Effective Lagrangian Approach to HVP: HLS Model & Global Fit Methods

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LPNHE Paris 6/7

(thanks to P. David, L. Delbuono, F. Jegerlehner)

OUTLINE

- The HLS Model
- Breaking HLS : BKY & (ρ,ω,ϕ) mixing
- Global Fit Scope: τ vs $e^+ e^-$
  $[e^+e^- \to \pi\pi / K^+K^- / K_L K_S / \pi\gamma / \eta\gamma / \pi\pi\pi]$
- Global fits with only scan data
- Global fit with τ, scan and ISR data → HVP/g-2
- Conclusions

The HLS Model I

- Define

M.Bando *et al.* Phys. Rep. 164 (1988) 217

M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

$$\xi_{L/R} = e^{[\mp i P / f_\pi]}$$

PS field matrix

- The covariant derivative

V Field Matrix

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - i g V_\mu \xi_{L/R} + i \xi_{L/R} G_{L/R}$$

- with

$$G_R = eQA_\mu \,, \qquad G_L = eQA_\mu + \frac{g_2}{\sqrt{2}}\left(W_\mu^{+} T_{+} + W_\mu^{-} T_{-} \right)$$

The unbroken HLS Model II

- Define $$L/R = (D_\mu \xi_{L/R})\, \xi^\dagger_{L/R}$$

- Non-anomalous HLS pieces

$$L_{A/V} = -\frac{f_\pi^2}{4} Tr\left[L \mp R\right]^2$$

- Full HLS Lagrangian

$$L_{HLS} = L_A + a\, L_V + L_{FKTUY}\left[+L_{YM}\right]$$

The HLS Model III

- The anomalous (FKTUY) Lagrangian pieces

T. Fujiwara *et al*. Prog. Theor. Phys. 73 (1985) 926

$$L_{VVP} = -\frac{N_c g^2}{4\pi^2 f_\pi} c_3 \varepsilon^{\mu\nu\alpha\beta} Tr\left[\partial_\mu V_\nu \partial_\alpha V_\beta P \right] \qquad L_{AVP} \approx \left(c_4 - c_3 \right)$$

$$L_{AAP} = -\frac{N_c e^2}{4\pi^2 f_\pi} (1 - c_4) \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_\nu \partial_\alpha A_\beta Tr\left[Q^2 P \right]$$

$$L_{VPPP} = -i \frac{N_c g}{4\pi^2 f_\pi^3} (c_1 - c_2 - c_3) \varepsilon^{\mu\nu\alpha\beta} Tr\left[V_\mu \partial_\nu P \partial_\alpha P \partial_\beta P \right]$$

$$L_{APPP} = -i \frac{N_c e}{3\pi^2 f_\pi^3} \left[1 - \tfrac{3}{4}(c_1 - c_2 + c_4) \right] \varepsilon^{\mu\nu\alpha\beta} A_\mu Tr\left[Q \partial_\nu P \partial_\alpha P \partial_\beta P \right]$$

The HLS Model IV

- **Most general unbroken HLS Lagrangian :**

$$L_{HLS} = L_A + a\, L_V + L_{FKTUY}$$

- **Limited parameter freedom :**

$$g\,,a\,,c_3\,,[\,c_4\,],c_1 - c_2$$

- ⟶ **Unable to really account for a large amount of precise data**
- ⟶ **Should be broken**

Breaking the HLS Model (V)

- **Breaking through several mechanisms**
  - **The BKY mechanism**

M.Bando *et al*. Nucl. Phys. B259 (1985) 493
A. Bramon *et al*. Phys. Lett. B345 (1995) 263
M.Benayoun *et al*. Phys. Rev. D58 (1998) 074006

  - **Vector Meson Mixing**

M.Benayoun *et al*. EPJ C55 (2008) 199
M.Benayoun *et al*. EPJ C65 (2010) 211

  - **Additional breaking & mixing in PS sector**

G. 't Hooft , Phys. Rept. 142 (1986) 357

The BKY Mechanism

- **BKY breaking :**

$$\mathcal{L}_{A/V} = -\frac{f_\pi^2}{4} Tr\left[L \mp R\right]^2 \longrightarrow \mathcal{L}_{A/V} = -\frac{f_\pi^2}{4} Tr\{\left[L \mp R\right] X_{A/V}\}^2$$

✓ **Breaking matrix (BKY extended to Isospin)**

$$X_{A/V} = \text{Diag}\left\{q_{A/V},\ y_{A/V},\ z_{A/V}\right\}$$

SU(3) Brk

Isospin Brk

M.Benayoun *et al*. EPJ C17 (2000) 593

M. Hashimoto, Phys. Rev. D54 (1996) 5611

M.Benayoun *et al*. EPJ C72 (2012) 1848

Isospin Breaking : Vector Field Mixing I

- After BKY breaking, vector mass term ~~diagonal~~

  ideal vector fields ≠ mass *eigenstates* (tree level)

- Vector Mass Term Diagonalization:

$$\begin{bmatrix} \rho_I \\ \omega_I \end{bmatrix} = \begin{bmatrix} \rho_{R1} \\ \omega_{R1} \end{bmatrix} - \Delta_V \begin{bmatrix} h_V \, \omega_{R1} \\ (1-h_V)\, \rho_{R1} \end{bmatrix} \quad \& \quad \varphi_I = \varphi_{R1} \qquad (\Delta_V = q_V - y_V)$$

F. Jegerlehner & R. Szafron EPJ C71 (2011) 1632

C. Wolfe&K. Maltman, Phys. Rev. D80 (2009)114024

- Should be combined with Nonet Sym. Brk in V sector

Vector Field Mixing II

- **s-dependent** transitions among vector fields generated at one loop by HLS Lag.

$$\left(\rho_{R1}+\omega_{R1}-\sqrt{2}\,z_V\,\varphi_{R1}\right)K^{-}\overleftrightarrow{\partial}K^{+}+\left(\rho_{R1}-\omega_{R1}+\sqrt{2}\,z_V\,\varphi_{R1}\right)K^{0}\overleftrightarrow{\partial}\overline{K}^{0}$$

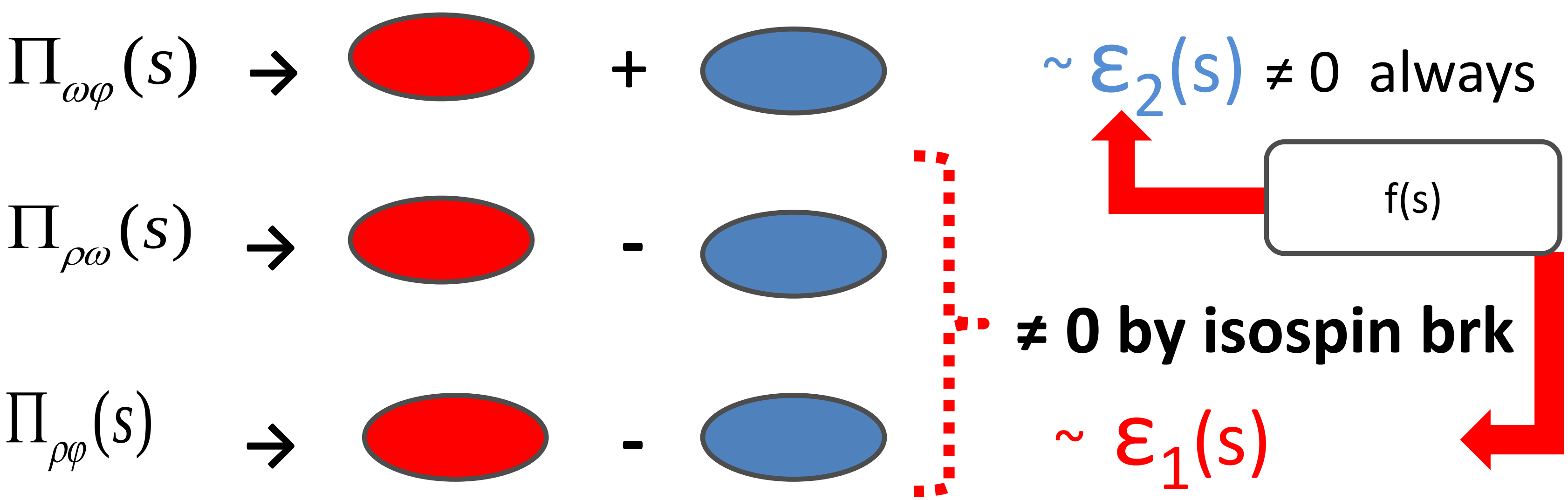
- [K $\overline{\text{K}}$ loops ++ K* $\overline{\text{K}}$ (VVP) & K* $\overline{\text{K}}$* (YM)]

ideal fields no longer mass eigenstates

PS mesons :: isospin symmetry breaking : $m_{K^{\pm}} \neq m_{K^{0}}$

Transitions at one loop

- Kaon loops : = $K^+ K^-$, = $K^0 \bar{K}^0$
- contribute to Vector meson self-masses
- Generate transitions among Vector meson:



The Mass Matrix Eigen System

M.Benayoun *et al.* EPJ C72 (2012) 1848

At one loop (No BKY IB)**:**

$$M^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) + \varepsilon_2(s) & \varepsilon_1(s) & -\sqrt{2} z_V \, \varepsilon_1(s) \\ \varepsilon_1(s) & m^2 + \varepsilon_2(s) & -\sqrt{2} z_V \, \varepsilon_2(s) \\ -\sqrt{2} z_V \, \varepsilon_1(s) & -\sqrt{2} z_V \, \varepsilon_2(s) & z_V m^2 + 2 z_V^{\,2} \varepsilon_2(s) \end{pmatrix}$$

As :

$$\left(m^2, \Pi_{\pi\pi}(s)\right) >> \varepsilon_2(s) >> \varepsilon_1(s)$$

Solve perturbatively

$$M^2(s) = M_0^2(s) + \delta M^2(s)$$

From **Ideal** To **Physical** Fields

$$\begin{pmatrix} \rho^0_{R1} \\ \omega_{R1} \\ \varphi_{R1} \end{pmatrix} = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & \gamma \\ -\beta & -\gamma & 1 \end{bmatrix} \begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix}$$

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon)=1$$

Physical Fields

R_1 Fields (X_V)

$$\begin{bmatrix} \alpha(s) \\ \beta(s) \\ \gamma(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi^\rho_{\pi\pi}(s) - \left[m_\omega^{HK}\right]^2} \\ \dfrac{\sqrt{2}\, z_V\, \varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi^\rho_{\pi\pi}(s) - \left[m_\varphi^{HK}\right]^2 + (1-2z_V^2)\,\varepsilon_2(s)} \\ \dfrac{\sqrt{2}\, z_V\, \varepsilon_2(s)}{\left[m_\omega^{HK}\right]^2 - \left[m_\varphi^{HK}\right]^2 + (1-2z_V^2)\,\varepsilon_2(s)} \end{bmatrix}$$

Isospin Breaking in the τ Sector

IB effects in dipion spectrum (as usual)

- **Short Range & Long Range IB corrections**

$$B_{\pi\pi} \frac{1}{N}\frac{dN}{ds} = \frac{1}{\Gamma_\tau}\frac{d\Gamma_{\pi\pi}(s)}{ds} S_{EW} G_{EM}(s)$$

W. Marciano & A. Sirlin, PRL 71 (1993) 3629.
V. Cirigliano *et al.* PL B 513 (2001) 361
& JHEP 08 (2002) 002

- **$\pi^\pm\pi^0$ Phase space factor**
- **No significant sensitivity to $\rho^\pm - \rho^0$ mass/width diff. within global fits**

IB in τ & IB in e^+e^- :: unrelated
but both involved in global fits

The PS Sector

Diagonalization of the PS Kinetic Energy Term

- **Rescaling of Kaon fields**
- **Mixing of the π^0, η, η'** system because of :

❖ **BKY IB & SU(3) breakings**

❖ **Nonet symmetry breaking :** $L_{'tHooft} \Rightarrow \frac{\lambda^2}{2} \partial^\mu \eta_0 \partial^\mu \eta_0$

π^0, η, η' : 2-angle mixing scheme

→ **Phenomenology favors $\theta_0=0$ [$\longrightarrow \theta_P \approx \theta_8/2$]**

M.Benayoun *et al.* EPJ C72 (2012) 1848

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M.Benayoun *et al.* EPJ C17 (2000) 593

HLS : A VMD Model

- The (Broken) Hidden Local Symmetry model :
  - is a unified VMD framework encompassing
    $e^+e^- \to \pi\pi$ /KKbar /$\pi\gamma$ /$\eta\gamma$ /$\pi\pi\pi$ & $\tau \to \pi\pi\nu_\tau$ & PVγ, P$\gamma\gamma$ decays & $\eta/\eta' \to \gamma\pi\pi/\gamma\gamma$ & ……
  - Has few basic parameters : e, f_π ,V_{ud},V_{us}, a , g ,…
  - (+ breaking parameters)
  - Present Limit : One vector nonet
  - ✓ Up to the ≈ ϕ mass region (≈ 1.05 GeV)
  - ✓ No scalars mesons, no ρ', no ρ'' ……

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& PVγ, Pγγ decays & η/η' → γππ/γγ & ……

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g-2 & Global Models/Fits

- **Hadronic VP contributions to g-2**

$$a_\mu(H_i) = \frac{1}{4\pi^3}\int_{s_{th}}^{s_{cut}} ds\, K(s)\,\sigma(e^+e^- \to H_i, s)$$ ⟸ **Measured** Xsection

$$a_\mu^{HVP} = \sum_{H_i} a_\mu(H_i)$$ :: Xsections **uncorrelated**

- **VMD : $\Rightarrow$ Underlying physics correlations -> HLS cross-sections derived through a global fit (param. values & covariance matrix) :**

Measured Xsection ⟶ **Model** Xsection

g-2 & Global Models/Fits

- **Hadronic VP contributions to g-2**

$$a_\mu(H_i) = \frac{1}{4\pi^3} \int_{s_{th}}^{s_{cut}} ds\, K(s)\, \sigma(e^+e^- \to H_i, s)$$ ⟸ **Model** Xsection

$$a_\mu^{HVP} = \sum_{H_i} a_\mu(H_i)$$ :: Xsections **(VMD) correlated**

- **VMD : ⟹ Underlying physics correlations ->**
HLS cross-sections derived through a global fit
(param. values & covariance matrix) :

Measured Xsection ⟶ **Model** Xsection

HLS→Correlated Physics Channels

- Non-Anomalous annihilations : $e^+e^- \rightarrow \pi^+\pi^-$ / $K^+K^-/K_L K_S$
- Decay spectra : $\tau^\pm \rightarrow \pi^\pm\pi^0 \nu_\tau$ (**later** also : $\eta/\eta' \rightarrow \pi^+\pi^-\gamma$)
- Anomalous Processes : $e^+e^- \rightarrow \pi^0 \gamma/\eta\gamma/ \pi^+\pi^- \pi^0$
- Radiative decays widths ($VP\gamma$, $\pi^0/\eta/\eta' \rightarrow \gamma\gamma$)

- **$\phi \rightarrow \pi\pi$ (Br ratio and phase)**

  ([$Br(\phi \rightarrow \pi^+ \pi^-)$ x$Br(\phi \rightarrow e^+ e^-)$])

HLS MODEL & Breaking Scheme
OVERCONSTRAINED
by more than 40 data sets

VMD Strategy for g-2 Estimate

GLOBAL FIT of the largest collection of data sets

Examine the Global Fit Prob. + some tags (χ^2/N_i)

- IF satisfactory:

1/ HLS form factors & fit param. & cov. produce improved/motivated HVP contributions up to 1.05 GeV

from $\pi^+\pi^-$/ K^+K^- /K_L K_S /$\pi\,\gamma$ /$\eta\,\gamma$ /$\eta'\,\gamma$ /$\pi\,\pi\,\pi$

- IF poor :

2/ discard poorly described data sets & restart

3/ BUT : Each channel should stay covered by data

Fit Strategies with τ & scan data

Fit 6 annihilation channels (+**VPγ, P→γ γ, ϕ→ππ**):

- ✓ Minimize the **true** χ2 (**Full** Exp. Error Cov. Matrix **valid**)
- ✓ $\tau^\pm \to \pi^\pm \pi^0 \nu_\tau$ spectra : **constraints** on Model params.
- ➢ **Config. A : Fit all scan data spectra (except for SND K^+K^- : χ^2/N_i =59/26)**
- ➢ **Config. B : Conf. A & excl. $\pi^+\pi^-\pi^0$ s> 1GeV^2 (c_1-c_2 issue)**

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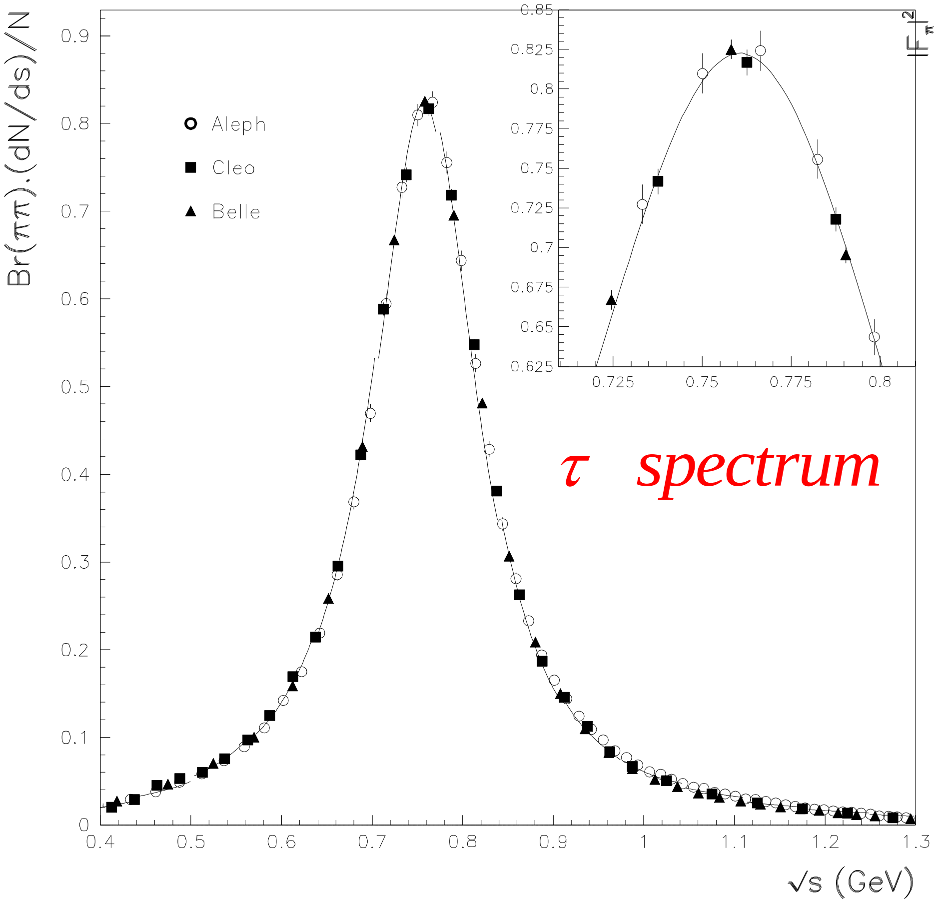
- ✓ $\eta/\eta' \to \pi^+\pi^-\gamma$: checks or **constraints (as τ spectra)**

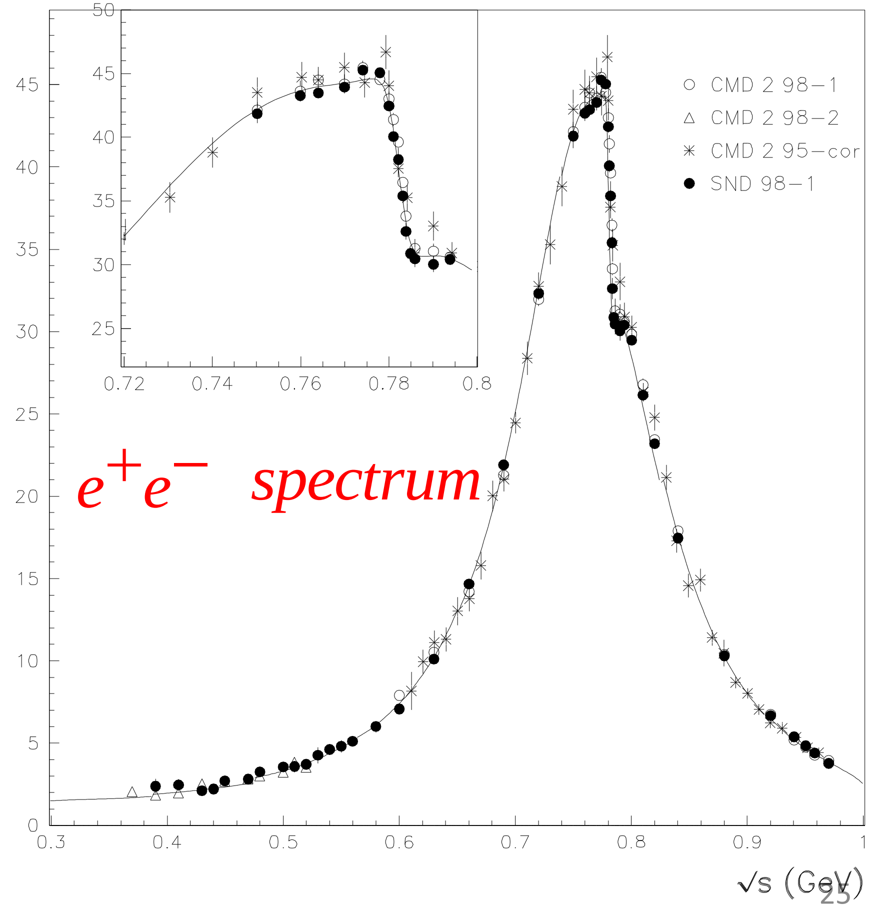
Global Fit Results (τ & scan data)

