Hadronic light-by-light from Dyson-Schwinger equations

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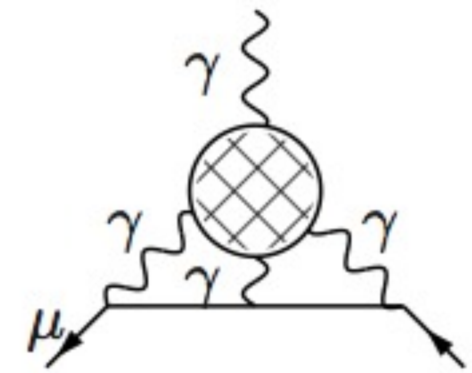
Deutsche Forschungsgemeinschaft

Together with
Richard Williams, Gernot Eichmann, Tobias Goecke, Jan Haas

Overview

1. Hadronic vacuum polarisation



2. Hadronic light by light

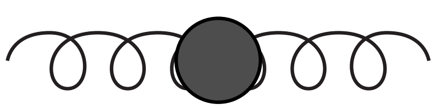
3. Outlook: towards the photon four-point function

QCD in covariant gauge

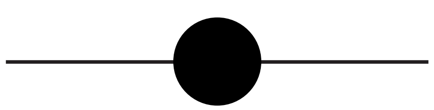
Quarks and Gluons

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \; \exp\left\{ - \int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\,(F^a_{\mu\nu})^2 + \text{gauge fixing} \right) \right\}$$

Landau gauge propagators in momentum space,



$$D^{Gluon}_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$



$$S^{Quark}(p) = Z_f(p^2)[-i\not{p} + M(p^2)]^{-1}$$

The Goal: gauge invariant information in a gauge fixed approach.

Nonperturbative QCD: Complementary approach

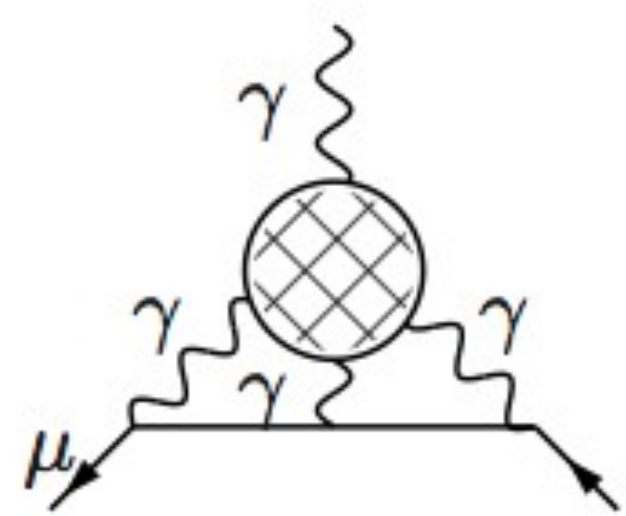
Quarks and gluons

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Dyson-Schwinger Equations
  - Physical quark masses
  - Full momentum dependencies
  - Multi-scale problems feasible

Hadrons

- Effective theories and models (χPT, chiral models,...)
- Dispersive approach
  - Physical degrees of freedom



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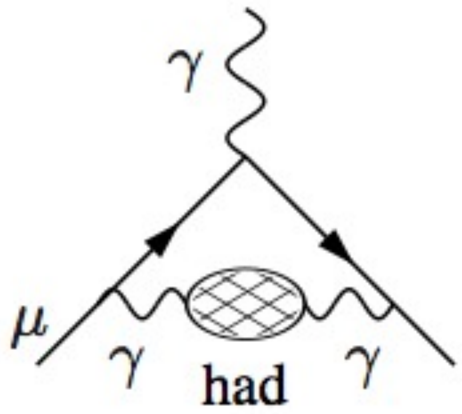
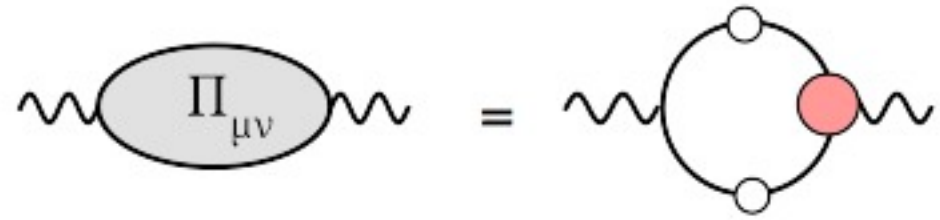
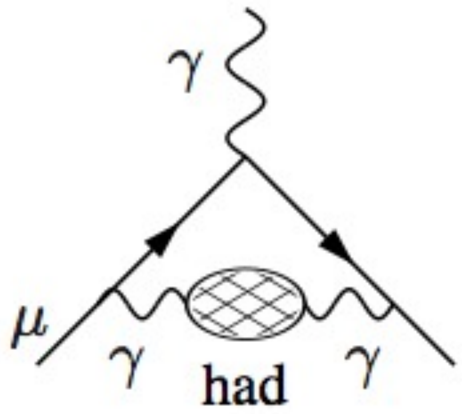
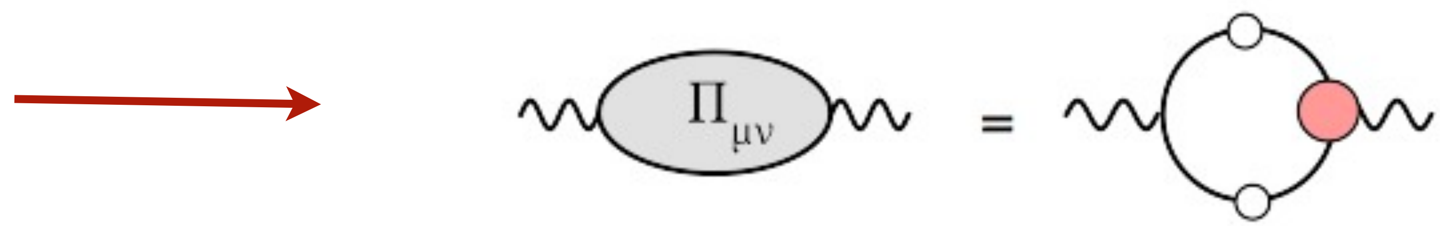




Evaluated at spacelike momenta!

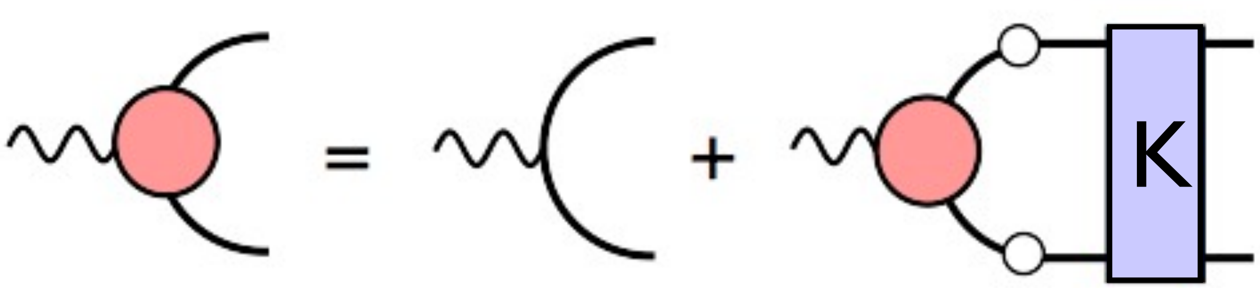
Hadronic vacuum polarization





Evaluated at spacelike momenta!

Need to determine quark propagator and quark-photon vertex:



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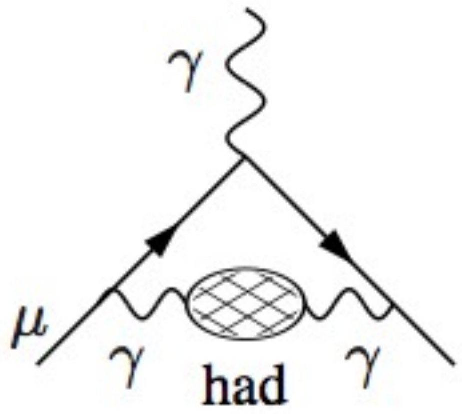

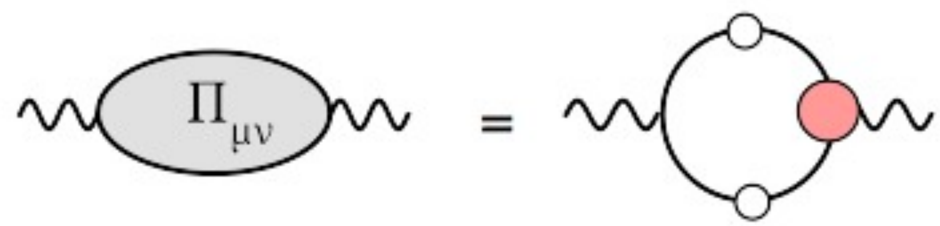

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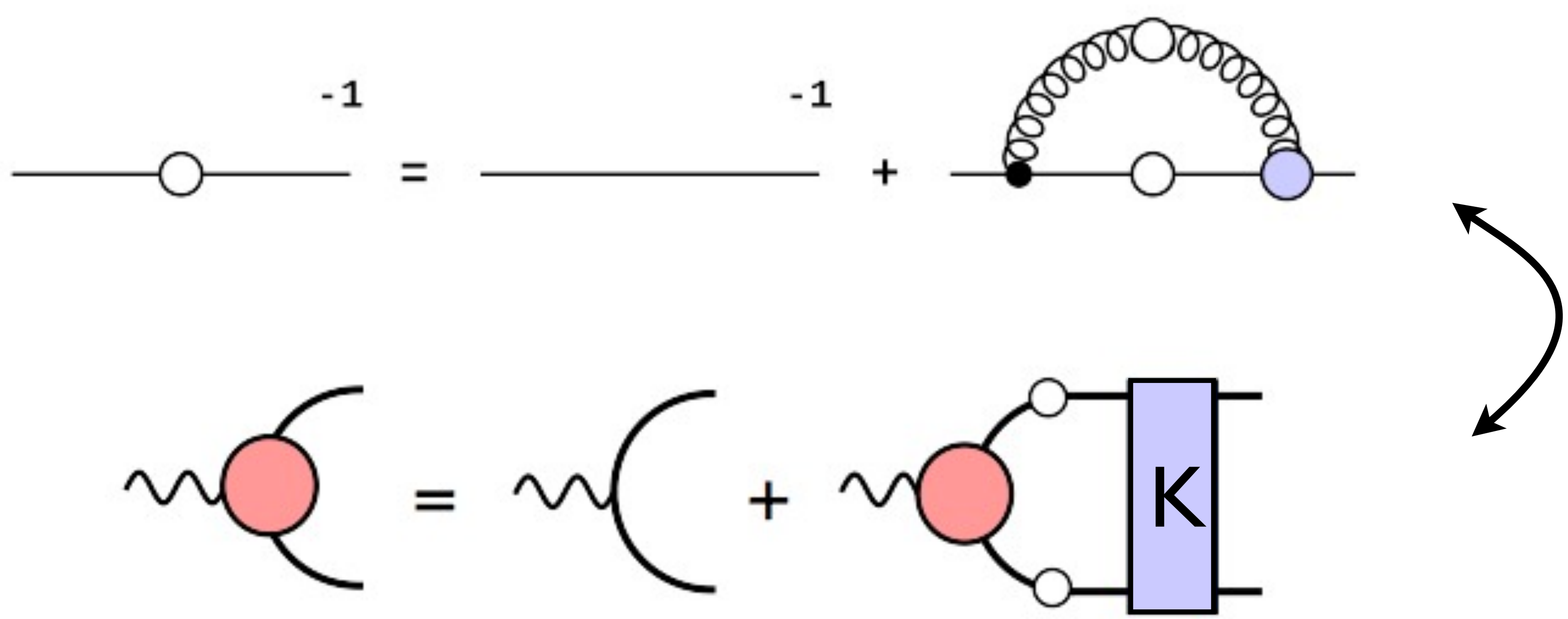
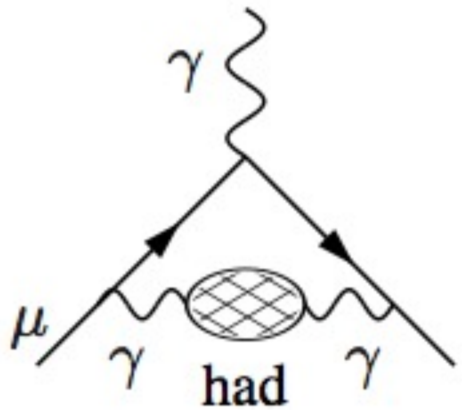
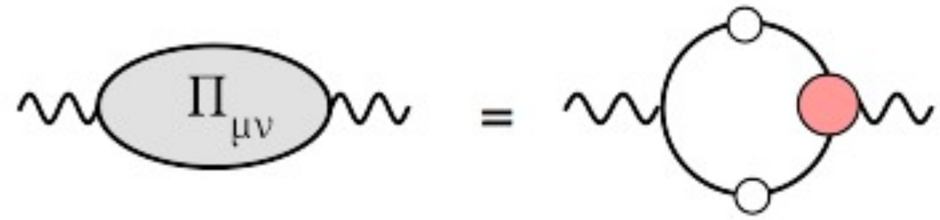


