Dispersive approach to the hadronic LbL contribution to $(g-2)_\mu$

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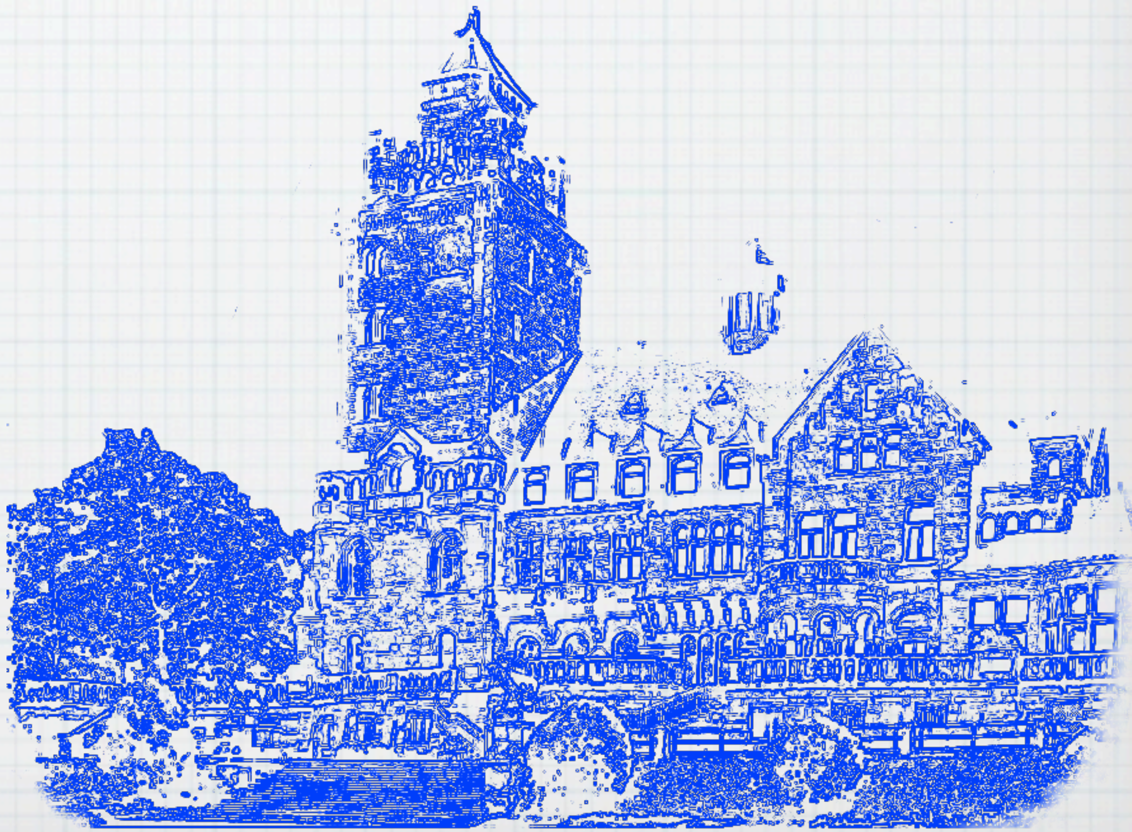
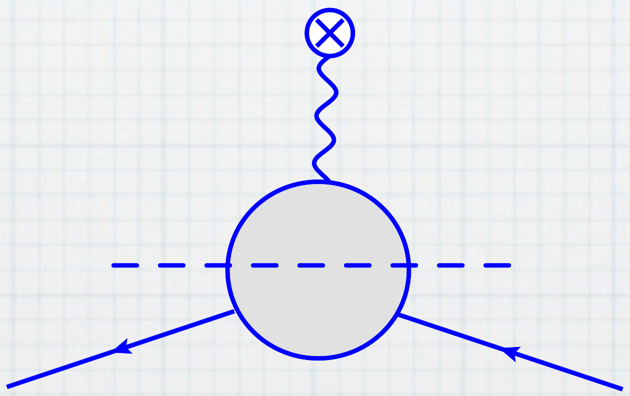
(g-2) MITP Workshop

Schloss Waldthausen, Mainz, Germany

April 1-5, 2014



$F_2(t)$ and dispersion relations



a_μ from a dispersion relation

$$a_\mu = \lim_{k\to 0} F_2(k^2, (p+k)^2, p^2)$$

Pauli form factor
in the limit of static electromagnetic field

a_μ from a dispersion relation

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on-shell conditions
for a muon

$$(p+k)^2 = m^2$$
$$p^2 = m^2$$

$$\longrightarrow \quad (p\cdot k) = -k^2/2$$

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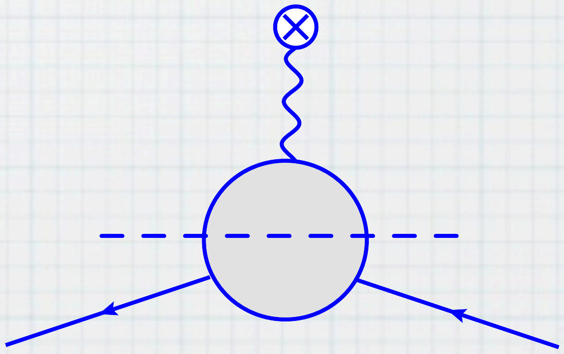
$$a_\mu = F_2(0)$$

$$F_2(0) = \frac{1}{2\pi i}\int \frac{\mathrm{d}k^2}{k^2}\,\mathrm{Abs}\,F_2(k^2)$$

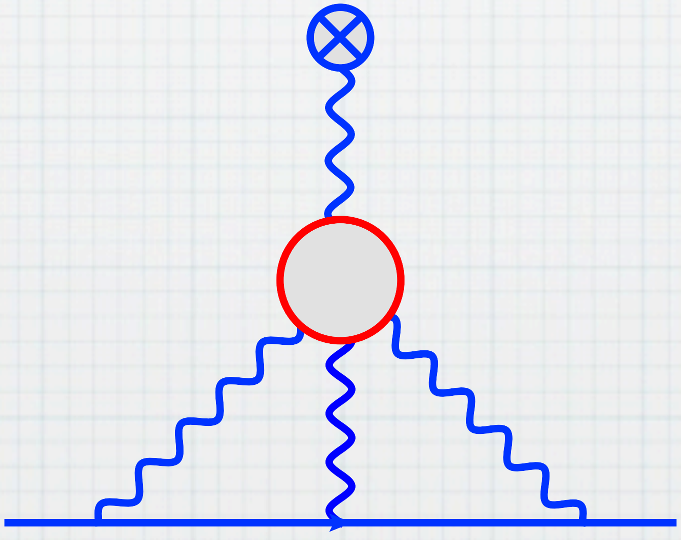


HLbL contribution to (g-2)$_\mu$

projection technique

$$F_2(k^2) = \mathrm{Tr}\left[(\not{p}+m)\Lambda_\nu(p',p)(\not{p}'+m)\Gamma^\nu(p',p)\right]$$

$$(-ie)\Gamma_{1\rho}(p',p) = \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{(-i)^3}{q_1^2 q_2^2 (k-q_1-q_2)^2} \frac{i^2}{[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$
$$\times\, (-ie)^3 \gamma^\lambda(\not{p}+\not{k}-\not{q}_2+m)\gamma^\nu(\not{p}+\not{q}_1+m)\gamma^\mu (ie)^4 \Pi_{\mu\nu\lambda\rho}(q_1, k-q_1-q_2, q_2, k)$$



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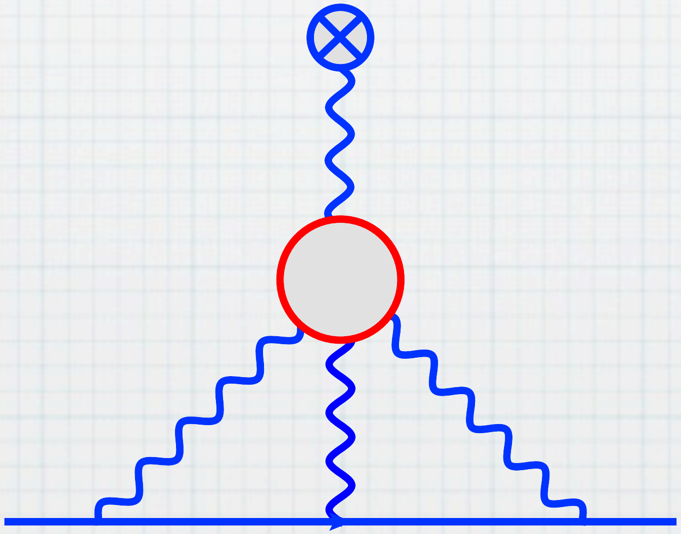
$$F_2(k^2) = e^6\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\int\frac{\mathrm{d}^4q_2}{(2\pi)^4}\frac{\Pi_{\mu\nu\lambda\rho}(q_1,k-q_1-q_2,q_2,k)L^{\mu\nu\lambda\rho}(p,p',q_1,k-q_1-q_2,q_2)}{q_1^2q_2^2(k-q_1-q_2)^2\,[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$



HLbL contribution to (g-2)$_\mu$

projection technique

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completeness relation ⟶ helicity amplitudes

$$\sum_\lambda (-1)^\lambda \varepsilon^{\mu *}(q,\lambda)\varepsilon^\nu(q,\lambda) = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

$$\Pi_{\mu\nu\lambda\rho} L^{\mu\nu\lambda\rho} = \sum_{\lambda,\lambda_1,\lambda_2,\lambda_3} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} L_{\lambda_1\lambda_2\lambda_3\lambda} \Pi_{\lambda_1\lambda_2\lambda_3\lambda}$$

HLbL contribution to $(g-2)_\mu$

projection technique

$$F_2(k^2) = \mathrm{Tr}\left[(\not{p}+m)\Lambda_\nu(p',p)(\not{p}'+m)\Gamma^\nu(p',p)\right]$$

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completeness relation → helicity amplitudes

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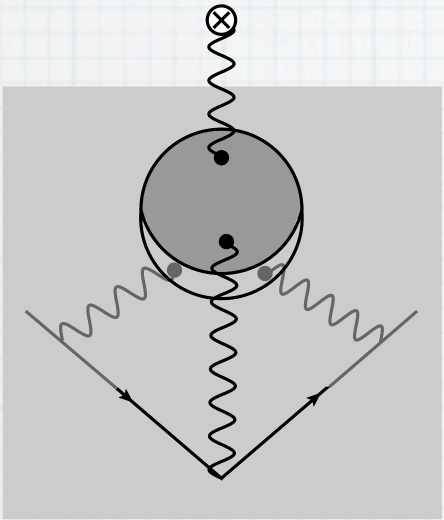
weighting functions (entire)

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p,p',q_1,k-q_1-q_2,q_2)$$

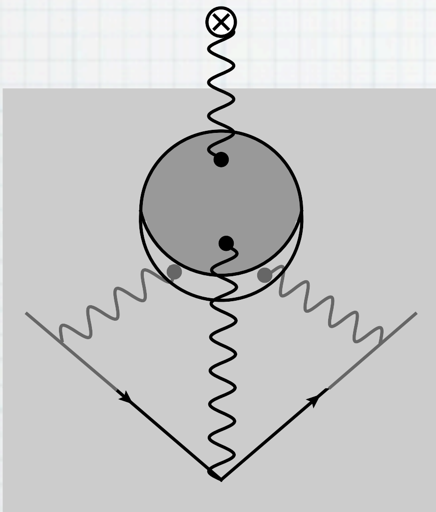
analytic structure →

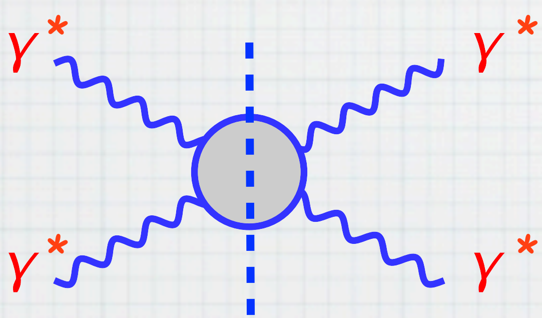
$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1,k-q_1-q_2,q_2,k)}{q_1^2 q_2^2 (k-q_1-q_2)^2\,[(p+q_1)^2-m^2]\,[(p+k-q_2)^2-m^2]}$$

Discontinuity



Discontinuity



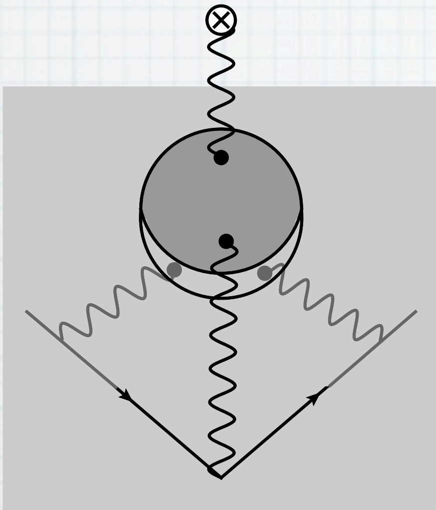


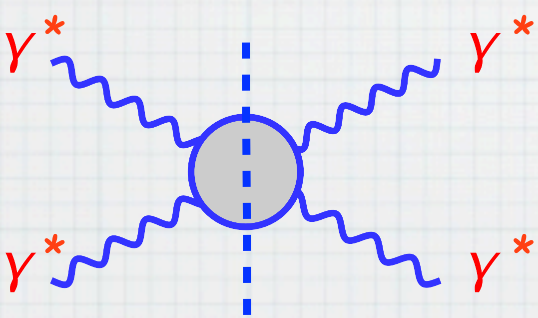
Discontinuity

$$D_{2\mu H^+} = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^2 \delta\left((p+q_1)^2 - m^2\right) \delta\left((p+k-q_2)^2 - m^2\right)$$

$$\times \frac{L_{\lambda_1\lambda_2\lambda_3\lambda}(p,p',q_1,k-q_1-q_2,q_2)}{q_1^2 q_2^2 (k-q_1-q_2)^2}$$

$$\times \, \mathrm{Disc}_{(q_1+q_2)^2} \Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)$$





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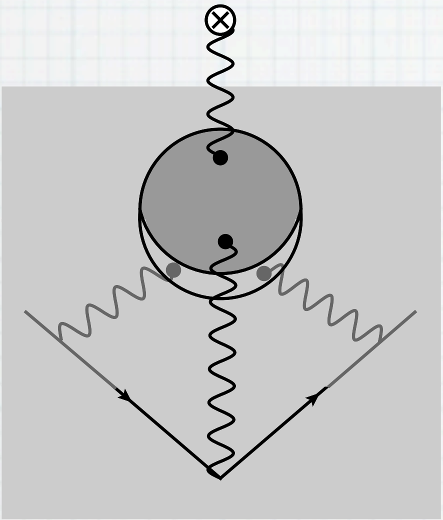

