

Dispersion theory to connect

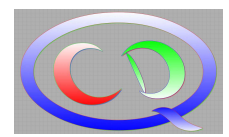
$$\eta \rightarrow \pi\pi\gamma \text{ to } \eta \rightarrow \gamma^*\gamma$$

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Standard treatment: **sum of Breit-Wigners**

Propagator: $iG_k(s) = \overline{\text{---}}_k = i/(s - M_k^2 + iM_k\Gamma_k)$

Scattering:

$$\sum_k = \sum_k ig_k^2 G_k(s)$$

Production:

$$\sum_k \otimes = \left(\sum_k ig_k G_k(s) \alpha_k \right) + i\beta$$

Problems:

- Wrong threshold behavior (cured by $\Gamma = \Gamma(s)$)
- Violates unitarity → **wrong phase motion**
- Parameters reaction dependent
only pole positions and residues universal!

single channel:

$$\text{Disc}(F)/2i = \text{Im}(F) = e^{-i\delta(s)} \sin(\delta(s)) F(s)$$

→ **Watson theorem:** $F(s) = |F(s)|e^{i\delta(s)}$ and **Omnès function**

$$\Omega(s) = \exp \left(\frac{s}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s' - s - i\epsilon)} \right)$$

such that, for **negligible left-hand cuts**

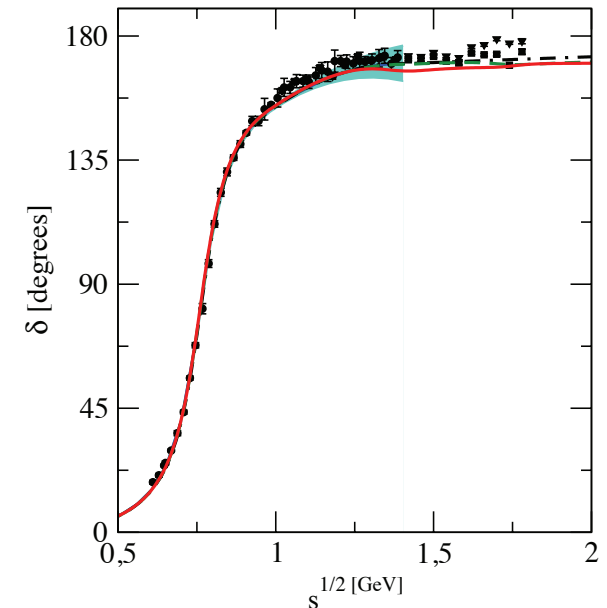
$$F(s) = P(s)\Omega(s)$$

→ $\Omega(s)$ is **universal and fixed** in elastic regime

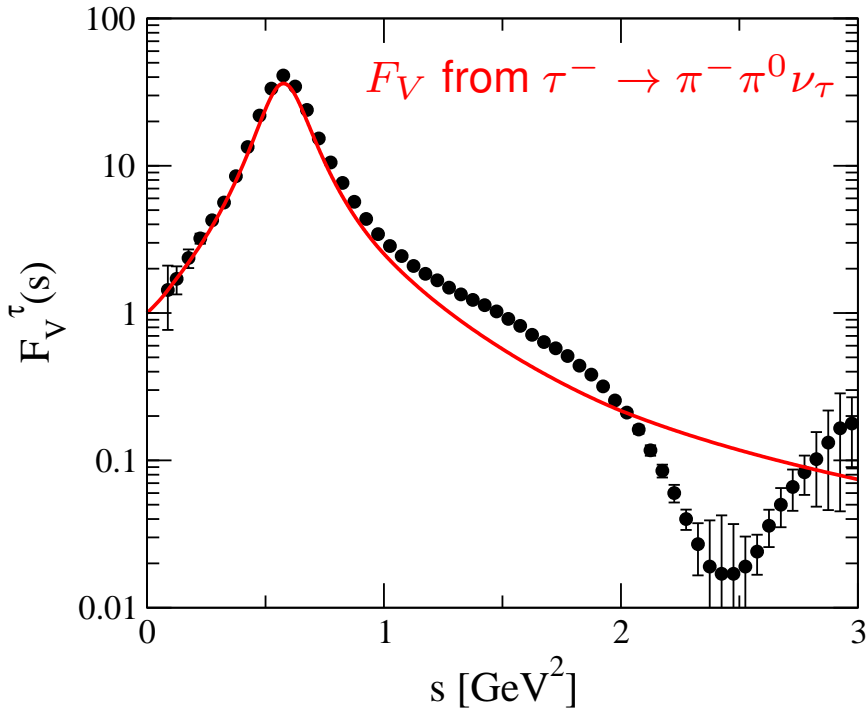
→ $P(s)$ **reaction specific** and contains e.g.