Kernel K uniquely related to quark-DSE via axWTI

Hadronic vacuum polarization





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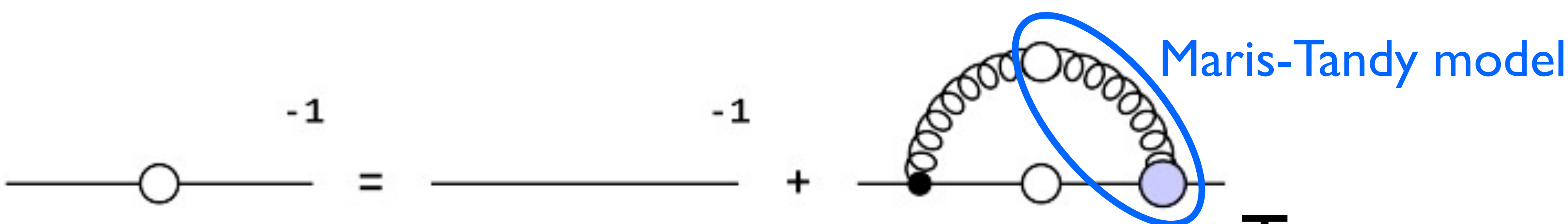

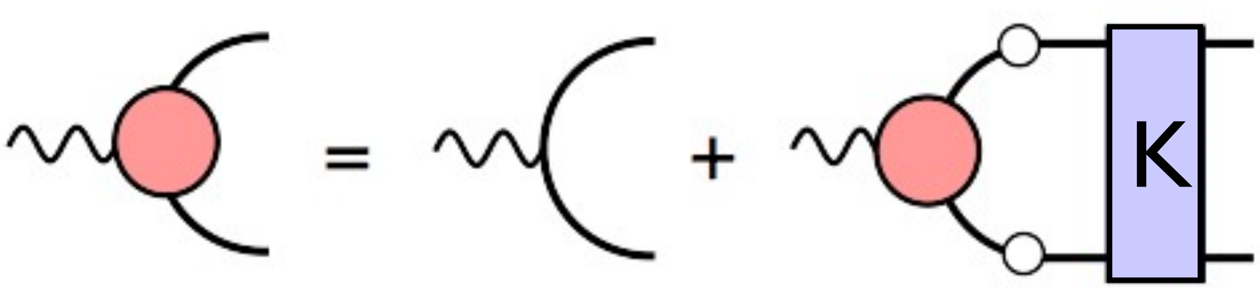

Two strategies:

I. Calc. gluon and vertex from their DSEs

II. Combine gluon and vertex in model

Model for quark-gluon interaction

$$\boxed{\alpha(k^2) = \pi\eta^7\left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)}$$

Maris, Tandy, 1999

- two (related) parameters η and Λ from f_π
- α_{UV} from perturbation theory
- masses $m_u = m_d$ from m_π or m_ρ
- Renormalizable and momentum dependent !

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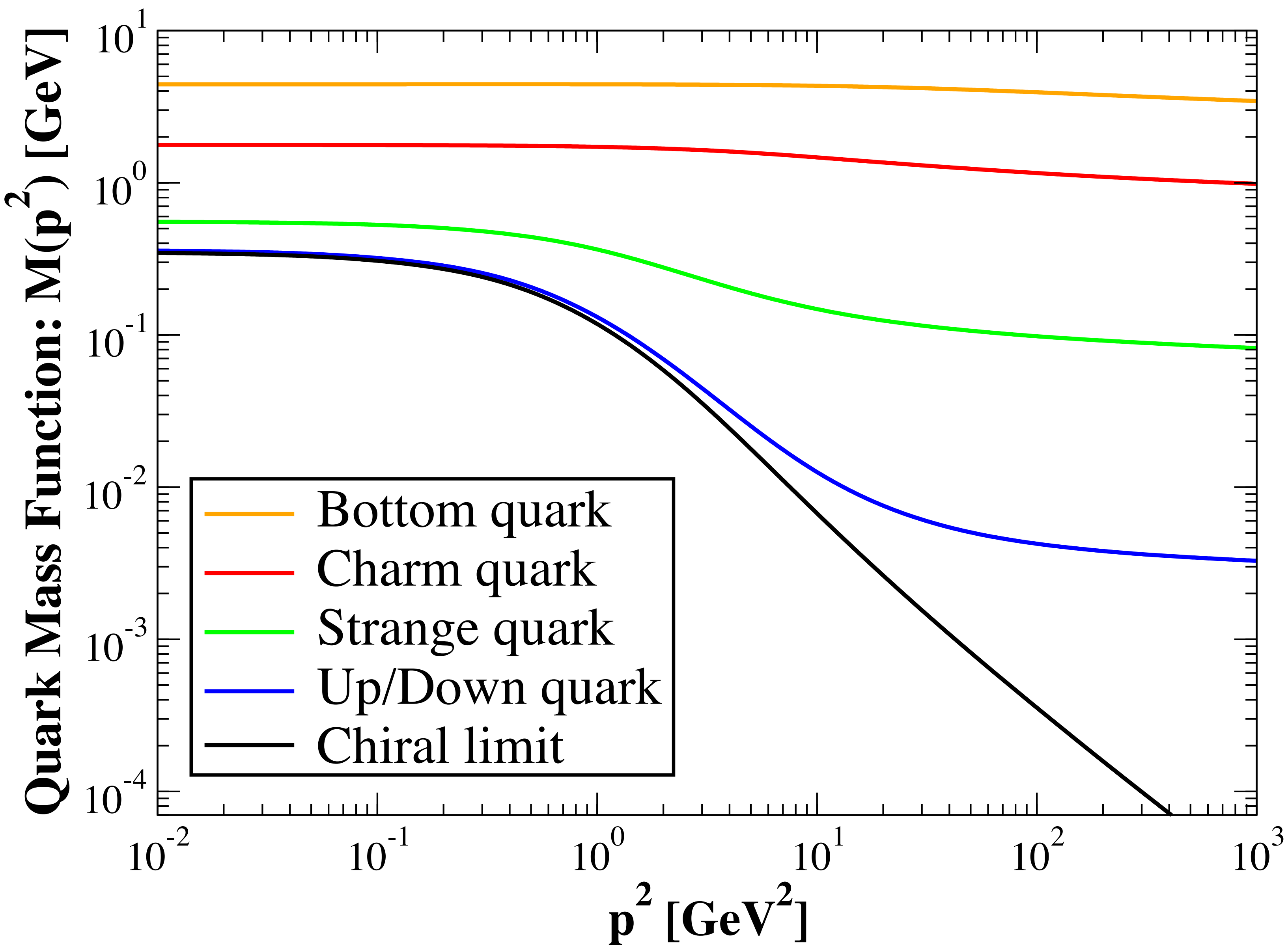
NJL-model

$$\alpha(k^2) = \alpha\ \Theta(\Lambda^2 - k^2)$$

Quark mass: momentum and flavor dependence

Typical solution:



- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of M_{weak}
- Chiral condensate:

$$\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$$

Friday, April 4, 2014

Phenomenology from Maris-Tandy interaction

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ \mathrm{s}^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ \mathrm{s}^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

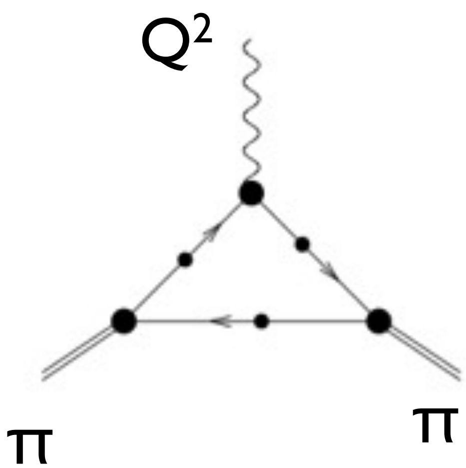
Scattering length (PM, Cotanch, PRD66, 116010)