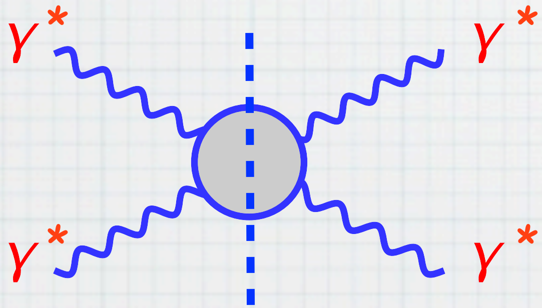

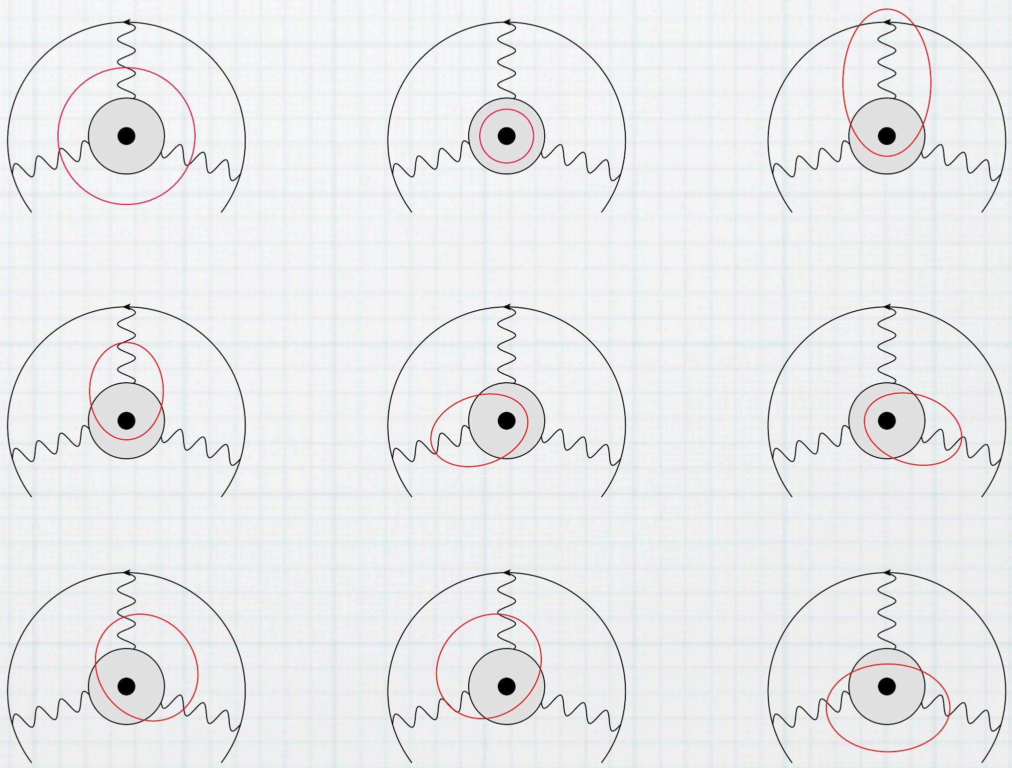

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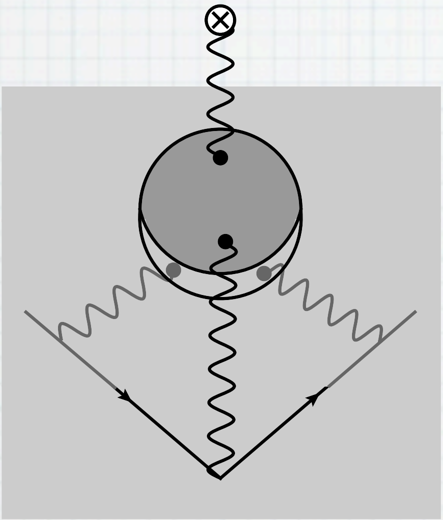
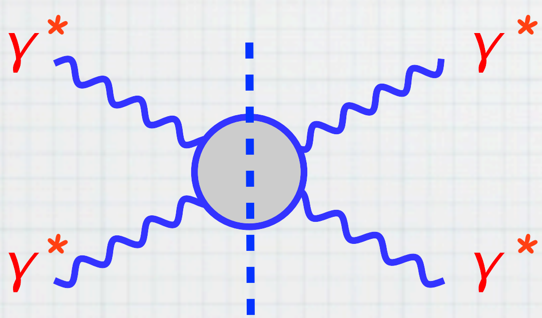
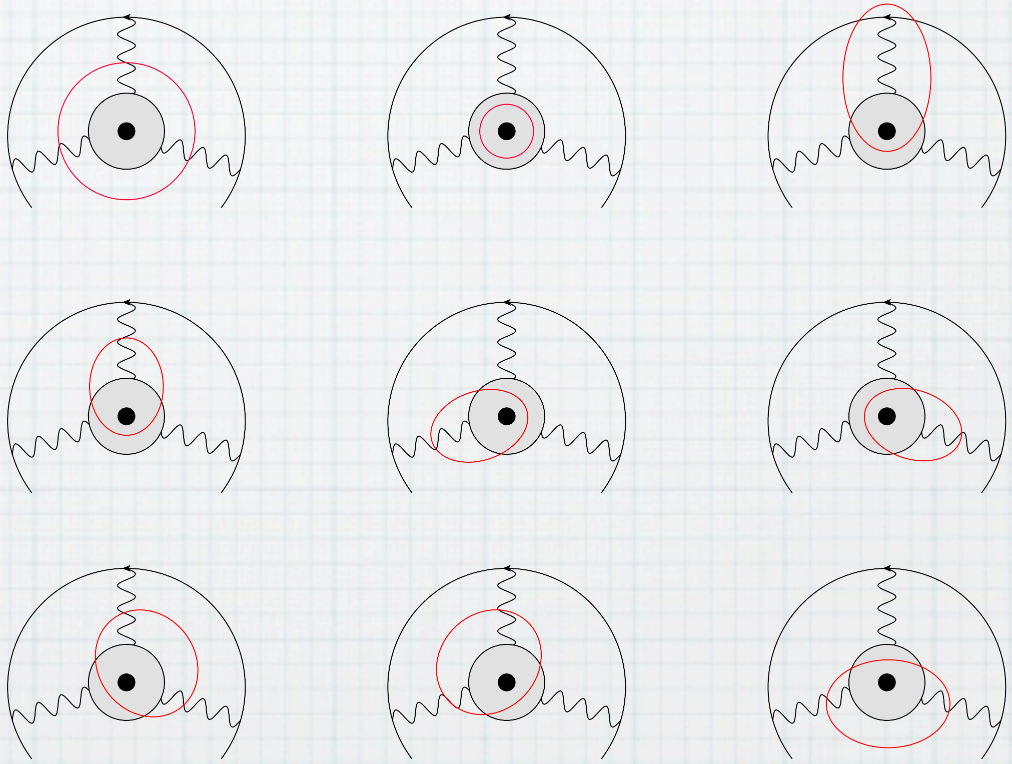
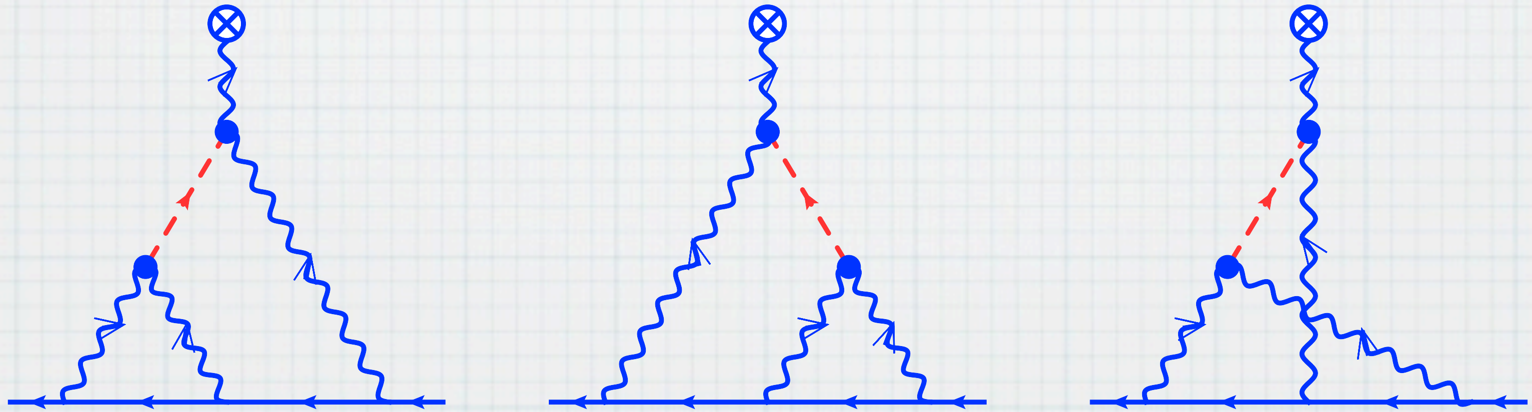






$$\mathrm{Abs} F_2(k^2) = D_{3\gamma} + D_{H^-} + D_{2\mu H^+} + D^{(1)}_{\gamma H^+} + D^{(2)}_{\gamma H^+} + D^{(3)}_{\gamma H^+} + D^{(1)}_{2\gamma H^-} + D^{(2)}_{2\gamma H^-} + D^{(3)}_{2\gamma H^-}$$

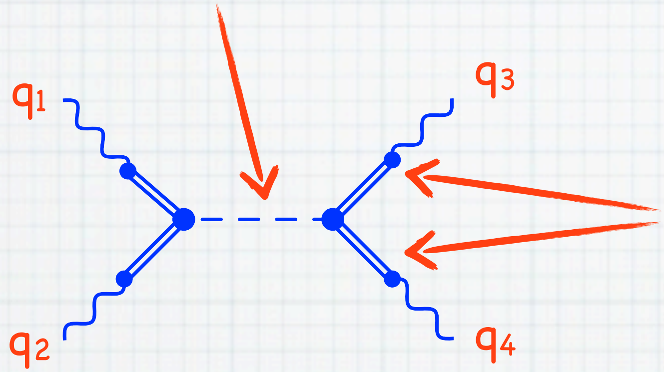
Meson pole contributions to $(g\text{-}2)_\mu$



Pole contributions

π^0 - pole in $(q_i+q_j)^2$

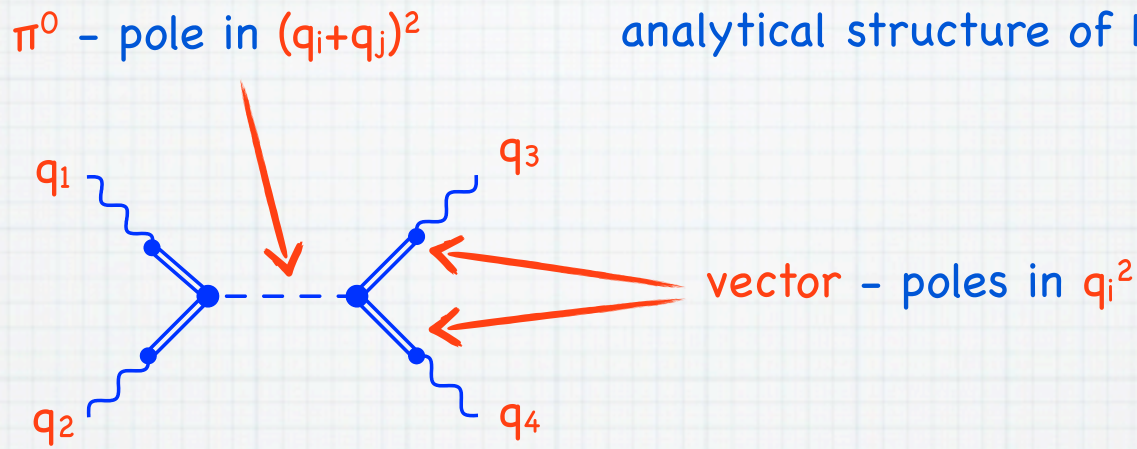
analytical structure of LbL amplitude



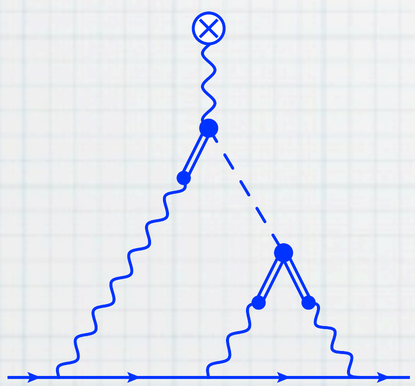
vector - poles in q_i^2

Pole contributions

analytical structure of LbL amplitude

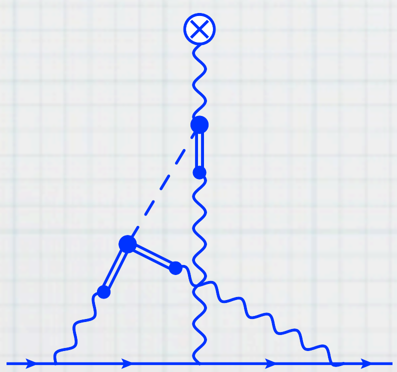


(1)



$$F_2^{(1)}(t) = e^6\Lambda^6|F_{P\gamma^*\gamma^*}(0,0,M^2)|^2 \int \frac{\mathrm{d}^4q_1}{(2\pi)^4}\int \frac{\mathrm{d}^4q_2}{(2\pi)^4}$$
$$\times \frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}\frac{1}{q_2^2}\frac{1}{q_2^2-\Lambda^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(k-q_1-q_2)^2-\Lambda^2}$$
$$\times \frac{1}{(p+q_1)^2-m^2}\frac{1}{(p+k-q_2)^2-m^2}\frac{1}{(k-q_1)^2-M^2}T_1(q_1,q_2,p,k)$$

(2)



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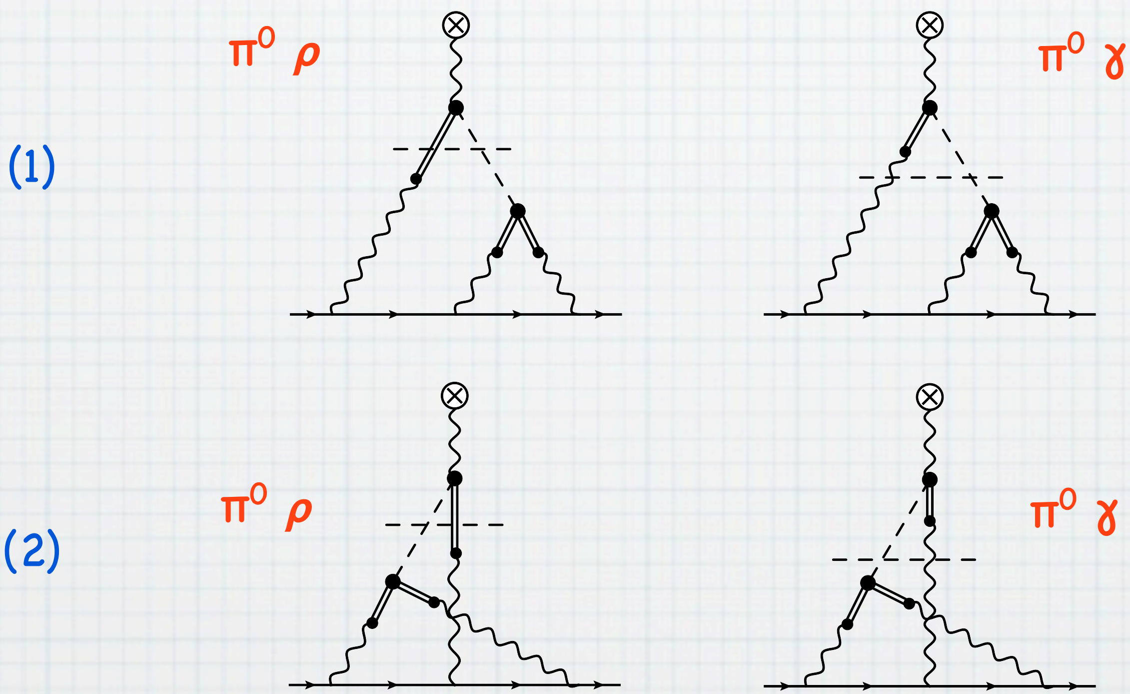
$$\times \frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}\frac{1}{q_2^2}\frac{1}{q_2^2-\Lambda^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(k-q_1-q_2)^2-\Lambda^2}$$
$$\times \frac{1}{(p+q_1)^2-m^2}\frac{1}{(p+k-q_2)^2-m^2}\frac{1}{(q_1+q_2)^2-M^2}T_2(q_1,q_2,p,k)$$

2-particle discontinuities

$$\text{Disc}^{(2)}\Gamma_i^{\rho}(t) = \text{Disc}_{\pi^0\rho}\Gamma_i^{\rho}(t) + \text{Disc}_{\pi^0\gamma}\Gamma_i^{\rho}(t)$$



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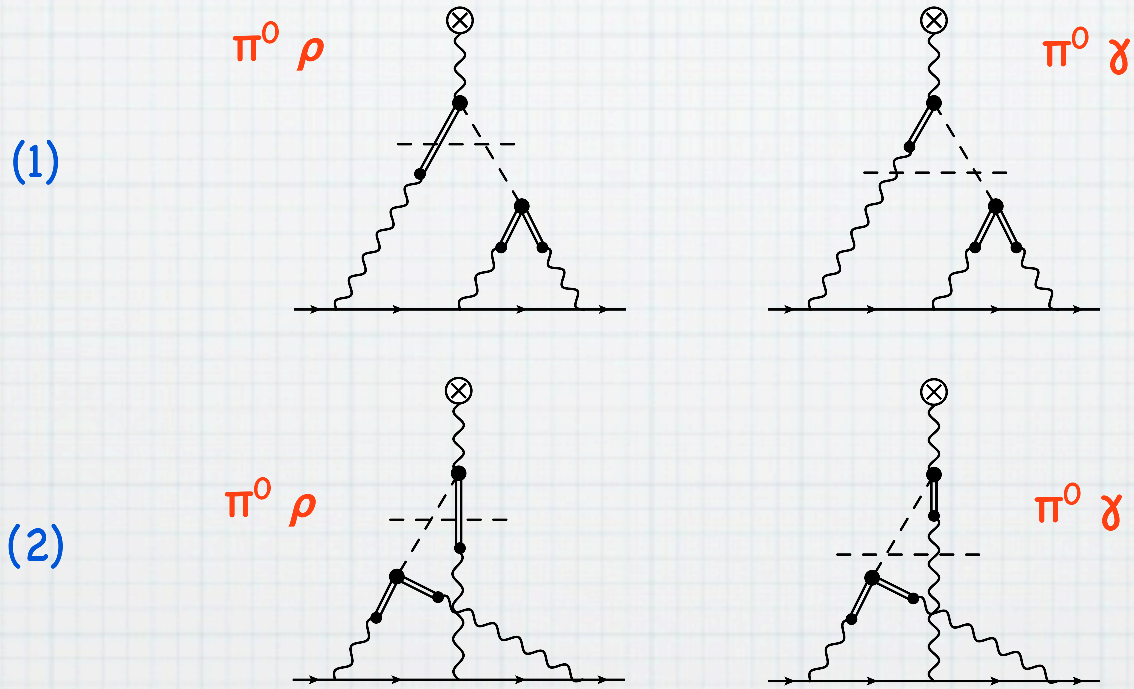
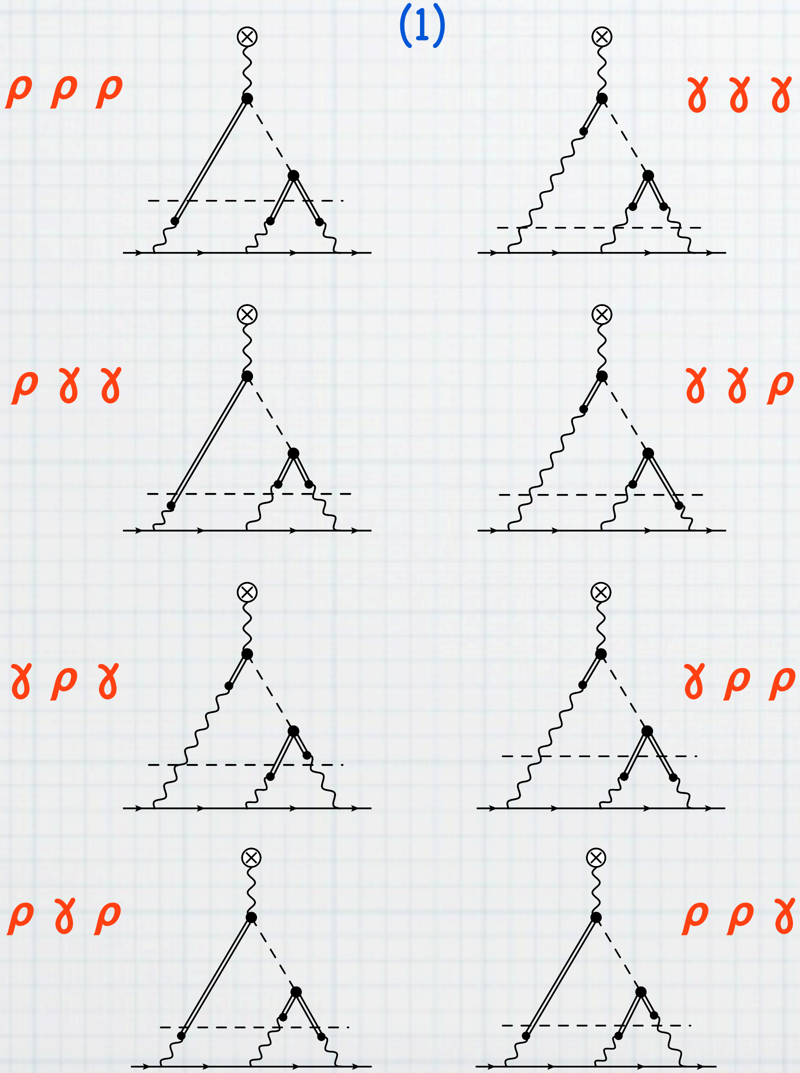


3-particle discontinuities

$$\mathrm{Disc}^{(3)}\Gamma_1^\rho(t) = \mathrm{Disc}_{\gamma\gamma\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\gamma\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\rho\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\gamma\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\gamma\rho\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\gamma\rho}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\rho\gamma}\Gamma_1^\rho(t) + \mathrm{Disc}_{\rho\rho\rho}\Gamma_1^\rho(t)$$

$$\mathrm{Disc}^{(3)}\Gamma_2^\rho(t) = \mathrm{Disc}_{\gamma\gamma\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\gamma\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\rho\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\gamma\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\gamma\rho\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\gamma\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\rho\gamma}\Gamma_2^\rho(t) + \mathrm{Disc}_{\rho\rho\rho}\Gamma_2^\rho(t) + \mathrm{Disc}_{\mu\mu\pi}\Gamma_2^\rho(t)$$

(1)



