Form factors for semileptonic B-meson decays into light hadrons from lattice QCD

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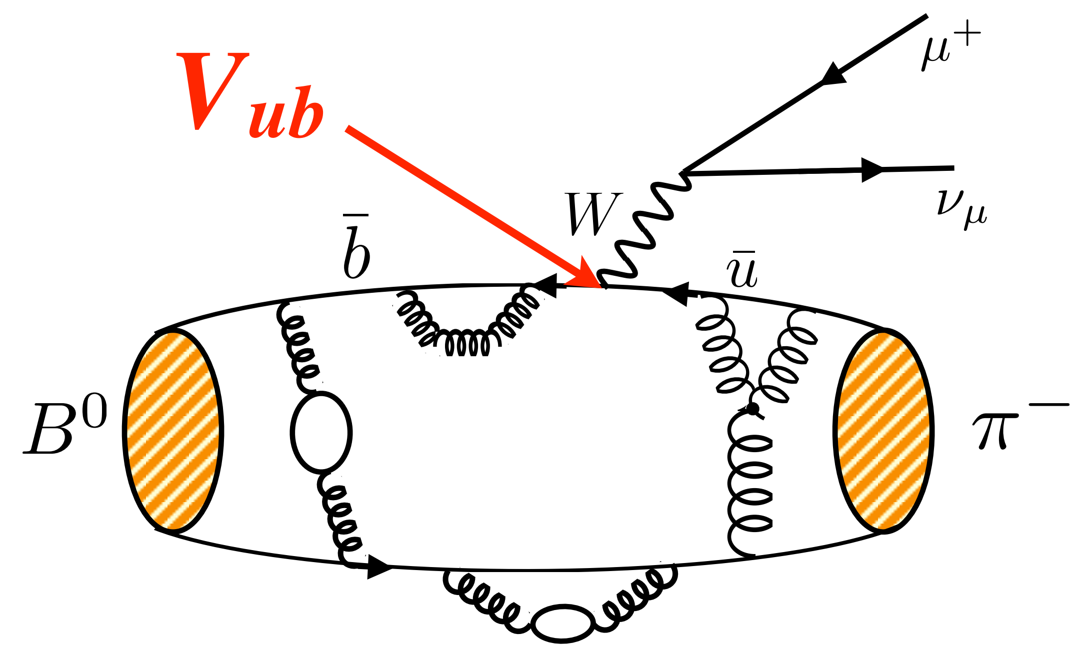
Challenges in Semileptonic B Decays,
MITP, JGU, Mainz, 09-13 April 2018

Outline

- Motivation and Introduction
  - ★ Introduction to LQCD
- CC form factors for $B \to \pi, K$
  - ★ Implications
- FCNC form factors
  - ★ into pseudoscalar final states
  - ★ into vector final states
- Summary and Outlook

Introduction and Motivation

example: $B^0 \to \pi^- \ell^+ \nu_\ell$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \to K \ell^+ \ell^-)}{dq^2}, \dots$$

$$\frac{d\Gamma(B \to D \ell \nu)}{d\omega}, \frac{d\Gamma(B \to D \tau \nu)}{d\omega}, \dots$$

$$\Delta m_{d(s)}$$

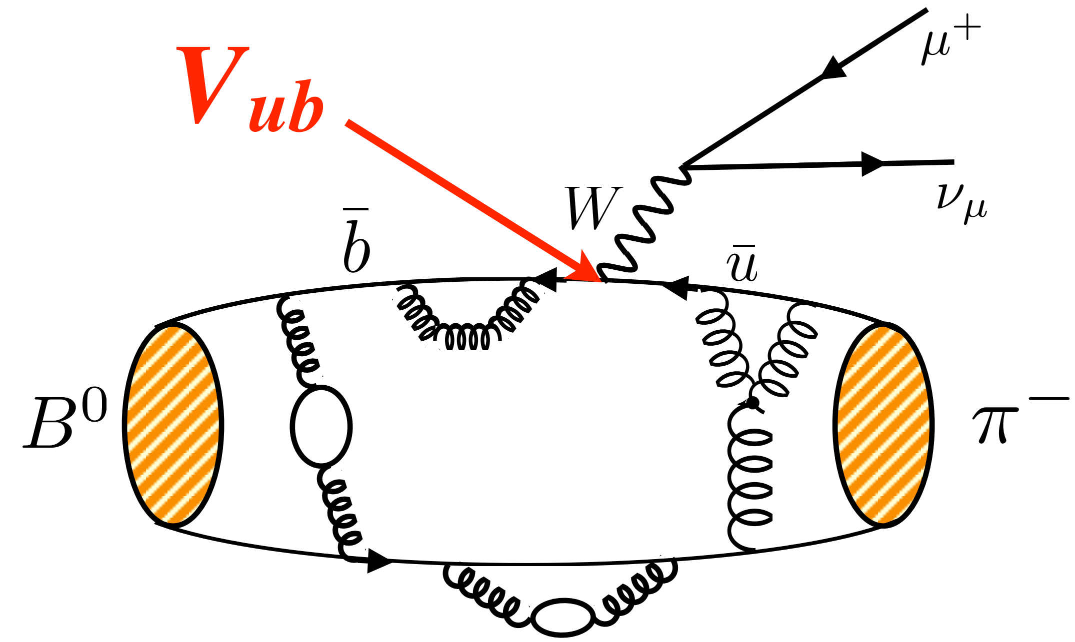
$$\vdots$$

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Introduction and Motivation

example: $B^0 \to \pi^- \ell^+ \nu_\ell$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

Two main purposes:

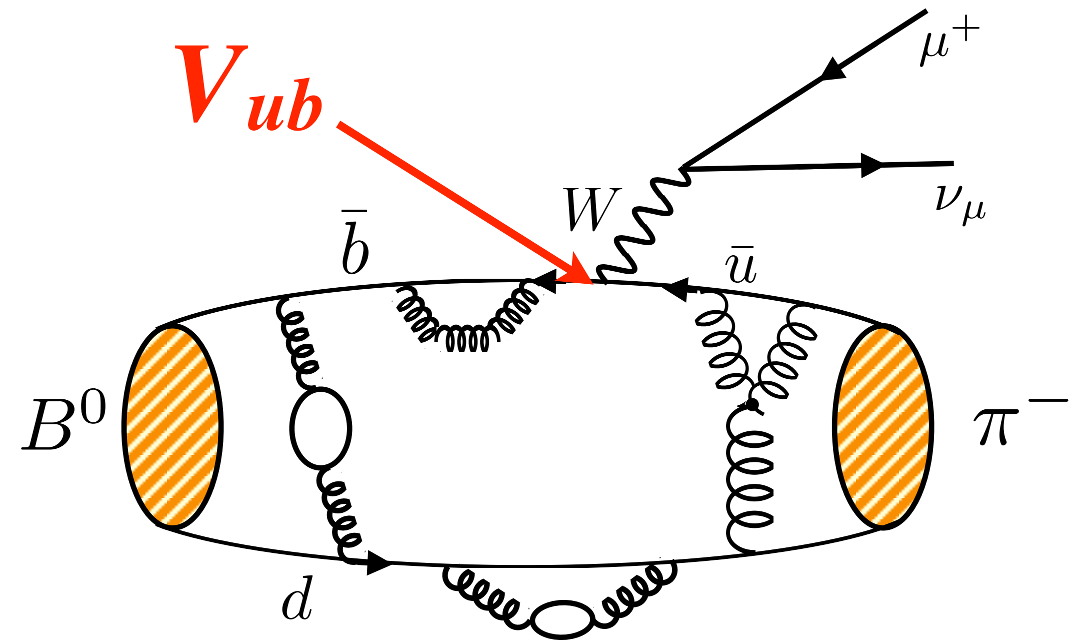
- ✦ combine experimental measurements with LQCD results to determine CKM parameters.
- ✦ confront experimental measurements of rare processes or lepton flavor (universality) violating observables with SM theory using LQCD inputs.

⋮

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

form factors for $B \to \pi \ell \nu$ & V_{ub}



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2 \qquad q^2 = (p_B - p_\pi)^2$$

★ calculate the form factors in the low recoil (high q^2) range.

★ use model-independent parameterization of q^2 dependence.

★ calculate the complete set of form factors, $f_+(q^2), f_0(q^2)$ and $f_T(q^2)$.

★ for $f_+(q^2)$ compare shape between experiment and lattice.

form factors for $B \to \pi \ell \nu$

$$\langle\pi|V^\mu|B\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2}q^\mu$$

$$\langle\pi|S|B\rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_u}f_0(q^2)$$

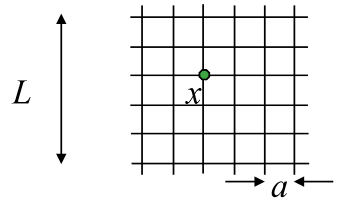
$$\langle\pi|T^{\mu\nu}|B\rangle = 2\frac{p_B^\mu p_\pi^\nu - p_B^\nu p_\pi^\mu}{M_B + M_\pi}f_T(q^2)$$

Convenient for lattice QCD:

$$\langle\pi|V^\mu|B\rangle = \sqrt{2M_B}\left[v^\mu f_\parallel(E_\pi) + p_\perp^\mu f_\perp(E_\pi)\right] \quad v = p/M_B$$

$$f_\parallel(E_\pi) = \frac{\langle\pi|V^4|B\rangle}{\sqrt{2M_B}}, \qquad p_\perp^\mu = p^\mu - (p\cdot v)v^\mu$$

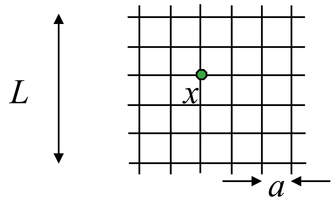
$$f_\perp(E_\pi) = \frac{\langle\pi|V^i|B\rangle}{\sqrt{2M_B}}\frac{1}{p^i}$$



Lattice QCD Introduction

The State of the Art

- ☆ Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with
  - sub percent precision.
  - total errors that are commensurate (or smaller) than the corresponding experimental uncertainties.
- ☆ Scope of LQCD calculations is increasing due to continual development of new methods:
  - baryons
  - nonleptonic decays ($K \to \pi\pi$, …)
  - resonances, scattering, long-distance effects,
  - QED effects
  - …



Lattice QCD Introduction

[inspired by A. Kronfeld]

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \quad \ldots$

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi} \quad f_{+,0,T}^{B\to\pi} \quad \ldots$

$f_{K^\pm} \quad f_{B_{(s)}} \quad \ldots \quad \langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\Delta M_K, \, \epsilon_K$

$\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$B \to K^* \ell\ell \to K\pi \, \ell\ell \ldots$

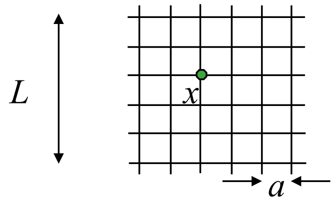
$K^+ \to \ell^+ \nu \, (\gamma) \quad \ldots$

$K^+ \to \pi^+ \ell^+ \ell^-$

$K^+ \to \pi^+ \nu \bar{\nu} \quad \ldots$

Complexity

LQCD flagship results	First complete LQCD results, large(ish) errors	First results, physical params, incomplete systematics	new methods, pilot projects, unphysical kinematics



Lattice QCD Introduction

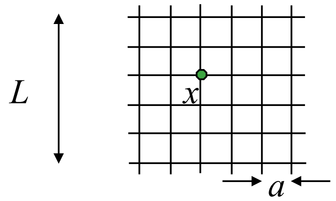
combined chiral-continuum interpolation/extrapolation

When

$$m_{\text{light}} \lesssim 1/2\,(m_u + m_d)_{\text{phys}}$$

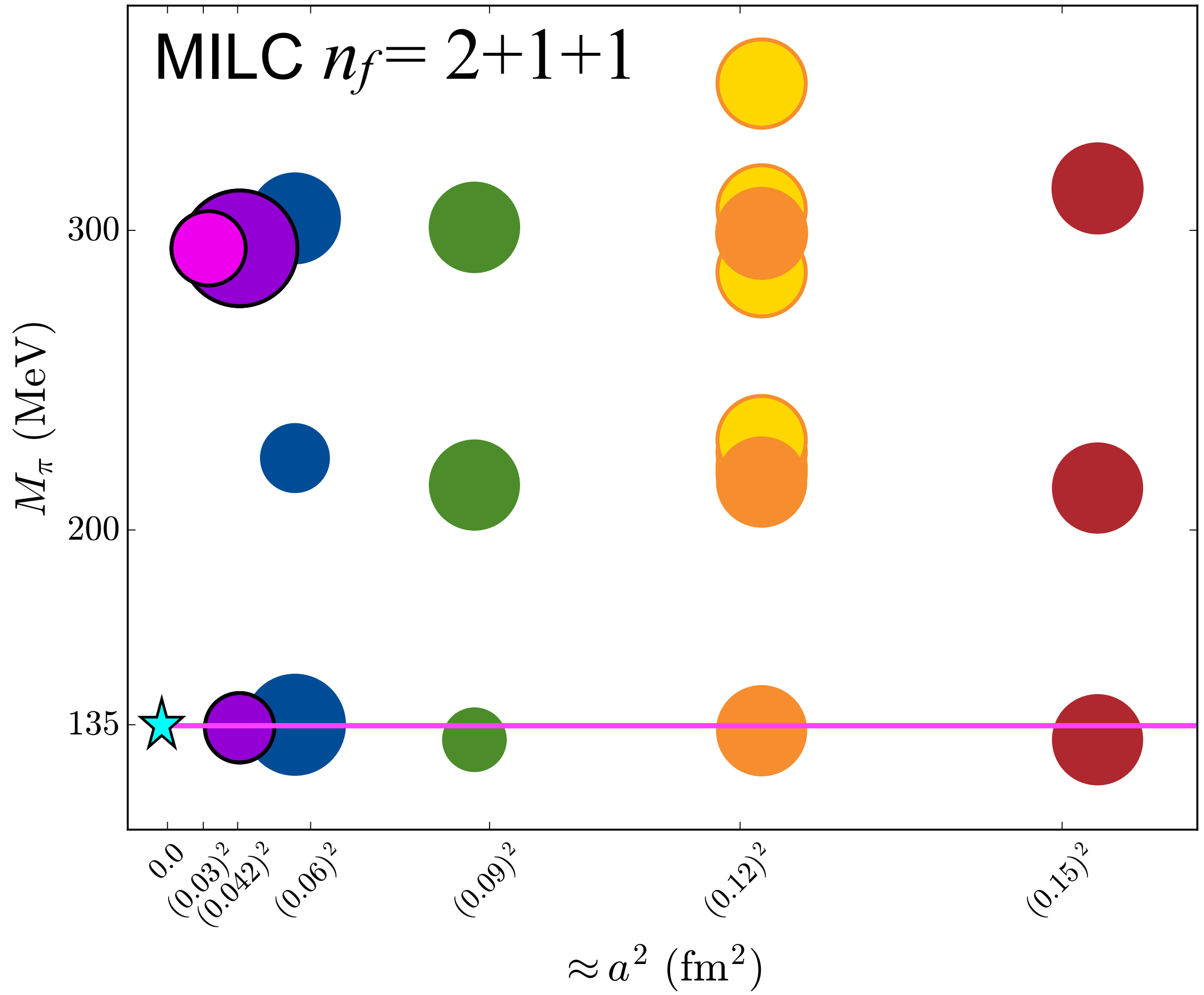
χPT guides the interpolation/extrapolation to the physical point.

- combined chiral-continuum interpolation/extrapolation
- include (light quark) discretization effects (for example, staggered χPT)
- can also add HQ discretization terms to chiral-continuum fits
- for B,D meson processes use Heavy Meson χPT: $\chi\text{PT} + 1/M$ expansion

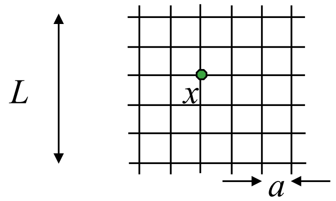


Lattice QCD Introduction

combined chiral-continuum interpolation/extrapolation



Growing number of collaborations have generated sets of ensembles that include sea quarks with physical light-quark masses: PACS-CS, BMW, MILC, RBC/UKQCD, ETM,…

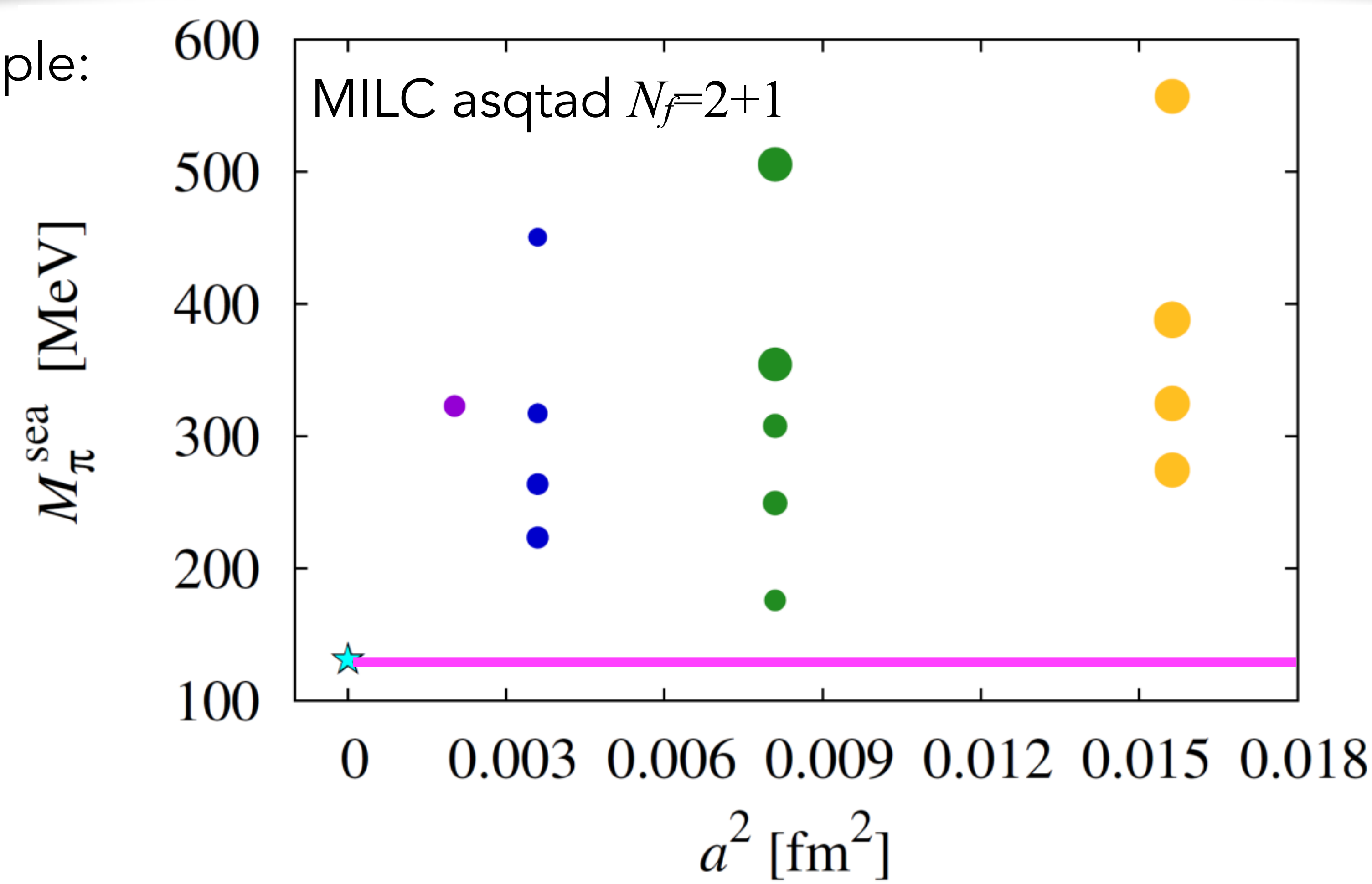


Lattice QCD Introduction

combined chiral-continuum extrapolation

Not all LQCD calculations include physical mass ensembles …

Example:



…especially for B,D meson quantities.

Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\rm QCD}$), leading discretization errors ~ $\alpha_s^k (a\Lambda_{\rm QCD})^n$
- For heavy quarks, leading discretization errors $\sim \alpha_s^k (am_h)^n$

  for b quarks: need $a \lesssim 0.04\mathrm{fm}$ so that $am_b \lesssim 1$

  not a problem for charm with improved action: $a \lesssim 0.12$ fm

  ➠ need effective field theory methods for b quarks

  for charm can use light quark methods, with improved action

- avoid errors of $(am_b)^n$ in the action by using EFT:
  - ✦ relativistic HQ actions (Fermilab, Columbia [aka RHQ],…)
  - ✦ HQET
  - ✦ NRQCD

or

- use an improved light quark action (HISQ, tmWilson, DWF, NP imp. Wilson,….)
  - ✦ use same LQ action for heavy quarks, but keep $am_h < 1$
  - ✦ use HQET to extrapolate/interpolate to b quark mass

Existing/ongoing/planned form factor calculations

Semileptonic B decays to light pseudo scalars

- HPQCD:
  NRQCD b quarks+ HISQ light valence
  1. $B \to K$ form factors (rare decay) on MILC asqtad (2+1) ensembles
     [arXiv:1306.0434, PRL 2013; arXiv:1306.2384, PRD 2013]
     5 ensembles, $a \approx 0.09,\ 0.12$ fm, $m_\pi = 270\text{-}400$ MeV, 1-loop PT matching
  2. $B_s \to K$ form factors on MILC asqtad (2+1) ensembles
     [arXiv:1406.2279, PRD 2014]
     5 ensembles, $a \approx 0.09,\ 0.12$ fm, $m_\pi = 175\text{-}300$ MeV, 1-loop PT matching
  3. $B \to \pi$ form factor at zero recoil on MILC HISQ (2+1+1) ensembles
     [arXiv:1510.07446, PRD 2016]
     8 ensembles, $a \approx 0.09,\ 0.12{,}0.15$ fm, $m_\pi = 135\text{-}300$ MeV, 1-loop PT matching
     soft pion theorem confirmed

Existing/ongoing/planned form factor calculations

- RBC/UKQCD:
  RHQ action for b quark + DWF light on DWF (2+1) ensembles
  $B_s \to K$ and $B \to \pi$ form factors
  [arXiv:1501.05373, PRD 2015]
  5 ensembles, $a \approx 0.09, 0.11$ fm, $m_\pi = 290\text{-}420$ MeV,
  mostly nonperturbative renormalization, aka mNPR (with 1-loop PT matching)
  ongoing effort to add more ensembles, including at physical mass

Existing/ongoing/planned form factor calculations

- FNAL/MILC:
  1. Fermilab b quark +
     (a) asqtad light on MILC asqtad (2+1) ensembles
     form factors for
     $B \to \pi$ [arXiv:1503.07839, PRD 2015; arXiv:1507.01618, PRL 2015] and
     $B \to K$ [arXiv:1509.06235, PRD 2016]
     - 12 ensembles, $a \approx 0.045, 0.06, 0.09, 0.12$ fm, $m_\pi = 175\text{-}420$ MeV
     - mNPR (with 1-loop PT matching)

     $B_s \to K$ [Y. Liu et al, Lattice 2017, arXiv:1711.08085] preliminary, **blinded**
     - 6 ensembles, $a \approx 0.06, 0.09, 0.12$ fm, $m_\pi = 175\text{-}350$ MeV
     - mNPR (with 1-loop PT matching)

     (b) HISQ light on MILC HISQ (2+1+1) ensembles
     $B \to \pi$, $B_s \to K$, $B \to K$ [Z. Gelzer et al, Lattice 2017, arXiv:1710.09442]
     5 ensembles, $a \approx 0.09, 0.12, 0.15$ fm, $m_\pi = 135\text{-}300$ MeV
     mNPR (with 1-loop PT matching), preliminary
  2. HISQ b quark + HISQ light (currently testing set-up)
     plan: $a \approx 0.03, 0.045, 0.06, 0.09, 0.12$ fm, $m_\pi = 135\text{-}300$ MeV
     nonperturbative renormalization (see Kronfeld talk on decay constants)

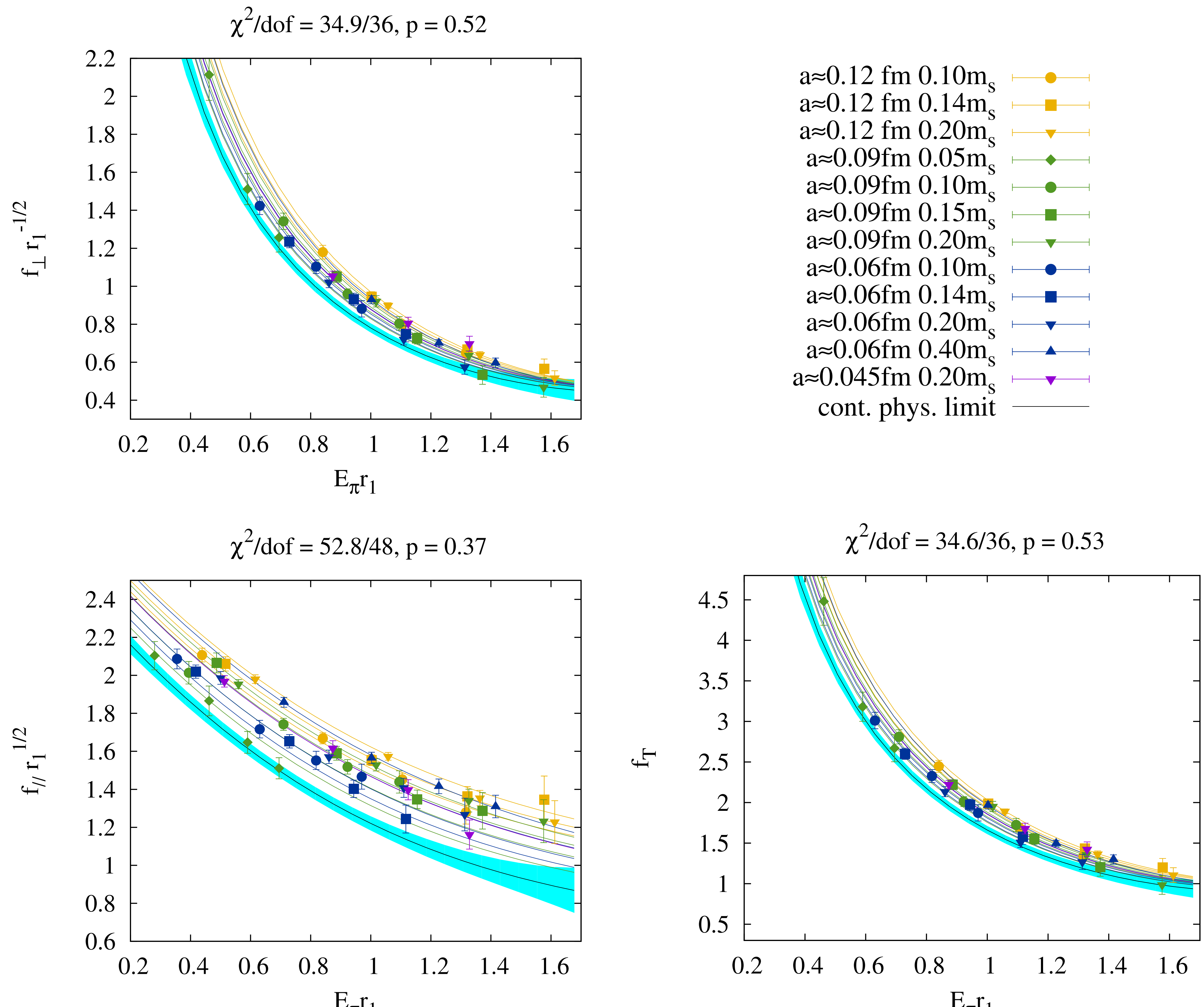
Existing/ongoing/planned form factor calculations

- ALPHA:
  HQET b quark + NP imp. Wilson on CLS NP imp. Wilson (2) ensembles
  $B_s \to K$ form factors
  [arXiv:1601.04277, PLB 2016]
  static limit, nonperturbative renormalization, aka NPR
  3 ensembles, $a \approx 0.05$, 0.065, 0.075 fm, $m_\pi = 310\text{-}330$ MeV

  preliminary results for the 1/m terms
  [M. Koren, Lattice 2017, arXiv:1711.01158]

- JLQCD:
  DWF b quark+ DWF light on DWF (2+1) ensembles
  $B \to \pi$ form factors
  [B. Colquhoun, Lattice 2017, arXiv:1710.07094]
  5 ensembles, $a \approx 0.044$, 0.055, 0.08 fm, $m_\pi = 300\text{-}500$ MeV
  use heavy quark mass $m_{\text{charm}} \leq m_{\text{heavy}} \leq 2.44 \times m_{\text{charm}}$
  and extrapolate to b quark mass
  preliminary results

form factors for $B \to \pi \ell \nu$



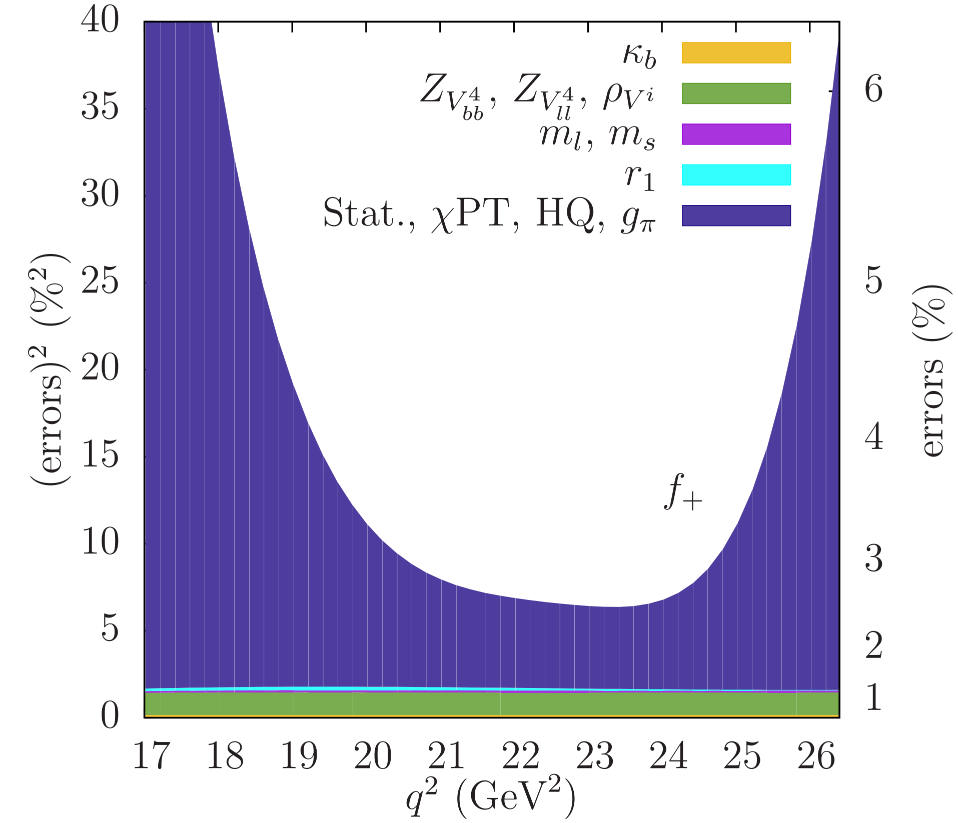
FNAL/MILC
[arXiv:1503.07839, PRD 2015]

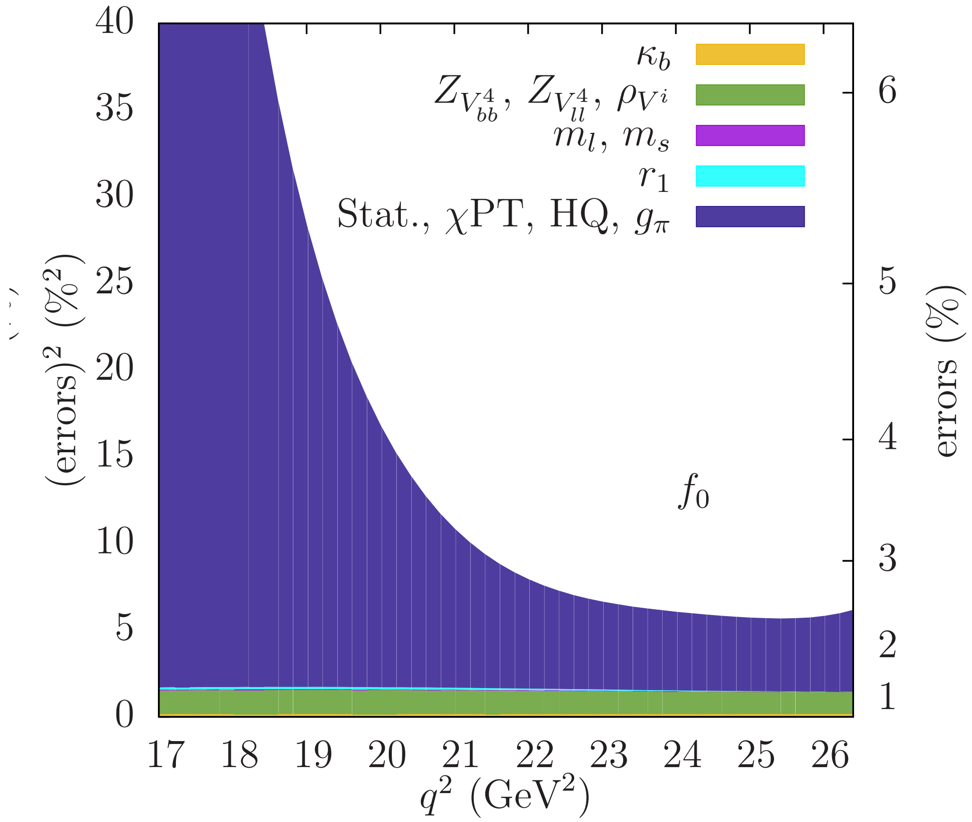
form factors for $B \to \pi \ell \nu$

FNAL/MILC
[arXiv:1503.07839, PRD 2015]

Table X. Error budgets of form factors f_+ and f_0 at $q^2 = 20\mathrm{GeV}^2$.

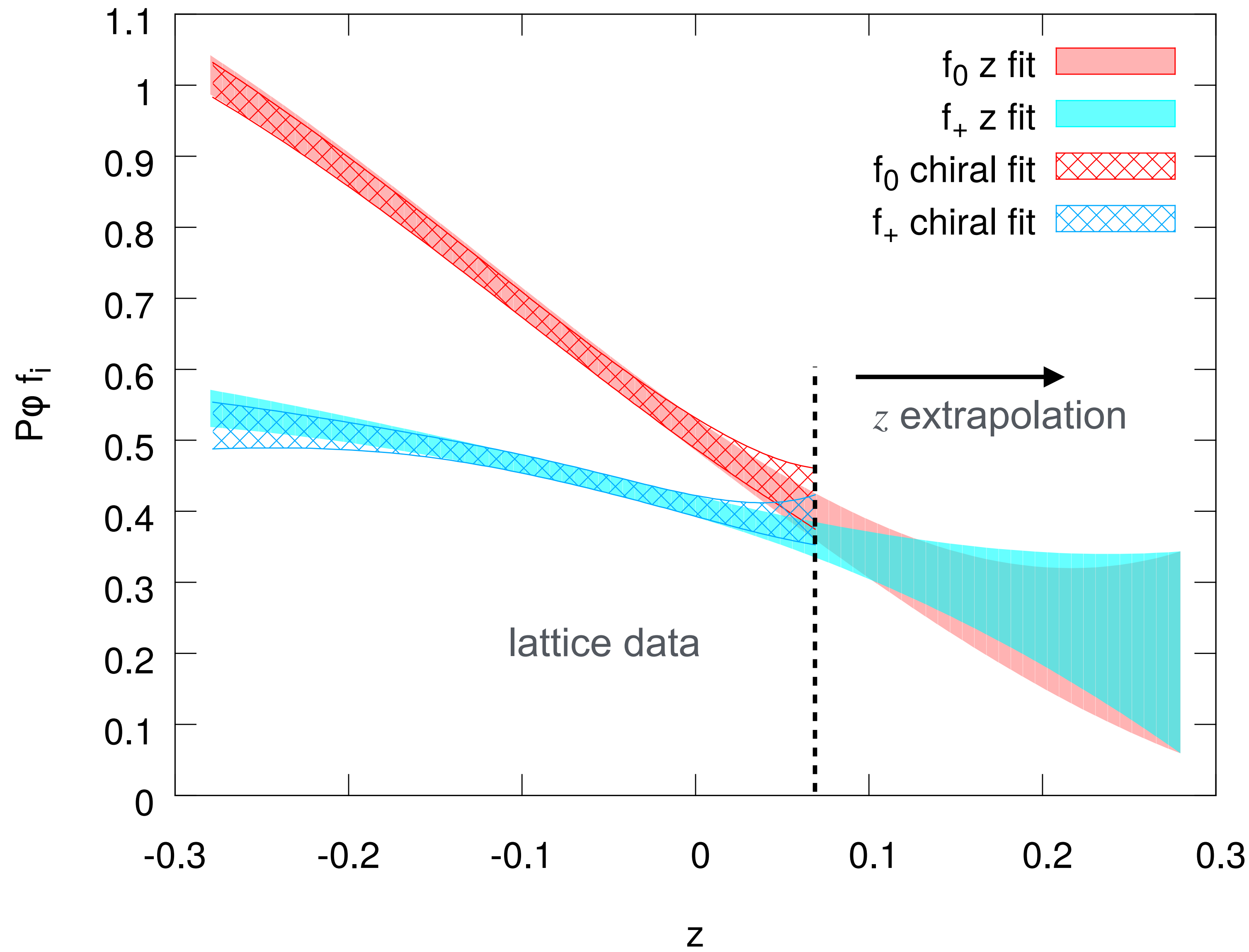
Uncertainty	δf_+	δf_0
Statistical+χPT+HQ+$g_{B^*B\pi}$	3.1	3.8
Scale r_1	0.5	0.7
Non-perturbative $Z_{V^4_{bb}}$	0.4	0.6
Non-perturbative $Z_{V^4_{ll}}$	0.4	0.4
Perturbative ρ	1.0	1.0
Heavy-quark mass mistuning	0.4	0.4
Light-quark mass tuning	0.4	0.2
Total	3.4	4.1