Data Set (#data points : NP)	Configuration A (χ^2) npar=26	Configuration B (χ^2) npar = 24	Approx. χ^2/ NP
Decays (10)	12.8	13.1	1.3
New Timelike (127)	120.9	122.3	0.95
Old Timelike (82)	52.9	51.8	0.63
$\pi^0\gamma$ (86)	68.0	67.5	0.79
$\eta\gamma$ (182)	123.9	122.5	0.68
$\pi^+\pi^-\pi^0$	210.2/179	120.0/99	1.20
K^+K^- (36)	29.1	29.0	0.81
K_LK_S (119)	120.4	119.5	1.00
τ ALEPH (37)	18.7	18.7	0.51
τ CLEO (29)	35.9	36.9	1.24
τ Belle (19)	28.9	29.0	1.52
X^2/dof / Probability	821.6/880 92.0%	730.4/802 96.3%	

Dipion Spectra (τ & scan)

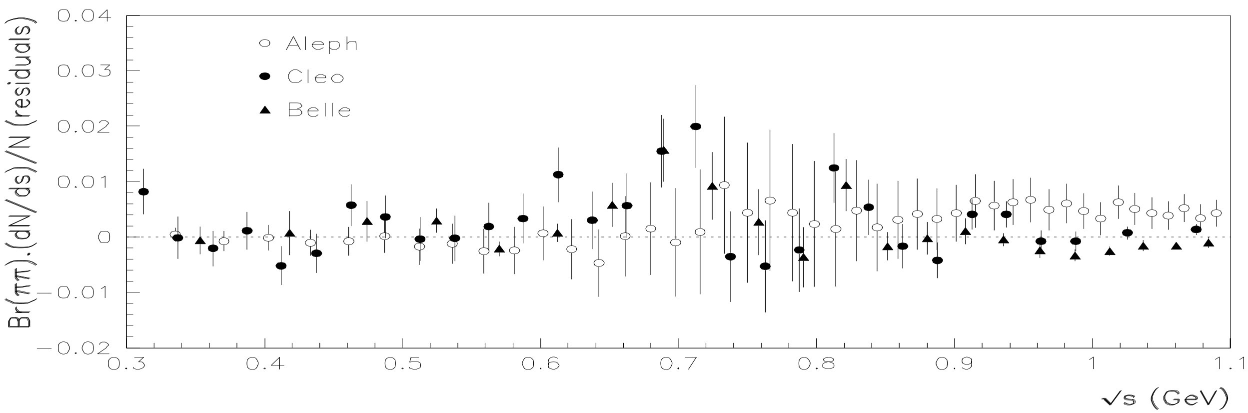
$\chi^2/N_p = 83.5/88$ (Prob ≈ 96%) $\chi^2/N_p = 122/127$



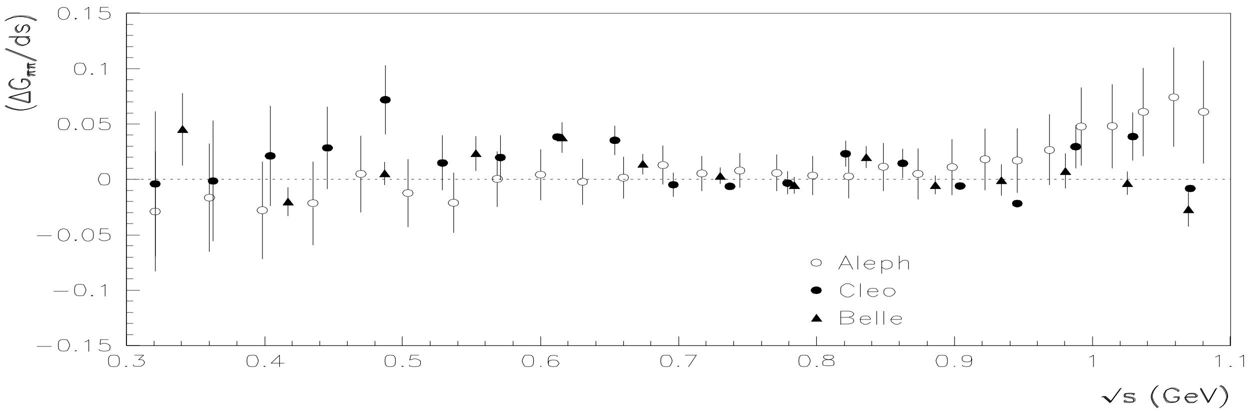


Tau Global Fit Residuals

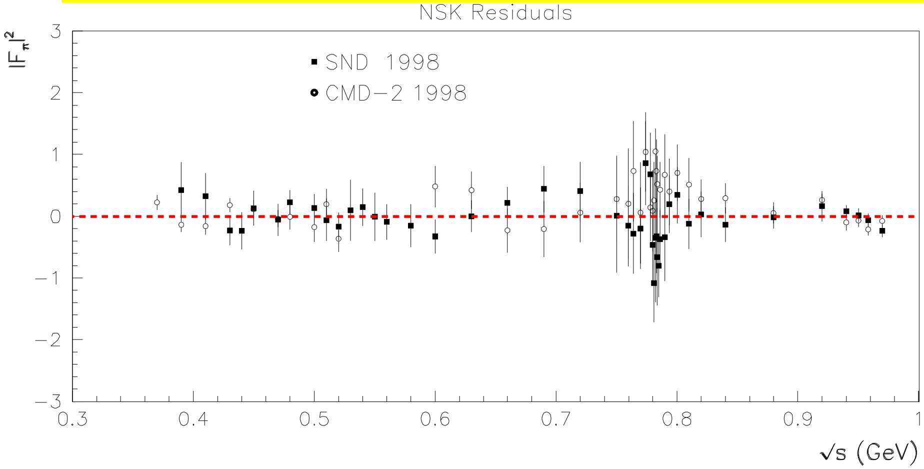
ΔF



ΔF/F

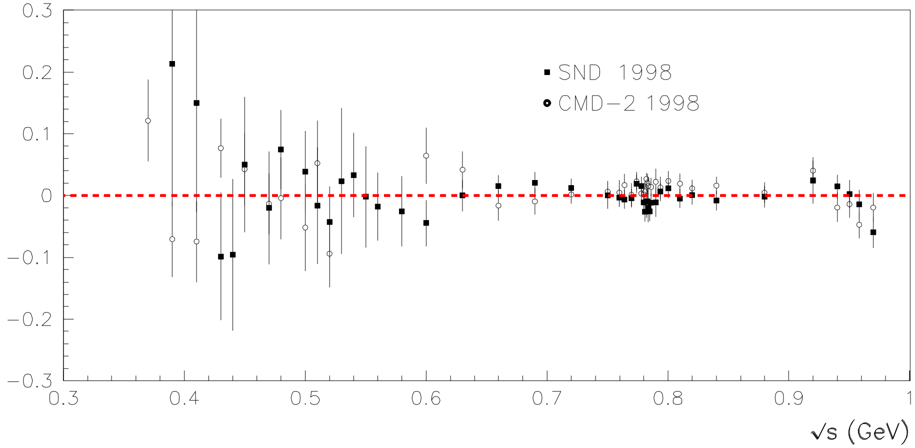


CMD2/SND Global Fit Residuals



ΔF

$$\chi^2 / N_p = 122.3 / 127$$



ΔF/F

e+e- vs τ : nothing puzzling

$\pi^+ \pi^-$ Spectra : NSK, KLOE, BaBar

- **Several measurements of the $\pi^+ \pi^-$ spectrum**

i. CMD2, SND

CMD2: Phys. Lett. **B648** (2007) 28, JETP Lett. **84** (2006) 413
SND: JETP **103** (2006) 380

ii. KLOE

KLOE08 : AIP Conf. Proc. **1182** (2009) 665 *

KLOE10: Phys. Lett. **B700** (2011) 102
KLOE12: Phys. Lett. **B720** (2013)336

iii. BaBar

BaBar : Phys. Rev. Lett. **103** (2009) 231801 *
Phys. Rev . **D86** (2012) 032013

exhibit conflicting behaviors within global fits

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GLOBAL FITS : Check ππ data sets

- Fits with each ππ data set in isolation (scan/KLOE's/BaBar) → **select on Prob.**

- **All other channels always included in fits**

  ($\tau \to \pi\pi \nu_\tau$, $e^+e^- \to$ K^+K^-/ $K_L K_S$/$\pi \gamma$/$\eta\gamma$/$\pi \pi \pi$)

Consistent $\pi^+\pi^-$ data sets for Global Treatment :

CMD2 & SND & KLOE10 & KLOE12

Fitting $\pi^+\pi^-$ Data within Global Fits I

Fit Cond. Conf. B	NSK & KLOE08 & KLOE10 & KLOE12 & BaBar with/without τ data

With $\pi^+\pi^-$	KLOE08(60)	KLOE10(75)	KLOE12(60)	NSK(127)	BaBar(270)
Single ($\chi^2/N_{\pi^+\pi^-}$)	1.74 [1.56] 21 [71] %	1.02 [0.94] 81 [96] %	1.08 [1.04] 87 [97] %	0.96 [1.0] 97 [99] %	1.26[1.24] 20 [33] %
Combined ($\chi^2/N_{\pi^+\pi^-}$)		1.09[1.03]	1.27[1.28]	1.07[1.11]	1.38[1.37]
Comb ($\chi^2/N_{\pi^+\pi^-}$) (Prob.) (%)	1.25[1.25] 5 [10] %				

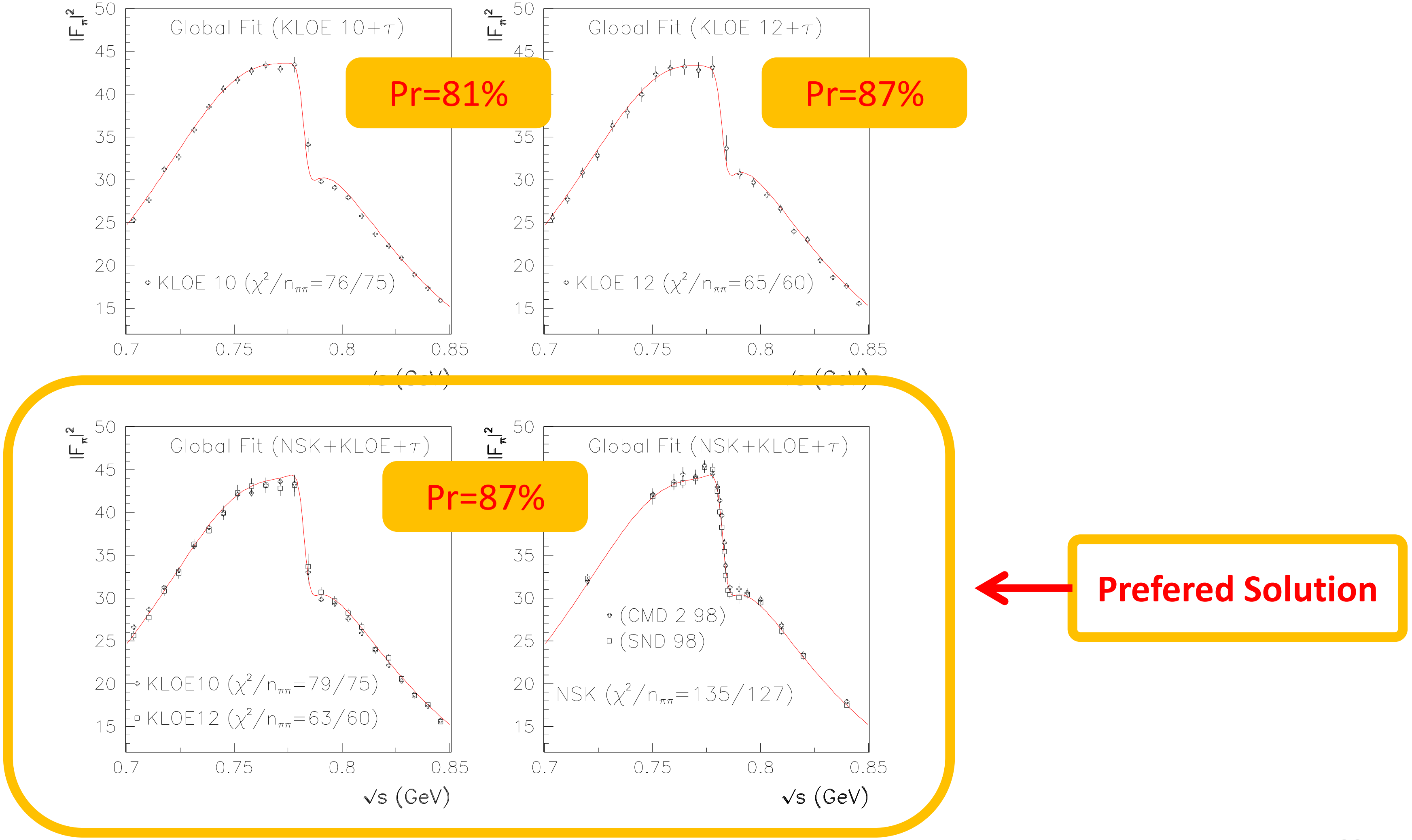
Fitting $\pi^+\pi^-$ Data within Global Fits I

Fit Cond. Conf. B	NSK & KLOE08 & KLOE10 & KLOE12 & BaBar with/without τ data				
With $\pi^+\pi^-$ +π -	KLOE08(60)	KLOE10(75)	KLOE12(60)	NSK(127)	BaBar(270)
Single ($\chi^2/N_{\pi+\pi-}$)	1.74 [1.56] 21 [71] %	1.02 [0.94] 81 [96] %	1.08 [1.04] 87 [97] %	0.96 [1.0] 97 [99] %	1.26[1.24] 20 [33] %
Combined ($\chi^2/N_{\pi+\pi-}$)		1.09[1.03]	1.27[1.28]	1.07[1.11]	1.38[1.37]
Comb ($\chi^2/N_{\pi+\pi-}$) (Prob.) (%)	1.25[1.25]			5 [10] %	

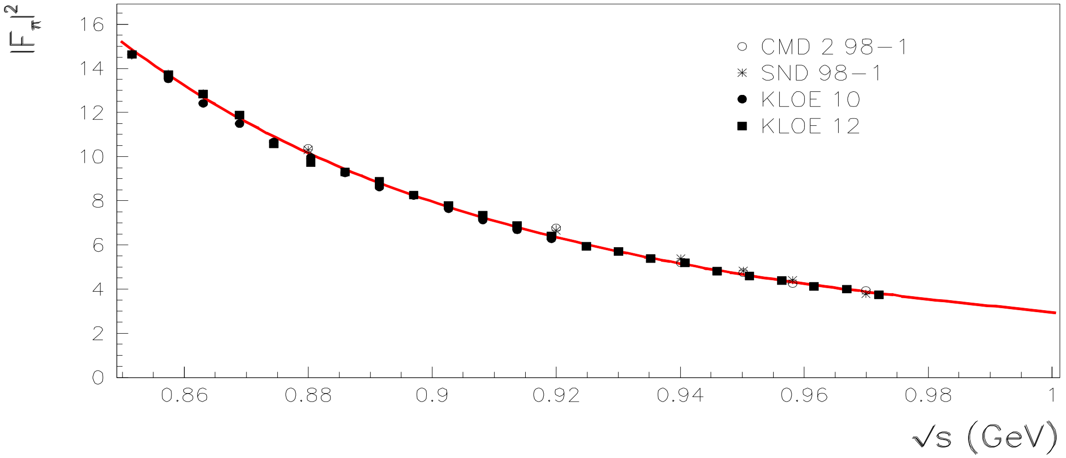
Fitting $\pi^+\pi^-$ Data within Global Fits II

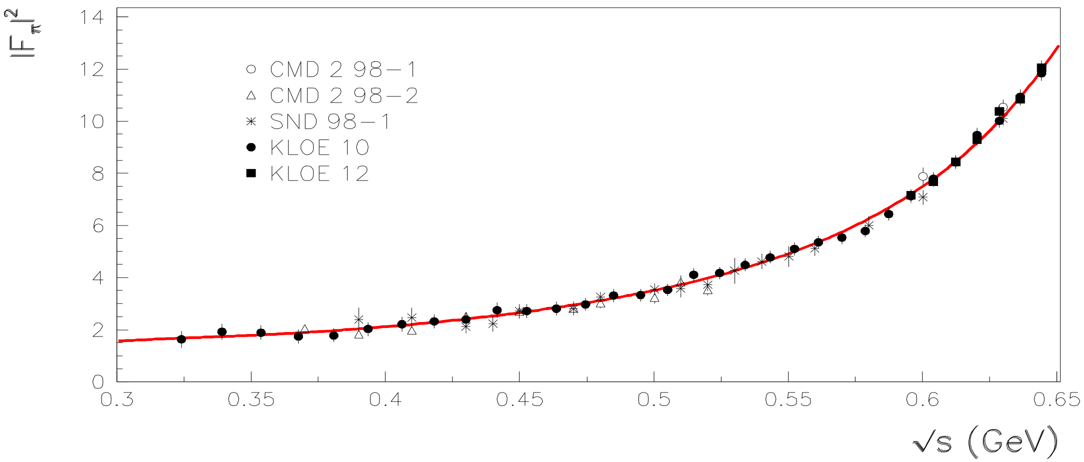
Fit Cond. Conf. B	NSK & KLOE10 & KLOE12 with/without τ data				
With $\pi^+\pi^-$ +π - from	KLOE10(75) alone	KLOE12(60) alone	KLOE(135) combined	NSK(127) alone	NSK+KLOE combined (262)
Single ($\chi^2/N_{\pi+\pi-}$) Prob.	1.02[0.94] 81 [96] %	1.08[1.04] 87 [97] %	1.01[0.99] 80 [96] %	0.96 [1.00] 97 [99] %	1.06 [1.07] 87 [96]%

