Light cone sum rules calculations for heavy to light form factors

Challenges in semi-leptonic B decays Mainz

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Introduction to exclusive V$_{ub}$

- Uncertainty on $|V_{ub}|^{\mathbf{incl}} \sim 7\%$ ($< 2\%$ on $|V_{cb}|^{\mathbf{incl}}$) due to large $b \to c\ell\nu$ background
- Competitive $|V_{ub}|^{\mathbf{excl}}$ from $B \to \pi\ell\nu$, depends on $f_+(q^2)$ (as $m_l \to 0$) from Lattice QCD ($q^2 \gtrsim 15\,\mathrm{GeV}^2$) or QCD sum rules on the light-cone (LCSR) ($q^2 \lesssim 6-7\,\mathrm{GeV}^2$)
- Also possible via other B decays, e.g. recent progress in $B \to \rho\ell\nu$, $\Lambda_b \to p\ell\nu$, $B_s \to K\ell\nu$

Obtaining the form factor in Light-cone sum rules:

$$\Pi_\mu = i\, m_b \int d^D x e^{-i\, p_B \cdot x} \langle \pi(p)| T\{\bar{u}(0)\gamma_\mu b(0) \bar{b}(x) i\gamma_5 d(x)\}|0\rangle,$$
$$= (p_B + p)_\mu \Pi_+(p_B^2, q^2) + (p_B - p)_\mu \Pi_-(p_B^2, q^2).$$

into

$B \to \pi$ transition ($f_+(q^2)$)

$$\langle \pi(p)|\bar{u}\gamma_\mu b|B(p_B)\rangle = (p_B + p)_\mu f_+(q^2) + (p_B - p)_\mu f_-(q^2)$$

B meson decay (f_B)

$$m_b \langle 0|\bar{d} i\gamma_5 b|B\rangle = m_B^2 f_B$$

Leading to:

$$\Pi_+(p_B^2, q^2) = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \int_{s>m_B^2} ds \frac{\rho_{\rm had}}{s - p_B^2},$$

($\rho_{\rm had}$ is spectral density of the higher-mass hadronic states)

On the other hand:

Light-cone expand about $x^2 = 0 \;\; \Rightarrow$

$\Pi_+(p_B^2, q^2) = \sum_n \int du\, \mathcal{T}_+{}^{(n)}(u, p_B^2, q^2, \mu^2)\phi^{(n)}(u, \mu^2) = \int ds \frac{\rho_{\rm LC}}{s - p_B^2},$

$\mathcal{T}_+^{(n)}(u, \mu^2)$: perturbatively calculable hard kernels

$\phi^{(n)}(u, \mu^2)$: non-perturbative LCDAs at twist n

e.g. n=2, $\langle \pi(p)|\bar{u}(0)\gamma_\mu\gamma_5\, d(x)|0\rangle = -if_\pi p_\mu \int_0^1 du\, e^{i\bar{u}p\cdot x}\phi(u, \mu^2) + \ldots,$

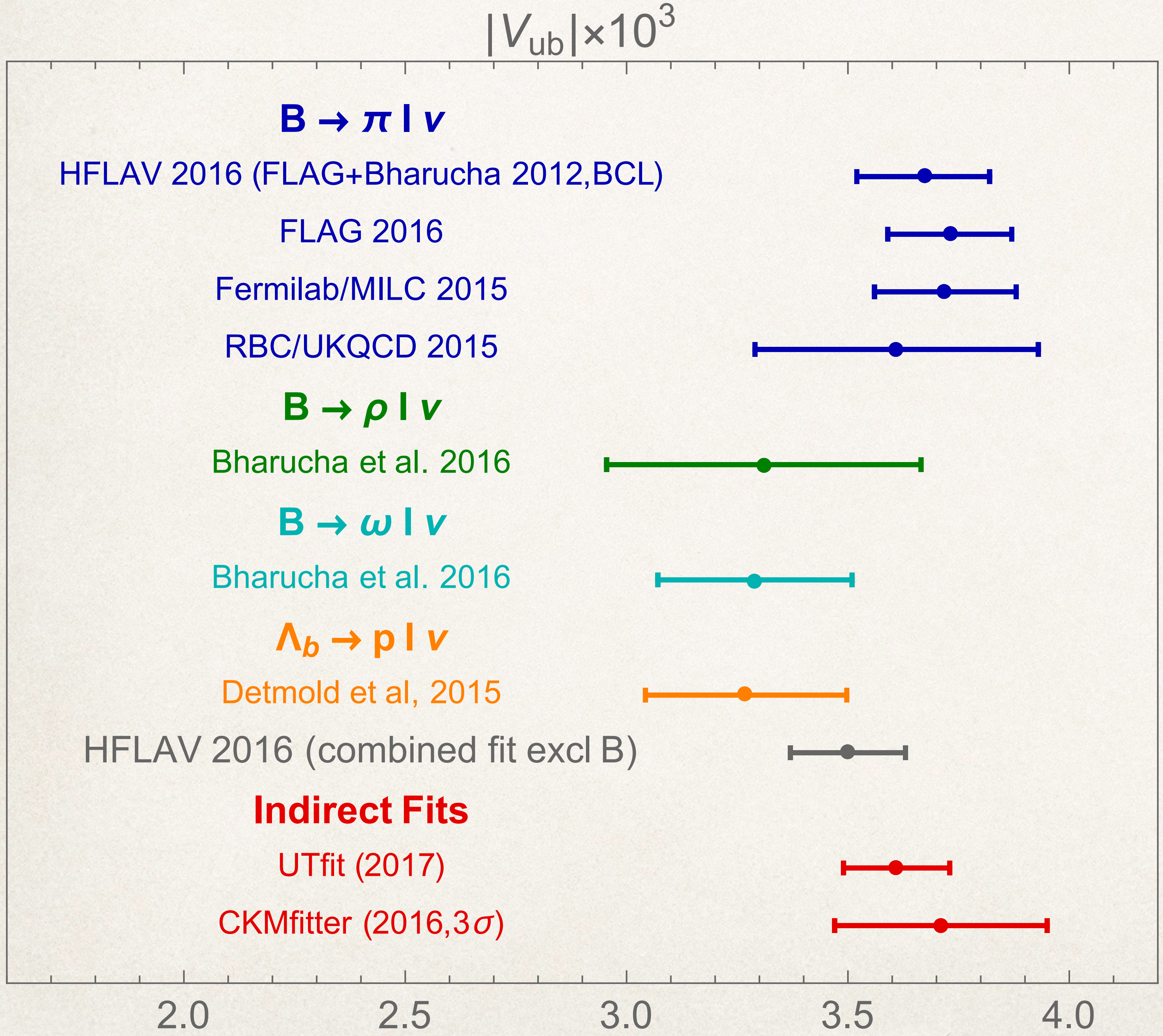
where $\phi(u, \mu^2) = 6u(1-u)\sum_{n=0}^{\infty} a_n(\mu^2) C_n^{3/2}(2u-1)$