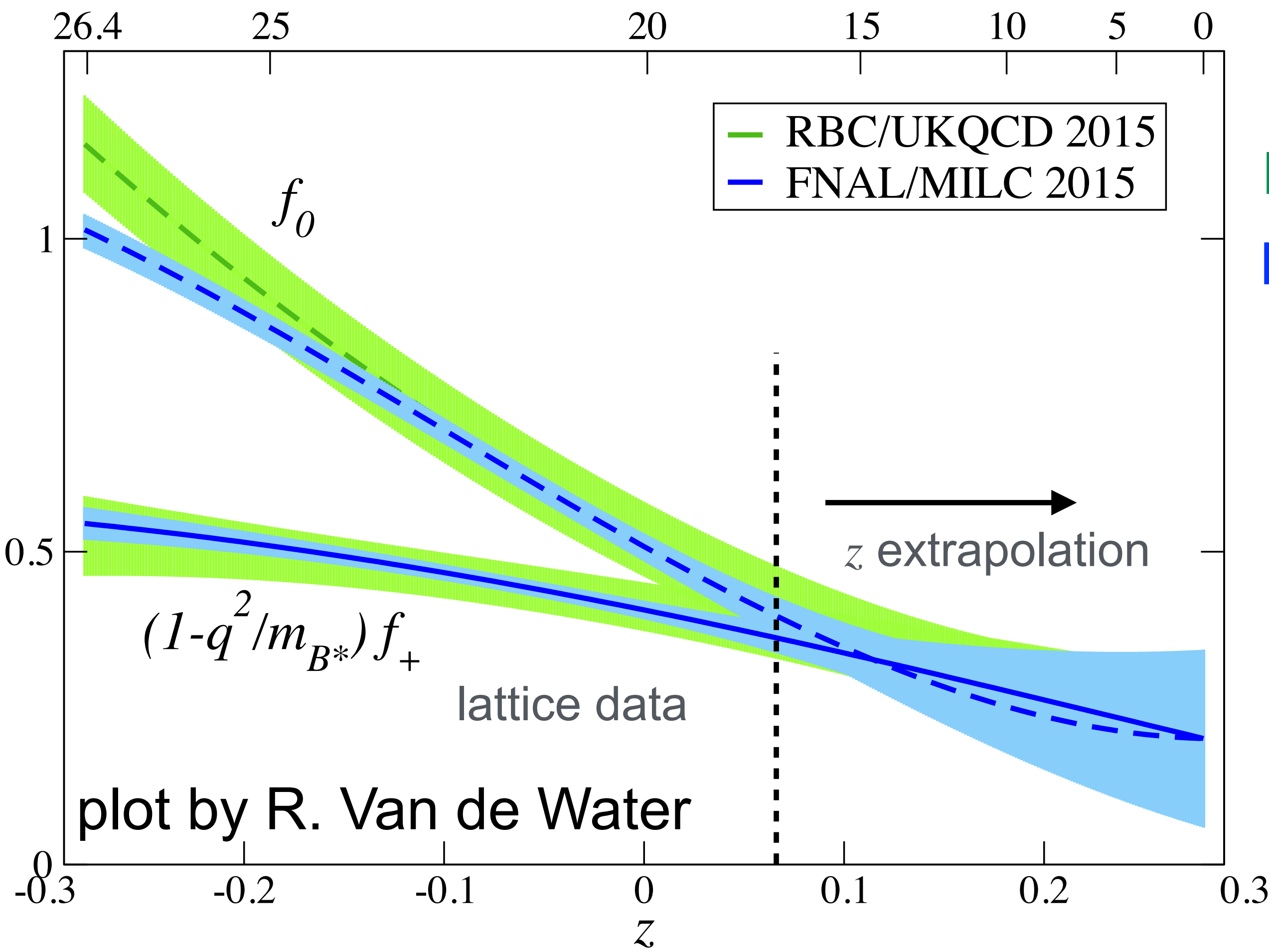


form factors for $B \to \pi \ell \nu$

FNAL/MILC [arXiv:1503.07839, PRD 2015]



form factors for $B \to \pi \ell \nu$ & V_{ub}



RBC/UKQCD [arXiv:1501.05373, PRD 2015]

FNAL/MILC [arXiv:1503.07839, PRD 2015]

FNAL/MILC & RBC/UKQCD form factors are in good agreement.

- Two independent LQCD **predictions** for $B_s \to K \ell \nu$ form factors

  [HPQCD, arXiv:1406.2279, PRD 2014; RBC, arXiv:1501.05373, PRD 2015]

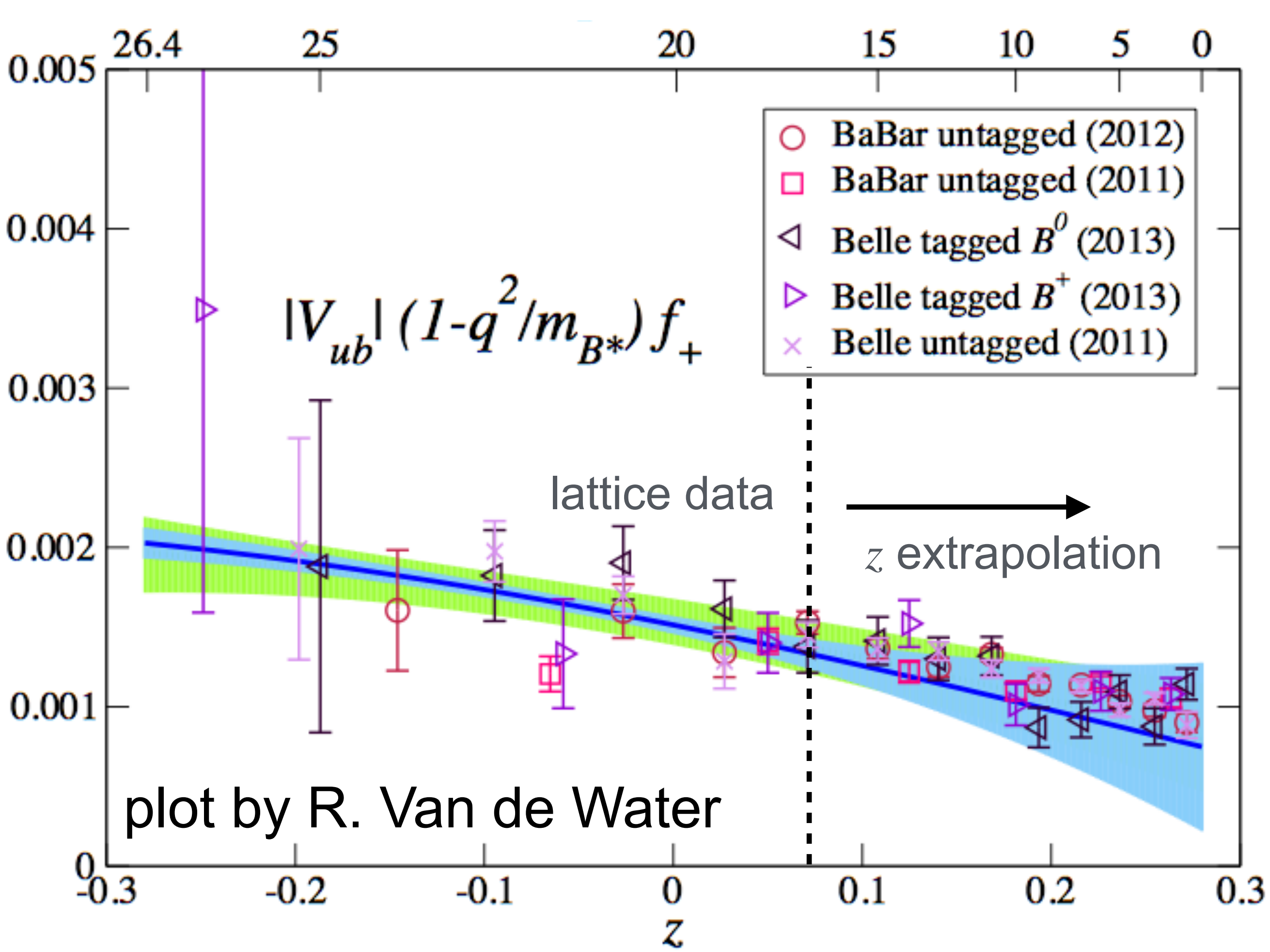
- ongoing work by:

  FNAL/MILC [Y. Liu (S. Gottlieb), Z. Gelzer (A. Kronfeld) @ Lattice 2017]

  RBC/UKQCD [O. Witzel @ Lattice 2017], ALPHA [M. Koren @ Lattice 2017],

  JLQCD [B. Colquhoun @ Lattice 2017]

form factors for $B \to \pi \, \ell \, \nu$ & V_{ub}



RBC/UKQCD [arXiv:1501.05373, PRD 2015]

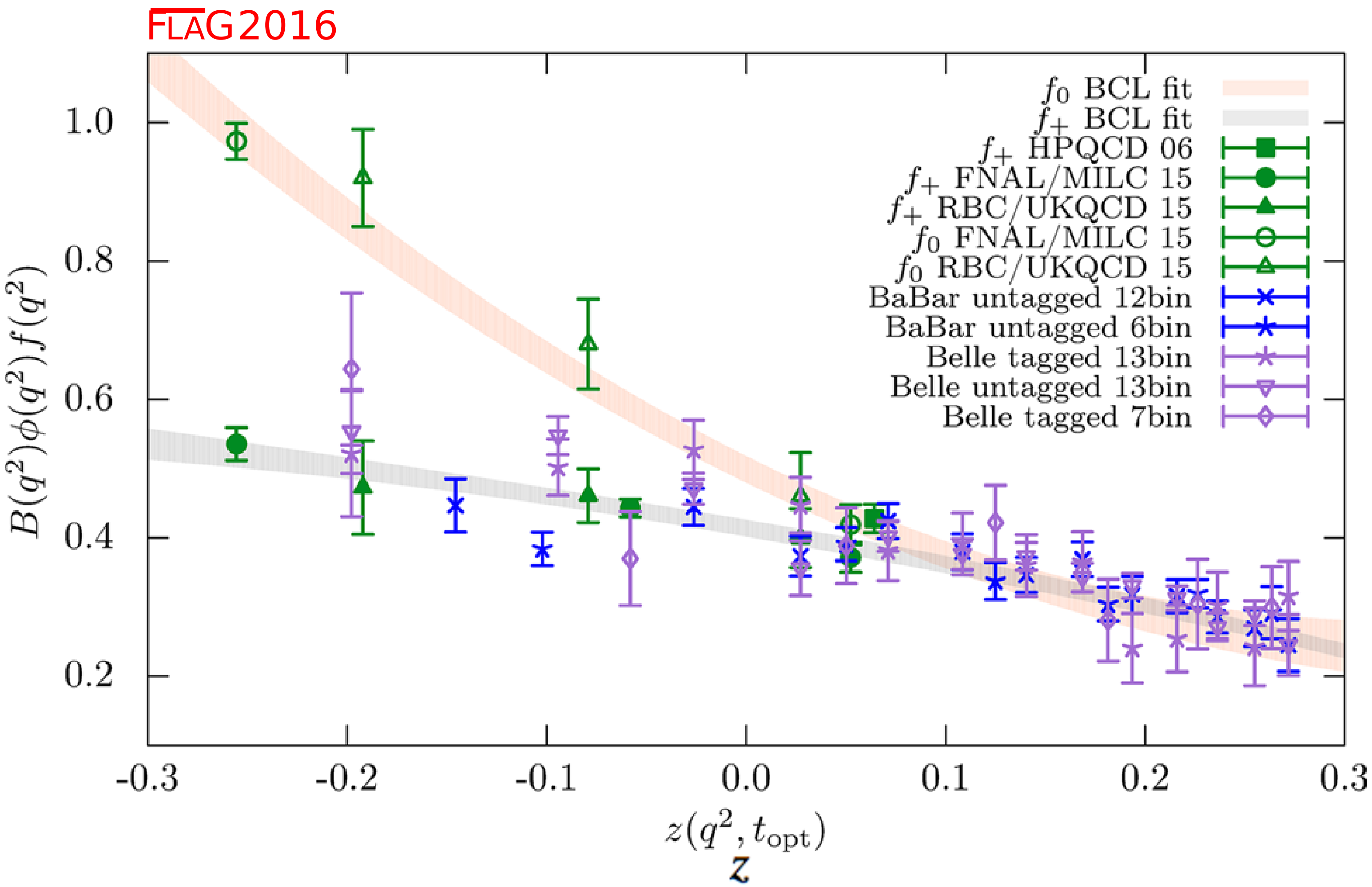
FNAL/MILC [arXiv:1503.07839, PRD 2015]

$|V_{ub}| = 3.72\ (16)\ 10^{-3}$

- shape of f_+ agrees with experiment and uncertainties are commensurate
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ and obtain form factors (f_+, f_0) with improved precision…
- determination of $|V_{ub}/V_{cb}|$ from Λ_b decay with LHCb [arXiv:1503.01421, PRD 2015; arXiv:1504.01568, Nature 2015]:

$$R_{FF} = \frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15\mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to p \mu \nu)}{dq^2} dq^2}{\int_{7\mathrm{GeV}^2}^{q^2_{\mathrm{max}}} \frac{d\Gamma(\Lambda_b \to \Lambda_c \mu \nu)}{dq^2} dq^2} = 1.471 \pm 0.094 \pm 0.109$$

form factors for $B \to \pi \ell \nu$ & V_{ub}



S. Aoki et al [FLAG-3 review, arXiv:1607.00299, EPJC 17, web update]

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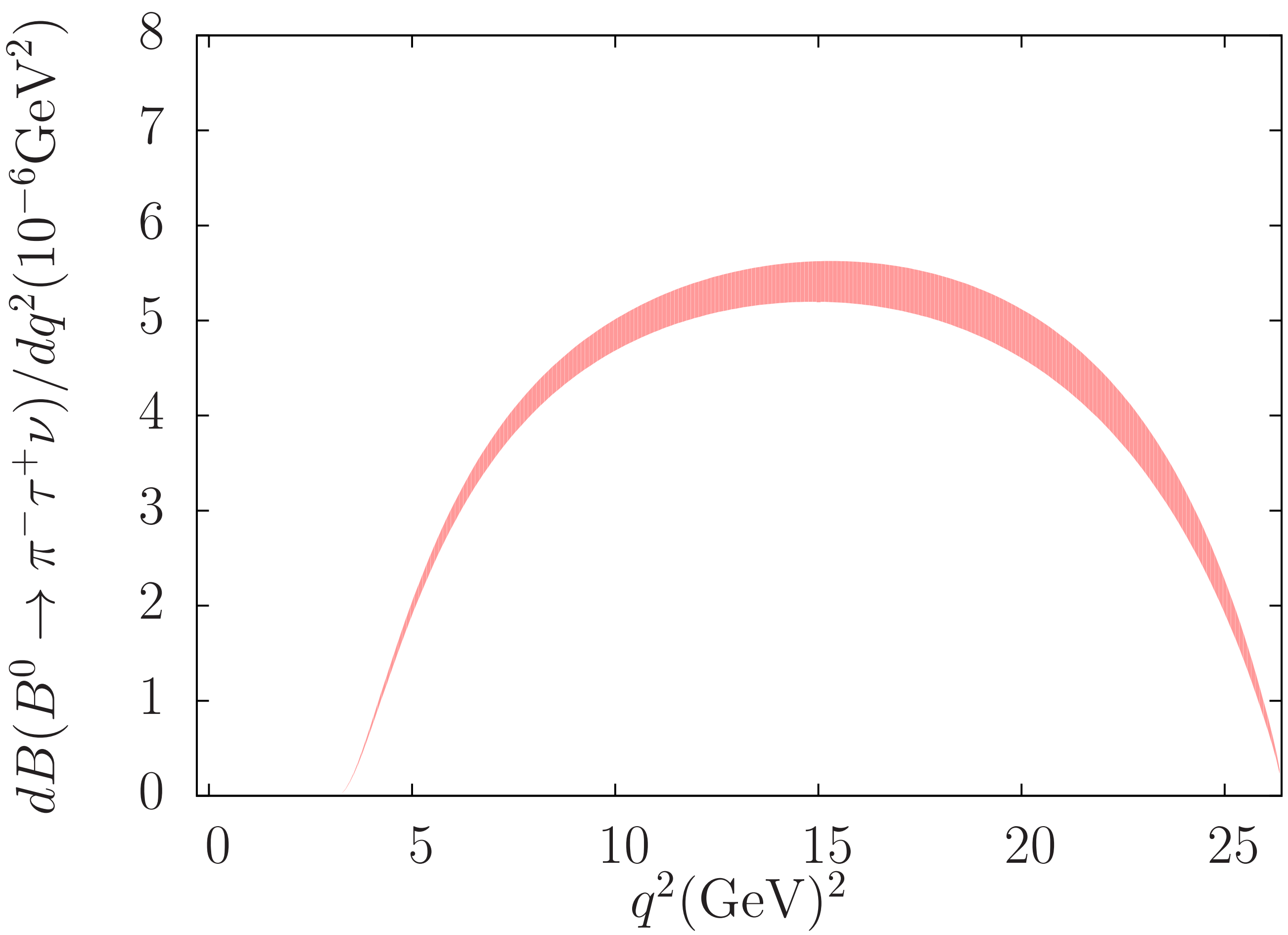
BSM phenomenology: LFU μ/e

D. Du et al [arXiv:1510.02349, PRD 2016]

SM prediction for $R(\pi) = \dfrac{\mathcal{B}(B \to \pi\,\tau\nu_\tau)}{\mathcal{B}(B \to \pi\,\ell\nu)} = 0.641(17)$

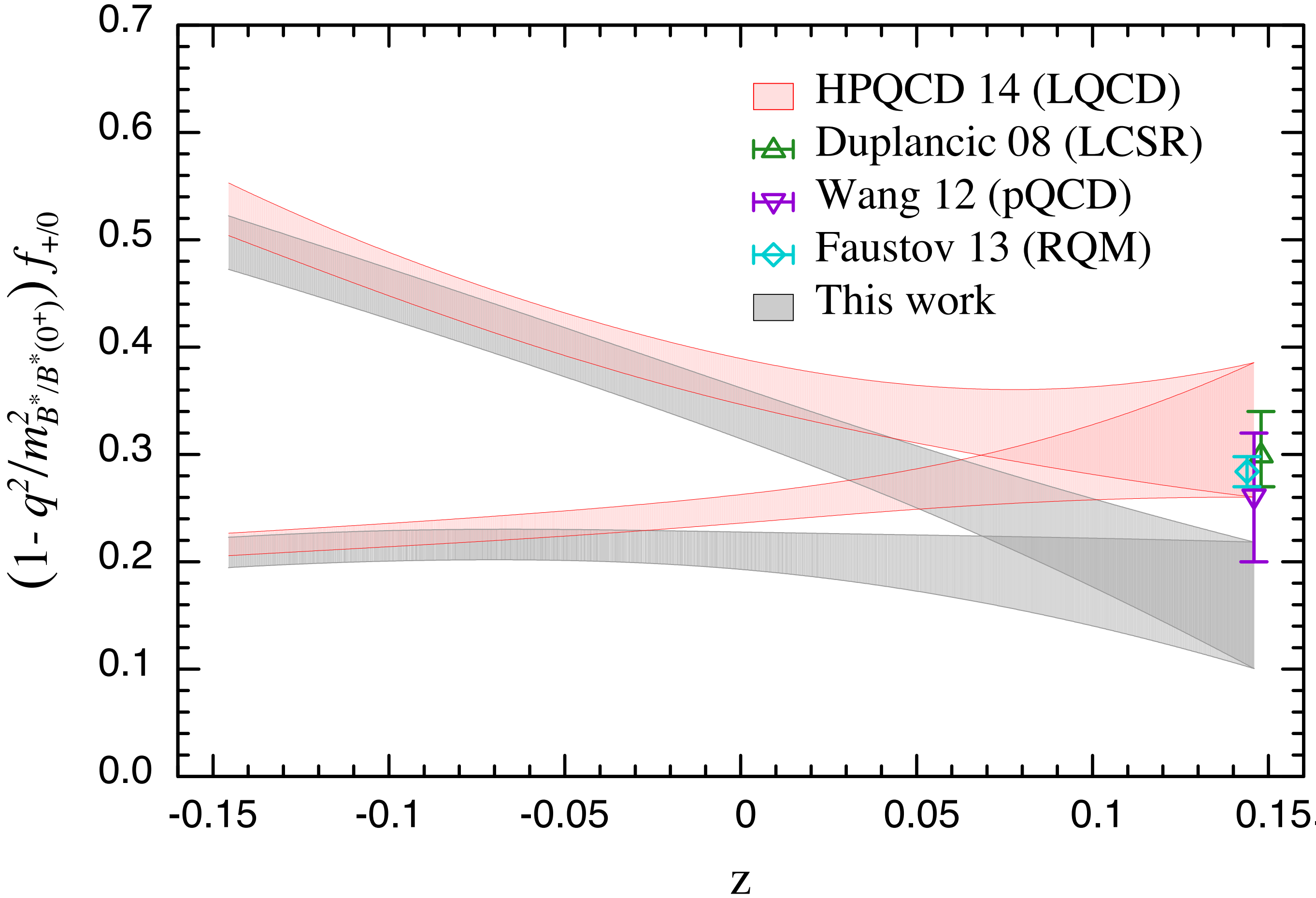


Uses the form factors from the combined LQCD + exp. fit to $d\mathcal{B}(B \to \pi\ell\nu)/dq^2$

