$B \to D^{(*)} \ell \nu$ form factors from Lattice QCD

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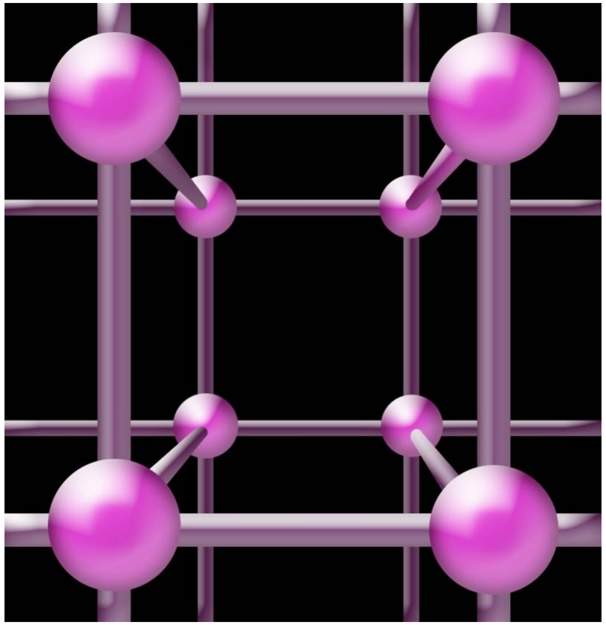
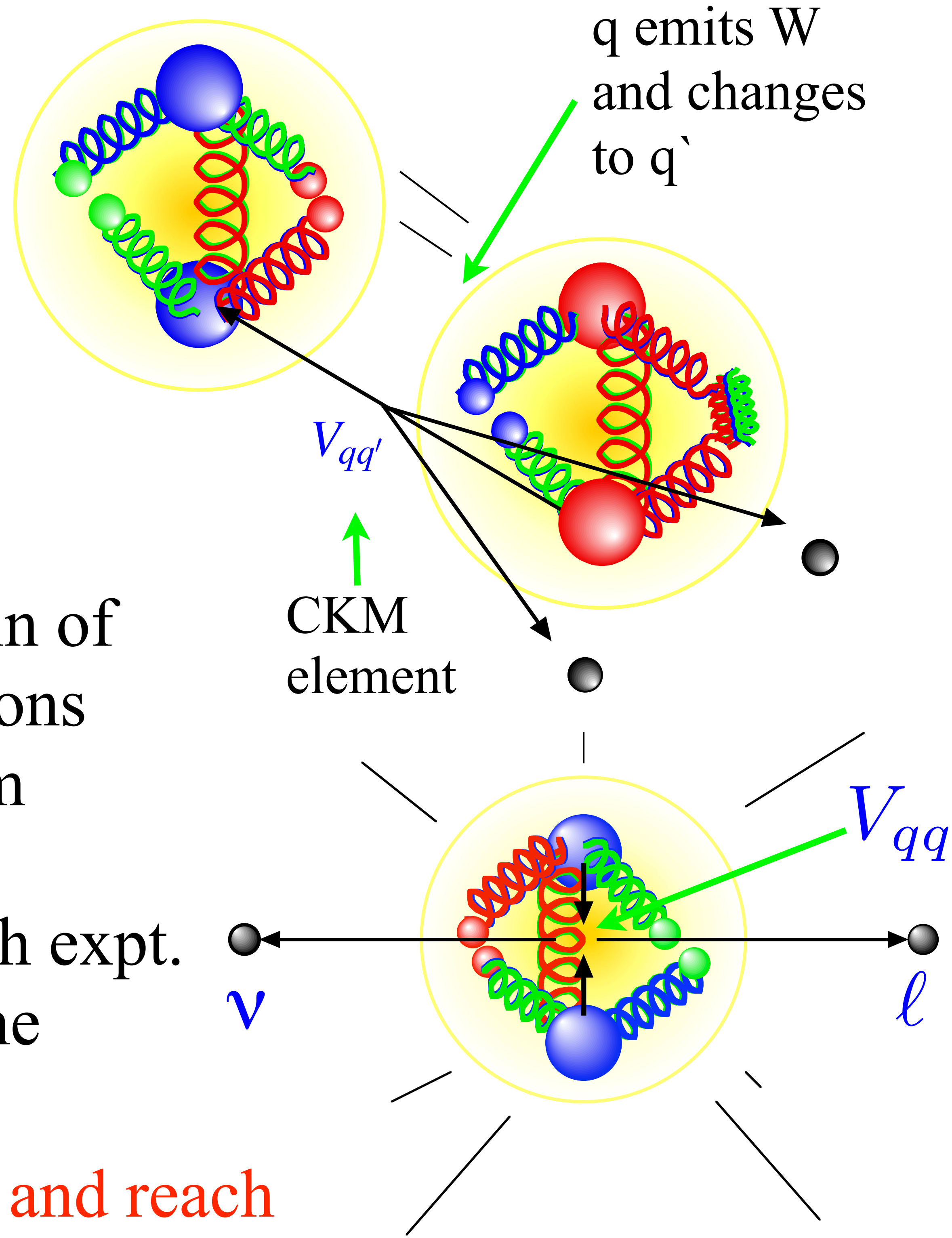
www.physics.gla.ac.uk/HPQCD