Sum rule for $f_+(q^2)$: $f_+(q^2) = \frac{1}{f_B m_B^2} \int_{m_b^2}^{s_0} ds\, \rho_{\rm LC}\, e^{-(s-m_B^2)/M^2}$

Status of $f_+(q^2)$ for $B \to \pi$ < 2012

- 1997: NLO twist-2 corrections were calculated (A. Khodjamirian et al, [arXiv:hep-ph/9706303]; E. Bagan, P. Ball and V. M. Braun, [arXiv:hep-ph/9709243])
- 2000: LO corrections up to twist-4 were calculated (A. Khodjamirian et al, [arXiv:hep-ph/0001297])
- 2004: NLO twist-3 corrections (P. Ball and R. Zwicky, [arXiv:hep-ph/0406232])
- 2008: $\overline{\rm MS}$ m_b is used in place of the pole mass (G. Duplancic et al, 2008)
- 2011: Use a_2, a_4 from F_π, LCSR+new JLab, Extrapolate by fitting to BCL q^2 parameterisation (A. Khodjamirian, T. .Mannel, N. Offen, Y. -M. Wang, [arXiv:1103.2655])

Summary of exclusive V_{ub}



In this talk:

2012 NNLO calculation $B \to \pi$ (AB)

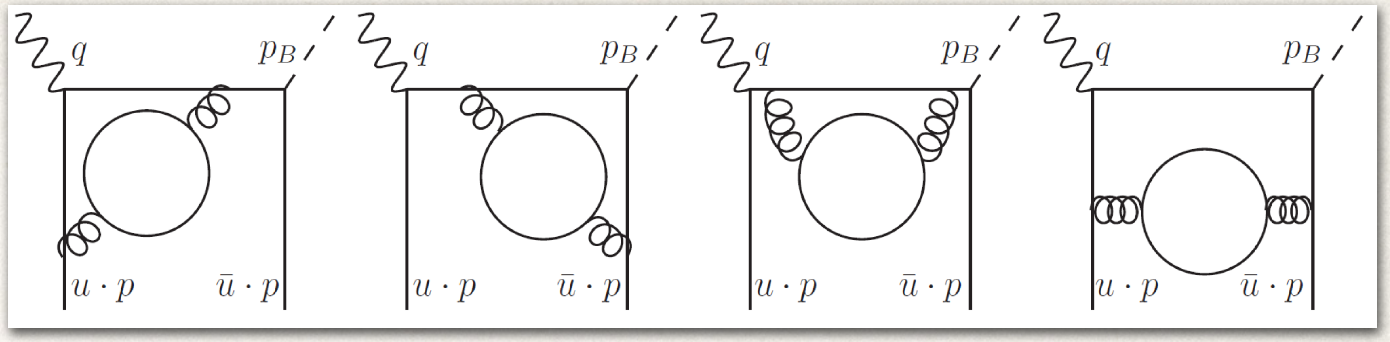
2014 Bayesian uncertainty analysis for the $B \to \pi$ form factor (Imsong, Khodjamirian, Mannel van Dyk)

2015 Update for B to V form factors (AB, Straub, Zwicky)

2017 Calculation of f_+ and f_T for $B_{(s)}$ to K form factors (Khodjamirian and Rusov)

Two-loop corrections

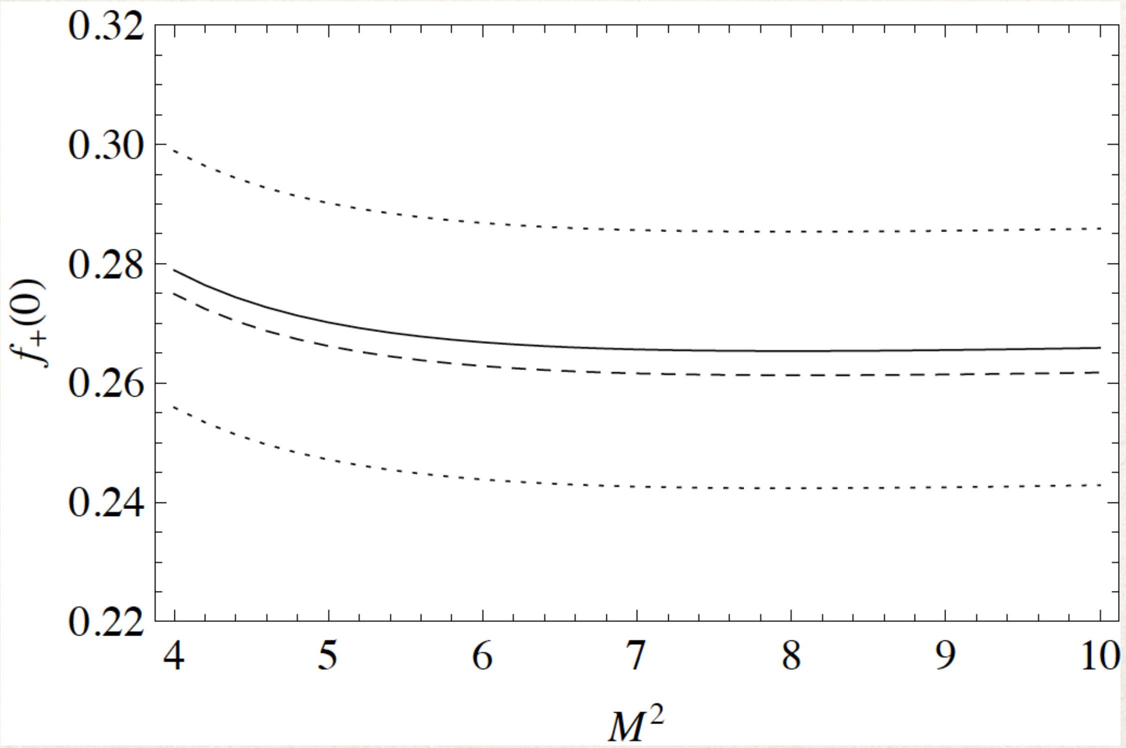
(AB 1203.1359)