▷ higher thresholds

▷ inelastic resonances

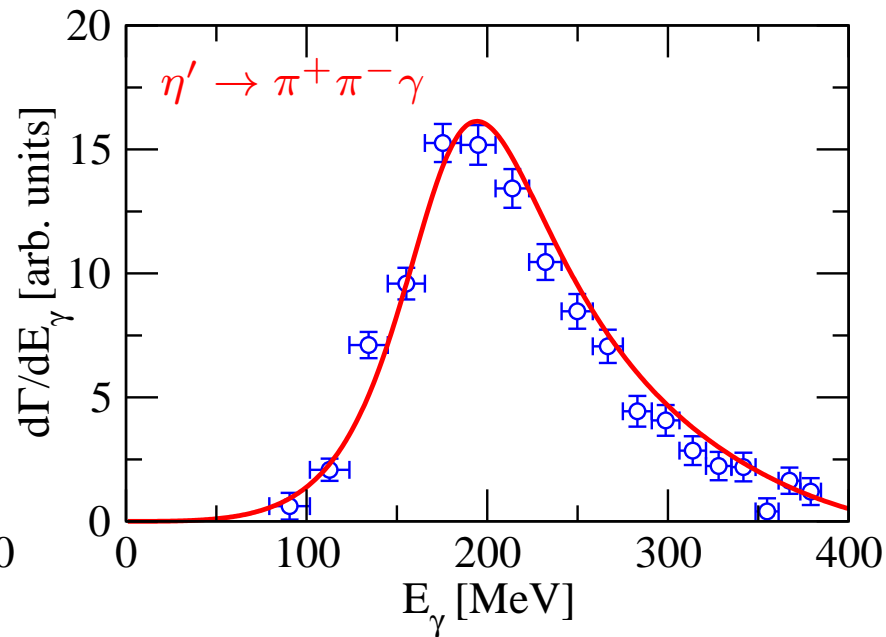
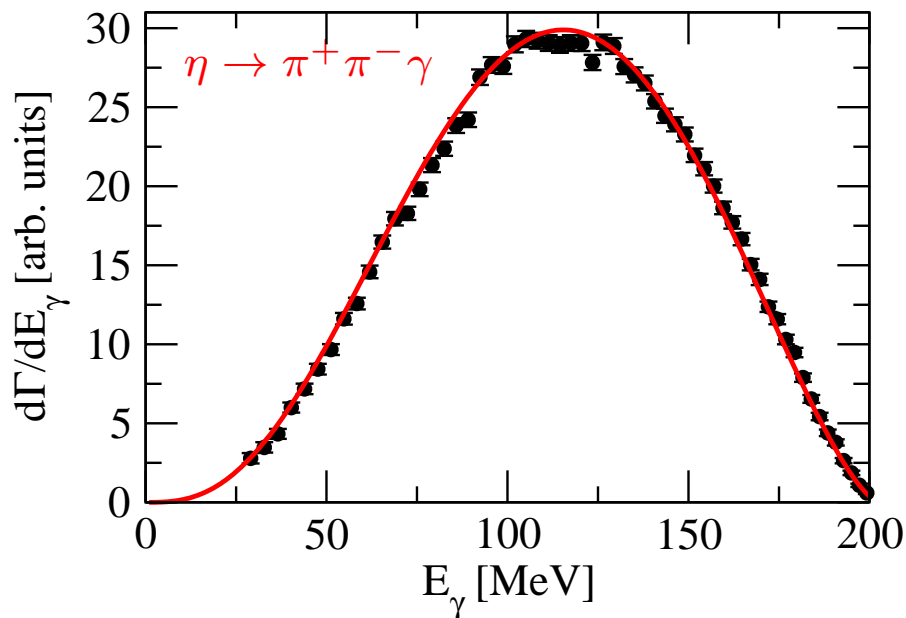