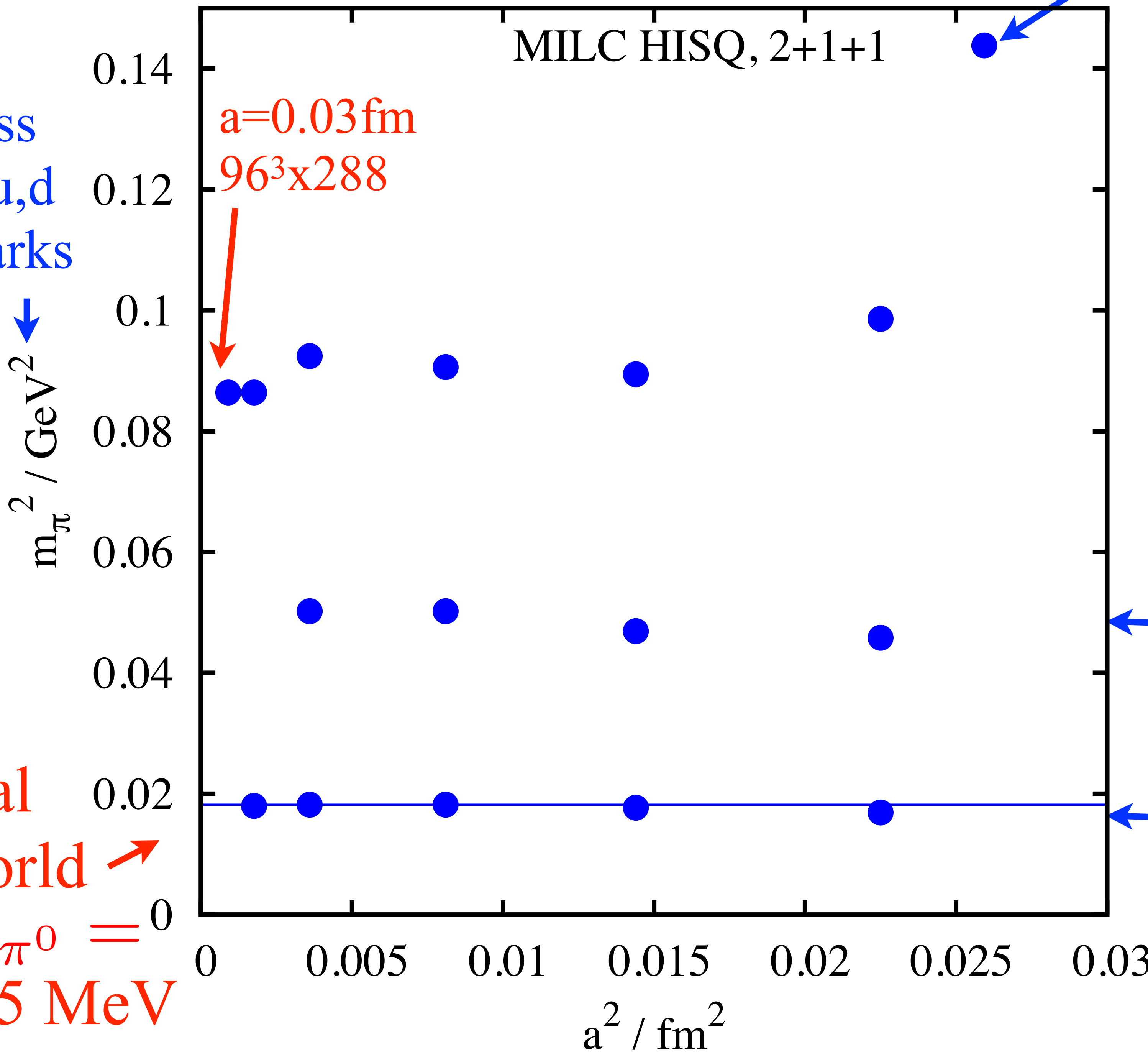


Lattice QCD allows calcln of hadron correlation functions that give masses and form factors for simple decay processes. Combined with expt. can test SM and determine parameters e.g. V_{CKM}

AIM: improved accuracy and reach

Parameters for gluon field configurations for state-of-the-art calculations

mass of u,d quarks



real world

$m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana et al, HPQCD, hep-lat/0610092.

1+1+1+1 also available

$m_{u,d} \approx m_s/10$

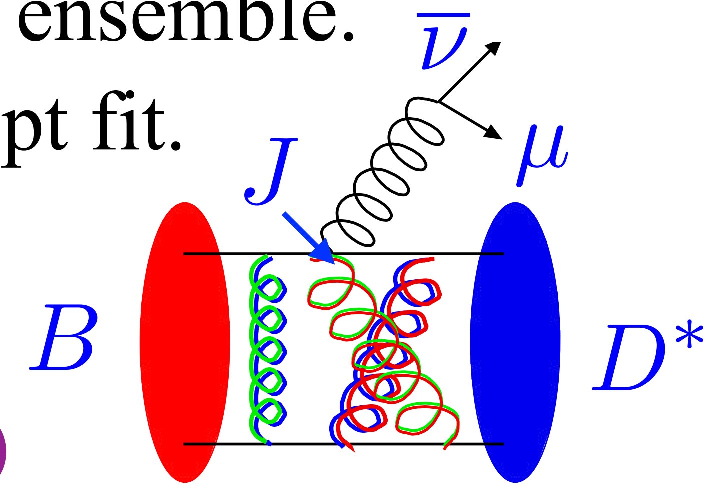
$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

Calculate quark propagators on background glue, combine into correlation functions. Average over ensemble.

Obtain $\langle D^{(*)}|J|B\rangle$ from combined 2pt/3pt fit.



Issue: how to handle b/c quarks

Current choices for $B \rightarrow D^{(*)}$:

c: Wilson-type ('Fermilab', RHQ, OK)

HISQ

twisted-mass, Mobius domain-wall …

b: Wilson-type ('Fermilab', RHQ, OK)

HISQ

NRQCD

twisted-mass

FNAL/MILC

LANL/SWME

ETMC

HPQCD

RBC/UKQCD

Systematic errors:

1) Discretisation - problem for relativistic actions

2) Operator matching effects - problem for nonrel. actions

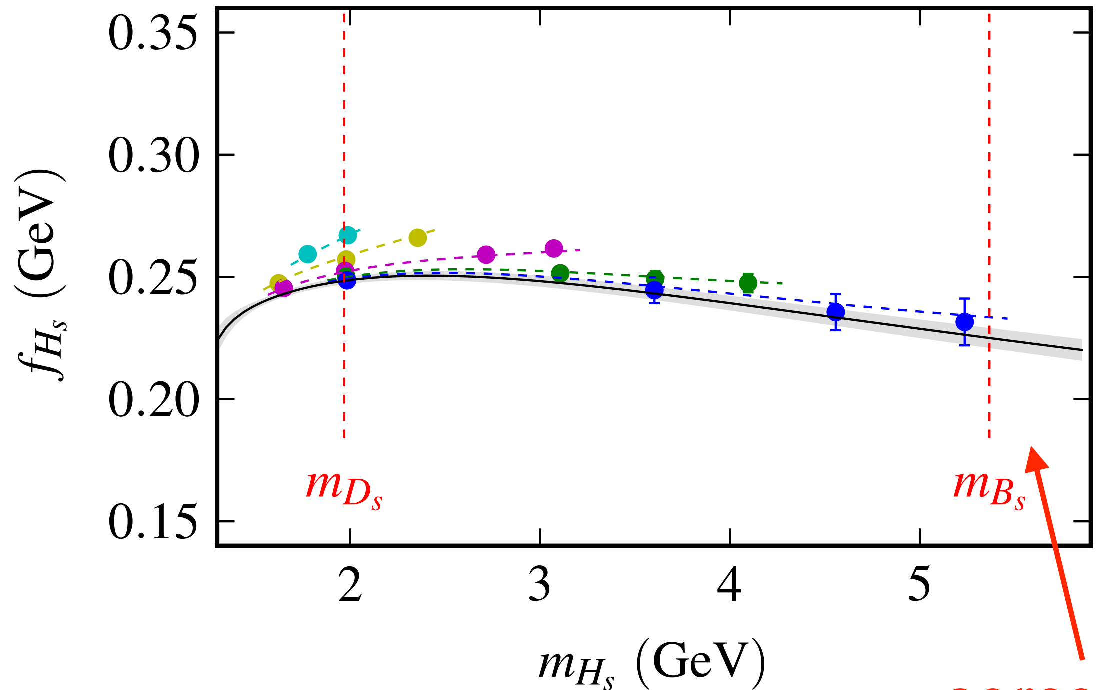
1) Discretisation - ‘seeing’ the lattice spacing

$$M(a) = M(0) \times (1 + c_1(\Lambda a)^2 + c_2(\Lambda a)^4 + \ldots)$$

For relativistic actions Λ can be m_q

Can be controlled up to b for HISQ

HPQCD: C. McNeile et al, 1110.4510



Some disc. effects suppressed by HQ effects, or can be cancelled between related quantities.