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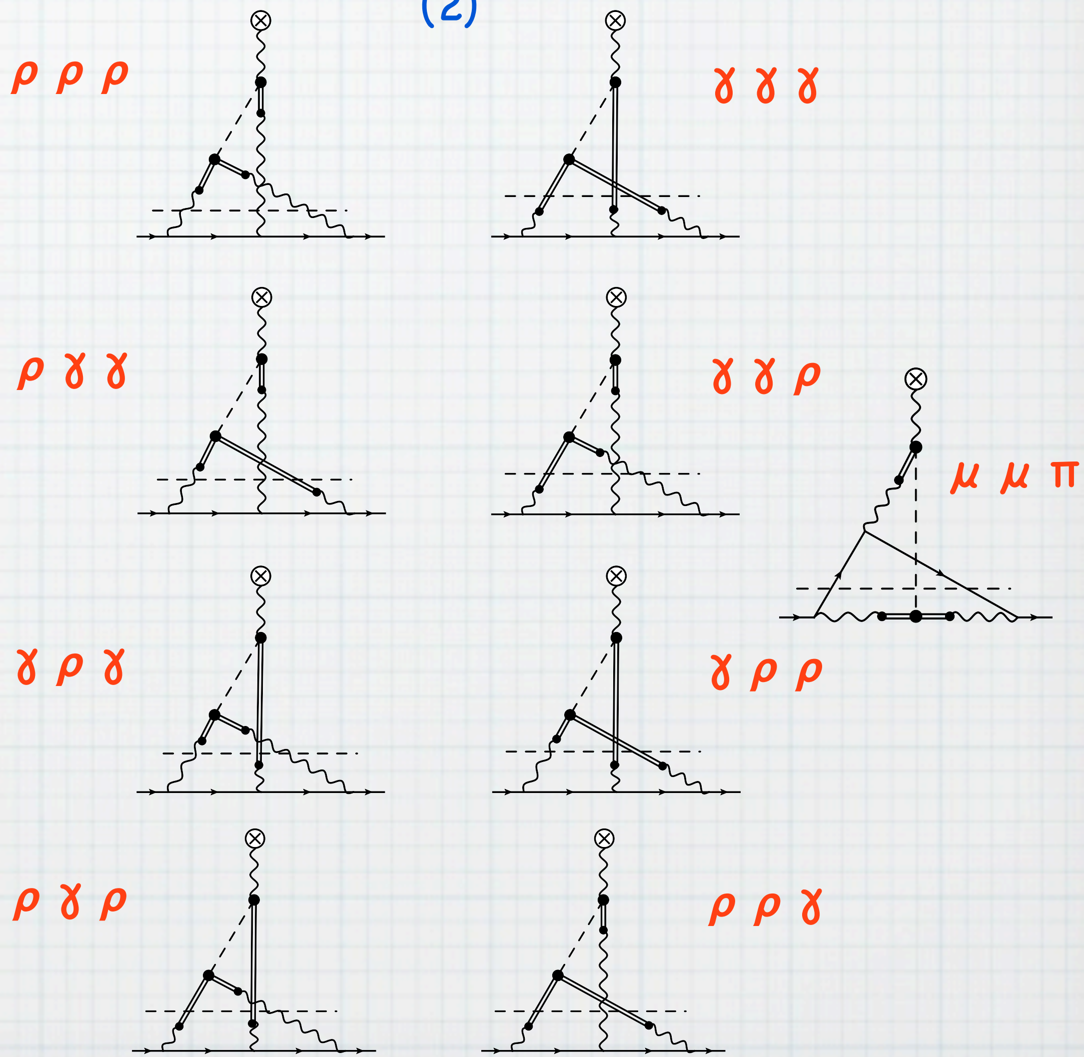

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$$\mathrm{Disc}^{(3)}\Gamma_1^{\rho}(t) = \mathrm{Disc}_{\gamma\gamma\gamma}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\gamma\gamma\rho}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\gamma\rho\gamma}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\rho\gamma\gamma}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\gamma\rho\rho}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\rho\gamma\rho}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\rho\rho\gamma}\Gamma_1^{\rho}(t) + \mathrm{Disc}_{\rho\rho\rho}\Gamma_1^{\rho}(t)$$

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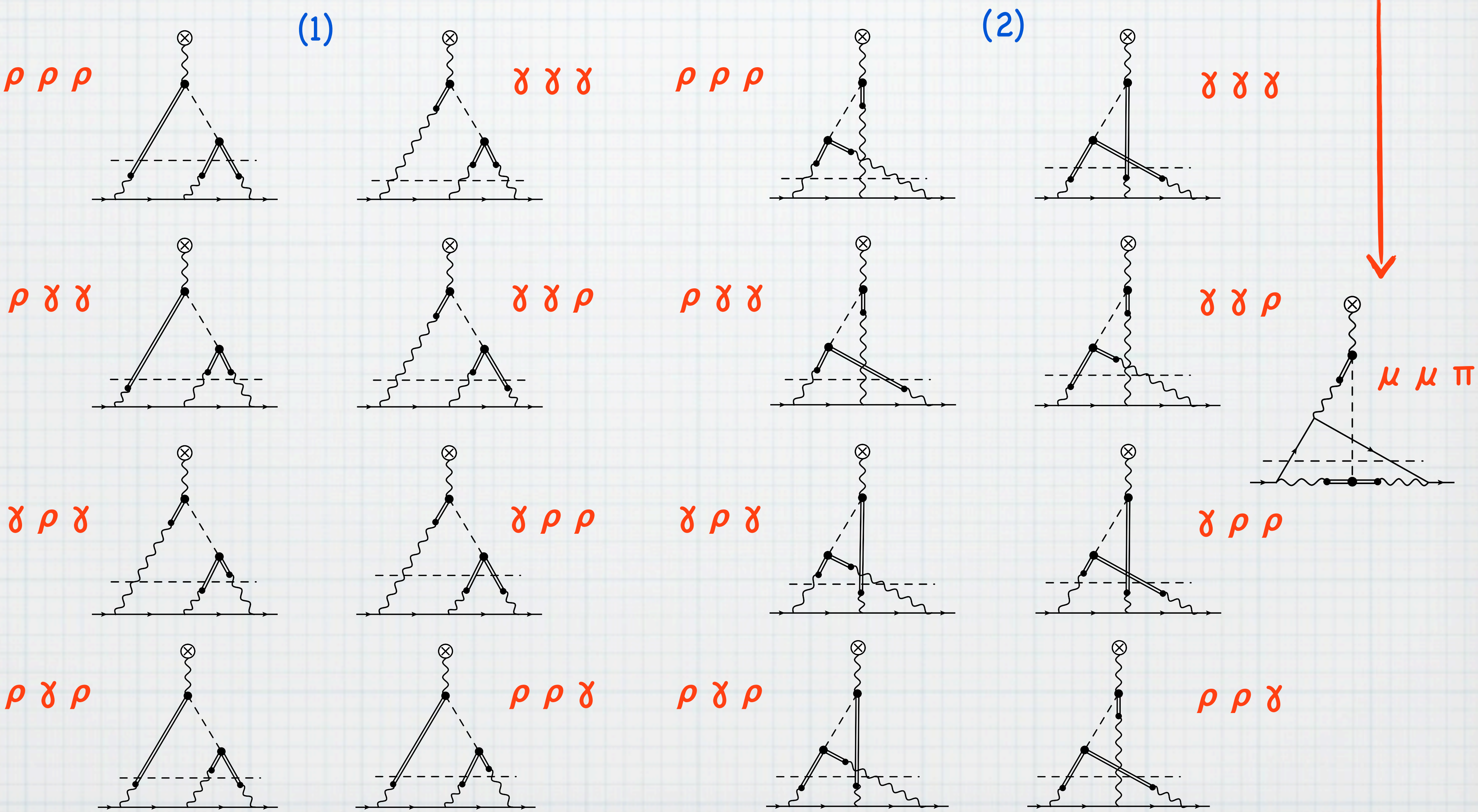


Reduction

rational fraction decomposition

$$\frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}=\frac{1}{\Lambda^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)$$

Reduction

rational fraction decomposition

$$\frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}=\frac{1}{\Lambda^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)$$

$$F_2^{(1)}(t)=e^6\Lambda^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(k-q_1)^2-M^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)$$
$$\times\left(S_3^{\Lambda\Lambda}-S_3^{\Lambda 0}-S_3^{0\Lambda}+S_3^{00}\right)$$

$$F_2^{(2)}(t)=e^6\Lambda^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(k-q_1)^2-M^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)\left(S_4^{\Lambda\Lambda}-S_4^{\Lambda 0}-S_4^{0\Lambda}+S_4^{00}\right)$$

Reduction

rational fraction decomposition

$$\frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}=\frac{1}{\Lambda^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)$$

$$F_2^{(i)}(t)=F_i^{\Lambda\Lambda\Lambda}(t)-F_i^{\Lambda\Lambda 0}(t)-F_i^{\Lambda 0\Lambda}(t)+F_i^{\Lambda 00}(t)-F_i^{0\Lambda\Lambda}(t)+F_i^{0\Lambda 0}(t)+F_i^{00\Lambda}(t)-F_i^{000}(t)$$

$$F_1^{\Lambda_1\Lambda_2\Lambda_3}(t)=e^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(k-q_1)^2-M^2}\frac{1}{q_1^2-\Lambda_1^2}S_3^{\Lambda_2\Lambda_3}$$

$$F_2^{\Lambda_1\Lambda_2\Lambda_3}(t)=e^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(k-q_1)^2-M^2}\frac{1}{q_1^2-\Lambda_1^2}S_4^{\Lambda_2\Lambda_3}$$

Reduction

rational fraction decomposition

$$\frac{1}{q_1^2}\frac{1}{q_1^2-\Lambda^2}=\frac{1}{\Lambda^2}\left(\frac{1}{q_1^2-\Lambda^2}-\frac{1}{q_1^2}\right)$$

$$F_2^{(i)}(t)=F_i^{\Lambda\Lambda\Lambda}(t)-F_i^{\Lambda\Lambda 0}(t)-F_i^{\Lambda 0\Lambda}(t)+F_i^{\Lambda 00}(t)-F_i^{0\Lambda\Lambda}(t)+F_i^{0\Lambda 0}(t)+F_i^{00\Lambda}(t)-F_i^{000}(t)$$

$$F_1^{\Lambda_1\Lambda_2\Lambda_3}(t)=e^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(k-q_1)^2-M^2}\frac{1}{q_1^2-\Lambda_1^2}S_3^{\Lambda_2\Lambda_3}$$

$$F_2^{\Lambda_1\Lambda_2\Lambda_3}(t)=e^6|F(0,0,M^2)|^2\int\frac{\mathrm{d}^4q_1}{(2\pi)^4}\frac{1}{(k-q_1)^2-M^2}\frac{1}{q_1^2-\Lambda_1^2}S_4^{\Lambda_2\Lambda_3}$$

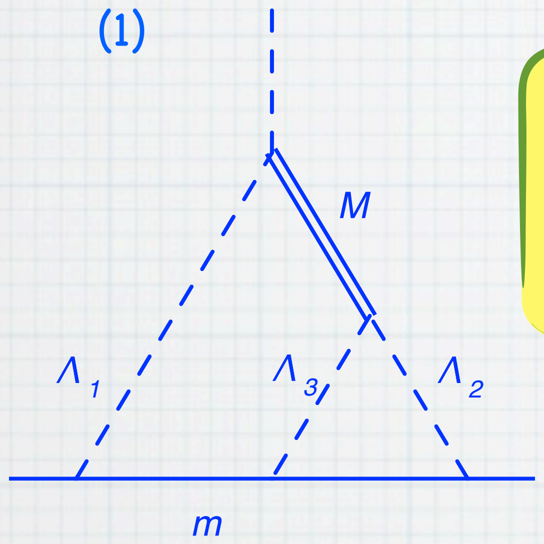
$$S_3^{\Lambda_2\Lambda_3}((p+q_1)^2,p^2,(k-q_1)^2)=\int\frac{\mathrm{d}^4q_2}{(2\pi)^4}\frac{T_1(q_1,q_2,p,k)}{[q_2^2-\Lambda_3^2]\,[(k-q_1-q_2)^2-\Lambda_2^2]\,[(p-q_2+k)^2-m^2]}$$

$$S_4^{\Lambda_1\Lambda_2}\left((p+q_1)^2,k^2,q_1^2,(k-q_1)^2,p^2,(p+k)^2\right)=\int\frac{\mathrm{d}^4q_2}{(2\pi)^4}\frac{1}{[(k-q_1-q_2)^2-\Lambda_1^2]\,[q_2^2-\Lambda_2^2]}\times\frac{T_2(k-q_1-q_2,q_2,p,k)}{[(p+k-q_1-q_2)^2-m_1^2]\,[(p+k-q_2)^2-m_2^2]}$$

Scalar two-loop amplitudes in the dispersive approach

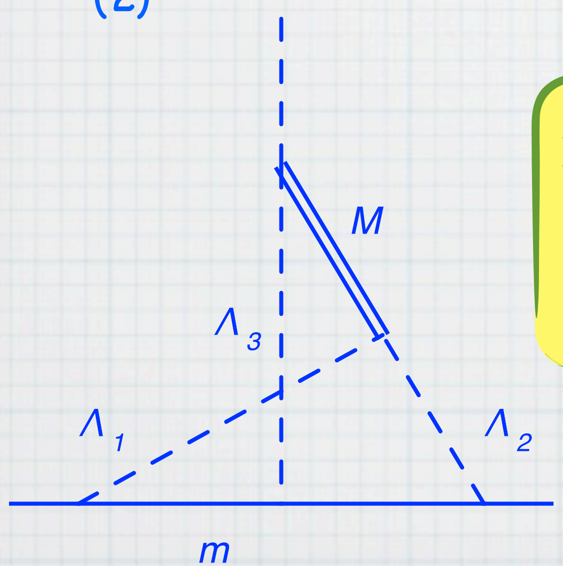
Scalar two-loop vertex functions

(1)



$$\Gamma_1(t) = \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{(k-q_1-q_2)^2 - \Lambda_3^2} \frac{1}{q_1^2 - \Lambda_1^2} \frac{1}{q_2^2 - \Lambda_2^2} \frac{1}{(k-q_1)^2 - M^2} \times \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

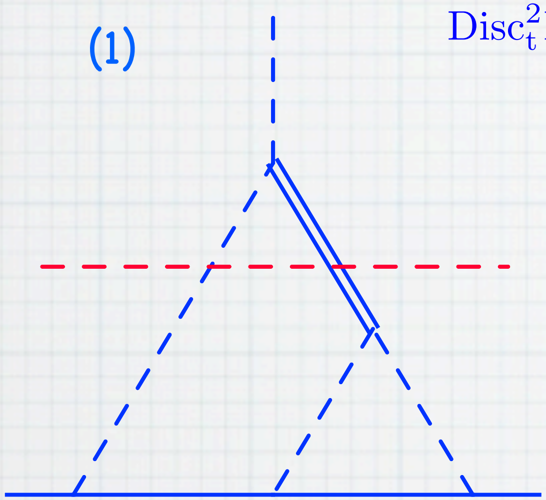
(2)



$$\Gamma_2(t) = \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{(k-q_1-q_2)^2 - \Lambda_3^2} \frac{1}{q_1^2 - \Lambda_1^2} \frac{1}{q_2^2 - \Lambda_2^2} \frac{1}{(q_1+q_2)^2 - M^2} \times \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

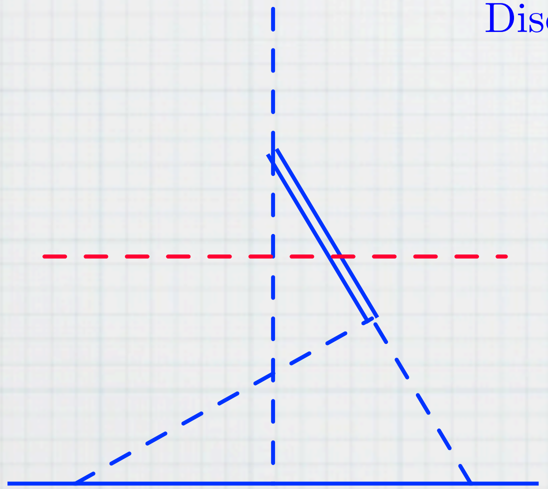
2-particle cuts

(1)



$$\mathrm{Disc}_t^2\Gamma_1(t) = \int \frac{d^4q_1}{(2\pi)^4}(-2\pi i)^2\delta(q_1^2-\Lambda_1^2)\delta((k-q_1)^2-M^2)$$
$$\times \frac{1}{(p+q_1)^2-m_1^2} M_3\left((k-q_1)^2,(p+q_1)^2,m^2,\Lambda_2^2,\Lambda_3^2,m^2\right)$$

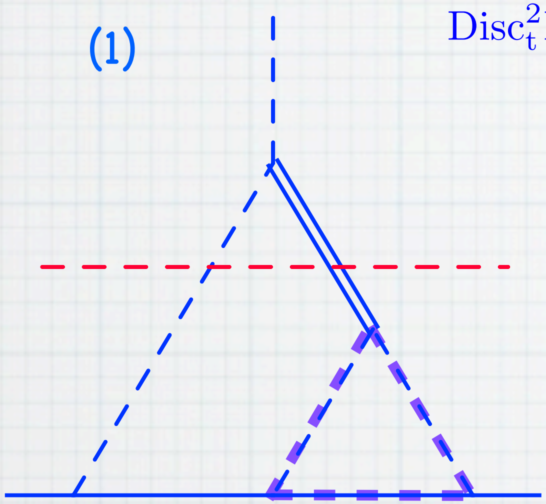
(2)



$$\mathrm{Disc}_t^2\Gamma_2(t) = \int \frac{d^4q_1}{(2\pi)^4}(-2\pi i)^2\delta((q_1^2-\Lambda_3^2)\delta((k-q_1)^2-M^2)$$
$$\times M_4\left((k-q_1)^2,p^2,q_1^2,(p+k)^2,(p+k-q_1)^2,(p+q_1)^2,\Lambda_2^2,\Lambda_1^2,m^2,m^2\right)$$

2-particle cuts

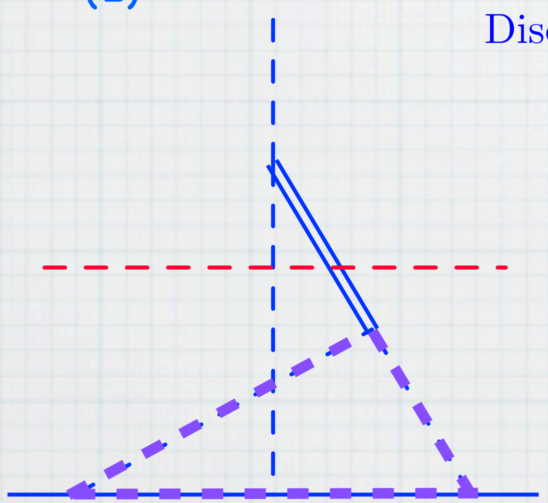
(1)



$$\mathrm{Disc}_t^2\Gamma_1(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4}(-2\pi i)^2 \delta(q_1^2 - \Lambda_1^2)\delta((k-q_1)^2 - M^2)$$
$$\times \frac{1}{(p+q_1)^2 - m_1^2} M_3\left((k-q_1)^2, (p+q_1)^2, m^2, \Lambda_2^2, \Lambda_3^2, m^2\right)$$

$$M_3\left((k-q_1)^2, (p+q_1)^2, m^2, \Lambda_2^2, \Lambda_3^2, m^2\right)$$
$$= \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} \frac{1}{q_2^2 - \Lambda_2^2} \frac{1}{(k-q_1-q_2)^2 - \Lambda_3^2} \frac{1}{(p+k-q_2)^2 - m^2}$$