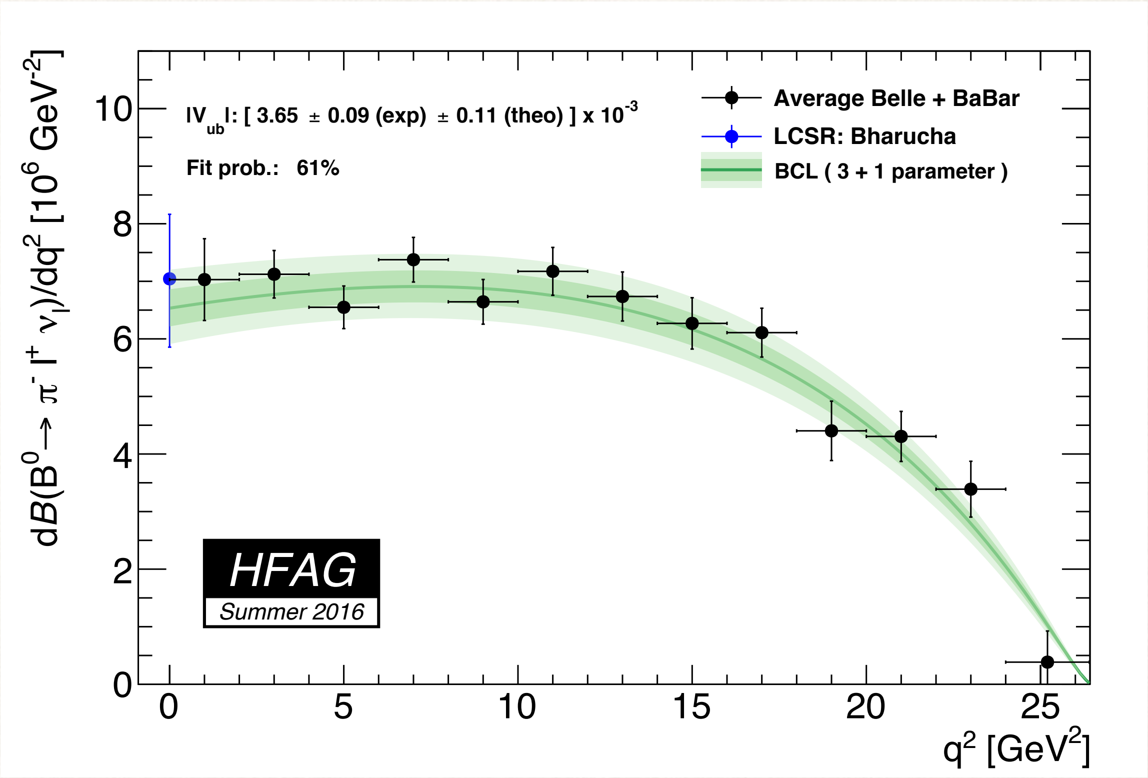


- Test argument that radiative corrections to $f_+ f_B$ and f_B should cancel when both calculated in sum rules (2-loop contribution to f_B in QCDSR sizeable) $\Rightarrow$ *Calculate subset of two-loop radiative corrections for twist-2 contribution to* $f_+(0) \propto \beta_0$

- $f_+(0)$ $(0.262^{+0.020}_{-0.023})$ at $\mathcal{O}(\alpha_s^2\beta_0)$ (solid) with uncertainties$\lesssim 9\%$ (dotted), compared to $\mathcal{O}(\alpha_s)$ result (dashed), as a function of Borel parameter M^2

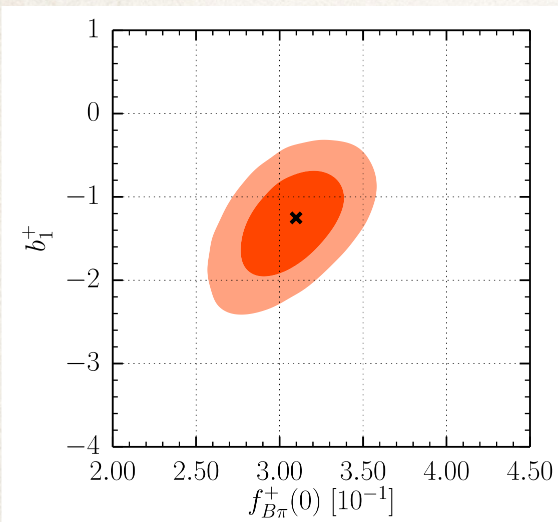
- Despite $\sim 9\%$ $\mathcal{O}(\alpha_s^2\beta_0)$ corrections to f_B, change in $f_+(0)$, only $\sim 2\%$