agrees w. NRQCD b

For NRQCD disc. effects $\alpha_s(\Lambda a)^2$ or higher

For Fermilab disc. effects $\alpha_s(\Lambda a)$ or $\alpha_s(ma)$

2) Operator matching

NRQCD

$\overline{\psi}_c \Gamma \Psi_b$

$$J = (1 + \alpha_s z_0 + \ldots) \times [J^{(0)} +$$

rel. corrns

$$(1 + \alpha_s z_1 + \ldots) J^{(1)} +$$

$\overline{\psi}_c \Gamma \gamma \cdot \vec{\nabla} \Psi_b / m_b$

$z_i \equiv$
$z_i(am_b, am_c)$

$$(\alpha_s z_2 + \ldots) J^{(2)} + \ldots]$$

$\overline{\psi}_c \gamma \cdot \overleftarrow{\nabla} \gamma_0 \Gamma \Psi_b / m_b$

z_0 (only) known for NRQCD $b \to c$

Fermilab

tree-level field 'rotation'

$$J = (1 + \alpha_s z_0 + \ldots)[\overline{\Psi}_c \Gamma \Psi_b + \ldots]$$

$$\Psi_x = (1 + d_1 \gamma \cdot \nabla) \psi_x$$

matched to O(α_s) after
normalising by c and b vector currents

$d_1 \equiv d_1(ma)$
$\to 0$
as
$ma \to 0$

HISQ action allows absolute current normalisation
TM. uses ratios where normln. cancels

$B \to D^* \ell \nu$

In zero-recoil limit, only A_1 form factor contributes.
Calculate in lattice QCD:

$$\langle D^*(\vec{p}=0)|\bar{c}\gamma^j\gamma_5 b|B(\vec{p}=0)\rangle = (M_B + M_{D^*})A_1(q^2_{max})\epsilon^j$$

$$h_{A_1}(1) = \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} A_1(q^2_{max})$$

Extrapolation of exptl rate gives $\overline{\eta}_{EW} h_{A_1}(1)|V_{cb}|$

Combine lattice and expt. to get Vcb

HQS (Luke's theorem) : result not sensitive to first-order rel. corrns., gives confidence in robustness

Fermilab/MILC 1403.0635

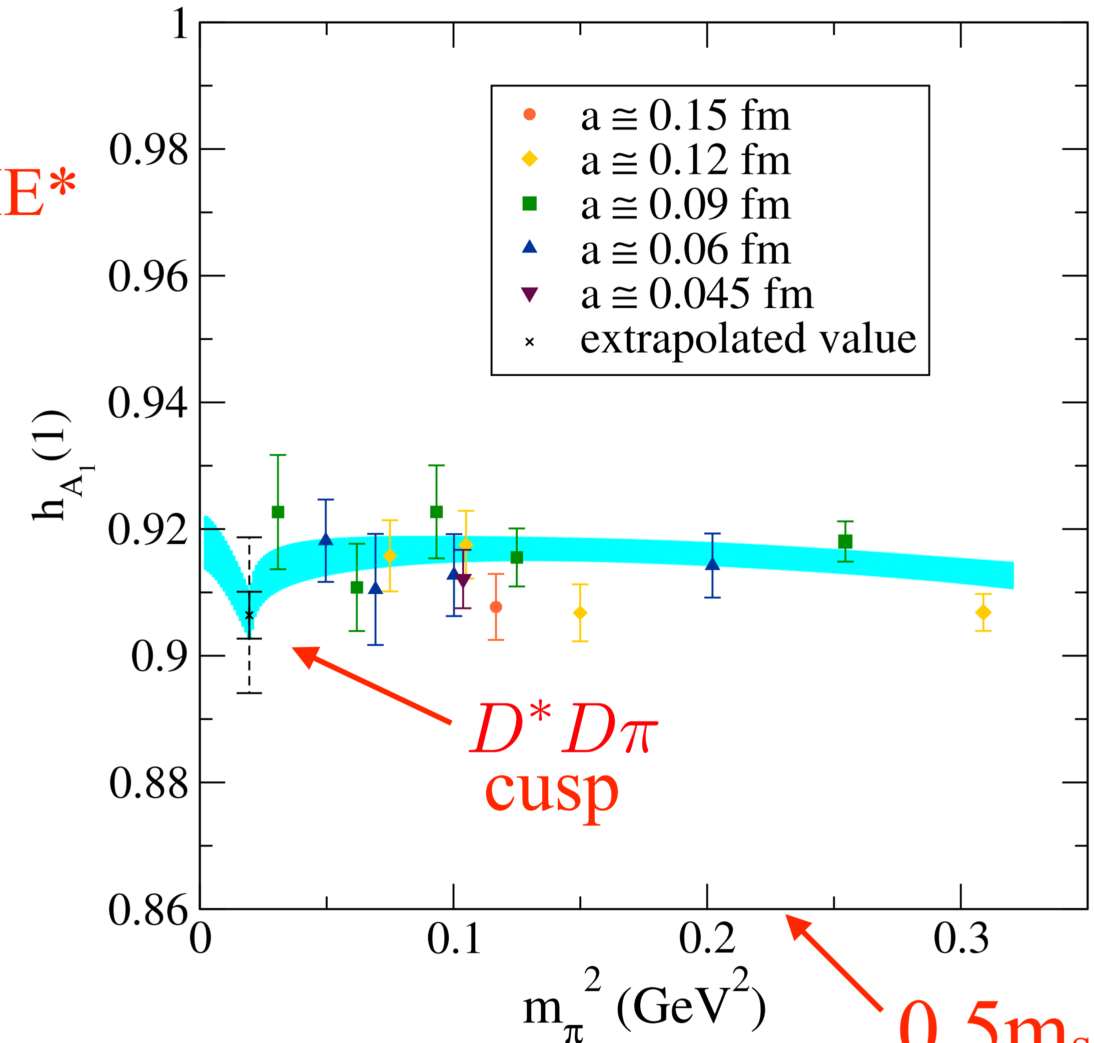
clover action b and c *SAME*

asqtad light on $n_f = 2+1$

asqtad cfgs

'Ratio trick' improves stat. error, cancels systs.

$$R = \frac{\langle D^*|A_j|B\rangle\langle B|A_j|D^*\rangle}{\langle D^*|V_4|D^*\rangle\langle B|V_4|B\rangle} = |h_{A_1}|^2$$