Global Fits : Top

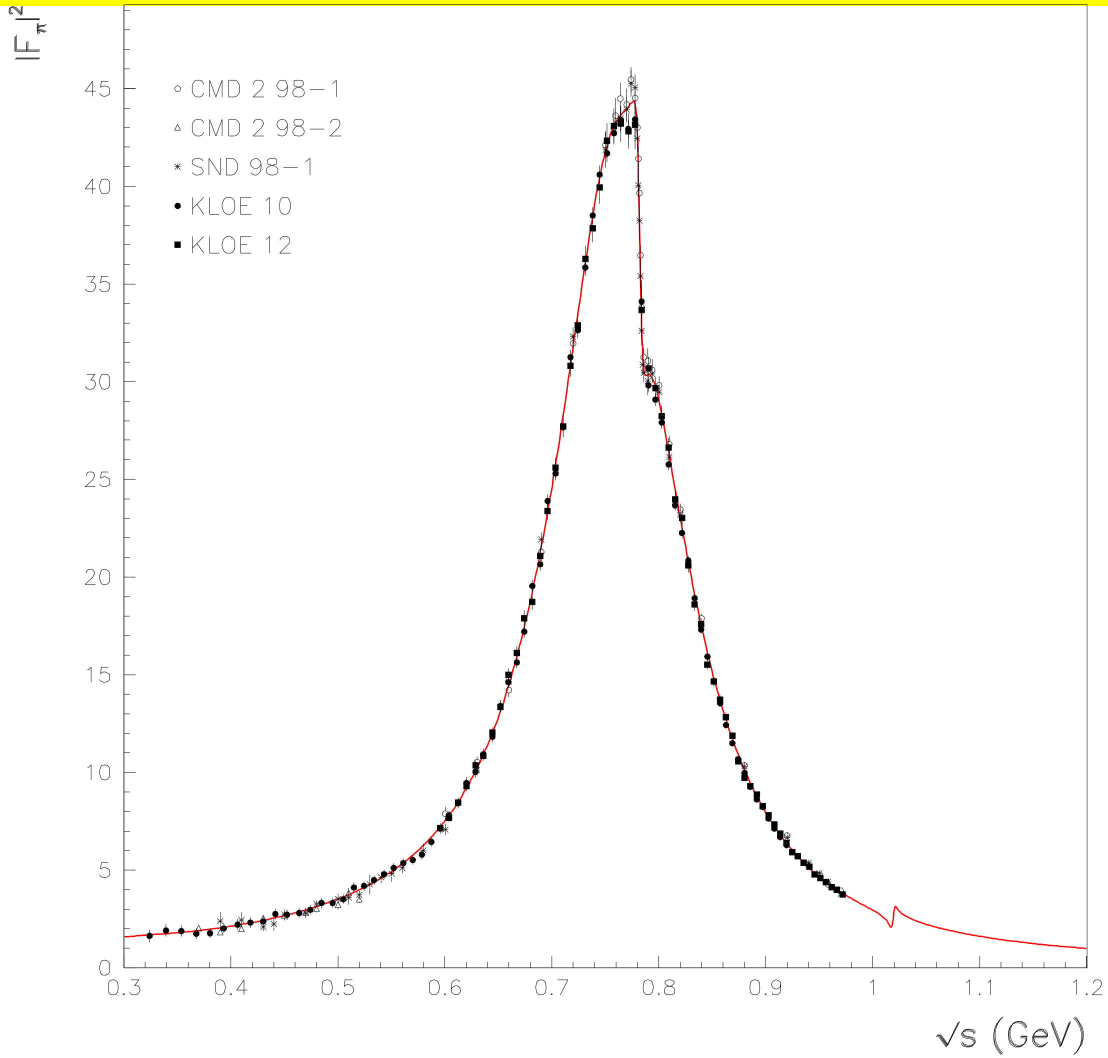


Global Fits : Sides

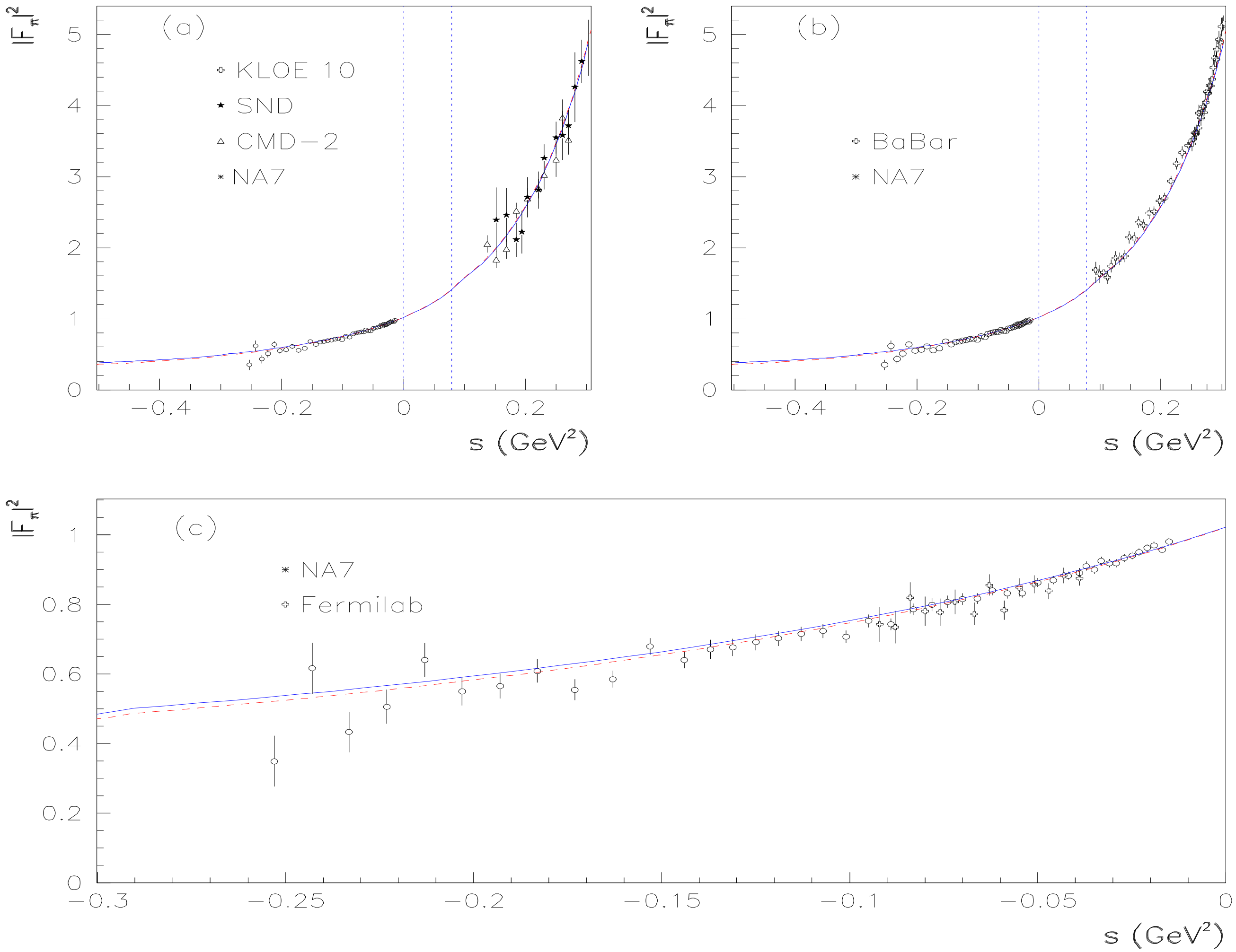




GLOBAL FIT : All ππ spectrum



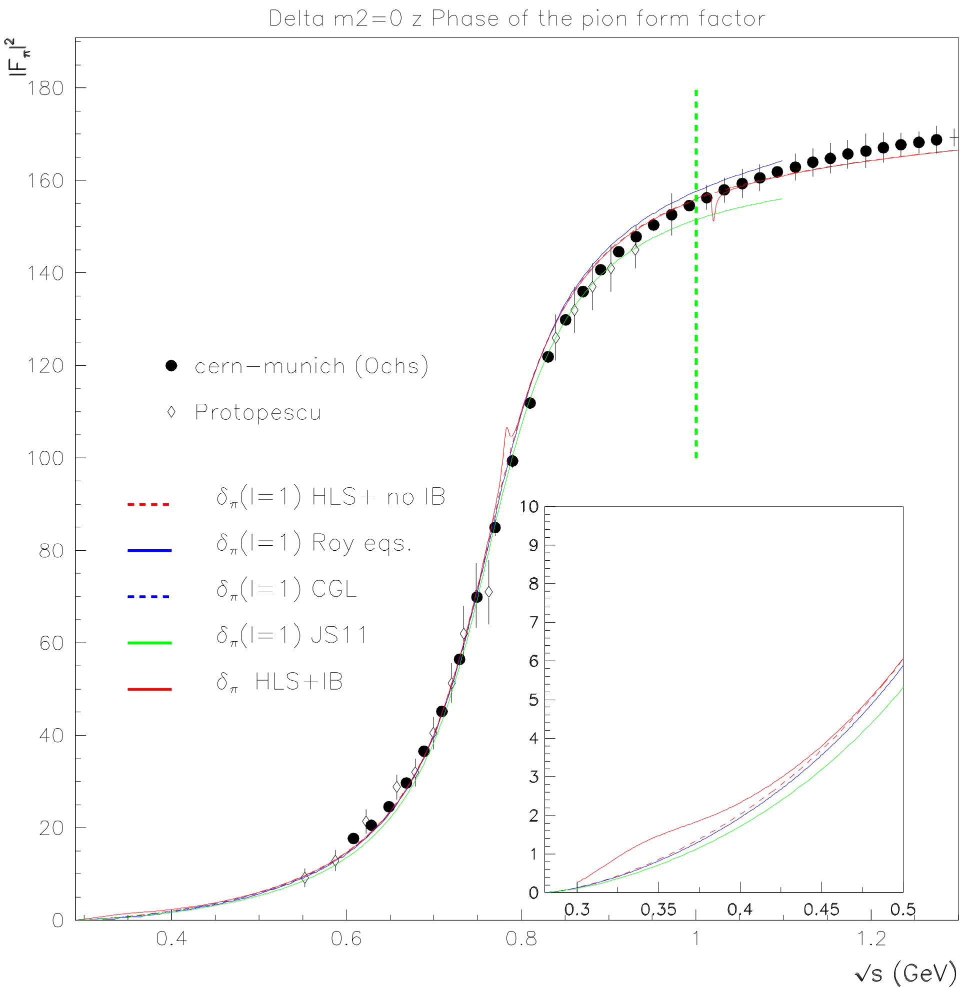
Spacelike & Threshold Regions



Connection at threshold

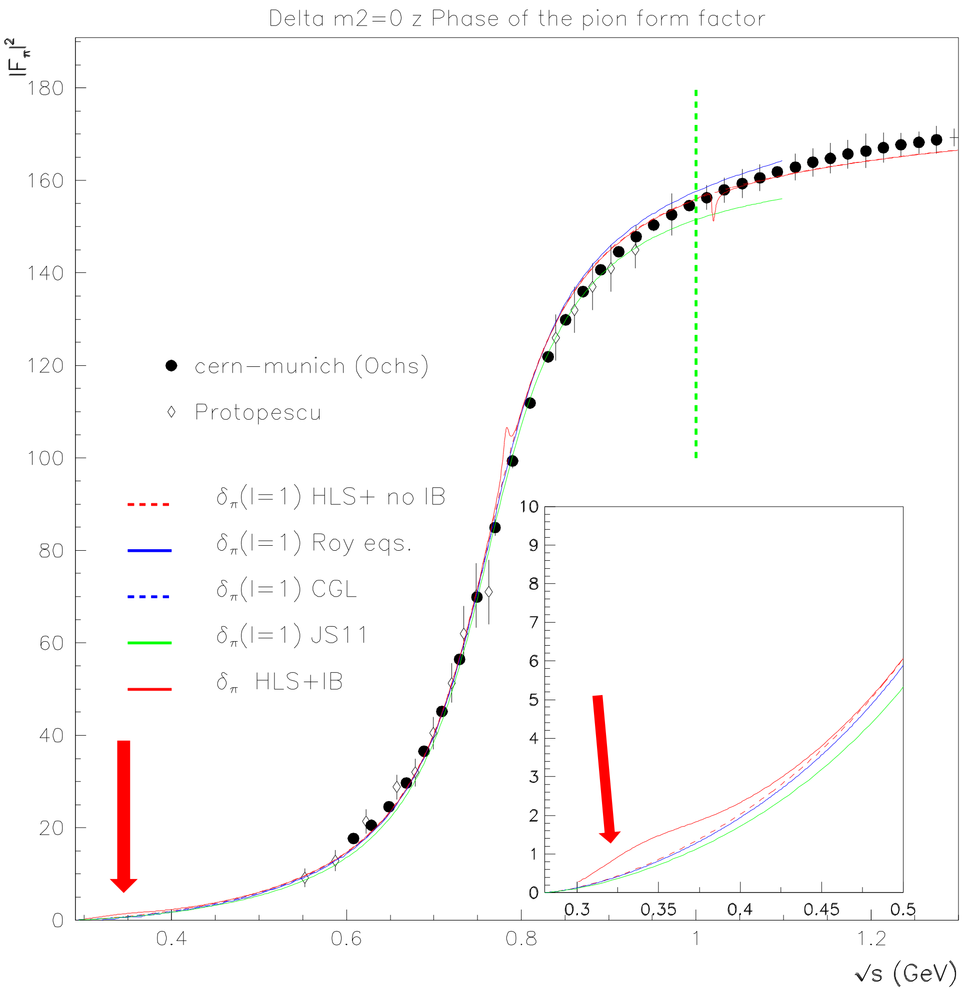
Extrapolation to s<0

Predicted Phase shift (I)



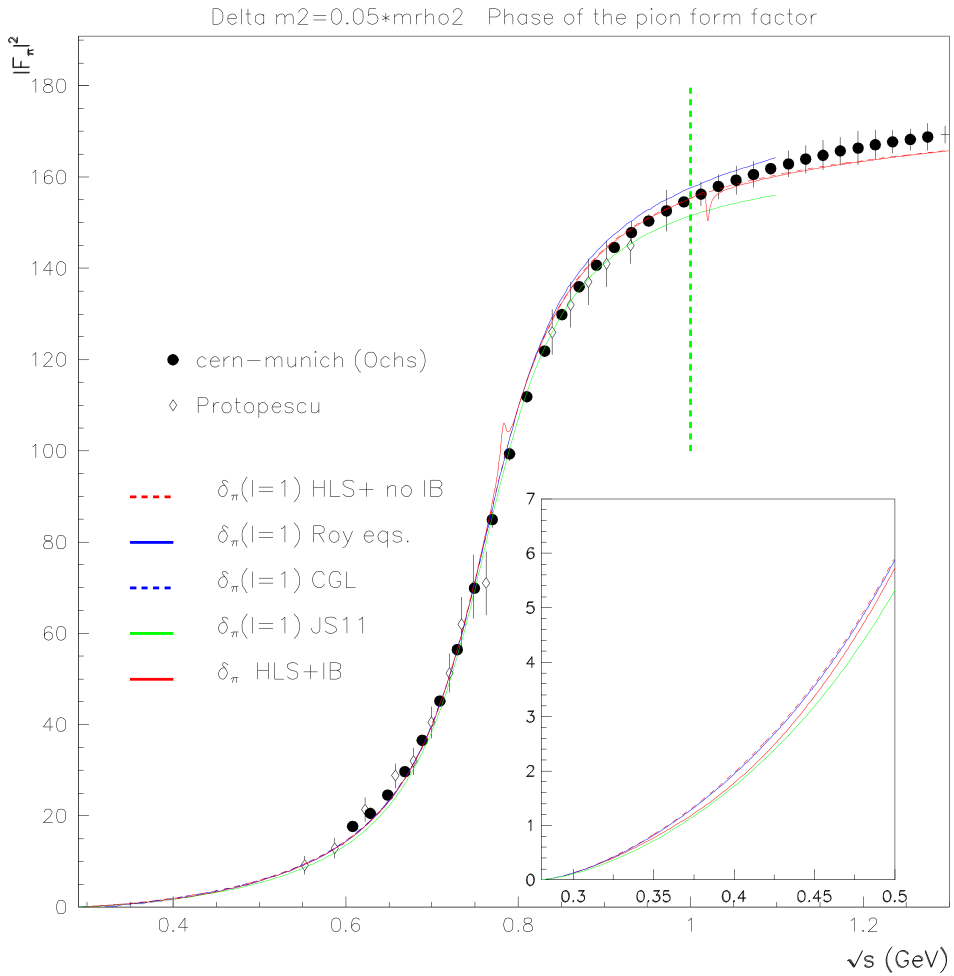
$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.00$$

Predicted Phase shift (I)



$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.00$$

Predicted Phase shift (II)



$$\left[\frac{m_\omega^{HK}}{m_\rho^{HK}}\right]^2 = 1.05$$

Vector Nonet Symmetry Breaking

- The mixing angles γ and β vanish at s=0
- Not α

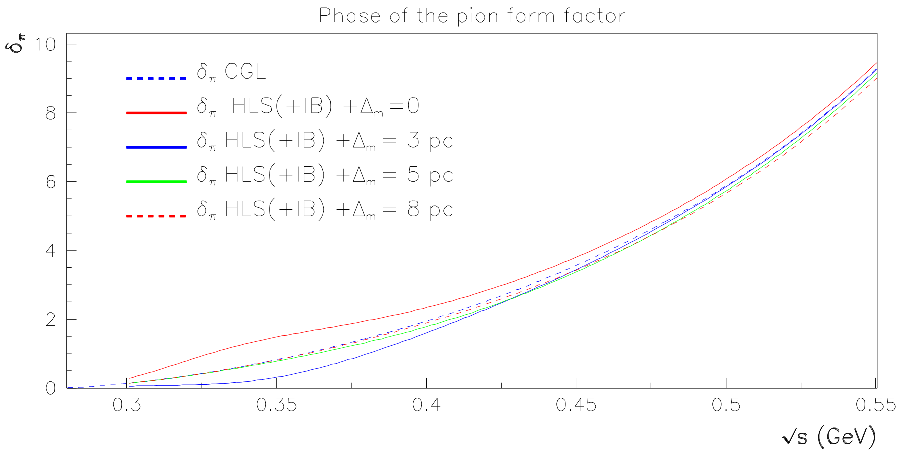
$$\alpha(s) = \frac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi_{\pi\pi}^{\rho}(s) - \left[m_\omega^{HK}\right]^2} \quad \textbf{as} \quad m_\rho^{HK} = m_\omega^{HK}$$

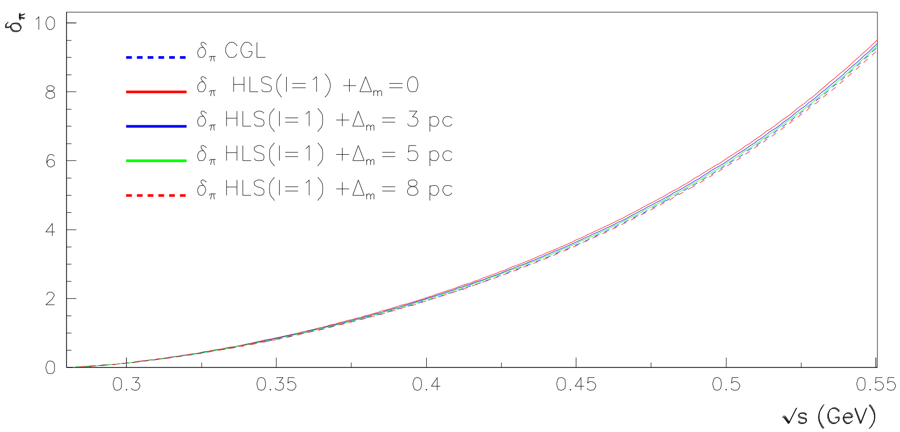
- α(0)=0 -> NSB in Vector Sector :

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - ig(V_\mu + \delta V_\mu)\xi_{L/R} + i\xi_{L/R} G_{L/R}$$

- With : $\delta V_\mu = \xi \frac{\omega_I \sqrt{2} + \phi_I}{3} Diag[1,1,1]$

Threshold Behavior -> NSB





$$\alpha(s) = \frac{\varepsilon_1(s)}{\left[m_\rho^{HK}\right]^2 + \Pi_{\pi\pi}^{\rho}(s) - \left[m_\omega^{HK}\right]^2}$$

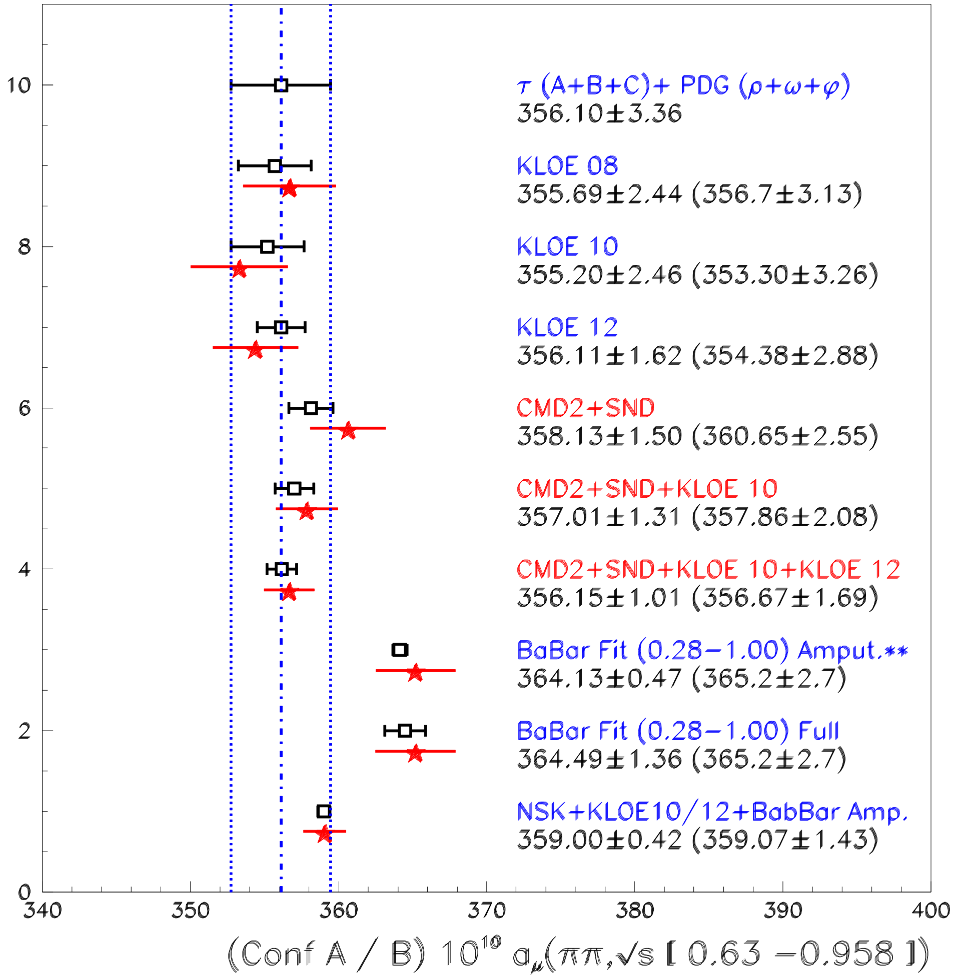
$$\Delta_m = \frac{\left[m_\omega^{HK}\right]^2}{\left[m_\rho^{HK}\right]^2} - 1$$

From Fits to HVP up to 1.05 GeV (B)

- **Uncertainties improved by ≈2**

Channel	NSK +τ	NSK+KLOE+τ	Direct Estimate
$\pi^+\pi^-$	496.67 ± 2.13	495.22 ±1.45	498.53 ± 3.73 (497.72 ± 2.12)
$\pi^0\gamma$	4.53 ± 0.04	4.54 ± 0.04	3.35 ± 0.11
$\eta\gamma$	0.63 ± 0.01	0.63 ± 0.01	0.48 ± 0.01
$\eta'\gamma$	0.00 ± 0.00	0.00 ± 0.00	---
$\pi^+\pi^-\pi^0$	39.50 ± 0.58	39.07 ± 0.58	43.24 ± 1.47
$K_L K_S$	11.54 ± 0.08	11.54 ± 0.08	12.31 ± 0.33
K^+K^-	16.94 ± 0.21	16.95 ± 0.21	17.88 ± 0.54
Total up to 1.05 GeV	569.81 ± 2.02	567.94 ± 1.56	575.79 ± 4.06 (574.98 ± 2.66)

