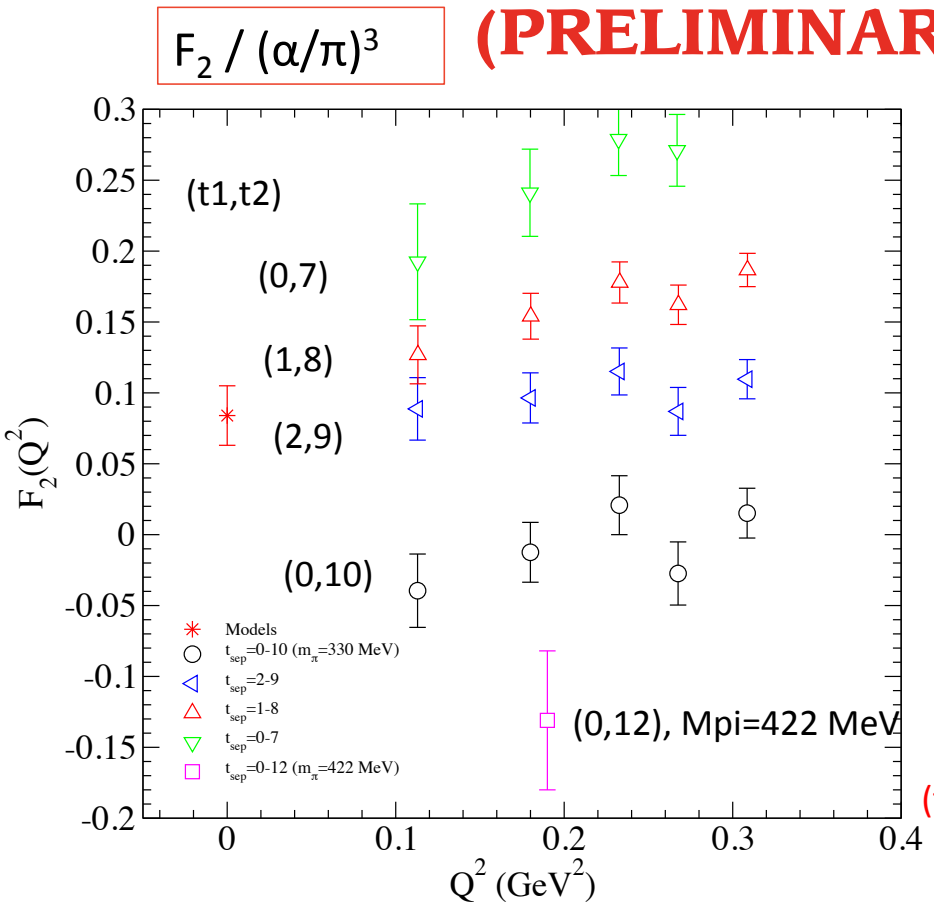
- $O^{(\mathrm{imp})}$ has smaller error
  $O^{(\mathrm{appx})}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply





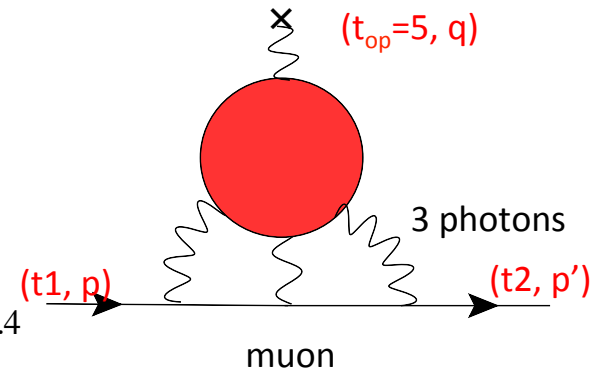
T. Izubuchi (hadronic light-by-light)

HLBL from Nf=2+1 DWF (PRELIMINARY)