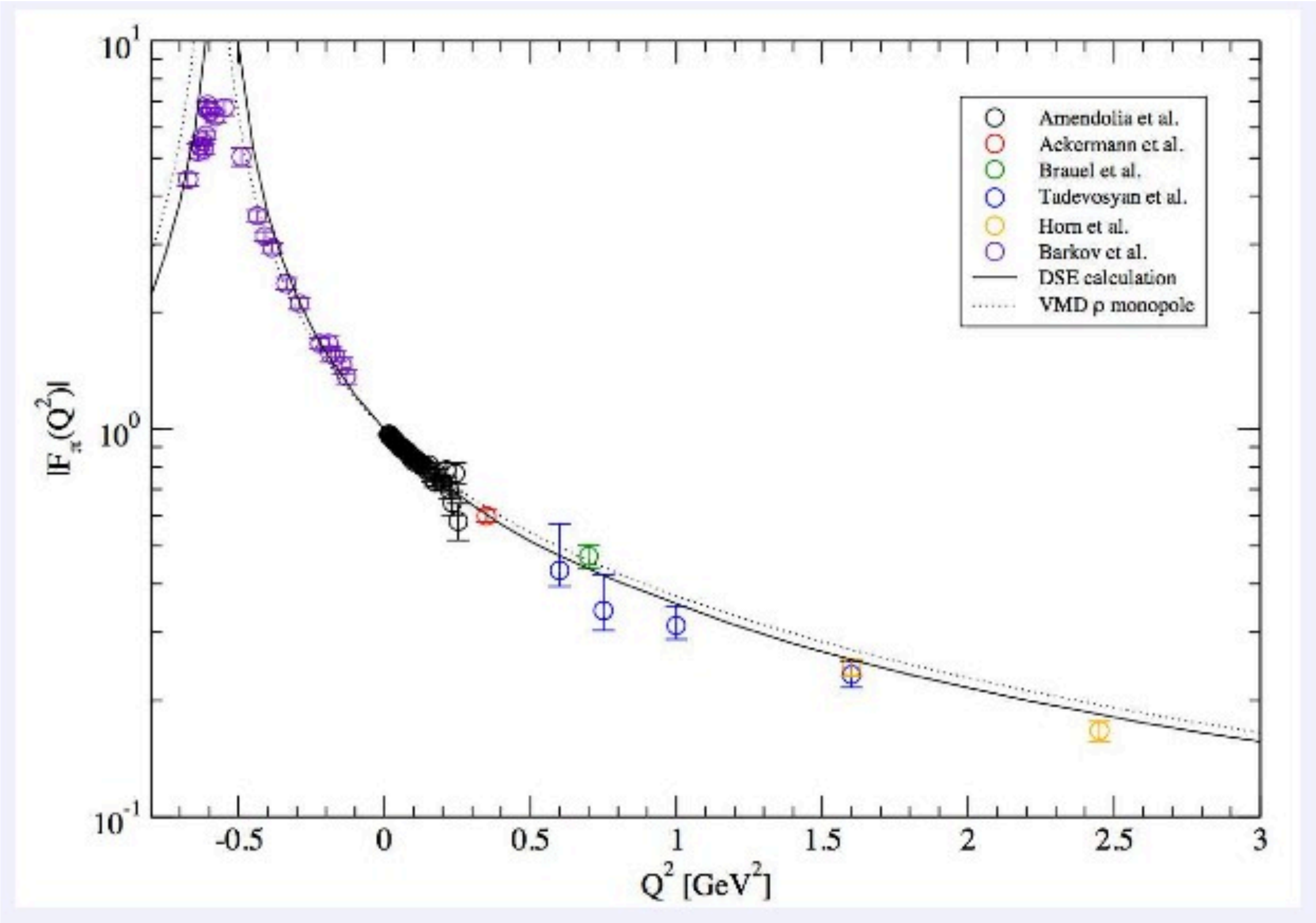
a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

M_ρ, M_ϕ, M_{K^*} good to 5%, f_ρ, f_ϕ, f_{K^*} good to 10%

Slide from Pieter Maris

Pion form factor:

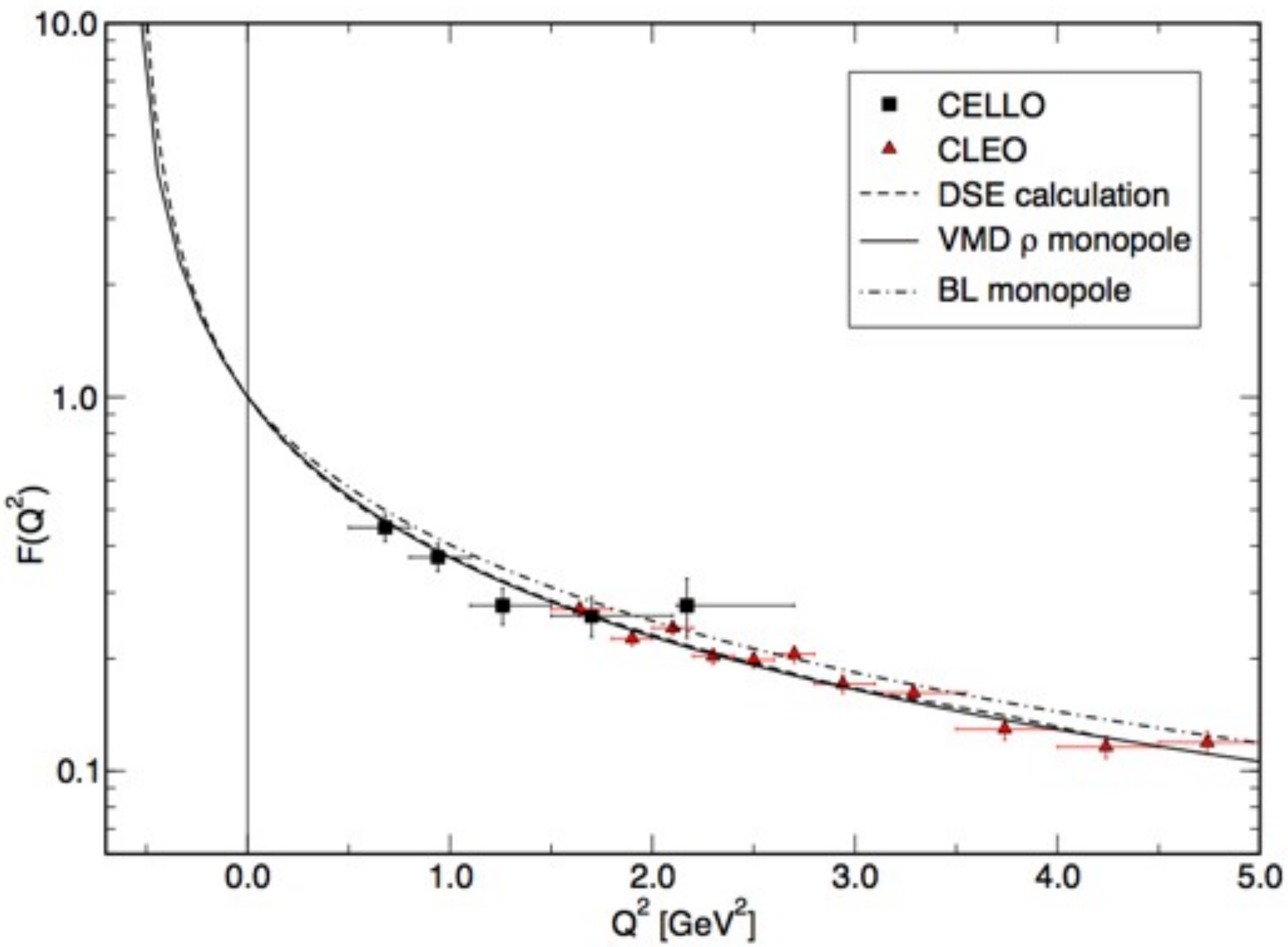


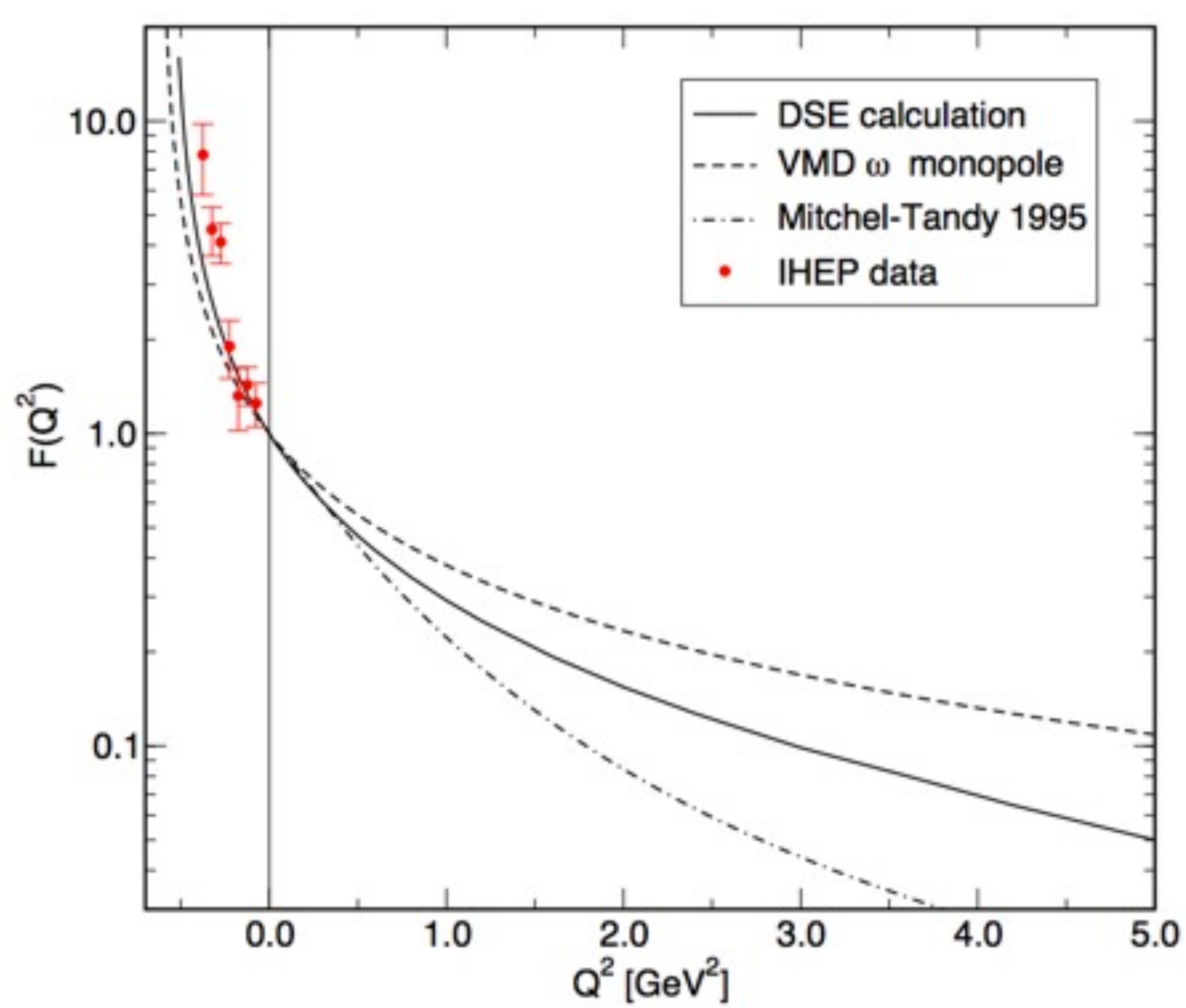


Krassnigg, Schladming 2011;
Maris, Tandy NPPS 161, 2006

Vector meson poles dynamically generated!