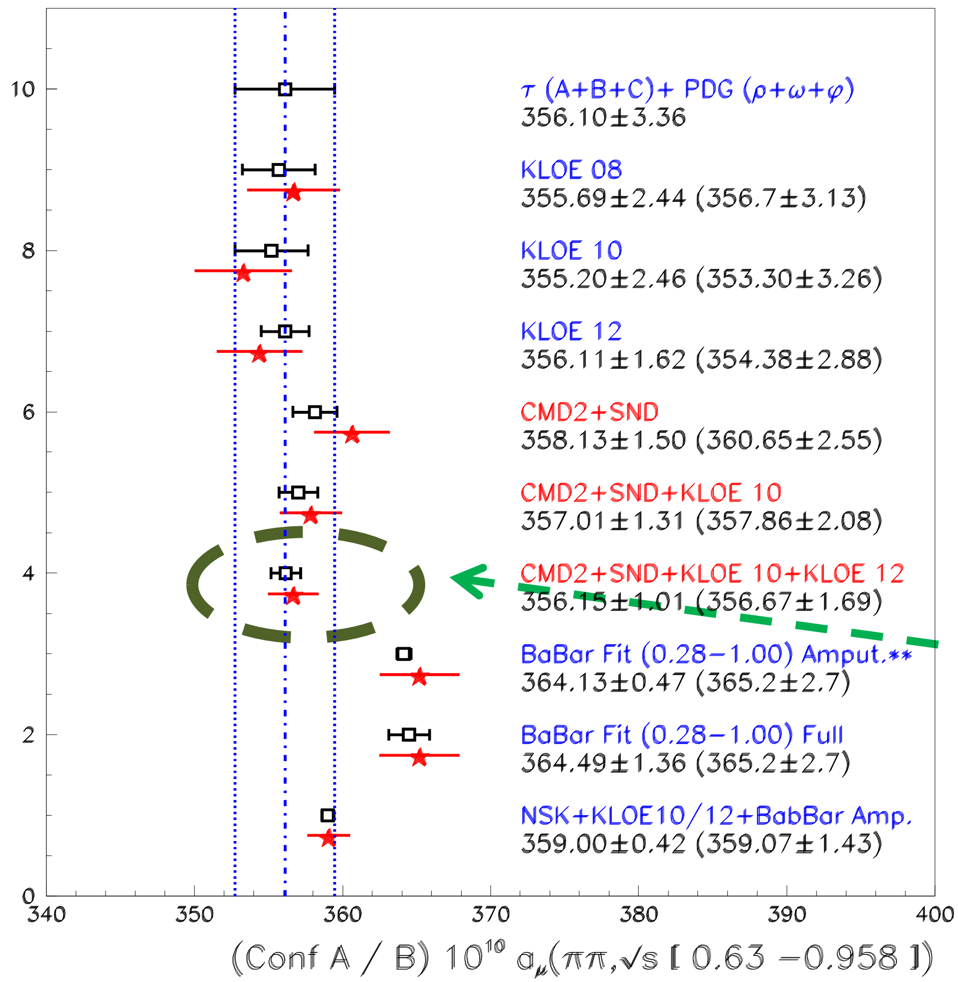
$a_\mu(\pi\pi)$ over $m_{\pi\pi}$=[0.63,0.958] GeV (Conf B)



Squares : fit values (incl. τ)
Stars : exp. estimates

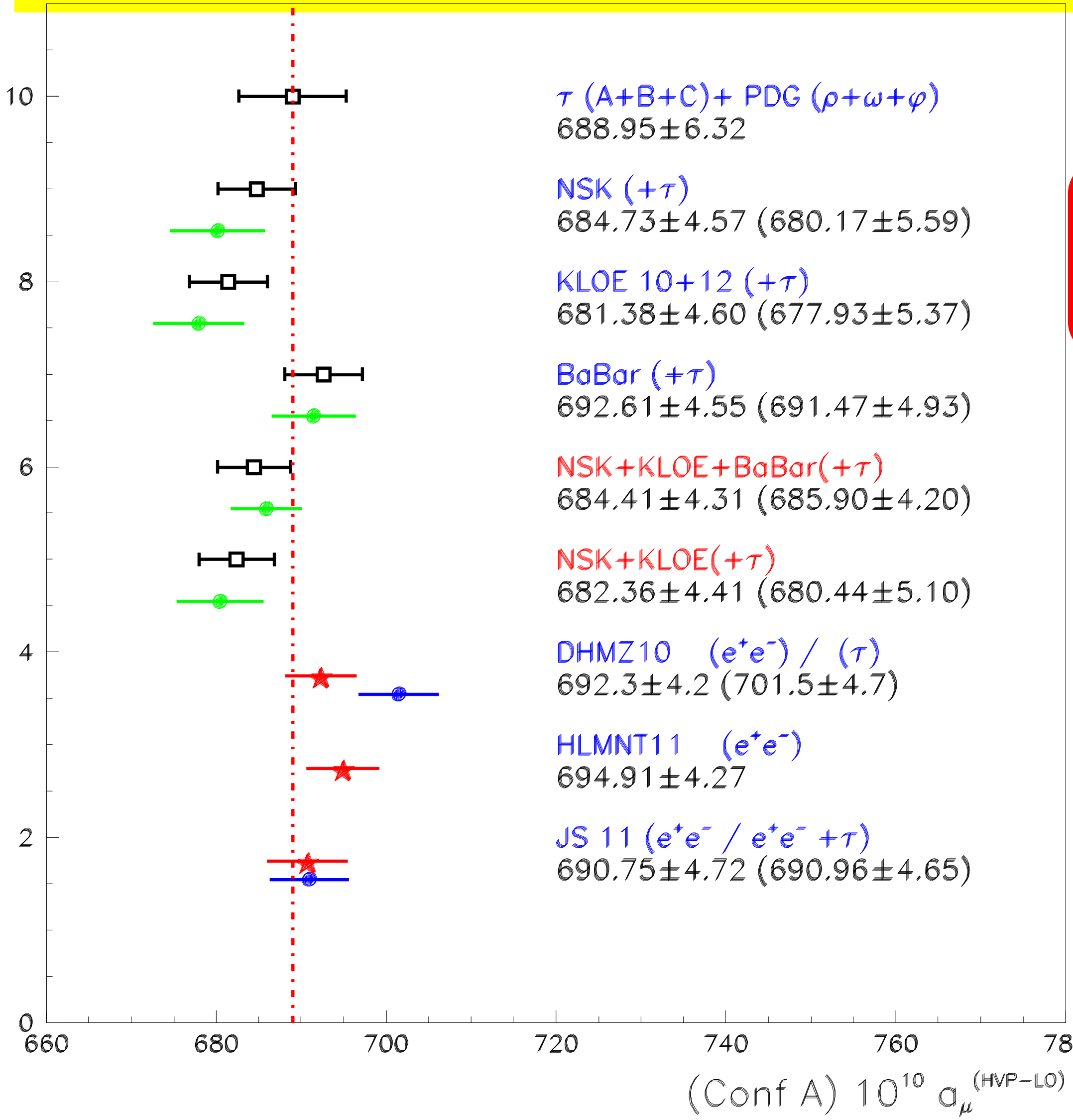
Uncertainties improved by ≈40%

$a_\mu(\pi\pi)$ over $m_{\pi\pi}$=[0.63,0.958] GeV (Conf B)



Accord with Exp. expectations

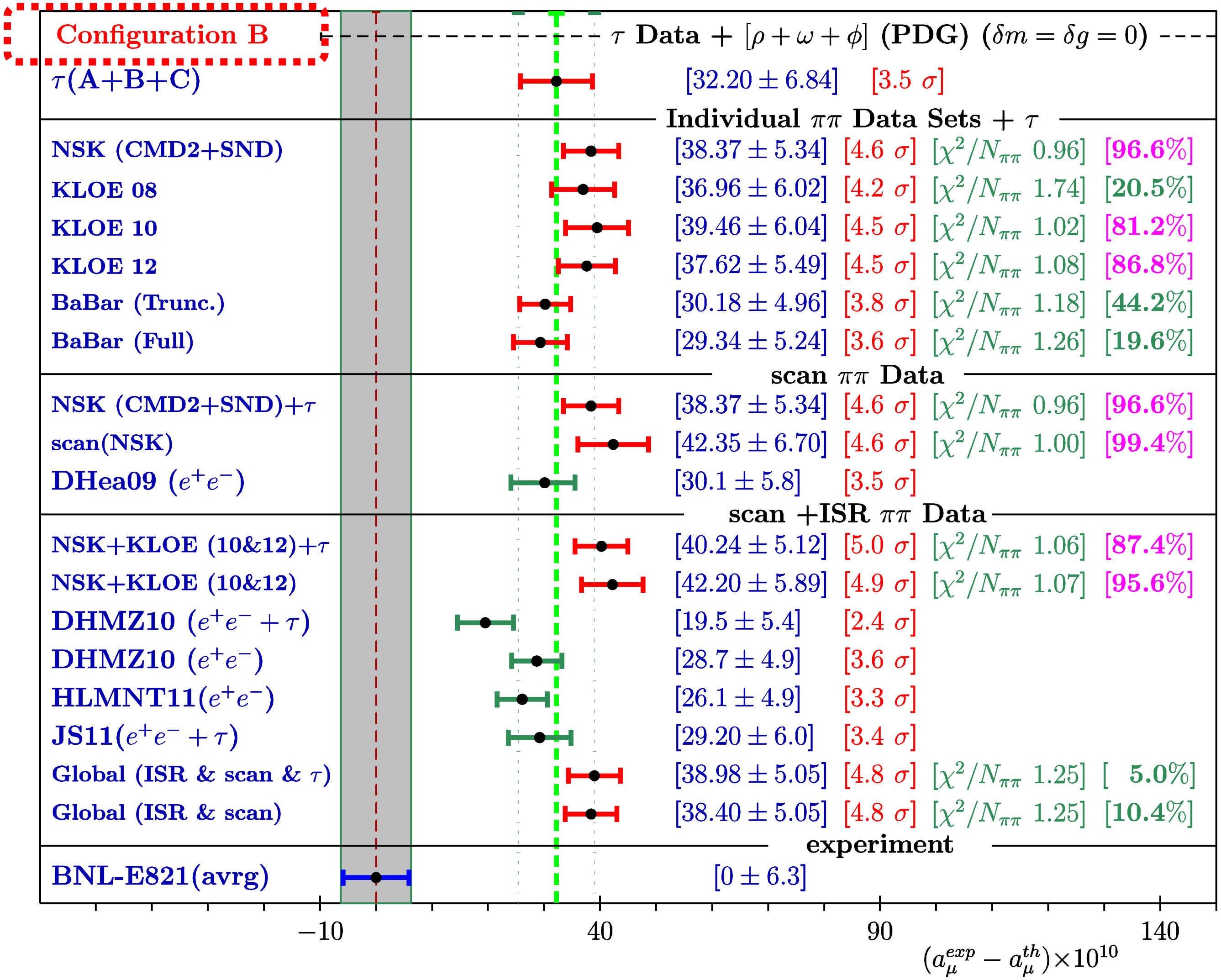
HVP LO



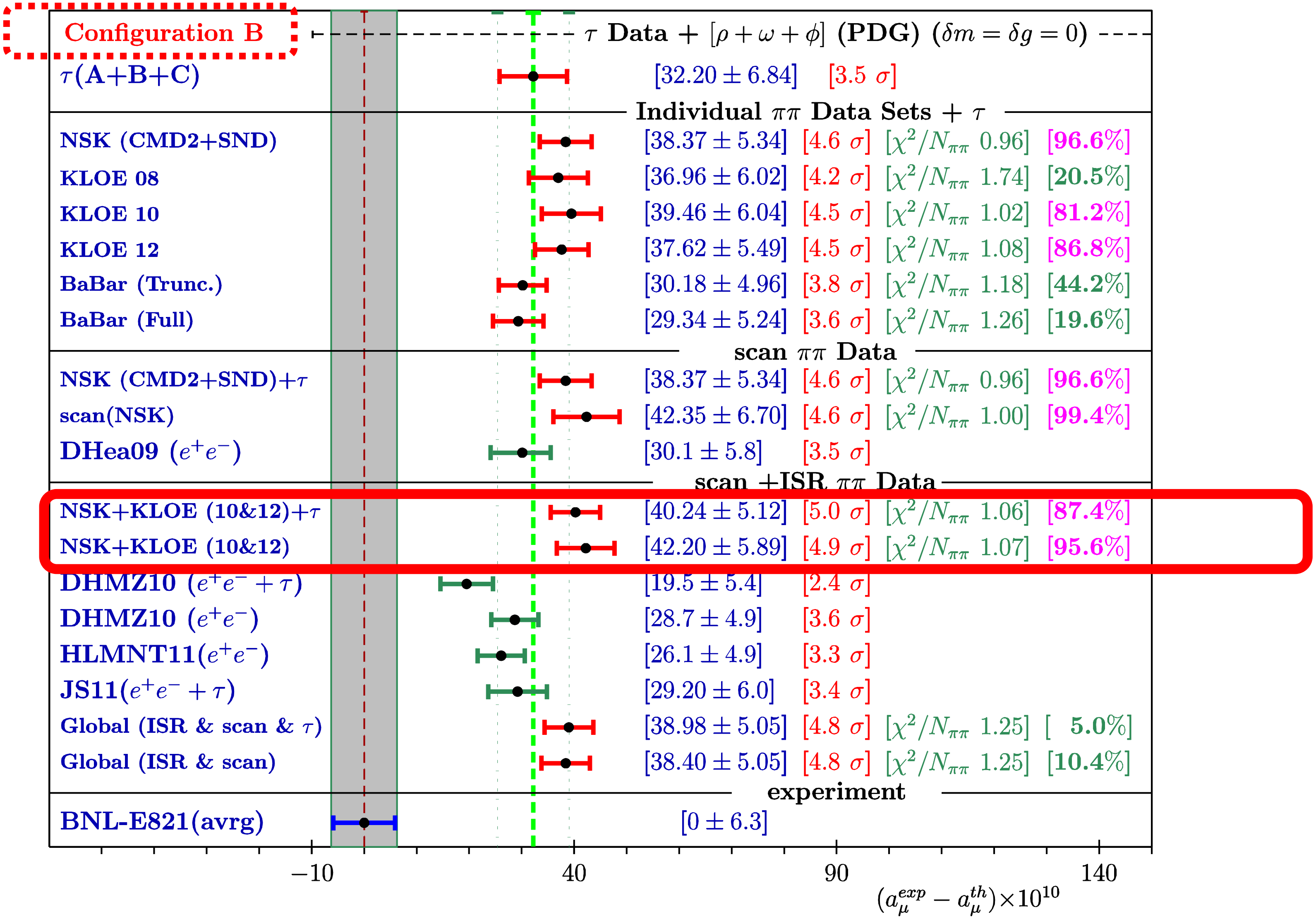
Black : incl. τ
Geen : excl. τ

Red : excl. τ

g-2 Estimates & Discrepancy



g-2 Estimates & Discrepancy



++ Behavior at s=0 (II)

$$F_\pi(s) \underset{s\to 0}{\approx} 1 + a\,s + b\,s^2$$

a = 1.8 GeV^{-2} b = 4.2 GeV^{-4}

add. syst. : shift $a_\mu(\pi\pi)$ => +[1.4, 2.2] 10^{-10}

NSK+KLOE+ τ
(B) prob. 87%

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right] = 40.24 + \begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi + \begin{bmatrix}+2.0\\-0.0\end{bmatrix}_\tau + \begin{bmatrix}+0.0\\-2.2\end{bmatrix}_{s=0} \pm 5.12_{th} \pm 6.30_{\exp}$$

NSK+KLOE +BaBar+ τ
(B) prob .5%

$$10^{10}\times\left[a_\mu^{\exp}-a_\mu^{th}\right] = 38.98 + \begin{bmatrix}+0.6\\-1.3\end{bmatrix}_\phi + \begin{bmatrix}+2.0\\-0.0\end{bmatrix}_\tau + \begin{bmatrix}+0.0\\-2.2\end{bmatrix}_{s=0} \pm 5.05_{th} \pm 6.30_{\exp}$$

significance for Δ a_μ > 4.5 σ (4.0 σ :: JN LbL)

Conclusions

1/ The upgraded HLS model → good simultaneous fit of

$e^+e^- \to \pi\,\pi$ /K^+K^-/ K_LK_S/ $\pi\,\gamma$/$\eta\gamma$/$\pi\,\pi\,\pi$ ($\sqrt{s} \leq 1.05$ GeV) →

→ allows to address consistency issues

2/ $e^+e^- \to \pi\,\pi$ & $\tau \to \pi\pi\,\nu$ spectra :

→ NO obvious signal of (e^+e^- vs τ) puzzle

3/ π+ π- data from CMD2,SND, KLOE 10 & KLOE12

consistent with K^+K^-(CMD2)/ K_LK_S/ $\pi\,\gamma$/$\eta\gamma$/$\pi\,\pi\,\pi$ data

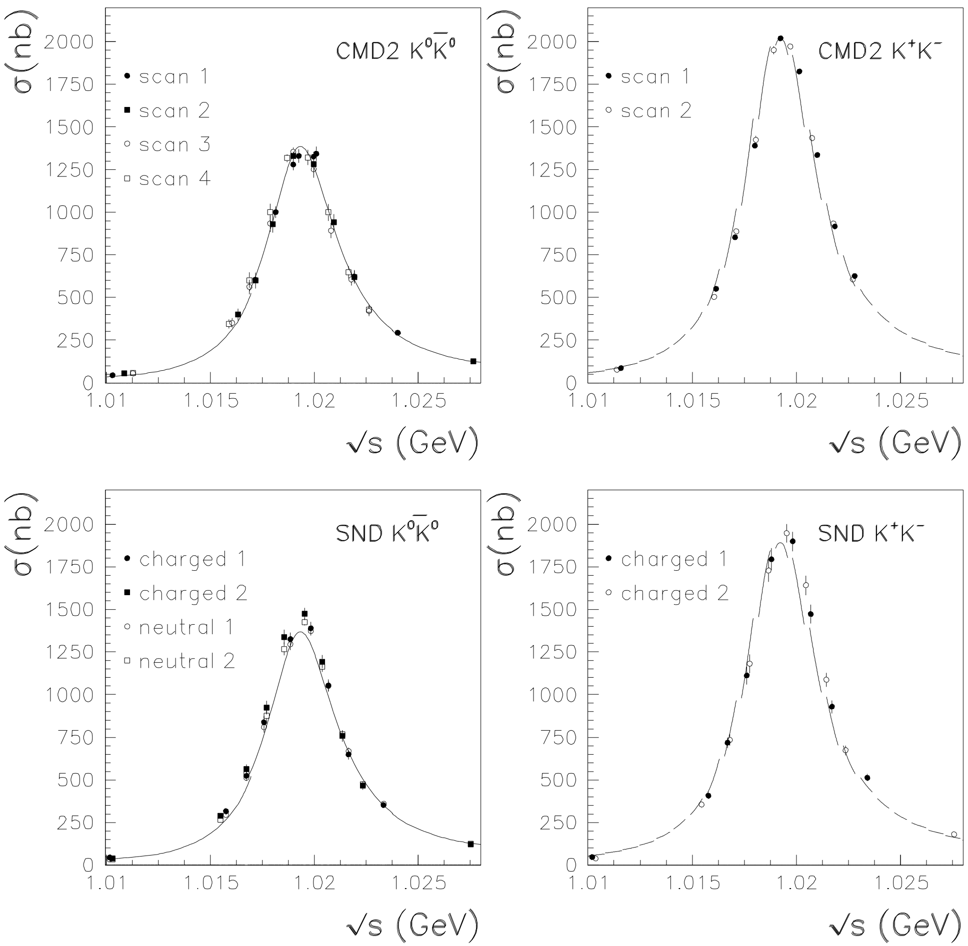
4/ The discrepancy with BNL g-2 value is $\Delta a_\mu > 4.5(/4.0)\,\sigma$