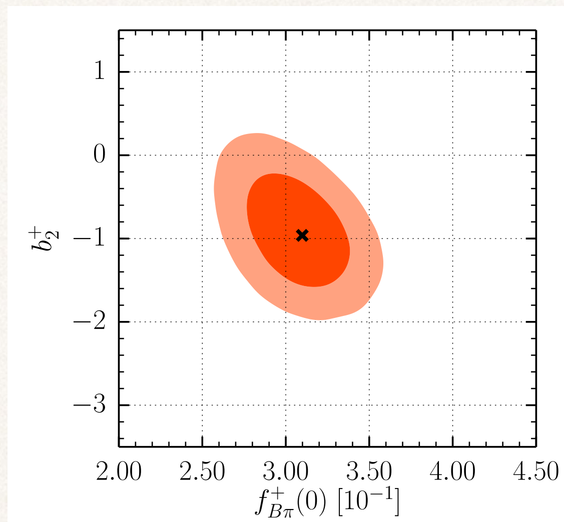




Extrapolation and unitarity bounds for the B → π form factor

(I. S. Imsong, A. Khodjamirian, T. Mannel, D. van Dyk, 1409.7816)





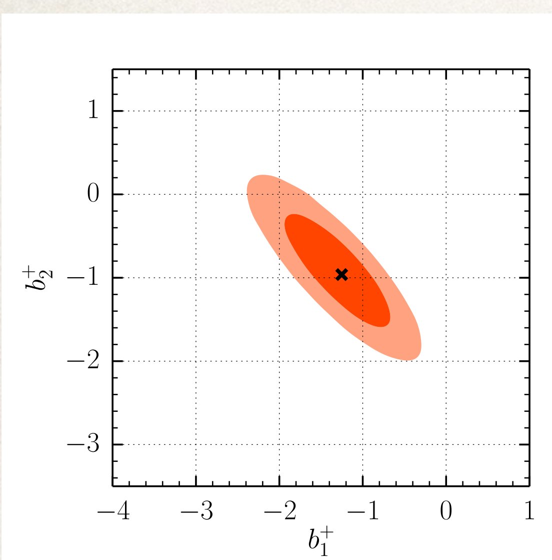


Figure 1. The regions with 68% probability (red) and 95% probability (orange) for all two-dimensional marginalisations of the posterior $P(\vec{\lambda}|\mathrm{LCSR})$. The cross marks the best-fit point.

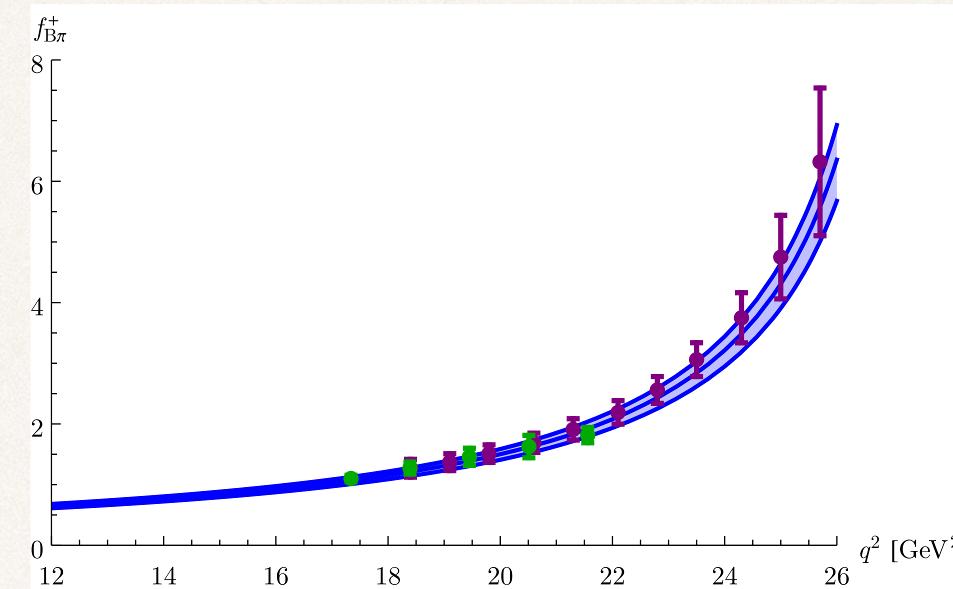


Figure 2. Form factor $f^+_{B\pi}(q^2)$ obtained at $q^2 < 12\mathrm{GeV}^2$ from the statistical analysis of LCSR, fitted to z-series representation and extrapolated to large q^2. The solid lines correspond to the 68% probability envelope and the best fit curve. The green (magenta) points are HPQCD [7] (Fermilab-MILC [8]) lattice QCD results.

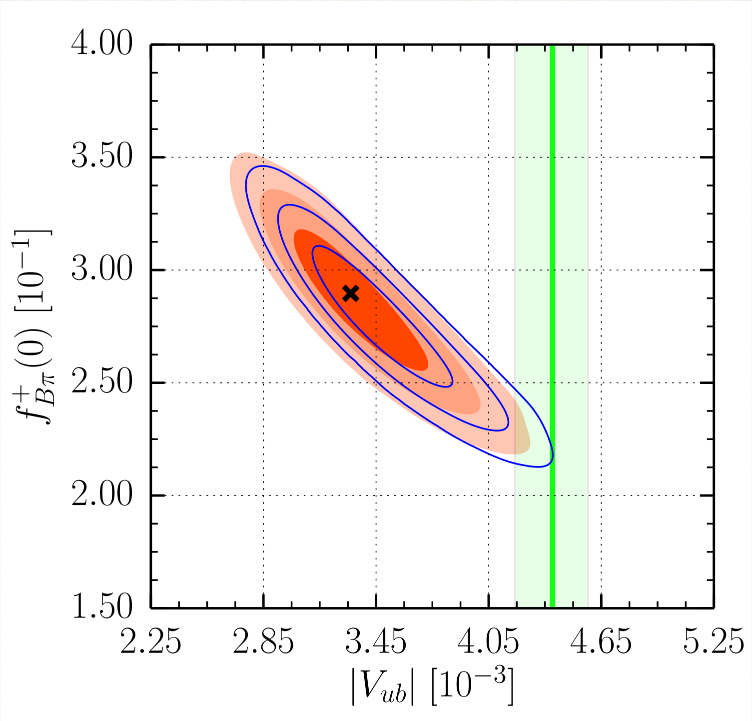
- Use Bayesian analysis: prior distributions for inputs, construct likelihood function based on SR fulfilling m_B to 1%, obtain posterior distributions using Bayes theorem
- Posterior distributions of inputs only different for s_0 : (41±4) GeV^2(~gaussian)
- Fit to BCl exp, find central value of $f_+(0) = 0.31 \pm 0.02$: raised due to value m_b, s_0, μ
- Obtaining $f_+(q_2)$ and first two derivatives at 0 and 10 GeV^2 allowed extrapolation to high q^2 using improved unitarity bounds