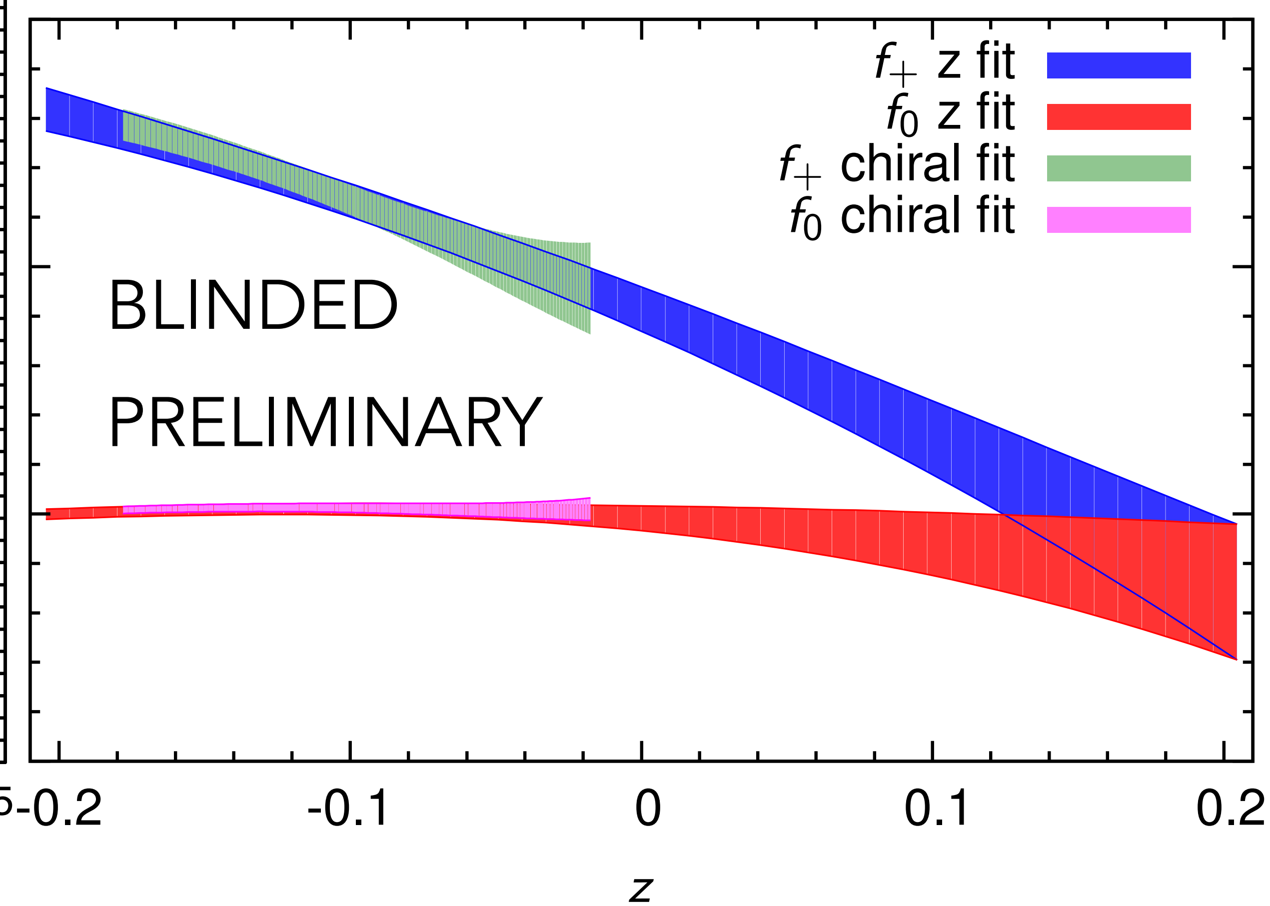
Expect Belle II to measure this ratio.

form factors for $B_s \to K$

RBC/UKQCD [arXiv:1501.05373, PRD 2015]



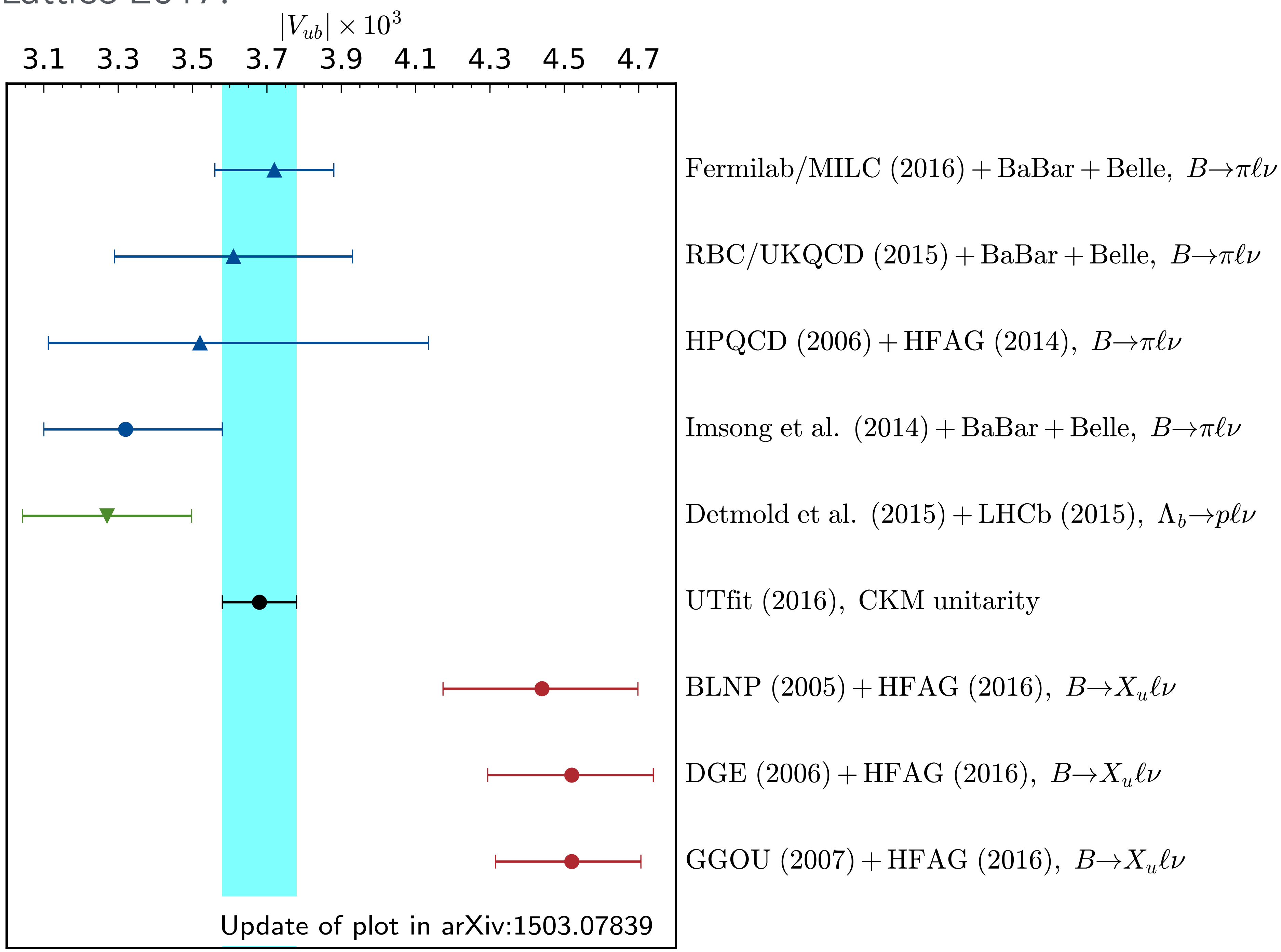
Y. Liu et al, Lattice 2017, arXiv:1711.08085



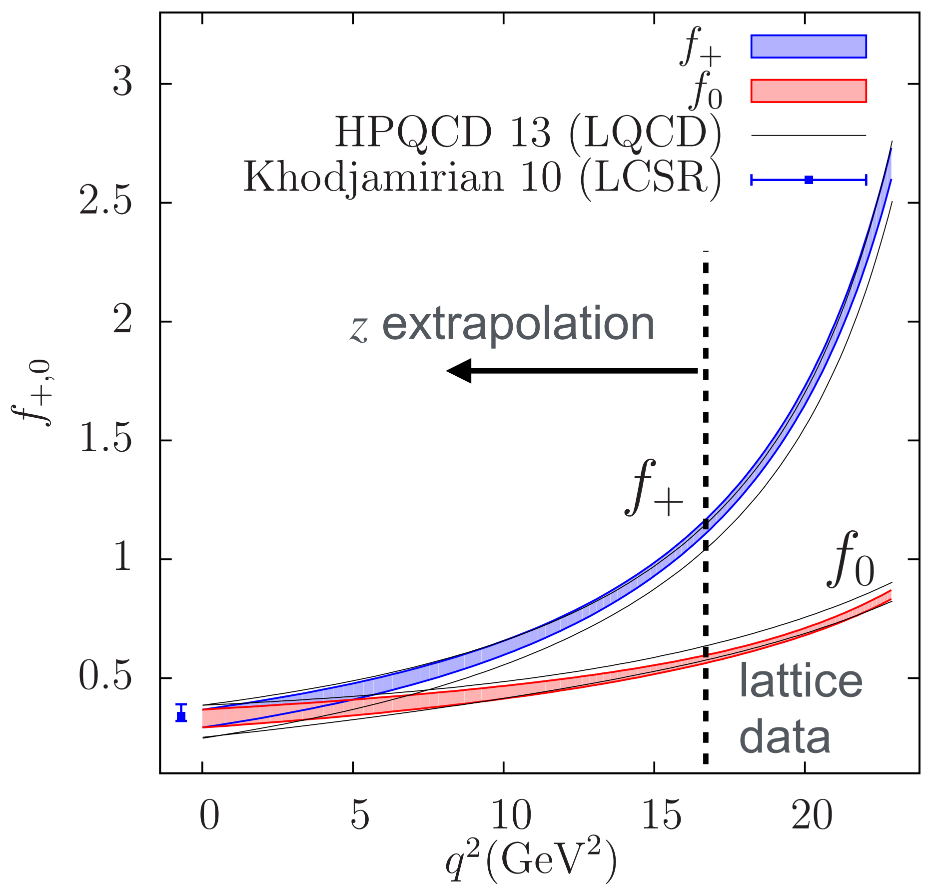
- ⭐ waiting for experimental measurement (LHCb, see talk by M. Calvi)
- ⭐ Lattice form factors consistent
  (some tension at large recoil between RBC/UKQCD and HPQCD)
- ⭐ ongoing work by FNAL/MILC, ALPHA, JLQCD

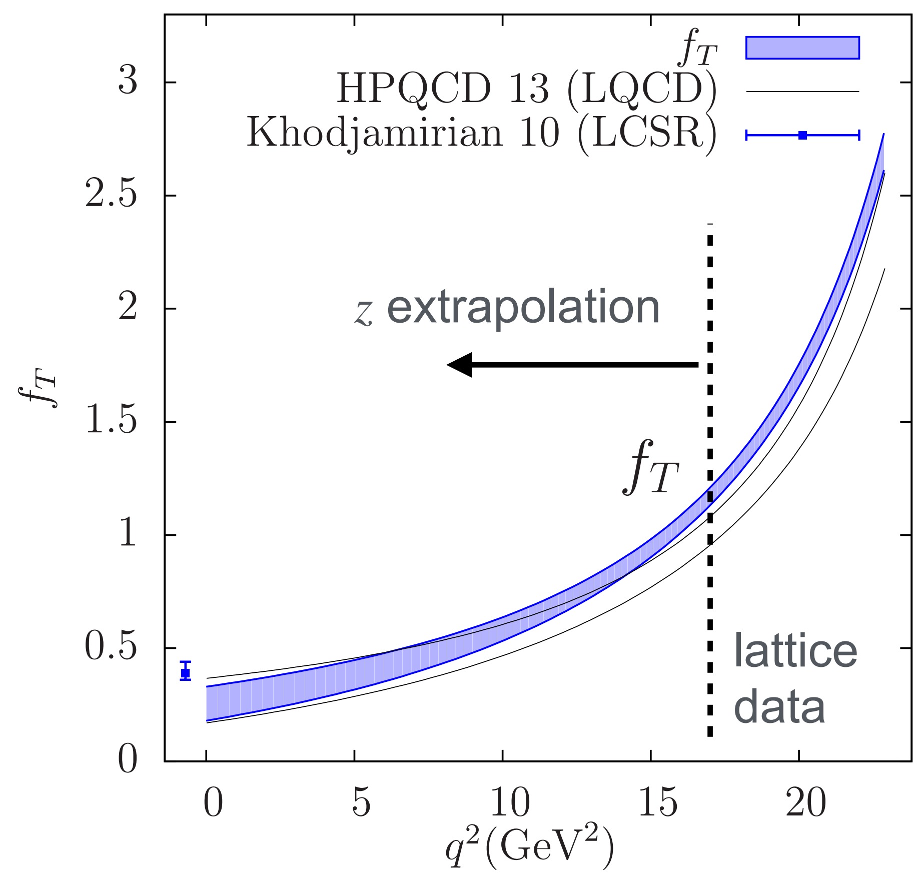
Exclusive $|V_{ub}|$

A. Kronfeld @ Lattice 2017:



form factors for $B \to K\,\ell\ell$



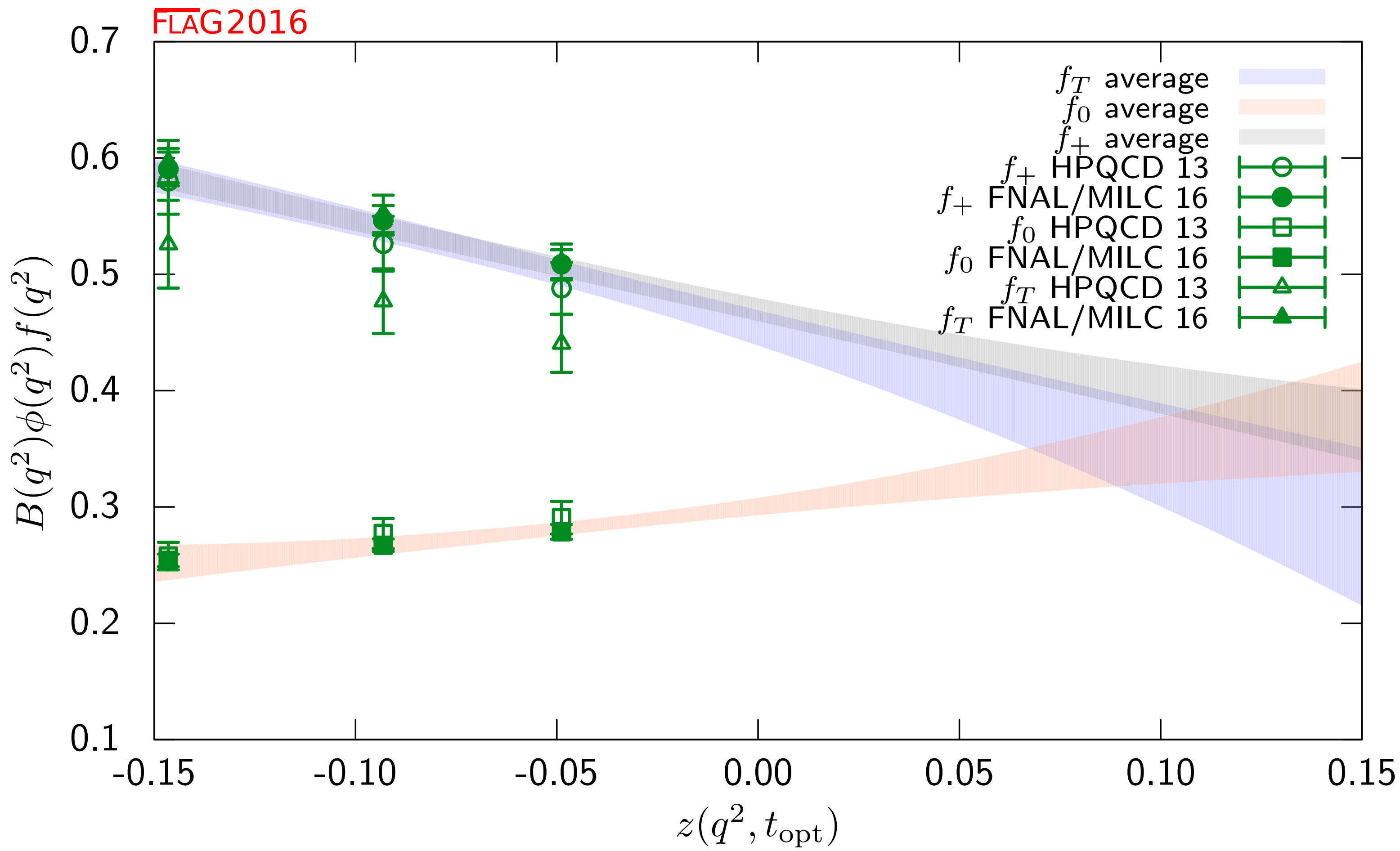


HPQCD [arXiv:1306.0434, 1306.2384, PRL 2013]

FNAL/MILC [arXiv:1509.06235, PRD 2016]

- Two LQCD calculations (on overlapping ensemble sets, different valence actions): HPQCD (NRQCD b + HISQ), FNAL/MILC (Fermilab b + asqtad)
- consistent results for all three form factors
- consistent with LCSR [Khodjamarian et al, arXiv:1006.4945, JHEP 2010]
- Note: First LQCD calculation of $\Lambda_b \to \Lambda\,\ell^+\ell^-$ form factors (10 total) [Detmold & Meinel, arXiv:1602.01399, 2016 PRD].