(2)

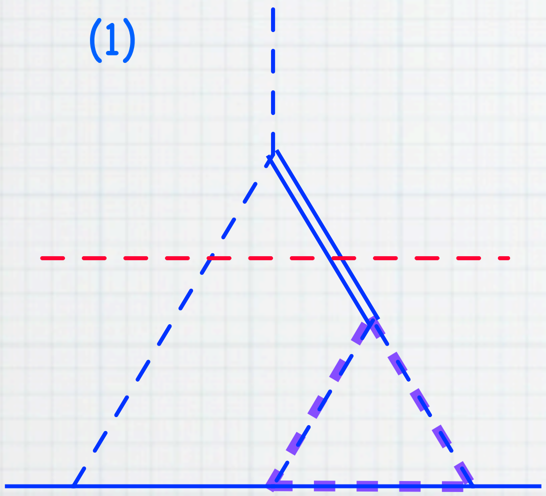


$$\mathrm{Disc}_t^2\Gamma_2(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4}(-2\pi i)^2 \delta((q_1^2 - \Lambda_3^2)\delta((k-q_1)^2 - M^2)$$
$$\times M_4\left((k-q_1)^2, p^2, q_1^2, (p+k)^2, (p+k-q_1)^2, (p+q_1)^2, \Lambda_2^2, \Lambda_1^2, m^2, m^2\right)$$

$$M_4\left((k-q_1)^2, p^2, q_1^2, (p+k)^2, (p+k-q_1)^2, (p+q_1)^2, \Lambda_2^2, \Lambda_1^2, m^2, m^2\right)$$
$$= \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} \frac{1}{q_2^2 - \Lambda_2^2} \frac{1}{(k-q_1-q_2)^2 - \Lambda_1^2} \frac{1}{(p+k-q_1-q_2)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

2-particle cuts

(1)



$$\mathrm{Disc}_t^2\Gamma_1(t) = -\frac{1}{8\pi}\int \mathrm{d}\cos\theta_1 \frac{\beta_1 M_3(M^2, s_-, m^2, \Lambda_2^2, \Lambda_3^2, m^2)}{2m^2 - 2m_1^2 + q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$

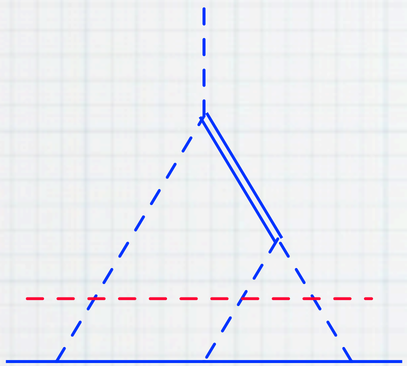
$$M_3\left((k-q_1)^2, (p+q_1)^2, m^2, \Lambda_2^2, \Lambda_3^2, m^2\right)$$
$$= \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4}\frac{1}{q_2^2-\Lambda_2^2}\frac{1}{(k-q_1-q_2)^2-\Lambda_3^2}\frac{1}{(p+k-q_2)^2-m^2}$$

(2)

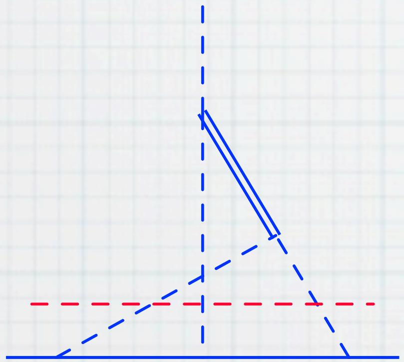
$$\mathrm{Disc}_t^2\Gamma_2(t) = -\frac{1}{16\pi}\int \mathrm{d}\cos\theta_1\ \beta_1 M_4\left(M^2, m^2, \Lambda_3^2, m^2, s_+, s_-, t, \Lambda_2^2, \Lambda_1^2, m^2, m^2\right)$$

$$M_4\left((k-q_1)^2, p^2, q_1^2, (p+k)^2, (p+k-q_1)^2, (p+q_1)^2, \Lambda_2^2, \Lambda_1^2, m^2, m^2\right)$$
$$= \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4}\frac{1}{q_2^2-\Lambda_2^2}\frac{1}{(k-q_1-q_2)^2-\Lambda_1^2}\frac{1}{(p+k-q_1-q_2)^2-m_1^2}\frac{1}{(p+k-q_2)^2-m_2^2}$$

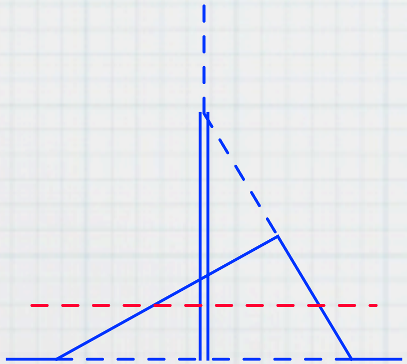
3-particle cuts



$$\mathrm{Disc}_{\mathrm{t}}^{3}\Gamma_1(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2)$$
$$\times \frac{1}{(k-q_1)^2 - M^2} \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

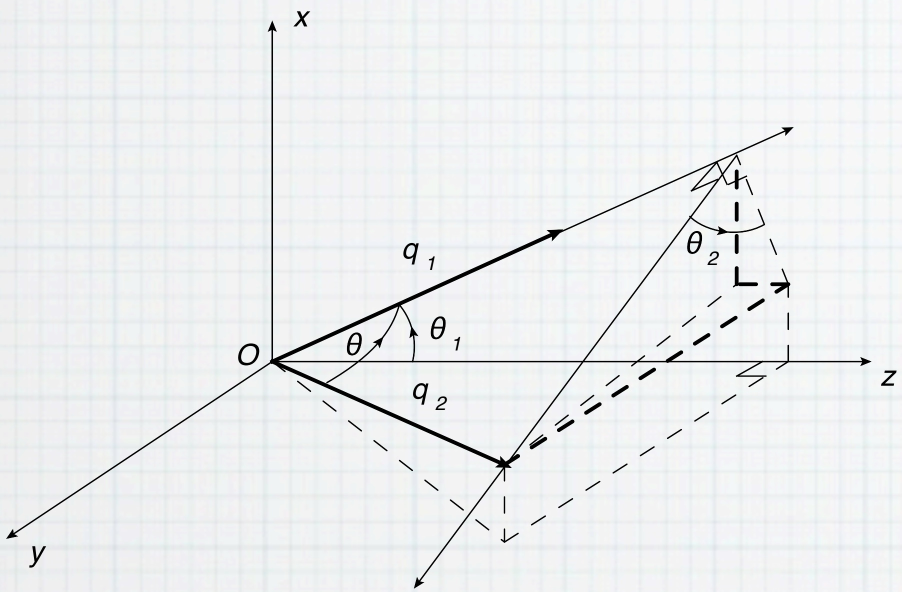


$$\mathrm{Disc}_{\mathrm{t}}^{3,1}\Gamma_2(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2)$$
$$\times \frac{1}{(q_1+q_2)^2 - M^2} \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$



$$\mathrm{Disc}_{\mathrm{t}}^{3,2}\Gamma_2(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - m_1^2)\delta(q_2^2 - m_2^2)\delta((k - q_1 - q_2)^2 - M^2)$$
$$\times \frac{1}{(q_1+q_2)^2 - \Lambda_3^2} \frac{1}{(p+q_1)^2 - \Lambda_1^2} \frac{1}{(p+k-q_2)^2 - \Lambda_2^2}$$

Angular parametrization



$$k = \left(\sqrt{t}, 0, 0, 0\right),$$

$$p = \frac{\sqrt{t}}{2}\,(-1, 0, 0, \beta),$$

$$q_1 = \frac{\sqrt{t}}{2}\beta_1\left(\frac{t - t_1 + q_1^2}{t\beta_1}, \sin\theta_1, 0, \cos\theta_1\right),$$

$$q_2 = \frac{\sqrt{t}}{2}\beta_2\left(\frac{t - t_2 + q_2^2}{t\beta_2}, -\cos\theta_1\cos\theta_2\sin\theta + \sin\theta_1\cos\theta,\right.$$
$$\left.\sin\theta\sin\theta_2, \sin\theta_1\cos\theta_2\sin\theta + \cos\theta_1\cos\theta\right),$$

$$\cos\theta = \frac{2t(q_1^2 + q_2^2 - t_{12}) + (t - t_1 + q_1^2)(t - t_2 + q_2^2)}{t^2\beta_1\beta_2}$$

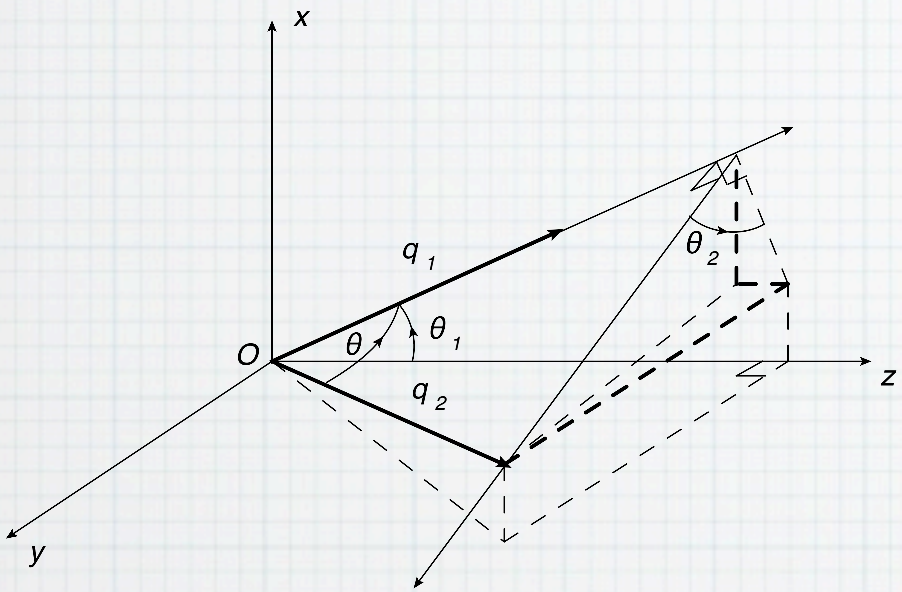
$$\int \mathrm{d}^4 q_1 = \int \mathrm{d}q_1^2 \int \mathrm{d}t_1 \int_0^\pi \mathrm{d}\cos\theta_1 \int \mathrm{d}\phi\, \frac{\beta_1}{8},$$

$$\int \mathrm{d}^4 q_2 = \int \mathrm{d}q_2^2 \int \mathrm{d}t_2 \int \mathrm{d}t_{12} \int \mathrm{d}\theta_2\, \frac{1}{4\beta_1 t}$$

$$\beta_i = \sqrt{\left(1 + \frac{q_i^2 - t_i}{t}\right)^2 - \frac{4q_i^2}{t}} \quad \text{and} \quad \beta = \sqrt{1 - \frac{4m^2}{t}}$$

$$\Omega(t_1, t_2, t_{12}, q_1^2, q_2^2, m_1^2, m_2^2) = \int_0^\pi \mathrm{d}\cos\theta_1 \int_0^{2\pi} \mathrm{d}\theta_2\, \frac{2}{2m^2 - 2m_1^2 + q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$
$$\times \frac{2}{2m^2 - 2m_2^2 + q_2^2 - t + t_2 + t\beta_2\beta(\sin\theta_1\cos\theta_2\sin\theta + \cos\theta_1\cos\theta)}$$

Angular parametrization



$$k = \left(\sqrt{t}, 0, 0, 0\right),$$

$$p = \frac{\sqrt{t}}{2}\,(-1, 0, 0, \beta),$$

$$q_1 = \frac{\sqrt{t}}{2}\beta_1 \left(\frac{t - t_1 + q_1^2}{t\beta_1}, \sin\theta_1, 0, \cos\theta_1\right),$$

$$q_2 = \frac{\sqrt{t}}{2}\beta_2 \left(\frac{t - t_2 + q_2^2}{t\beta_2}, -\cos\theta_1\cos\theta_2\sin\theta + \sin\theta_1\cos\theta, \right.$$
$$\left. \sin\theta\sin\theta_2, \sin\theta_1\cos\theta_2\sin\theta + \cos\theta_1\cos\theta\right),$$

$$\cos\theta = \frac{2t(q_1^2 + q_2^2 - t_{12}) + (t - t_1 + q_1^2)(t - t_2 + q_2^2)}{t^2\beta_1\beta_2}$$

$$\int \mathrm{d}^4 q_1 = \int \mathrm{d}q_1^2 \int \mathrm{d}t_1 \int_0^\pi \mathrm{d}\cos\theta_1 \int \mathrm{d}\phi\, \frac{\beta_1}{8},$$

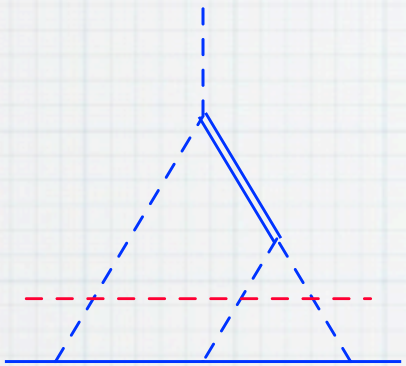
$$\int \mathrm{d}^4 q_2 = \int \mathrm{d}q_2^2 \int \mathrm{d}t_2 \int \mathrm{d}t_{12} \int \mathrm{d}\theta_2\, \frac{1}{4\beta_1 t}$$

$$\beta_i = \sqrt{\left(1 + \frac{q_i^2 - t_i}{t}\right)^2 - \frac{4q_i^2}{t}} \quad \text{and} \quad \beta = \sqrt{1 - \frac{4m^2}{t}}$$

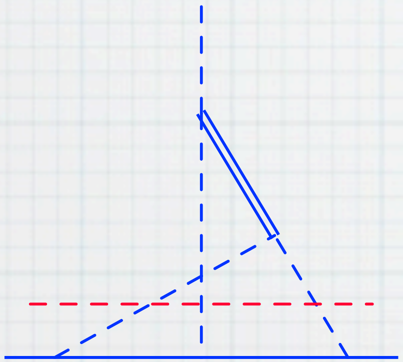
$$\Omega(t_1, t_2, t_{12}, q_1^2, q_2^2, m_1^2, m_2^2) = -\frac{4}{\beta_1\beta_2\beta^2 t^2}\,\frac{2\pi}{\sqrt{(a+b)^2 - 2ab(1+\cos\theta) - \sin^2\theta}}$$

$$\times \left[\log\frac{a-1}{a+1} + \log\frac{(a+b)(1-b) - b(1+\cos\theta) + ab(1+\cos\theta) + \sin^2\theta + (\cos\theta + b)c}{-(a+b)(1+b) + b(1+\cos\theta) + ab(1+\cos\theta) + \sin^2\theta - (\cos\theta - b)c}\right]$$

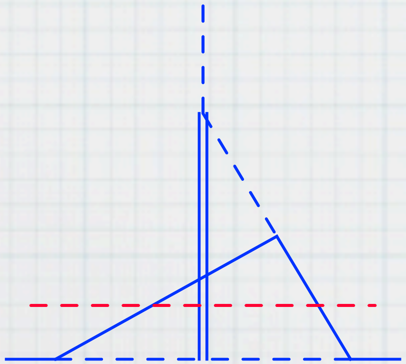
3-particle cuts



$$\mathrm{Disc}^3_{\mathrm{t}}\Gamma_1(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2) \times \frac{1}{(k-q_1)^2 - M^2} \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

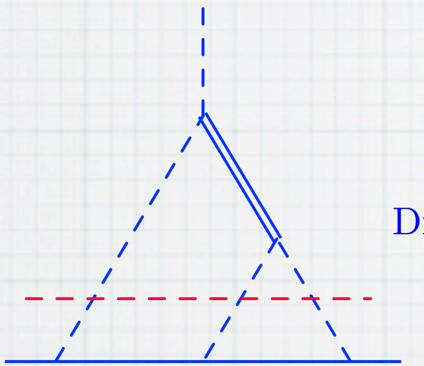


$$\mathrm{Disc}^{3,1}_{\mathrm{t}}\Gamma_2(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2) \times \frac{1}{(q_1+q_2)^2 - M^2} \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$

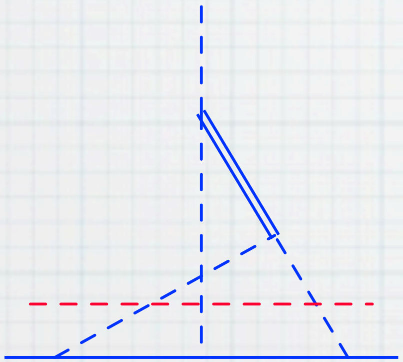


$$\mathrm{Disc}^{3,2}_{\mathrm{t}}\Gamma_2(t) = \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4} (-2\pi i)^3 \delta(q_1^2 - m_1^2)\delta(q_2^2 - m_2^2)\delta((k - q_1 - q_2)^2 - M^2) \times \frac{1}{(q_1+q_2)^2 - \Lambda_3^2} \frac{1}{(p+q_1)^2 - \Lambda_1^2} \frac{1}{(p+k-q_2)^2 - \Lambda_2^2}$$

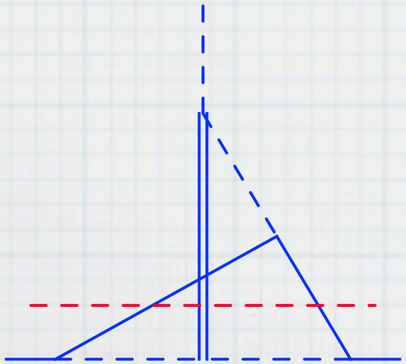
3-particle cuts



$$\mathrm{Disc_t}\Gamma_1^{(3)}(t) = i\frac{1}{2(4\pi)^4 t}\int \mathrm{d}t_1 \int \mathrm{d}t_2 \frac{1}{t_1 - M^2}\,\Omega(t_1, t_2, t + \Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2 - t_1 - t_2, \Lambda_1^2, \Lambda_2^2, m_1^2, m_2^2)$$



$$\mathrm{Disc_t}\Gamma_2^{(3,1)}(t) = i\frac{1}{2(4\pi)^4 t}\int \mathrm{d}t_1 \int \mathrm{d}t_2 \frac{1}{t + \Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2 - t_1 - t_2 - M^2} \times \Omega(t_1, t_2, t + \Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2 - t_1 - t_2, \Lambda_1^2, \Lambda_2^2, m_1^2, m_2^2)$$



$$\mathrm{Disc_t}\Gamma_2^{(3,2)}(t) = i\frac{1}{2(4\pi)^4 t}\int \mathrm{d}t_1 \int \mathrm{d}t_2 \frac{1}{t + m_1^2 + m_2^2 + M^2 - t_1 - t_2 - \Lambda_3^2} \times \Omega(t_1, t_2, t + m_1^2 + m_2^2 + M^2 - t_1 - t_2, m_1^2, m_2^2, \Lambda_1^2, \Lambda_2^2)$$