- Perform Bayesian analysis including experimental results to obtain $|V_{ub}|$

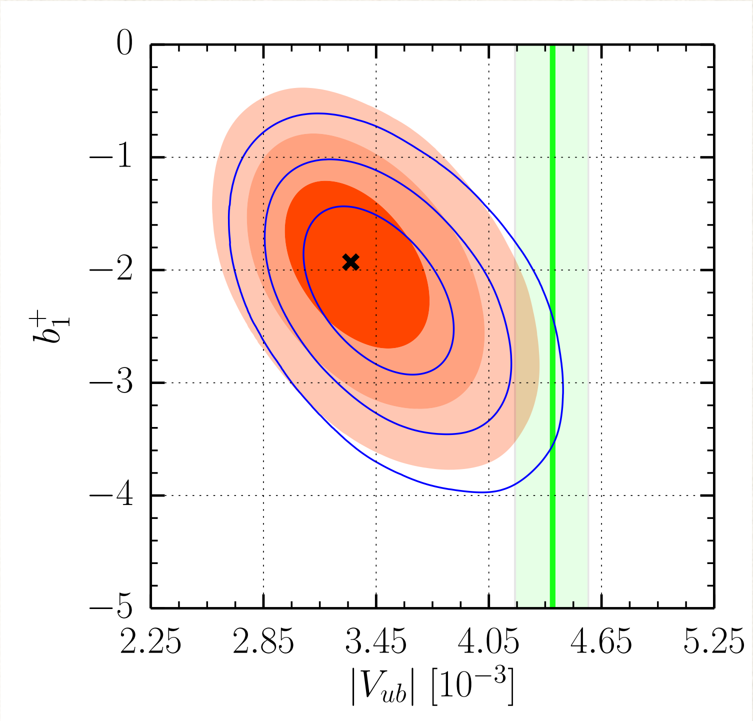
- Theory uncertainty on $|V_{ub}|$ obtained from analysis comparable to that of most accurate determinations from inclusive $b \to u$ transitions

- 2010 data set agrees better with inclusive than 2013

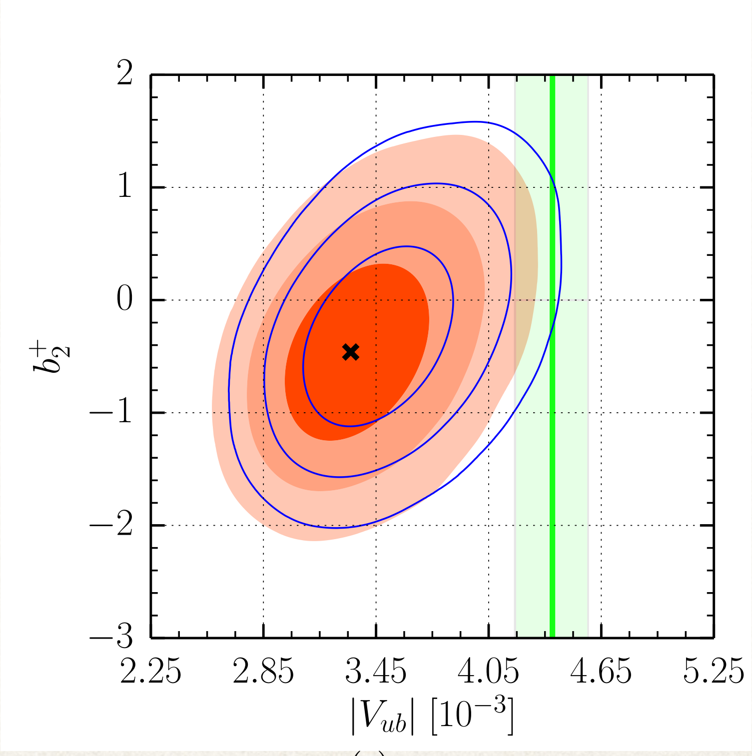
- Tension wrt GGOU determination seen beyond 99% C.L.



(a)



(b)



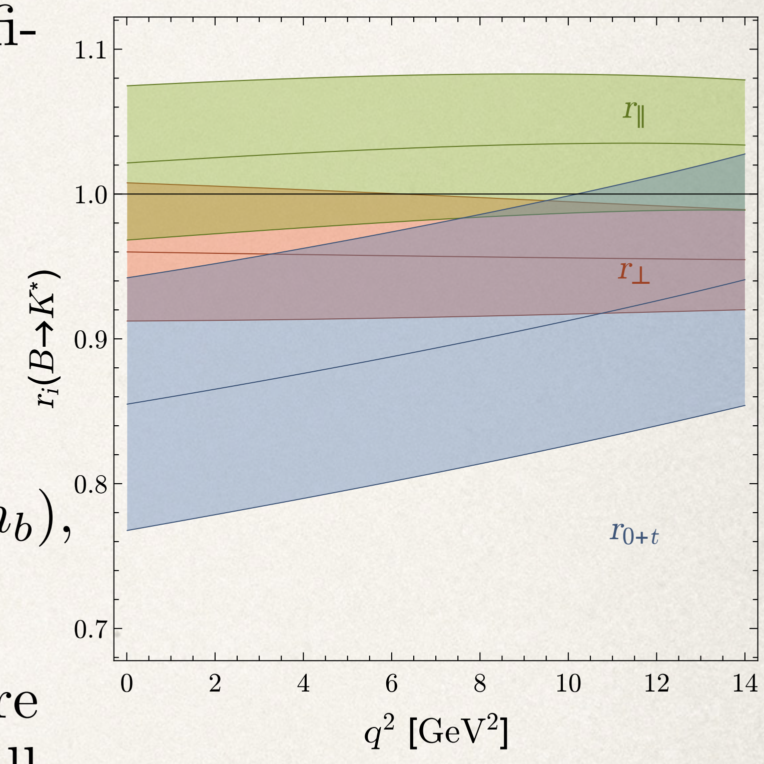
(c)