form factors for $B \to K\,\ell\ell$



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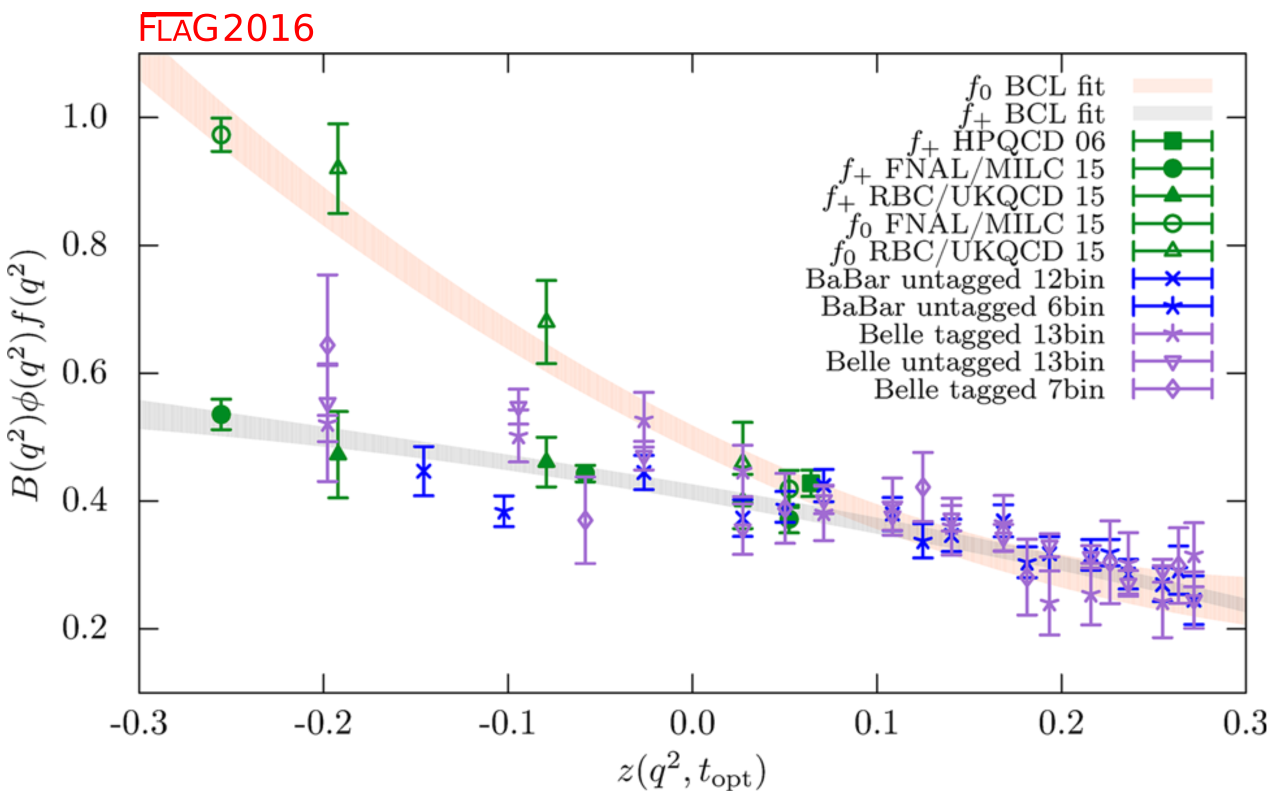
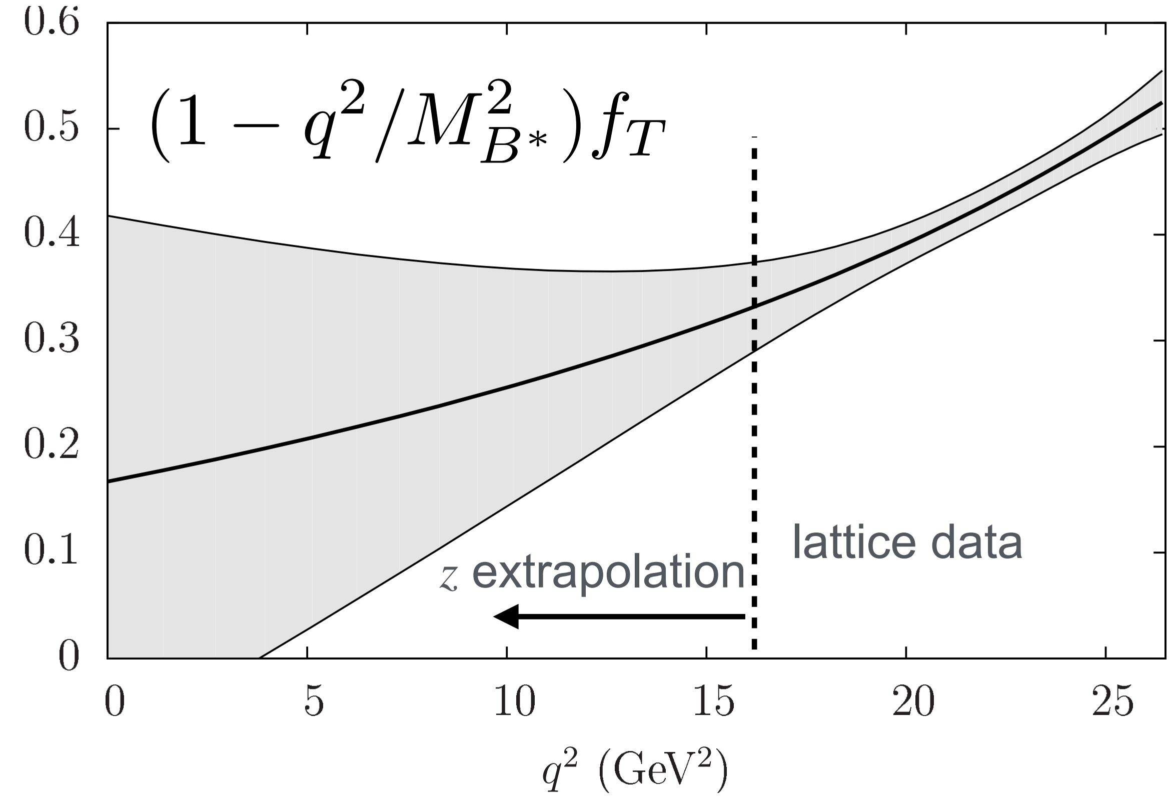
form factors for $B \to \pi \ell\ell$



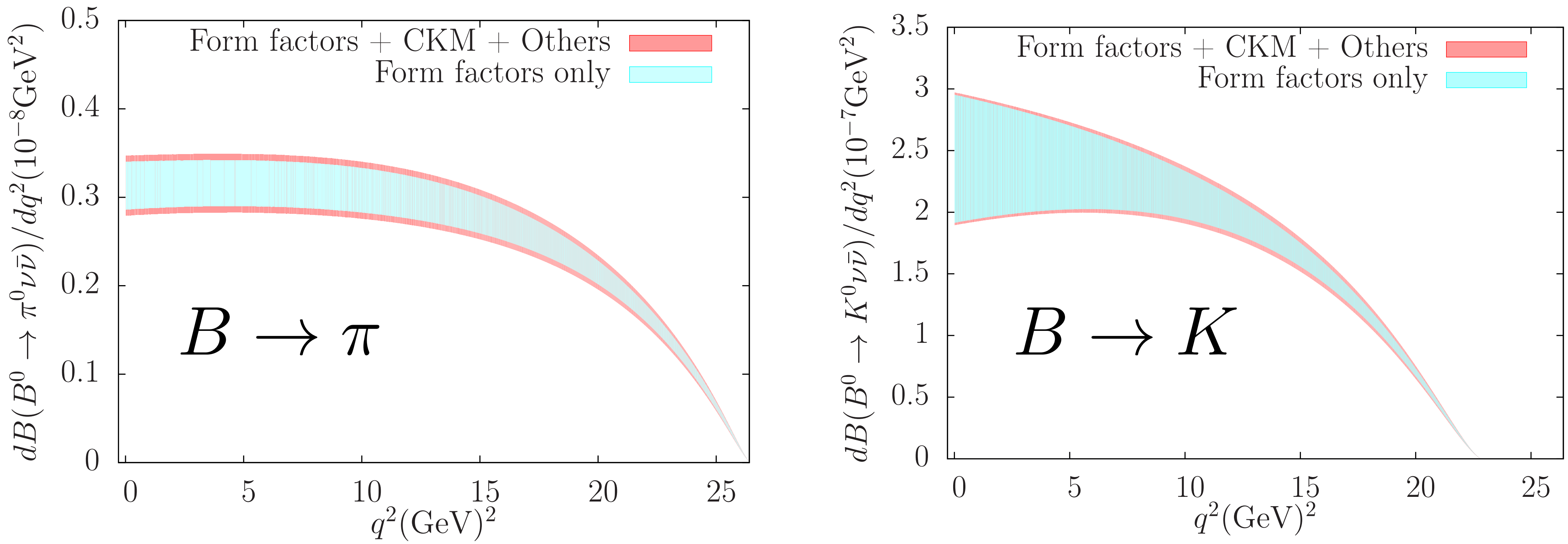
FNAL/MILC [arXiv:1507.01618, PRL 2015]



- LQCD calculation of f_T by FNAL/MILC
- Take f_+, f_0 from combined fit of lattice form factors + experimental data for $d\mathcal{B}(B \to \pi \ell \nu)/dq^2$

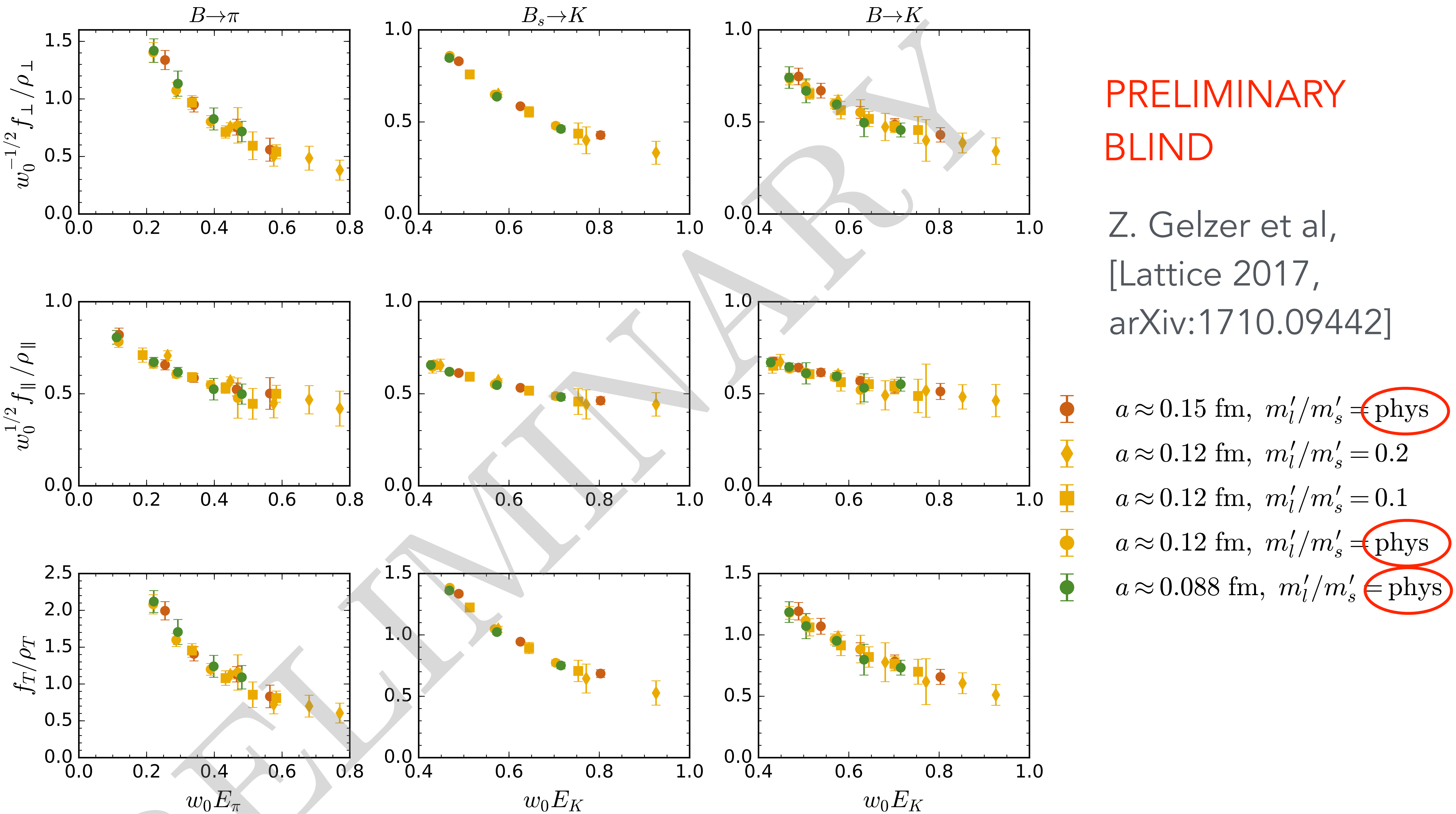
Rare semileptonic B decays to $\nu\bar{\nu}$ states

D. Du et al [arXiv:1510.02349, PRD 2016]



- theoretically clean, but difficult to measure experimentally
- Some modes will be accessible to Belle II

form factors for $B \to \pi, B_s \to K, B \to K$



PRELIMINARY
BLIND

Z. Gelzer et al,
[Lattice 2017,
arXiv:1710.09442]

Existing/ongoing/planned form factor calculations

Semileptonic B decays to light vector mesons

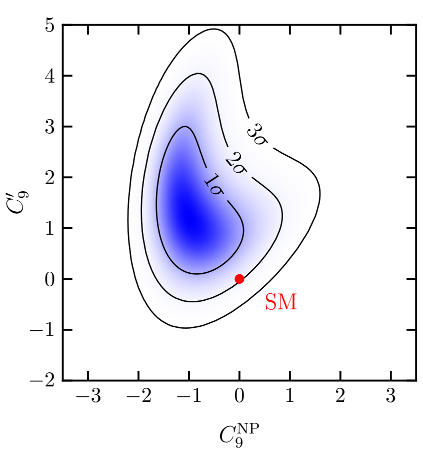
K^*, ϕ are treated as stable mesons

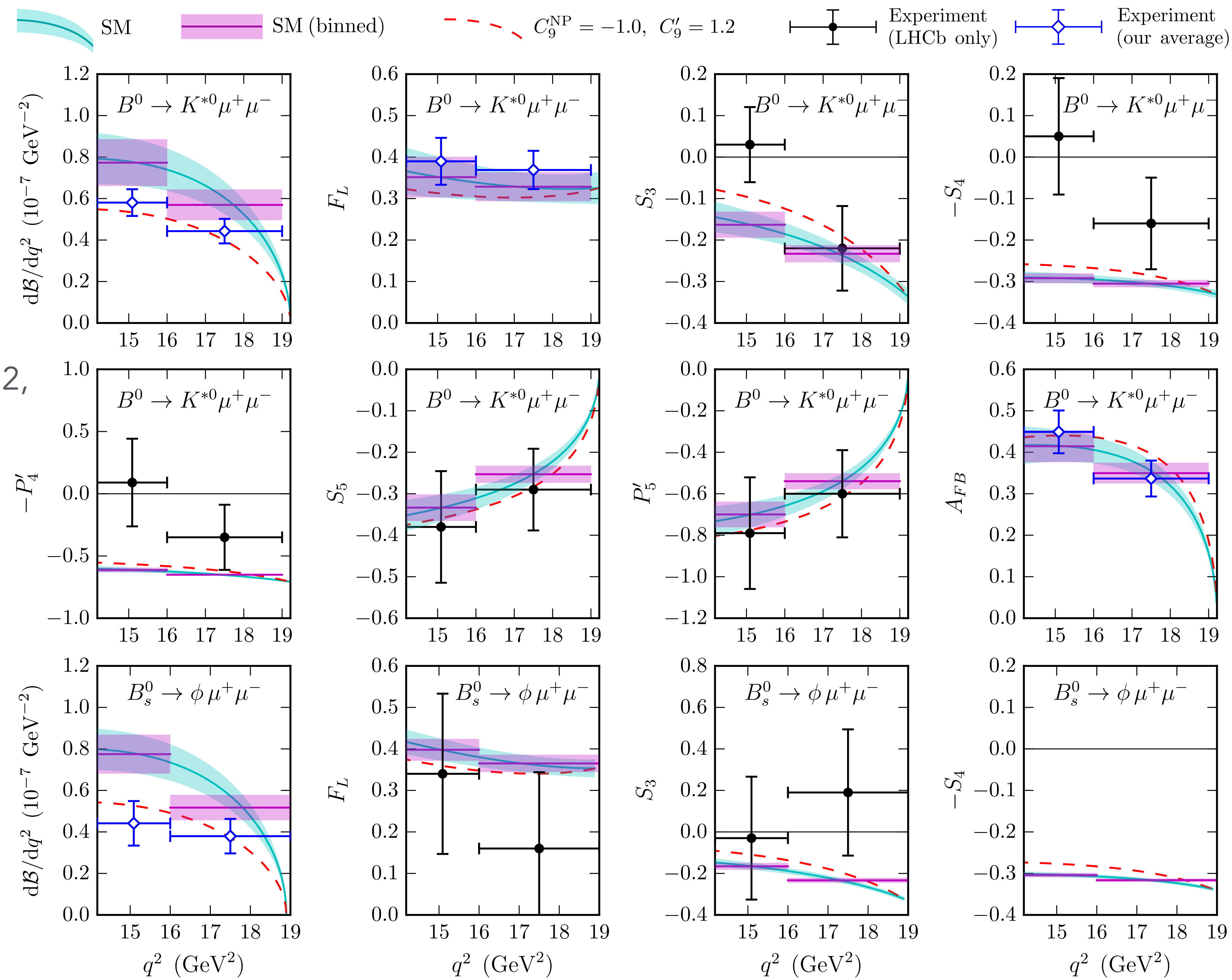
- Cambridge group (Horgan, Liu, Meinel, Wingate):
  NRQCD b quarks+ asqtad light valence
  - $B_s \to K^*$, $B \to K^*$, $B_s \to \phi$ form factors
  - MILC asqtad (2+1) ensembles
  - [arXiv:1310.3722, PRD 2014; arXiv:1310.3887, PRL 2014]
  - 3 ensembles, $a \approx 0.09, 0.12$ fm, $m_\pi = 270\text{-}400$ MeV, 1-loop PT matching

- RBC/UKQCD
  RHQ action for b quark + DWF light on DWF (2+1) ensembles
  - $B_s \to \phi$ form factors, preliminary results
  - [E. Lizarazo @ Lattice 2017; arXiv:1612.05112]
  - 7 ensembles, $a \approx 0.07, 0.09, 0.11$ fm, $m_\pi = 139\text{-}430$ MeV,
  - mostly nonperturbative renormalization, aka mNPR (with 1-loop PT matching)
  - ongoing effort to add more ensembles, including at physical mass

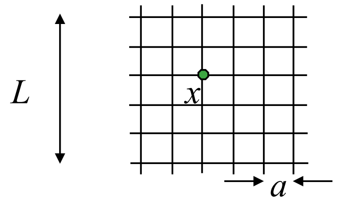
Rare semileptonic $B_{(s)}$ decay to vector states

Horgan et al
[arXiv:1310.3722, PRD 2014]





Semileptonic $B_{(s)}$ decays to vector states



Lattice QCD Introduction

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in χPT
- compare results at several Ls (with other parameters fixed)

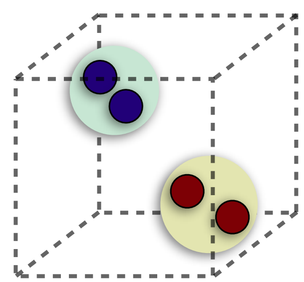
The story changes completely if there are two or more hadrons in the initial/final/intermediate state.