$D^* D\pi$ cusp

$0.5 m_s$

Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

$O(\alpha_s)$ matching coeff. (z_0) small

- argue good control from large m limit

- take error $0.1(\alpha_s)^2$

$$\frac{a\Lambda^2}{m_c}, a^2\Lambda^2$$

$$h_{A_1}(1) = 0.906(4)(12)$$

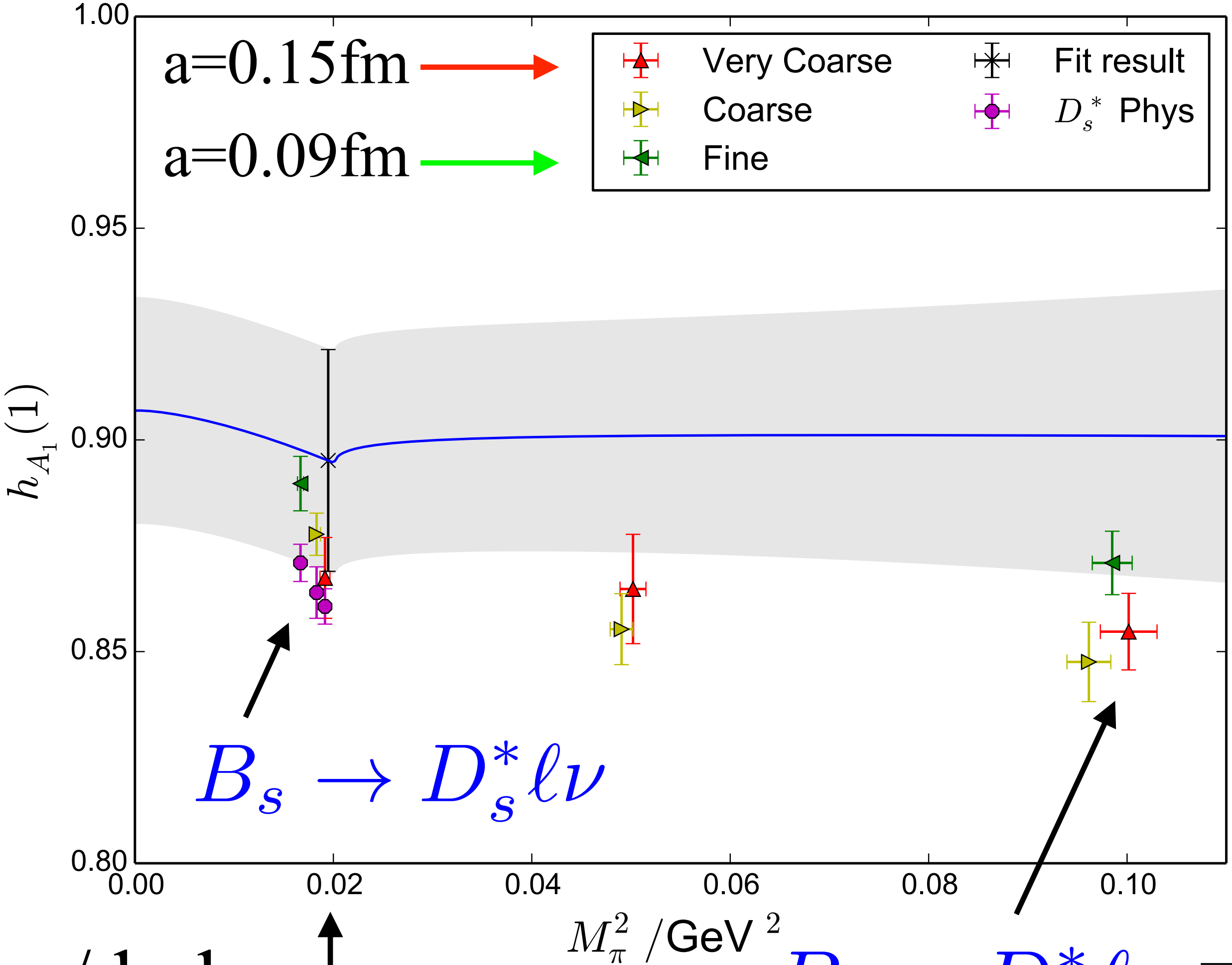
HPQCD: J. Harrison et al, 1711.11013

$n_f = 2+1+1$ cfgs

inc. phys u/d

imp. NRQCD b, ← $\alpha_s a^2$-accurate

HISQ u/d,s,c ← fully rel.



u/d phys mass

$B \to D^* \ell \nu$

$$h_{A_1} = 0.895(10)(24)$$

$$h^s_{A_1}(1) = 0.879(12)(26)$$

HQ limit not useful

$0.06 < z_0 < 0.24$

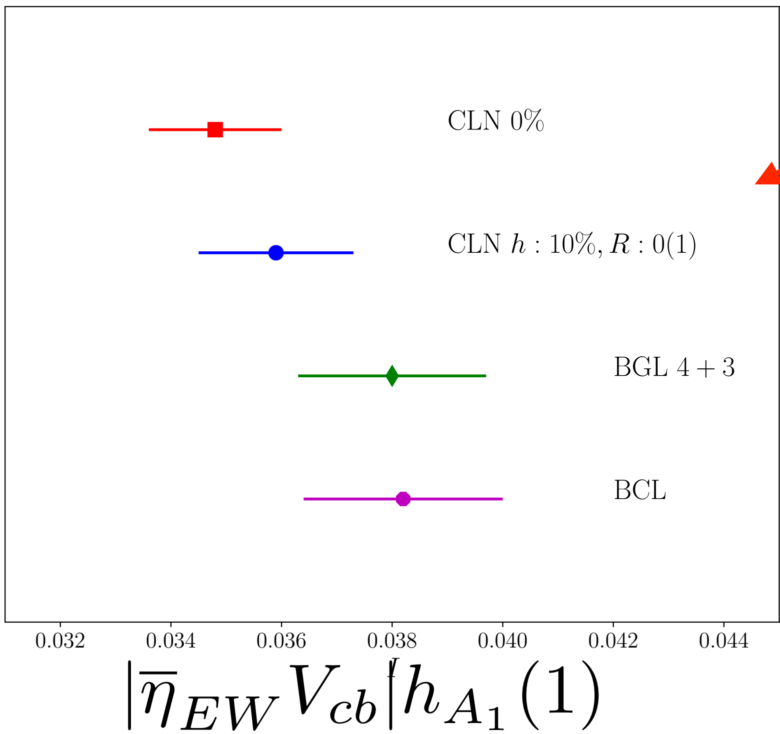
Allow $0.5(\alpha_s)^2$ in fit

test Λ/m_b current corrn MEs: J^1 (inc.) ~0.5%; J^2 (not inc.) ~1%

Uncertainty	$h_{A_1}(1)$	$h^s_{A_1}(1)$	$h_{A_1}(1)/h^s_{A_1}(1)$
α_s^2	2.1	2.5	0.4
$\alpha_s \Lambda_{\rm QCD}/m_b$	0.9	0.9	0.0
$(\Lambda_{\rm QCD}/m_b)^2$	0.8	0.8	0.0
a^2	0.7	1.4	1.4
$g_{D^* D\pi}$	0.2	0.03	0.2
Total systematic	2.7	3.2	1.7
Data	1.1	1.4	1.4
Total	2.9	3.5	2.2

Implications for Vcb

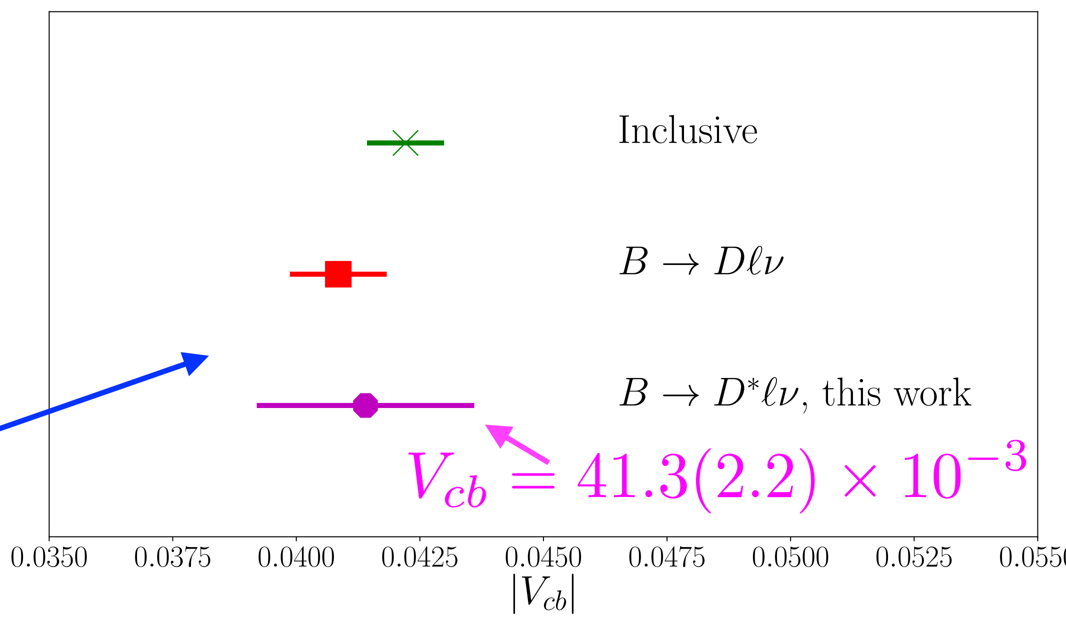
2017: exptl rate extrapolated to zero-recoil less clear. Seems likely that uncty from using HQET (neglecting $(\Lambda/m_c)^2$)was underestimated. 1703.05330, .06124, .08170