3-particle cut integration domain

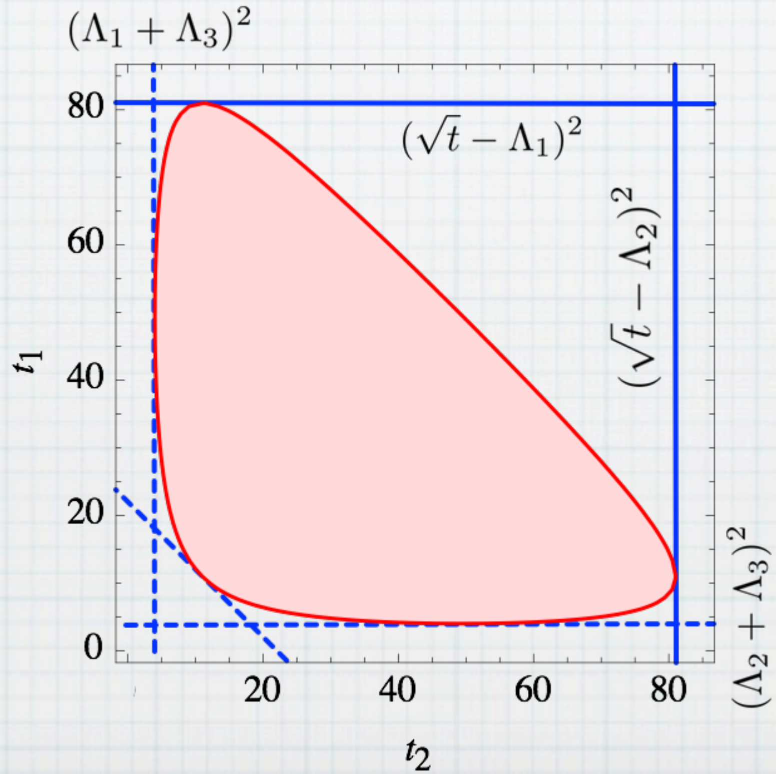
$$q_1^0 \geqslant \Lambda_1 \quad \Rightarrow \quad t_1 \leqslant (\sqrt{t} - \Lambda_1)^2,$$

$$q_2^0 \geqslant \Lambda_2 \quad \Rightarrow \quad t_2 \leqslant (\sqrt{t} - \Lambda_2)^2,$$

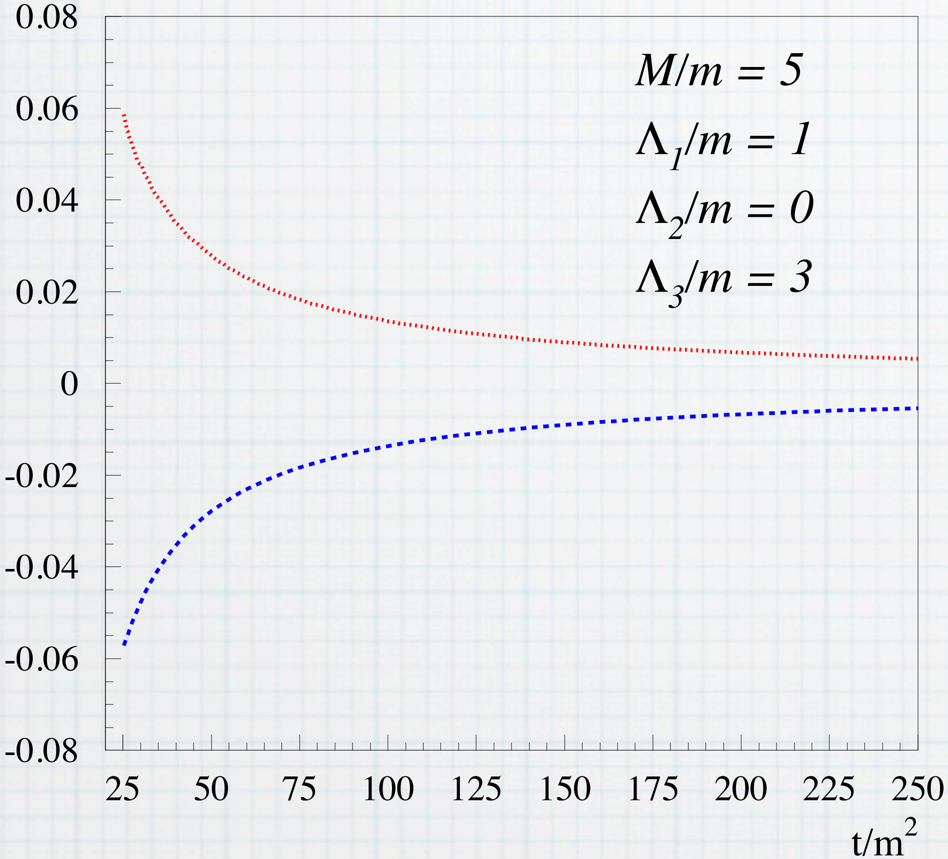
$$k^0 - q_1^0 - q_2^0 \geqslant \Lambda_3 \quad \Rightarrow \quad t_1 + t_2 \geqslant 2\sqrt{t}\Lambda_3 + \Lambda_1^2 + \Lambda_2^2$$

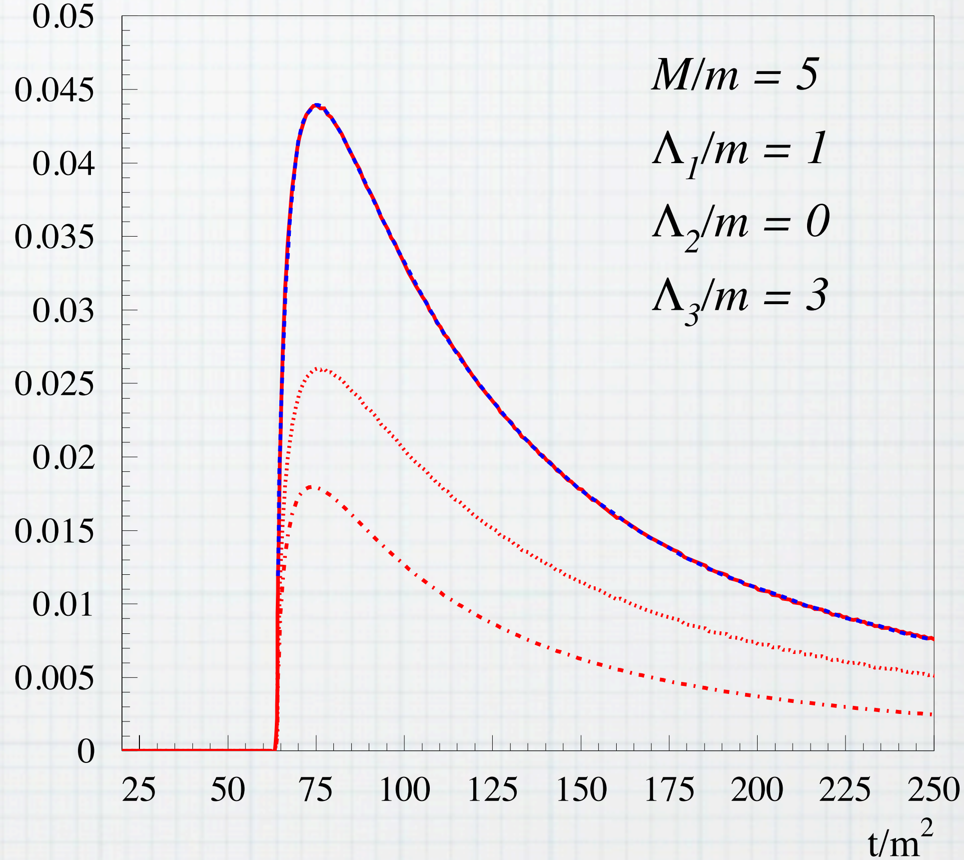
$$t_1 \geqslant (\Lambda_3 + \Lambda_2)^2, \qquad t_2 \geqslant (\Lambda_1 + \Lambda_3)^2$$

$$-1 \leqslant \frac{2t(q_1^2 + q_2^2 - t_{12}) + (t - t_1 + q_1^2)(t - t_2 + q_2^2)}{t^2 \beta_1 \beta_2} \leqslant 1$$

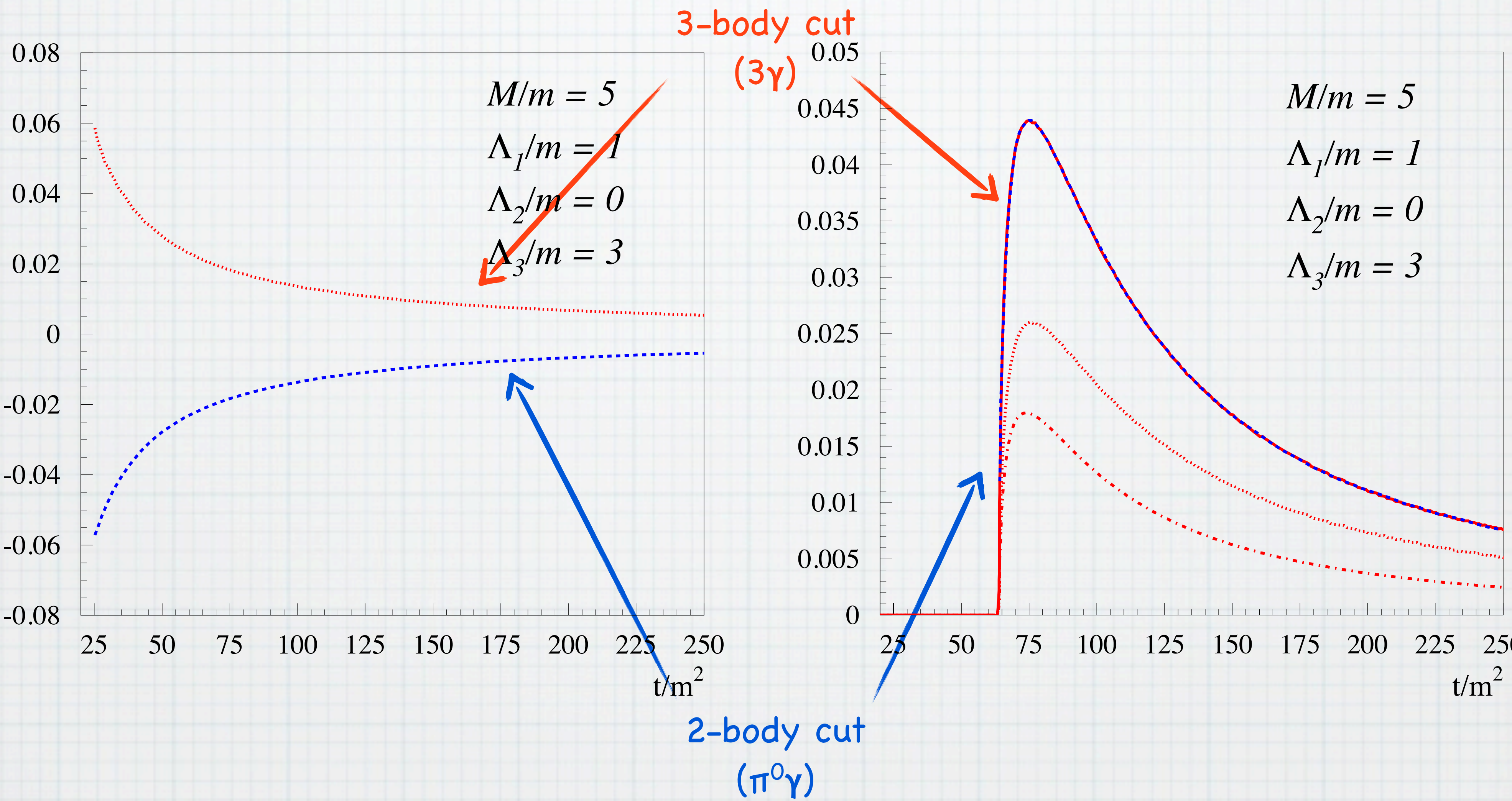


Discontinuity: imaginary parts

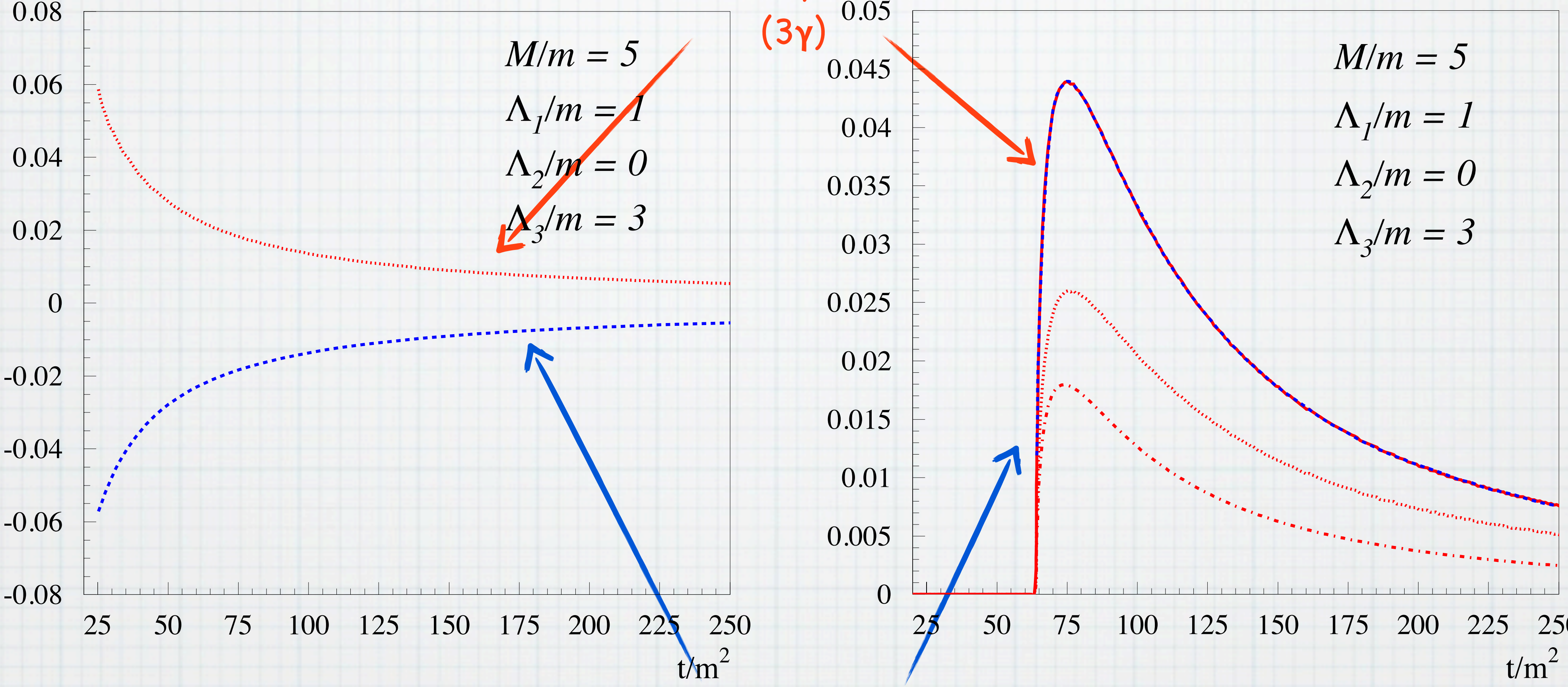




Discontinuity: imaginary parts



Discontinuity: imaginary parts

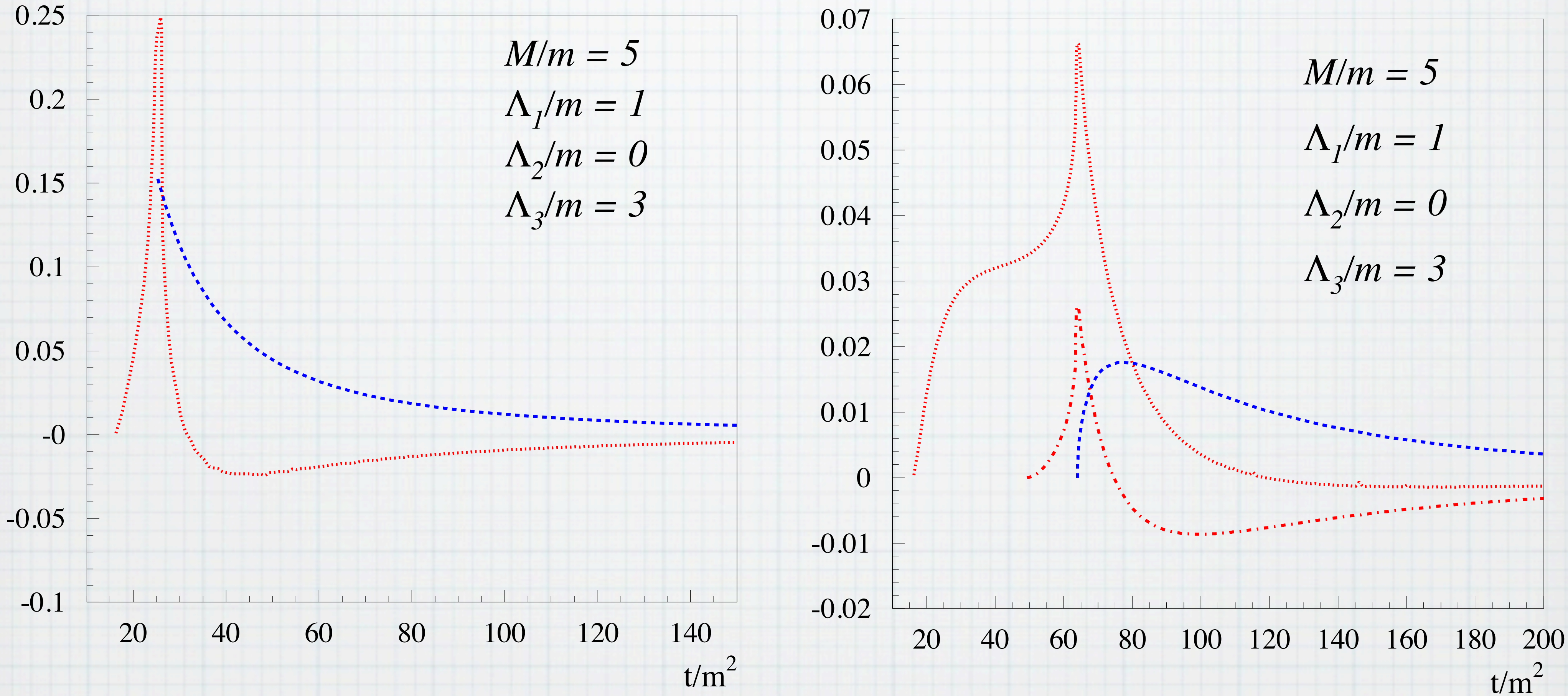


3-body cut (3γ)