Semileptonic $B_{(s)}$ decays to vector states

R. Briceno @ Lattice 2014 [arXiv:1411.6944]

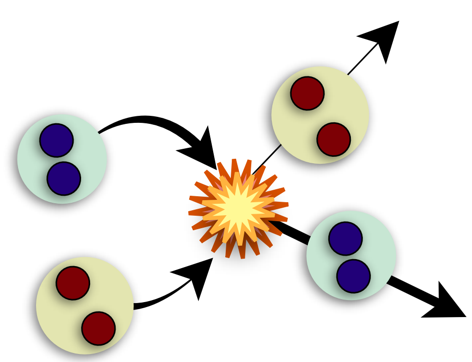
A roadmap towards physics

1. Calculate finite volume spectrum



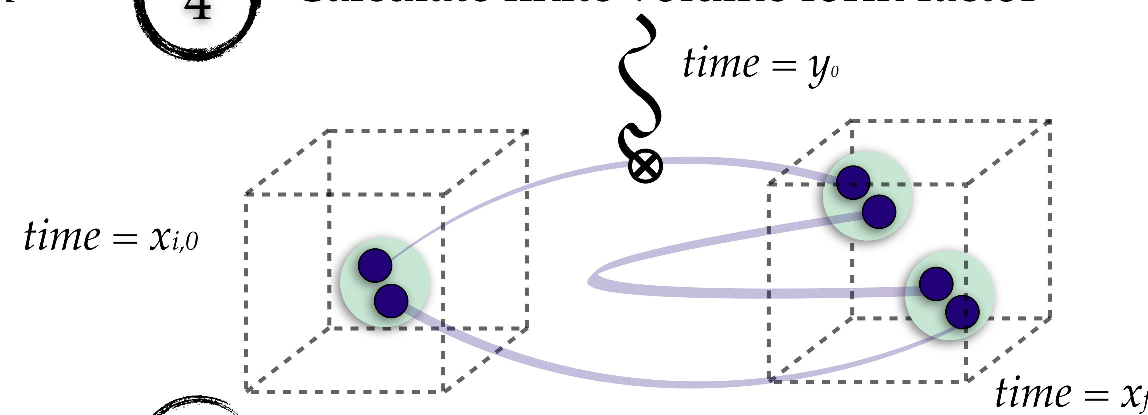
2. Plug into formalism

3. Out goes elastic & inelastic QCD scattering amplitudes



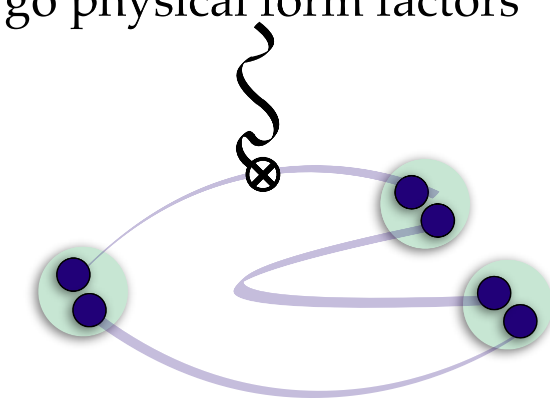
à la mode de **Lüscher (1986)**

4. Calculate finite volume form factor



5. Plug spectrum, scattering parameters and finite volume form factor into formalism

6. Out go physical form factors



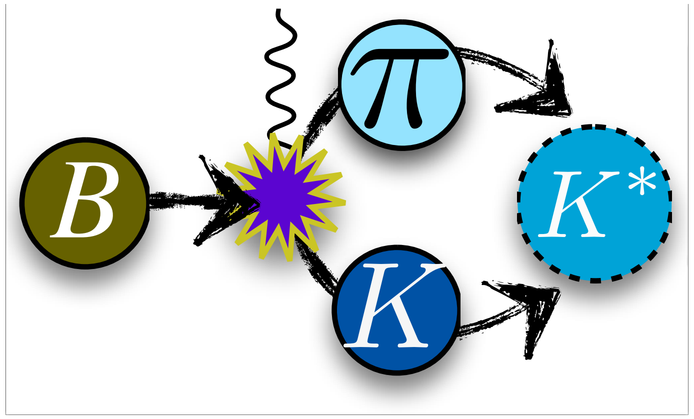
à la mode de **Lellouch & Lüscher (2000)**

Semileptonic $B_{(s)}$ decays to vector states

Formalism for multi-channel 1→ 2 transition amplitudes:

[Briceno, Hansen, Walker-Loud, arXiv:1406.5965, PRD 2015;1502.04314, PRD 2015]

weak current

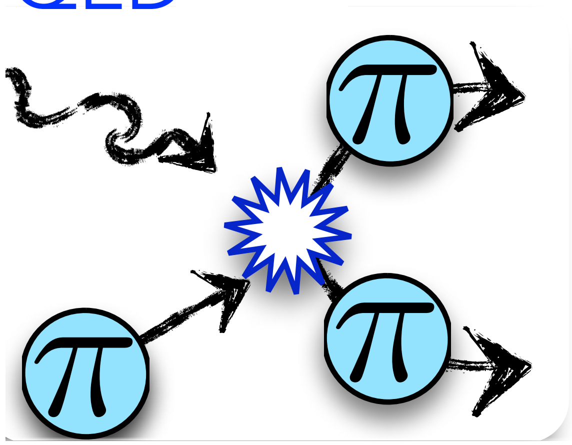


pilot study

[Agadjanov et al, arXiv:1605.03386, NPB 2016]

Same as for:

QED



studies of $\pi\gamma^{(*)}\to\pi\pi$

[Briceno et al arXiv:150706622, PRL 2015; arXiv:1604.03530, PRD 2016; Leskovec et al, arXiv: 1611.00282, Lattice 2016]

Summary

- ☆ LQCD results exist for complete set of $B \to \pi, K; B_s \to K$ form factors, including the tensor form factors
- ☆ Errors on $B \to \pi$ form factors are commensurate with current experimental uncertainties.
  - ➢ enable SM predictions of $R(X)$ and $|V_{ub}|$ determinations
  - ➢ constrain BSM theories
- ☆ Expect to see new LQCD results for $B_{(s)} \to \pi, K$ form factors with physical mass ensembles soon.
- ☆ Similar LQCD results for form factors for semileptonic $B_{(s)}$-meson decays to charm mesons.

Outlook

Amala Willenbrock

Outlook

- Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations.
- including charm sea quarks and adding isospin breaking effects ($m_u \neq m_d$) straightforward
- next milestone: Semileptonic B decay form factors to pseudo scalars with (sub)percent level precision
- structure-dependent QED effects:
  - ➢ program being developed for kaon decays, muon g-2 [D. Giusti et al, arXiv:1711.06537, PRL 2018; …]
  - ➢ need to extend to D,B-meson quantities
- First complete calculations of weak $\Lambda_{b,c}$ decay form factors
- theoretical framework for semileptonic B decays to vector meson final states exists [Briceño et al, arXiv:1406.5965, 2015 PRD; Agadjanov et al, arXiv:1605.03386].
  - ➢ enables LQCD calculations of form factors for $B_s \to K^* \ell\nu, \ B \to K^* \ell\ell, \ldots$ pilot studies are underway.

Thank you!

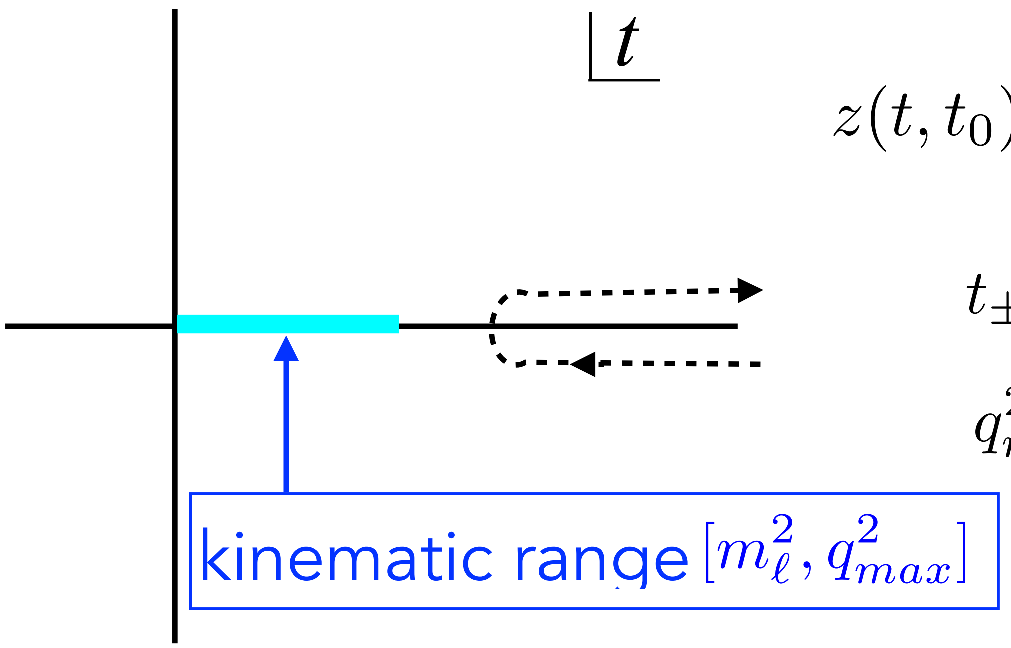
Farah Willenbrock

Appendix

Collaborators

- Fermilab Lattice Collaboration:
  AXK, Freeland, Gámiz, Gottlieb, Kronfeld, Laiho, Mackenzie,
  Neil, Simone, Van de Water
  Bailey, Bouchard, Chang, Gelzer, Liu
- MILC:
  Bazavov, Bernard, DeTar, Gottlieb, Heller, Sugar, Toussaint,
  Brown, Kim, Komijani, Li, Vacquero
- Computations done at
  NCSA (Blue Waters), ALCF (Argonne), NERSC, NCAR, TACC, USQCD clusters, ….

The z-expansion

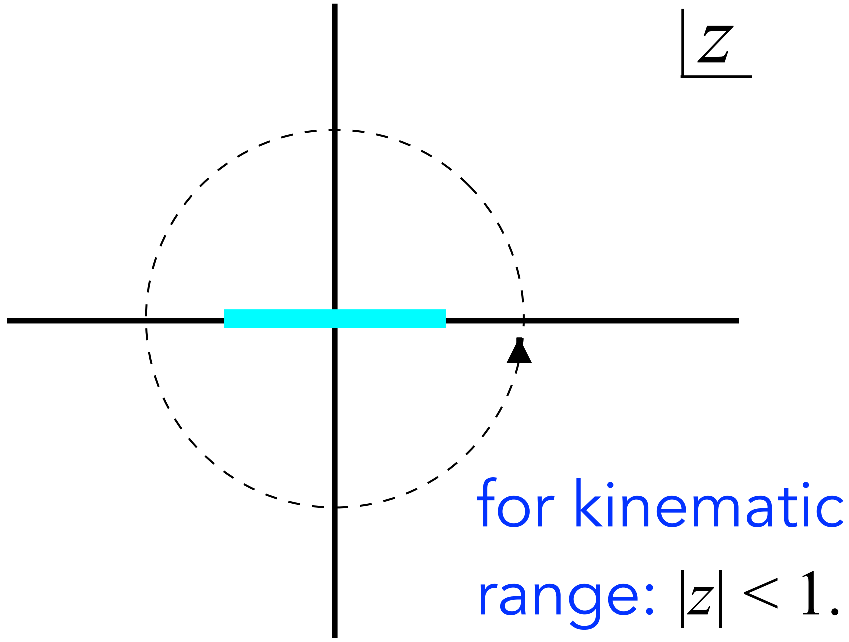


$$t = q^2$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$

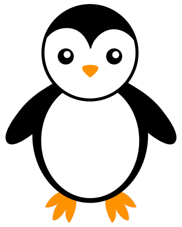


The form factor can be expanded as:

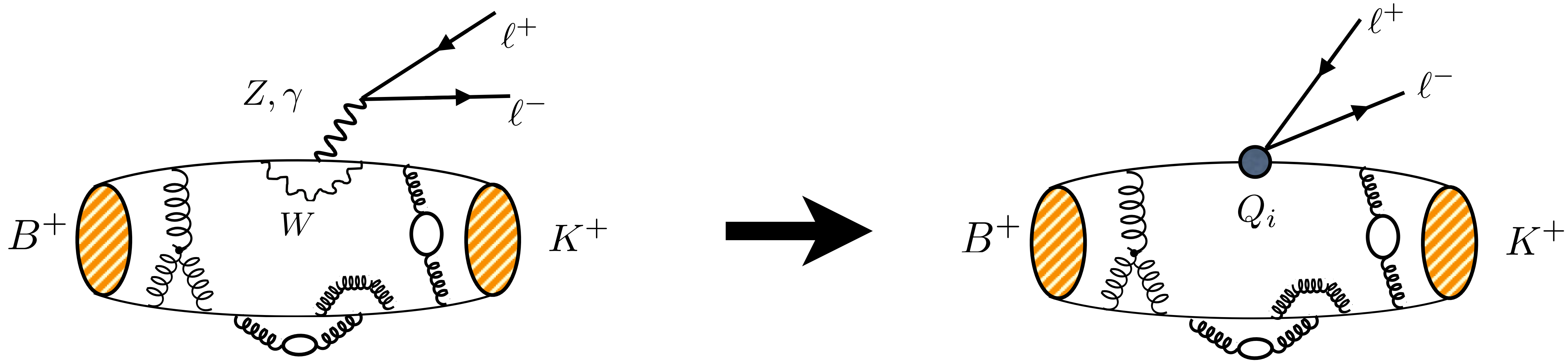
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0} a_k(t_0) z(t,t_0)^k$$

Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd, Grinstein, Lebed (hep-ph/9412324, PRL 95; hep-ph/9504235, PLB 95; hep-ph/9508211, NPB 96; hep-ph/9705252, PRD 97)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)
....

- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.



Rare semileptonic B decay



$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tq}^* V_{tb} \sum_i C_i(\mu) Q_i + \ldots$$

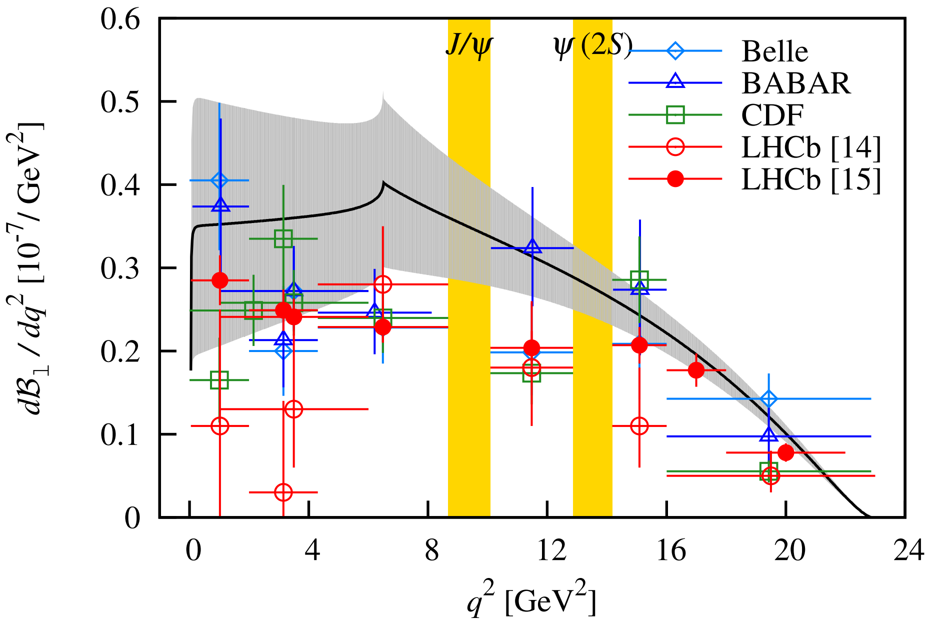
Parameterize the amplitude in terms of the three form factors $f_{+,0,T}(q^2)$:

$$A(B \to P\,\ell\ell) \sim C_7^{\text{eff}} f_T + (C_9^{\text{eff}} + C_{10}) f_+ + \text{nonfactorizable terms}$$

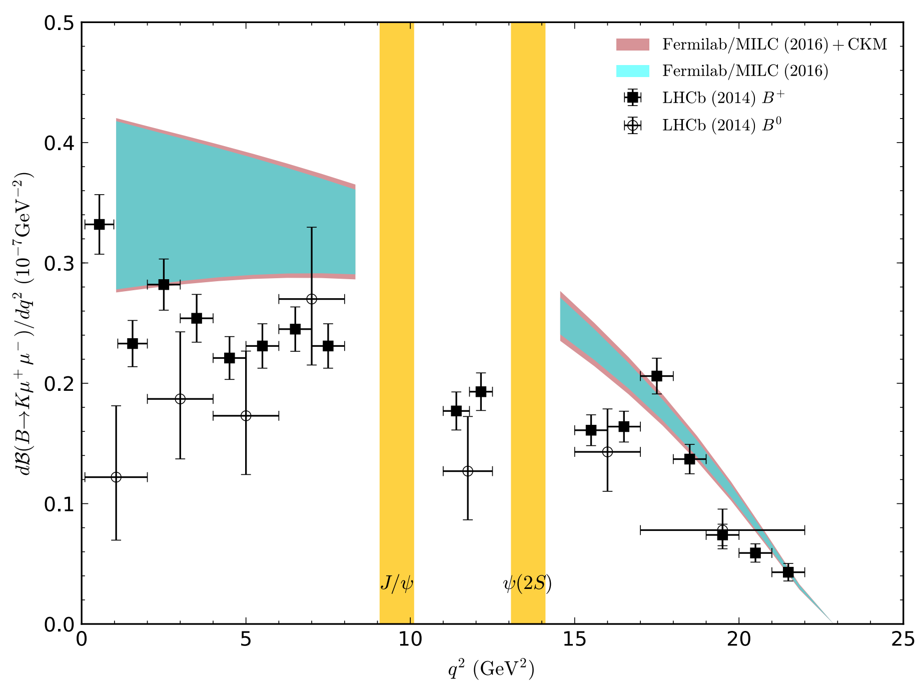
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