Figure 4. The two-dimensional marginal posteriors for $|V_{ub}|$ versus the BCL parameters (a) $f^+_{B\pi}(0)$, (b) b^+_1, and (c) b^+_2. The dark orange, orange, and light orange regions show, respectively, the 68%, 95% and 99% probability regions when using the "2013" data set. The blue contours delineate the corresponding probability regions of the "2010" data set. The green and light green vertical bands denote the central value and 68% CL interval of the HFAG world average [39] of the $|V_{ub}|$ determinations from inclusive decays $B \to X_u \ell \bar{\nu}$ according to the GGOU method [40].

(I. S. Imsong, A. Khodjamirian, T. Mannel, D. van Dyk, 1409.7816)

Update for B to V form factors (AB, D. Straub and R. Zwicky 1503.05534)

- Largest uncertainty in calculation is from form factors
- Best coverage in q^2: fit to LCSR/Lattice using series expansion, coefficients satisfy dispersive bounds.(AB, T. Feldmann, M. Wick, arXiv:1004.3249)
- **Our Aim:** improve uncertainty by making correlations available
- We obtain the four equation of motion relations:
  e.g. $T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0$
- Isgur-Wise relations at low recoil follow from $\mathcal{D}_\iota/(\mathcal{V}_\iota \text{ or } T_\iota) \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b)$, $\mathcal{D}_\iota$ is derivative FF, breaking of I-W relations.
- Certain combinations of $\mathcal{D}_\iota$'s may be small at large recoil: $\iota = 1, 2$ are direct candidates, and combinations of $\iota = 3, P$ result in potentially small ratio of $\mathcal{D}/T$



In order to fulfil EOM, V_1, T_1 and D_1 should have same s_0. As D_1 small, difficult to compensate different s_0^{T1} and s_0^{V1} via s_0^{D1}. For $s_0^{T1} = s_0^{V1} \pm 0.5$ GeV^2, a 5 GeV^2 change in s_0^{D1} is required. Therefore correlation between s_0^{V1} and s_0^{T1} seems reasonable, ensuring s_0^{V1}-s_0^{T1}<1 GeV^2. Apply same sum rules parameters for related FFs +correlations (7/8), less correlated for 0+t case (1/2)

Update for B to V form factors

(AB, D. Straub and R. Zwicky 1503.05534)

We carefully choose the sum rules parameters using the following:

- SR depends little on, but is clear extremum as fn of s_0, M^2, SR for m_B fulfilled;
- the continuum and higher twist contributions should be under control $\lesssim 30\%, 10\%$ respectively;
- Correlate s_0 for EOM related FFs, and M^2 for $FF \times f_B$ and f_B 50%.

Other improvements in the calculation:

- computation of full twist-4 (+partial twist-5) 2-particle DA contribution to FFs, plus determination of certain so-far unknown twist-5 DAs in the asymptotic limit
- discussion of non-resonant background for vector meson final states,
- determination and usage of updated hadronic parameters, specifically the decay constants
- fits with full error correlation matrix for the z-expansion coefficients, as well as an interpolation to the most recent lattice computation.

Update for the B$_{(s)}$ to K form factors

B$_{(s)}$ to K ll and B to π ll decays at large recoil and CKM matrix elements, Alexander Khodjamirian, Aleksey V. Rusov, arXiv:1703.04765 [hep-ph], JHEP 1708 (2017) 112.

The OPE result, schematically:

$$F^{(T)}_{B_sK}(q^2)_{\rm OPE} = (T_0^{(2)} + (\alpha_s/\pi)T_1(2)) \otimes \phi^{(2)} + \frac{\mu_K}{m_b}(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}) \otimes \phi_K^{(3)}$$
$$+ T_0^{(4)} \otimes \phi_K^{(4)} + \langle \bar{q}q\rangle \left(T_0^{(5)} \otimes \phi_k^{(2)} + \frac{\mu_K}{m_b} T_0^{(6)} \otimes \phi_K^{(3)}\right)$$

where $\phi_K(2,3,4) = \{\text{kaon DAs with non-asympt.terms}\}$, $\mu_K = \dfrac{m_K^2}{m_s + m_q}$

Include factorizable twist 5,6 contributions (Rusov 1705.01929) , find very small contribution