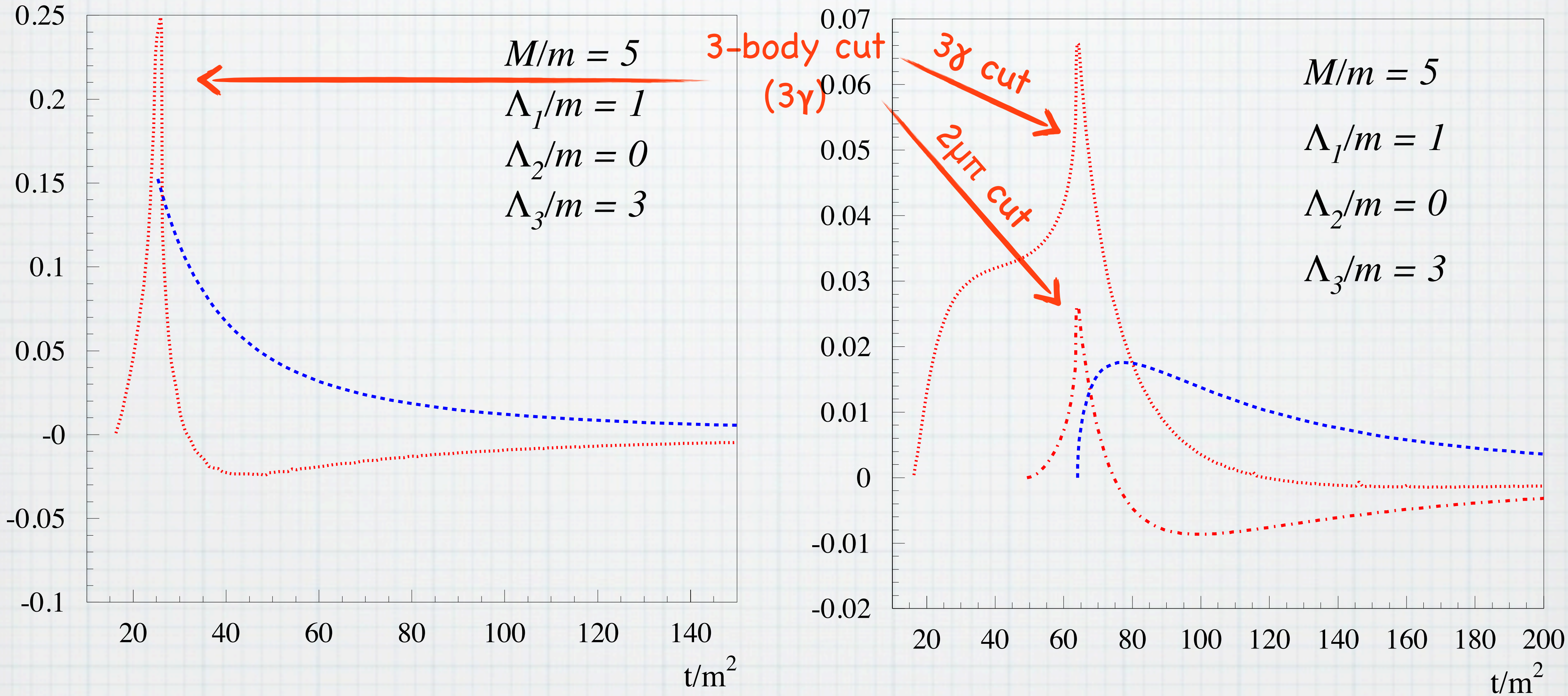
2-body cut ($\pi^0\gamma$)

Imaginary parts cancel: $\mathrm{Im}\left[\mathrm{Disc}_2 F(q'^2)\right] + \mathrm{Im}\left[\mathrm{Disc}_3 F(q'^2)\right] = 0$

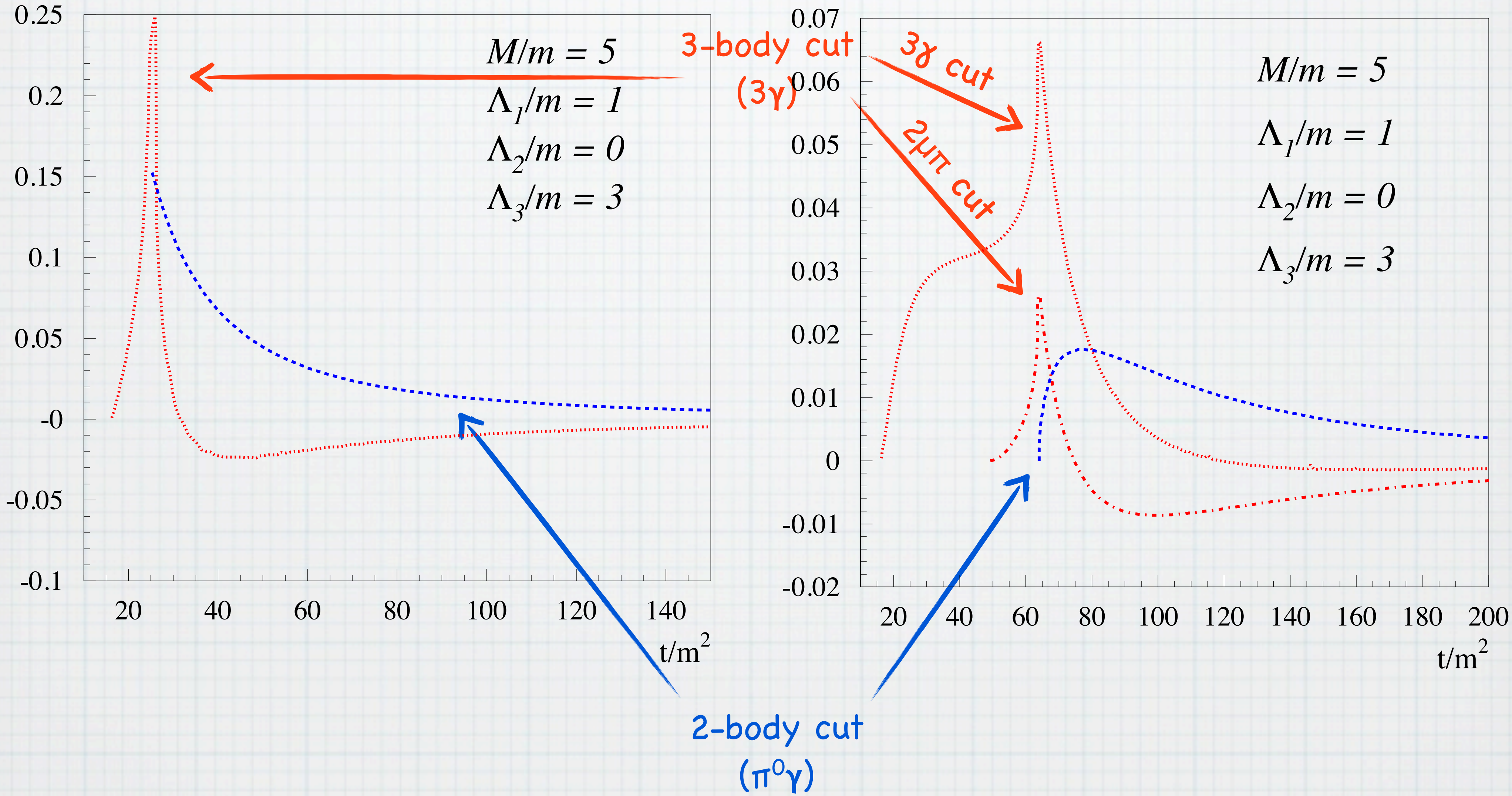
Discontinuity: results



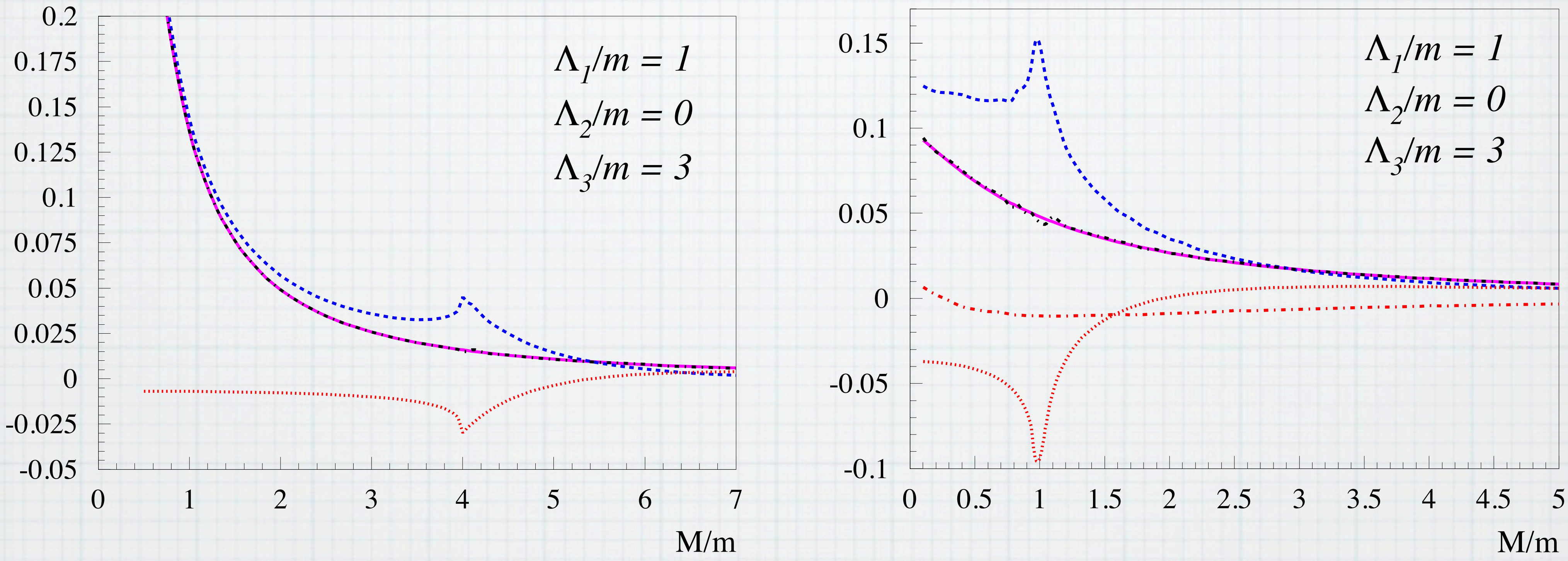
Discontinuity: results



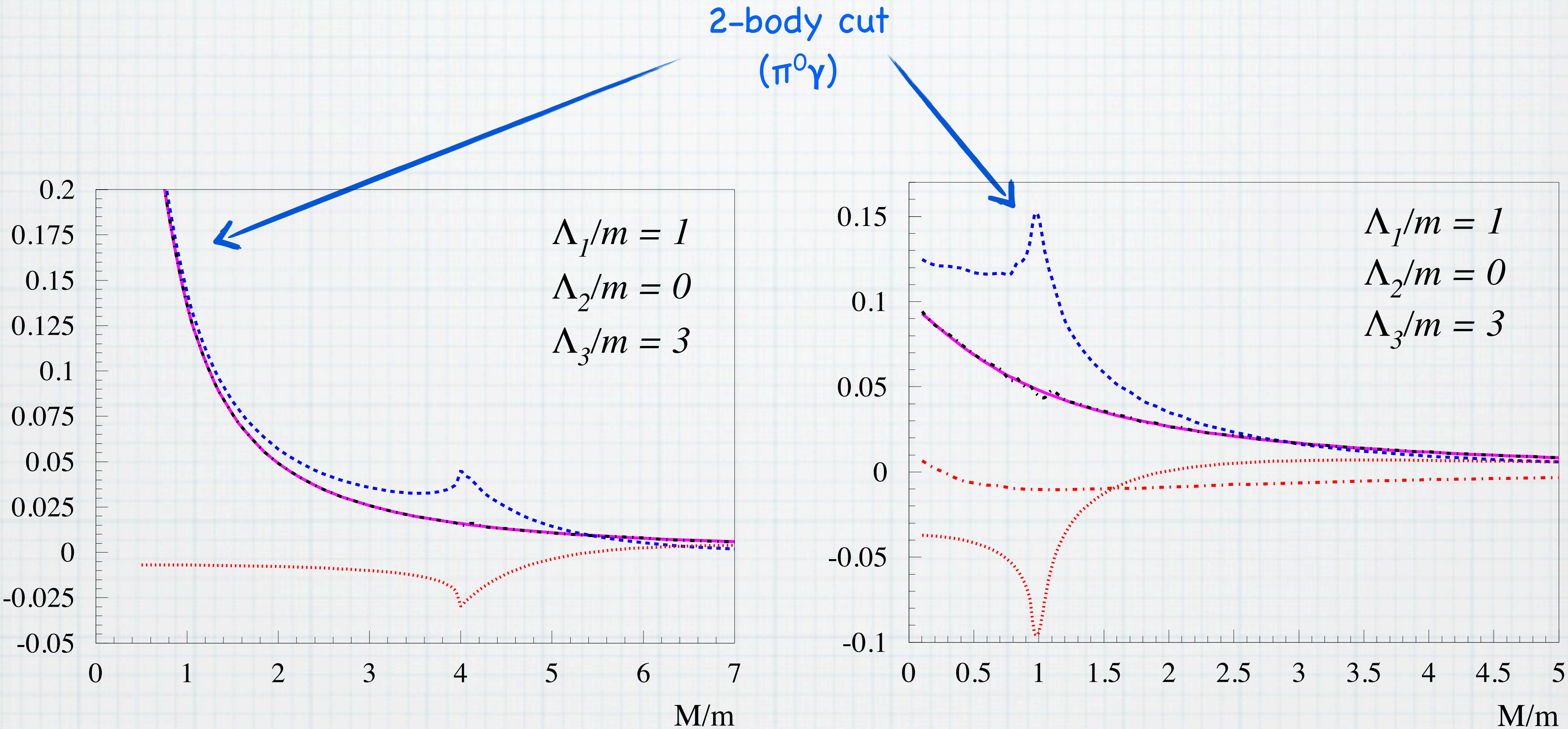
Discontinuity: results



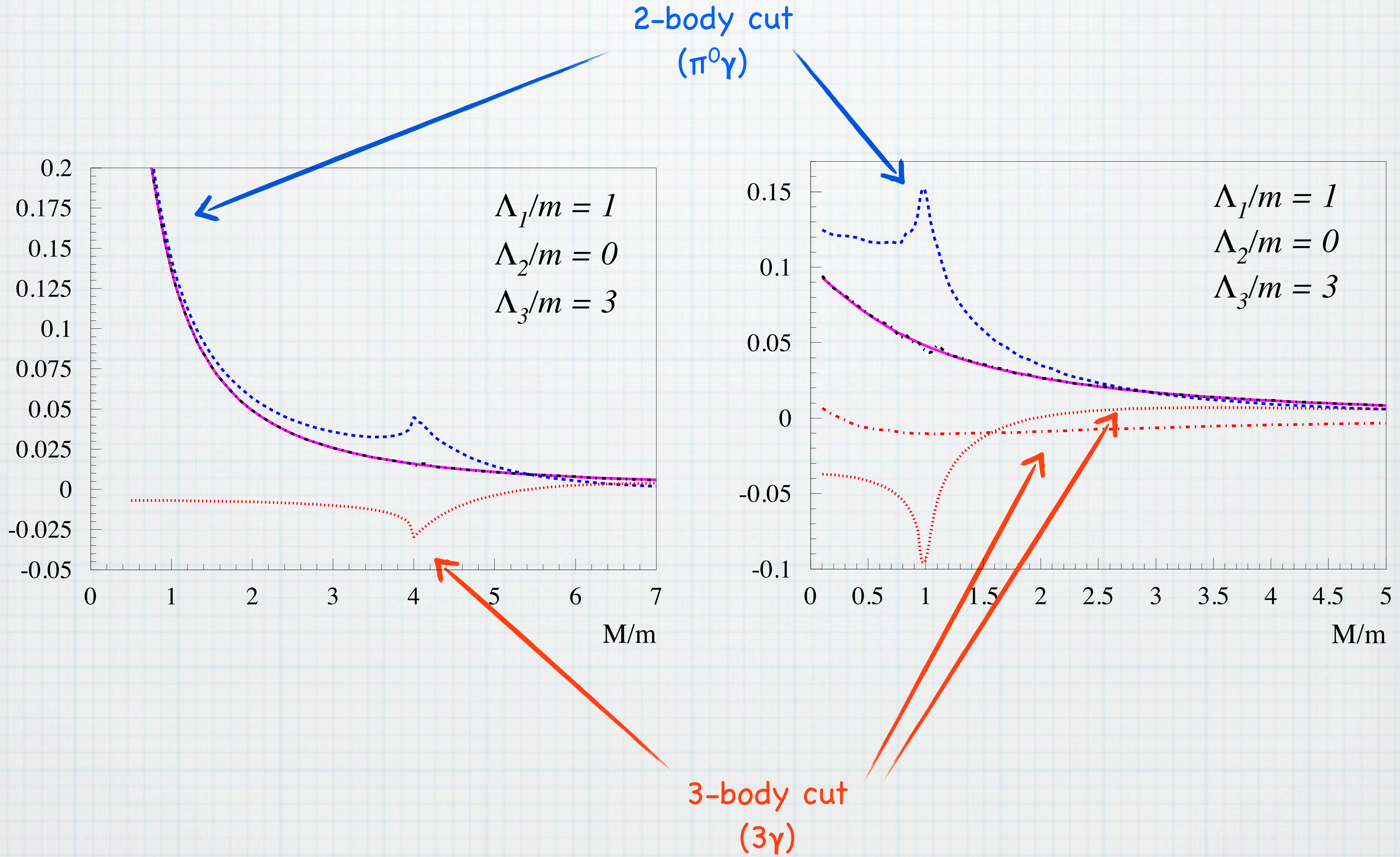
Real parts


0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
-0.025
-0.05
0 1 2 3 4 5 6 7
M/m
$\Lambda_1/m = 1$
$\Lambda_2/m = 0$
$\Lambda_3/m = 3$
0.15
0.1
0.05
0
-0.05
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
M/m
$\Lambda_1/m = 1$
$\Lambda_2/m = 0$
$\Lambda_3/m = 3$