Transition form factors



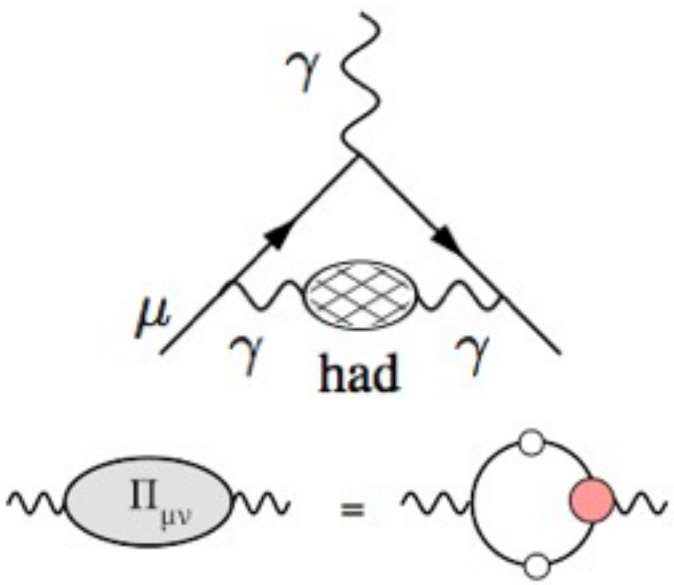


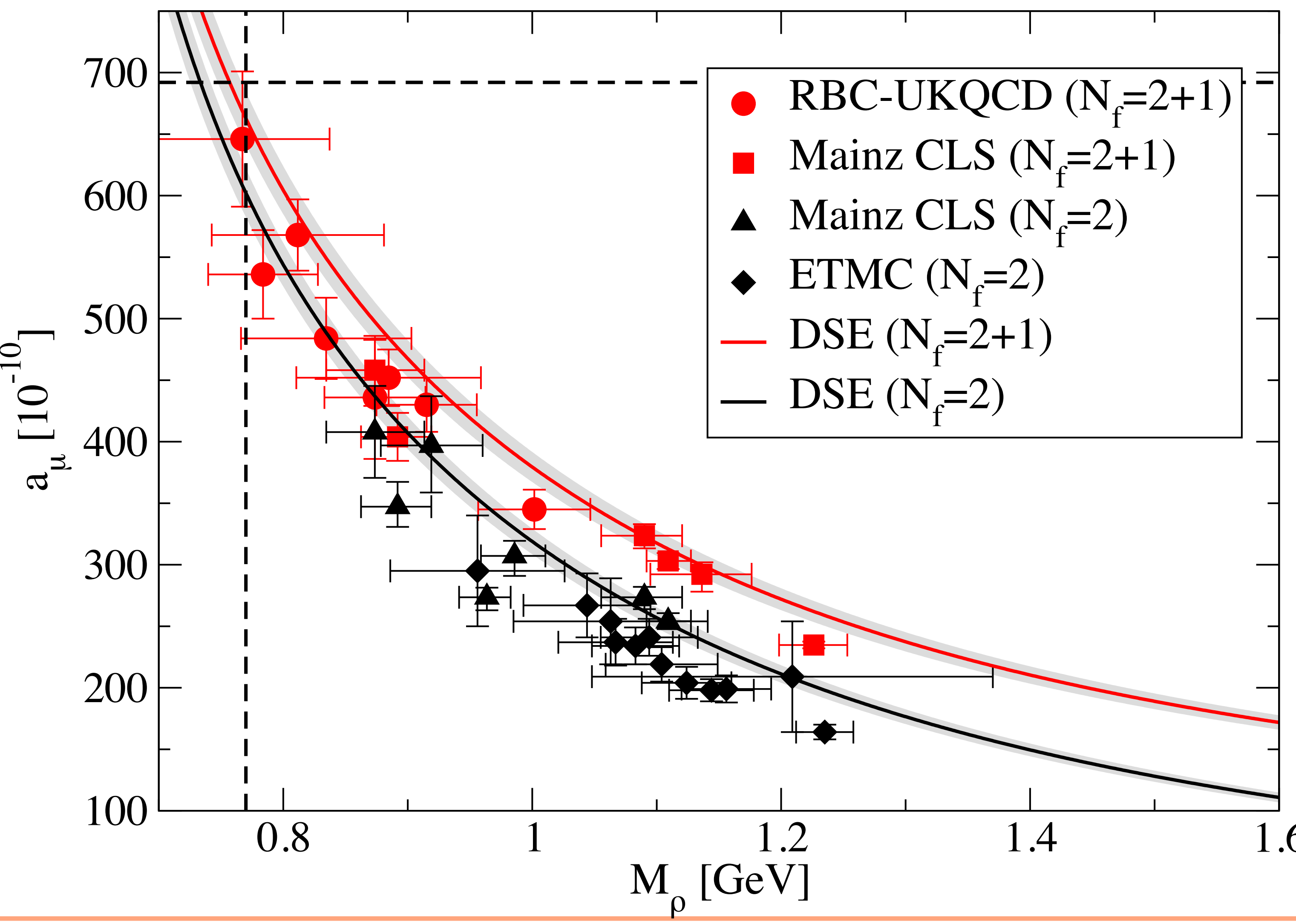
$\gamma^*\pi \rightarrow \gamma$

$\omega\pi\gamma^*$ and $\rho\pi\gamma^*$

- good agreement with data
- rho/omega pole generated dynamically

Maris, Tandy, Phys. Rev. C 65 045211 (2002)





10^{-10}	u/d	s	c	b
a_μ	600	60	15	1

DSE:

Goecke, CF, Williams, PLB 704 (2011)

$$a_\mu^{had.(1)} = (\boxed{744.0} \pm 2) \cdot 10^{-10} \qquad (m_\pi)$$

$$a_\mu^{had.(1)} = (\boxed{676.0} \pm 2) \cdot 10^{-10} \qquad (m_\rho)$$

Experiment:

$$a_\mu^{had.(1)} = (\boxed{692.1} \pm 5.6) \cdot 10^{-10}$$

Very reasonable agreement !

Overview

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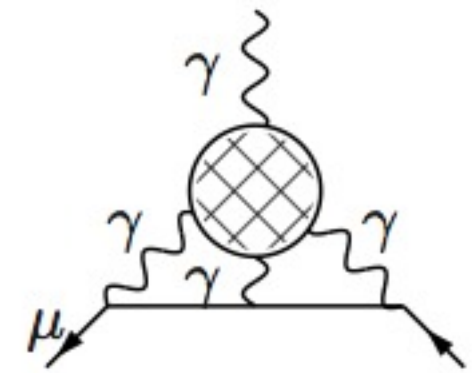
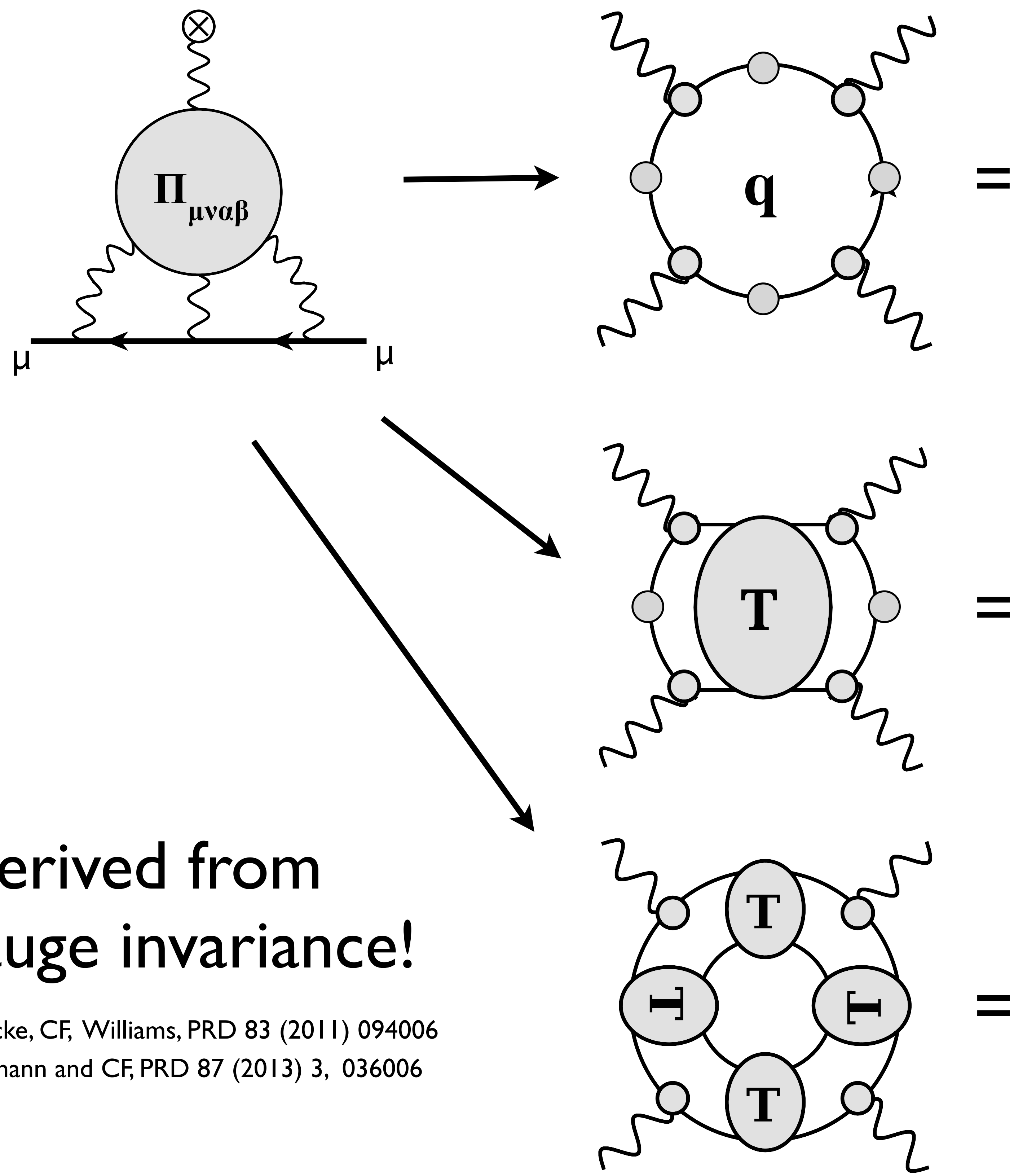


2. Hadronic light by light

3. Outlook: towards the photon four-point function

Hadronic Light-by-Light scattering



Derived from gauge invariance!