Our fits to unfolded Belle data

$$|\overline{\eta}_{EW} V_{cb}| h_{A_1}(1) = 0.038(2)$$

uncty 4x larger than HFAG 16



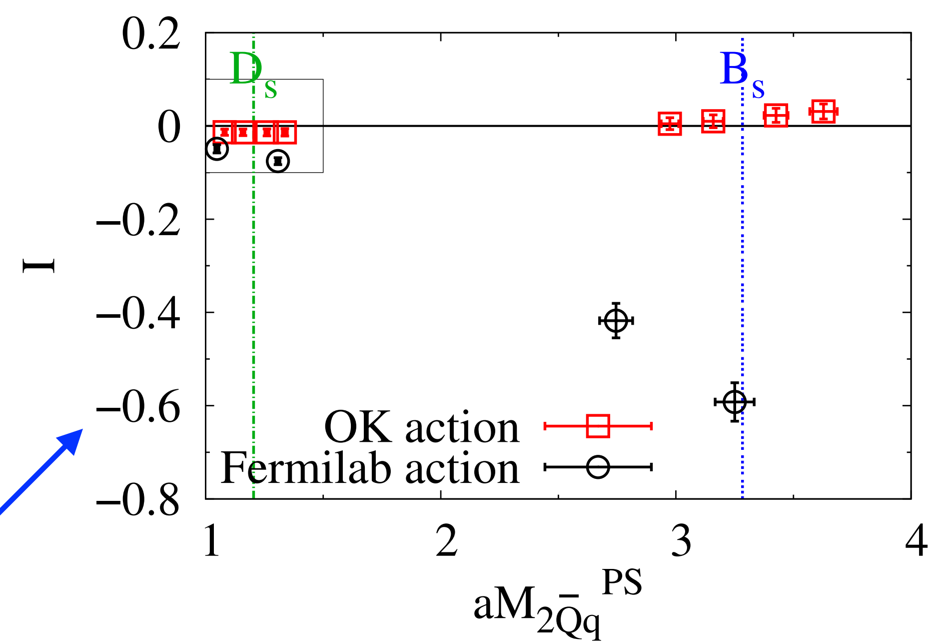
NOW: excl. and incl. *not* inconsistent for Vcb

improve by using non-zero recoil …

In progress: LANL/SWME calc. using improved (Oktay-Kronfeld) Fermilab action for b+c (+HISQ light)

Add higher dimension operators to action and current with tree-level m-dependent coefficients. `1711.01777, 01786`

	$h_{A_1}(w=1)$
source	error (%)
statistics	0.4
matching	0.4
χPT	0.5
$g_{D^*D\pi}$	0.3
c discretization	$1.0 \to (0.2)_{\mathrm{OK}}$
others	0.1
total	$1.4 \to (0.8)_{\mathrm{OK}}$



improvement demo - I is inconsistency between HH and HL masses from errors at p^4/m^3 terms

first runs on coarse lattices: extend to finer lattices.

still to do: perturbative matching of current (z_0)

W.Lee

In progress : $B \to D^*$ away from zero recoil, Fermilab/MILC, clover action on 2+1 asqtad cfgs.

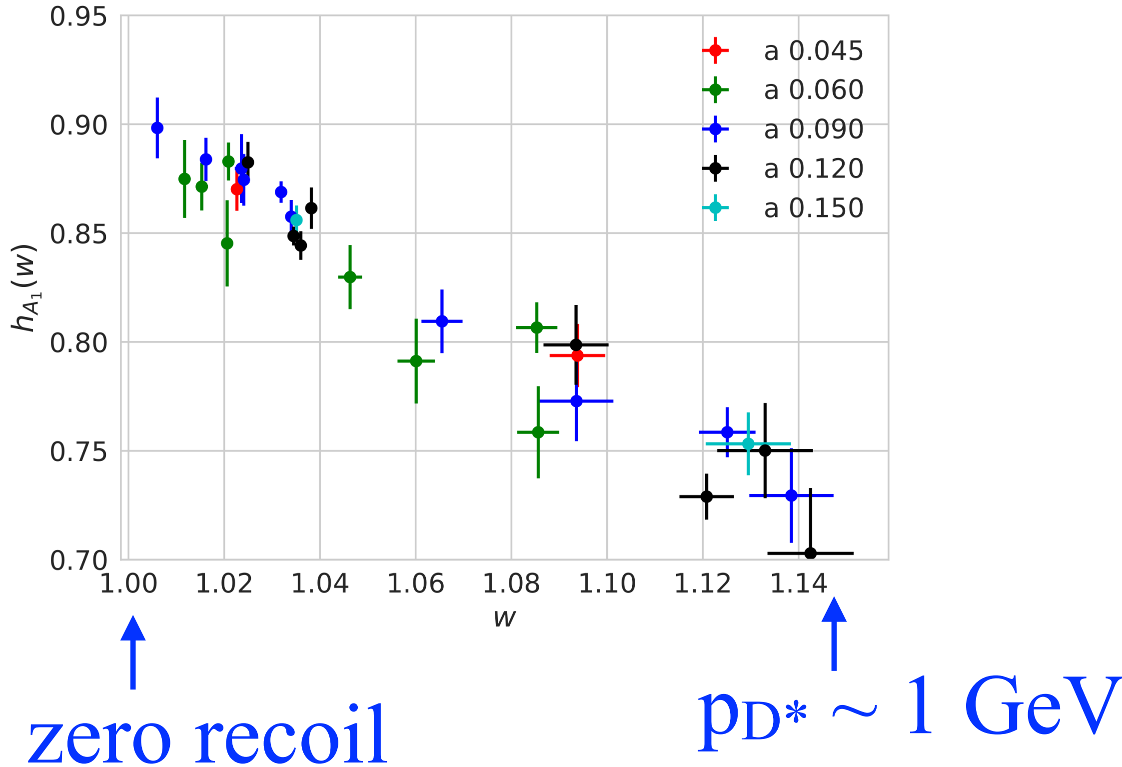
1710.09817

multiple form factors :
A_1, A_2, A_3, V
but A_1 dominates.

use double ratio for A_1

$$\frac{\langle D^*(\mathbf{p}_\perp)|A_1|B(0)\rangle\langle B(0)|A_1|D^*(\mathbf{p}_\perp)\rangle}{\langle D^*(0)|V_4|B(0)\rangle\langle B(0)|V_4|D^*(0)\rangle}$$

and ratio to A_1 for others.