Real parts

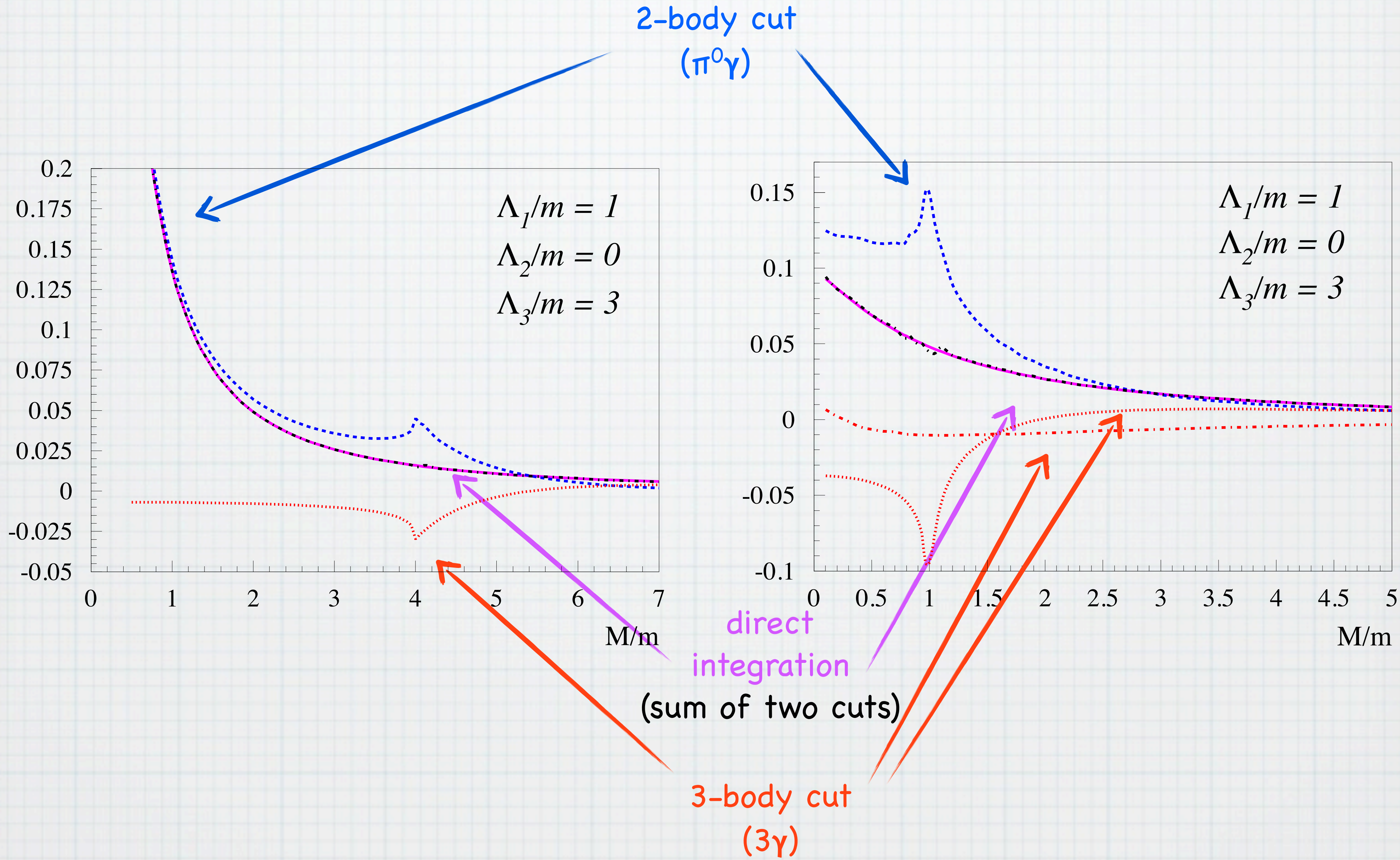

2-body cut
($\pi^0\gamma$)
$\Lambda_1/m = 1$
$\Lambda_2/m = 0$
$\Lambda_3/m = 3$
M/m
$\Lambda_1/m = 1$
$\Lambda_2/m = 0$
$\Lambda_3/m = 3$
M/m

Real parts

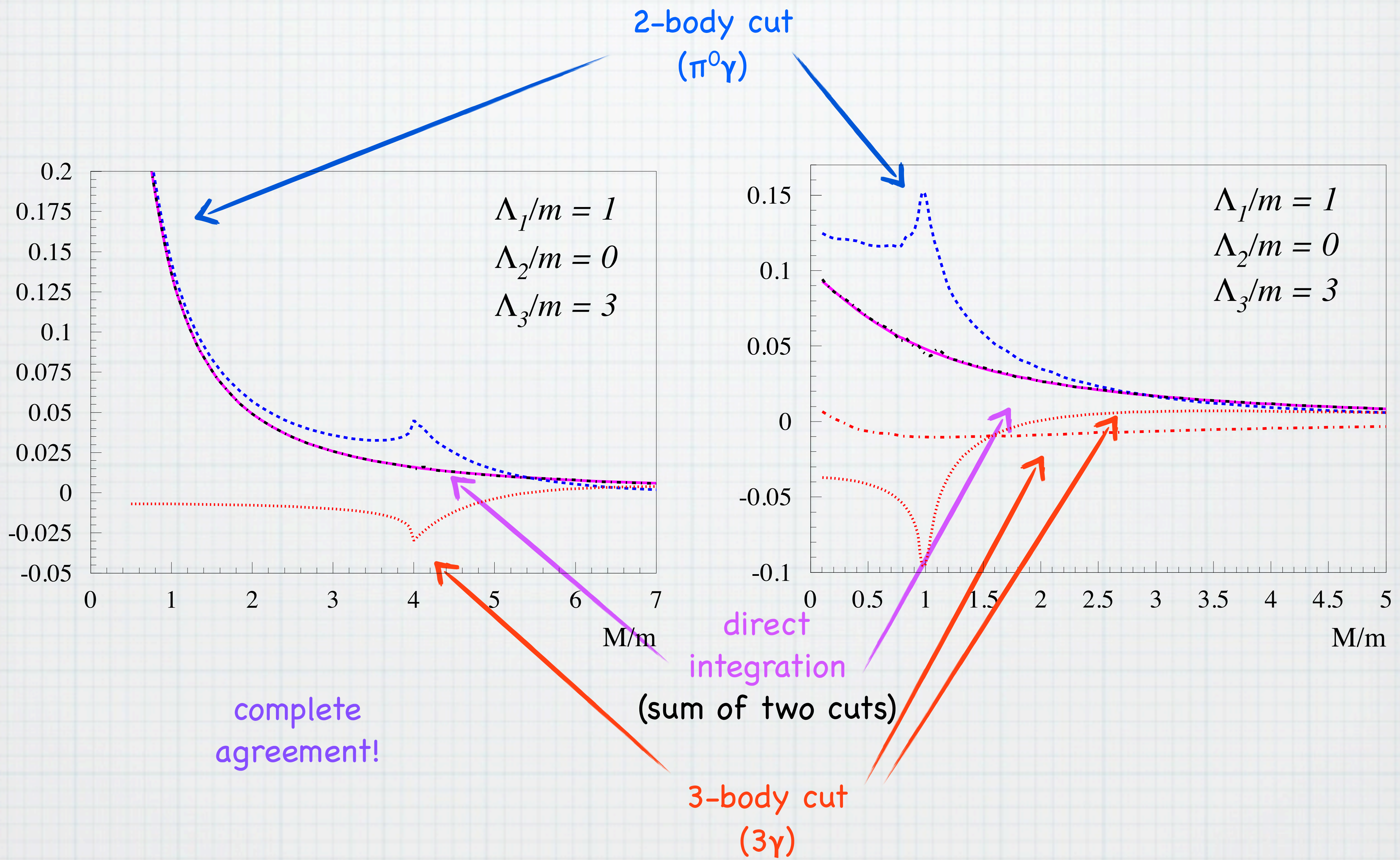


2-body cut
(π⁰γ)
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
-0.025
-0.05
0 1 2 3 4 5 6 7
M/m
Λ_1/m = 1
Λ_2/m = 0
Λ_3/m = 3
0.15
0.1
0.05
0
-0.05
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
M/m
3-body cut
(3γ)

Real parts


2-body cut
(π⁰γ)
Λ1/m = 1
Λ2/m = 0
Λ3/m = 3
M/m
direct
integration
(sum of two cuts)
3-body cut
(3γ)

Real parts



2-body cut
($\pi^0\gamma$)
$\Lambda_1/m = 1$
$\Lambda_2/m = 0$
$\Lambda_3/m = 3$
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
-0.025
-0.05
0 1 2 3 4 5 6 7
M/m
0.15
0.1
0.05
0
-0.05
-0.1
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
M/m
direct
integration
(sum of two cuts)
complete
agreement!
3-body cut
(3γ)

Meson pole contributions to $(g-2)_\mu$

in the dispersive approach

$(g-2)_\mu$: 2-particle cuts

$$\mathrm{Disc}_t^2 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = e^6 |F(0,0,M^2)|^2 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} (-2\pi i)^2 \delta\left((k-q_1)^2 - M^2\right) \delta\left(q_1^2 - \Lambda_1^2\right) \frac{S_3^{\Lambda_2\Lambda_3}}{(p+q_1)^2 - m^2}$$

(g-2)$_\mu$: 2-particle cuts

$$\mathrm{Disc}_t^2 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = -\frac{e^6|F(0,0,M^2)|^2}{8\pi}\beta_1 \int \mathrm{d}\cos\theta_1 \frac{N^{(1)}(q_1^2, m^2, t_1, t, \cos\theta_1)}{q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$

(g-2)$_\mu$: 2-particle cuts

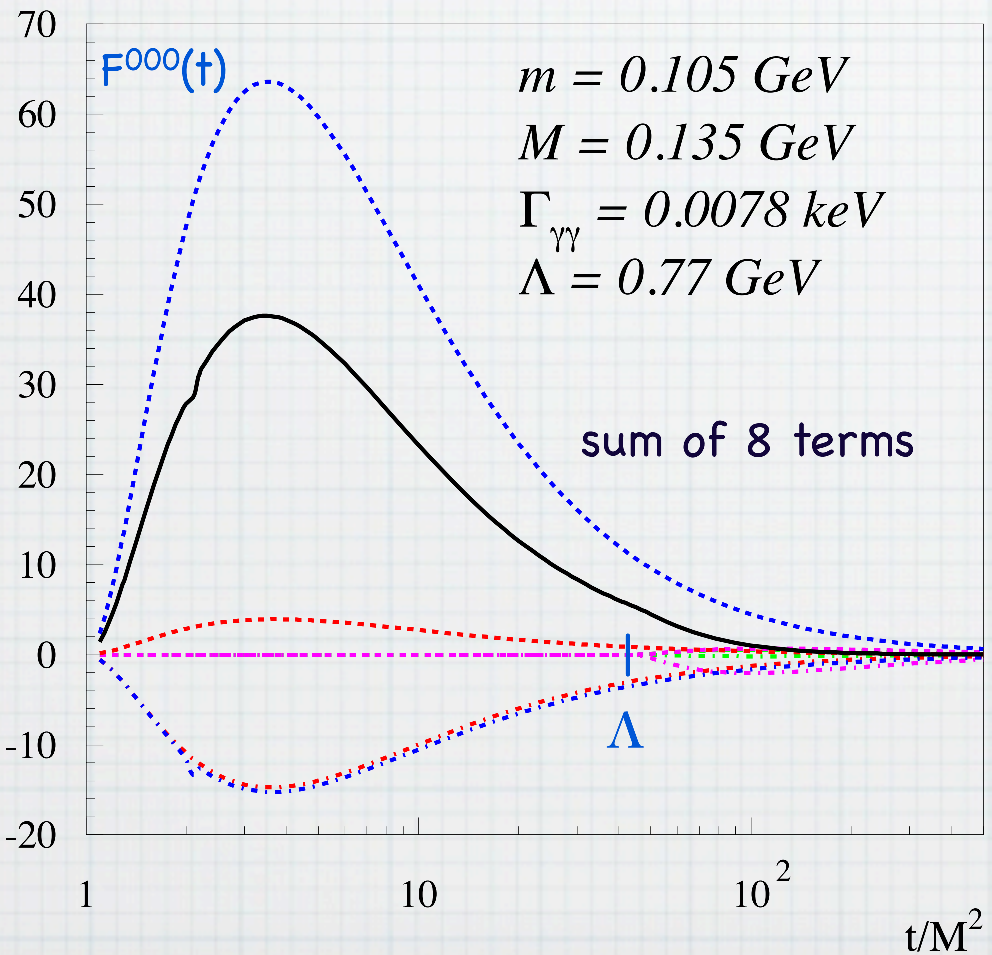
$$\mathrm{Disc}_t^2 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = -\frac{e^6|F(0,0,M^2)|^2}{8\pi}\beta_1 \int \mathrm{d}\cos\theta_1 \frac{N^{(1)}(q_1^2,m^2,t_1,t,\cos\theta_1)}{q_1^2+t_1-t-t\beta_1\beta\cos\theta_1}$$

$$\begin{aligned}
&N^{(1)}(q_1^2,m^2,t_1,t,\cos\theta_1) = \\
&M_3^{\Lambda_2\Lambda_3}(M^2,(q_1^2+t_1-t-t\beta_1\beta\cos\theta_1)/2,m^2) \\
&\times N_1(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&+M_2^{\Lambda_2\Lambda_3}(0)N_2(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&\quad + M_2^{\Lambda_2 m}(0)N_3(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&+M_2^{\Lambda_3 m}(0)N_4(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&\quad + M_2^{\Lambda_2 m}(m^2)N_5(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&+M_2^{\Lambda_2\Lambda_3}(M^2)N_6(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&+M_2^{\Lambda_3 m}(q_1^2+t_1-t-t\beta_1\beta\cos\theta_1)/2) \\
&\times N_7(q_1^2,m^2,t_1,t,\cos\theta_1) \\
&+N_8(q_1^2,m^2,t_1,t,\cos\theta_1)
\end{aligned}$$

(g-2)$_\mu$: 2-particle cuts

$$\mathrm{Disc}_t^2 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = -\frac{e^6|F(0,0,M^2)|^2}{8\pi}\beta_1 \int \mathrm{d}\cos\theta_1 \frac{N^{(1)}(q_1^2, m^2, t_1, t, \cos\theta_1)}{q_1^2+t_1-t-t\beta_1\beta\cos\theta_1}$$

Im $F_2(t)/t$ (in 10^{-10} GeV^{-2}): πγ cut, diagram a



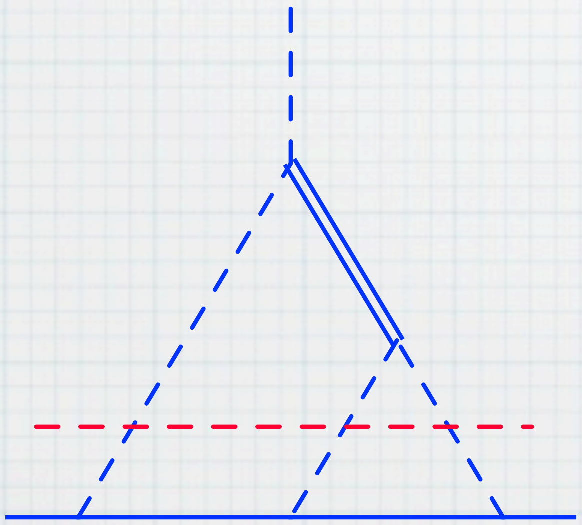
$$\begin{aligned}
&N^{(1)}(q_1^2, m^2, t_1, t, \cos\theta_1) = \\
&M_3^{\Lambda_2\Lambda_3}(M^2, (q_1^2+t_1-t-t\beta_1\beta\cos\theta_1)/2, m^2) \\
&\times N_1(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_2\Lambda_3}(0)N_2(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_2 m}(0)N_3(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_3 m}(0)N_4(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_2 m}(m^2)N_5(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_2\Lambda_3}(M^2)N_6(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+M_2^{\Lambda_3 m}(q_1^2+t_1-t-t\beta_1\beta\cos\theta_1)/2) \\
&\times N_7(q_1^2, m^2, t_1, t, \cos\theta_1) \\
&+N_8(q_1^2, m^2, t_1, t, \cos\theta_1)
\end{aligned}$$

3-particle cuts

$$\mathrm{Disc}_t^3 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = e^6 |F(0,0,M^2)|^2 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4}$$
$$\times (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2)$$
$$\times \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$
$$\times L(q_1, q_2, k, p)\Pi(q_1, q_2, k - q_1 - q_2, k)$$

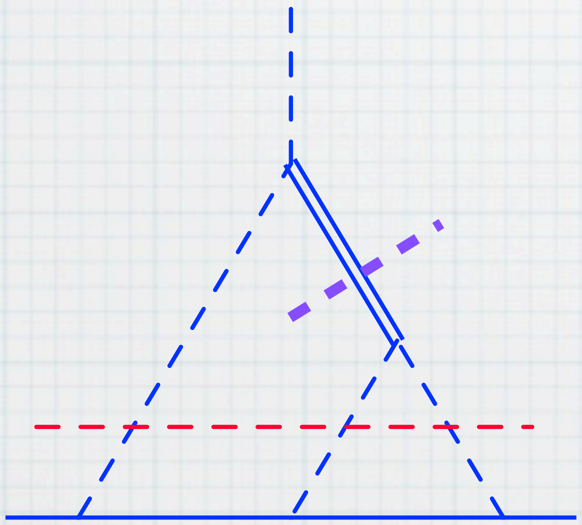
3-particle cuts

$$\mathrm{Disc}_t^3 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = e^6 |F(0,0,M^2)|^2 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4}$$
$$\times (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2)$$
$$\times \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$
$$\times L(q_1, q_2, k, p)\Pi(q_1, q_2, k - q_1 - q_2, k)$$



3-particle cuts

$$\mathrm{Disc}_t^3 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = e^6|F(0,0,M^2)|^2 \int \frac{\mathrm{d}^4 q_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 q_2}{(2\pi)^4}$$
$$\times (-2\pi i)^3 \delta(q_1^2 - \Lambda_1^2)\delta(q_2^2 - \Lambda_2^2)\delta((k - q_1 - q_2)^2 - \Lambda_3^2)$$
$$\times \frac{1}{(p+q_1)^2 - m_1^2} \frac{1}{(p+k-q_2)^2 - m_2^2}$$
$$\times L(q_1, q_2, k, p)\Pi(q_1, q_2, k - q_1 - q_2, k)$$

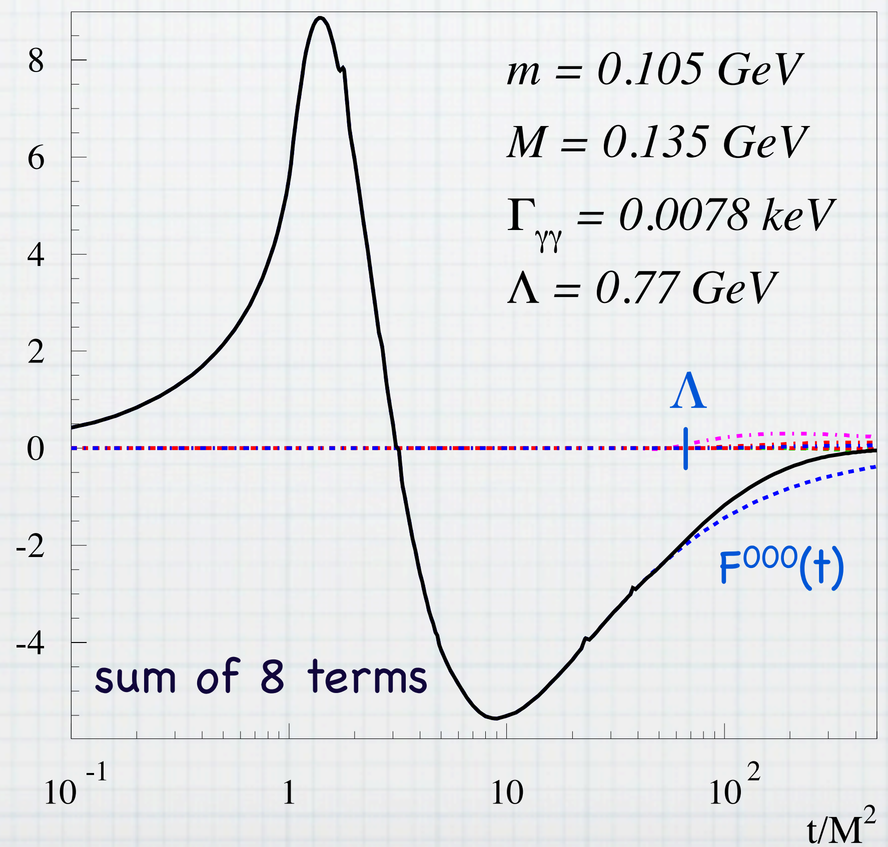


$$\Pi(q_1, q_2, k - q_1 - q_2, k) = \frac{P(q_1, q_2, k - q_1 - q_2, k)|_{(q_1-k)^2 = M^2}}{(q_1 - k)^2 - M^2}$$