Goecke, CF, Williams, PRD 83 (2011) 094006
Eichmann and CF, PRD 87 (2013) 3, 036006

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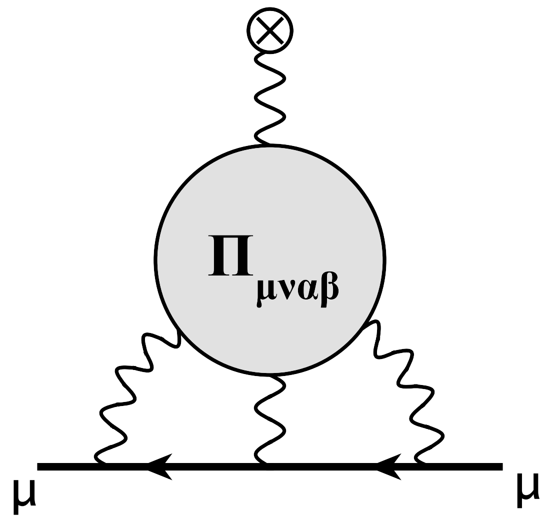

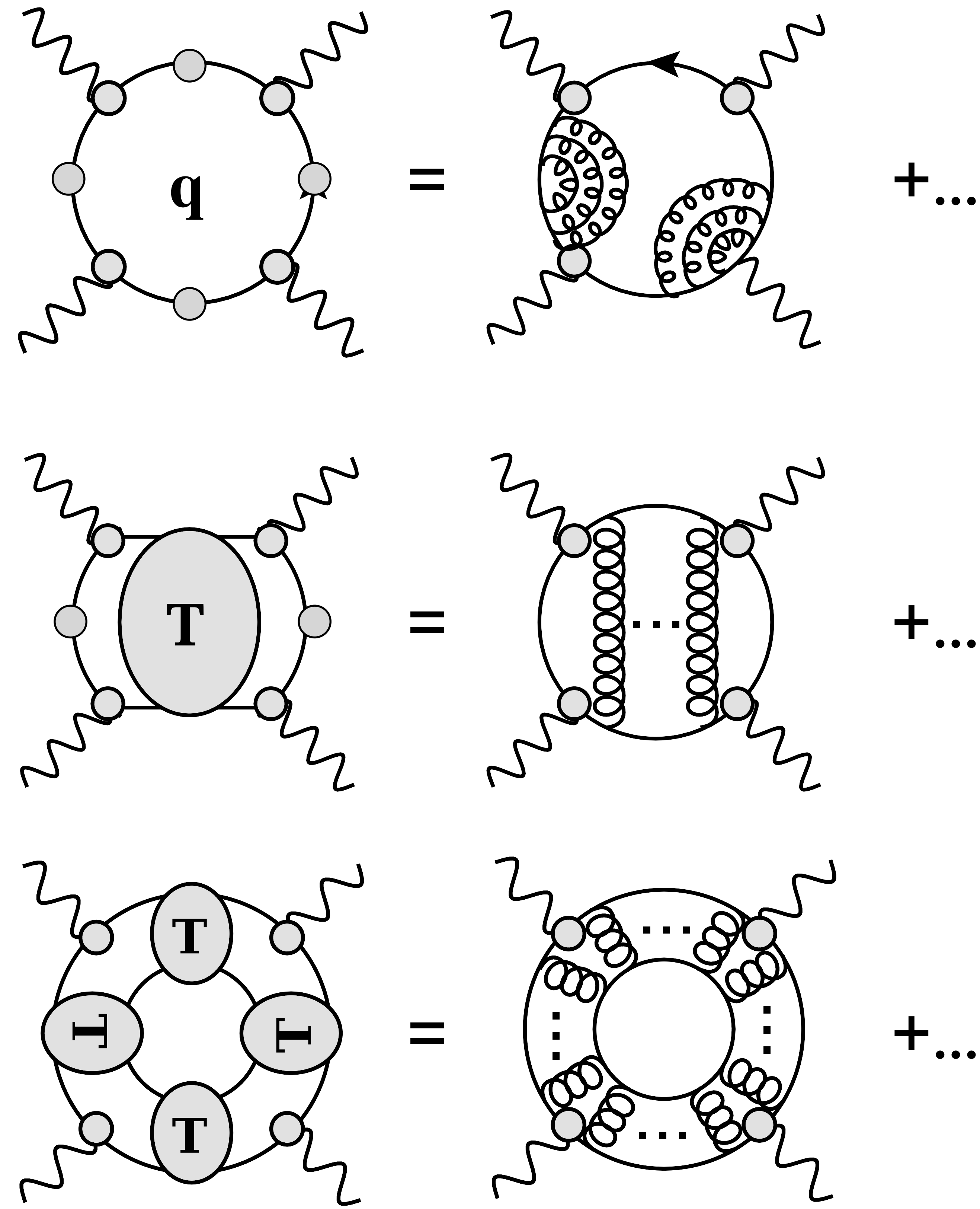

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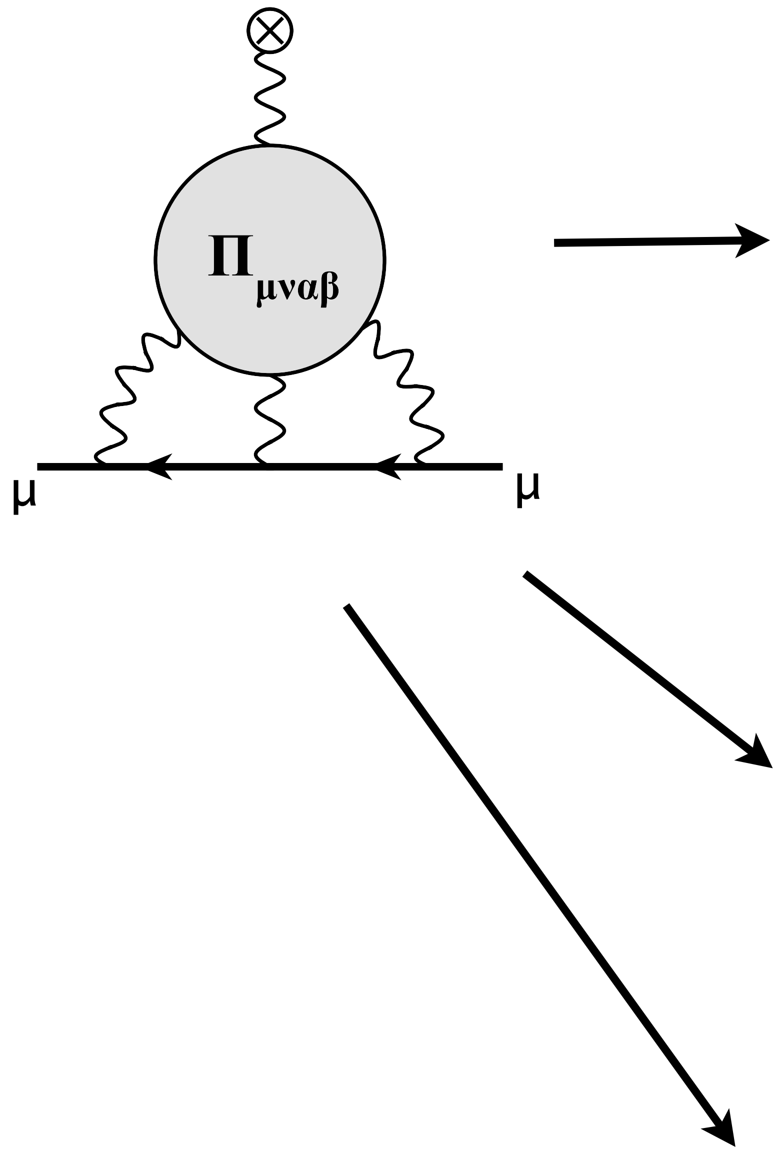