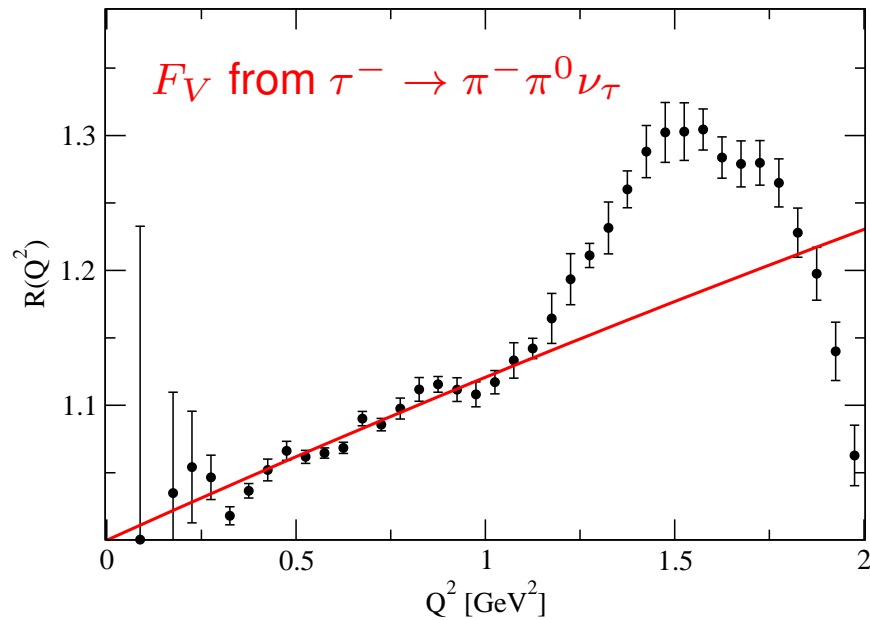
Universality of FSI



red lines: p-wave Omnes
 × kinematic factors

- bulk described properly
- there are deviations
 → $P(s)$ not constant!

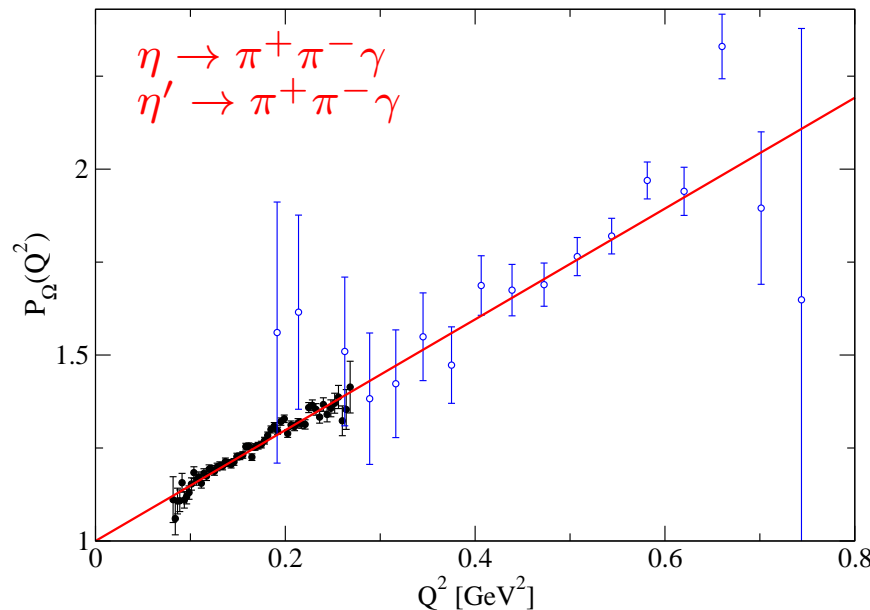




We had: $F(Q^2) = P(Q^2)\Omega(Q^2)$

We find for all 3 cases

- $P(Q^2)$ linear for $Q^2 < 1 \text{ GeV}^2$
- deviations in F_V by ρ' & ρ''