3-particle cuts

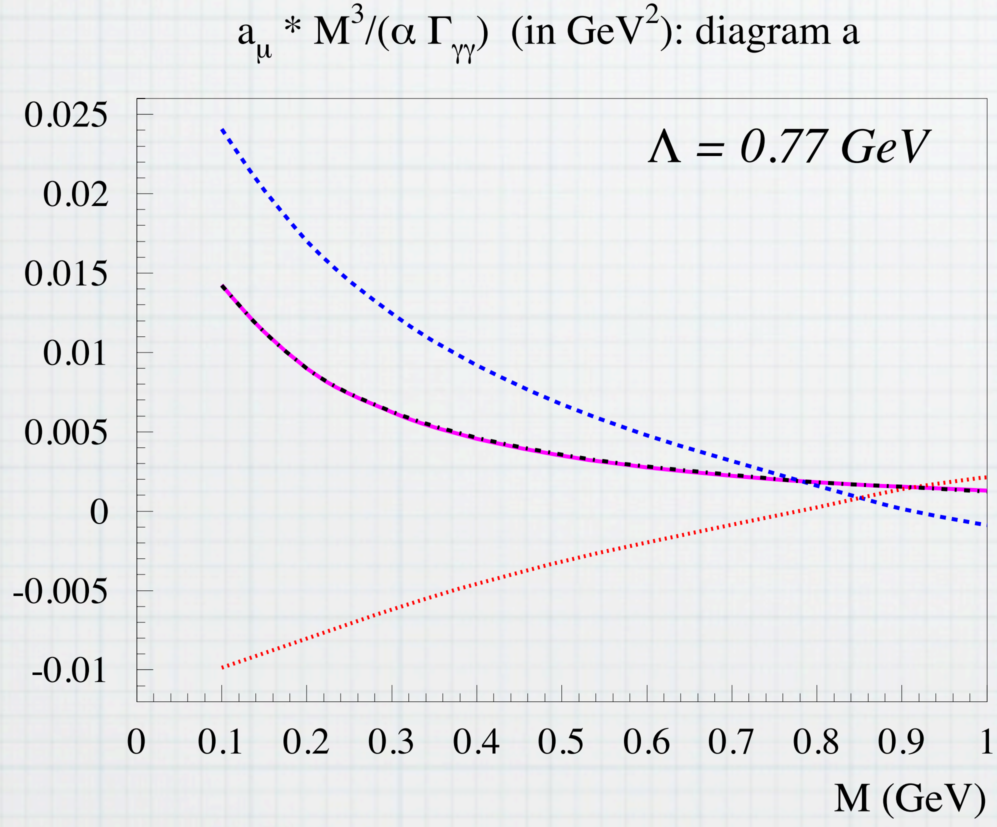
$$\mathrm{Disc}_t^3 F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) = \frac{ie^6|F(0,0,M^2)|^2}{(2\pi)^4 32t} \int \mathrm{d}t_1 \int \mathrm{d}t_2 \frac{1}{t_1 - M^2}$$
$$\times \int_0^\pi \mathrm{d}\cos\theta_1 \int_0^{2\pi} \mathrm{d}\theta_2 \; \frac{2}{2m^2 - 2m_1^2 + q_1^2 + t_1 - t - t\beta_1\beta\cos\theta_1}$$
$$\times \frac{2}{2m^2 - 2m_2^2 + q_2^2 - t + t_2 + t\beta_2\beta(\sin\theta_1\cos\theta_2\sin\theta + \cos\theta_1\cos\theta)} L(t_1,\ldots)P(M^2,\ldots)$$

Im $F_2(t)/t$ (in 10^{-10} GeV^{-2}): 3γ cut, diagram a



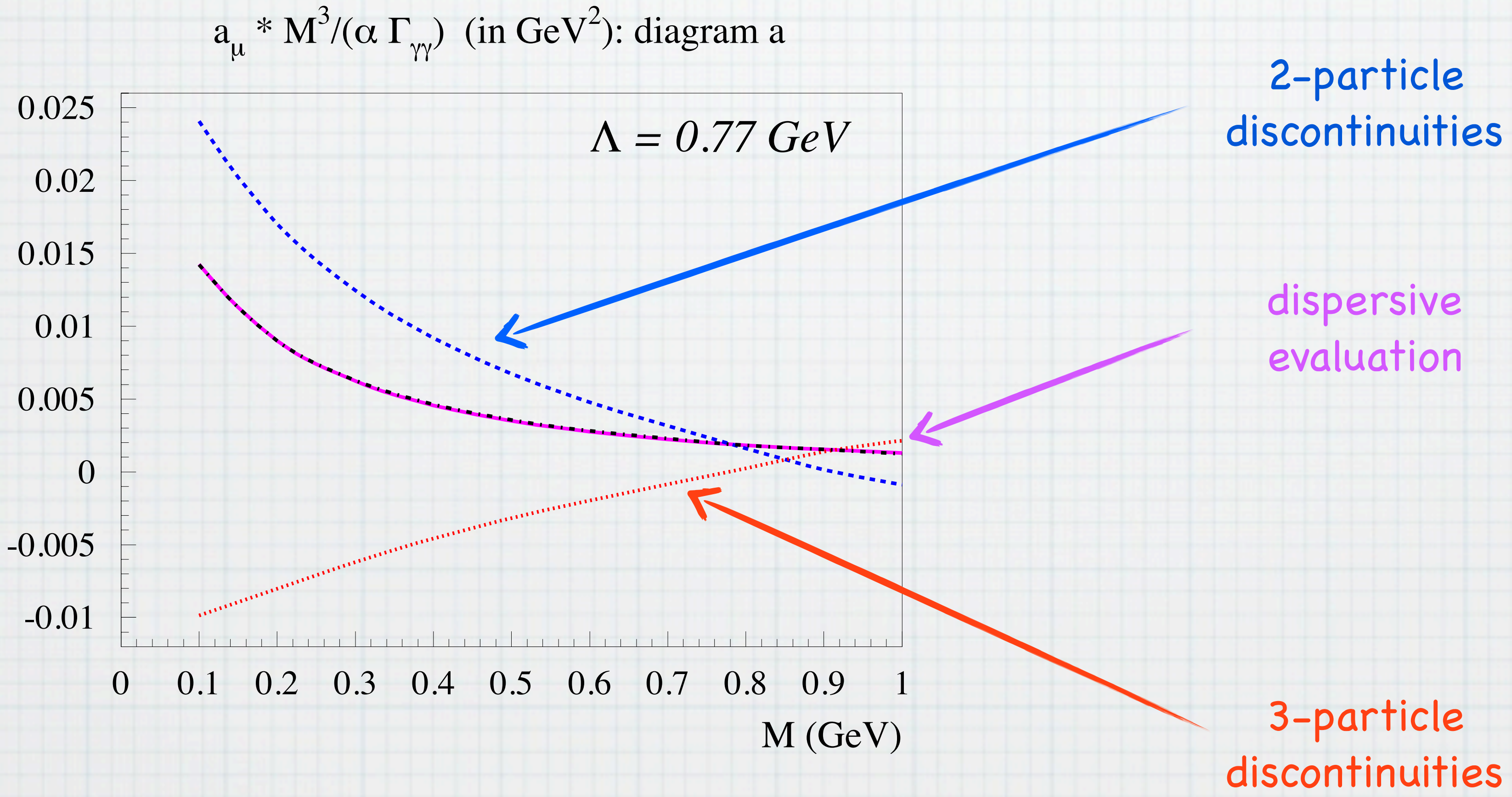
Real parts

$$F_1^{\Lambda_1\Lambda_2\Lambda_3}(0) = \frac{1}{2\pi i}\int_{(\Lambda_1+M)^2}^{\infty} \frac{dt}{t}\,\mathrm{Disc}_t^{(2)} F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) + \frac{1}{2\pi i}\int_{(\Lambda_1+\Lambda_2+\Lambda_3)^2}^{\infty} \frac{dt}{t}\,\mathrm{Disc}_t^{(3)} F_1^{\Lambda_1\Lambda_2\Lambda_3}(t)$$



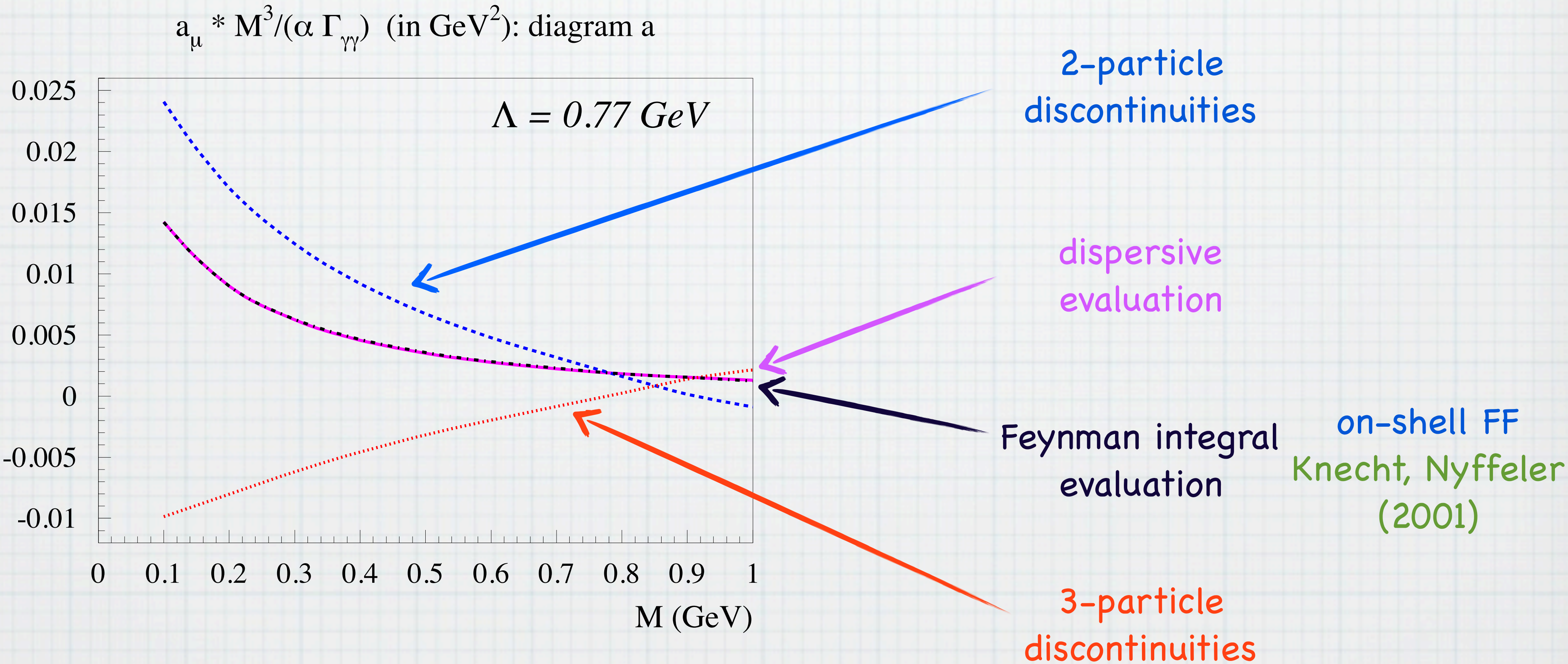
Real parts

$$F_1^{\Lambda_1\Lambda_2\Lambda_3}(0) = \frac{1}{2\pi i}\int_{(\Lambda_1+M)^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(2)}F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) + \frac{1}{2\pi i}\int_{(\Lambda_1+\Lambda_2+\Lambda_3)^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(3)}F_1^{\Lambda_1\Lambda_2\Lambda_3}(t)$$

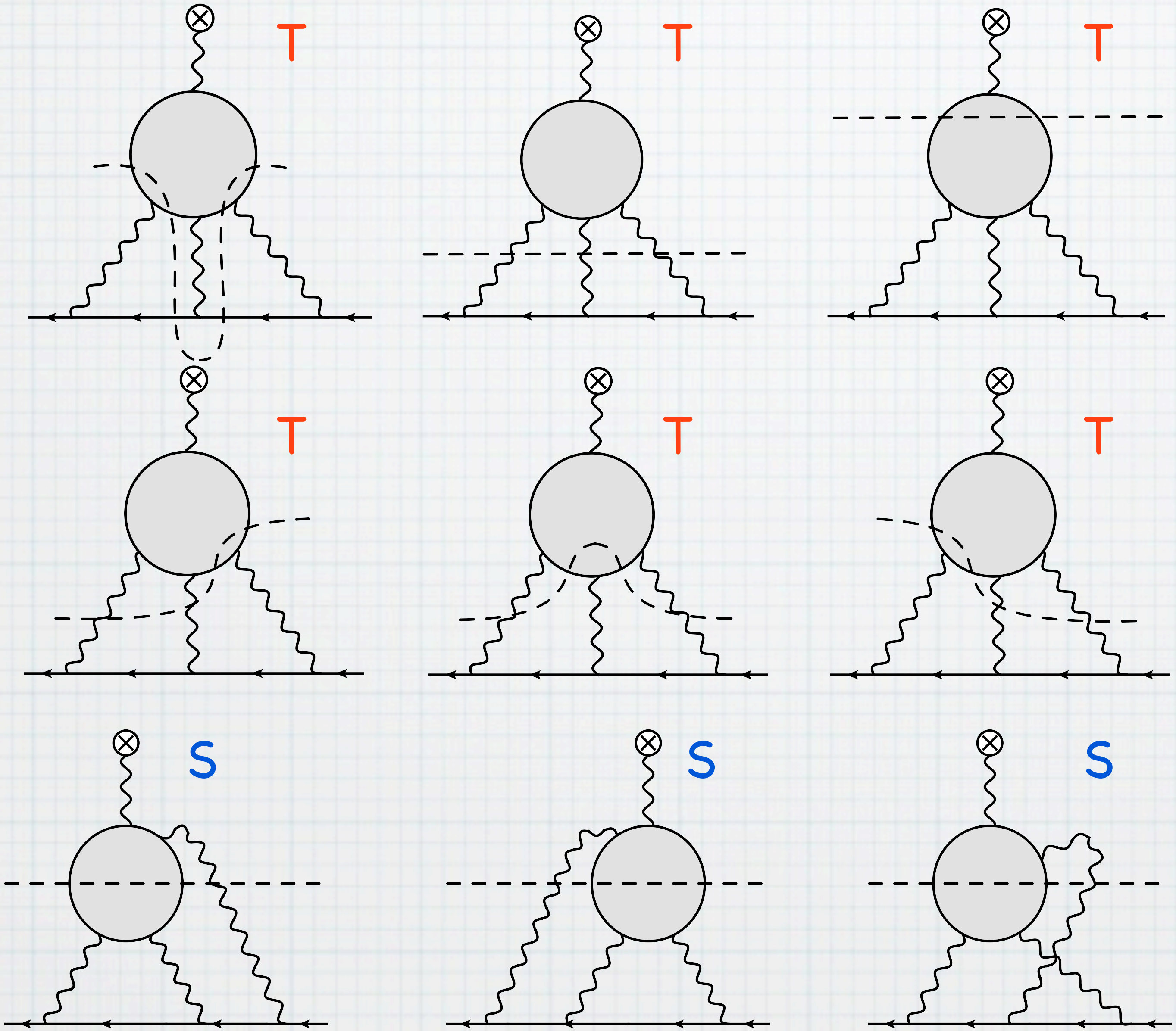


Real parts

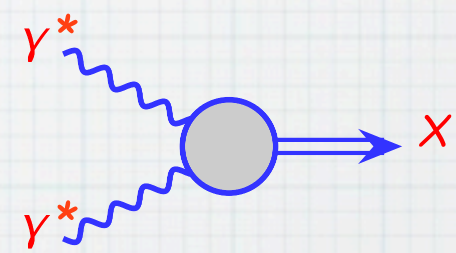
$$F_1^{\Lambda_1\Lambda_2\Lambda_3}(0) = \frac{1}{2\pi i}\int_{(\Lambda_1+M)^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(2)} F_1^{\Lambda_1\Lambda_2\Lambda_3}(t) + \frac{1}{2\pi i}\int_{(\Lambda_1+\Lambda_2+\Lambda_3)^2}^{\infty}\frac{dt}{t}\,\mathrm{Disc}_t^{(3)} F_1^{\Lambda_1\Lambda_2\Lambda_3}(t)$$



LbL discontinuity



T - time-like information



S - time-like information

Conclusions

Feynman integrals
(old approach)

Dispersive evaluation
(new approach)

Conclusions

Feynman integrals
(old approach)

static external field:
limit $k \to 0$

Dispersive evaluation
(new approach)

static external field:
evaluation of the dispersion integral at $k^2=0$

Conclusions

Feynman integrals (old approach) | Dispersive evaluation (new approach)

static external field:
limit $k \to 0$

evaluation of the loop integrals via
Wick rotation:
Euclidean correlation functions
→ space-like data

static external field:
evaluation of the dispersion integral at $k^2=0$

phase-space and dispersive integrals:
different options → space-, time-like and
on-shell data)

Conclusions

Feynman integrals (old approach)	Dispersive evaluation (new approach)
static external field: limit $k \to 0$	static external field: evaluation of the dispersion integral at $k^2=0$
evaluation of the loop integrals via Wick rotation: Euclidean correlation functions → space-like data	phase-space and dispersive integrals: different options → space-, time-like and on-shell data)
limited range of parametrizations for the non-PT part	factorization and analytical evaluation of the angular part of the phase space integral

Conclusions

Feynman integrals (old approach)	Dispersive evaluation (new approach)
static external field: limit $k \to 0$	static external field: evaluation of the dispersion integral at $k^2=0$
evaluation of the loop integrals via Wick rotation: Euclidean correlation functions → space-like data	phase-space and dispersive integrals: different options → space-, time-like and on-shell data)
limited range of parametrizations for the non-PT part	factorization and analytical evaluation of the angular part of the phase space integral
off-shell correlation functions; dispersive representation possible? (see Colangelo's talk)	direct expression through the physical phase-space integrals