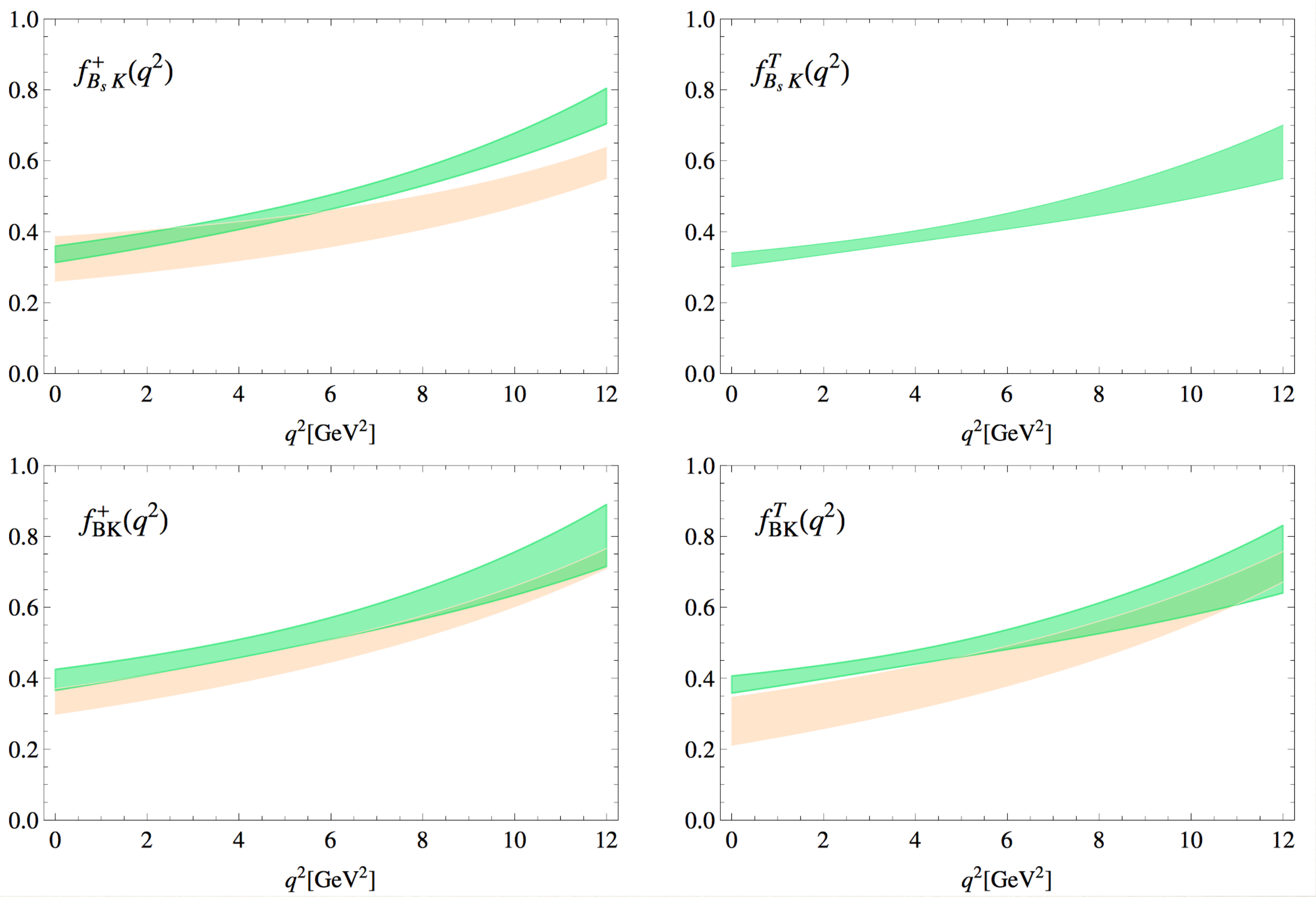
Additional improvements:

- Corrected subheading twist 3/4 contributions
- Use updated (smaller) QCDSR result for $f_{B(s)}$ from 2013
- Important update from LCSR for B_s to K

$f^+_{B_sK}(0)$	0.336
Tw2 LO	47.0%
Tw2 NLO	8.8%
Tw3 LO	47.1%
Tw3 NLO	−3.9%
Tw4 LO	1.0%
Tw5 LO-fact	−0.039%
Tw6 LO-fact	−0.005%

Results for $B_s \to K$ and $B \to K$ form factors and observables

Alexander Khodjamirian, Aleksey V. Rusov, arXiv: 1703.04765 [hep-ph], JHEP 1708 (2017) 112.



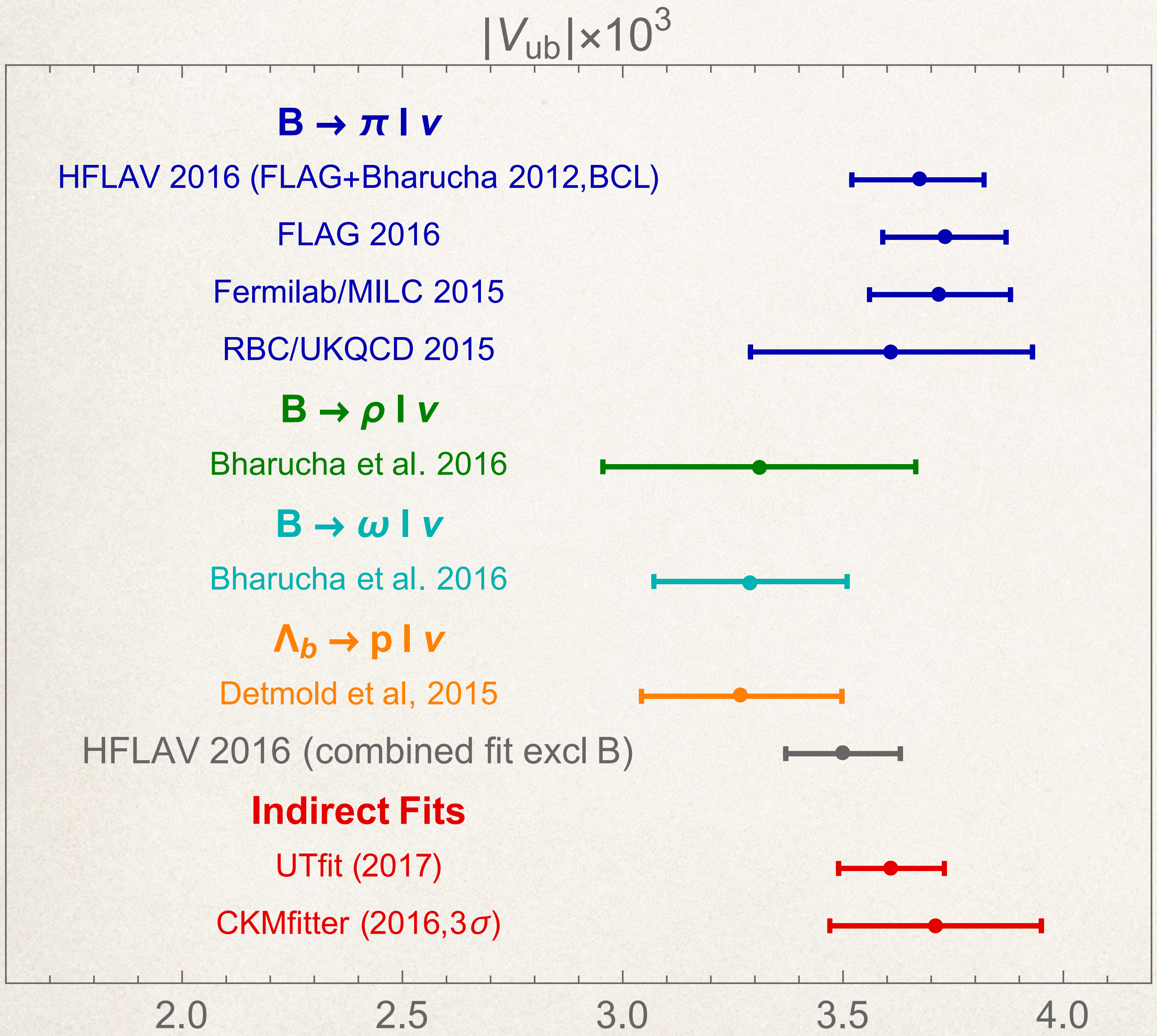
The vector (tensor) form factors of $B_s \to K$ and $B \to K$ from LCSRs with the dark-shaded (green) bands. Extrapolations of the lattice QCD results for $B_s \to K$ (HPQCD) and $B \to K$ (FermiLAB/MILC) form factors are shown with the light-shaded (orange) bands.