$$P(Q^2) = A_0(1 + \alpha Q^2)$$

→ α reaction specific

→ $\alpha[\eta] = \alpha[\eta']$ understood

1-loop ChPT + large N_c

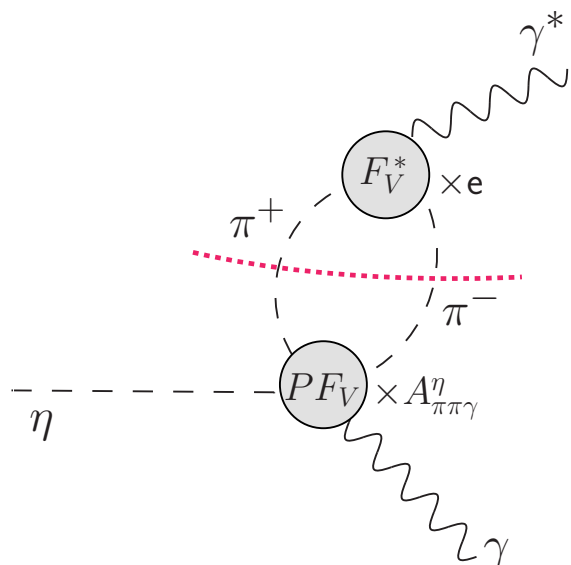
→ $\alpha = 1.3 \pm 0.1 \text{ GeV}^{-2}$ understood

box vs. triangle anomaly + large N_c

F. Stollenwerk et al., PLB 707, 184 (2012)

Going to $\eta \rightarrow \gamma\gamma^*$

Allows for parameter free prediction for **isovector** part of slope



$$F_{\eta\gamma^*\gamma}(Q^2, 0) \equiv 1 + \Delta F_{\eta\gamma^*\gamma}^{(I=1)}(Q^2, 0) + \Delta F_{\eta\gamma^*\gamma}^{(I=0)}(Q^2, 0)$$

$$\Delta F_{\eta\gamma^*\gamma}^{(I=1)} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2} \right) \int_{4m_\pi^2}^{\infty} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

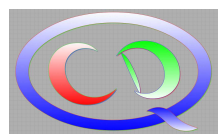
$$\Delta F_{\eta\gamma^*\gamma}^{(I=0)} \text{ needs to be modeled}$$

We thus demonstrated:

- $\eta \rightarrow \gamma\gamma^*$ and $\eta \rightarrow \pi^+\pi^-\gamma$ closely connected
- FSI ($\Omega(Q^2)$) universal; driven by ρ -pole and residues
- $P(Q^2)$ (reaction dependent!) also modifies $\eta \rightarrow \gamma^*\gamma$

C. H. et al. Eur.Phys.J. C73 (2013) 2668.

The isoscalar contribution



Modeled using VMD (okay, since resonances are narrow)

$$\Delta F_{\eta\gamma^*\gamma}^{(I=0)}(Q^2, 0) = \frac{w_{\eta\omega\gamma} Q^2}{m_\omega^2 - Q^2 - im_\omega\Gamma_\omega} + \frac{w_{\eta\phi\gamma} Q^2}{m_\phi^2 - Q^2 - im_\phi\Gamma_\phi}.$$

using 'data' (isoscalar contribution isolated in exp. analysis) via

$$\sigma(e^+e^- \rightarrow \eta\gamma) \Big|_{s=m_V^2}^{\text{isoscalar}} = \frac{12\pi \text{BR}(V \rightarrow \eta\gamma) \text{BR}(V \rightarrow e^+e^-)}{m_V^2},$$

we find

$$w_{\eta\omega\gamma} \approx (0.78 \pm 0.04) \times 1/8 \quad \text{and} \quad w_{\eta\phi\gamma} \approx (0.75 \pm 0.03) \times (-2/8)$$

where $1/8$ and $-2/8$ are the standard VMD values.