5/ Refine BHLS & More Data & other modellings

6 / HLS Fit → P $\gamma^*\gamma^*$ couplings for $-0.5 \leq s \leq 1.1$ GeV^2

Backup Slides I : Global Fit Plots

$e^+e^- \to$ K Kbar



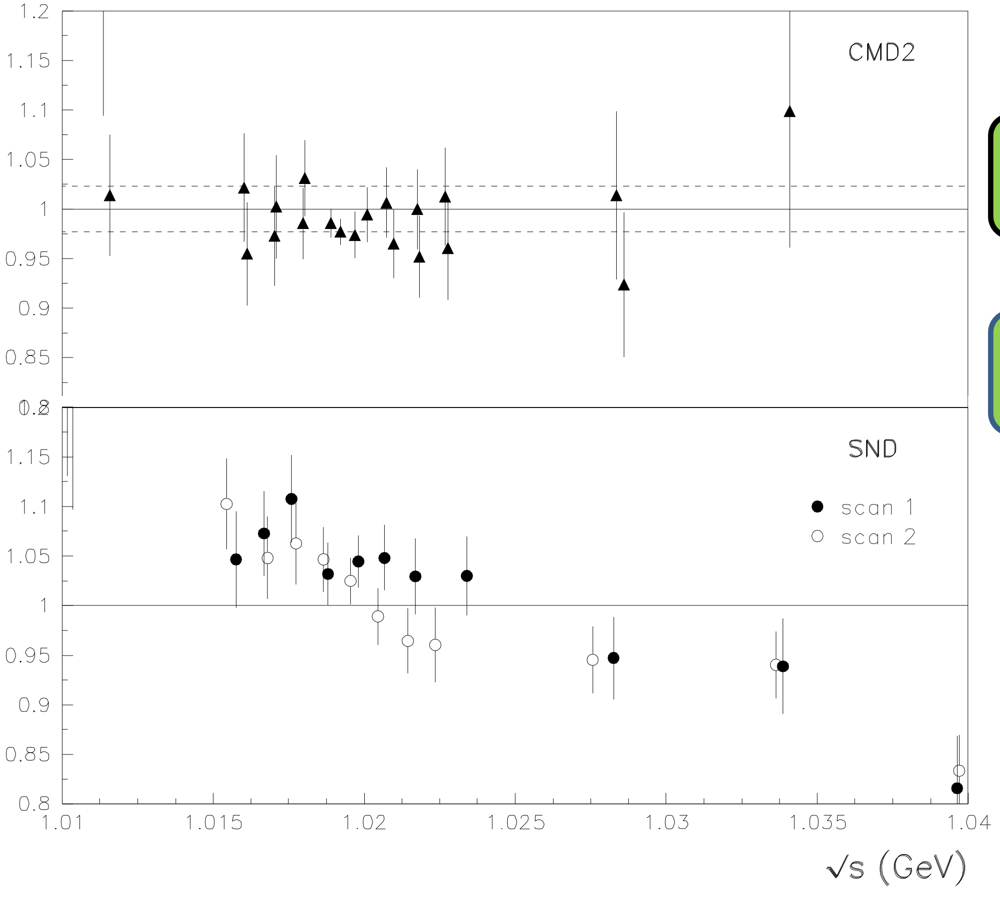
$K^+ K^-$

$$\chi^2 / N_p = 29/36$$

$K_L K_S$

$$\chi^2 / N_p = 120/119$$

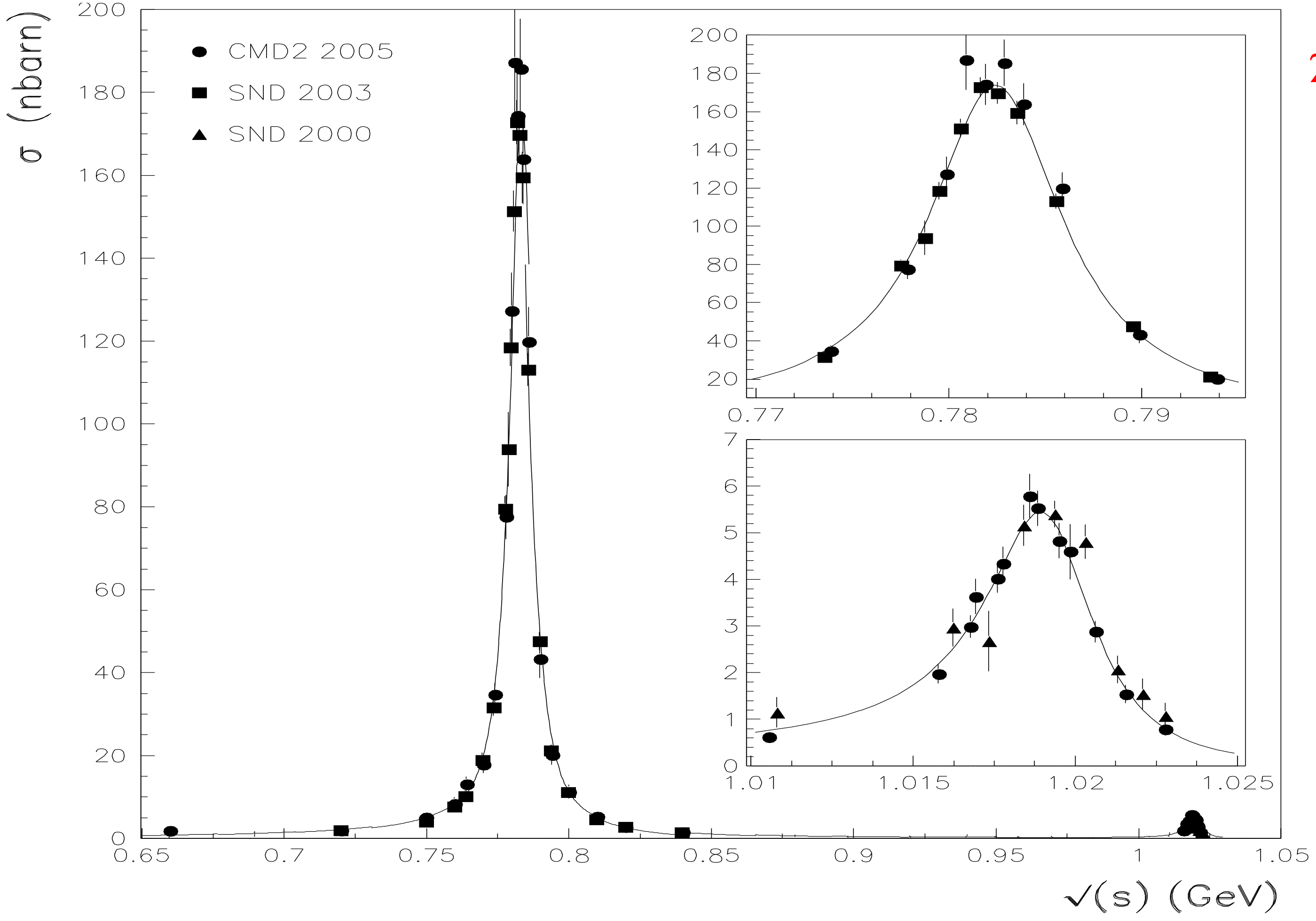
$e^+e^- \rightarrow$ K Kbar Ratios



$K^+ K^-$ $\quad \chi^2 / N_p = 29/36$

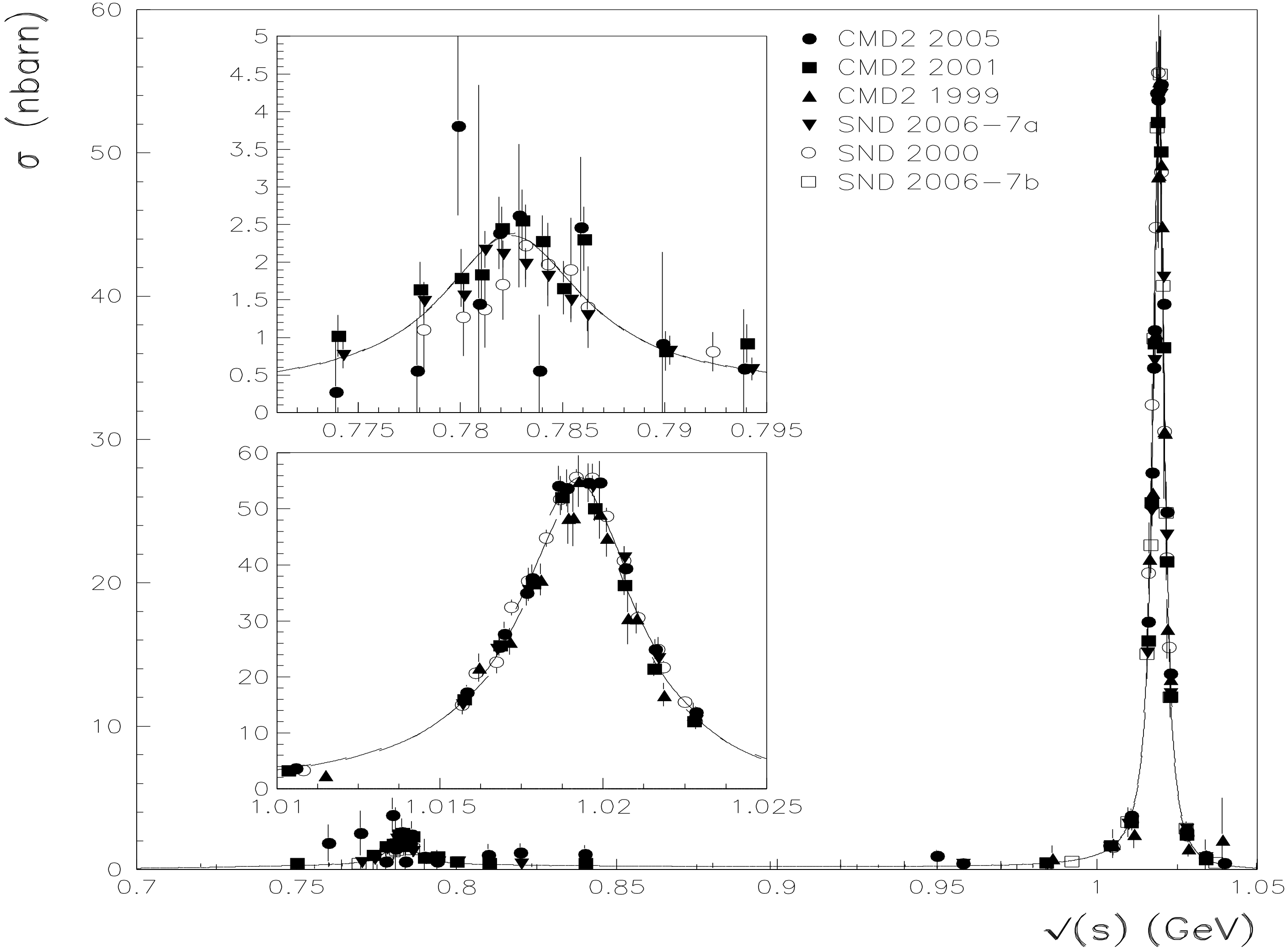
$K_L K_S$ $\quad \chi^2 / N_p = 120/119$

$e^+e^- \rightarrow \pi^0 \gamma$



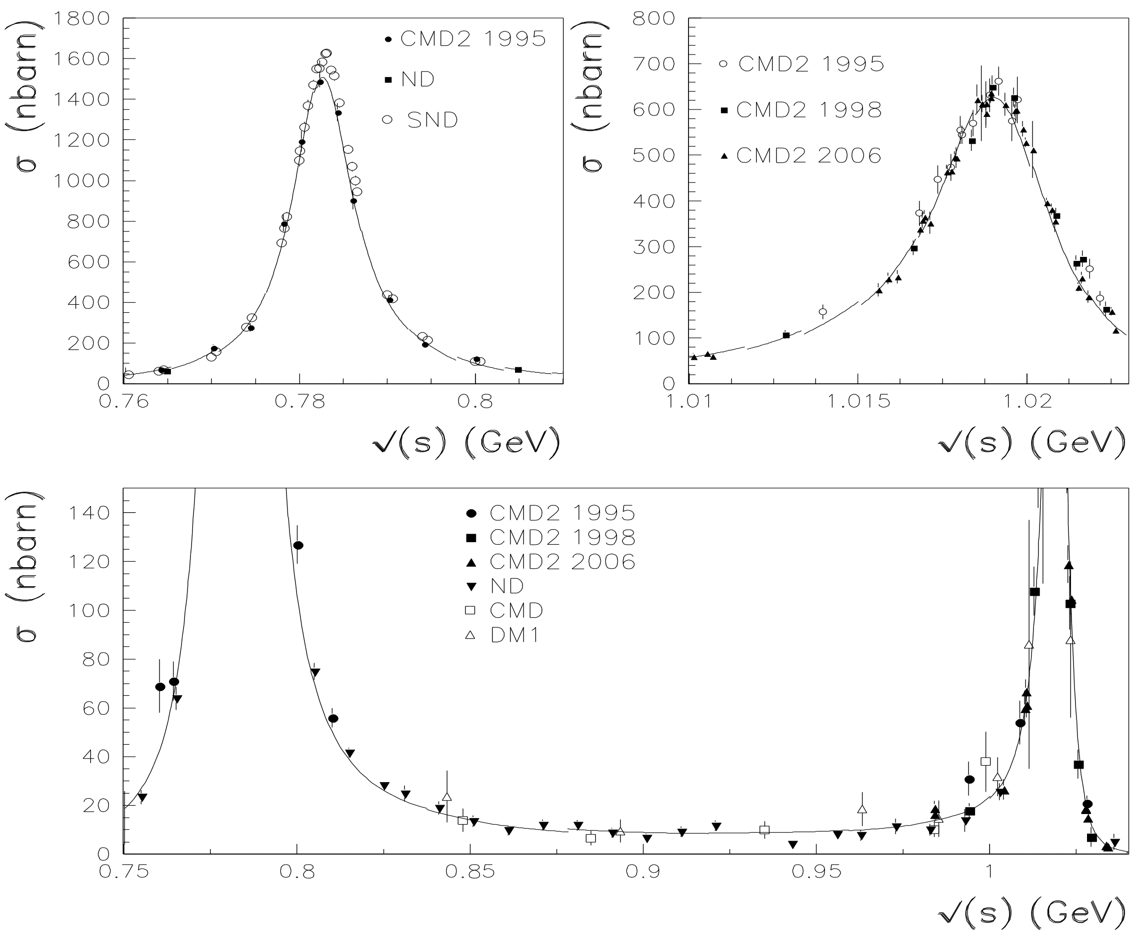
$$\chi^2 / N_p = 68/86$$

$e^+e^- \to \eta\gamma$



$$\chi^2/N_p = 123/182$$

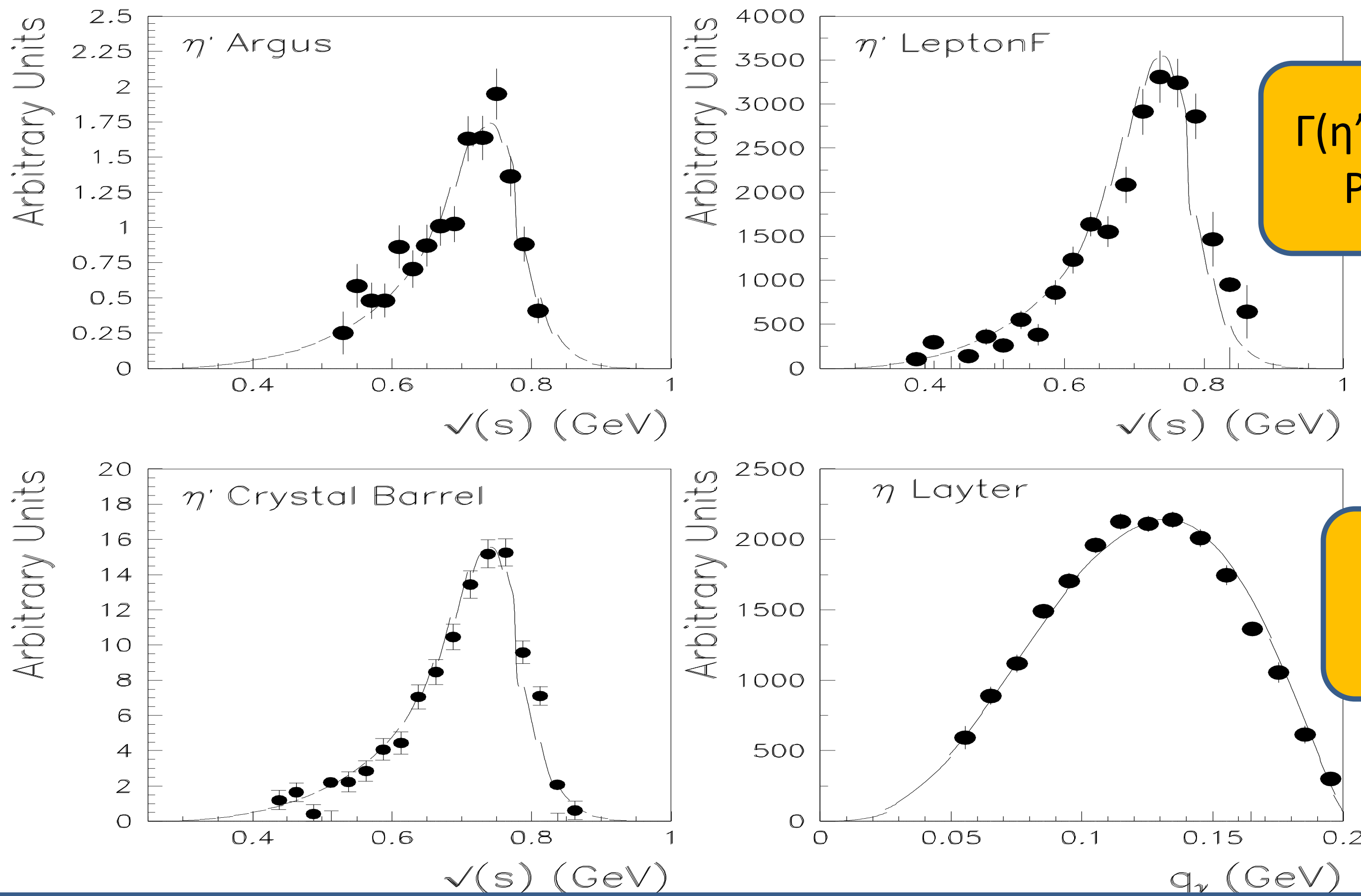
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$



$$\chi^2 / N_p = 210/179$$

Former : Prediction for η/η'→ ππ γ

M. Benayoun *et al.* ArXiv 0907.4047

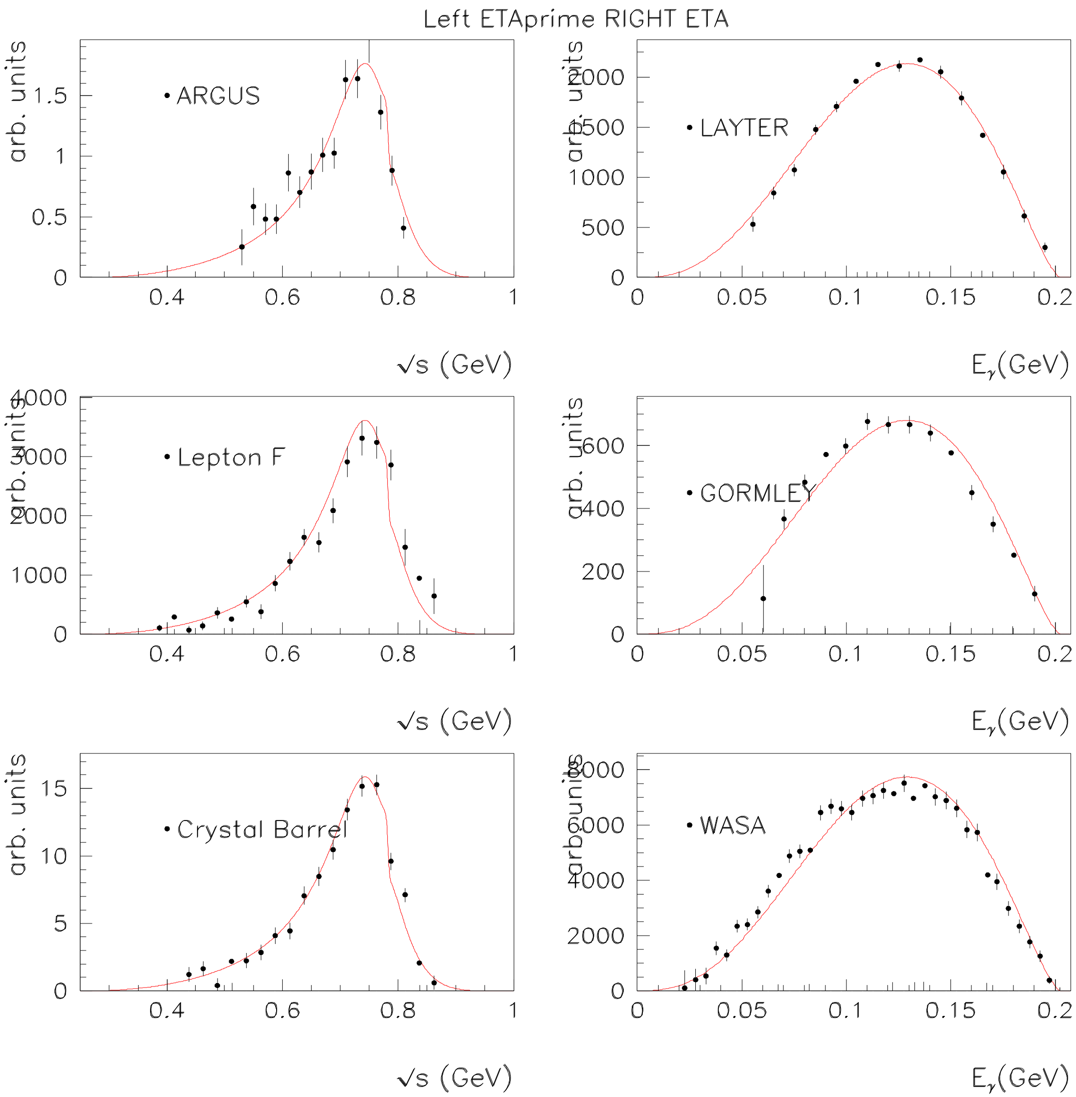


Γ(η'→ ππ γ)=53.11±1.47
PDG : 58.3±2.8 keV

Γ(η→ ππ γ)=55.82±0.83
PDG : 60±4 eV

Lineshapes and yields : **tests** for **g** value and ρ^0 **lineshape**

Present : Predictions for $\eta/\eta' \rightarrow \pi\pi\gamma$

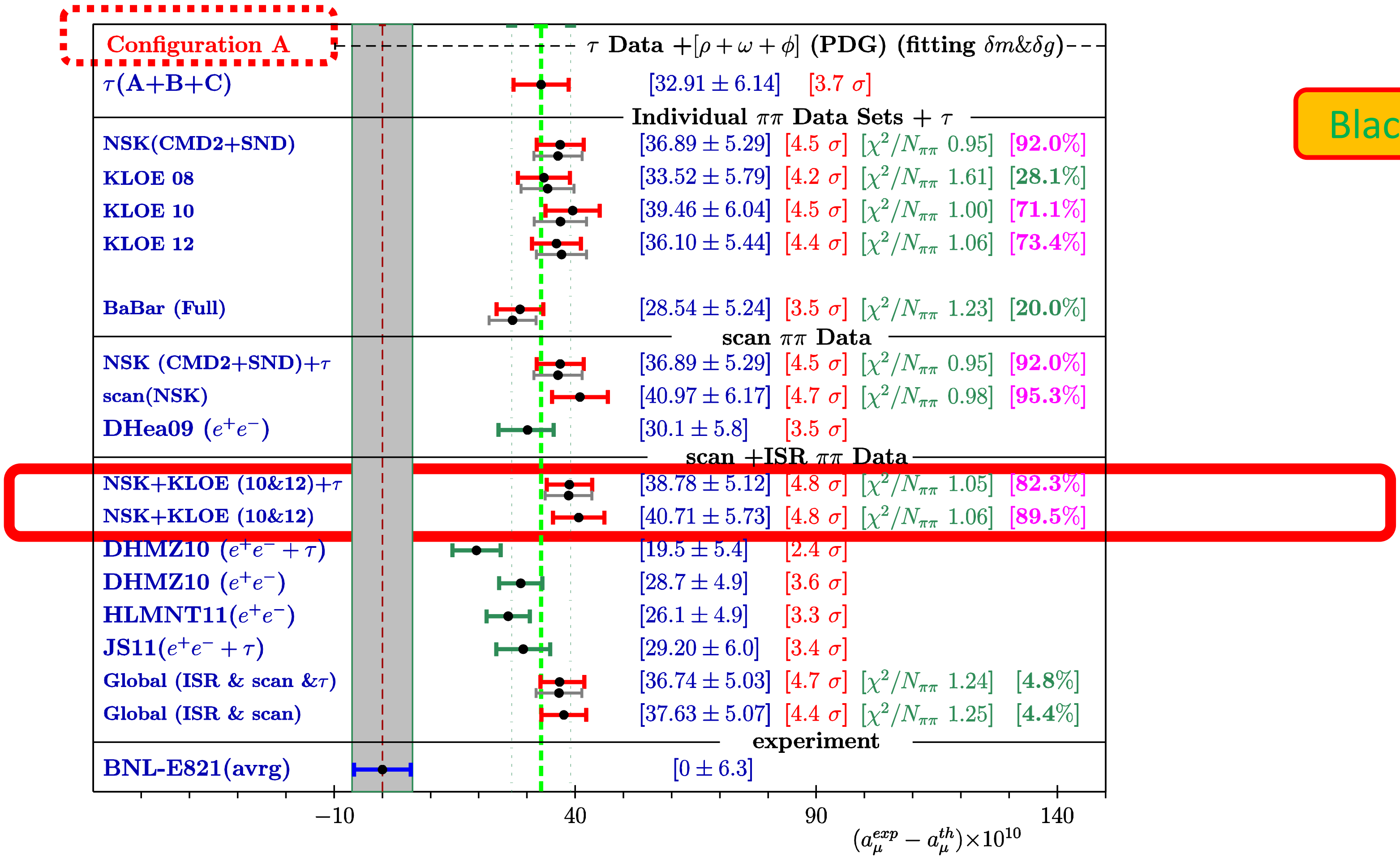


HVP Results incl. τ (A)