Now systematic errors larger from missing current corrections …

Test HQET relations between form factors

Bernlochner et al, 1708.07134

$B_{(s)} \to D_{(s)} \ell \nu$

For light leptons, only $f_+(q^2)$ contributes to rate

$$\sqrt{\frac{d\Gamma}{d\omega}} \propto |\overline{\eta}_{EW}||V_{cb}|(\omega^2 - 1)^{3/4}\mathcal{G}(\omega)$$

$$f_+(\omega) = \frac{1+r}{2\sqrt{r}}\mathcal{G}(\omega)$$

$$r = m_D/m_B = 0.354$$

$$0 < q^2 < 11.6\,\mathrm{GeV}^2$$

$$1 < \omega = v_B \cdot v_D < 1.59$$

kinematics makes zero recoil less useful for combn with expt. - need to cover more of q^2 range. Map q^2 to z for fitting/comparison

$$z(\omega) = \frac{\sqrt{1+\omega} - \sqrt{2}}{\sqrt{1+\omega} + \sqrt{2}}$$

$$0 < z < 0.0644$$

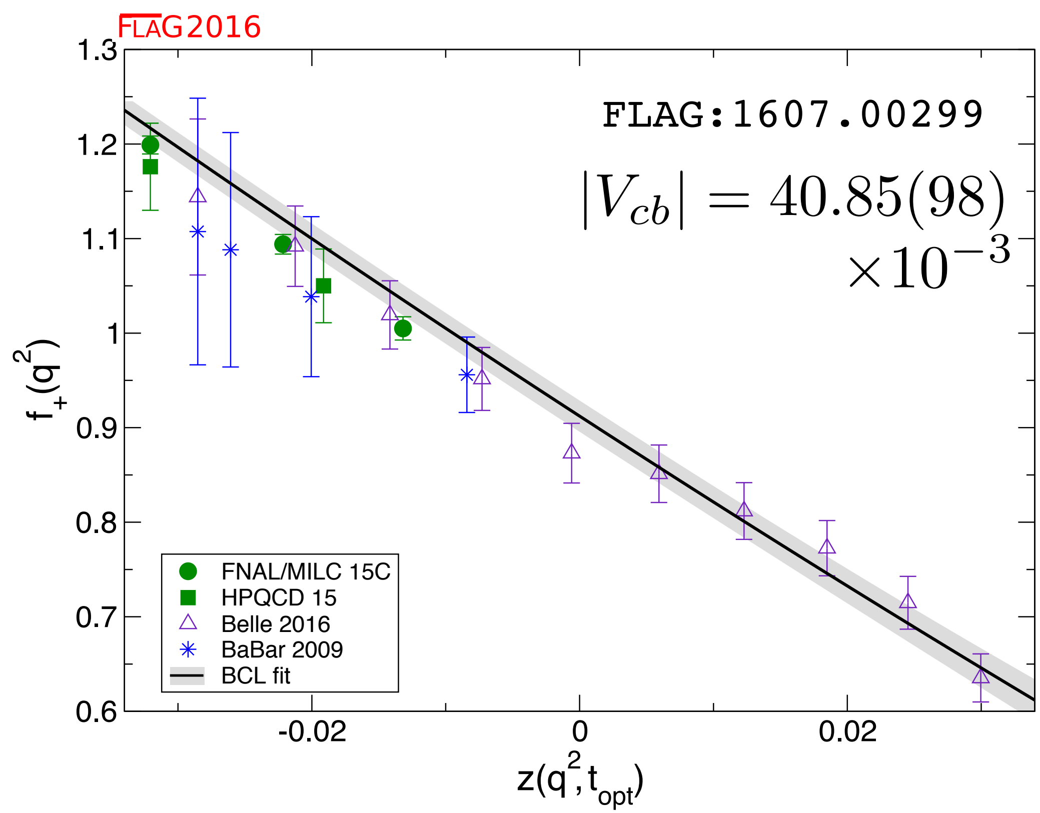
Lattice calcs. so far:
small recoil, simple current ops, $O(\alpha_s)$ renormln.

Fermilab/MILC 1503.07237 $\mathcal{G}^{B\to D}(1) = 1.054(4)(8)$

clover action b+c +asqtad on n_f=2+1, a= 0.12 - 0.045 fm.
Use 3pt ratios. Take non-zero recoil matching error < 1%

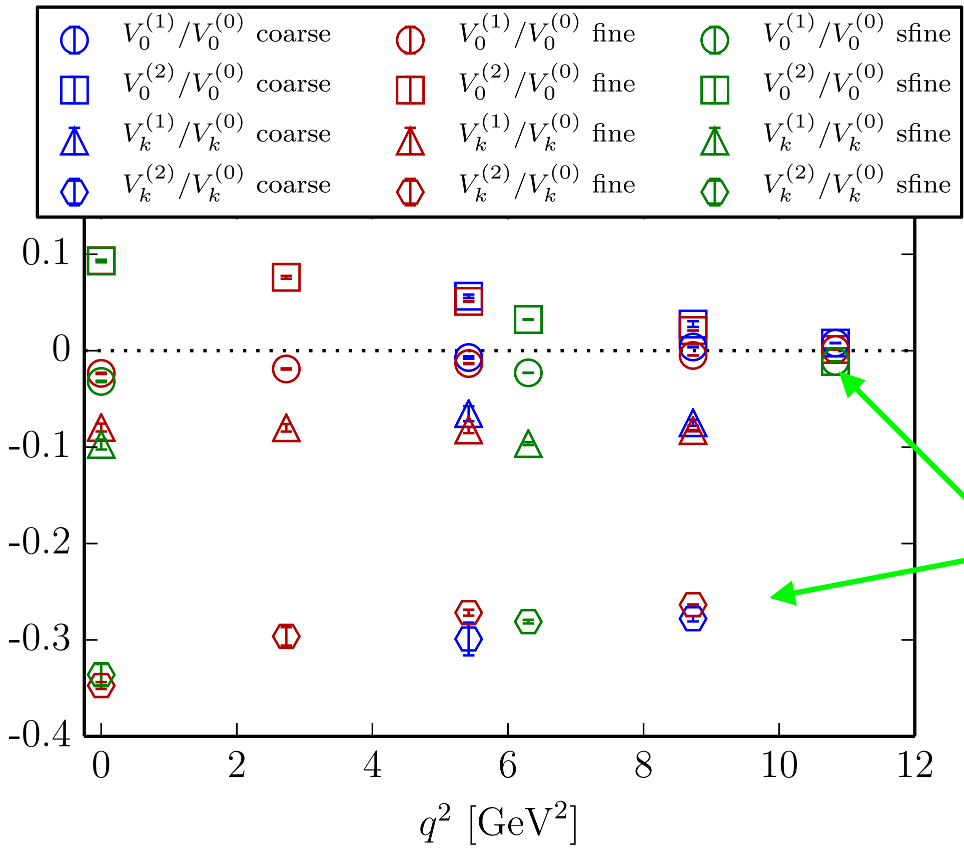
HPQCD 1505.03925, 1703.09728 NRQCD b, HISQ c, on n_f=2+1, a=0.12 - 0.09 fm . Matching error 2% [inc. $O(\Lambda/m_b)$ corrns only]

$\mathcal{G}^{B\to D}(1) = 1.035(40)$



ETM, 1310.5238
twisted-mass on n_f=2, with mass ‘step-scaling’ up to b.