Isoscalar contribution small for η ($\sim 2\%$)

$$\Delta F_{\eta\gamma^*\gamma}^{(I=1)} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2} \right) \int_{4m_\pi^2}^{\infty} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

→ $P(s') = (1 + \alpha s')$ with

$\alpha = (1.32 \pm 0.13) \text{ GeV}^{-2}$ from KLOE

Babusci et al., PLB718(2013)910

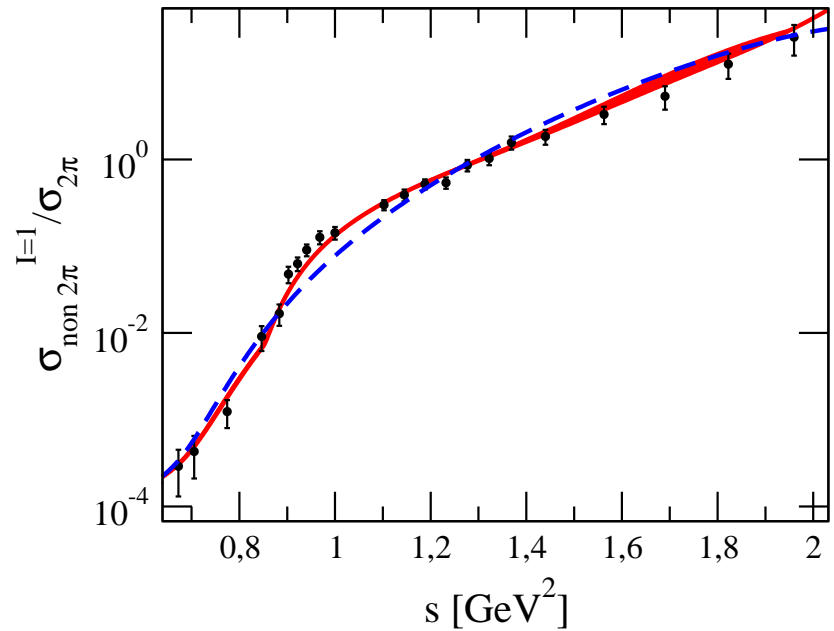
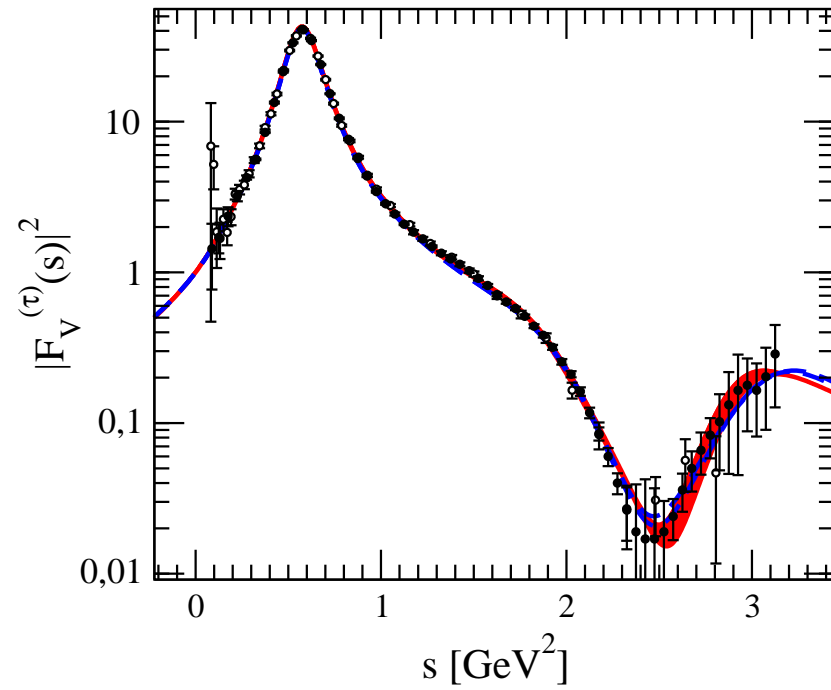
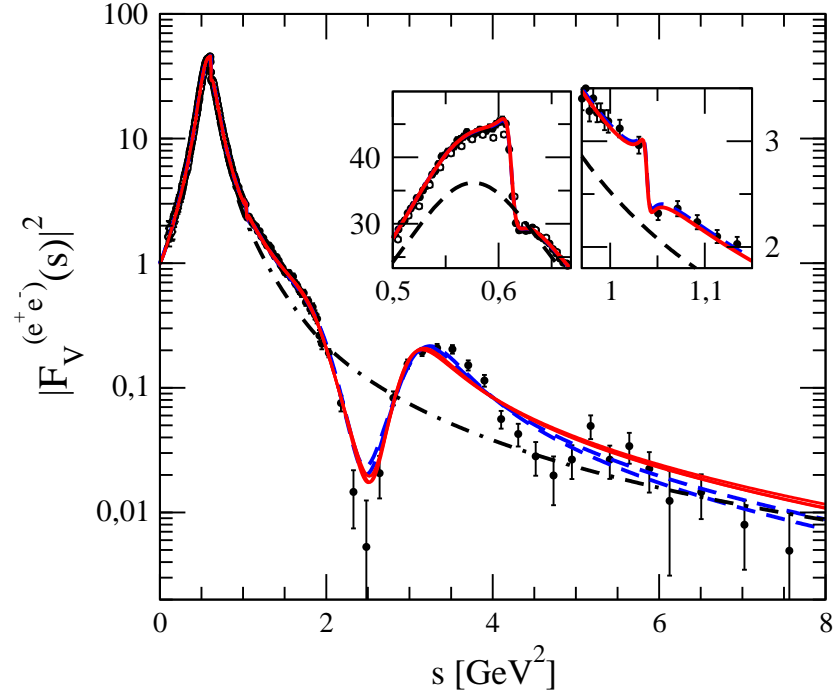
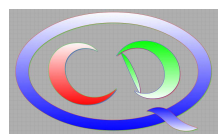
→ $\kappa_\eta \equiv e A_{\pi\pi\gamma}^\eta f_\pi^2 / A_{\gamma\gamma}^\eta$ from data