- Up to 1.05 GeV, uncertainties improved by ≈2

Channel	NSK +τ	NSK+KLOE+τ	Direct Estimate
$\pi^+\pi^-$	496.89 ± 1.96 (493.01 ± 3.46)	494.95 ±1.44	498.53 ± 3.73 (497.72 ± 2.12)
$\pi^0\gamma$	4.52 ± 0.04	4.52 ± 0.04	3.35 ± 0.11
$\eta\gamma$	0.63 ± 0.01	0.63 ± 0.01	0.48 ± 0.01
$\eta'\gamma$	0.00 ± 0.00	0.00 ± 0.00	---
$\pi^+\pi^-\pi^0$	40.77 ± 0.53	40.80 ± 0.54	43.24 ± 1.47
$K_L K_S$	11.61 ± 0.08	11.62 ± 0.08	12.31 ± 0.33
K^+K^-	16.89 ± 0.18	16.87 ± 0.18	17.88 ± 0.54
Total up to 1.05 GeV	571.29 ± 2.04 (567.21 ± 3.52)	569.40 ± 1.53	575.79 ± 4.06 (574.98 ± 2.66)

g-2 Estimates & Discrepancy



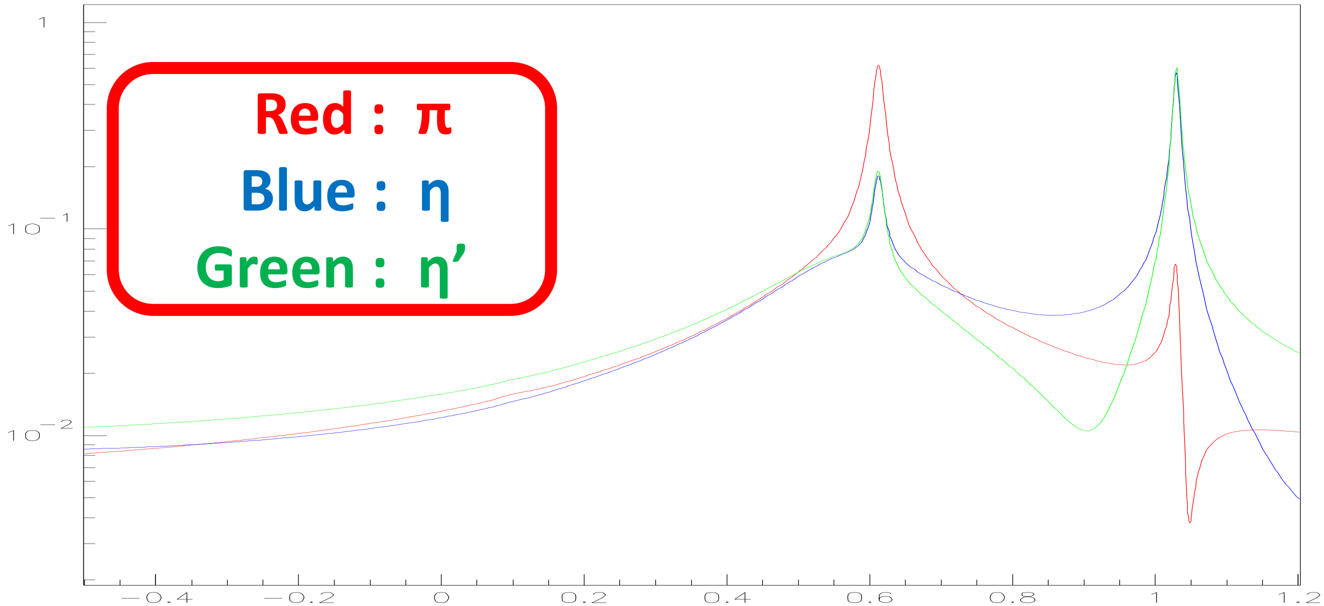
Black : δm=δg=0

Backup Slides II : P $\gamma^*\gamma^*$ couplings

HLS Global Fits : P $\gamma^*\gamma^*$ couplings

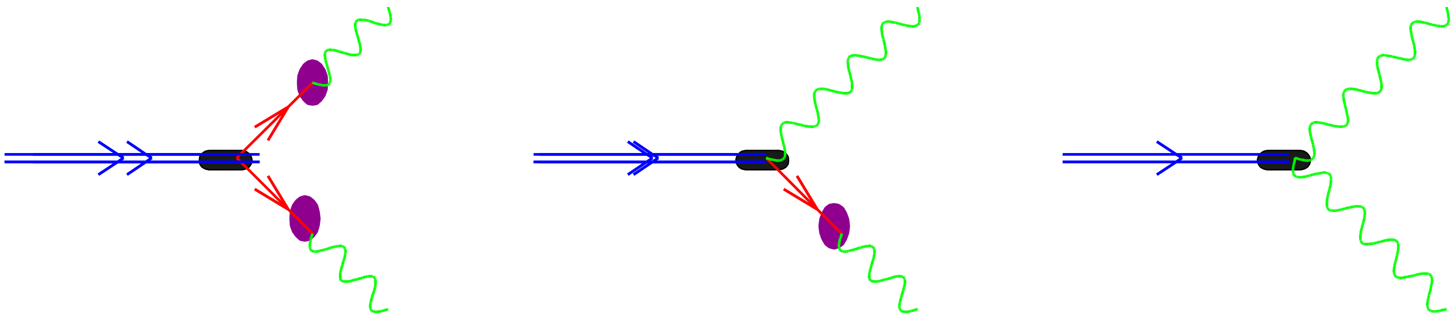
- The HLS global fit to
- $\tau \to \pi\pi \nu_\tau$, $e^+e^- \to \pi^+\pi^-$/ K^+K^-/ $K_L K_S$/$\pi \gamma$/$\eta\gamma$/$\pi \pi \pi$

→ vertices for P γ γ* for γ* on [-0.5,1.1] GeV²



HLS Global Fits : P γ*γ* couplings

- **Amplitudes/couplings can be derived from :**

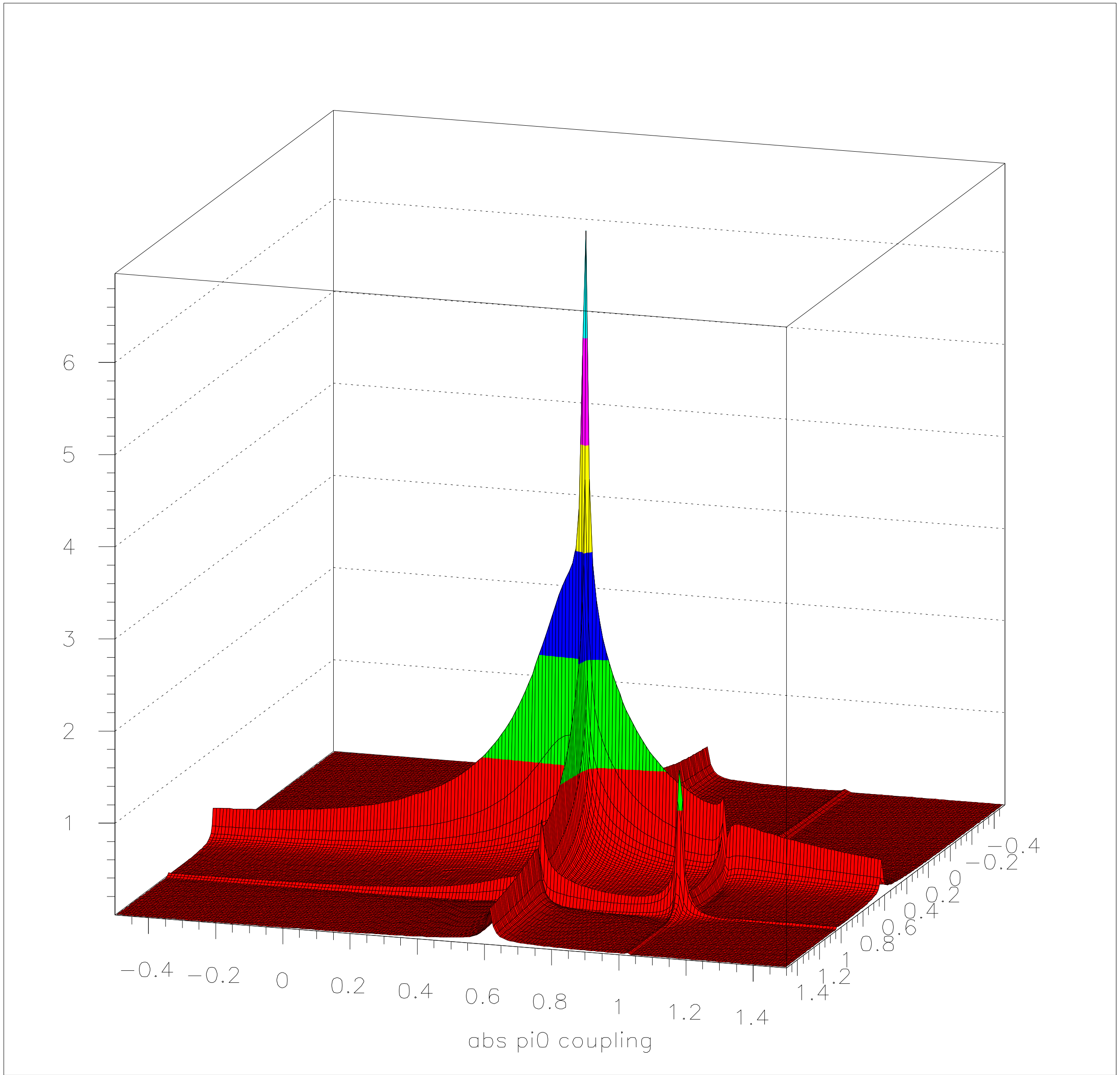


$$L_{AAP}(c_3, s_1, s_2) = c_P(c_3, s_1, s_2)\,\varepsilon^{\mu\nu\alpha\beta}\,\partial_\mu A_\nu(s_1)\,\partial_\alpha A_\beta(s_2)\,P$$

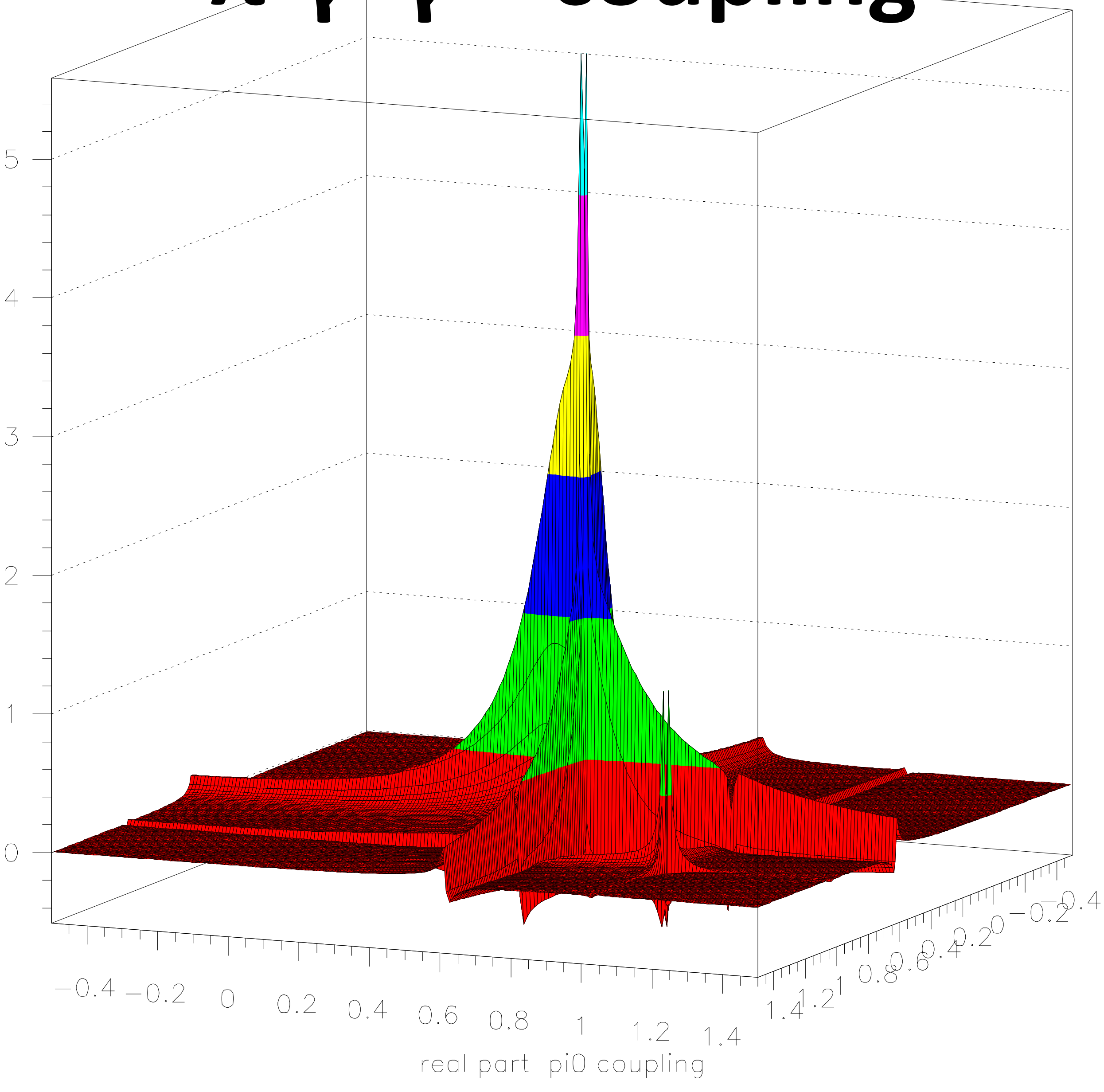
- **And the limit for $s_1=s_2=0$ is just WZW**