$B \to K$ Experiment vs. Theory $B \to \pi$

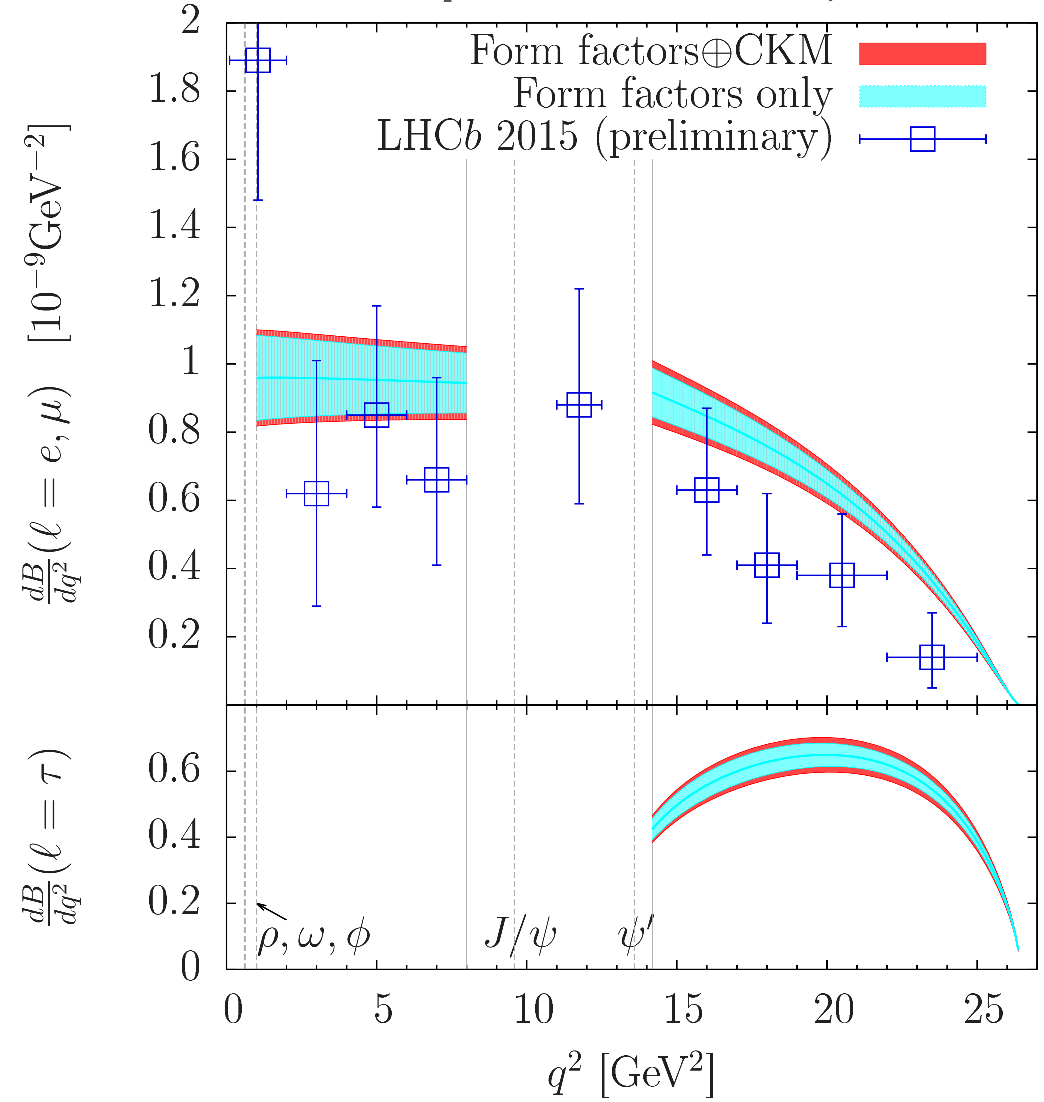
HPQCD [arXiv:1306.0434, 2013 PRL]



FNAL/MILC [arXiv:1509.06235, 2016 PRD



FNAL/MILC [arXiv:1507.01618, 2015 PRL]



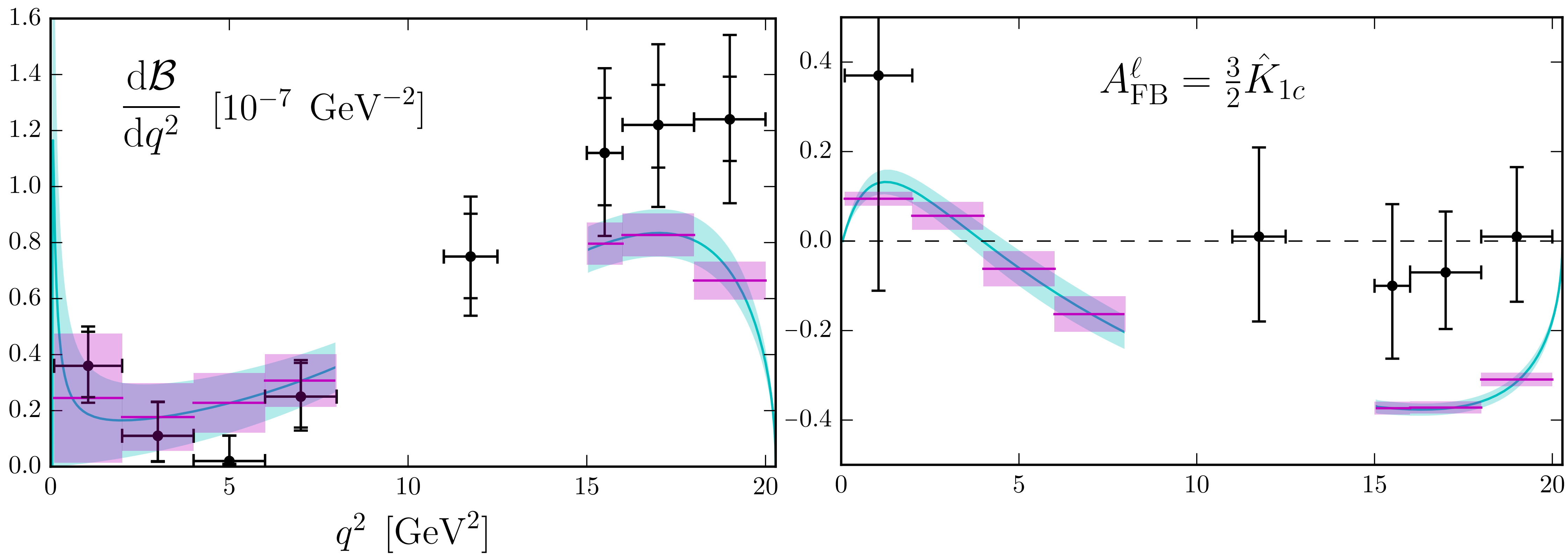
A. Kronfeld @ Lattice 2017
(omit older exp. data)

Phenomenology for $\Lambda_b \to \Lambda\, \ell^+ \ell^-$

Experiment vs. theory

- LHCb data + Detmold & Meinel form factors
  [arXiv:1503.07138, JHEP 2015]
- focus on regions above and below charmonium resonances
- exp. data lie above SM theory ~1-3σ tensions

Detmold & Meinel [arXiv:1602.01399, PRD 2016]

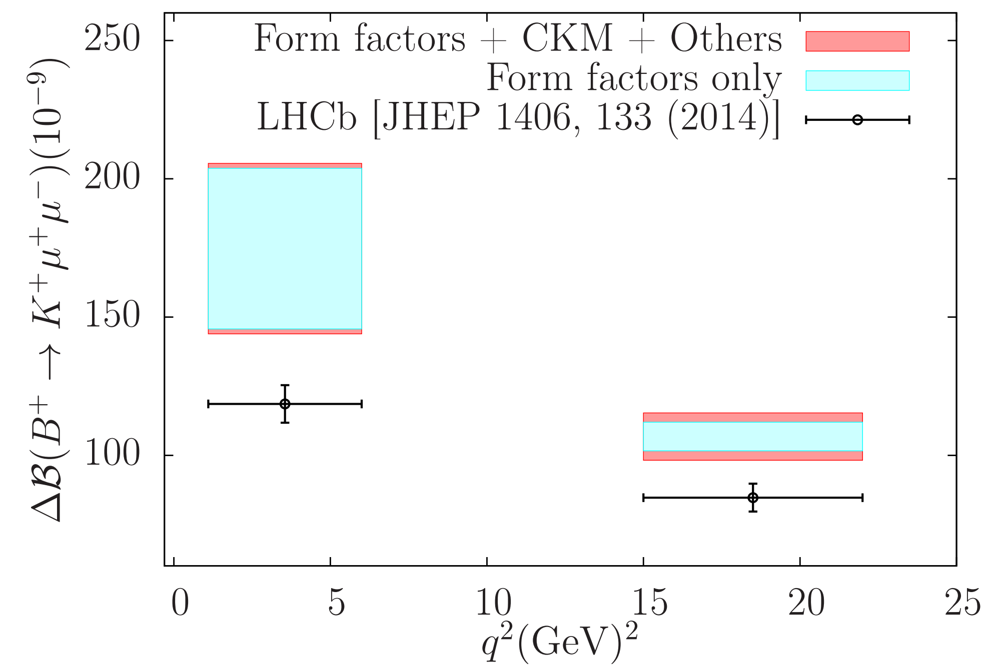


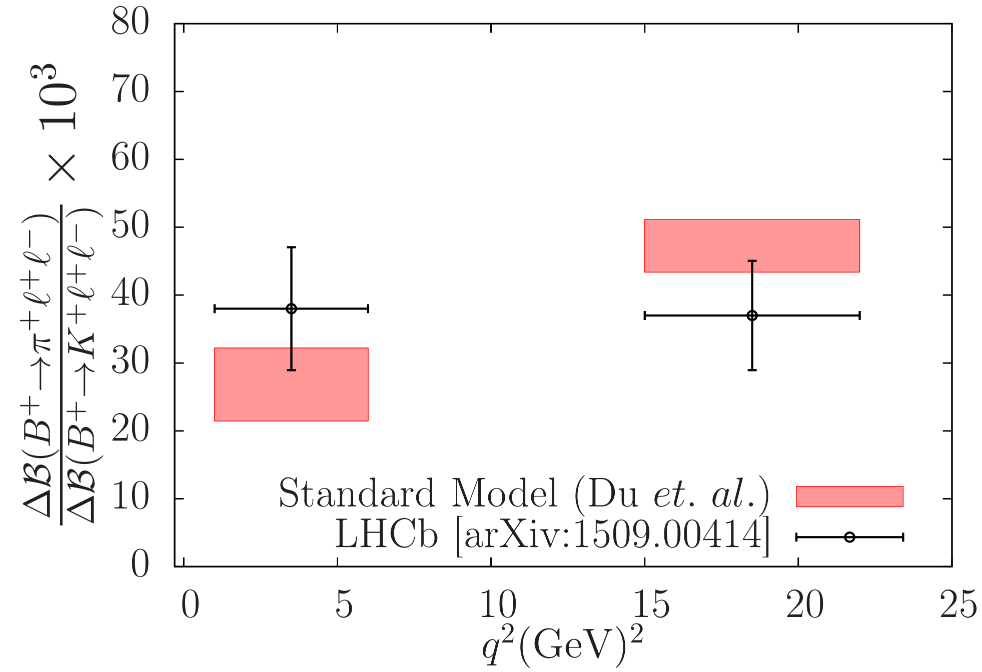
Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Experiment vs. theory

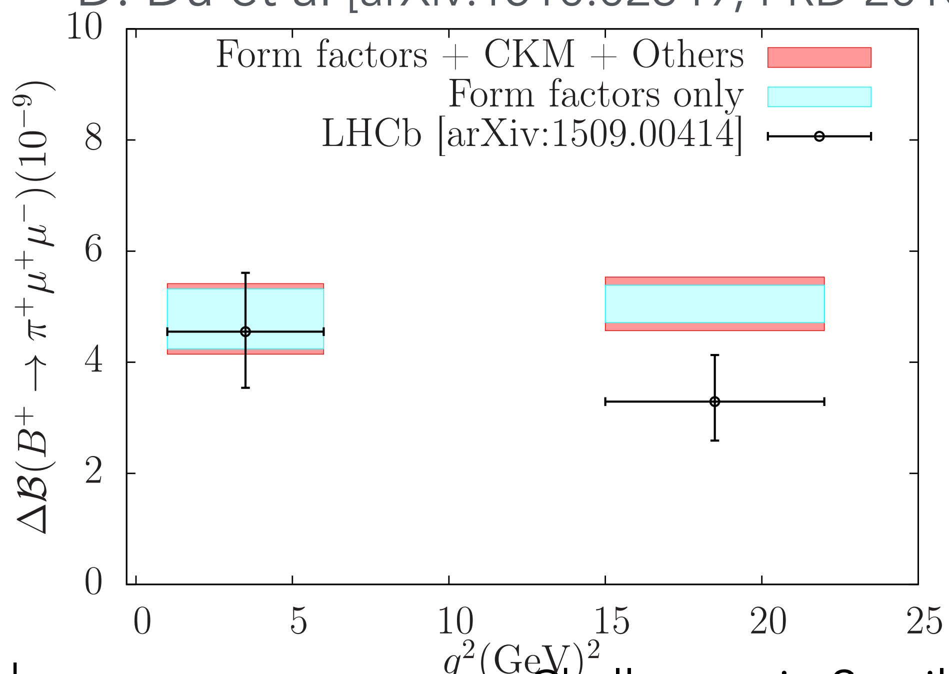
- LHCb data + FNAL/MILC form factors [arXiv:1509.00414, JHEP 2015;1403.8044, JHEP 2014]
- focus on large bins above and below charmonium resonances
- theory errors commensurate with experiment (but nonfactorizable contributions not under good control)
- yields ~1-2σ tensions
- $\Rightarrow$ determine $|V_{td}/V_{ts}|,|V_{td}|,|V_{ts}|$ or constrain Wilson coefficients

D. Du et al [arXiv:1510.02349, PRD 2016]





D. Du et al [arXiv:1510.02349, PRD 2016]



BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

D. Du et al [arXiv:1510.02349, PRD 2016]:

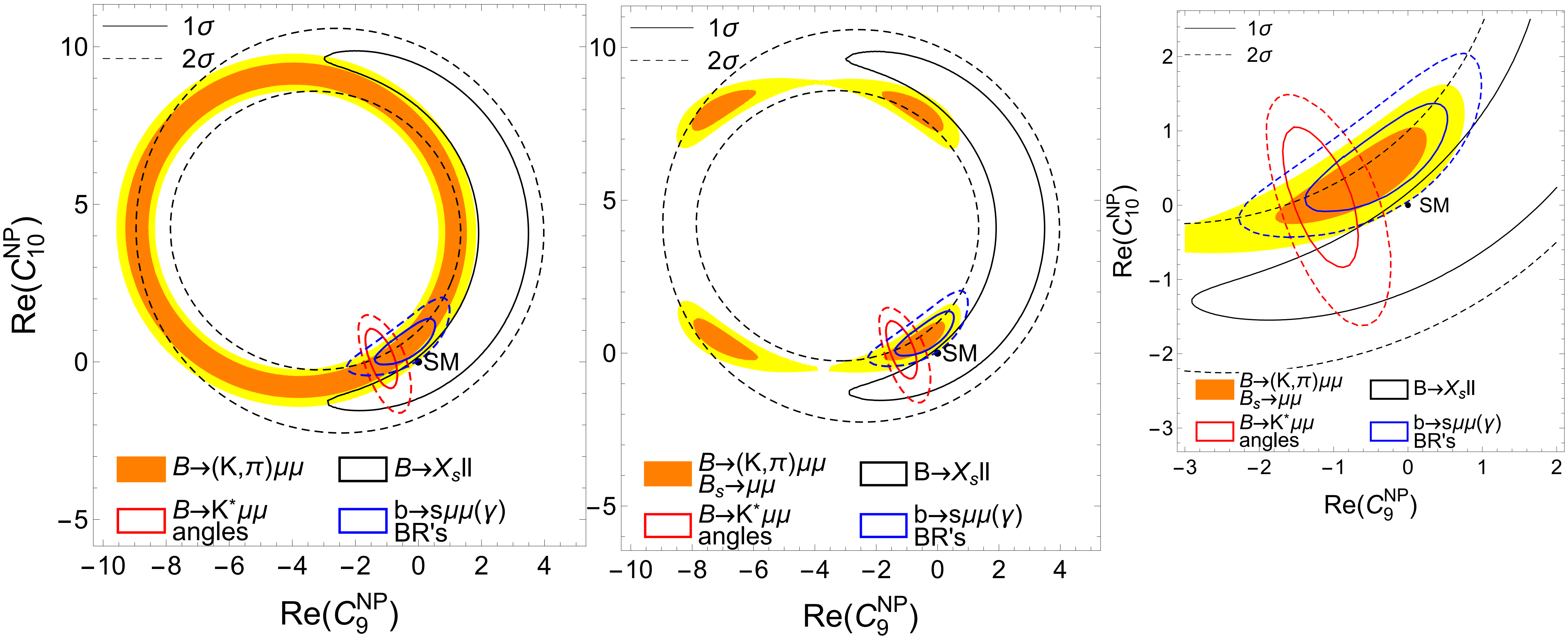
Constraints on Wilson coefficients (C_9, C_{10})

- New physics contributions modify the Wilson coefficients: $C_i \to C_i + C_i^{\rm NP}$ at the high scale, $\mu_0 = 120$ GeV
- use constraints from $B \to X_s \gamma$ to set $C_{7,8}^{\rm NP} = 0$
- assume MFV so that $C_i(b \to s\,\ell\ell) = C_i(b \to d\,\ell\ell)$
- assume $C_{9,10}^{\rm NP}$ are real (no new CP violating phases)
- take measured $\Delta\mathcal{B}(B \to K, \pi\, \mu^+\mu^-)$ in $\Delta q^2 = 1-6, 15-22\ {\rm GeV}^2$ together with FNAL/MILC form factors
- add $B_s \to \mu^+\mu^-$ constraint with lattice f_{Bs}

BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

D. Du et al [arXiv:1510.02349, PRD 2016]

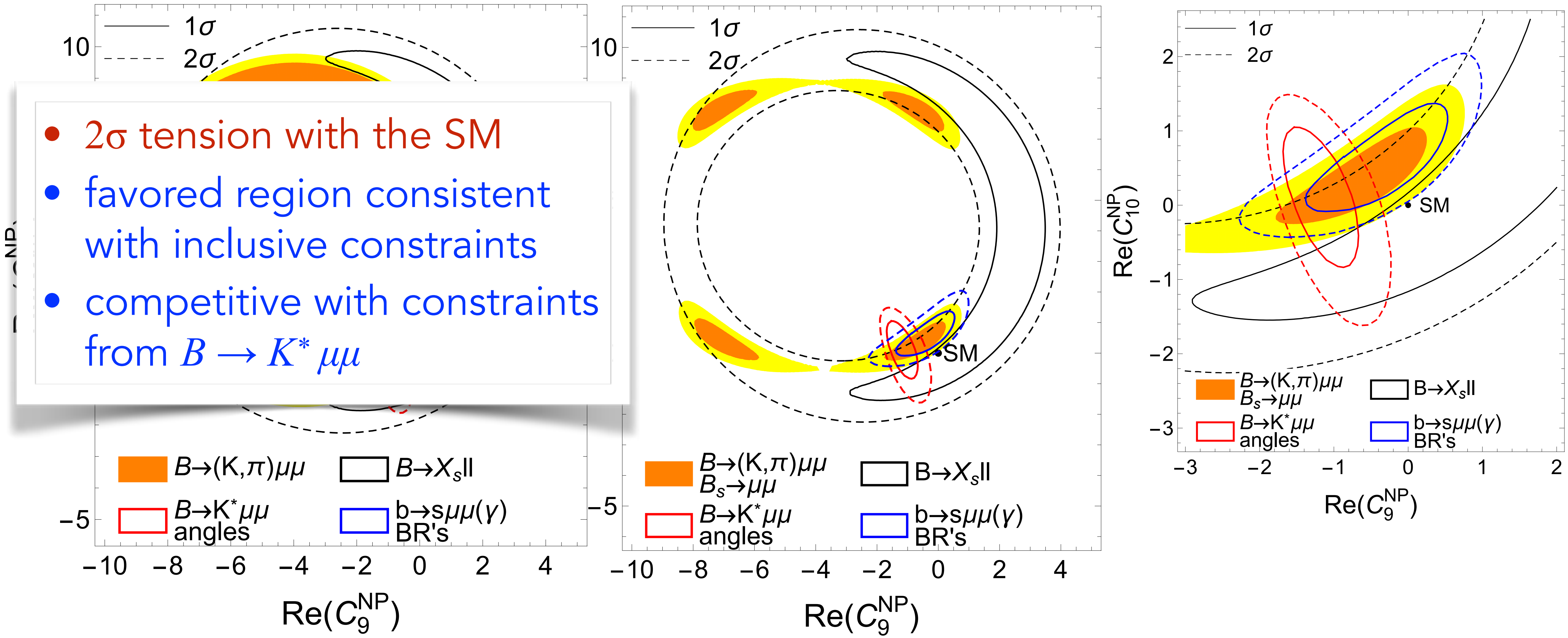


$\boxed{B \to K\,\mu\mu\text{, high } q^2 \text{ bin dominates constraint}}$

BSM Phenomenology for $B \to K, \pi\, \ell^+ \ell^-$

Constraints on Wilson coefficients (C_9, C_{10})