Review of Particle Properties (PDG)

→ $F_V(s')$ from recent parametrization

C.H., PLB715(2012)170

Extension to higher energies



Analytically improved K-matrix

C.H. PLB715(2012)170

- Including $\rho(770)$ via Omnes
- and ρ' and ρ'' via V_R
- unitary with 4π channels (via ρ' and ρ'')

$$\Delta F_{\eta\gamma^*\gamma}^{(I=1)} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2} \right) \int_{4m_\pi^2}^{\Lambda^2} ds' \sigma_\pi(s')^3 P(s') \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

→ Uncertainties in input (especially α)

→ **Isoscalar piece**: here we needed model (for phases)

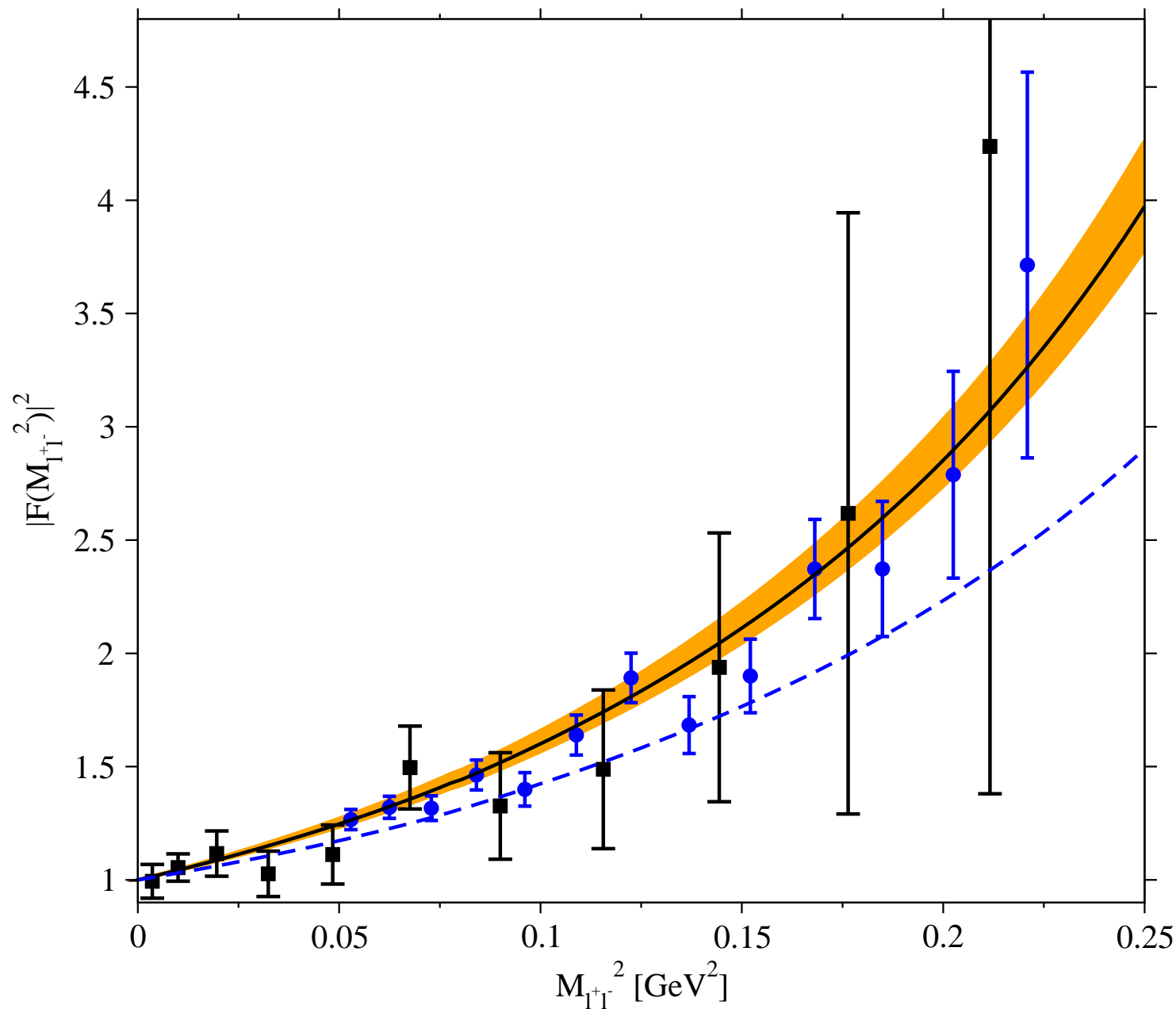
→ **Large s' behavior of $P(s')$ and $F_V(s')$:**

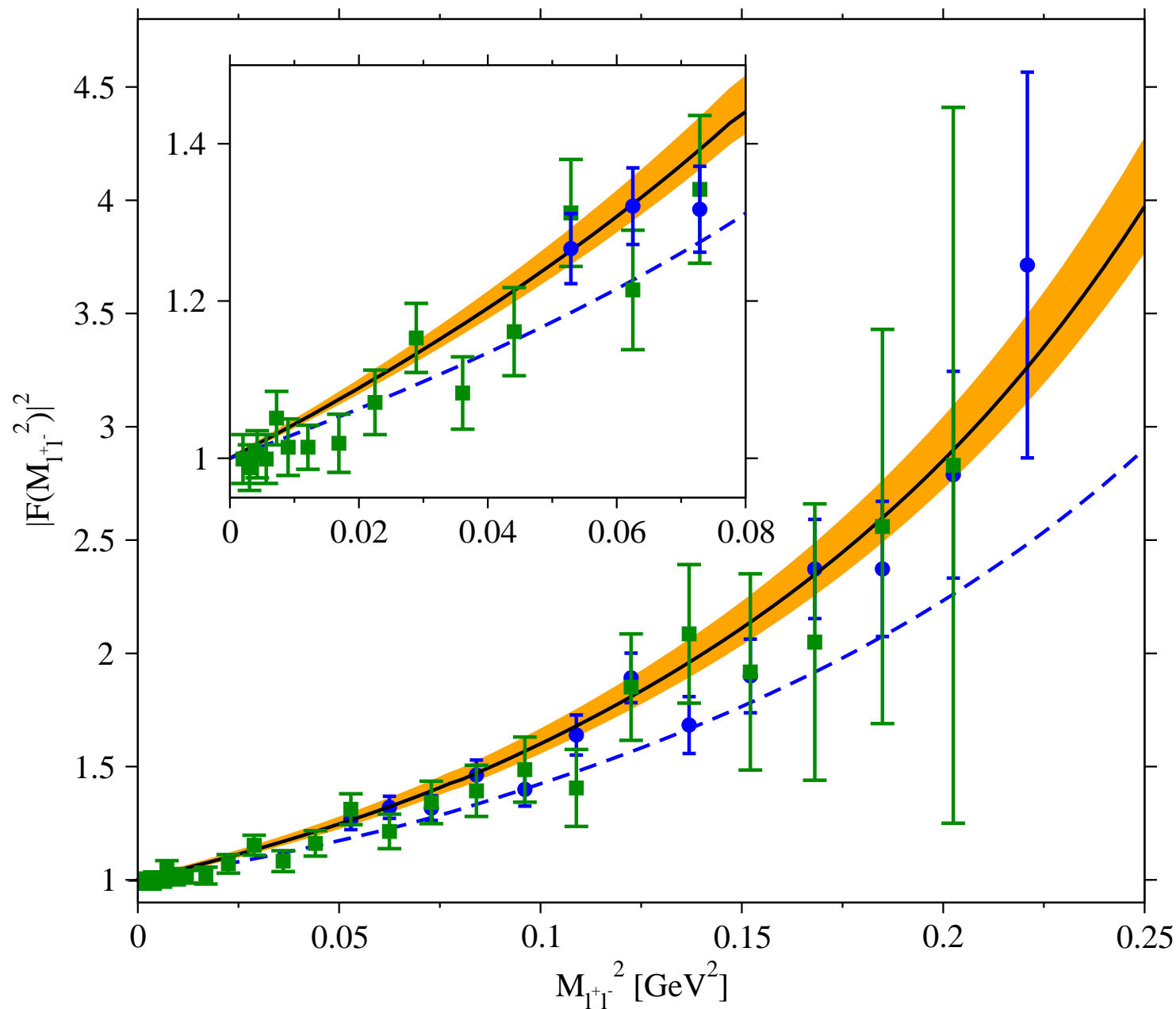
FSI in $e^+e^- \rightarrow \pi\pi$ and $\gamma\eta \rightarrow \pi\pi$ different in inelastic regime
(e.g. diff. couplings of ρ' and ρ'') → $\Lambda = m_{\eta'} \dots 2 \text{ GeV}$