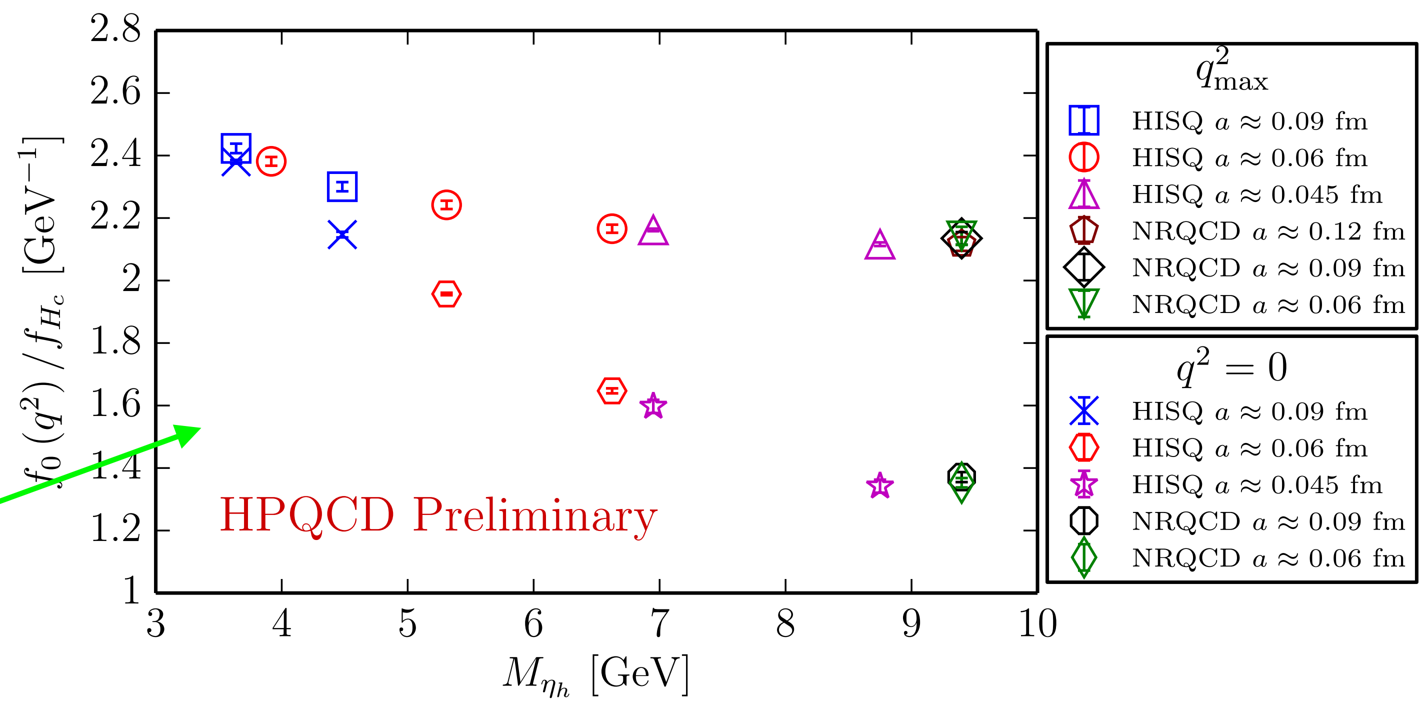
HPQCD: Lytle, Colquhoun, McLean



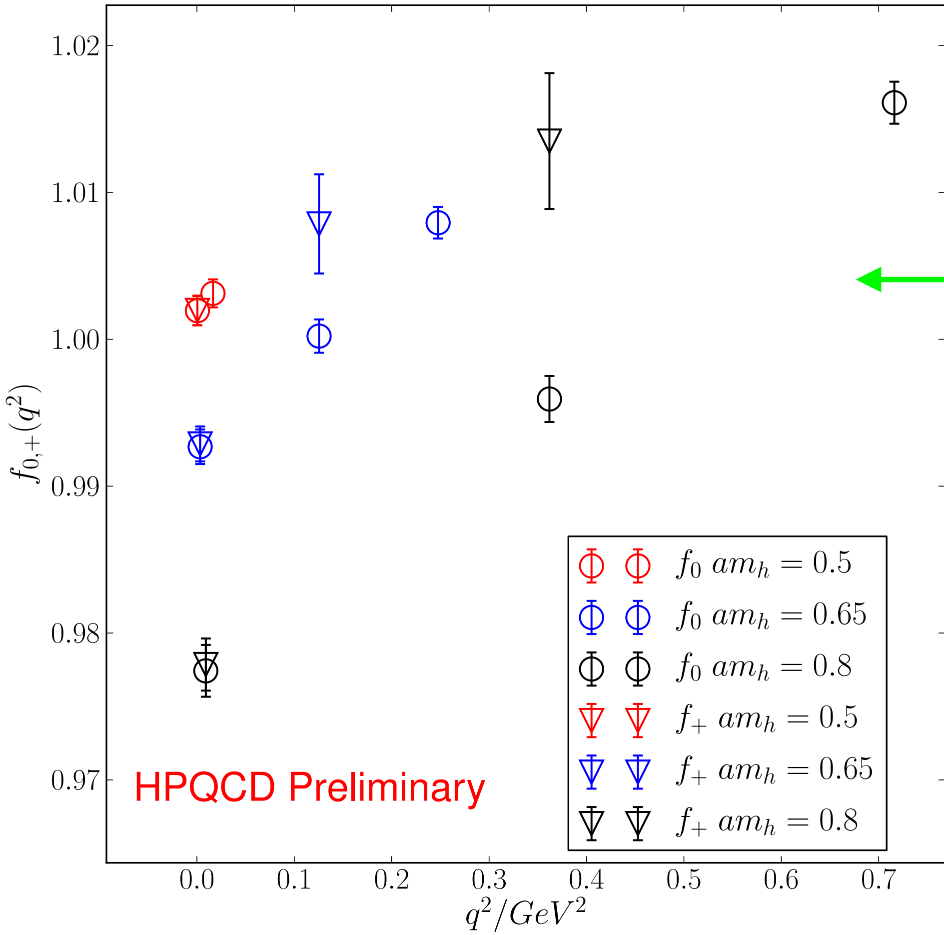
$B_c \to \eta_c \ell \nu$ as a testbed

NRQCD-HISQ: O(α_s) relativistic corrns from $\overline{\psi}_c \gamma \cdot \overleftarrow{\nabla} \gamma_0 \Gamma \Psi_b / m_b$ grow with c momentum - affect ff shape

Using HISQ for all quarks gives form factors with absolute normln. Test vs NRQCD at zero and max. recoil.