$$\Delta\zeta_{B_sK}\,[0, 12\,\mathrm{GeV}^2] \equiv \frac{1}{|V_{ub}|^2}\int_0^{12\,\mathrm{GeV}^2} dq^2\,\frac{d\Gamma(\bar{B}_s \to K^+\ell\bar{\nu}_\ell)}{dq^2} = 7.03^{+0.73}_{-0.69}\,\mathrm{ps}^{-1}$$

Decay mode	$B^- \to K^-\ell^+\ell^-$	$B^- \to \pi^-\ell^+\ell^-$	$\bar{B}_s \to K^0\ell^+\ell^-$
Measurement or calculation	$\mathcal{B}_{BK}[1.0\,,6.0]$	$\mathcal{B}_{B\pi}[1.0\,,6.0]$	$\mathcal{B}_{B_sK}[1.0, 6.0]$
Belle (2009)	$2.72\,^{+0.46}_{-0.42} \pm 0.16$	—	—
CDF (2011)	$2.58 \pm 0.36 \pm 0.16$	—	—
BaBar (2012)	$2.72\,^{+0.54}_{-0.48} \pm 0.06$	—	—
LHCb (2014,2015)	$2.42 \pm 0.7 \pm 0.12$	$0.091^{+0.021}_{-0.020} \pm 0.003$	—
HPQCD (2013)	3.62 ± 1.22	—	—
Fermilab/MILC (2015)	3.49 ± 0.62	0.096 ± 0.013	—
This work	$4.38^{+0.62}_{-0.57} \pm 0.28$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$0.154^{+0.018}_{-0.017} \pm 0.011$

Binned branching fractions in units of 10^{-8} GeV^2 for the 1-6 GeV^2 bin. The first (second) error is due to the uncertainty of the input (only of the CKM parameters).

Summary and Future Prospects



Future Prospects:

- Find higher twist (i.e. 5,6) terms in the factorizable approximation are small, but still would be good to check the full NNLO twist 2 and twist 3 contributions
- Bayesian uncertainty analysis of all $B \to P$, $D \to P$ LCSRs (for $B \to \pi$ in [Imsong,AK,Mannel,van Dyk (2013)])
- $B_s \to K l \nu$ measurement at LHCb/Belle II
- π, K DAs from LCSRs: BESS and Belle-2 data on $\gamma^* \gamma \to \pi_0$; JLab data on $F_{\pi/K}$
- Future Belle-2 data on the q^2-shape of $B \to \pi l \nu$ will provide additional constraints on the DA parameters

Back up slides

Form factors for B to V: Definitions

Express hadronic matrix elements via:

$$\langle K^*(p)|\bar{s}\gamma^\mu(1\mp\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,\mathcal{V}_1(q^2) \pm P_{2,3}^\mu\,\mathcal{V}_{2,3}(q^2) \pm P_P^\mu\mathcal{V}_P(q^2)$$

$$\langle K^*(p)|\bar{s}iq_\nu\sigma^{\mu\nu}(1\pm\gamma_5)b|\bar{B}(p_B)\rangle = P_1^\mu\,T_1(q^2) \pm P_{2,3}^\mu\,T_{2,3}(q^2)$$

where the Lorentz structures P_i^μ are

$$\begin{aligned}
P_P^\mu &= i(\eta^*\cdot q)q^\mu\,,\\
P_1^\mu &= 2\epsilon^\mu{}_{\alpha\beta\gamma}\eta^{*\alpha}p^\beta q^\gamma\,,\\
P_2^\mu &= i\{(m_B^2-m_{K^*}^2)\eta^{*\mu}-(\eta^*\cdot q)(p+p_B)^\mu\}\,,\\
P_3^\mu &= i(\eta^*\cdot q)\{q^\mu-\frac{q^2}{m_B^2-m_{K^*}^2}(p+p_B)^\mu\}
\end{aligned}$$

- Bjorken & Drell convention for the Levi-Civita tensor $\epsilon_{0123}=+1$
- η is the polarization of K^*
- Only 7 independent FFs

Results for the B to K* form factors