check: $A_{\gamma\gamma}^{\eta(I=1)} = (eA_{\pi\pi\gamma}^\eta/96\pi^2) \int_{4m_\pi^2}^{\Lambda^2} ds' \sigma_\pi(s')^3 P(s') |F_V(s')|^2$

overestimates exp. by $(7 \pm 5)\%$ for $\Lambda = 1 \text{ GeV}$

S.P. Schneider et al., PRD86(2012)054013; M. Hoferichter et al., PRD86(2012)116009





Next steps

→ Proposal to experiment: measure $e^+e^- \rightarrow \eta\pi^+\pi^-$

The method outlined here allows one to relate this to the isovector piece of $\eta \rightarrow \gamma^*\gamma^*$

→ Modify ρ' and ρ'' couplings such that at the same time get

▷ $F_{\eta\pi\pi\gamma}(s) = P(s)F_V(s)$ for $s < 1 \text{ GeV}^2$

▷ $F_{\eta\pi\pi\gamma}(s) \propto 1/s$ for $s \rightarrow \infty$

with this we may use unsubtracted dispersion integral

$$A_{\gamma\gamma}^{\eta(I=1)}(s) = \left(\frac{eA_{\pi\pi\gamma}^{\eta}}{96\pi^2} \right) \sum_c \int_{4m_{\pi}^2}^{\infty} \frac{s' ds'}{s' - s} \sigma_{\pi}(s')^3 F_{\eta(c)\gamma}(s')^* F_V^{(c)}(s')$$

constraints non-2- π channels! Uncertainty estimate?

- $\eta \rightarrow \pi\pi\gamma$ and $\eta \rightarrow \gamma^*\gamma$ are characterized by **two scales**
 - ▷ A **universal** one ($\sim m_\rho$) from $\pi\pi$ -interaction
 - ▷ A **reaction specific** one
- The latter for $\eta \rightarrow \pi\pi\gamma$ and $\eta \rightarrow \gamma^*\gamma$ are **related but not equal**
- The method outlined guarantees
 - ▷ Correct **pole positions**
 - ▷ Correct **phase motion**

Precondition also for CP studies

Thanks a lot for your attention