D. Du et al [arXiv:1510.02349, PRD 2016]

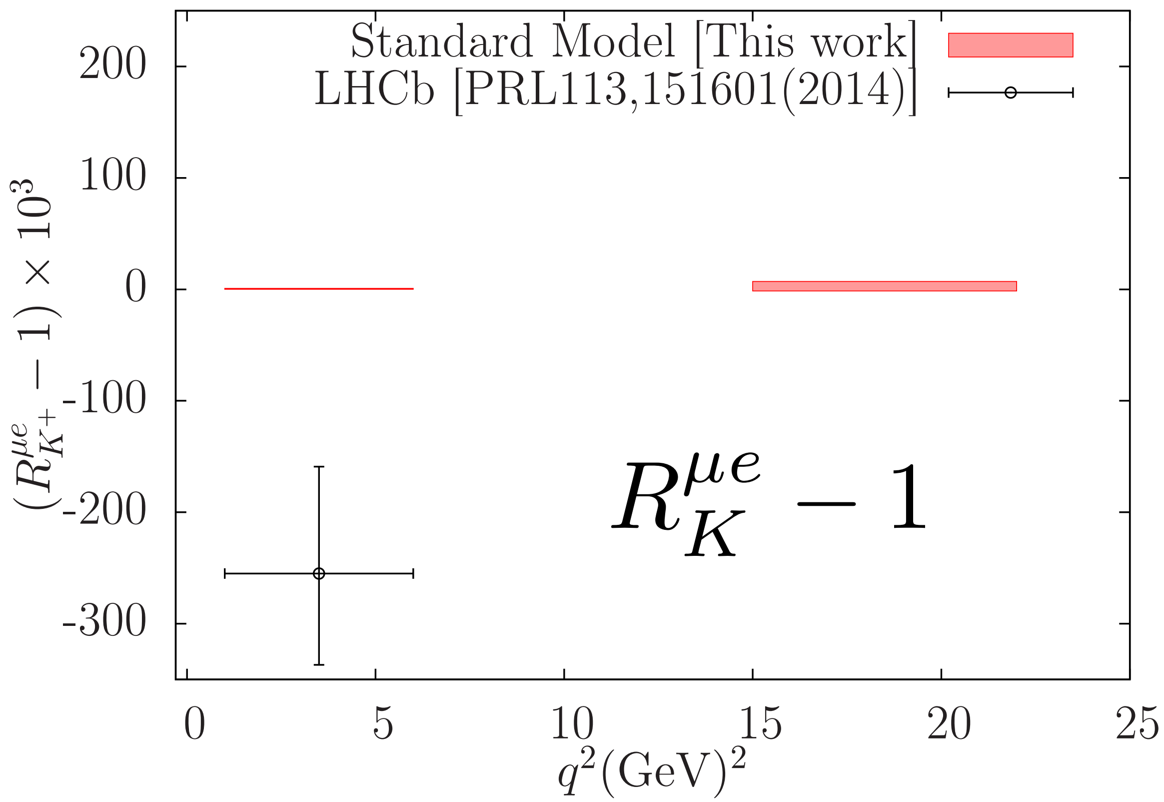


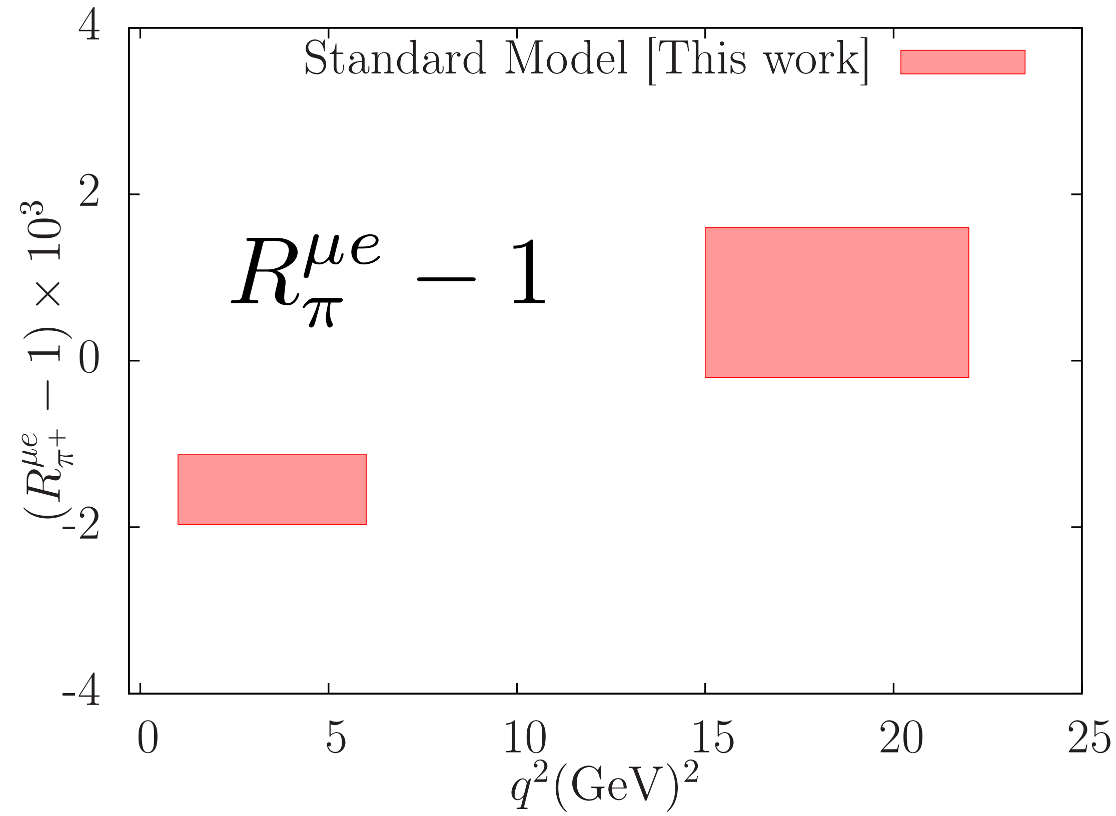
- 2σ tension with the SM
- favored region consistent with inclusive constraints
- competitive with constraints from $B \to K^* \mu\mu$

$B \to K\, \mu\mu$, high q^2 bin dominates constraint

BSM phenomenology: LFU μ/e

D. Du et al (arXiv:1510.02349, PRD 2016)

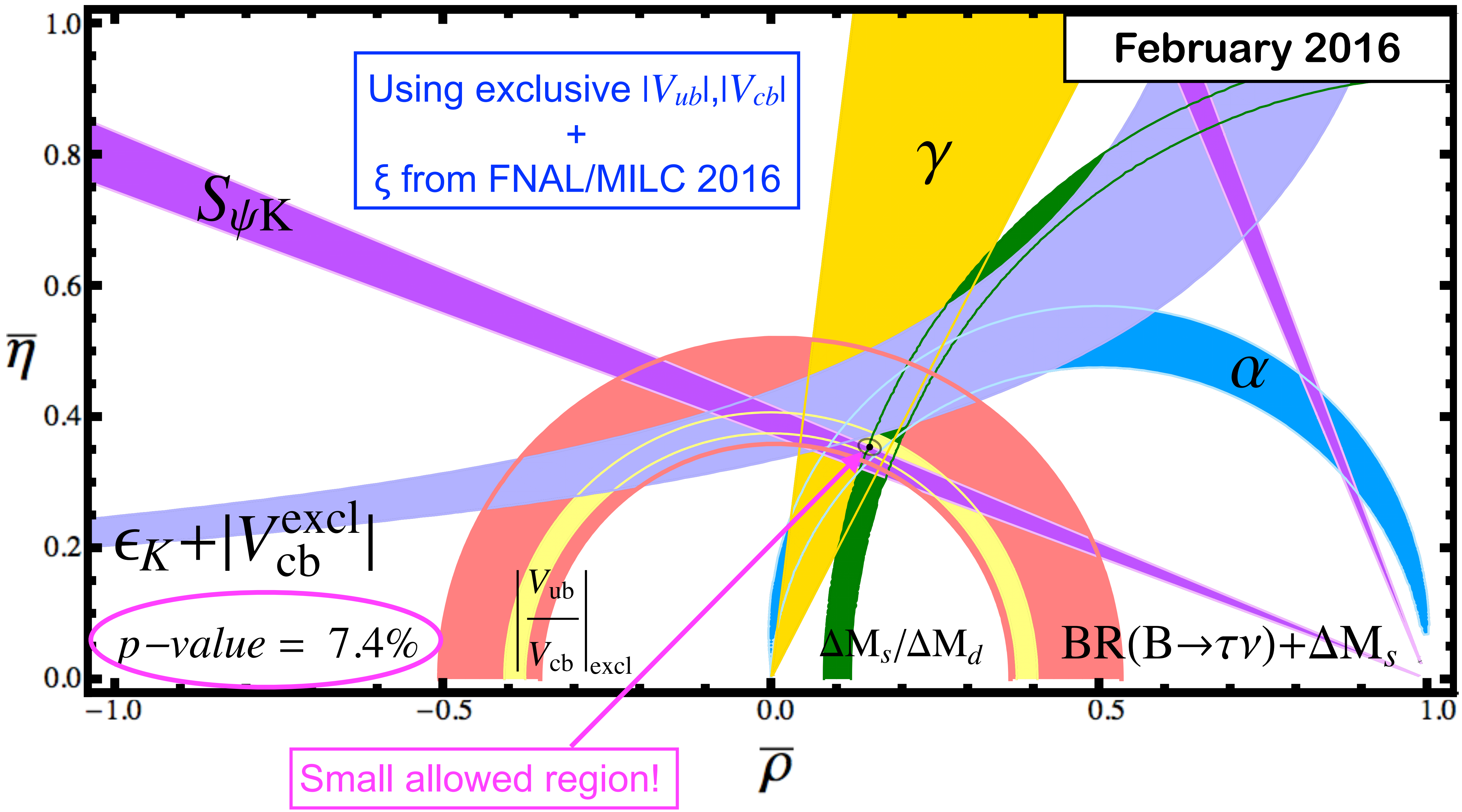




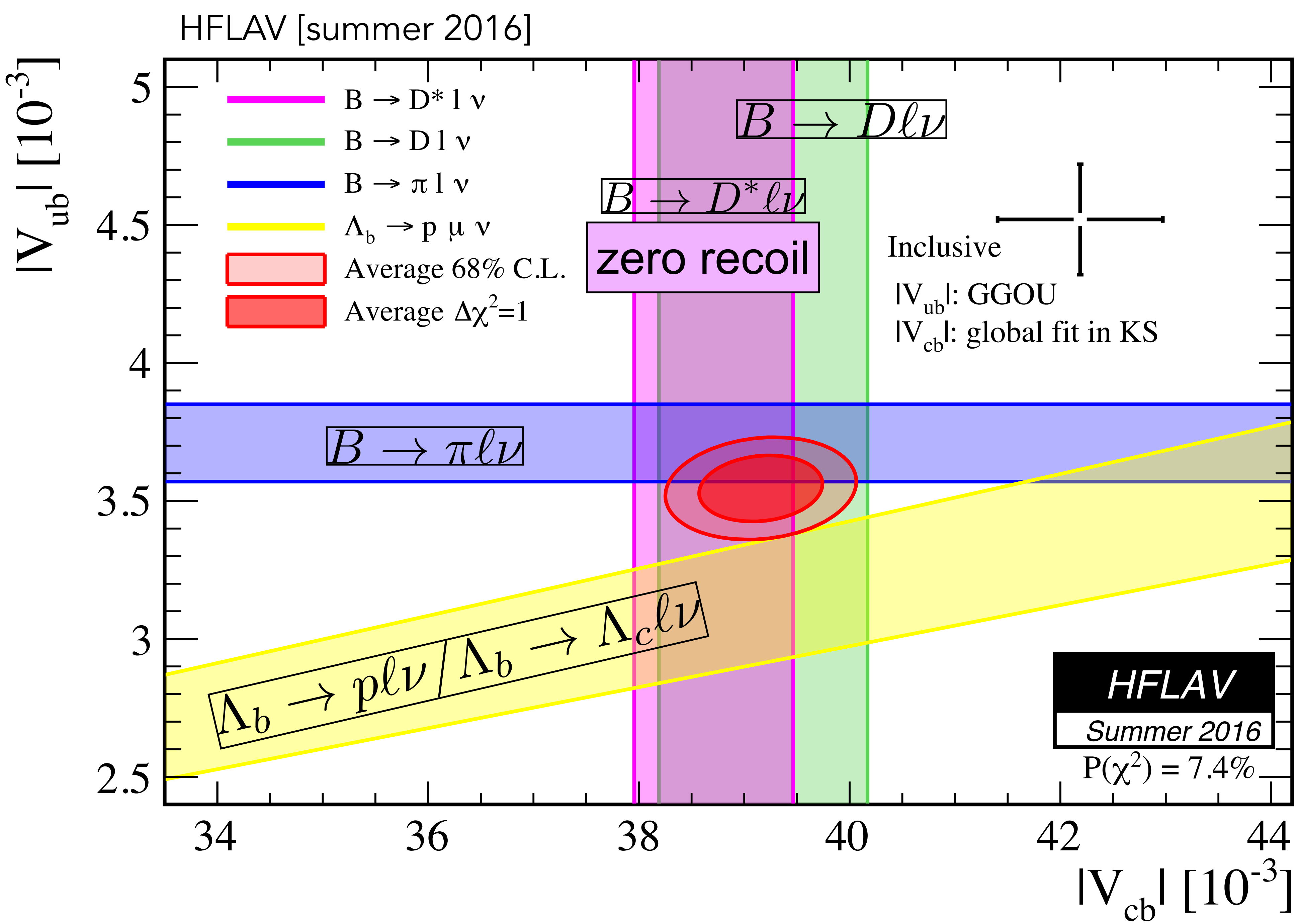
- ☆ $\sim 2.6\,\sigma$ tension between LHCb measurement and SM theory
- ☆ renewed interest due to recent LHCb measurement of R_{K^*}
  [R. Ajai et al, arXiv:1705.05802, JHEP 2017]
- ☆ The SM predictions of these ratios are insensitive to the form factors and nonfactorizable contributions.

UT analysis

Laiho, Lunghi, Van de Water [arXiv:0910.2928, arXiv:0910.2928, PRD 2010, E. Lunghi, priv. comm.]



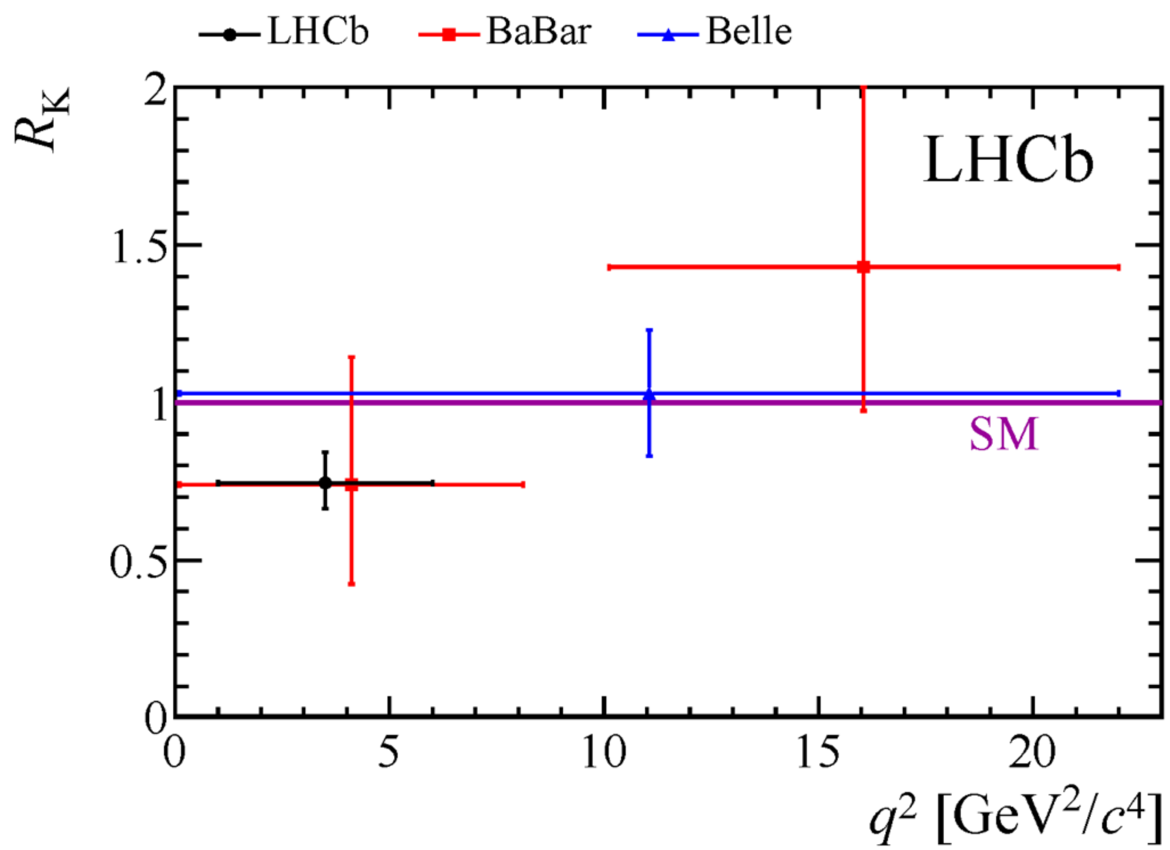
Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$



~3σ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

BSM phenomenology: LFU μ/e

Lepton universality test: $B \to K\mu^+\mu^- / B \to Ke^+e^-$



LHCb [arXiv:1406.6482, PRL 2014]:

$$\boxed{R_K = 0.745\,\left({}^{90}_{74}\right)(36)}$$

- ~2.6 σ tension between LHCb measurement and SM theory
- renewed interest due to recent LHCb measurement of R_{K^*} [R. Ajai et al, arXiv:1705.05802, JHEP 2017]
- The SM predictions of these ratios are insensitive to the form factors and nonfactorizable contributions.