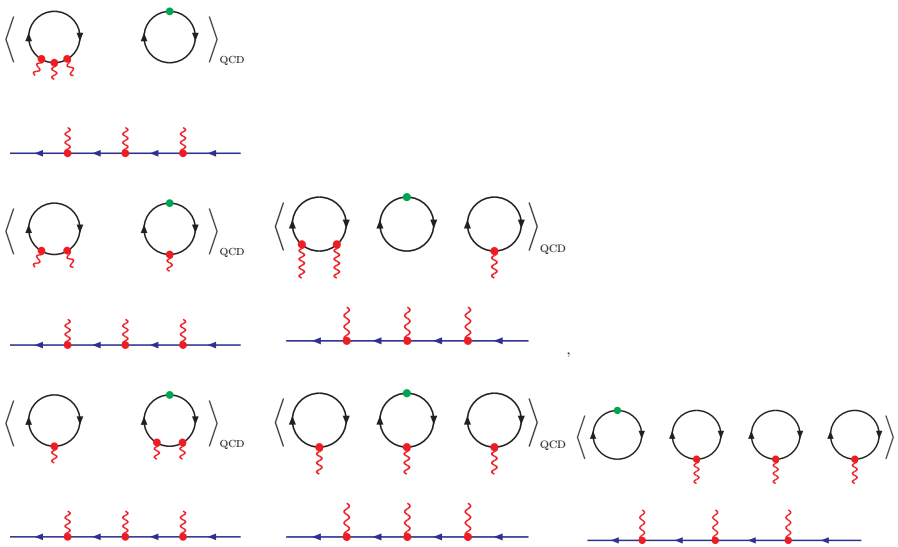
24^3 lattice size
$Q^2 = 0.11$ and 0.18 GeV^2
$m_\pi \approx 329$ MeV
$m_\mu \approx 190$ MeV

multiply
$(2/3)^4 + (-1/3)^4 + (-1/3)^4$

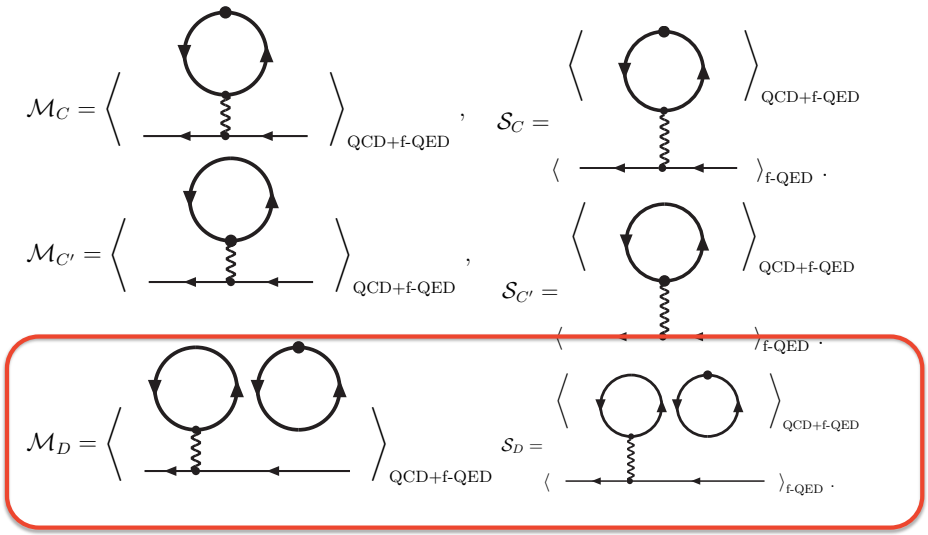


T. Izubuchi (hadronic light-by-light)

Disconnected diagrams



Similar strategy



Working Group Discussion (hadronic light-by-light)

- Still only one group calculating (FNAL, ...?)
- Potentially large finite volume effects for QCD+QED (massless photon) of concern
- Calculate 4 point function in QCD
- Calculate simpler quantities to check models (E. Shintani) and QCD+QED lattice calculations (*VVA* diagram (M. Knecht and others), mass splittings,...)

Outline

Summary/Outlook

- Lots of interest, work on hadronic contributions, esp. HVP
  - Statistical errors (sub) 1%
  - Several groups done/doing physical m_π (m_{quark}) simulations
  - Much effort on understanding systematics
  - Workshop very helpful, encouraging cooperation!
  - 2-3% total error on connected HVP in 2 years possible (my opinion)
  - May be achievable for disconnected too
- HLbL: QCD+QED promising (my opinion), but significant systematics. We are now running with $m_\pi = 170$ MeV and investigating excited state contamination
- Dynamical QED+QCD is coming too
- need more groups working on it (FNAL, ...)!
- Interest in 4pt function, $\pi \to \gamma^*\gamma^*$, other simpler quantities