$$L_{AAP}(c_3, 0, 0) = L_{WZW}$$

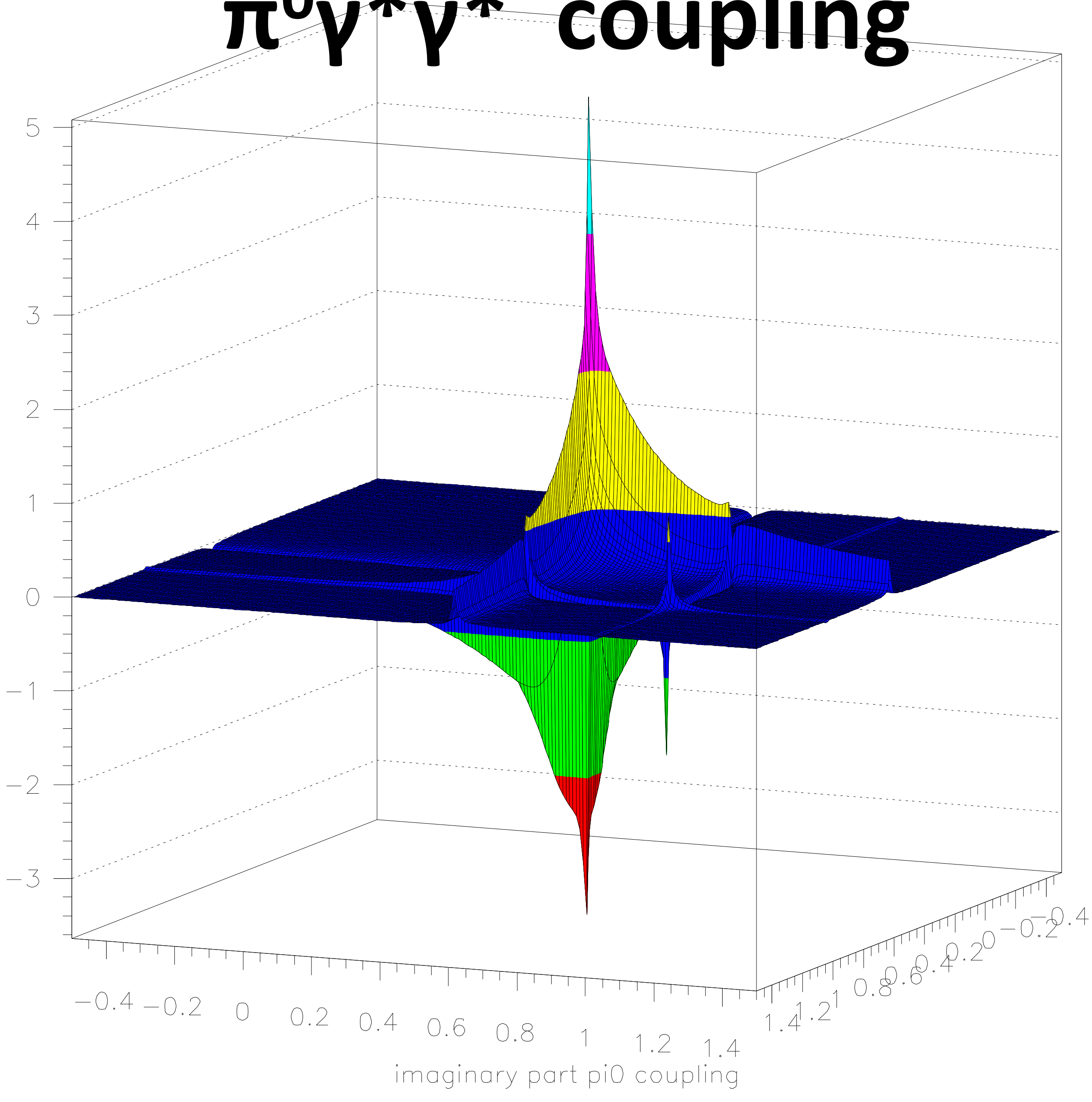
$\pi^0\gamma^*\gamma^*$ coupling



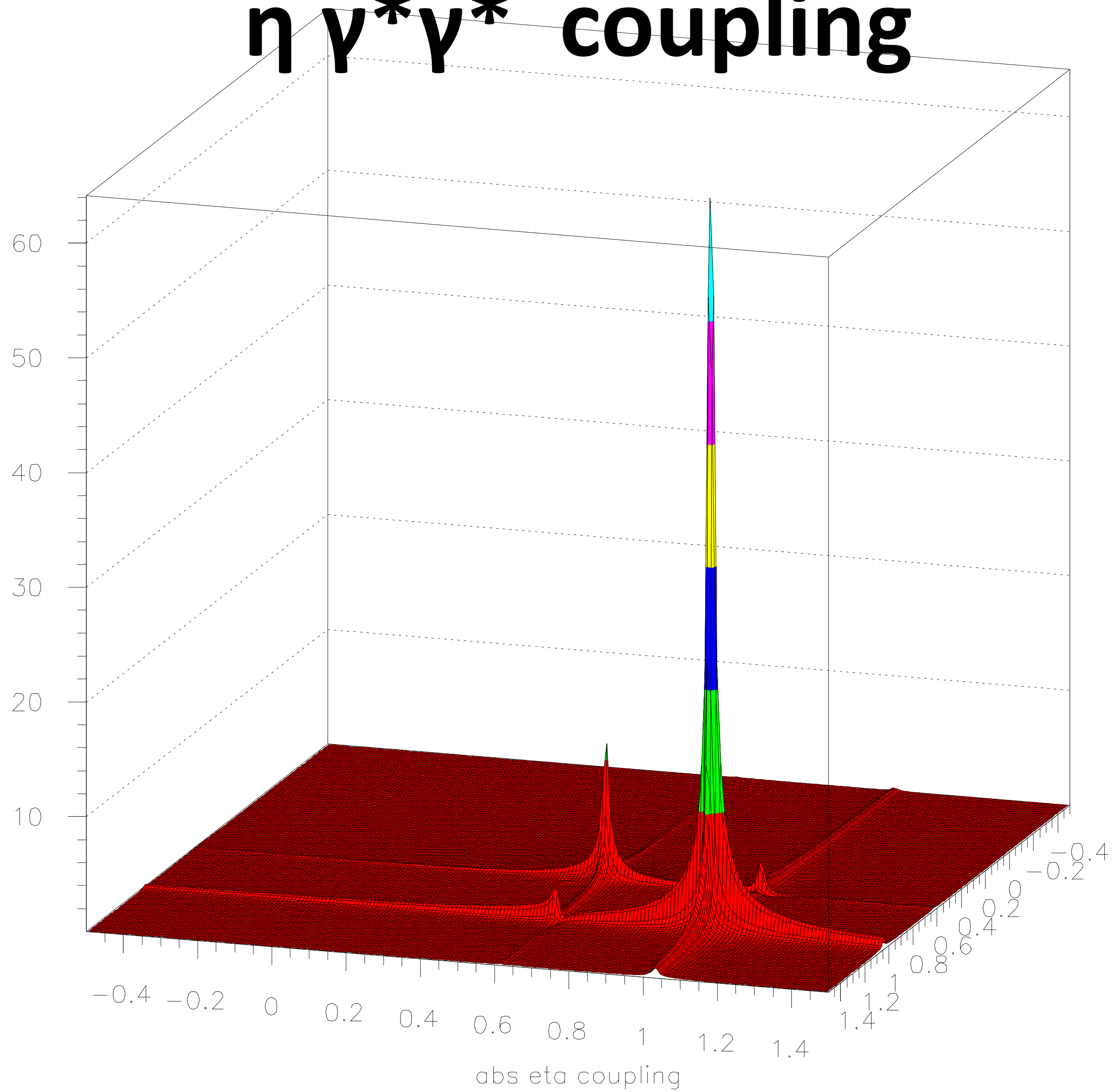
$\pi^0\gamma^*\gamma^*$ coupling



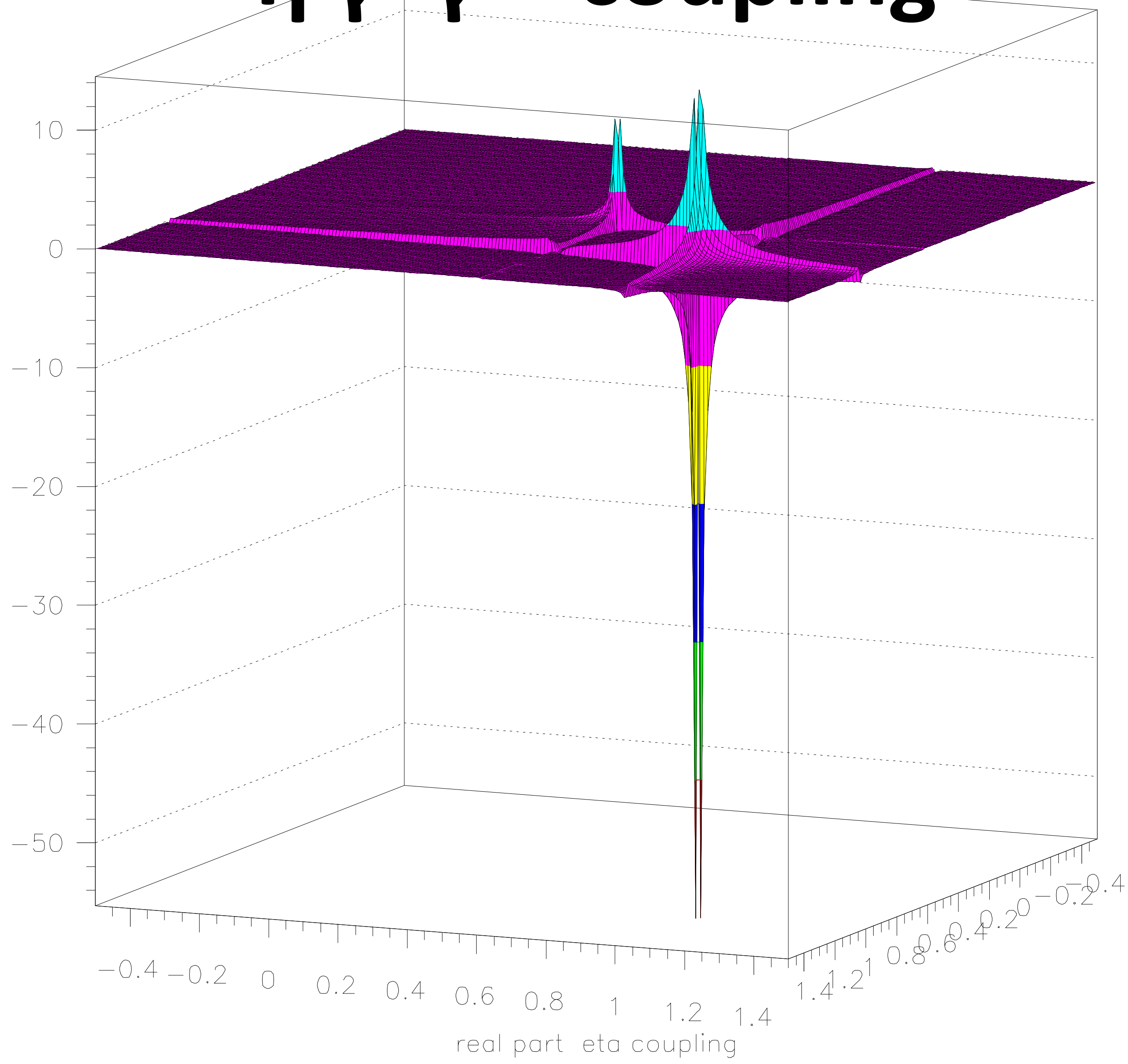
$\pi^0\gamma^*\gamma^*$ coupling



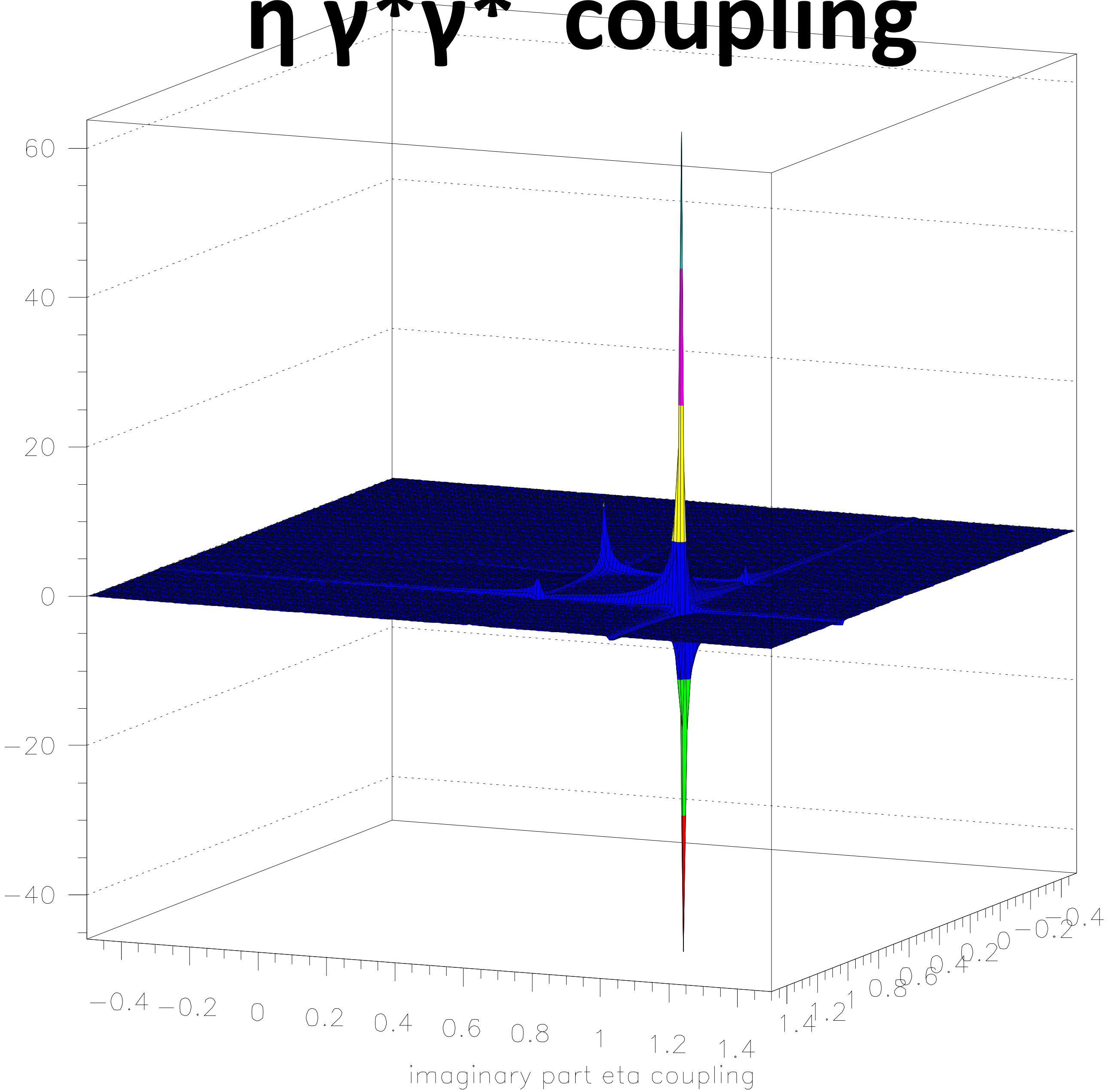
η γ*γ* coupling



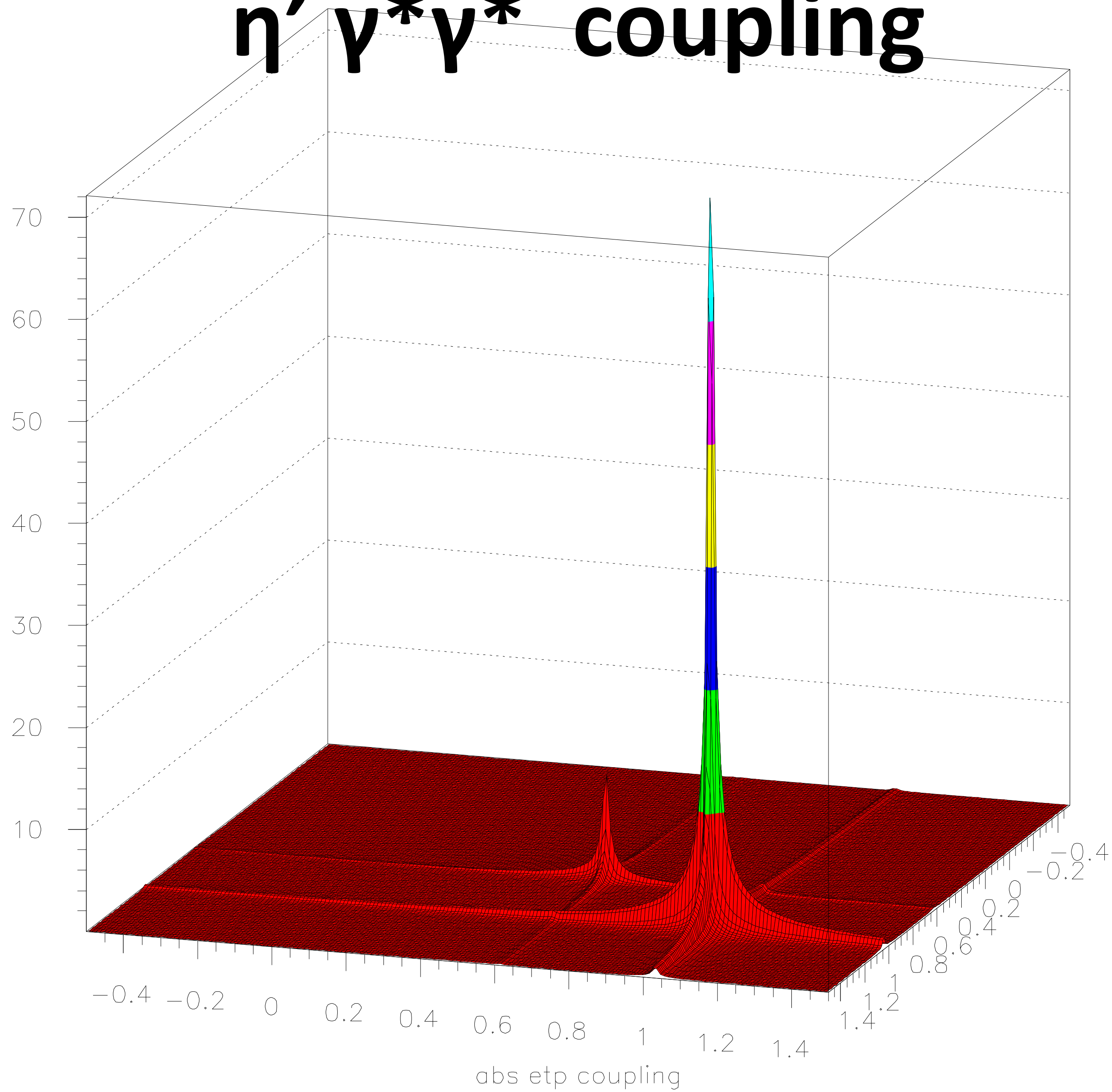
η γ*γ* coupling



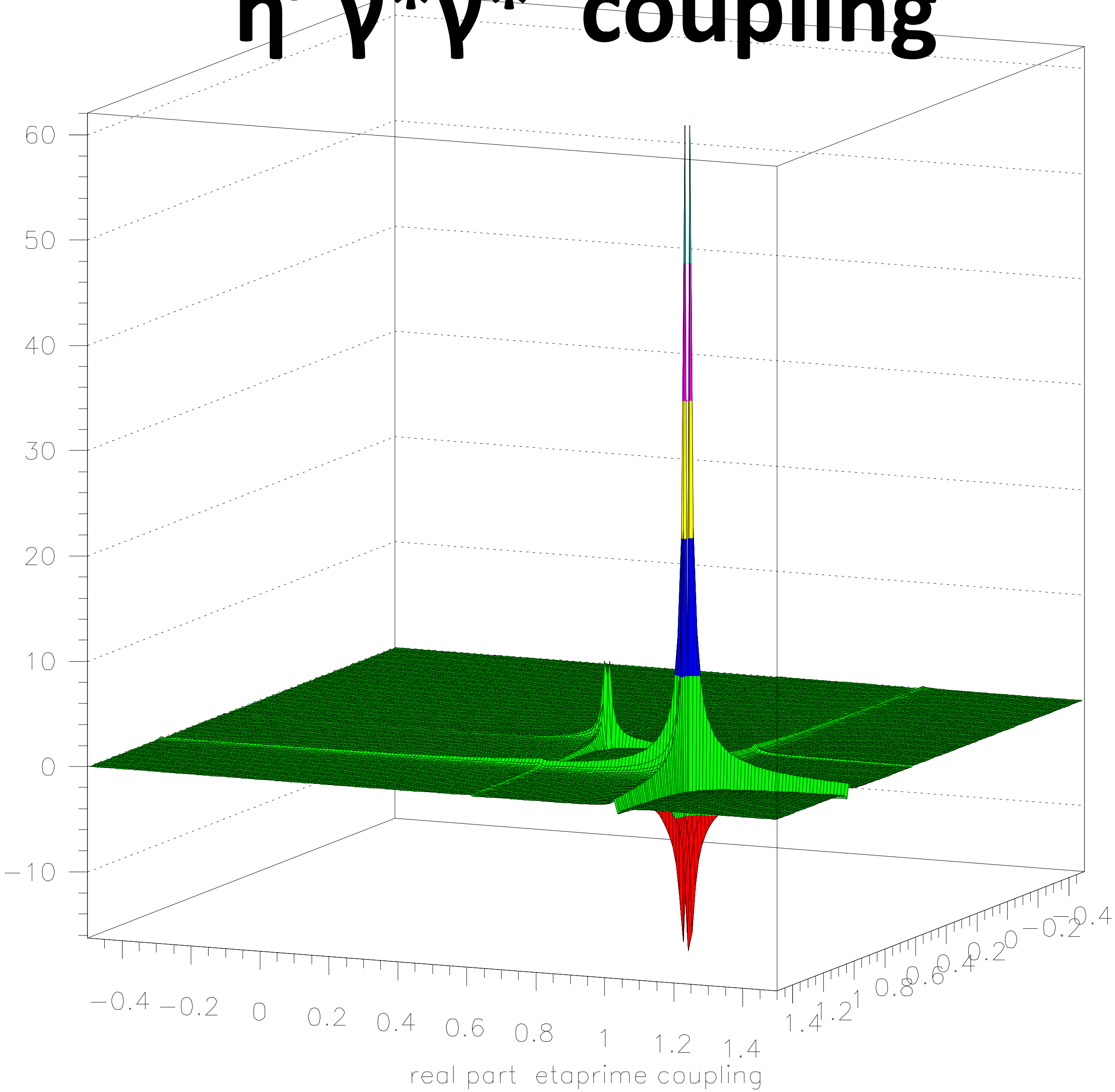
η γ*γ* coupling



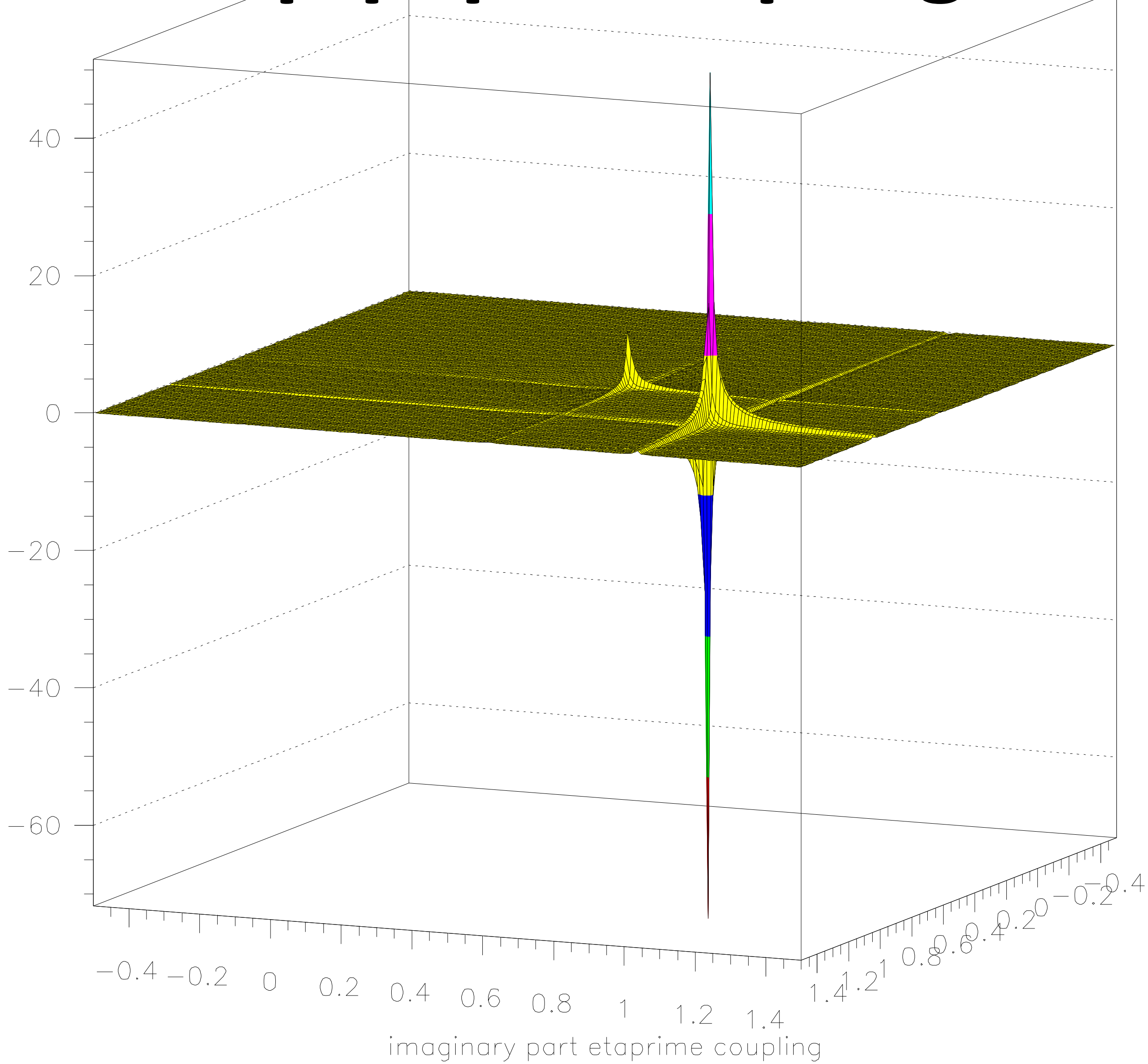
η’ γ*γ* coupling



η’ γ*γ* coupling



η’ γ*γ* coupling



Backup Slides III

Scale Uncertainty

- Minimize :

$$\chi^2 = [\, m - M - \lambda A \,]^T V^{-1} [\, m - M - \lambda A \,] + \lambda^2 / \sigma^2$$

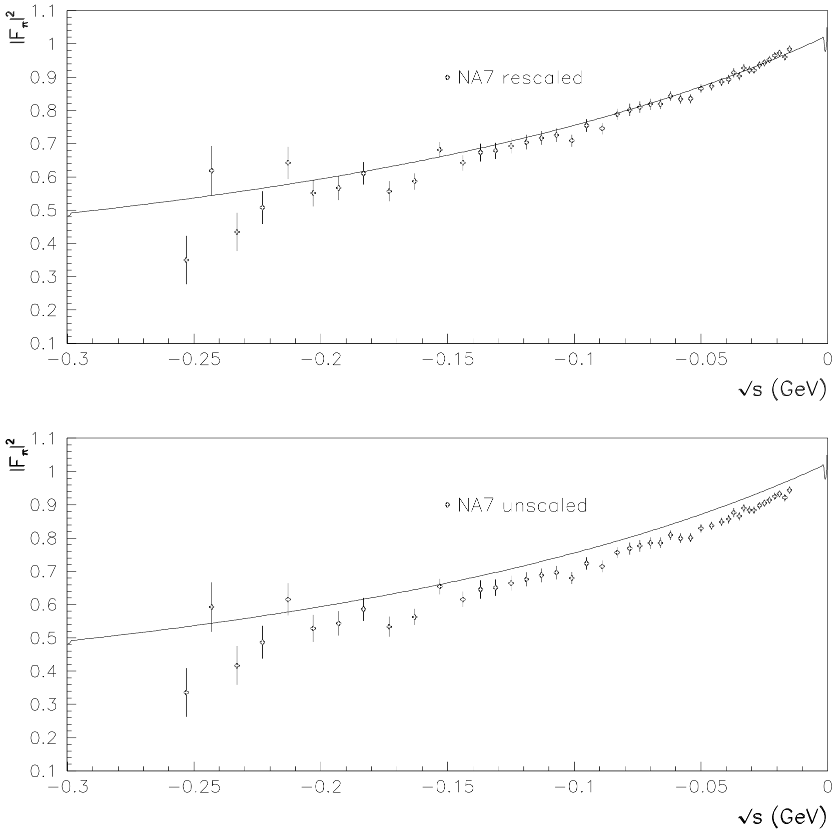
- Solve for λ **(A= m or M)** gives:

$$\chi^2 \equiv [\, m - M \,]^T \left[\, V + \sigma^2 A A^T \,\right]^{-1} [\, m - M \,]$$

- **And :**

$$\lambda = \left\{\, A^T V^{-1} [\, m - M \,] \right\} / \left\{\, A^T V^{-1} A + \frac{1}{\sigma^2} \,\right\}$$

NA7 Residuals ($\chi^2/N \approx 90/60$)!