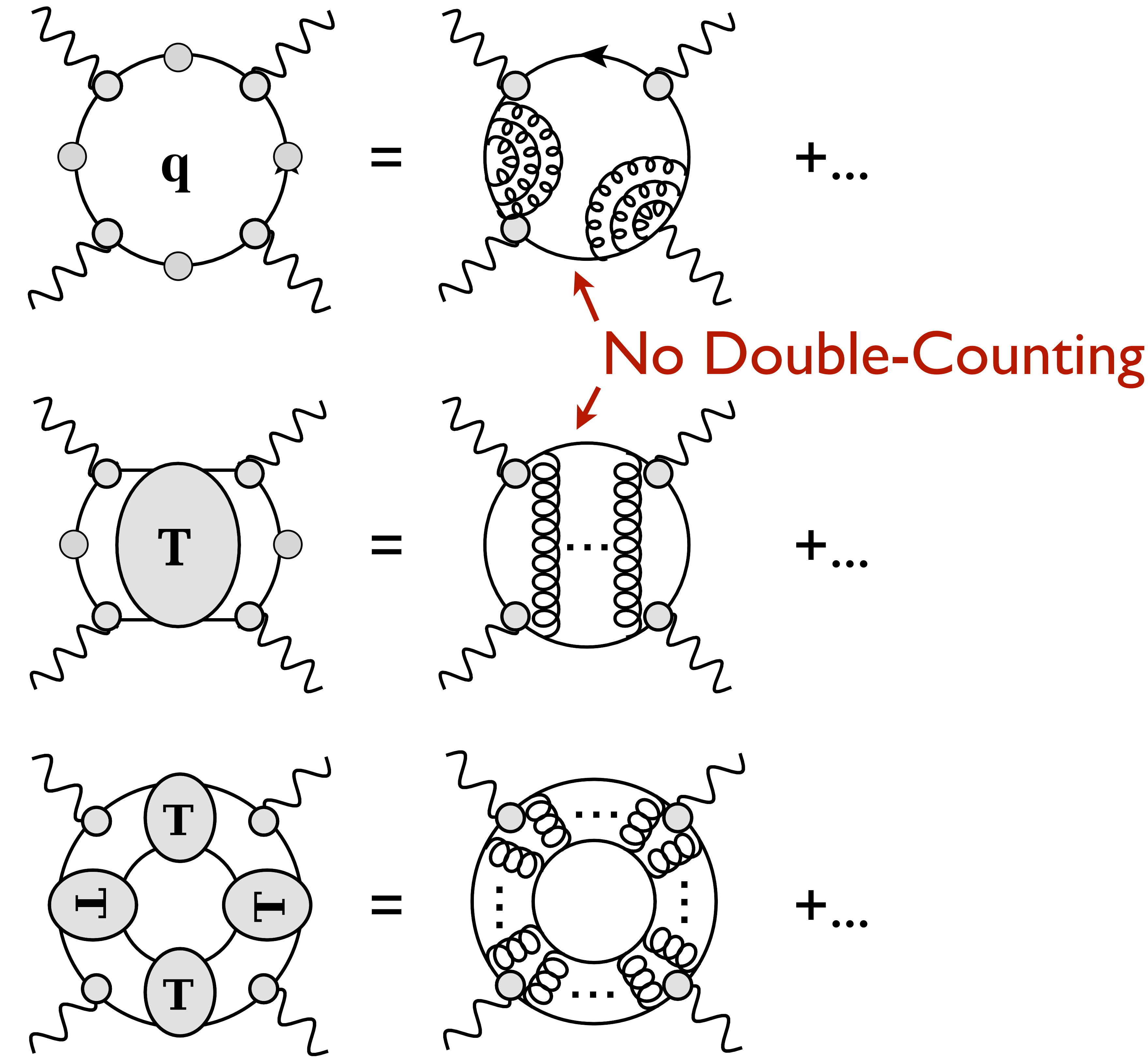

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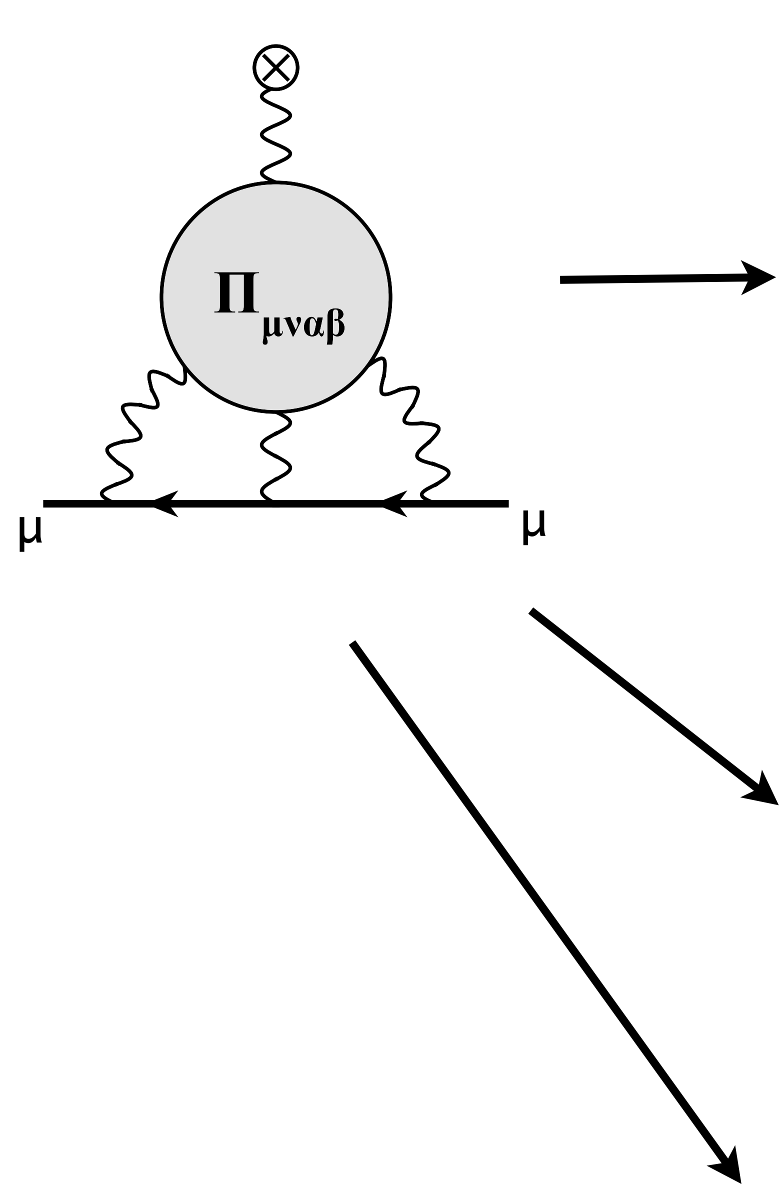

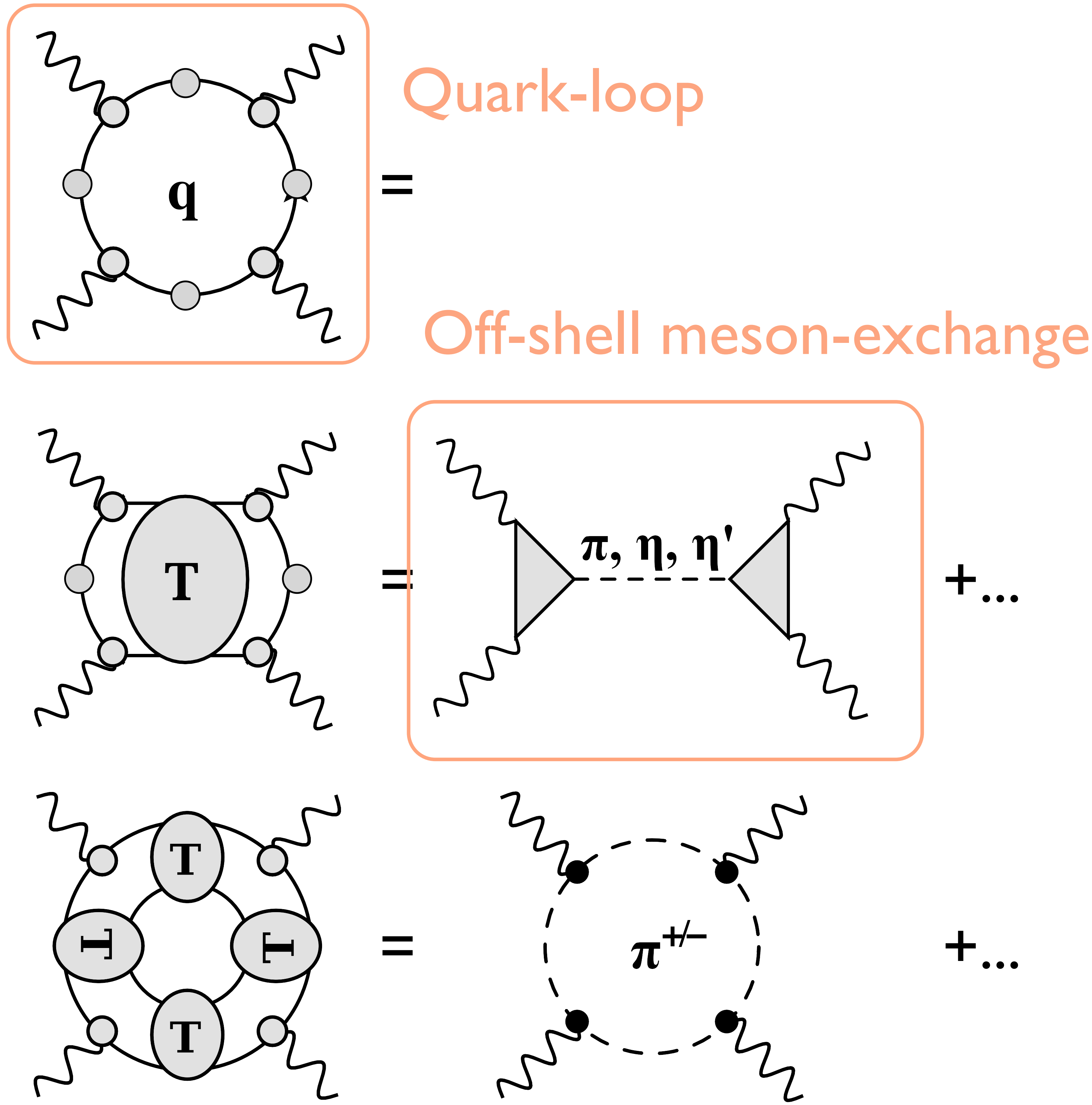

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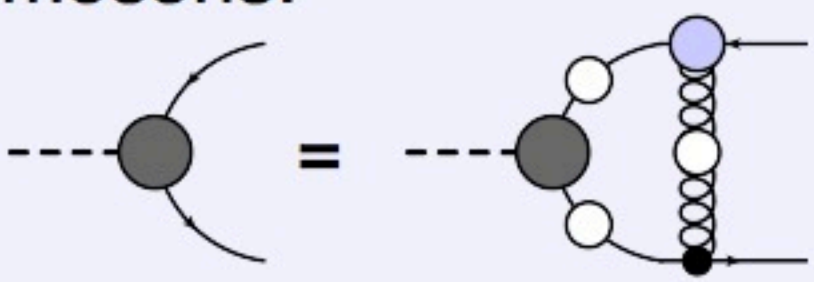
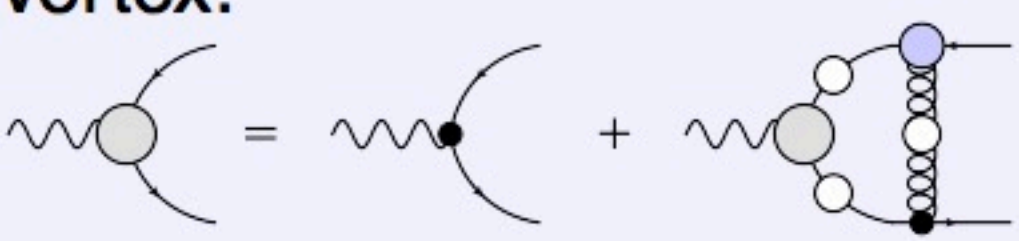
Meson-exchange contribution

Dressed quark propagator:

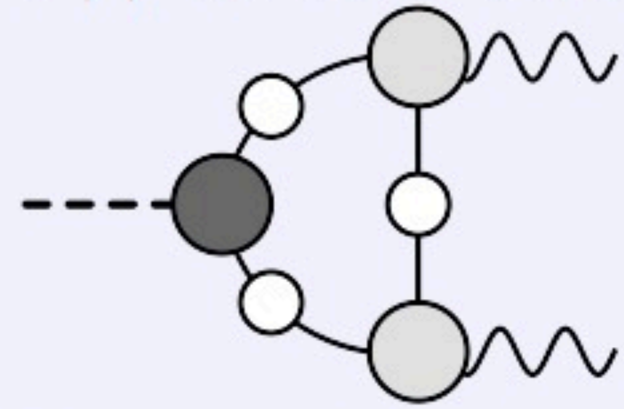
BSE for pseudoscalar mesons:



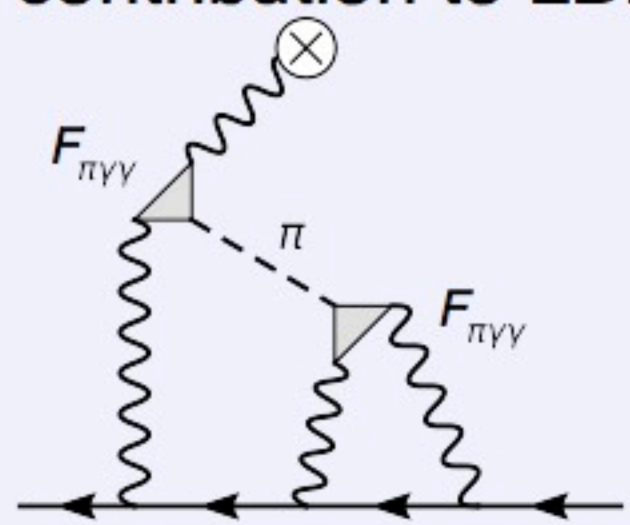
DSE/BSE for fermion-photon vertex:



$\pi\gamma\gamma$ 'off-shell form factor':



Meson exchange contribution to LBL:

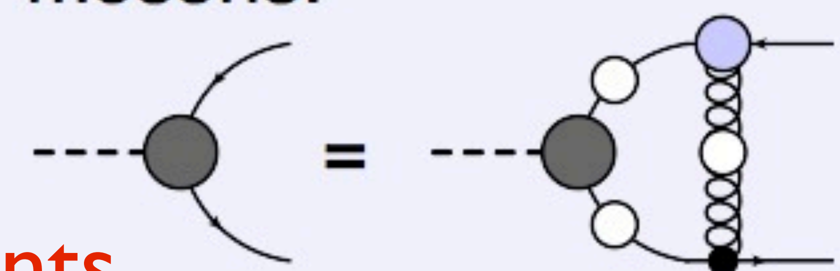


Meson-exchange contribution

Dressed quark propagator:

2 Components

BSE for pseudoscalar mesons:

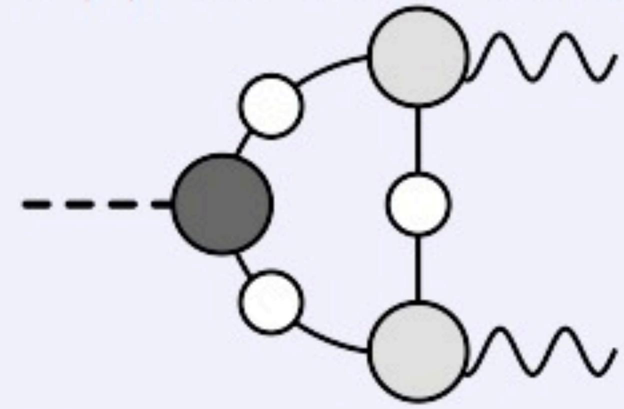


4 Components

DSE/BSE for fermion-photon vertex:

12 Components

$\pi\gamma\gamma$ 'off-shell form factor':



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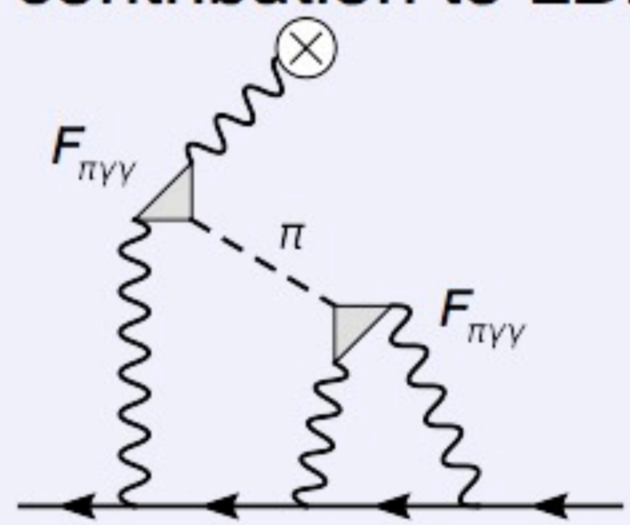
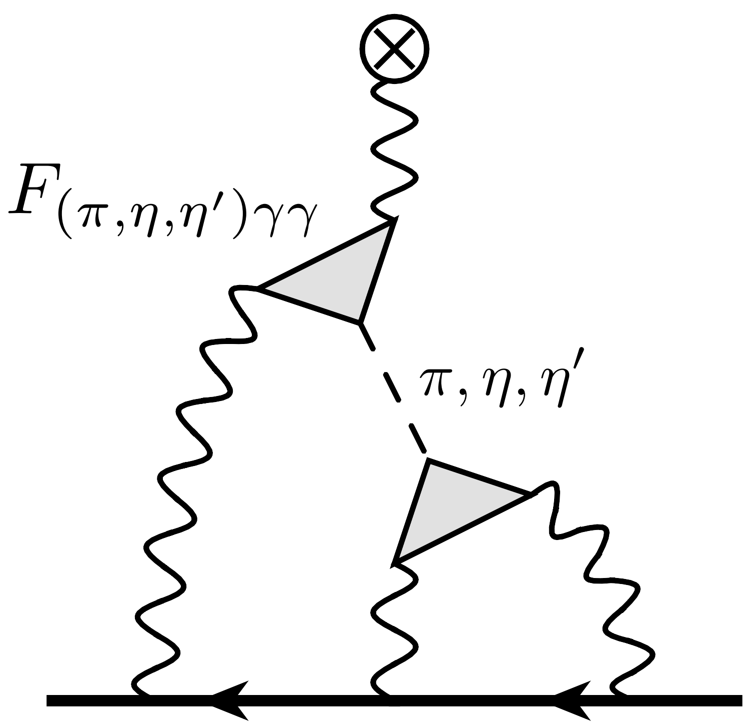
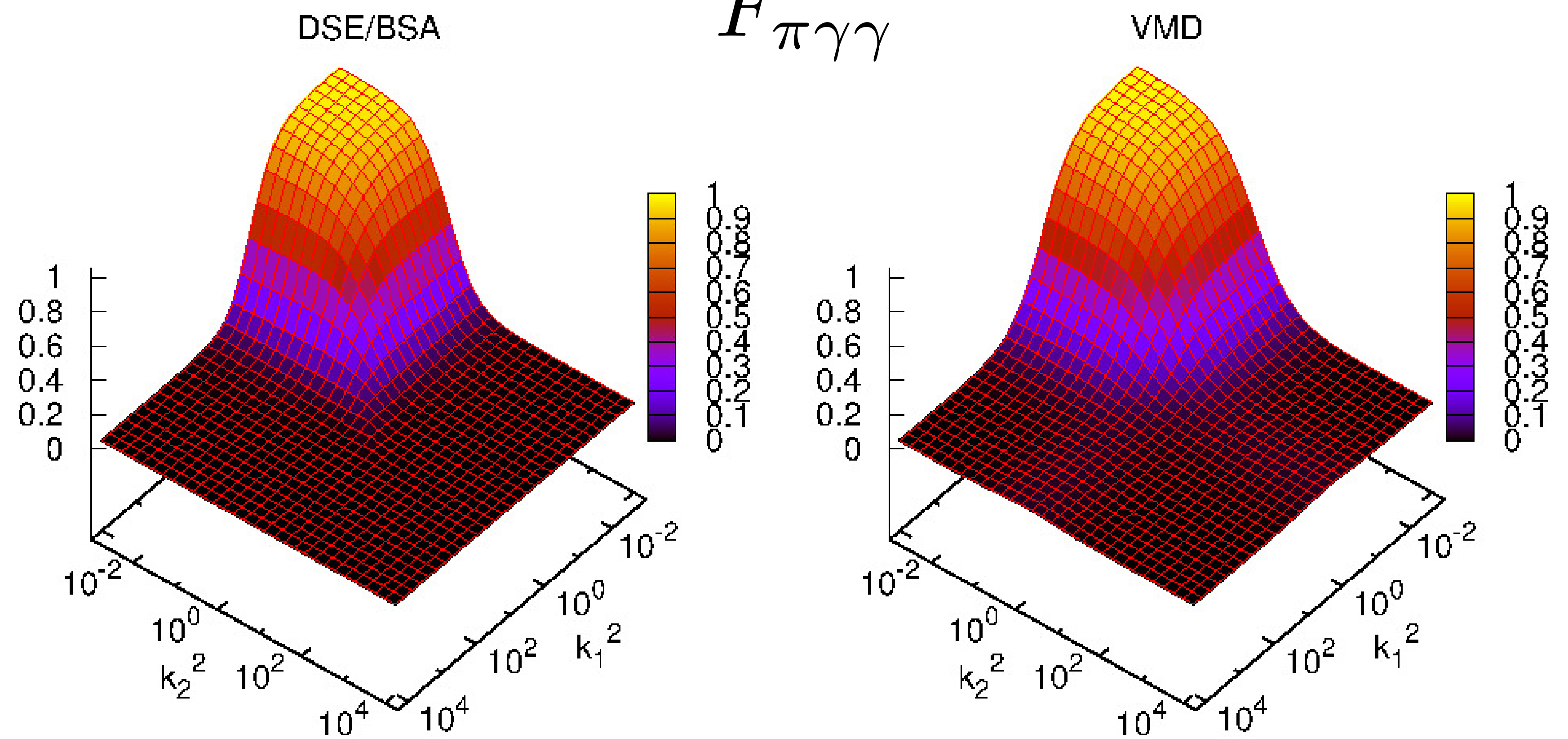


- full momentum dependencies
- no free parameters

Meson exchange contribution to LBL



$F_{\pi\gamma\gamma}$



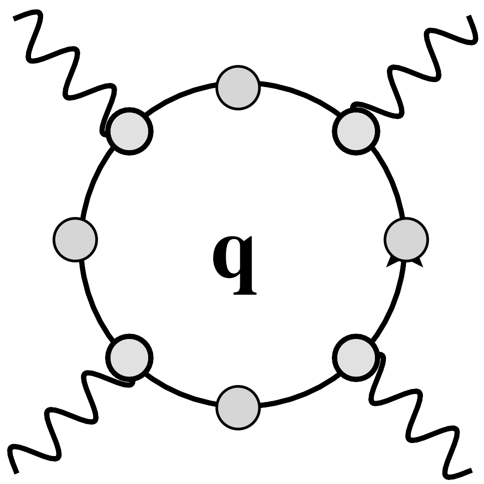
- $F_{\pi\gamma\gamma}$ overall similar to VMD-models
- our value: $$(a_\mu^{\pi,\eta,\eta'})_{DSE} = 8.1(1.3) \cdot 10^{-10}$$
- comparable with model calculations
- short distance constraints satisfied!

numerical: 0.2
systematic: 1.1

see also Dorokhov, Broniowski, PRD 78 (2008) 073011

Goecke, CF, Williams, PRD 83 (2011) 094006

Quark-loop contribution to LBL



- Numerically demanding due to superficial divergency
- Vertex contains 12 tensor structures

$$\Gamma^\mu = \sum_{i=1,4} BC_i \, L_i^\mu + \sum_{i=1,8} F_i \, T_i^\mu$$

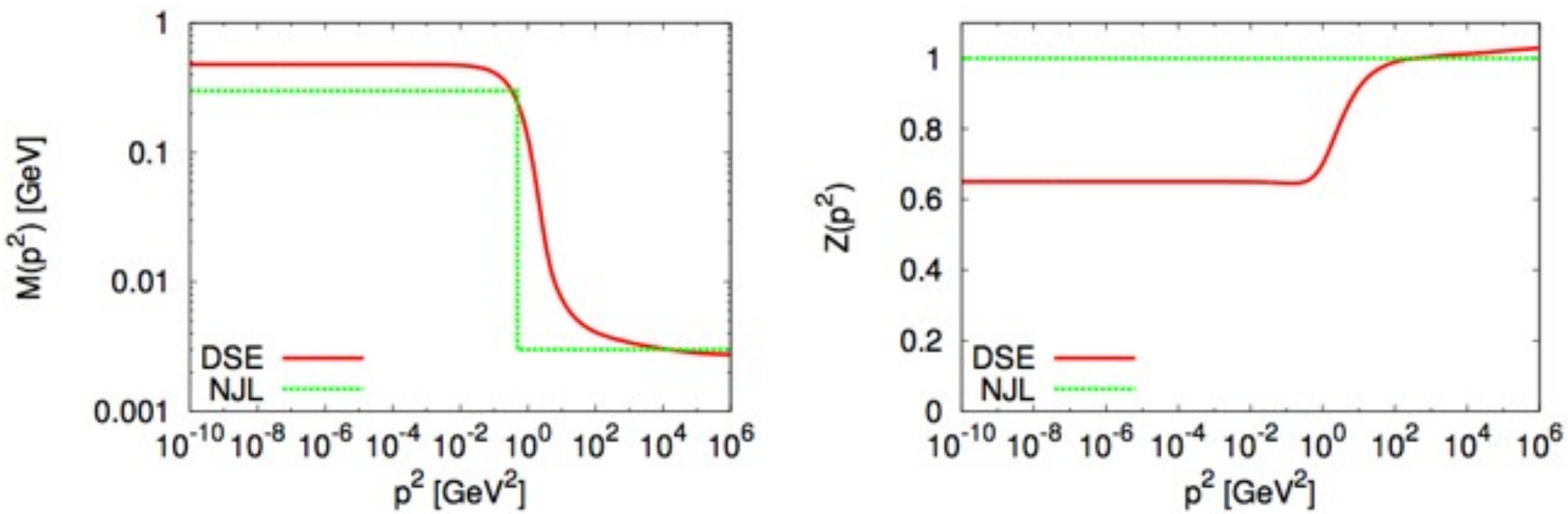
gauge part $\longrightarrow$ WTI

transverse part $\rightarrow$ vector-mesons

ENJL-model:	$(2.1 \pm 0.3) \cdot 10^{-10}$	Bijnens, Pallante and Prades PRL 75, (1995)
DSE:		
bare vertex	$(6.1 \pm 0.2) \cdot 10^{-10}$	CF, Goecke, Williams, EPJA 47 (2011) 28
BC_1 only	$(11.1 \pm 0.2) \cdot 10^{-10}$	
$BC_1 + F_1$	$(10.7 \pm 0.3) \cdot 10^{-10}$	Goecke, CF, Williams, PRD 87 (2013) 034013

Calculation not yet completed !