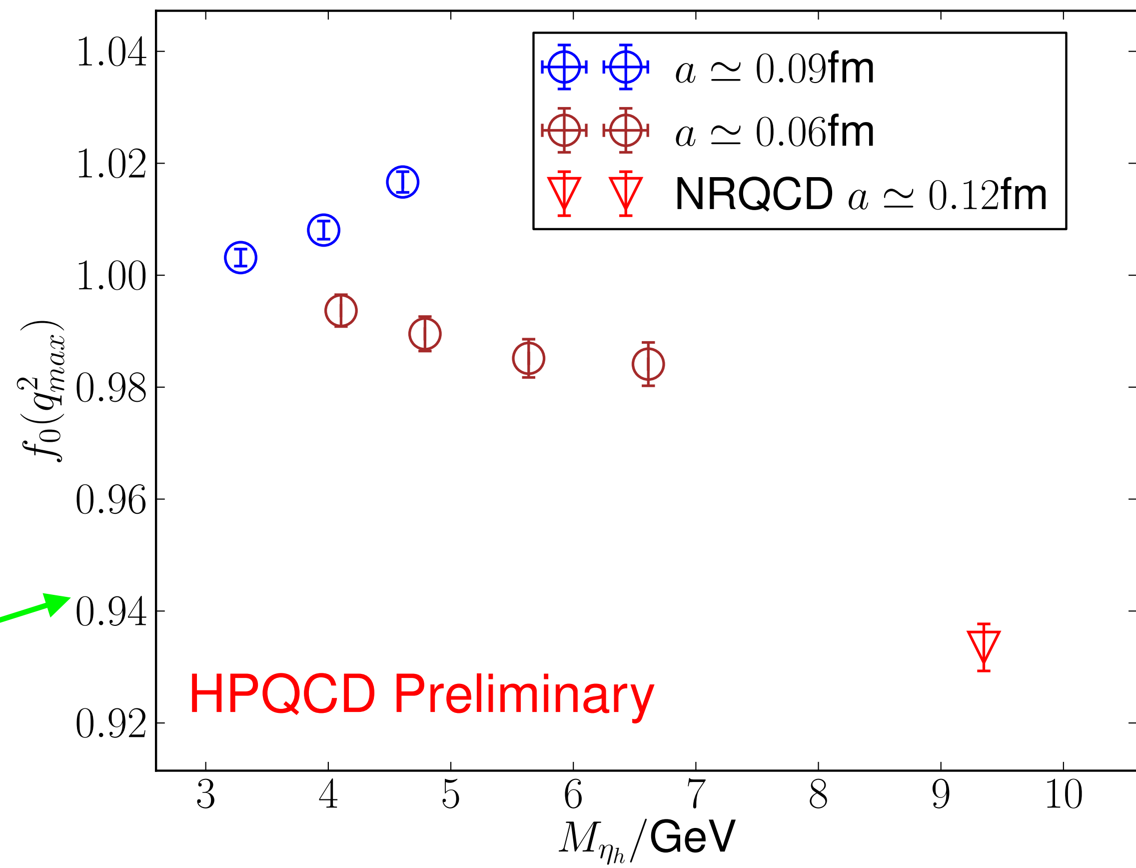


$B_s \to D_s \ell \nu$ preliminary, using all HISQ



cover full q^2 range for m_h values at each lattice spacing

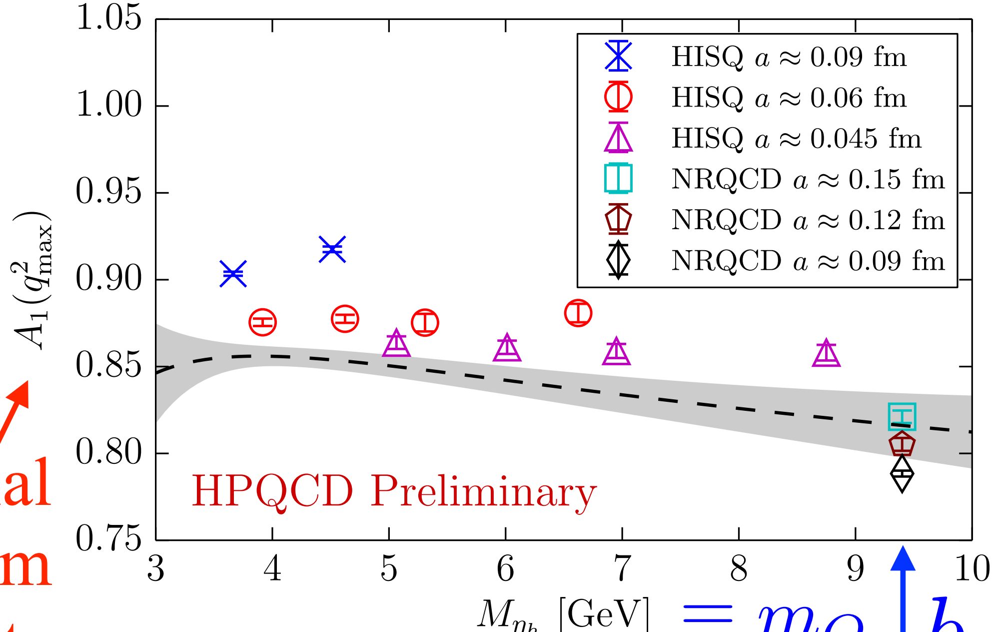
a=0.09 fm



On finer lattices, m_h inc. towards b . Dividing by f_{Bc} will reduce disc. effects

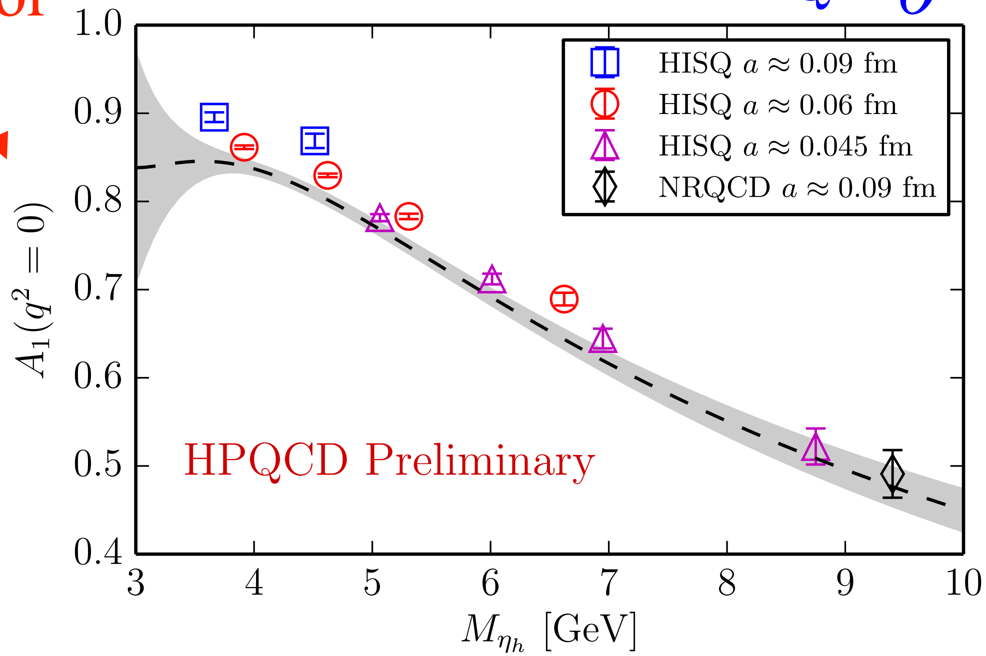
Ongoing/Future : provide lattice calculations for other form factors giving access to Vcb

e.g. $B_c \to J/\psi \ell \nu$



$\equiv m_Q$ b

axial form factor



can cover the full q² range for this decay accurately and compare relativistic and non relativistic approaches.

A. Lytle, B. Colquhoun et al, HPQCD

Lattice QCD form factors also needed for SM result for R(J/ψ)

$$R = \frac{\mathcal{B}(B_c \to J/\psi \tau \nu)}{\mathcal{B}(B_c \to J/\psi \mu \nu)}$$

LHCb, 1711.05623

tests lepton universality, cf R(D(*))

Conclusion

Lattice QCD form factors for $B_{(s)} \to D^{(*)}_{(s)} \ell\nu$ under good control (1.5-3%) at zero recoil.

Multiple methods agree

BUT results at non-zero recoil are needed for overlap with exptl data. This is harder and J in existing methods missing full shape info. from $\overline{\Psi}_b \vec{\nabla} \Psi_c$ operators

Relativistic methods for b quark e.g. using HISQ are possible on very fine lattices, underway.

Future

• lots still to do on semileptonic decays - extend processes studied (e.g. to B_c) and range in q^2.