V π π Couplings: **I=1 part of ω and φ**

$$\frac{iag}{2}\rho_I\,\pi^-\overleftrightarrow{\partial}\pi^+ \Rightarrow$$

$$\frac{iag(1+\Sigma_V)}{2}\Big[\rho^0+\big[(1-h_V)\Delta_V-\alpha(s)\big]\,\omega+\beta(s)\,\varphi\Big]\pi^-\overleftrightarrow{\partial}\pi^+$$

Mixing Angles I=1 terms

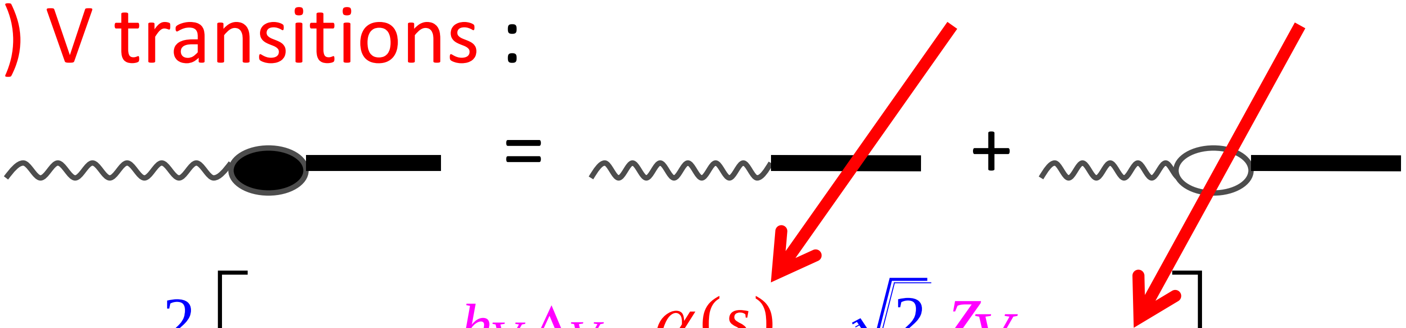
*At leading order : **ρ term unchanged** **(I=1 part)**

*small **s-dependent ω and φ couplings** **(I=1 part)**

* **Charged and neutral rho's: <u>same coupling</u> to pions**

Vector meson couplings to γ

- (γ) V transitions :



$$f_\rho^\gamma = agf_\pi^2\left[1+\Sigma_V+\frac{h_V\Delta_V}{3}+\frac{\alpha(s)}{3}+\frac{\sqrt{2}\,z_V}{3}\beta(s)\right]$$

$$f_\omega^\gamma = agf_\pi^2\left[1+\Sigma_V+3(1-h_V)\Delta_V-\alpha(s)+\frac{\sqrt{2}\,z_V}{3}\gamma(s)\right]$$

$$f_\varphi^\gamma = agf_\pi^2\left[-\frac{\sqrt{2}\,z_V}{3}+\beta(s)+\gamma(s)\right]$$

ρ couplings to γ/W

- (γ/W) V transitions :

$$\frac{f_\rho^{\gamma}}{f_\rho^{W}} = \left[1 + \frac{h_V \Delta_V}{3} + \frac{\alpha(s)}{3} + \frac{\sqrt{2}\, z_V}{3} \beta(s) \right] \Longleftarrow \left[f_\rho^{W} = ag\, f_\pi (1 + \Sigma_V) \right]$$

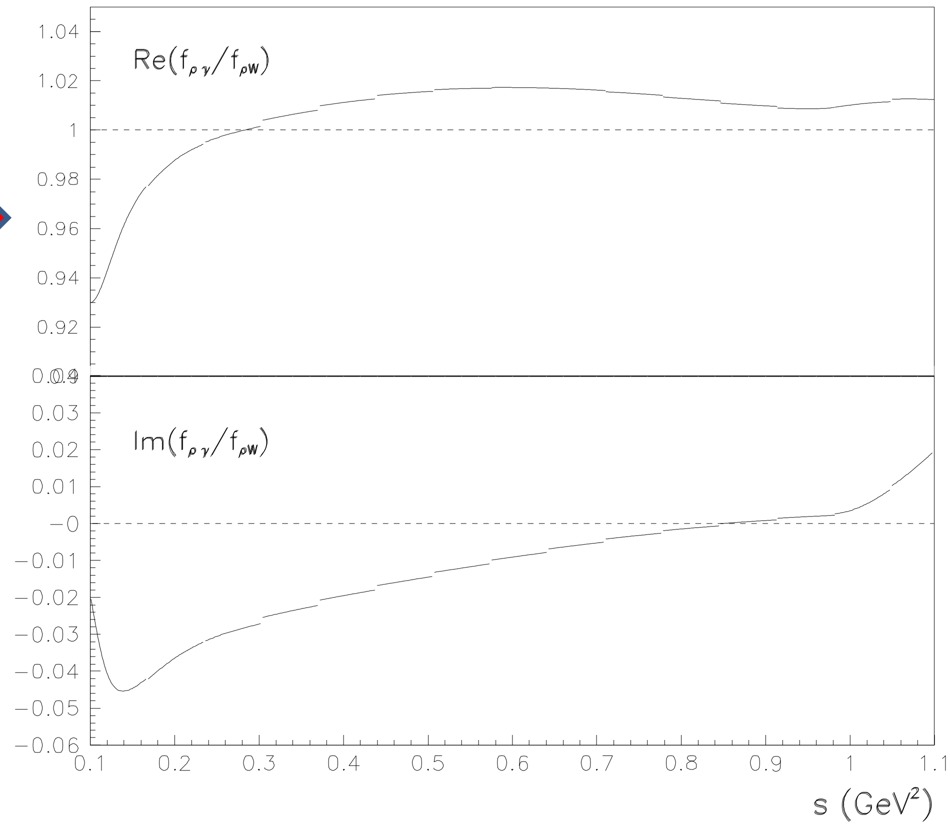
Full agreement between e^+e^- *and* τ **data**

ρ couplings to γ/W

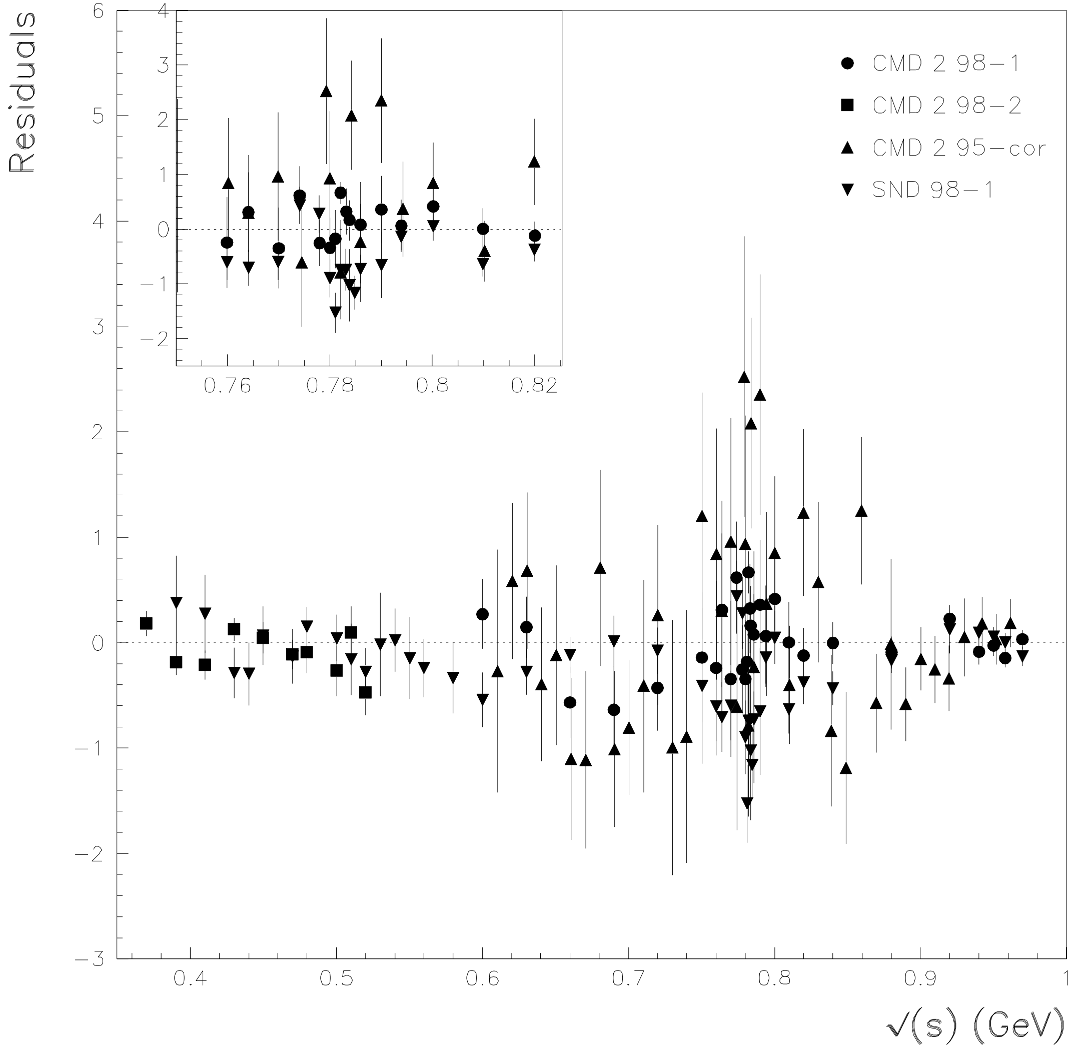
$$F(s) = \frac{f_\rho^\gamma}{f_\rho^W}$$

Re[F(s)]

Im[F(s)]

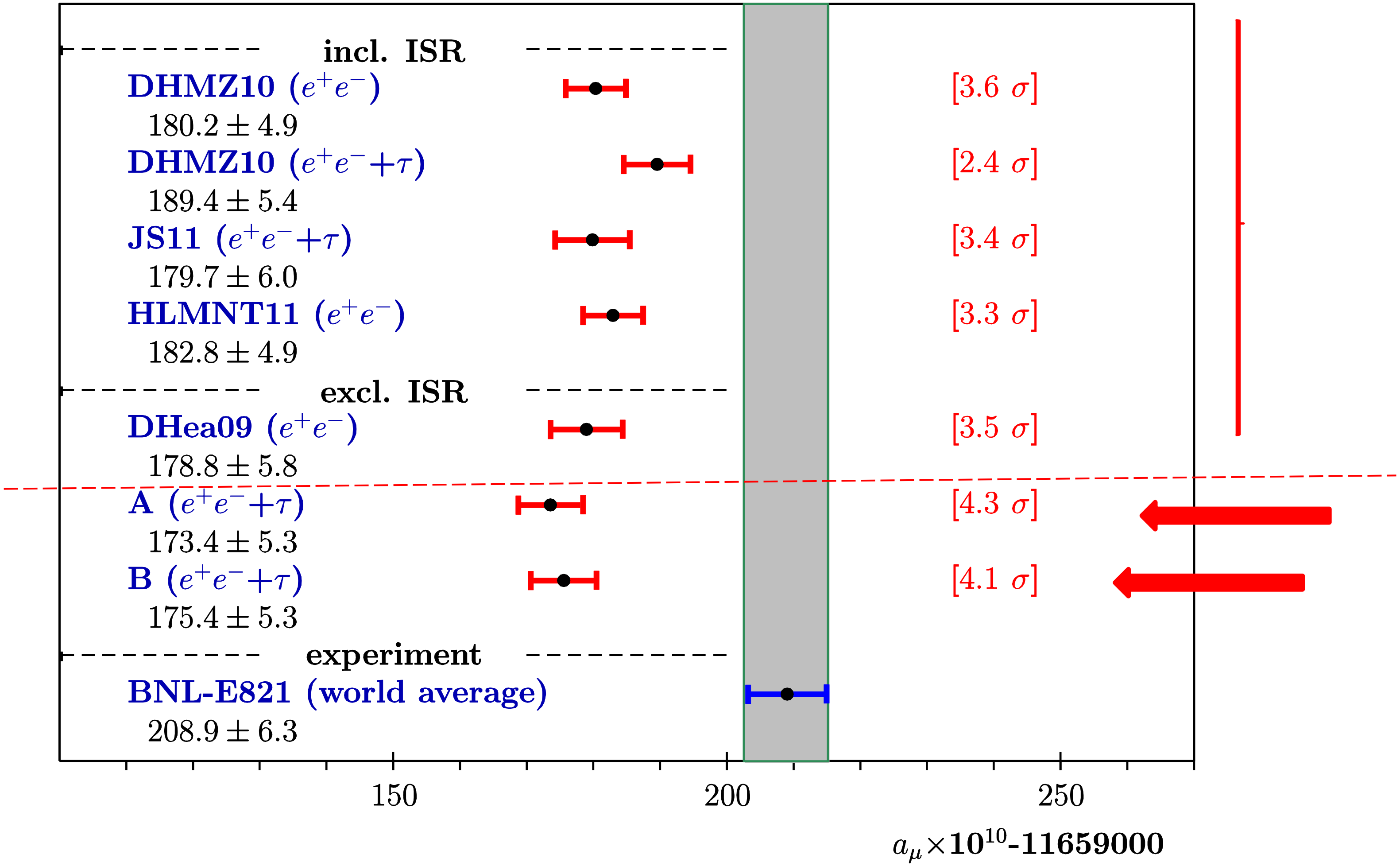


e^+e^- Fit Residuals

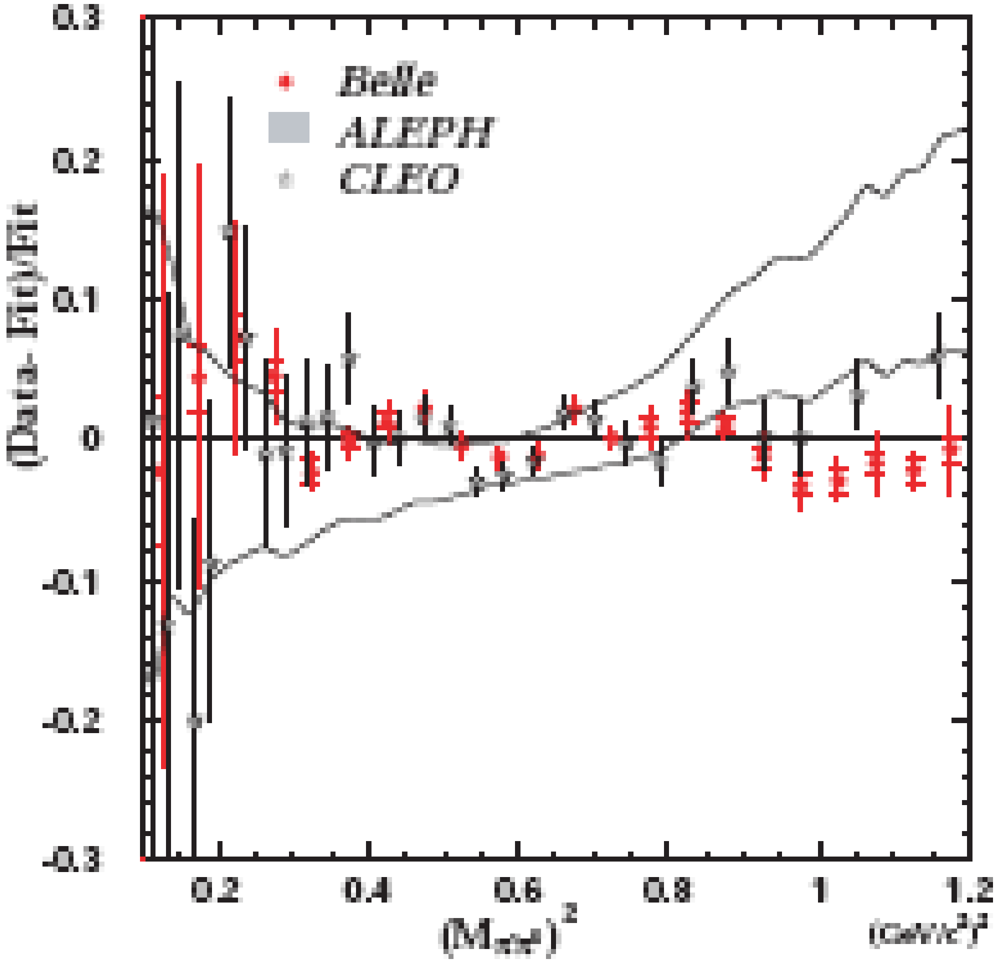


e^+e^- Data

VMD estimate of g-2

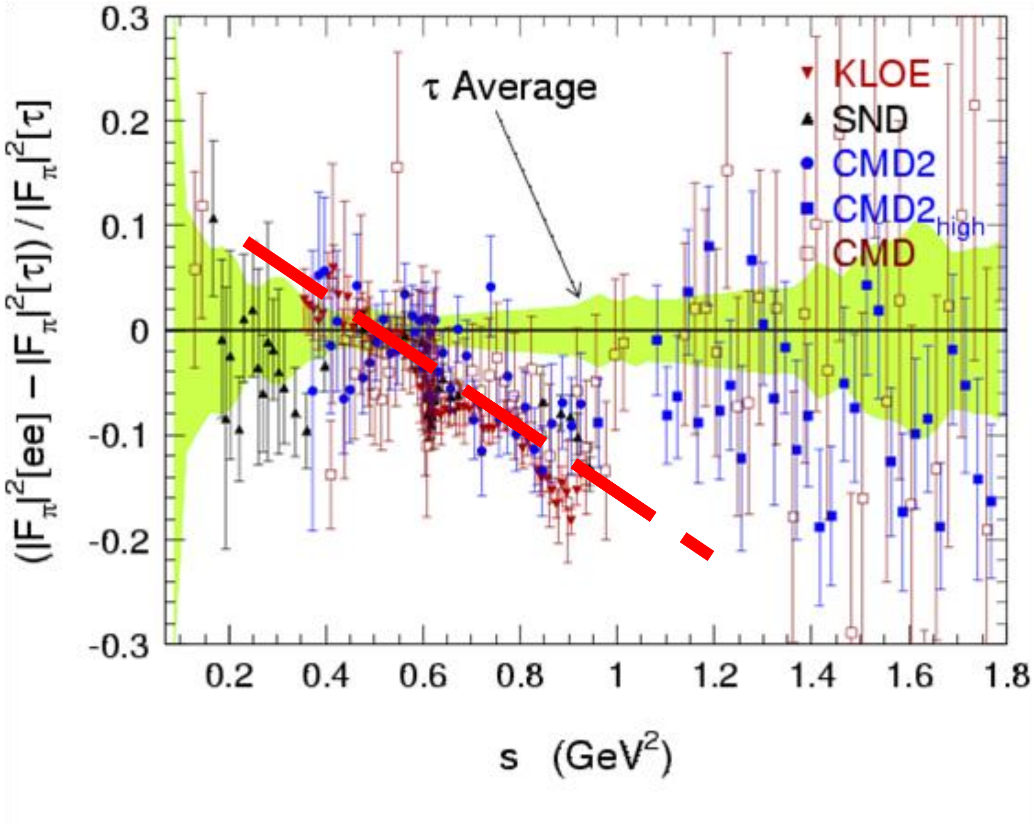


BELLE/ALEPH/CLEO



Dipion Mass Spectrum 2007

- M. Davier NP Proc. Supp. 169 (2007) 288



- **s-dependent (missing) effect !**