Comparison ENJL vs. DSE



- NJL: no momentum dependence in quark
- NJL: no relative momenta in quark-photon vertex $\Gamma_\mu(P, \cancel{p, p.P})$
- artificial suppression of quark-loop contribution

ENJL-model:	$6.0 \cdot 10^{-10}$	$\rightarrow$	$2.1 \cdot 10^{-10}$
DSE (BC_I + F_I):			
$N_f = 2$	$10.0 \cdot 10^{-10}$	$\rightarrow$	$9.6 \cdot 10^{-10}$
$N_f = 4$	$11.1 \cdot 10^{-10}$	$\rightarrow$	$10.7 \cdot 10^{-10}$

Goecke, CF, Williams, Phys. Rev. D 87 (2013) 034013

Hadronic LBL contributions - DSE + Models

Group	Tool	π_0,η,η'	quark-loop	π_{+-}	SUM
BPP	ENJL	8.5(1.3)	2.1(0.3)	-1.9(1.3)	8.3(3.2)
HKS	HLS	8.3(0.6)	1.0(1.1)	-0.4(0.8)	8.9(1.6)
PdRV		11.4(1.3)	0.2	-1.9(1.3)	10.5(2.6)
GFW	DSE	8.1(0.2)	10.7(0.3)		18.8(0.5)

preliminary!

numerical

systematic ?

BPP	J. Bijnens, E. Pallante and J. Prades Phys. Rev. Lett. 75, 1447 (1995)
HKS	M. Hayakawa, T. Kinoshita and A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995)
PdRV	J. Prades, E. de Rafael and A. Vainshtein, arXiv:0901.0306
GFW	T. Goecke, CF, R. Williams, Phys. Rev. D 87 (2013) 034013

Overview

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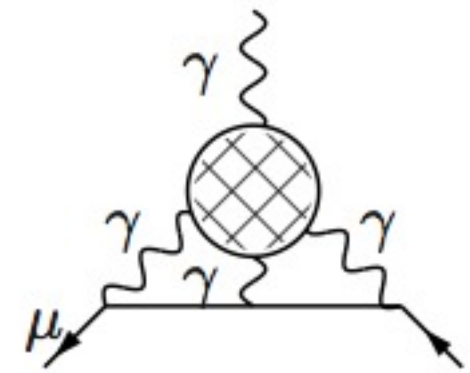
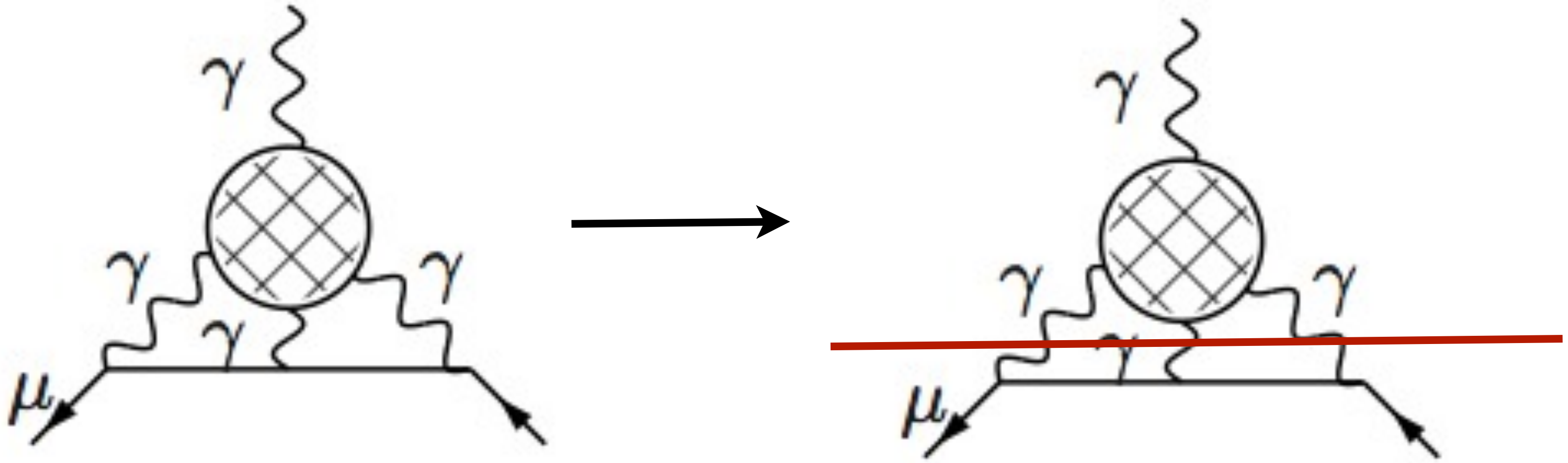


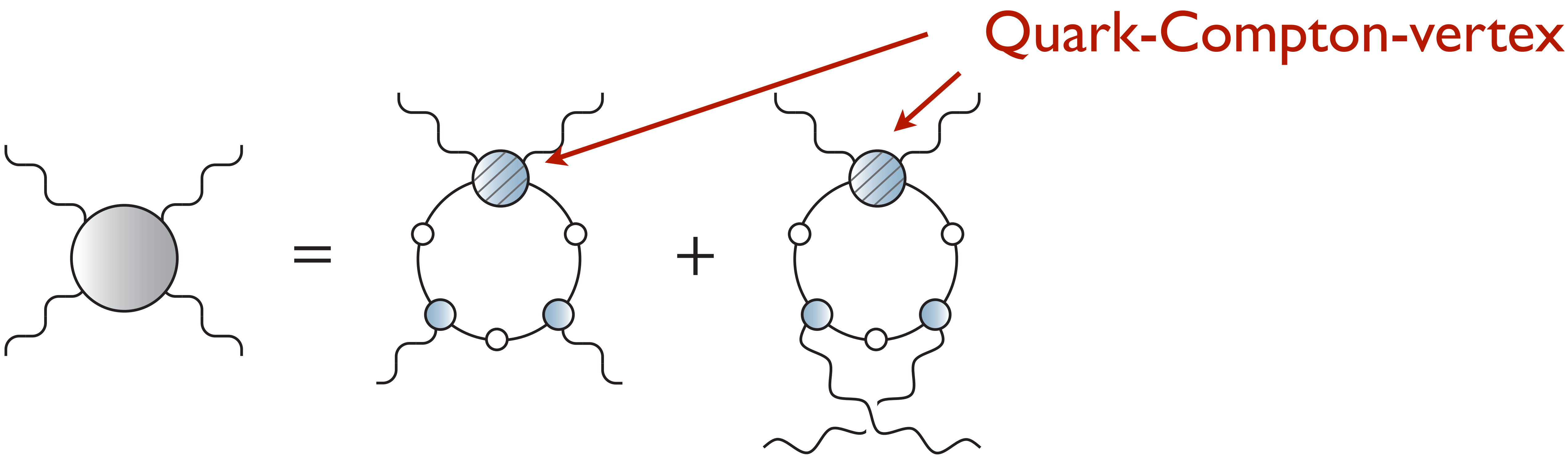
2. Hadronic light by light

3. Outlook: towards the photon four-point function

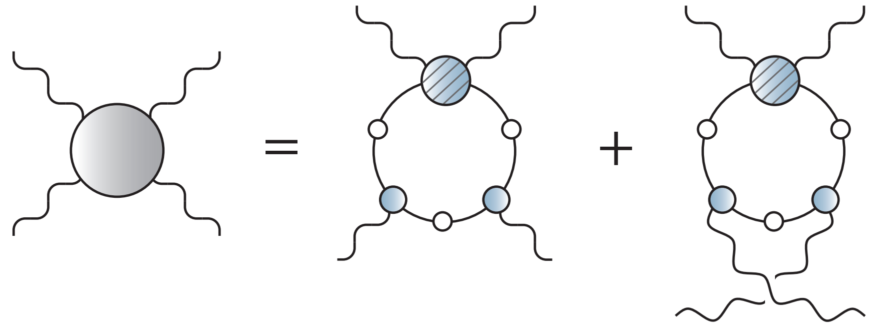
The four-photon amplitude



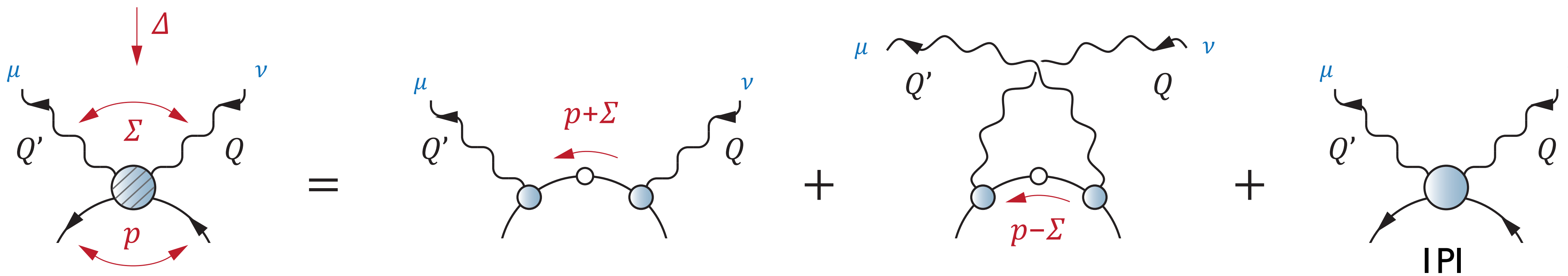
- Orthogonal basis constructed: 136 elements (41 after gauge inv.)
- four-photon amplitude satisfies (exact within RL):



Quark-Compton-vertex



Compton-vertex:



Quark-Loop incl. "meson-exchange"

- basis constructed (128 elements)
- meson exchange contributions isolated and $F_{\pi\gamma\gamma}$ recovered

Eichmann and CF, PRD 87 (2013) 3, 036006

Next steps:

- calc. full quark-loop contribution
- calc. full 1PI contribution and systematically compare with "off-shell meson approximation"

Eichmann, CF, Williams, work in progress

Reducing systematic errors in LBL

Step by step process:

I. Complete rainbow-ladder calc. of four-photon amplitude

- Avoid systematic errors of off-shell meson exchange approx.
- From HVP, form factors, etc.: expect error of <10 %
- Can be checked systematically by comparison with lattice at selected kinematical points

II. Include contributions beyond rainbow-ladder

- Pion-loop effects

Eichmann, CF. ,Williams, Haas, work in progress

Summary

- (Euclidean) QCD based approach
  - renormalizable
  - UV $\rightarrow$ IR: dynamical quark mass generation
- Hadronic vaccuum polarisation
  - agreement with experiment on 5 % level
- Light-by-light
  - large effects due to momentum dependencies
  - full calculation w.o. off-shell meson approximation under way

Highly desirable:
Systematic comparison with lattice results at
physical and unphysical pion and/or myon masses !